How damaging are environmental policy targets in terms of welfare?⋆

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Abstract

International environmental agreements translate in environmental policy targets for individual countries. To reach these targets, corresponding governments must stimulate the private sector to do the right investments, for instance in renewable energy. This paper studies the effect of a subsidy on the probability of reaching the policy target and on the level of social welfare.

The subsidy in the form of a fixed price support accelerates investment and increases the investment size. As such it helps to reach a policy target in time, but also has its own welfare effects. The paper defines a new welfare measure, "the expected Welfare corresponding to the Policy Target" abbreviated by WPT, that takes all these effects into account, including the penalty that is incurred upon not reaching the policy target. Based on the WPT we determine the optimal subsidy size. We find that a policy target increases the optimal subsidy size. An international policy target can cause a tradeoff in the sense that a large investment is required to achieve the target, while at the same time such a large investment is bad for welfare.

Keywords: Investment Analysis, Uncertainty, Subsidy, Policy Target, Welfare Analysis

JEL classification: C61, D81, H23, Q58

1 Introduction

This paper considers the problem of stimulating firms to undertake green investments in a dynamic model of investment under uncertainty. Intertemporal issues are crucial in designing a proper policy towards climate change (see Stern (2018)). As world infrastructure will double in the next 15-20 years, irreversibility of investment requires immediate action in advocating green investments in order to fulfill future environmental policy targets. In maximizing profits or pursuing growth, due to the large investment cost, market and technology uncertainty, energy producers from themselves are not incentivized enough to invest in renewable energy. Instead they use fossil fuel and emit greenhouse gases to a too large extent (Eichner and Runkel,

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2014). For this reason, in reaching an environmental policy target that arises from international environmental agreements, governments have to stimulate the private sector to sufficiently invest in green energy. Such a policy target is for instance set by the EU, who demands that in 2030 27% of total energy consumption should be produced by renewable energy. Moreover, according to the regulations of the EU parliament, if the member states fall short in meeting their national reference points (target), they either have to take national measures to catch up or to make a financial donation (EU, 2018).

We consider a framework where the government of a country has to deal with an environmental policy target, resulting from an international environmental agreement. The target consists of investing a certain amount in green energy before a certain point in time. The government employs a subsidy to stimulate energy producers to undertake green energy investments. Of course, the resulting investments influence social welfare and the principal aim of this paper is to investigate this effect. When the target is not realized, the government incurs a certain penalty. We mainly consider a subsidy in the form of fixed price support, but we check whether our results are robust in case of flexible price support or a subsidy in the form of reimbursed investment cost. We take into account that the energy market is a market with uncertain future demand, in which the energy producer is constantly forecasting demand and balancing the value of investing now and delaying investment. To do so we employ the real options approach to determine the optimal investment decision.

In general it holds that the difference in objective between profit and welfare maximization poses a coordination problem and requires governmental regulation (Rodrik, 1992). Policy instruments such as taxation has been proposed in the literature and they are not exclusive to the energy sector. Pennings (2000) studies taxation and investment subsidies to influence the instant investment. Hassett and Metcalf (1999) consider uncertainty in the tax policy, such as the U.S. investment tax credits that have been changed on many occasions since being introduced in 1964. They show that for a relatively low tax rate, more tax-policy uncertainty speeds up irreversible investment because the firm inclines to invest at a low tax rate.

Subsidy support is commonly implemented in the field of green energy (see e.g., Abrell et al. (2019)). With the subsidy instrument the regulator can influence the firm's investment decisions in order to meet socially optimal goals or to realize the assigned policy targets. The subsidy support can take several forms such as feed-in premiums (FIP), reimbursed investment costs, feed-in tariffs (FIT), tradable green certificates (TGC), and quota obligations. The European Commission (2014) suggests to adopt FIP because it is a market-based approach and some risk can be shared between investors and consumers.

Our paper belongs to the research field of governmental green energy policies and their effect on the investment timing decisions of private firms. Boomsma et al. (2012) take subsidy payments as a volatile process. The support schemes considered include FIT, FIP, and renewable energy certificates. Boomsma and Linnerud (2015) focus on the uncertainty to introduce or retract subsidy schemes and its effect on the firm's investment timing. They find that the risk of subsidy termination speeds up investment. This result is supported by Adkins and Paxson (2015), who provide the intuition that the firm wants to catch the subsidy before it is gone. Similarly, future provision of a subsidy delays investment because the firm wants to wait for the subsidy. This influence of subsidy retraction and provision is further studied by Chronopoulos et al. (2016). Besides investment timing, they also consider the influence of policy uncertainty on the investment capacity/size. They find that future subsidy retraction lowers the amount of installed capacity, whereas future subsidy provision raises the incentive to install a larger capacity.

The present paper first considers the investment problem without the policy target. We find that from a welfare perspective a profit maximizing firm invests too late in the right amount. Introducing a subsidy in the form of a fixed price support stimulates the firm to invest earlier, which increases welfare, but also more, which reduces welfare. Then we introduce a policy target in the form of a required investment of a certain size before a certain deadline, where a penalty has to be paid upon not fulfilling the target. This could introduce a tradeoff in the following form: fulfilling the target could involve a too large investment being bad for welfare but on the other hand prevents paying the penalty.

When determining the optimal subsidy rate all these welfare effects should be taken into account. For this reason we introduce a new welfare measure, "the expected Welfare corresponding to the Policy Target" abbreviated by WPT. Employing the WPT we obtain that the policy target raises the optimal subsidy rate, and also that the firm needs additional stimulus in the form of a higher subsidy when the economic environment is more uncertain, the market trend is low, and the discount rate is high.

We apply our theoretical model to the Italian electricity market. We find that a fixed price support of 4% is optimal in order to realize the 2030 target set by the EU, which is that 30% of the energy consumption comes from renewable resources.

As such this paper adds to the literature studying the effect of subsidies. Sheriff (2008) claims that politically motivated subsidies can have undesired environmental consequences, and he analyzes the welfare-maximizing policy under firm's productivity information constraints in a static setting. Pineda et al. (2018) find that the FIT and FIP are more efficient than TGC in realizing a given renewable investment quantity target when the power producers are risk averse. More information about effects of subsidies in the renewable energy field can be found in Koltsaklis and Dagoumas (2018). As a policy instrument, subsidy is also used to realize certain policy target. In particular, Bigerna et al. (2019) investigates the scenario where a single-purposed regulator realizes the targeted investment timing and size by means of a subsidy policy, regardless of its welfare implications. The present research extends Bigerna et al. (2019) by focusing on the welfare measurement of a policy target by WPT, and determines the subsidy policy of a regulator, who not only tries to realize a policy target, but also to maximize the corresponding WPT. To the best of our knowledge, there is no literature investigating welfare effects of environmental policy targets. This is an important topic, which deserves to be analyzed, as the present paper does.

This paper is organized as follows. Section 2 builds the theoretical model and analyzes the profit and welfare maximizing investment decisions. Section 3 analyzes the welfare-maximizing subsidy with and without policy target constraint. Section 4 conducts an empirical analysis for the Italian electricity market. Section 5 carries out a comparative statics analysis with special emphasis on the implications when the policy target is adjusted. Section 6 concludes.

2 Model

This section consists of three parts. First we present the model framework. We proceed by analyzing the optimal investment decision from a profit maximizing perspective. Finally, we consider the same decision from the point of view of the social planner.

2.1 Model Setup

Consider a framework where a renewable energy producing firm can undertake an irreversible investment to enter the market and serve the market demand for renewable energy. The investment decision involves choosing the time and the size, K, of the investment, where the size is the firm's production capacity. The firm is assumed to be sufficiently large to exert market power. This is to cover the fact that, the EU electricity industry shows a high degree of concentration on national and regional scales (European Commission (2011)), which is also true for the Italian electricity market, see e.g., Bigerna et al. (2016a,b). In that light market power will play an important role, and to limit the complexities of our model, we let one representative producer serve the whole market.

Following Borenstein and Holland (2005); Borenstein (2005); Spees and Lave (2008); Kopsakangas Savolainen and Svento (2012), we employ an iso-elastic demand function. The two main arguments are the following. First, abundant research work has estimated the price elasticity for electricity to be a constant or lying in a narrow range, see for instance Lijesen (2007). Second, the iso-elastic demand function is linear in logs, which makes it easily tractable from an econometric point of view. Still, the existing literature is not consistent on how to model the electricity market demand. For instance, Bigerna et al. (2019); Pineda et al. (2018) and Özge. Özdemir et al. (2016) consider a linear demand specification with the obvious advantage of analytical tractability.

According to our assumption, it follows that the market price, p_t, is equal to

$$p_t(X_t) = X_t K^{-\gamma}, \tag{1}$$

where $0 < \gamma < 1$ is the inverse of demand elasticity. It is important to note that with such constant demand elasticity, it is always optimal to use all available capacity. Given an elasticity level and a capacity size, X_t measures the market demand level at time t. To capture the characteristic of an energy market with uncertain demand shocks overtime, the process X_t is a geometric Brownian motion (GBM) process given by

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}\omega_t,$$

in which μ is the drift rate, $\mathrm{d}\omega_t$ is the increment of a Wiener process, and $\sigma > 0$ is the volatility parameter.[1]

The firm is assumed to be risk neutral and discounts against rate r. We also assume $r > \mu$, as otherwise it would always be optimal for the firm to delay the investment (Dixit and Pindyck, 1994, p. 140). The investment costs take the form $\delta_0 + \delta_1 K$ with $\delta_0 \geq 0$ and $\delta_1 > 0$.

2.2 Profit-maximizing investment decision

To obtain the investment decision that maximizes the expected profit stream, the firm has to solve the following optimization problem:

$$F(X) = \max_{K \geq 0, T \geq 0} \mathbb{E}\left(\int_{t=T}^{\infty} X_t K^{1-\gamma} \exp(-rt)\mathrm{d}t - (\delta_0 + \delta_1 K)\exp(-rT) \mid X_0 = X \right),$$

where $F(X)$ is the value of the firm's option to invest, and T is the moment of investment.

The optimal investment decision involves finding the optimal size and the optimal timing. For a given value of X, we derive the optimal value of the investment size $K_F(X)$ by solving

$$\max_{K \geq 0} \mathbb{E}\left(\int_{t=0}^{\infty} X_t K^{1-\gamma} e^{-rt}\mathrm{d}t - \delta_0 - \delta_1 K \mid X_0 = X \right) = \max_{K \geq 0}\left(\frac{XK^{1-\gamma}}{r-\mu} - \delta_0 - \delta_1 K \right). \tag{2}$$

[1]Note that renewable energy generation is intermittent because of the influence from weather conditions, which makes production partly predictable and highly variable at short and medium time scales. However, production is more predictable and less variable on the long run, e.g, on yearly time scales. In the context of long term investment decisions, we do not consider intermittency and simplify the framework for our analysis, where we follow Boomsma et al. (2012); Boomsma and Linnerud (2015) and Dalby et al. (2018).

This gives

$$K_F(X) = \left(\frac{X(1-\gamma)}{(r-\mu)\delta_1}\right)^{1/\gamma}. \tag{3}$$

Second, we consider the optimal investment timing. To do so we determine the investment threshold X_F at which it holds that the firm is indifferent between investing and not investing. In the scenario where $X_0 < X_F$ it is optimal for the firm to invest when the process X reaches X_F for the first time. Otherwise, it is optimal to invest immediately. Following the standard real options analysis (Dixit and Pindyck, 1994, chapter 4 & 5), we obtain that for $X \leq X_F$ the value of the option to invest can be expressed as

$$F(X) = AX^{\beta},$$

where A is a positive constant and β is the positive root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0. \tag{4}$$

In what follows we impose that

$$\beta\gamma > 1.$$

As in Huisman and Kort (2015), this assumption is needed to obtain a positive threshold and corresponding investment size (see Proposition 1 below). If it does not hold, then it implies that the uncertainty parameter σ is so large (note that β decreases with σ (Dixit and Pindyck, 1994, p. 144)) that the investment option is too valuable, and keeping it alive is always optimal. In other words, the firm never invests.

When the firm invests, we denote the value of the investment project by $V(X, K)$. The following value matching and smooth pasting conditions can then be employed to determine the optimal investment threshold X_F for a given capacity size K :

$$\begin{aligned} F(X_F) &= V(X_F, K), \\ \left.\frac{\partial F(X)}{\partial X}\right|_{X=X_F} &= \left.\frac{\partial V(X,K)}{\partial X}\right|_{X=X_F}. \end{aligned}$$

This yields

$$X_F(K) = \frac{\beta(r-\mu)(\delta_0 + \delta_1 K)}{(\beta-1)K^{1-\gamma}} \tag{5}$$

Based on (3) and (5) we can develop the following proposition.

Proposition 1 *The optimal investment threshold X_F^* and the corresponding optimal capacity level K_F^* are given by*

$$\begin{aligned} X_F^* &= \frac{r-\mu}{1-\gamma}\delta_1 \left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma}, \\ K_F^* \equiv K_F(X^*) &= \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}. \end{aligned}$$

Note that if at time $t = 0$ it already holds that $X_0 \geq X_F^*$, then the firm invests immediately at X_0 to attract a capacity of size $K_F(X_0)$.

