Effectiveness of Monitoring, Managerial Entrenchment, and Corporate Cash Holdings

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Abstract

We develop a dynamic model of a firm in which cash management is partially delegated to a self-serving manager. Shareholders trade off the cost of dismissing the manager with the cost of managerial discretion over the use of liquid funds. An improvement in corporate governance quality may have a positive or a negative effect on levels and values of cash balances, depending on the source of the improvement. While a reduction of managerial entrenchment results in lower cash balances and mostly higher marginal cash values, we demonstrate that the opposite is true when the monitoring of managerial actions becomes more effective. A managerial asset substitution problem produces a novel hump-shaped relation between the firm’s liquidity levels and the collective propensity of shareholders and managers to reduce cash flow risk.

Keywords: corporate governance, payout policy, risk management

1. Introduction

Economists as far back as Adam Smith (1776) have identified issues relating to the decision-making process of firms where agents do not bear the full extent of the wealth consequences of their actions. Corporate governance mechanisms have emerged to protect shareholders’ interests against decisions that could be at the detriment to their wealth. Sufficient protection from foul play has enabled the separation of ownership and control, which – in turn – has led to the emergence and prevalence of corporations in modern times.

Although it may be evident to many scholars that a firm’s corporate governance influences its decision-making principles, it is sometimes less apparent how certain corporate policies are affected. In this paper, we seek to understand how the quality of a firm’s corporate governance relates to the liquidity and risk

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management policies of a firm. We do this in an attempt to shed light onto conflicting evidence in the empirical literature investigating the afore-mentioned relations. Notably, although several studies report a negative relation between corporate governance and corporate cash holdings (Dittmar, Mahrt-Smith, and Servaes, 2003; Kalcheva and Lins, 2007; Chen, Chen, Schipper, Xu, and Xue, 2012), others document a positive relation (Harford, Mansi, and Maxwell, 2008) or, even, the inability to establish one or the other (Opler, Pinkowitz, Stulz, and Williamson, 1999; Bates, Kahle, and Stulz, 2009). Studies examining the effect of corporate governance on corporate risk management activities yield fairly mixed results too (Haushalter, 2000; Aretz and Bartram, 2010).

To elucidate these conflicting results, we develop a dynamic model of cash management in which we disentangle two key elements of corporate governance, managerial entrenchment and the shareholders’ ability to monitor managerial actions. We use the model to examine their implications on cash holdings and risk-taking strategies of a firm. We also analyze effects that these two separate facets of corporate governance have on the value of corporate cash.

In the presented model, we draw a distinction between two discrete mechanisms through which shareholders can protect their investment: a) the limitation of managerial actions that directly destroy shareholder value (e.g. through the board’s non-ratification of a negative-NPV investment or acquisition) and b) the ultimate disciplining action of replacing the company’s management team (e.g. by enabling the market for corporate control). A deficiency in the ability to exercise any of these two mechanisms exacerbates the agency costs of managerial discretion. We relate the insufficiency of the former to ineffective monitoring, while we attribute the lack of credibility of the latter to managerial entrenchment. Although these two facets of governance are often hard to distinguish, they are explicitly separated from each other in the model to allow us to capture the complex effect that corporate governance can have on corporate policies.\textsuperscript{1}

We embed inefficient monitoring and managerial entrenchment in a stylized continuous-time model of a firm facing the least amount of frictions required to justify positive but finite cash reserves: external financing costs and a cash return shortfall. We recognize two sets of reasons why cash reserves suffer a return shortfall; the first one represents a systematic liquidity discount related to market imperfections, while the second one is related to the effectiveness of a firm’s board monitoring and reflects the ability of its manager to tunnel corporate funds to her own benefit. Consistent with the findings by Bertrand and Schoar (2003) of the active role that managers play in setting corporate dividend policies, we assume that the payout decision is delegated to the firm’s management. At the same time, shareholders maintain the equity issuance decision

\textsuperscript{1}The distinction that we draw bears similarities with the contrast between internal and external control mechanisms (Gillan, 2006), but the overlap is not perfect. For instance, internal mechanisms (e.g., the board of directors) may ultimately be used to materialize the threat of replacing incumbent management, while the inefficiency of external mechanisms (e.g. the mergers and acquisitions market) can facilitate the inefficient empire-building tendency of managers.
to allow for the disciplining role of the issuance process (Rozeff, 1982; Easterbrook, 1984) to be reflected in the amount raised from the market. As an ultimate control mechanism, shareholders hold the option to replace the manager, with the cost of doing representing managerial entrenchment. The parsimonious but explicit incorporation of these two agency cost manifestations allows us to provide an explanation for the conflicting results found in the empirical literature on the effect of corporate governance on cash holding policies.