2.3 Welfare-maximizing investment decision

The regulator maximizes total social surplus consisting of the sum of producer and consumer surplus. Concerning the latter we follow (Huisman and Kort, 2015, expressions (B11) and (B12)). The instantaneous consumer surplus $cs(X_t, K)$ is equal to

$$\int_{p}^{\infty} \left(\frac{X_t}{p}\right)^{1/\gamma} \mathrm{d}p = \frac{\gamma X_t^{1/\gamma}}{1-\gamma} p^{\frac{\gamma-1}{\gamma}} \bigg|_{X_t K^{-\gamma}}^{\infty} = \frac{\gamma X_t}{1-\gamma} K^{1-\gamma}.$$

Given $X_0 = X$ at time 0 and capacity level K, the total expected consumer surplus $CS(X, K)$ is equal to

$$CS(X,K) = \mathbb{E}\left[\int_{t=0}^{\infty} \frac{e^{-rt}\gamma X_t K^{1-\gamma}}{1-\gamma} \mathrm{d}t \,\bigg|\, X_0 = X\right] = \frac{\gamma}{1-\gamma} \frac{XK^{1-\gamma}}{r-\mu}.$$

Accounting for the expected producer surplus $PS(X, K)$, which is equal to the value of the project for the firm, given by (2), i.e.

$$PS\,(X,K) = \frac{XK^{1-\gamma}}{r-\mu} - \delta_0 - \delta_1 K,$$

the total expected surplus is given by

$$W(X,K) = CS(X,K) + PS(X,K) = \frac{XK^{1-\gamma}}{(1-\gamma)(r-\mu)} - \delta_0 - \delta_1 K. \tag{6}$$

Following the same steps as in Section 2.2 we can then derive that for a given GBM level X, the corresponding social optimal investment capacity is equal to

$$K_W(X) = \left(\frac{X}{\delta_1(r-\mu)}\right)^{1/\gamma}.$$

For a given capacity size K, the socially optimal investment threshold reads

$$X_W(K) = \frac{\beta(r-\mu)(1-\gamma)(\delta_0+\delta_1 K)}{(\beta-1)K^{1-\gamma}}.$$

Combining $K_W(X)$ and $X_W(K)$ yields the socially optimal investment timing and capacity presented in Proposition 2.

Proposition 2 *The socially optimal investment threshold X_W^* and the corresponding optimal investment capacity K_W^* are given by*

$$\begin{aligned} X_W^* &= X_F^*(1-\gamma) = \delta_1(r-\mu)\left(\frac{\beta\delta_0(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma}, \\ K_W^* &= K_F^* = \frac{\beta\delta_0(1-\gamma)}{\delta_1(\beta\gamma-1)}. \end{aligned}$$

Comparing the regulator's and the firm's optimal investment, we find that given the same realization of the stochastic process X_t, investment takes place earlier in the social optimum, but with the same capacity size. Note that this is different than in Huisman and Kort (2015). That paper employs a linear demand function to find that the firm invests at the socially optimal time but with a capacity size that is half of the socially optimal capacity. However, under iso-elastic demand the profit maximizing firm thus invests too late from a social planner perspective. The resulting welfare loss discounted to the investment moment of the profit maximizing firm is given by the following expression:

$$\left(\frac{X_F^*}{X_W^*}\right)^\beta W(X_W^*, K_W^*) - W(X_F^*, K_F^*) = \left(\left(\frac{1}{1-\gamma}\right)^\beta - \frac{\beta\gamma + 1 - \gamma}{1-\gamma}\right)\frac{\delta_0}{\beta\gamma - 1} > 0.^2$$

3 Subsidy

Denote the subsidy flow as $s(X_t, K, S_i),\ i \in \{P, F\}$ for a given capacity level K and a subsidy rate parameter S_i. This flow can be implemented in the form of FIP. On the one hand it can be a flexible price support, where it is a proportional add-on to the market price, i.e. $s(X_t, K, S_P) = p_t(X_t)KS_P$. On the other hand it can be a fixed price support, where it is a fixed add-on to the market price, i.e. $s(X_t, K, S_F) = KS_F$. These types of subsidies have been predominantly used in European countries in the previous two decades in an attempt to encourage investment in renewables. The regulator can also subsidize through a lump sum transfer, which often reimburses part of the investment costs (a one-time remuneration transfer presented as a fraction of investment costs), i.e. $s(K, S_G) = S_G(\delta_0 + \delta_1 K)$. Here we assume no distribution costs associated with the subsidy transfer and analyze mainly the subsidy scheme that is applied in Italy, which is a fixed add-on S_F to the market price. This type of subsidy scheme is applied in several other countries as well[3].

Subsidies can be implemented for two reasons. The first reason is to increase welfare. From this perspective, in this section we first determine the subsidy level that changes the investment decision of a profit maximizing firm in such a way that welfare is increased as much as possible. We check whether the first-best solution is obtainable in this way, i.e. whether the obtained investment decision mimics the one that should be chosen by the social planner.

The second reason is to change the firm's investment behavior in such a way that a certain policy target is reached. Such a policy target implies that the firm should invest in a certain minimal capacity size within a certain time frame. Changing the firm's investment behavior has welfare consequences, which is the main topic of our study.

A policy target frequently results from an international agreement to solve or mitigate some environmental problem. Countries sign the corresponding contract and then have to incentivize firms to undertake the necessary investments. They can do so by implementing a subsidy. However, a firm is not obliged to invest, and will especially refrain from doing so if the demand realization is such that the process X stays below the threshold level. This can also happen even if a subsidy makes an investment more attractive. Therefore, despite offering a certain subsidy, a country could still fail to reach the policy target. If this happens, the country usually has to pay a fine. This section finally studies the implications of such a fine for subsidy implementation and resulting welfare consequences.

This section is organized as follows. In Subsection 3.1 we refrain from introducing the policy target to obtain the welfare maximizing subsidy and the corresponding welfare level. Subsection 3.2 considers the problem including the policy target and establishes its effect on subsidy and welfare.

[2] The welfare difference can be treated as a function of γ. The first order derivative with respect to γ is positive. This expression is positive for $\gamma = 0$, so $\gamma > 0$ would also make it positive.

[3] As of 2017 16 European countries have provided support for renewables in the form of Feed-in-premiums (CEER, 2018; IRENA, 2018; IRENA, 2019)).

3.1 The welfare maximizing subsidy

As the firm maximizes profit and thus does not take into account the consumer surplus, the firm's investment generates an externality and does not lead to the first-best outcome. In an attempt to align the firm's investment with the social optimum, a regulator can implement a policy instrument with the aim that the firm internalizes this externality when undertaking the investment. To determine to what extent this is possible, we start out by presenting the two objective functionals of the firm and the regulator. Where the profit-maximizing firm has the objective to maximize the producer surplus:

$$\max_{T\geq 0, K\geq 0} \mathbb{E}\left[\int_{t=T}^{\infty} p(X_t,K)K\exp(-rt)\mathrm{d}t - (\delta_0+\delta_1 K)\exp(-rT)\,\middle|\, X_0 = X\right],$$

the regulator's objective is to maximize the sum of producer and consumer surplus, i.e.

$$\max_{T\geq 0, K\geq 0} \mathbb{E}\left[\int_{t=T}^{\infty} \big(p(X_t,K)K + cs(X_t,K)\big)\exp(-rt)\mathrm{d}t - (\delta_0+\delta_1 K)\exp(-rT)\,\middle|\, X_0 = X\right].$$

From these two expressions it follows that if a subsidy flow $s(X_t,K)$ satisfies $s(X_t,K) = cs(X_t,K)$ for all $t \in [T,\infty)$, it aligns the decisions of the social planner and the firm. The subsidy as such does not influence social welfare directly since it is added to the producer surplus and at the same time has to be deducted as subsidy costs. Instead the subsidy affects social welfare indirectly by influencing the timing and size of the investment of the profit-maximizing firm.

Let $\tilde{S}(X,K)$ be the expected discounted future subsidy transfer from government to firm, and $CS(X,K)$ the expected consumer surplus. In order to align the firm's investment decision with the social optimum, the subsidy needs to be set such that the following conditions hold:

$$\tilde{S}(X_W^*,K_W^*) = CS(X_W^*,K_W^*), \tag{7}$$

$$\frac{\partial\tilde{S}(X_W^*,K_W^*)}{\partial X} = \frac{\partial CS(X_W^*,K_W^*)}{\partial X}, \tag{8}$$

$$\frac{\partial\tilde{S}(X_W^*,K_W^*)}{\partial K} = \frac{\partial CS(X_W^*,K_W^*)}{\partial K}. \tag{9}$$

These conditions are straightforward in the sense that, to get to the same investment decision, the value matching and smooth pasting conditions, i.e., (7) and (8), as well as the first order condition with respect to the investment size, i.e., (9), need to be similar. In particular, equation (7) indicates that the expected discounted future subsidy transfer equals the expected consumer surplus at optimal demand and capacity. Expressions (8) and (9) guarantee that when the firm makes optimal investment decisions, the effect of the subsidy transfer on the firm is indeed the same as the effect of the consumer surplus on the regulator. It follows that if the subsidy can be set such that these three conditions hold, the investment decision of the profit maximizing firm results in the first-best welfare outcome.

Let the expected discounted producer surplus be $PS_F(X,K,S_F)$ for a given GBM level X and a capacity size K under the subsidy rate S_F. The firm's optimal investment decision is $(X^*(S_F),K^*(S_F))$. The corresponding social welfare is $W(X^*(S_F),K^*(S_F))$. If S_F makes the firm's optimal decision to be such that $(X^*(S_F),K^*(S_F)) = (X_W^*,K_W^*)$, then the subsidy scheme is optimal and denoted by S_F^*, which maximizes welfare $W(\cdot,\cdot)$.

We proceed by analyzing how the subsidy influences the firm's investment decision. If the regulator provides a subsidy rate S_F at the beginning of the firm's planning period the firm will take it into account when deciding about its investment. If the firm invests at GBM level X in capacity size K, and the subsidy is provided in the form of a fixed price support, the expected discounted producer surplus can be expressed as

$$PS_F(X,K) = \frac{XK^{1-\gamma}}{r-\mu} + \frac{S_F K}{r} - \delta_0 - \delta_1 K.$$

Maximizing $PS_F(X,K)$ with respect to K yields that the optimal capacity is given by

$$K(X,S_F) = \left(\frac{X(1-\gamma)}{(r-\mu)(\delta_1 - S_F/r)}\right)^{1/\gamma}.$$

The investment threshold for a given capacity size K and subsidy rate S_F is equal to

$$X(K,S_F) = \frac{\beta(r-\mu)(r\delta_0 + r\delta_1 K - S_F K)}{r(\beta-1)K^{1-\gamma}}.$$

Combining these two equations results in the following proposition.

Proposition 3 *When the subsidy flow is $s(X_t, K, S_F) = S_F K$, and provided that $S_F < r\delta_1$,*[4] *the firm's optimal investment threshold $X^*(S_F)$ and corresponding investment capacity $K^*(S_F)$ are given by*

$$X^*(S_F) = \frac{r-\mu}{1-\gamma}\left(\delta_1 - \frac{S_F}{r}\right)\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma-1)}\right)^{\gamma},$$

$$K^*(S_F) = \frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma-1)}.$$

In the previous section we had the same result but then for $S_F = 0$ (see Proposition 1). After comparing Propositions 1 and 2 we learned that, compared to the social optimum, in case of no subsidy the profit maximizing firm invests too late in the right capacity size. Proposition 3 shows that introducing a subsidy has two effects. On the one hand it speeds up the investment, which is thus a good thing from a welfare perspective. However, on the other hand it raises the investment size, which then grows beyond the socially optimal level.[5] We conclude that implementing this subsidy S_F will never result in a first best solution, i.e. it will not align the firm's and the socially optimal investment decision. The intuition is that at the same time one instrument cannot steer two different investment dimensions, timing and size, such that *both* admit their socially optimal level.

Figure 1 depicts the social welfare $w(X^*(S_F), K^*(S_F))$, generated by the profit-maximizing investment decision, as a function of the subsidy rate S_F. The underlying parameter values for the numerical results and illustrations in this section are taken from the empirical analysis provided in Section 4. In particular, $w(X^*(S_F), K^*(S_F))$ stands for the expected stream of social welfare discounted to the initial point in time, where the GBM process admits the value X_0. Therefore, $w(X^*(S_F), K^*(S_F))$ is equal to

$$w(X^*(S_F), K^*(S_F)) = \left(\frac{X_0}{X^*(S_F)}\right)^{\beta} W(X^*(S_F), K^*(S_F)),$$

and it reaches its maximum[6] when the subsidy rate is equal to

$$\hat{S}_F = \frac{r\delta_1\beta\gamma}{\beta\gamma^2 + (\beta+1)(1-\gamma)}. \tag{10}$$

[4]Note that S_F/r is the discounted marginal support from the government by investing one unit and δ_1 is the marginal cost of investing one unit. So $S_F/r < \delta_1$ implies that the firm incurs at least part of the marginal cost.

[5]Our results differ from Bigerna et al. (2019) where a subsidy leads to a firm investing earlier and in a smaller capacity. The difference is due to the difference in demand functions. Bigerna et al. (2019) has a linear inverse demand function of the form $p_t = X_t - \eta K$.

[6]$w(\cdot,\cdot)$ is a convex function of S_F when $S_F < \big(r\delta_1(\beta\gamma+\gamma-1)\big)/\big(\beta\gamma^2+(\beta+1)(1-\gamma)\big)$ and concave otherwise. On the convex part, $w(\cdot,\cdot)$ increases with S_F, so that $w(\cdot,\cdot)$ will always admit its maximum value in the concave domain of S_F.

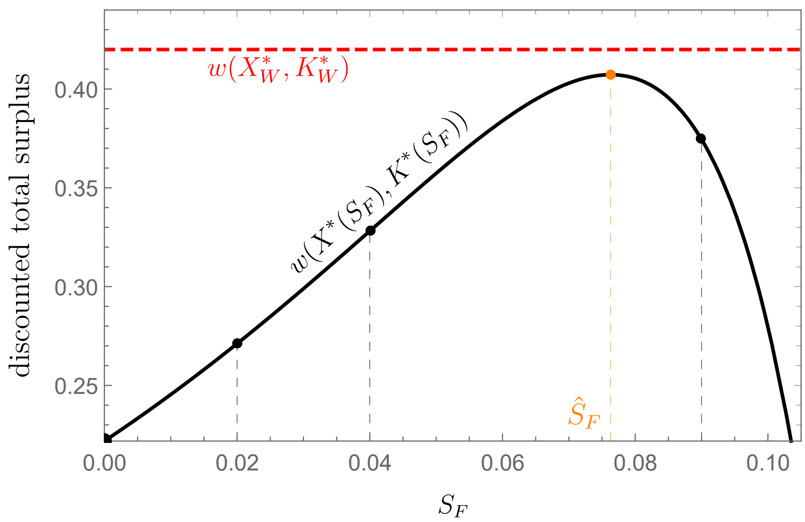


Figure 1: Illustration of the welfare as a function of subsidy rate S_F. Parameter values: $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$, $\delta_1 = 2.365$ and $X_0 = 0.25$.

In Figure 1 this corresponds to $\hat{S}_F = 0.091$. At the same time, indeed the figure clearly shows that the first best solution, represented by the red dashed line, is out of reach.

Figure 2 depicts iso-welfare curves in the (X, K) plane. Every curve corresponds to a subsidy rate S_F and connects points (X, K) resulting in the same welfare level for investments taking place when the current GBM level is X and the investment size is K. One of these points, denoted by a large dot on the corresponding curve, is the optimal investment decision $(X^*(S_F), K^*(S_F))$ corresponding to this subsidy level. More specifically, for a given subsidy rate S_F, all the points on the corresponding iso-welfare curve satisfy the following equation,[7]

$$\frac{XK^{1-\gamma}}{(r-\mu)(1-\gamma)} - \delta_0 - \delta_1 K - \left(\frac{X}{X^*(S_F)}\right)^{\beta} W(X^*(S_F), K^*(S_F)) = 0.$$

A curve represents a higher total surplus if it is closer to the red node (X_W^*, K_W^*), which represents the socially optimal decision. Note that in Figure 2, $(X^*(0), K^*(0))$ and (X_W^*, K_W^*) are on the same horizontal level, confirming that the firm installs the socially optimal capacity size when $S_F = 0$. As S_F gradually increases, $K^*(S_F)$ increases and therewith, deviates from the socially optimal capacity size K_W^*, while at the same time the optimal investment threshold $X^*(S_F)$ gets closer to the socially optimal investment threshold X_W^*. Along the welfare dimension the combination $(X^*(S_F), K^*(S_F))$ is getting closer to the socially optimal point (X_W^*, K_W^*) for increasing S_F up until $\hat{S}_F$. As S_F continues to increase from $\hat{S}_F$ on, $(X^*(S_F), K^*(S_F))$ gets further away from (X_W^*, K_W^*), implying a decrease in the generated social welfare.

Finally, let us do some comparative statics analysis, where we depart from the situation that we want to keep the investment size K fixed. This is relevant if a certain investment target needs to be reached. Consider an increase of the uncertainty parameter σ. From Proposition 3 we learn that, given the subsidy level, it raises K^*. This implies that an increase of σ has to be accompanied by a reduction in the subsidy level S_F to keep the investment size K^* at the same level. The increase of σ then has two effects on the

[7] On an iso-welfare curve, the welfare generated by any point is compared at a reference time that corresponds to the GBM level $X^*(S_F)$.

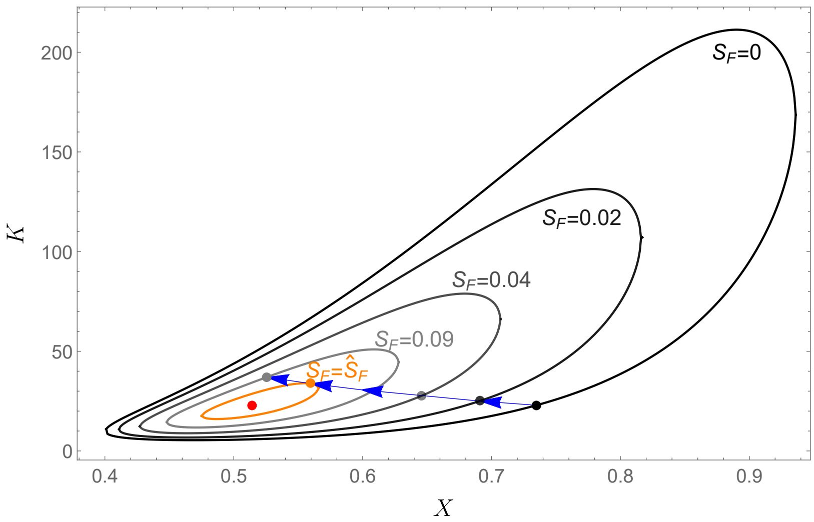


Figure 2: Illustration of the iso-welfare as a function of the subsidy rate S_F for the set of parameter values: $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$ and $\delta_1 = 2.365$.

investment threshold $X^*(S_F)$. First there is a *direct effect*, indicating that an increase of σ raises $X^*(S_F)$. Second there is an *indirect effect,* representing the mechanism that an increase of σ is accompanied by a decrease in S_F, where the latter also raises X^*. We conclude that in case of the uncertainty parameter σ both the direct and the indirect effect result in an increase of X^*, implying that the investment will be delayed. The following corollary gives a complete overview of the comparative statics results.

Remark 1 *For an increase in the market uncertainty σ and the fixed investment cost δ_0, $X^*(S_F)$ increases under both the direct effect and the indirect effect, so the investment is delayed. For changes in the market trend μ and the discount rate r, the influence of the direct effect is not straightforward, which also holds for the total effect.*

3.2 The subsidy and the policy target

The regulator is considered to apply a subsidy policy while taking into account that there exists a policy target regarding investment. In particular, the policy target involves installing a minimally desired level of investment capacity $\bar{K}$ within a designated period of time from $t = 0$ to $\bar{T}$. [8] In case the target is not reached in time, a penalty, say a monetary transfer of C, is levied on the regulator. For instance, in 2014 the EU brought a case against Ireland for failing to fully implement the 2009 Renewable Energy Directive, recommending a penalty payment of €25,445.505 for each day that Ireland had not fully implemented the directive.[9] Although the subsidy is intended to be there to reach the target, still it will have its own welfare implications. The latter also holds for the target itself. Below we establish these welfare effects.

Let us depart from the reasonable scenario that $\bar{K} > K_F^* = K_W^*$, i.e. the international treaty requires a larger investment than that the firm or the social planner would undertake by itself. From Proposition 3 we

[8] Note that we interpret $\bar{K}$ as the expected percentage of demand that will be served by the new investment in RE industry until time $\bar{T}$.

[9] https://ec.europa.eu/commission/presscorner/detail/en/IP_14_44.

obtain that the subsidy level needed to incentivize the firm to invest in a capacity of size $\bar{K}$, is equal to

$$S_F(\bar{K}) = r\delta_1 - \frac{r\delta_0\beta(1-\gamma)}{\bar{K}(\beta\gamma-1)}, \tag{11}$$

whereas the corresponding investment threshold level is given by

$$X(\bar{K}) := X(\bar{K}, S_F(\bar{K})) = \frac{\beta(r-\mu)}{\bar{K}^{1-\gamma}(\beta-1)}\left(\delta_0 + \delta_1\bar{K} - \frac{S_F(\bar{K})\bar{K}}{r}\right) = \frac{\beta\delta_0(r-\mu)}{(\beta\gamma-1)\bar{K}^{1-\gamma}}. \tag{12}$$

Note that this investment threshold decreases with the capacity policy target $\bar{K}$. To explain this result, we have to recognize that there is a direct and an indirect effect. The *direct effect* is known from the literature (e.g. Dangl (1999) or Huisman and Kort (2015)) indicating that for a larger investment to be optimal, better market conditions are needed, which translates in the investment threshold to be higher. The *indirect effect* is due to the subsidy rate S_F, which, according to (11), increases with the capacity target $\bar{K}$, and, therefore, makes a larger investment relatively cheaper for the firm. This reduces the investment threshold $X(\bar{K})$. Our result in (12) shows that the *indirect effect* dominates the *direct effect.* The fact that a larger capacity $\bar{K}$ corresponds to a smaller investment threshold is thus contradictory to the standard real options literature.

The policy target also involves that the investment should take place before a certain time $\bar{T}$. We know that the investment time is time $\bar{\tau}$, where $\bar{\tau} := \inf\{t \geq 0 | x_t \geq X(\bar{K})\}$, implying that $\bar{\tau}$ is stochastic. The policy target is thus reached when $\bar{\tau} \leq \bar{T}$, in which case the subsidy policy works as planned and a capacity size $\bar{K}$ is installed in time. In the other case, thus when $\bar{\tau} > \bar{T}$, however, the necessary investment is not undertaken in time, implying that the regulator faces a penalty.

Until now we analyzed a subsidy level exactly corresponding to an investment of size $\bar{K}$. However, the regulator has an alternative in announcing a higher subsidy rate. This results in the firm investing more, so that still the target is reached upon investment, but also earlier, as our above equation (12) learns. Hence, the advantage of a subsidy rate being higher is that the probability goes up that the investment policy target is reached in time. Denoting the investment time by τ^* for a given subsidy rate S_F and the corresponding investment threshold $X^*(S_F)$ as defined in Proposition 3, so that $\tau^* := \inf\{t \geq 0 | x_t \geq X^*(S_F)\}$, this probability can be expressed as

$$\begin{aligned}\text{Prob}(\tau^* \leq \bar{T}) = \phi &\left(\frac{-\ln\left(\frac{X^*(S_F)}{X_0}\right) + (\mu - \frac{1}{2}\sigma^2)\bar{T}}{\sigma\sqrt{\bar{T}}}\right) \\ &+ \left(\frac{X^*(S_F)}{X_0}\right)^{2\frac{\mu}{\sigma}-1} \phi\left(\frac{-\ln\left(\frac{X^*(S_F)}{X_0}\right) - (\mu - \frac{1}{2}\sigma^2)\bar{T}}{\sigma\sqrt{\bar{T}}}\right),\end{aligned}$$

where $\phi(\cdot)$ denotes the cumulative distribution function of a standard normal random variable.

In deciding about the subsidy rate, not only the probability of reaching the policy target is an important input, but also the effect on welfare should be taken into account. Taking a welfare perspective, we learned from the previous section that without a subsidy the firm is too late in investing in the right size. Then a gradual increase of the subsidy rate makes the firm investing earlier in a larger capacity, where the latter is thus bad for welfare. The fact that it accelerates the investment timing has a positive welfare effect as long as $X^*(S_F) \geq X^*(\hat{S}_F)$.

To quantify the effects of the policy target $(\bar{K}, \bar{T})$ and the subsidy rate S_F on welfare, we first define a welfare function denoted by $\text{WPT}(S_F, \bar{K}, \bar{T})$, where WPT stands for "*the expected Welfare corresponding to the Policy Target*". This function can be expressed as

$\text{WPT}(S_F, \bar{K}, \bar{T}) =$

$$
\begin{cases}
\text{Prob}(\tau^* \le \bar{T}, X^*(S_F)) \left(\frac{X_0}{X^*(S_F)}\right)^\beta W(X^*(S_F), K^*(S_F)) & \text{if } X^*(S_F) > X_0 \text{ and } K^*(S_F) < \bar{K}, \\
-e^{-r\bar{T}}C, & \\
\text{Prob}(\tau^* \le \bar{T}, X^*(S_F)) \left(\frac{X_0}{X^*(S_F)}\right)^\beta W(X^*(S_F), K^*(S_F)) & \text{if } X^*(S_F) > X_0 \text{ and } K^*(S_F) \ge \bar{K}, \\
-\left(1 - \text{Prob}(\tau^* \le \bar{T}, X^*(S_F))\right) e^{-r\bar{T}}C, & \\
\frac{X_0 K^{1-\gamma}(X_0, S_F)}{(1-\gamma)(r-\mu)} - \delta_0 - \delta_1 K(X_0, S_F) - e^{-r\bar{T}}C, & \text{if } X^*(S_F) \le X_0 \text{ and } K(X_0, S_F) < \bar{K}, \\
\frac{X_0 K^{1-\gamma}(X_0, S_F)}{(1-\gamma)(r-\mu)} - \delta_0 - \delta_1 K(X_0, S_F), & \text{if } X^*(S_F) \le X_0 \text{ and } K(X_0, S_F) \ge \bar{K}.
\end{cases} \tag{13}
$$

Equation (13) distinguishes that essentially four different outcomes are possible, depending on whether the current demand is such that immediate investment is optimal, and whether the resulting optimal investment size for the given subsidy is large enough to satisfy the capacity target $\bar{K}$. If immediate investment is not optimal, i.e. $X^*(S_F) > X_0$, we distinguish two cases. First, if the resulting investment size is smaller than the capacity target, the penalty needs to be paid anyhow. Then the WPT is equal to the welfare resulting from the capacity investment if the firm invests before time $\bar{T}$, minus the imposed penalty at time $\bar{T}$.