We show that, in the presence of a self-serving manager, the marginal value of a dollar of corporate cash is not monotonically decreasing in the level of cash reserves nor does it take values exclusively greater or equal to one, in contrast to the case of a firm free of agency concerns. Instead, the deviation from typical optimality conditions due to the shareholder-manager conflict results in a U-shaped cash value function with respect to the level of liquidity. As with prior contributions, liquidity is most valuable close to refinancing because this is when its function as a loss-absorbing buffer matters the most. Indeed, a positive amount of cash could be helpful for the firm to endure a temporary negative cash flow shock and prevent the incurrence of external financing costs. As cash reserves accumulate and the risk of (costly) refinancing recedes, the marginal value of cash drops. The manager’s empire-building disposition induces hoarding of cash beyond shareholders’ optimal disbursement point and eventually leads to an additional dollar of cash held inside the firm being valued below par. However, as the cost of keeping the manager at the helm of the firm mounts with excess cash, so does the moneyness of shareholders’ option to replace her. The threat of dismissal forces the manager to ultimately pay funds out to shareholders, which restores the marginal value of cash back up to par right before payout. This non-monotonicity of the value of cash is a key feature of our model and generates novel testable implications for a firm’s liquidity and risk management policies, as well as for the valuation of corporate cash holdings.

Our contribution to the literature is threefold. First, we isolate two key aspects underlying the shareholder-manager conflict and highlight the ambiguous relation between corporate governance and corporate cash levels. In our framework, both managerial entrenchment and the effectiveness of monitoring managerial actions, contrasting notions of the protection of shareholders’ rights, result in larger cash balances. On one hand, an increase in the costs of searching for and implementing a change of the incumbent manager shields the latter from challenges unless the potential gain in value exceeds these costs. The heightened tolerance towards the firm’s management incubates a delay in cash distributions to investors. On the other hand, the reduction in a manager’s perceived tunneling activities as a result of more effective monitoring ensues an increase of her relative contribution to firm value compared to shareholders’ next best alternative, which she exploits by hoarding more cash inside the firm. Taking into account the marked entanglement of these governance facets, our model allows to explain the mixed results of empirical studies investigating the relationship between the overall quality of corporate governance and holdings of liquid assets.
Second, our paper demonstrates that the source of an improvement in a firm's governance matters when it comes to assessing the impact on the valuation of its corporate reserves. In particular, an extra dollar of cash is on average more valuable in firms with lower managerial entrenchment, e.g. those facing more credible external threats. In the model, the effect of managerial entrenchment on the value of cash is purely indirect and a result of the suspension of payout until cash holdings accumulate at higher levels. In option pricing terms, lower managerial entrenchment implies a lower exercise price of the shareholders' option to replace incumbent management and therefore higher marginal values of cash, all else equal. In contrast, a unit of cash is less valuable in firms with tighter internal monitoring processes. This is not immediately expected because more effective monitoring mechanisms reduce the fraction of cash reserves being wasted in non-value increasing activities. As the complementing fraction of each cash dollar that remains in the firm's reserves is higher, this could be reflected in an increase in the marginal value of cash. This is, however, not the only consequence of a decrease in tunneling that is relevant for valuation purposes. Intuitively, it also involves an increase in the expected instantaneous cash flow to the firm and a delay in paying back funds to shareholders. Both of these effects of more effective monitoring diminish the contribution of a dollar of cash to avoiding external financing costs and to reaching the payout threshold respectively, and hence its value. Consequently, although both a decrease in managerial entrenchment and a decrease in tunneling activities lead to higher firm values, the former is largely associated with higher cash values while the opposite is true for the latter.

Third, our setup enables the analysis of the relation between the levels of corporate risk attitudes and cash holdings. Due to the suboptimality of the payout decision, the shareholders’ value function is concave-convex over the range of cash holdings. In risk management terms, shareholders would prefer a reduction in the cash flow risk at lower levels of corporate cash, but would benefit from increased risk-taking at higher levels of cash. At the same time, managers would rather intensify risk-reducing activities as cash reserves grow due to the higher extraction of perks when the firm is cash rich. Interestingly, when managerial entrenchment is sufficiently high for managers to avoid replacement, they would even benefit from a temporary increase in the riskiness of a firm’s cash flows when the cash balances are close to depletion. The mismatch in the convexities of the utility functions of the parties involved, and hence in their risk preferences, constitutes an expression of the shareholder-manager conflict and yields a rich set of results and implications.