Second, if the resulting investment size is sufficiently high, i.e. $K^*(S_F) \ge \bar{K}$, the penalty only needs to be paid if the investment does not take place before time $\bar{T}$. It follows that the WPT is equal to the welfare resulting from installing capacity size $K^*(S_F)$ multiplied by the probability that the investment threshold is reached in time, minus the cost of the penalty times the probability that the investment threshold is not reached in time.

If it is optimal for the firm to invest immediately, i.e. $X^*(S_F) \le X_0$, we also have to distinguish two cases depending on whether the installed capacity size at $t = 0$, i.e. $K(X_0, S_F)$, is sufficient to reach the capacity target. If the capacity size is not sufficient, the WPT is equal to the welfare resulting from immediate investment in capacity $K(X_0, S_F)$ reduced by the penalty incurred at the policy deadline $\bar{T}$. In case the capacity installed is large enough, the WPT is just equal to the welfare resulting from immediate investment in capacity $K(X_0, S_F)$.

A higher subsidy rate results in the firm investing more but also earlier. It follows that in the initial situation the firm will only invest immediately if the subsidy rate is large enough. In both situations the subsidy needs to be large enough for the investment size to exceed the capacity target. Hence, for the subsidy rate four different regions could possibly exist that are associated with the four different expressions of $\text{WPT}(S_F, \bar{K}, \bar{T})$ in equation (13). In particular we have

Case 1: $S_F < S_F^{1,2}$: $X^*(S_F) > X_0$ and $K^*(S_F) < \bar{K}$,

Case 2: $S_F^{1,2} \le S_F < S_F^{2,3}$: $X^*(S_F) > X_0$ and $K^*(S_F) \ge \bar{K}$,

Case 3: $S_F^{2,3} \le S_F < S_F^{3,4}$: $X^*(S_F) \le X_0$ and $K^*(S_F) < \bar{K}$,

Case 4: $S_F \ge S_F^{3,4}$: $X^*(S_F) \le X_0$ and $K^*(S_F) \ge \bar{K}$,

in which

$$
\begin{aligned}
S_F^{1,2}(\bar{K}) &= r\delta_1 - \frac{r\delta_0\beta(1-\gamma)}{(\beta\gamma - 1)\bar{K}}, \\
S_F^{2,3} &= r\delta_1 - \frac{r\beta\delta_0(1-\gamma)}{\beta\gamma - 1} \left(\frac{X_0(\beta\gamma - 1)}{\delta_0\beta(r-\mu)}\right)^{\frac{1}{1-\gamma}}, \\
S_F^{3,4}(\bar{K}) &= r\delta_1 - \frac{rX_0(1-\gamma)}{(r-\mu)\bar{K}^\gamma}.
\end{aligned}
$$

However, not all four cases can occur within the same scenario. To see this define the capacity level[10]

$$\tilde{K} = \frac{1}{r}\left(\frac{\delta_0\beta(r-\mu)}{X_0(\beta\gamma-1)}\right)^{\frac{1}{1-\gamma}}.$$

Straightforward calculations reveal that for a capacity target level $\bar{K} = \tilde{K}$, it holds that $S_F^{1,2} = S_F^{2,3} = S_F^{3,4}$, and thus

$$\begin{aligned}
&S_F < S_F^{1,2} : X^*(S_F) > X_0, K^*(S_F) < \bar{K},\\
&S_F = S_F^{1,2} : X^*(S_F^{2,3}) = X_0, K^*(S_F^{2,3}) = \bar{K},\\
&S_F > S_F^{1,2} : X^*(S_F) < X_0, K^*(S_F) > \bar{K}.
\end{aligned}$$

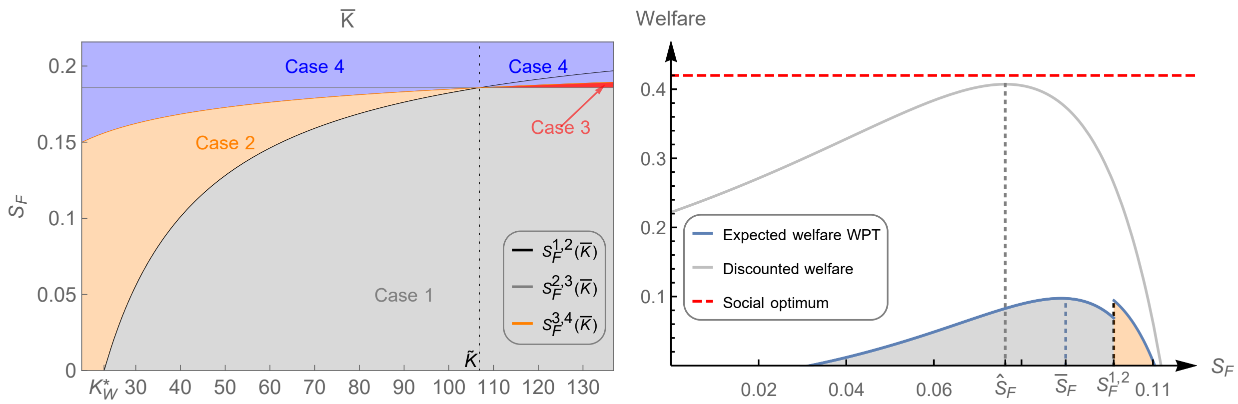
If the capacity target is relatively small, i.e. $\bar{K} < \tilde{K}$, it follows that $S_F^{1,2} < S_F^{3,4} < S_F^{2,3}$, implying

$$\begin{aligned}
&S_F < S_F^{1,2} : X^*(S_F) > X_0, K^*(S_F) < \bar{K},\\
&S_F^{1,2} \le S_F < S_F^{3,4} : X^*(S_F) \ge X_0, K^*(S_F) \ge \bar{K},\\
&S_F^{3,4} \le S_F : X^*(S_F) \le X_0, K^*(S_F) \ge \bar{K}.
\end{aligned} \tag{14}$$

Finally, if the capacity target is relatively large, $\bar{K} > \tilde{K}$, we get that $S_F^{1,2} > S_F^{3,4} > S_F^{2,3}$, which results into

$$\begin{aligned}
&S_F < S_F^{2,3} : X^*(S_F) > X_0, K^*(S_F) < \bar{K},\\
&S_F^{2,3} \le S_F < S_F^{3,4} : X^*(S_F) \le X_0, K^*(S_F) < \bar{K},\\
&S_F^{3,4} \le S_F : X^*(S_F) \le X_0, K^*(S_F) \ge \bar{K}.
\end{aligned}$$

This is all summarized in Figure 3a.



(a) Illustration of different cases

(b) Influence of the subsidy rate on the social welfare

Figure 3: Illustration of different cases and the welfare function. The parameter values are $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$, $\delta_1 = 2.365$ and $C = 2$. It is assumed that $X_0 = 0.25$ and $T = (2030 - 2018) \times 4 = 48$.

For a given policy target, specified by the capacity size $\bar{K}$ and the deadline $\bar{T}$, the regulator has to set the subsidy rate $\bar{S}_F$ to maximize the expected social welfare WPT. Let us take, for example, $\bar{K} = 40$ for

[10]When $X_0 < \frac{\beta\delta_0(r-\mu)}{\beta\gamma-1}\left(\frac{\delta_1(\beta\gamma-1)}{\beta\delta_0(1-\gamma)}\right)^{1-\gamma}$, it holds that $\tilde{K} > K_W^*$; Otherwise, it holds that $\tilde{K} \le K_W^*$.

which it holds that $\bar{K} = 40 < \tilde{K} = 106.8$. This implies that the capacity target is relatively small so that we are in scenario (14), as confirmed in Figure 3a. This example with $\bar{K} = 40$ is illustrated in Figure 3b, where three different social welfare as functions of the subsidy rate S_F are demonstrated: the expected social welfare WPT, the discounted welfare generated by subsidy from the previous section thus ignoring the policy deadline and without the fine C, i.e., $\left(\frac{X_0}{X^*(S_F)}\right)^\beta W(X^*(S_F), K^*(S_F))$, and the socially optimal welfare, i.e. $\left(\frac{X_0}{X_W^*}\right)^\beta W(X_W^*, K_W^*)$.

It turns out that immediate investment in a large capacity, $K^*(S_F) \geq \bar{K}$, is not profitable (for $S_F^{3,4} = 0.168$ the corresponding WPT is negative), so this implies that the welfare maximizing subsidy rate is either Case 1 or Case 2. This means that investment will not take place immediately. Only when the subsidy is large enough the investment size will be such that $K \geq \bar{K}$. The result is that WPT is maximized by setting a subsidy level $\bar{S}_F < S_F^{1,2}$, so that we are in Case 1. It follows that, even when the firm invests in time, still the policy target is not reached, because $K^*(\bar{S}_F) < \bar{K}$. Hence, it will always be the case that the penalty has to be paid. Apparently the fine C is not big enough to avoid this outcome.

Note that, when ignoring the policy target, the optimal fine would be smaller, i.e. $\hat{S}_F < \bar{S}_F$. This is because in the WPT only investments before the policy deadline $\bar{T}$ are counted. Therefore, there is additional stimulus for the firm to invest sooner, which explains the higher subsidy level.

4 Empirical estimation for the 2030 EU policy: The case of Italy

We conduct an empirical analysis to study the policy for renewable energy sources (RES) of Italy, in light of the new EU target for the year 2030. We use official data from the International Energy Agency (IAE, 2018), the International Renewable Energy Agency (IRENA, 2018), the Italian Government (Italian Government, 2018 and 2019) and other technical relevant sources referenced below.

This section is organized as follows. First, we estimate the model parameters, and then we do the simulations.

4.1 Parameter estimation

The policy strategy of Italy is to design measures for the electricity sector focusing on the support of construction of new plants and revamping of existing ones, also with specific incentives. The typical FIP, which depends on size and type and vintage according to the Italian regulatory framework, can be estimated on average to be around 8-10% of the electricity market price in 2017. According to the 2030 EU target, the target share of energy from renewable sources in gross final consumption of energy in 2030 should increase from 17.4% in 2016 to 30% in 2030. This implies a 55.4% RES share in the electricity sector. We have taken the consensus forecast of total final consumption of energy in 2030 from the International Energy Agency Report (IEA, 2019) and we computed the necessary level of RES capacity to satisfy the 55.4% target of RES share in the total electricity in 2030. This new level of RES capacity is expected to reach 93.2 GW then. Given the initial level of existing capacity of 53.2 GW in 2017, this implies an almost 40 additional GW of investment in RES capacity in the period.

The investment cost worldwide ranges between 0.9 and 6 USD/W in the present period. This depends on the location and the technology. With respect to the latter it is known that PV and on shore wind are less costly, while concentrated solar and off-shore wind are more costly (IEA, 2018 and IRENA, 2018). For our calculations we take a prudent value of 2.51 USD/W (Table 1), which is consistent with the new indicator released by the International Energy Agency (2019), the value-adjusted levelized cost of energy (VALCOE) that is in the range of 150-160 USD/MWh (IEA, 2018 and EEA, 2017).

	Cost USD/W	Average plant size MW	Capacity investment per year GW	Price elasticity
world range	0.9–6	3–64	85	-0.3 – -0.05
United States	1.64	11.2	15	-0.11
Asia	1.41	22.5	45	-0.09
Europe	2.51	25	20	-0.20

Table 1: Main economic variables for the calibration of the demand parameters (2017-2018). Sources: own computations based on data from IRENA (2018), Elshurafa et al. (2018), Eurostat (2018), and Ilas et al. (2018).

We know that the current total capacity installed in Italy is 53.2 GW in 2018. Departing from the currently estimated investment cost in 2018 (2.51 USD/W), we can estimate the value of the existing capacity in 2018 to amount to 133.6 billion USD ($133.6 = 2.512 \times 10^9 \times 53.2$).