The contrasting attitudes towards cash flow risk ascribe recognized agency problems to distinct sections of the cash holdings distribution. On the one hand, the risk-increasing tendency of managers at lower levels of cash is at odds with the shareholders’ aversion for cash flow risk when the probability of refinancing is high. This managerial asset substitution problem mirrors managerial hubris behavior, manifestations of which include overconfidence of entrepreneurs (Hayward, Shepherd, and Griffin, 2006) and CEOs (Malmendier and Tate, 2005) as well as heightened corporate risk taking (Li and Tang, 2010). On the other hand, at
higher levels of cash shareholders exhibit a preference towards risk taking while the manager’s aversion to
cash flow risk reaches its highest. This is in line with the managerial “quiet life” hypothesis (Bertrand
and Mullainathan, 2003), according to which managers exhibit excessively cautious behavior such as, for
example, undertaking diversifying acquisitions at the detriment of shareholder value (Gormley and Matsa,
2016). Hence, we find that a firm with entrenched management could experience either of these agency
problems depending on the level of its cash reserves.

In the paper, we show that the change in the attitude towards risk from the two parties does not happen at
the same level of cash holdings. In fact, there is a substantial intersection area where both shareholders and
managers would benefit from mitigating the riskiness of cash flows. While the cash flow volatility depends to
a great extent on the allocation of decision rights regarding the investment mix, we posit that an assumption
of collective decision-making on risk setting results in an untested, to the best of our knowledge, hump-
shaped relation between the level of liquid assets and corporate attitude towards risk taking. On the basis
of a consensual risk management policy, our numerical implementation reveals that while the intersection
interval, and hence the likelihood of observing active risk-reducing activities, shrinks with the effectiveness of
monitoring, it grows with the restriction of managerial entrenchment. Similar to the result on cash holding
levels, the conflicting effects of these two corporate governance facets supports the mixed empirical evidence.

Our modeling follows the literature on safety stock inventory models applied to liquid assets. The earliest
contributions in the field include the discrete-time models by Baumol (1952), Tobin (1956), and Miller and
Orr (1966), while more recently continuous-time models are proposed by Gryglewicz (2011), Bolton, Chen,
and Wang (2011), Dé camps, Mariotti, Rochet, Villeneuve (2011), and Anderson and Carverhill (2012). In all
these articles, the payout and issuance decisions are complying with optimality conditions derived from a firm
value maximization problem. In our framework, this is equivalent to a special case of perfect human capital
market competition, i.e. where equally-skilled agents compete in a world with non-existent replacement costs.
We relax this implied assumption by introducing two agent-specific imperfections, which from a modeling
perspective corresponds to suboptimal decision-making, and analyze their impact on the implications derived
from past contributions.

Our work advances the strand of research that connects agency conflicts to financing, payout, and risk
management decisions. Prior contributions include Jensen (1986), Zwiebel (1996), Morellec (2004), and
Lambrecht and Myers (2008). All of these focus mostly on the leverage decision, but the underlying mechan-
isms bear similarities with our framework. Jensen (1986) argues that managers enjoy higher utility from
managing firms with high cash flows, as they leave more room for managerial discretion over how these
cash flows will be allocated. Increasing a firm’s leverage restricts the agency costs of cash flow and thereby
increases firm value. According to Zwiebel (1996), managers self-constrain their discretion by increasing the
firm’s debt levels to avoid challenges to their tenure. In a similar setup to ours, Morellec (2004) shows that managers use their discretion to overinvest while keeping debt levels low. In our model, managerial discretion is channeled through a retention of higher cash balances and a delay of payouts. As in Lambrecht and Myers (2008), the agency-free outside option of shareholders determines the firm’s incumbent payout policy. Although we examine the same shareholder-manager conflict as these contributions, our paper focuses on the cash holding decision and the effect it has on refinancing and marginal values of cash.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the issuance and payout policies of a firm under the helm of an entrenched self-serving manager. Section 4 derives comparative statics results on the levels and values of cash holdings. Section 5 sketches the implications of our model on corporate risk management policies. Section 6 concludes.

2. Model

We consider a firm operating a single project which generates continuous and random cash flows. The cumulative operating cash flow process \( Y_t \) is modeled as an arithmetic Brownian Motion, such that

\[
dY_t = \mu dt + \sigma dW_t, \tag{1}
\]

where \( \mu \) represents the expected cash flows per period of time, \( \sigma > 0 \) the standard deviation of these cash flows, and \( dW_t \) the increment of a standard Wiener process defined over a probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \) with the filtration \( \{\mathcal{F}_t; t \geq 0\} \) satisfying the usual conditions of completeness and right-continuity. The firm is all-equity financed, has no growth opportunities and both the mean and standard deviation of cash flows are constant over time. All parties are risk neutral and discount cash flows at a rate \( r > 0 \).