We define the units as follows. We assume that p is expressed in USD /KWh and K is expressed in GW, so that the policy target corresponds to $\bar{K} = 40$ GW. We use a unit conversion factor from GW to KW (10^{-6}) to maintain coherence in the units of measurement. In order to calibrate the parameter of the demand function for Italy we take that $p_t = X_t K_t^{-\gamma}$ and use $\gamma = .3$, which is an elasticity consistent with the range in Table 1 and with previous findings for Italy (Elshurafa et al., 2018; Bigerna et al., 2019). In this way, from the above formula with the data available in 2018 for the unit energy price, 0.0759 USD/KWh, we get that $0.0759 = X_0 \times 53.2^{-0.3}$, i.e., $X_0 = 0.25$.

The investment cost function per GW is calibrated in the initial year 2018, using the estimated value of the existing capacity (133.6 billion USD), and using engineering and other costs (Ilas et al., 2018) as a proxy for the fixed component δ_0. To do so, we estimate the fixed component at about 5.8% of the total cost (value taken from the report IRENA (2012)). Then we obtain that $\delta_0 = 7.74$ and $\delta_1 = 2.365$, i.e. $133.6 = 7.74 + 2.365 \times 53.2$.

Concerning the discount rate, we use the computation of the Italian Industry Association published in a White book on renewable investment to 2030. Knowing that the so called balance-of-system costs (technical installation, administrative and bureaucratic costs) are relevant in Italy (Andreuzzi et al., 2017), we get to a value for the discount rate $r = 10\%$. To estimate the parameters μ and σ in the GBM process, we assume that X_t is expected to increase with the increase in consumers' willingness to pay for RES, $\phi = 0.0135$, which we estimate as the combination of two effects in the period 2010–2017, namely, the income growth and the expected reduction of the unit cost (similar to Finjord et al. (2018) and Bigerna et al. (2019)). From the historic data of price changes, we estimate the sample variance of log change of X_t as $\sigma^2 = 0.00245$. Based on the GBM relation $\phi = \mu - \sigma^2/2$, we can recover the values (rounded off): $\alpha = 0.015$ and $\sigma = 0.05$. The parameters for the simulations are summarized in Table 2.

Straightforward calculations learn that the firm's and the socially optimal investment decisions without subsidy correspond to investment thresholds of $X_F^* = 0.735$, $X_W^* = 0.514$, and capacity size $K_F^* = K_W^* = 22.91$ GW. Hence, to reach the political target of $\bar{K} = 40$ GW, a subsidy policy is necessary to incentivize the firm to invest more. The target of 40 GW can be reached with a subsidy of about $S_F = 0.101$, which is about 4% of the price (0.101/2.512). Based on the European Commission (2019) report, a rough estimation of the subsidies in 2017 is that S_F is around 10% of the price, and also for Italy it is around 8-10%.

r	μ	σ	δ_0	δ_1	γ	K_0	X_0
0.1	0.015	0.05	7.74	2.365	0.30	53.2	0.25

Table 2: Estimation of parameter values in 2017 for the EU 28 member states. Sources: own computations based on data from Elshurafa et al. (2018), Eurostat (2018), Ilas et al. (2018), and Andreuzzi et al. (2017).

However, the policy target is not only about size but also about timing: the required investment should be undertaken before 2030. In that respect, the previous section has shown that a higher subsidy not only increases the size of the investment but also accelerates it, which thus raises the probability of reaching the target in time. We conclude that it could be optimal to have a subsidy larger than $S_F = 0.101$. This is taken into account in Sections 4.2 and 5.

4.2 Model simulation 2018-2030

We simulate the model for the 2018-2030 period to investigate the probability of reaching the 2030 target. The simulation of the geometric Brownian motion is carried out with 10000 replications.

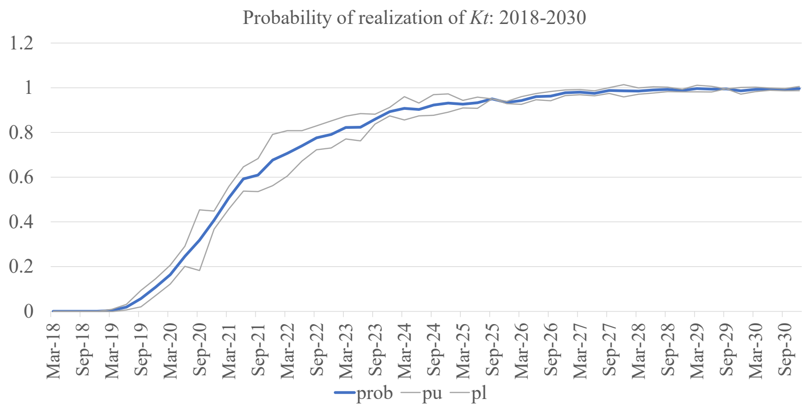
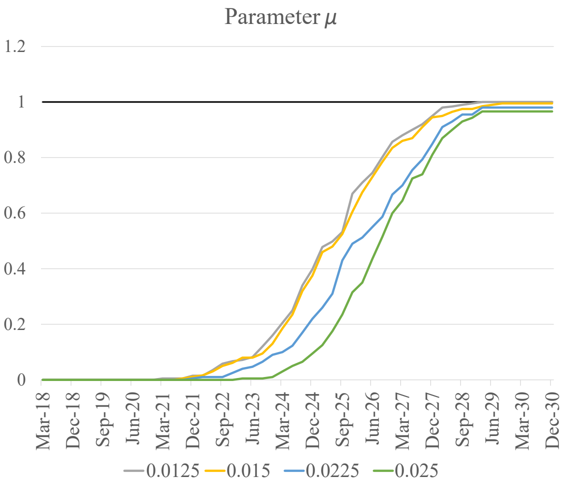


Figure 4: Probability of realization of K_t over the period of 2018-2030.

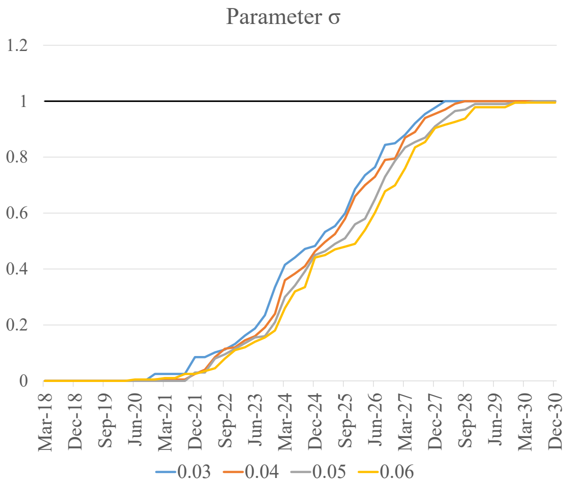
Figure 4 illustrates the average probability that the investment threshold has been reached. As we know, the capacity is equal to the target $\bar{K} = 40$ GW, which corresponds to a subsidy level $S_F = 0.101$, and the investment threshold equals $X^*(S_F) = 0.497$. Figure 4 also includes error bands pu (upper) and pl (lower) of two standard deviations.[11] We determine for each simulation $j \in [1, 10000]$ the value $X_{t,j}$. As long as $X_{t,j}$ stays below $X^*(S_F)$ the firm does not invest so that $K_{t,j} = 0$. As soon as $X_{t,j}$ hits $X^*(S_F)$ from below, the firm invests and $K_{t,j} = \bar{K}$ from that moment on. We then average $K_{t,j}$, $t \in [\text{March } 2018, \text{December } 2030]$, over all 10000 simulations. The result is that the sample probability of reaching the target $\bar{K}$ is approximately 95% at the end of 2029 and 100% at the end of 2030.

We perform a sensitivity analysis of the parameters μ, σ and r, and take four different values for each of them. The other parameter values are the ones reported in Table 2. Concerning the GBM trend μ it can be concluded from Figure 5a that for two out of four values it is almost sure that the capacity target is reached in time. When μ becomes larger, e.g., $\mu = 0.0225$ or $\mu = 0.025$, a capacity target of $\bar{K} = 40$ GW corresponds

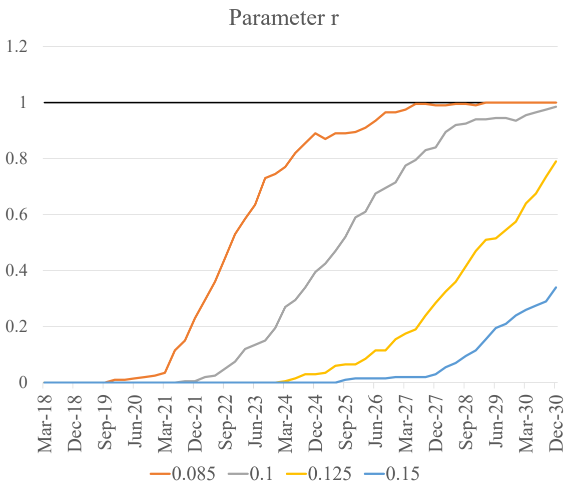
[11] The error bands are empirical confidence intervals of 95% of the probability mass of a normal distribution. It relies on the idea of large numbers, given that we make 10000 simulations.



(a) μ



(b) σ



(c) r

Figure 5: Sensitivity analysis for parameter changes in μ, σ and r influencing the probability of realization.

to a smaller subsidy rate, which results in late investment with positive probability that the capacity target is not realized in time. In such a situation a raise of the subsidy rate may be desirable so that the firm is incentivized to invest earlier.

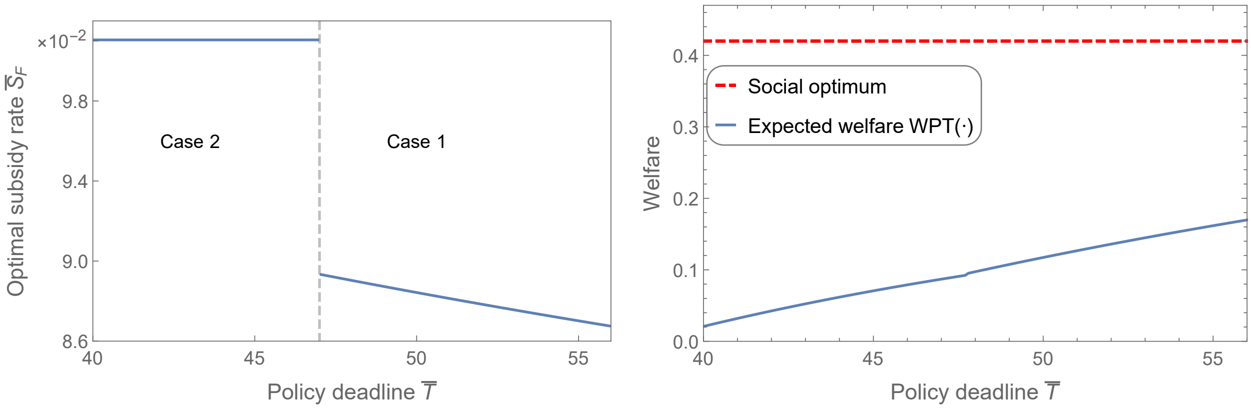
We know from standard real options analysis (e.g. Dixit and Pindyck (1994)) that a more uncertain economic environment delays investment. This explains that Figure 5b shows that for a high uncertainty parameter value, i.e. $\sigma = 0.06$, it takes longer for the probability to reach the 100% level. We conclude that more uncertainty could make that a higher subsidy is needed to tempt the firm to undertake the necessary investment earlier.

A low discount rate goes along with a large net present value of the firm's investment. This is the reason why for higher values of r the probability that the policy target is reached can be quite low (see Figure 5c). For such cases an increase of the subsidy rate can still incentivize the firm to invest in time.

5 Comparative Statics Analysis: the Policy Target and the Subsidy Type

The policy target consists of three parameters, which we consecutively analyze in this section. We start out with the deadline $\bar{T}$, then followed by the required investment size $\bar{K}$ and the penalty C, respectively. The section finishes off by considering other subsidy types than the fixed price support S_F.

5.1 The deadline $\bar{T}$



(a) $\bar{S}_F$ that maximizes the expected welfare

(b) The expected welfare generated by $\bar{S}_F$

Figure 6: Influence of the policy deadline adjustment on the optimal subsidy rate $\bar{S}_F$ and the corresponding expected social welfare. The parameter values are $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$, $\delta_1 = 2.365$ and $C = 2$. It is assumed that $X_0 = 0.25$ and $\bar{K} = 40$.

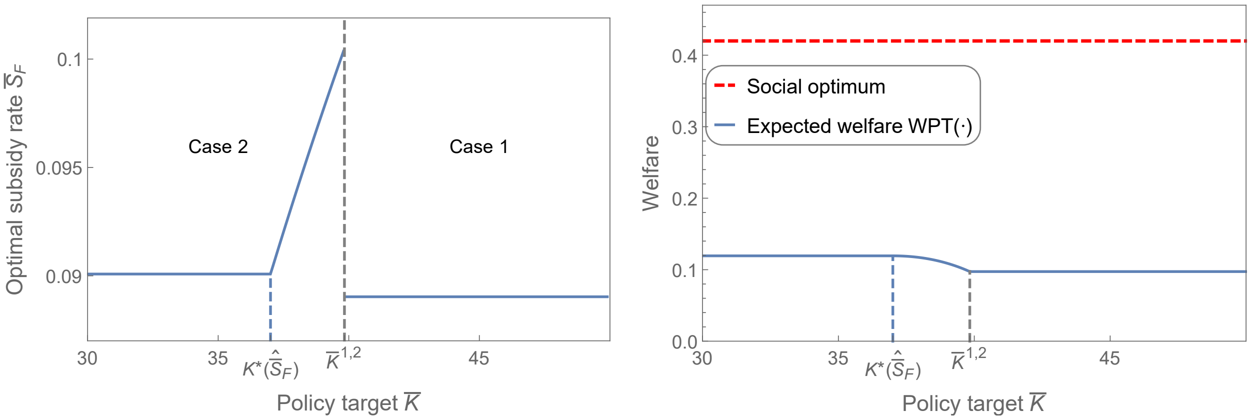
Figure 6a shows how the optimal subsidy rate depends on the policy deadline $\bar{T}$. From Figure 3b we already know that either Case 1 or Case 2 applies, i.e. in any case there is no immediate investment. If the policy deadline is sufficiently tight, $\bar{T} \leq 47$, the optimal subsidy is a constant subsidy level exactly corresponding to an investment of size $\bar{K}$. This implies that it is given by expression (11) and equals $S_F(\bar{K}) = 0.101$ for the parameter values of Table 2.

When $\bar{T} \geq 48$ the policy deadline lies far in the future, so that, whenever a penalty needs to be paid, the corresponding discounted cost is less. Therefore, it is less important for the regulator to reach the target. The regulator prefers to set a subsidy level such that it corresponds to an investment size lower than the target $\bar{K}$, which is closer to the unconstrained welfare maximizing investment size K_F, and leads to larger welfare as shown in Figure 6b. The implication is that, independent of whether the investment will take place before the deadline $\bar{T}$ or not, the country has to pay the penalty in any case.

Figure 6a also shows that the subsidy level is decreasing when the deadline becomes less tight. The reason is that in the WPT the investment payoff only counts when the investment takes place before the deadline is over. A larger subsidy rate accelerates investment and therefore subsidy is large for small $\bar{T}$ in this domain.

5.2 The Capacity Target $\bar{K}$

Setting a subsidy accelerates investment, which is good for welfare, because without a subsidy the firm invests too late from a welfare perspective. Without a policy target the optimal subsidy rate is given by (10). With a policy target of $\bar{K} = 40$ there is an additional incentive to accelerate investment, leading to a higher subsidy rate than (10) as illustrated in Figure 3b.



(a) $\bar{S}_F$ that maximizes the expected welfare

(b) The expected welfare generated by $\bar{S}_F$

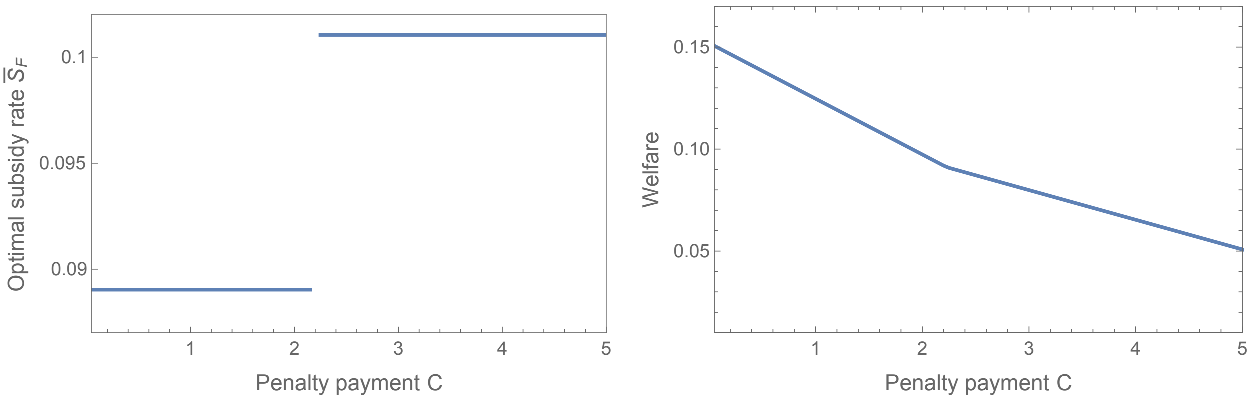
Figure 7: Influence of policy capacity target adjustment on the optimal subsidy rate $\bar{S}_F$ and the corresponding expected social welfare. The parameter values are $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$, $\delta_1 = 2.365$ and $C = 2$. It is assumed that $X_0 = 0.25$ and $\bar{T} = 48$.

Figure 7a shows that for low target levels the subsidy level is constant, say $\hat{\hat{S}}_F$, which, via (11), corresponds to $K^*(\hat{\hat{S}}_F)$. As long as the target level remains below $K^*(\hat{\hat{S}}_F)$, the penalty does not need to be paid upon investment. When the target level raises beyond the level $K^*(\hat{\hat{S}}_F)$ the corresponding subsidy is increasing too, and well in such a way that the firm invests exactly the amount corresponding to the target level. In other words, the subsidy increases with $\bar{K}$ such that expression (11) is satisfied.

If the target level increases beyond a level denoted by $\bar{K}^{1,2}$, the required capacity size deviates too much from the unconstrained socially optimal level K_F. Consequently, for a target level $\bar{K} > \bar{K}^{1,2}$ the regulator gives up reaching the target. In particular, it sets the subsidy at a lower level but still higher than the unconstrained level (10), because there is an incentive to speed up firm investment, due to the fact that the investment payoff only counts for WPT if the investment takes place before the policy deadline. The firm's investment size is such that it is lower than the policy target level $\bar{K}$. It follows that the regulator for sure knows it has to pay the penalty at the deadline $\bar{T}$, even if the investment takes place before this time.

In Figure 7b the corresponding welfare levels are depicted. For $\bar{K} \leq K^*(\hat{\hat{S}}_F)$ the subsidy as well as the firm's investment decision and also the probability of reaching the policy target are constant, which then also holds for WPT. In the capacity target domain $(K^*(\hat{\hat{S}}_F), \bar{K}^{1,2})$ satisfying the policy target requires the firm to invest in a capacity level that deviates more and more from the socially optimal level. Consequently, WPT decreases. For a policy target even larger, subsidy is constant and set at a relatively low level, and the resulting firm's investment decision remains constant there. This then also holds for the resulting WPT that is diminished by the penalty to be paid at the policy deadline. As a result, Figure 7b depicts that social welfare can be damaged by an increasing capacity target level.

5.3 The Penalty C



(a) $\bar{S}_F$ that maximizes the expected welfare

(b) The expected welfare generated by $\bar{S}_F$

Figure 8: Influence of the penalty punishment adjustment on the optimal subsidy rate $\bar{S}_F$ and the corresponding expected social welfare. The parameter values are $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$ and $\delta_1 = 2.365$. It is assumed that $X_0 = 0.25$, $\bar{T} = 48$ and $\bar{K} = 40$.

Figure 8 shows how changes in the penalty payment C influences the optimal subsidy rate $\bar{S}_F$ and the corresponding expected welfare. Figure 8a shows that there are essentially two domains of penalty levels. For C low the regulator is not incentivized enough to satisfy the policy target. Therefore it sets a relatively low subsidy level. Consequently, the firm invests too less to satisfy the target level and the regulator pays the (low) penalty for sure at the policy deadline.

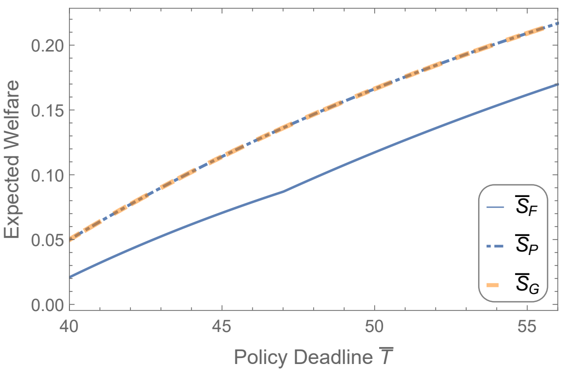
For C large the subsidy rate is higher. This is because the regulator does not want to pay a large penalty. For this reason it sets the subsidy such that the associated investment size is exactly equal to the target level $\bar{K}$. As we know this requires that the subsidy rate corresponds to the target level $\bar{K}$ via expression (11).

It can be expected that in both domains WPT is decreasing in C, because of the larger penalty that needs to be incurred. This also holds for the area where the firm invests according to target level, because then there is a positive probability that the investment is not undertaken before the deadline. Figure 8b confirms.

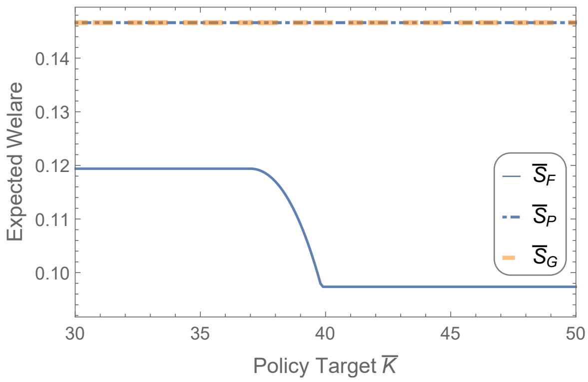
5.4 A Different Subsidy Type

This section considers a different type of feed-in-premium subsidy S_P, which stands for flexible price support rather than the fixed price support S_F, and a lump-sum transfer subsidy S_G. The theoretical derivation of the optimal investment decision and the expected social welfare for these two subsidies can be found in Appendix B. The first insight is that subsidy rates S_P and S_G do not affect the firm's investment capacity, but accelerate the timing of the investment. Considering the situation without the policy target, and given that without subsidy the firm invests too late in the right size, this makes it possible to align the private firm's and the social optimal investment decision, where the required subsidy rates are $S_P^* = \gamma/(1-\gamma)$ and $S_G^* = \gamma$, respectively (see Appendix B).

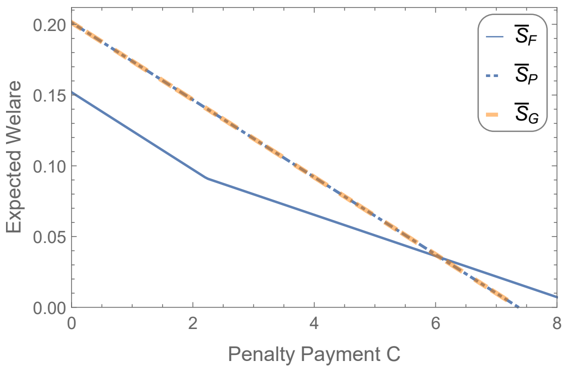
To compare the welfare effect by the optimal subsidy rate $\bar{S}_P$ and $\bar{S}_G$, which maximizes the expected social welfare $\mathrm{WPT}(S_i)$ with $i \in \{P, G\}$ as in equation (19), we conduct a similar numerical analysis as for $\bar{S}_F$, which is shown in Figure 9. When adjusting the policy target, the welfare generated by $\bar{S}_P$ and $\bar{S}_G$



(a) Welfare when adjusting the policy deadline



(b) Welfare when adjusting the policy target



(c) Welfare when adjusting the penalty payment

Figure 9: Effect of the policy adjustment on the expected welfare under subsidy P and G. The parameter values are $\mu = 0.015$, $r = 0.1$, $\sigma = 0.05$, $\gamma = 0.3$, $\delta_0 = 7.74$ and $\delta_1 = 2.365$. The default policy is $\bar{T} = 48$, $\bar{K} = 40$ and $C = 2$.

is identical, where we get that $K_i^*(S_i) = 22.9$, $i = S, G$, $X_P^*(S_P) > X_0$ if $S_P < 1.93$ and $X_G^*(S_G) > X_0$ if $S_G < 0.659$. Given the capacity target $\bar{K} = 40$, the policy target will never be reached. Hence, the penalty has to be paid at the deadline, and the expected welfare function is defined either in Case 1 or Case 3. This is because, as just noted, the subsidy rates S_P and S_G do not influence the investment size. In that sense they are not the suitable policy instruments to incentivize the firm to invest more in order to reach the policy target.

Figure 9a basically shows that, regarding the welfare effects, the conclusions drawn from the analysis with subsidy S_F carry over to subsidies S_P and S_G. A stricter deadline as well as a higher penalty have a negative effect on the WPT. Since S_P and S_G are unable to influence investment size, the capacity target $\bar{K}$ will never be reached. Consequently, the penalty has to be incurred for sure and WPT is not affected for different values of $\bar{K}$ as long as $\bar{K}$ exceeds $K_i^*(S_i)$, $i = S, G$.

6 Conclusion

As a result of international environmental agreements countries have to fulfill policy targets, for instance in the form of commitments regarding green investments. In that light the EU wants from its member states that in 2030 30% of the energy consumption comes from renewable resources. To deal with such a target a country has the possibility to stimulate firm investments by offering subsidies. The aim of this paper is to disentangle the corresponding welfare effects. To reach that aim, this paper develops a new welfare measure, "the expected Welfare corresponding to the Policy Target", abbreviated by WPT. The WPT takes into account all welfare effects of a subsidy, including the fine a country needs to incur upon not reaching the target. This enables us to determine the optimal subsidy rate.