Our model allows the firm to hold a non-negative amount of liquid assets in the form of cash reserves, the level of which at any point in time \( t \) is denoted by \( C_t \). Changes in the cash balance follow the realizations of operating cash flows \( dY_t \). Note from (1) that the project may generate not only operating profits \( dW_t > -\mu/\sigma dt \), but also operating losses \( dW_t < -\mu/\sigma dt \). In the former case, a fraction of cash flows may be paid out to investors, with the remainder being retained inside the firm as liquid assets. In the latter case, negative cash flows may be funded either through internal (cash delpetion) or external financing.

We assume that holding cash inside the firm is costly for two reasons. We denote by \( \lambda \) the return shortfall on cash compared to the discount rate \( r \) used by agents. This shortfall may be a result of market imperfections, such as tax distortions (Faulkender and Wang, 2006; Riddick and Whited, 2009) or transaction costs giving rise to liquidity premia (Kim, Mauer, and Sherman, 1998; Vayanos and Vila, 1999). We denote the second component of the cost of carrying cash by \( \theta \) and we attribute it to agency costs of cash stock. In
the spirit of Jensen’s (1986) free cash flow theory, these costs represent the degree of managerial discretion over the use of the firm’s cash reserves, allowing for the manager’s extraction of private benefits of control to the expense of shareholders. Similar to existing cash accumulation models (Décamps et al., 2011; Bolton et al., 2011; Nikolov and Whited, 2014), we assume that the level of resources that the manager can tunnel out of the firm is proportional to the level of the firm’s cash reserves. Given that our model’s main mechanism is based on the comparison of shareholders’ value under incumbent management relative to their outside option, $\lambda$ and $\theta$ may alternatively be both interpreted on the basis of agency costs. Under such an interpretation, $\theta$ would be the tunneling differential between the two alternatives.

Denoting by $U_t$ the cumulative payout process and by $I_t$ the cumulative net issuance process, the corporate cash inventory evolves thus according to

$$dC_t = dY_t + (r - \lambda - \theta) C_t \, dt + dI_t - dU_t,$$

which is the sum of the operating cash flow and the return generated on cash reserves in a time interval $dt$, plus the amount of external financing obtained, less the payout to shareholders during the same time interval.

Firms may raise funds to replenish their cash balance. Raising external financing in the capital markets is typically costly. Following Décamps et al. (2011) and Décamps et al. (2017), we assume that these costs consist of a proportional component amounting to $p - 1 \geq 0$ for each dollar raised, as well as a fixed component $\phi > 0$ that is incurred by the firm each time financing occurs. For example, in order to finance a cash deficit of $d$ dollars, the firm will need to issue securities for $p(d + \phi)$ dollars.\(^2\) Hence, at any time $t$, the cumulative gross security issuance process $J_t$ evolves according to

$$dJ_t = p \left( dI_t + \phi \, 1_{dI_t > 0} \right),$$

where $1_{dI_t > 0}$ is a indicator taking a value of 1 if financing takes place at time $t$ and 0 otherwise.

Fixed and marginal costs have distinct roles in inventory control models. Positive fixed costs ensure that restocking happens in lumpy amounts while marginal costs introduce a wedge between the resetting targets of the state variable. For cash management models in particular, their inclusion is sufficient for reproducing respectively the stylized facts of intermittent financing and non-trivial time lags between financing and payout (see Décamps et al., 2011). The justification of these costs is not necessarily restricted to the obvious

\(^2\)A variant of this design would be to assume instead that the fixed cost is paid directly by the financier rather than the issuer, such that the fixed cost component is not subject to proportional costs, as in, e.g., Bolton et al. (2011). In the example above, this would mean that the firm needs to issue securities only for $(p \cdot d + \phi)$ dollars. Altering the modeling of issuance costs has no qualitative impact on our findings.
transaction costs associated with external financing, but also typically extends, especially in the case of marginal costs, to a reduced-form equivalent of adverse selection costs (Hennessy and Whited, 2007; Riddick and Whited, 2009) that would prevent investors from replenishing cash reserves to the full. The absence of marginal issuance costs from a traditional cash management model would result in a very high probability of a payout taking place immediately after a new security issuance, conflicting thus with empirical findings (Leary and Roberts, 2005).

By introducing a conflict of interest between shareholders and managers, even in a parsimonious fashion, we avoid the need for the proportional cost parameter to reflect agency considerations, confining thus its scope to capturing the effect of direct marginal issuance costs. Indeed, in our framework, unlike in previous contributions, a positive marginal cost is not necessary for a material time interval between refinancing and payout to exist. That is, the probability of a share repurchase immediately after new financing has been attracted is zero even in the absence of marginal brokerage fees, i.e. for \( p = 1 \). As further discussed in Section 3.2.1 below, this result is directly linked to the U-shape of the marginal value of cash in the model.

We model the manager-shareholder conflict by allowing the firm’s liquidity policy to be partially delegated to a self-serving manager who is able to tunnel the agency component of the cost of carrying cash, \( \theta C_t dt \), to her own benefit. As a counterbalance, shareholders hold the right to replace her whenever they see it fit. Upon managerial dismissal, i.e. at time \( \tau_L \), the firm is liquidated for an equivalent of \( L(C_{\tau_L}) \), the supremum value among the subset of alternatives that shareholders have regarding their option value on the firm’s assets. That is, the liquidation function \( L(\cdot) \) represents the value of shareholders’ best outside option.

Following Lambrecht and Myers (2008), we assume that replacing the manager eliminates the agency problem, yet entails costly collective action. Upon liquidation, shareholders receive the payoff they would be entitled to if they were shareholders of an equivalent firm where the manager’s interests are fully aligned with theirs – there is no tunneling of cash (\( \theta = 0 \)) and liquidity policy decisions are maximizing shareholder value; yet, the value of operations is reduced by a fraction \( \gamma \). To illustrate this point, consider the payoff to shareholders of the agency-free firm as the sum of the value of the cashless firm and the value of its cash reserves. Denoting the former by \( a^L \) and the latter by \( m^L(c) \), the liquidation function can be expressed as

\[
L(c) = (1 - \gamma) a^L + m^L(c).
\]  

\( (4) \)

\(^3\)As made clear further on, the mechanism of our model does not rely on the manager’s dismissal as such, but rather on the threat thereof. Whether the firm will continue its operations under new management or will be liquidated is of little importance in our setup, and hence, the terms “managerial dismissal”, “managerial replacement” and “liquidation” are used interchangeably.

\(^4\)This assumption implies that, in this paper, shareholders’ best alternative option is to take the firm private, i.e. operate the firm themselves. An extension allowing for a choice among a continuum of intermediate options between the incumbent management and full alignment is within the bounds of our framework, but would introduce unnecessary complexity here with no real benefit to the scope of this study.

\(^5\)Given their limited liability, shareholders also hold an abandonment option they would exercise as soon as their payoff
The loss in value generated by the parameter $\gamma$ need not be restricted to the cost of collective action; alternative interpretations would include a severance package or a loss of human capital due to the manager’s dismissal.$^6$ What is of essence for our model is the existence of a wedge between the shareholders’ payoff under incumbent management and the value of their outside option, which creates a space for managerial rents to materialize. We refer to any such cost of managerial replacement as “managerial entrenchment”.

Importantly, managerial entrenchment is a theoretical foundation that allows for the partial transfer of decision rights to a self-serving manager leading to liquidity policies that do not necessarily maximize shareholder’s value. The allocation of decision rights in the presented setup stems from the implicit contract between the two parties. The manager holds control rights over the firm’s cash with the shareholders maintaining the respective residual rights. This implies that while shareholders cannot explicitly force the manager to pay out, they can threaten to dismiss her. The manager does not release funds to shareholders unless she knows that they may act upon this threat, i.e. they have an at least equally valuable outside option. Equivalently, the manager cannot force refinancing nor has any control on the amount of cash to be injected in the firm when refinancing occurs. The implicit assumption is that the manager does not have equally valuable alternatives for her human capital and thus cannot credibly threaten to terminate her contract with the shareholders. Given her set of outside options, her optimal strategy simplifies to ensuring that she remains the shareholders’ best option.

We set the objective of the manager as the choice of the payout policy that maximizes the present value of her rents. Recall that these are equal to a proportion $\theta$ of cash reserves that she is able to tunnel to her own benefit for as long as she remains in office. After her dismissal, she is assumed to remain unemployed ever after. The manager’s problem can thus be expressed as

$$\sup_{u \in \mathcal{U}|(i^* \in \mathcal{I}, \tau^{L*})} \mathbb{E} \left[ \int_{0}^{\tau^{L*}} \theta C_t e^{-rt} dt \right],$$

where $C_t$ is the regulated cash process described by (2), $\mathcal{U}$ is a set of admissible cash distribution policies $(u)$, $i^* \in \mathcal{I}$ is the value-maximizing shareholders’ issuance policy from the set of admissible policies and $\tau^{L*}$ is the shareholders’ optimal timing of managerial removal.

Existing shareholders’ objective is to choose the refinancing ($i$) and liquidation ($\tau^L$) policies that maximize

$$L(c) = \max \left[(1 - \gamma) a^L + m^L(c), 0\right].$$

The exercise of this abandonment option is of limited importance to the scope of this study and, therefore, in both the theoretical solution and the numerical implementation that follow, we limit the analysis to parameter values for which the liquidation function is strictly positive.