The paper mostly concentrates on a FIP subsidy in the form of a fixed price support, which is applied in Italy with regard to the 2030 EU target. Implementing this subsidy lets the firm invest earlier, which is good for welfare, but also more, where the latter is too much from a welfare perspective. In that light an international policy target can cause a tradeoff in the sense that a large investment is required to achieve the target, while at the same time such a large investment is bad for welfare. Taking into account the penalty a country needs to incur upon not reaching the 2030 EU target, we obtain that a fixed price support of 4% is optimal in the Italian situation.

The paper builds on the real options framework where a risk-neutral decision maker can only invest once. Future work could focus on relaxing these assumptions, i.e. allow for the decision maker to invest multiple times and consider risk-averse preferences. It also seems relevant to take into account technological progress due to which later investments are more efficient. Given that other policy instruments such as investment cost reimbursement, tradable green certificate, and quota obligations can also be employed in the energy market, determining their welfare effects is also a relevant extension. Combining one of these instruments with a subsidy could maybe align firm behavior with the social optimum.

References

Jan Abrell, Mirjam Kosch, and Sebastian Rausch. Carbon abatement with renewables: Evaluating wind and solar subsidies in germany and spain. *Journal of Public Economics*, 169:172 – 202, 2019.

Roger Adkins and Dean Paxson. Subsidies for renewable energy facilities under uncertainty. *The Manchester School*, 2015.

Andrea Andreuzzi, Barbara Marchetti, Stefania Nardone, and Rodà Massimo. Libro bianco per uno sviluppo efficiente delle fonti rinnovabili al 2030. Technical report, Confindustria, Roma, Italy, 2017.

Simona Bigerna, Carlo Andrea Bollino, and Paolo Polinori. Market power and transmission congestion in the italian electricity market. *The Energy Journal*, 37:133 – 154, 2016a.

Simona Bigerna, Carlo Andrea Bollino, and Paolo Polinori. Renewable energy and market power in the italian electricity market. *The Energy Journal*, 37(Bollino-Madlener Special Issue), 2016b.

Simona Bigerna, Xingang Wen, Verena Hagspiel, and Peter M. Kort. Green electricity investments: Environmental target and the optimal subsidy. *European Journal of Operational Research*, 279(2):635 – 644, 2019.

Trine K. Boomsma and Kristin Linnerud. Market and policy risk under different renewable electricity support schemes. *Energy*, 89:435–448, 2015.

Trine K. Boomsma, Nigel Meade, and Stein-Erik Fleten. Renewable energy investments under different support schemes: A real options approach. *European Journal of Operational Research*, 220(1):225–237, 2012.

Severin Borenstein. The long-run efficiency of real-time electricity pricing. *The Energy Journal*, 26(3), 2005.

Severin Borenstein and Stephen Holland. On the efficiency of competitive electricity markets with time-invariant retail prices. *RAND Journal of Economics*, 36(3):469–493, 2005.

Michail Chronopoulos, Verena Hagspiel, and Stein-Erik Fleten. Stepwise green investment under policy uncertainty. *Energy Journal*, 37(4):87 – 108, 2016.

Council of European Energy Regulators. Status review of renewable support schemes in europe for 2016 and 2017. Technical report, Dec 2018. URL `https://www.ceer.eu/documents/104400/-/-/80ff3127-8328-52c3-4d01-0acbdb2d3bed`. [Online; accessed 12. Dec. 2019].

Peder A.O. Dalby, Gisle R. Gillerhaugen, Verena Hagspiel, Tord Leth-Olsen, and Jacco J.J. Thijssen. Green investment under policy uncertainty and bayesian learning. *Energy*, 161:1262 – 1281, 2018.

Thomas Dangl. Investment and capacity choice under uncertain demand. *European Journal of Operational Research*, 117(3):415–428, 1999.

Avinash K. Dixit and Robert S. Pindyck. *Investment under Uncertainty.* Princeton University Press, Princeton, 1994.

Thomas Eichner and Marco Runkel. Subsidizing renewable energy under capital mobility. *Journal of Public Economics*, 117:50 – 59, 2014.

Amro M. Elshurafa, Shahad R. Albardi, Simona Bigerna, and Carlo Andrea Bollino. Estimating the learning curve of solar pv balance-of-system for over 20 countries: Implications and policy recommendations. *Journal of Cleaner Production*, 196:122 – 134, 2018.

European Commission. 2009–2010 Report on Progress in Creating the Internal Gasand Electricity Market – Technical Annex. Technical report, Brussels, 2011.

European Commission. Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions: A policy framework for climate and energy in the period from 2020 to 2030. Technical report, Brussels, 2014.

European Commission. Commission staff working document accompanying the document report from the commission to the european parliament, the council, the european economic and social committee and the committee of the regions energy prices and costs in europe {COM(2019) 1 final}. Technical report, European Commission, Brussels, 2019.

European Environment Agency. Renewable energy in Europe 2017 Recent growth and knock-on effects. Technical report, Copenhagen, Denmark, 2017. URL `https://www.eea.europa.eu/publications/renewable-energy-in-europe-2017/download`.

Eurostat. Renewable energy statistics. `http://ec.europa.eu/eurostat/statistics-explained/index.php/Renewable_energy_statistics`, 2018. Online; accessed 30-01-2018.

Fredrik Finjord, Verena Hagspiel, Maria Lavrutich, and Marius Tangen. The impact of norwegian-swedish green certificate scheme on investment behavior: A wind energy case study. *Energy policy*, 123:373–389, 2018.

Kevin A. Hassett and Gilbert E. Metcalf. Investment with uncertain tax policy: Does random tax policy discourage investment. *The Economic Journal*, 109(457):372–393, 1999.

Kuno J. M. Huisman and Peter M. Kort. Strategic capacity investment under uncertainty. *The RAND Journal of Economics*, 46(2):376–408, 2015.

Andrei Ilas, Pablo Ralon, Asis Rodriguez, and Michael Taylor. Renewable power generation costs in 2017. Technical Report 978-92-9260-040-2, International Renewable Energy Agency (IRENA), Abu Dhabi, UAE, 2018.

International Energy Agency. World Energy Outlook 2018. Technical report, 2018.

International Energy Agency. World energy outlook 2019. Technical report, Paris, France, 2019.

International Renewable Energy Agency. Renewable Energy Technologies: Cost Analysis Series. Technical report, June 2012.

International Renewable Energy Agency. Renewable Power Generation Costs in 2017. Technical report, Abu Dhabi, 2018.

International Renewable Energy Agency. Renewable energy auctions: Status and trends beyond price, International. Technical report, Abu Dhabi, 2019.

Nikolaos E. Koltsaklis and Athanasios S. Dagoumas. State-of-the-art generation expansion planning: A review. *Applied Energy*, 230:563 – 589, 2018.

Maria Kopsakangas Savolainen and Rauli Svento. Real-time pricing in the nordic power markets. *Energy Economics*, 34(4):1131–1142, 2012. ISSN 0140-9883.

Mark G. Lijesen. The real-time price elasticity of electricity. *Energy Economics*, 29(2):249 – 258, 2007.

European Parliament and the Council of the European Union. Regulation (eu) 2018/1999 of the european parliament and of the council. *Official Journal of the European Union*, 2018. URL `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018R1999&from=EN`.

Enrico Pennings. Taxes and stimuli of investment under uncertainty. *European Economic Review*, 44(2):383 – 391, 2000.

Robert S Pindyck. The long-run evolution of energy prices. *The Energy Journal*, pages 1–27, 1999.

Salvador Pineda, Trine K. Boomsma, and Sonja Wogrin. Renewable generation expansion under different support schemes: A stochastic equilibrium approach. *European Journal of Operational Research*, 266(3): 1086–1099, 2018.

Dani Rodrik. Political economy and development policy. *European Economic Review*, 36(2):329 – 336, 1992.

Glenn Sheriff. Optimal environmental regulation of politically influential sectors with asymmetric information. *Journal of Environmental Economics and Management*, 55(1):72 – 89, 2008.

Kathleen Spees and Lester Lave. Impacts of responsive load in pjm: Load shifting and real time pricing. *The Energy Journal*, 29(2), 2008.

Nicholas Stern. Public economics as if time matters: Climate change and the dynamics of policy. *Journal of Public Economics*, 162:4 – 17, 2018.

Xingang Wen. *Investment under uncertainty: Timing and capacity optimization.* PhD thesis, Tilburg University, 2017.

Özge. Özdemir, Francisco D. Munoz, Jonathan L. Ho, and Benjamin F. Hobbs. Economic analysis of transmission expansion planning with price-responsive demand and quadratic losses by successive LP. *IEEE Transactions on Power Systems*, 31(2):1096–1107, 2016.

Appendix

A Proofs

Proof of Proposition 1 and 2: The calculation and derivation of the firm's optimal investment decision is similar as the derivation given in Pindyck (1999) and Huisman and Kort (2015).

Proof of Proposition 3: For a given capacity size K and $X_t = X$, the value of the discounted profit flow at X is equal to

$$V(X, K, S_F) = \frac{XK^{1-\gamma}}{r-\mu} + \frac{S_F K}{r} - \delta_0 - \delta_1 K.$$

Maximizing $V(X, K, S_F)$ with respect to K yields that the optimal capacity for a given X is given by

$$K(X, S_F) = \left(\frac{X(1-\gamma)}{(r-\mu)(\delta_1 - S_F/r)}\right)^{1/\gamma}.$$

Substituting $K(X, S_F)$ into $V(X, K, S_F)$ gives the expected value as a function of X and S_F, i.e., $V(X, S_F)$. Let the value before investment threshold X^* be AX^β. Then the value matching and smooth pasting conditions at X^* yield

$$X^*(S_F) = \frac{r-\mu}{1-\gamma}(\delta_1 - S_F/r)\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma - 1)}\right)^\gamma.$$

From the optimal investment threshold $X^*(S_F)$, we can get that the optimal investment capacity $K^*(S_F)$ is equal to

$$K^*(S_F) \equiv K^*(X^*(S_F)) = \frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma - 1)}.$$

Proof of Remark 1: According to $K(X, S_i)$ in Proof of Proposition 3, if there are changes in the uncertainty parameter σ, the regulator can adjust the subsidy rate S_F and still attain the target size of investment such that

$$\frac{\mathrm{d}K^*}{\mathrm{d}\sigma} = \frac{\partial K^*}{\partial S_F}\frac{\partial S_F}{\partial \sigma} + \frac{\partial K^*}{\partial \beta}\frac{\partial \beta}{\partial \sigma} = \frac{\beta(\beta\gamma - 1)}{r}\frac{\partial S_F}{\partial \sigma} - \left(\delta_1 - \frac{S_F}{r}\right)\frac{\partial \beta}{\partial \sigma} = 0.$$

There are two effects shown in the above equation. One effect, denoted by $(\partial K^*/\partial\beta)(\partial\beta/\partial\sigma)$, is the standard real options result that the increase of uncertainty parameter σ makes the firm invest more because $\partial\beta/\partial\sigma < 0$. In order to maintain the investment capacity level, the other effect, denoted by $(\partial K^*/\partial S_F)(\partial S_F/\partial\sigma)$, has to balance the first effect. This implies the adjustment of S_F should be given by

$$\frac{\partial S_F}{\partial \sigma} = \frac{r\delta_1 - S_F}{\beta(\beta\gamma - 1)}\frac{\partial \beta}{\partial \sigma} < 0.$$

S_F decreasing with σ implies that the regulator has to lower the subsidy rate in order to discourage the firm to invest more. This adjustment of subsidy also influences the firm's optimal investment timing, and the effect on X^* is equal to

$$\frac{\mathrm{d}X^*}{\mathrm{d}\sigma} = \frac{\partial X^*}{\partial S_F}\frac{\partial S_F}{\partial \sigma} + \frac{\partial X^*}{\partial \beta}\frac{\partial \beta}{\partial \sigma} = -\frac{r-\mu}{r(1-\gamma)}\frac{r\delta_1 - S_F}{\beta(\beta\gamma - 1)}\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma - 1)}\right)^\gamma \frac{\partial \beta}{\partial \sigma} > 0.$$

This implies that the increases in uncertainty would delay the firm's investment threshold,[12] when the regulator adjusts the subsidy rate to maintain the same level of investment capacity. There are two effects caused by changes in σ, one direct effect represented by $(\partial X^*/\partial\beta)(\partial\beta/\partial\sigma)$, and one indirect effect represented by $(\partial X^*/\partial S_F)(\partial S_F/\partial\sigma)$. In our non-linear model, both effects are positive and an increase in σ delays the investment. Note that this result is different from Bigerna et al. (2019), where as the uncertainty increases, the adjustment of subsidy rate actually decreases the investment threshold, because there a negative indirect effect dominates the positive direct effect.