$^6$In fact, our interpretation of $\gamma$ is related to CEO power and, as such, is not limited to direct costs of managerial replacement, but also indirect ones, such as the loss of managerial talent (Song and Wan, 2019).
the value of their shares. Their optimization problem can thus be expressed as

\[
\sup_{(i \in I, \tau^L) | u^* \in U} \mathbb{E} \left[ \int_0^{\tau^L} (dU_t - dJ_t) e^{-rt} + L(C_{\tau^L}) e^{-r \tau^L} \right],
\]  

(6)

where \( u^* \in U \) is the optimal manager’s payout policy. The first term inside the brackets represents the present value of their payoffs under incumbent management, i.e. the firm’s total payout net of the claim of new investors. The second term represents the present value of their outside option.

3. Model solution

In order to solve the problem described in Section 2 and summarized by equations (5) and (6), we need first to determine the value of shareholders’ outside option \( L(c) \). Recall that the liquidation function is the value of an agency-free but otherwise identical firm, adjusted for the cost of collective action \( \gamma \). Hence, the valuation of \( L(c) \) is closely related to the value of an agency-free firm, which we denote by \( V^L(c) \). Determining \( V^L(c) \) follows the solution in Décamps et al. (2011), a summary of which is reproduced below.

Fixed costs of financing induce shareholders to delay refinancing as much as possible.\(^7\) In the absence of a credit line, external funding is not pursued until cash reserves are fully depleted.\(^8\) Costly refinancing adds value to cash and shareholders refrain from paying it out until the marginal benefit of doing so is higher than the marginal cost. In this “inaction” region, i.e. where neither refinancing nor payout occurs, the cash accumulation process of the agency-free firm evolves according to

\[
dC_t = dY_t + (r - \lambda) C_t \ dt.
\]  

(7)

Standard dynamic programming arguments yield the following differential equation:

\[
r V^L(c) = [\mu + (r - \lambda) c] V^L_c(c) + \frac{\sigma^2}{2} V^L_{cc}(c),
\]  

(8)

where \( V^L_c \) and \( V^L_{cc} \) denote respectively the first- and second-order derivatives of \( V^L \) with respect to \( c \).

Since the proportional cost \( \lambda \) of holding cash is constant, any cash exceeding a threshold level \( \overline{C}^L \) where the marginal benefit and cost of holding cash equate, is paid out to shareholders. Hence, for any level of cash \( c \geq \overline{C}^L \), the value function \( V^L \) satisfies

\[
V^L(c) = V^L(\overline{C}^L) + c - \overline{C}^L.
\]  

(9)

\(^7\)The original model in Décamps et al. (2011) also allows for the liquidation of the firm if financing costs are prohibitively expensive. Consistently with footnote 5, we do not treat here cases that lead to the exercise of this abandonment option.

\(^8\)For the treatment of credit lines in dynamic cash management problems, the reader is referred to Bolton et al. (2011) and Décamps et al. (2017).
At the payout threshold $C^L$, $1 of cash inside the firm is worth to shareholders the same as $1 of cash outside the firm. Hence,

$$V^L_c (C^L) = 1.$$  \(10\)

The maximization of shareholder value at the payout threshold implies the following super-contact condition:

$$V^L_{cc} (C^L) = 0.$$  \(11\)

Turning to the refinancing policy, recall that refinancing costs consist of a fixed and a proportional component. Due to the former, it is beneficial for shareholders to issue lump sums of new equity whenever financing is needed. At the time of issuance, i.e. at $c = 0$, the value-matching condition is

$$V^L (0) = V^L (\tilde{C}^L) - p (\tilde{C}^L + \phi),$$  \(12\)

where $\tilde{C}^L$ is the refinancing target of the agency-free firm. When choosing the target level of refinancing, shareholders weigh the benefit from delaying the next equity issuance against the direct proportional cost of the current issuance. The optimal amount of new securities issued is given by the following smooth-pasting condition:

$$V^L_c (\tilde{C}^L) = p.$$  \(13\)

The value function satisfying conditions (8) to (13) is an increasing and concave function of the level of cash reserves $c$, where the marginal value of cash is strictly greater than one for $c < \tilde{C}^L$ and equal to one at $c = \tilde{C}^L$. Given that the value of cash is zero at $c = 0$, i.e. shareholder value is equal to the value of the cashless firm as defined in (4), the liquidation function $L(c)$ is given by

$$L(c) = (1 - \gamma) \frac{V^L_c (0)}{V^L_c (0)} + \frac{[V^L_c (c) - V^L_c (0)]}{\text{Value of corporate cash, } m^L(c)}.$$  \(14\)

Having determined shareholders’ liquidation function, we now turn to the cash policy and shareholder value for a firm controlled by a self-serving manager. The intuition of the model is that shareholders will exercise their option to replace the manager as soon as the value of their outside option exceeds the value of their claim under incumbent management. We denote the value of shareholders’ claim in the agency-bound firm by $V(c) (\geq L(c))$. The shape of $V(c)$ depends on shareholders’ policy regarding the dismissal of incumbent management.

Given the lack of an equally good outside option for herself, the manager will act in a way that maximizes

\footnote{See Dumas (1991).}
her rents while delaying her replacement as much as possible. Because of her inability to credibly precommit to subsequent dividend payments, the manager has no incentive to pay out to shareholders before it becomes necessary for retaining her tenure. On the other hand, and similar to the agency-free firm, shareholders are better off delaying the incurrence of refinancing costs until cash reserves are completely depleted. Hence, in this case too, there is an inaction region where cash reserves accumulate according to

\[ dC_t = dY_t + (r - \lambda - \theta) C_t \, dt \]  

(15)

and shareholders’ value satisfies the following ordinary differential equation (ODE):

\[ r V(c) = \left[ \mu + (r - \lambda - \theta) c \right] V_c(c) + \frac{\sigma^2}{2} V_{cc}(c). \]  

(16)

Note that the difference between the two processes described by (7) and (15) lies in the manager’s tunneling activity, resulting in a slower cash accumulation (by \( \theta C_t \, dt \)) for the agency-bound firm. Recall that managerial entrenchment creates a wedge between the shareholder’s expected value under incumbent management and the value of their outside option, which results in shareholders’ tolerance of the manager’s adverse spending. The perceived tunneling activity of the manager determines the rate at which this wedge thins with liquid assets and essentially sets out the conditions of her tenure. Naturally, there is a critical level of perceived tunneling \( \tilde{\theta} \) above which, or a degree of managerial entrenchment \( \gamma \) below which, the cost of keeping the manager in charge of the firm becomes so high, or the cost of dismissal so low, that she is unable to maintain her position no matter the payout policy she implements. Although these cases are also of interest and may be worth exploring further, we focus our analysis to values of the tunneling parameter that allow for at least one payout policy guaranteeing the manager’s tenure. For completeness, we outline the conditions for the solution of cases where \( \theta > \tilde{\theta} \) at the end of this section.

For \( \theta \leq \tilde{\theta} \), we conjecture that the manager is able to exercise a payout policy that permanently secures her position inside the firm. To illustrate this, consider the possibility that the manager exercises the payout policy that maximizes shareholder value, as described for the case of the agency-free firm above, but for a cost of carry equal to \( (\lambda + \theta) \). For such a policy and for a given managerial entrenchment parameter \( \gamma \), there is a \( \theta \) sufficiently small such that the shareholder value for the agency-bound firm exceeds the value of the outside option, i.e. the cost of collective action outweighs the loss due to the extra cost of carrying cash. As long as this is the case, there is room for the manager to extract additional perks from her position.

Given that the amount she can expropriate at a given instant is linear in cash balance, the manager is better off delaying payout, i.e. she has incentives to initiate payout at the level of cash that is suboptimally high from shareholders’ perspective. As the payout trigger deviates from the optimal level, the cost of holding an extra dollar of cash inside the firm exceeds the benefit of doing so, and thus the marginal value of cash
drops below one. This means that shareholders’ claim grows at a lower rate than the value of their outside option with respect to cash, and hence, there is a level of cash holdings above which keeping the manager in charge of the firm is so costly that her dismissal becomes optimal.\textsuperscript{10} To maintain her position, the manager needs to pay out as much as necessary so that cash never exceeds shareholders’ indifference level, i.e. the level of cash at which their value in the agency-bound firm is equal to the value of their outside option. Denoting the payout trigger by $C$, this managerial constraint can be expressed as
\begin{equation}
V(C) = L(C).
\end{equation}

Denoting the level of cash after payout by $C - b$, the shareholders’ value function satisfies
\begin{equation}
V(C) = V(C - b) + b.
\end{equation}

Given that the payout threshold $C$ is the highest possible cash stock that will ensure her tenure, it is the point at which her managerial rents are at their peak and, in the absence of a fixed cost of payout, paying out a lump sum to shareholders will only reduce her perks in the next time increment.