Similar analysis can be conducted on other parameters. For changes in the fixed investment cost parameter δ_0, in order to maintain the same level of investment size, it holds that

$$\frac{\mathrm{d}K^*}{\mathrm{d}\delta_0} = \frac{\partial K^*}{\partial S_F}\frac{\partial S_F}{\partial \delta_0} + \frac{\partial K^*}{\partial \delta_0} = \frac{\beta(1-\gamma)}{(\beta\gamma-1)(\delta_1 - S_F/r)^2}\left(\delta_1 - \frac{S_F}{r} + \frac{\delta_0}{r}\frac{\partial S_F}{\partial \delta_0}\right) = 0,$$

which yields that the adjustment on the subsidy rate satisfies

$$\frac{\partial S_F}{\partial \delta_0} = -\frac{r\delta_1 - S_F}{\delta_0} < 0.$$

Because the increase in the fixed investment costs makes the firm invest more as implied by $\partial K_F^*/\partial\delta_0 > 0$, the regulator has to discourage the firm to invest less by lowering the subsidy rate S_F to maintain the same investment capacity size. The corresponding influence on X^* is equal to

$$\frac{\mathrm{d}X^*}{\mathrm{d}\delta_0} = \frac{\partial X^*}{\partial S_F}\frac{\partial S_F}{\partial \delta_0} + \frac{\partial X^*}{\partial \delta_0} = -\frac{r-\mu}{r(1-\gamma)}\frac{\partial S_F}{\partial \delta_0}\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma-1)}\right)^{\gamma} > 0.$$

Similar as the influence of σ, the direct effect represented by $\partial X^*/\partial\sigma > 0$, and the indirect effect by $(\partial X^*/\partial S_F)(\partial S_F/\partial\delta_0)$ is also positive. So we observe the effect that an increase in δ_0 delays the firm's investment.

For the changes in the unit cost parameter δ_1, in order to keep the target size of investment, it holds that

$$\frac{\mathrm{d}K^*}{\mathrm{d}\delta_1} = \frac{\partial K^*}{\partial S_F}\frac{\partial S_F}{\partial \delta_1} + \frac{\partial K^*}{\partial \delta_1} = -\frac{\delta_0\beta(1-\gamma)}{(\beta\gamma-1)(\delta_1 - S_F/r)^2}\left(1 - \frac{\partial S_F/\partial\delta_1}{r}\right) = 0.$$

Thus, the adjustment of the subsidy with respect to the unit investment cost is equal to $\partial S_F/\partial\delta_1 = r$. Because the increase in δ_1 makes the firm invest less, the regulator has to encourage the firm's investment by increasing the subsidy rate in order to main the same level of capacity size. Moreover, the provision of the subsidy is based on the installed capacity size, and S_F/r can be treated as the marginal support for one unit of the installed capacity. $\partial S_F/(r\partial\delta_1) = 1$ implies the changes in the marginal support offsets the changes in the marginal cost for the investment capacity. Thus, the adjustment of subsidy rate S_F with respect to δ_1 does not influence the investment threshold X^* and $\mathrm{d}X^*/\mathrm{d}\delta_1 = 0$.

When there are changes in the demand trend parameter μ, in order to keep the desired level of investment, the regulator adjusts S_F in such a way that

$$\frac{\mathrm{d}K^*}{\mathrm{d}\mu} = \frac{\partial K^*}{\partial \beta}\frac{\partial \beta}{\partial \mu} + \frac{\partial K^*}{\partial S_F}\frac{\partial S_F}{\partial \mu} = \frac{\delta_0(1-\gamma)}{(\delta_1 - S_F/r)^2(\beta\gamma-1)^2}\left(\frac{\beta(\beta\gamma-1)}{r}\frac{\partial S_F}{\partial \mu} - \left(\delta_1 - \frac{S_F}{r}\right)\frac{\partial \beta}{\partial \mu}\right) = 0.$$

[12] This can be derived from the expected hitting time of a geometric Brownian motion, i.e.,

$$\mathbb{E}[T(X = X^*)] = \begin{cases} \frac{1}{\mu - \sigma^2/2}\ln\left(X^*/X_0\right) & \text{if } \mu > \sigma^2/2, \\ \infty & \text{if } \mu \le \sigma^2/2. \end{cases}$$

Given that $\partial\beta/\partial\mu < 0$, it holds $(\partial K^*/\partial\beta)(\partial\beta/\partial\mu) > 0$, which is the standard real options result that the firm invests more if the trend of the market demand grows larger. To maintain the same size of investment, this effect has to be offset by a lowered subsidy rate S_F, which discourages the firm to invest more. More specifically, the adjustment of S_F with respect to changes in μ is equal to

$$\frac{\partial S_F}{\partial\mu} = \frac{r\delta_1 - S_F}{\beta(\beta\gamma-1)}\frac{\partial\beta}{\partial\mu} < 0.$$

The total influence of changes in μ and adjustment of S_F on investment timing is given by

$$\begin{aligned}\frac{\mathrm{d}X^*}{\mathrm{d}\mu} &= \frac{\partial X^*}{\partial\mu} + \frac{\partial X^*}{\partial\beta}\frac{\partial\beta}{\partial\mu} + \frac{\partial X^*}{\partial S_F}\frac{\partial S_F}{\partial\mu} \\ &= -\frac{\delta_1 - S_F/r}{1-\gamma}\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma-1)}\right)^\gamma \left(1 + \frac{r-\mu}{\beta(\beta\gamma-1)}\frac{\partial\beta}{\partial\mu}\right).\end{aligned}$$

The sign of $\mathrm{d}X^*/\mathrm{d}\mu$ depends on the other parameters. The direct effect of μ on X^* is denoted by $\partial X^*/\partial\mu + (\partial X^*/\partial\beta)(\partial\beta/\partial\mu)$, where the first term is negative and the second term is positive. So the sign of this direct effect is positive depends on the parameter values, which makes the overall effect of μ on X^* not straightforward, even though the indirect effect $(\partial X^*/\partial S_F)(\partial S_F/\partial\mu)$ is positive.

Similar analysis can be done on the discount rate r. We can first derive that the adjustment of S_F satisfies

$$\begin{aligned}\frac{\mathrm{d}K^*}{\mathrm{d}r} &= \frac{\partial K^*}{\partial r} + \frac{\partial K^*}{\partial\beta}\frac{\partial\beta}{\partial r} + \frac{\partial K^*}{\partial S_F}\frac{\partial S_F}{\partial r} \\ &= \frac{\delta_1(1-\gamma)}{(\delta_1 - S_F/r)^2(\beta\gamma-1)^2}\left(\frac{\beta(\beta\gamma-1)}{r}\left(\frac{\partial S_F}{\partial r} - \frac{S_F}{r}\right) - \left(\delta_1 - \frac{S_F}{r}\right)\frac{\partial\beta}{\partial r}\right) = 0.\end{aligned}$$

The adjustment of S_F with respect to r is given by

$$\frac{\partial S_F}{\partial r} = \frac{r\delta_1 - S_F}{\beta(\beta\gamma-1)}\frac{\partial\beta}{\partial r} + \frac{S_F}{r} > 0,$$

because $\partial\beta/\partial r > 0$. The corresponding influence on the optimal investment timing is

$$\begin{aligned}\frac{\mathrm{d}X^*}{\mathrm{d}r} &= \frac{\partial X^*}{\partial r} + \frac{\partial X^*}{\partial\beta}\frac{\partial\beta}{\partial r} + \frac{\partial X^*}{\partial S_F}\frac{\partial S_F}{\partial r} \\ &= \frac{\delta_1 - S_F/r}{1-\gamma}\left(\frac{\delta_0\beta(1-\gamma)}{(\delta_1 - S_F/r)(\beta\gamma-1)}\right)^\gamma \left(1 - \frac{r-\mu}{\beta(\beta\gamma-1)}\frac{\partial\beta}{\partial r}\right).\end{aligned}$$

The sign of $\mathrm{d}X^*/\mathrm{d}r$ depends on the comparison between $\partial\beta/\partial r$ and $\frac{r-\mu}{\beta(\beta\gamma-1)}$. In the direct effect, it can be derived that $(\partial X^*/\partial\beta)(\partial\beta/\partial r) < 0$, but the sign of $\partial X^*/\partial r$ is not straightforward. The indirect effect is $(\partial X^*/\partial S_F)(\partial S_F/\partial r) < 0$.

B Subsidy P and subsidy G

Under the non-linear demand structure, for the subsidy flow $p_t(X_t)KS_P$ and lump sum subsidy $S_G(\delta_0+\delta_1 K)$, Wen (2017) calculates the firm's investment decision and the corresponding social welfare.

B.1 Subsidy P

When the subsidy is such that $s(X_t, K, S_P) = p_t(X_t)KS_P$, the optimal capacity for a given X is given by

$$K_P(X, S_P) = \left(\frac{X(1-\gamma)(1+S_P)}{\delta_1(r-\mu)}\right)^{1/\gamma}.$$

The firm's investment threshold $X_P^*(S_P)$, given that $X_P^*(S_P) > X_0$, and capacity $K_P^*(S_P)$ for a given subsidy rate S_G read

$$X_P^*(S_P) = \frac{\delta_1(r-\mu)}{(1-\gamma)(1+S_P)}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma} . \tag{15}$$

$$K_P^*(S_P) = \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)} . \tag{16}$$

The corresponding social welfare generated by S_P is given by

$$W_P(S_P) \equiv W(X^*(S_P), K^*(S_P)) = -\frac{\big(1+S_P-\beta S_P+(1+S_P)(\beta-1)\gamma\big)\delta_0}{(1+S_P)(1-\gamma)(1-\beta\gamma)} .$$

Because $X_P^*(S_P=0) > X_W^*$, and the subsidy makes firm invest earlier, it is possible to align firm's and social optimal investment threshold by choosing appropriate subsidy rate S_P^*. This implies that the regulator can align the firm's decision to the social optimal investment with a subsidy rate S_P^* such that,

$$S_P^* = \frac{\gamma}{1-\gamma} .$$

If $X_0 \geq X_P^*(S_P)$, the firm invests at X_0 with capacity $K_P(X_0, S_P)$, which leads to a welfare level $W(X_0, K_P(X_0, S_P))$.

B.2 Subsidy G

When the subsidy is a lump-sum transfer $S_G(\delta_0+\delta_1 K)$ to the investing firm, the investment capacity for a given GBM level X and subsidy rate S_G is equal to

$$K_G(X, S_G) = \left(\frac{X(1-\gamma)}{(r-\mu)(1-S_G)\delta_1}\right)^{1/\gamma} .$$

The investment decision when $X_G^*(S_G) > X_0$ for the subsidy rate S_G is given by

$$X_G^*(S_G) = \frac{\delta_1(r-\mu)(1-S_G)}{1-\gamma}\left(\frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)}\right)^{\gamma} , \tag{17}$$

$$K_G^*(S_G) = \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)} . \tag{18}$$

The regulator can align the firm's and the social optimal investment decision by implementing a subsidy rate $S_G^* = \gamma$. The corresponding social welfare generated by the subsidy rate S_G is given by

$$W_G(S_G) \equiv W(X^*(S_G), K^*(S_G)) = \frac{\big(\beta S_G - 1 + \gamma - \beta\gamma\big)\delta_0}{(1-\gamma)(1-\beta\gamma)} .$$

If $X_0 \geq X_G^*(S_G)$, the firm invests at X_0 with capacity $K_G(X_0, S_G)$, which yields the welfare level $W(X_0, K_G(X_0, S_G))$.

Because neither S_P nor S_G influences the firm's investment capacity, under a RES policy specified by $\bar{T}$, $\bar{K}$ and C, the expected social welfare generated by S_i with $i \in \{P, G\}$ is equal to

$$WPT(S_i, \bar{K}, \bar{T}) =$$
$$\begin{cases} \text{Prob}(\tau^* \leq \bar{T}, X_i^*(S_i))\left(\frac{X_0}{X_i^*(S_i)}\right)^{\beta} W_i(S_i) - \left(\frac{X_0}{X_{\bar{T}}}\right)^{\beta} C & \text{if } X_i^*(S_i) > X_0 \text{ and } \bar{K} > \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)} , \\ \text{Prob}(\tau^* \leq \bar{T}, X_i^*(S_i))\left(\frac{X_0}{X_P^*(S_i)}\right)^{\beta} W_i(S_i) & \\ \quad - \big(1 - \text{Prob}(\tau^* \leq \bar{T}, X_i^*(S_i))\big)\left(\frac{X_0}{X_{\bar{T}}}\right)^{\beta} C & \text{if } X_i^*(S_i) > X_0 \text{ and } \bar{K} \leq \frac{\delta_0\beta(1-\gamma)}{\delta_1(\beta\gamma-1)} , \\ \frac{X_0 K_i^{1-\gamma}(X_0,S_i)}{(r-\mu)(1-\gamma)} - \delta_0 - \delta_1 K_i(X_0, S_i) - \left(\frac{X_0}{X_{\bar{T}}}\right)^{\beta} C & \text{if } X_i^*(S_i) \leq X_0 \text{ and } K_i(X_0, S_i) < \bar{K} , \\ \frac{X_0 K_i^{1-\gamma}(X_0,S_i)}{(r-\mu)(1-\gamma)} - \delta_0 - \delta_1 K_i(X_0, S_i) & \text{if } X_i^*(S_i) \leq X_0 \text{ and } K_i(X_0, S_i) \geq \bar{K} . \end{cases} \tag{19}$$