Therefore, the manager value maximizing policy is to distribute any cash flow which, added to the existing cash reserves, exceeds this payout threshold, i.e., $b = 0$. Taking the limit of (18) as $b$ approaches to zero yields
\begin{equation}
V^M_c(C) = 1.
\end{equation}

When the firm’s cash reserves are depleted, i.e., at $c = 0$, shareholders decide how much cash is going to be invested in the firm by choosing the amount of new equity to be issued. Recall that the refinancing policy is chosen unilaterally by shareholders and that their payoff is diluted every time that an issuance occurs due to the costs of external financing. As long as the manager remains in her position after refinancing, the shareholders’ payoff function satisfies
\begin{equation}
V(0) = V(C) - p(C + \phi),
\end{equation}
where $\tilde{C}$ represents the net issuance proceeds. These are optimally chosen by shareholders to maximize the value of their claim and therefore satisfy the first-order condition
\begin{equation}
V_c(\tilde{C}) = p.
\end{equation}

The critical level of tunneling, $\theta$, for which the ODE (16) and conditions (17) to (21) are satisfied is
\begin{footnote}
\textsuperscript{10} Recall that the marginal value of cash in the agency-free firm is bounded downwards at one.
\end{footnote}
defined as the level for which the payout threshold $C$ equals the optimal payout threshold that would have been chosen by shareholders running an agency-free firm with a cost of carry ($\lambda + \tilde{\theta}$), denoted by $\bar{C}$. As long as $\theta \leq \tilde{\theta}$ (or, equivalently, $C \geq \bar{C}$), the manager is able to retain her position indefinitely and $V(c) = V^M(c)$. Proposition 1 summarizes the results:

**Proposition 1.** For a firm run by a sufficiently entrenched self-serving manager, it holds that

1. The value of the firm, $V(c)$, is an increasing function of its cash stock $c$, concave for $c \in [0, C^*)$ and convex for $C \in (C^*, \bar{C}]$, where $0 < C^* \leq \bar{C}$.

2. The marginal value of cash, $V^M(c)$, is strictly higher than one in the interval $[0, C_1)$, strictly lower than one in the interval $(C_1, \bar{C})$ and equal to one for $c = C_1 < C^*$ and $c = \bar{C}$.

3. Payout occurs when the accumulated cash reaches the payout threshold $C$, which acts as a reflecting barrier.

4. Equity issuance occurs whenever the firm runs out of cash, i.e. at $c = 0$, at which point shareholders replenish cash reserves up to $\bar{C} \leq C_1$.

Although further developed below, it is worth noting a key result of Proposition 1. Unlike previous contributions of cash accumulation models (Bolton et al., 2011; Décamps et al., 2011; Décamps et al., 2017), the value to shareholders is not strictly concave throughout its support $(0, \bar{C})$. Instead, it is concave for low values of cash, but convex for high values of cash. This in turn implies that although shareholders would not benefit by an increase in the riskiness of cash flows when the firm is running low on liquid assets, they would welcome such an increase in risk when liquidity exceeds the level that maximizes the value of their share. Hence, our framework allows shareholders to gain from policies that decrease cash flow risk, but from also ones that increase it, depending on the level of cash. The risk management implications of this result are further discussed in Section 5.

Another key result, related to the ensuing $U$-shape of the marginal value of cash, is that the value of one dollar of cash inside the firm drops below its face value for high levels of cash. In our framework, a unit of cash in corporate reserves does not always add value to shareholders as a marginal value below one indicates that this unit of cash would be more valuable if it were in their personal reserves (gaining the opportunity rate of return $r$) instead. In Section 4.3 below, we decompose the value of cash and derive empirical implications regarding the impact of firm-specific and economy-wide attributes on the valuation of corporate cash.

Before turning to the analysis of our model, we outline below the conditions that solve for the firm value $V(c)$ in the range of values where conditions (17) to (21) are not satisfied. As already mentioned, Proposition 1 holds as long as the manager’s tunneling activity is sufficiently low to allow for $C \geq \bar{C}$. If not, there is no payout policy that would guarantee her position in the firm. Indeed, for $\theta > \tilde{\theta}$, the manager’s constraint (17) cannot be satisfied as paying out suboptimally early, i.e. at a cash level below $\bar{C}$, would only
decrease the value to shareholders even further. This means that the manager is not able to indefinitely delay her dismissal and shareholders exercise their option as soon as cash holdings reach a level of cash $C^l$ where
\[ V(C^l) = L(C^l). \] (22)
Shareholders exercise their option to dismiss the manager in a way that maximizes their payoff and therefore $C^l$ satisfies the following first-order condition:
\[ V_c(C^l) = L_c(C^l). \] (23)
As long as $C^l$ is higher than $\bar{C}$ as determined by the remaining conditions (20) and (21), the manager holds her position for any cash level $c < C^l$. Note that any attempt to pay out to shareholders triggers an immediate dismissal.

Lastly, when $\theta$ exceeds the level for which $C^l \geq \bar{C}$, the manager is also dismissed when cash reserves are depleted. In such cases of extreme tunneling, conditions (20) and (21) are substituted by
\[ V^M(0) = L(0). \] (24)
Exploring the solution of $V(c)$ that satisfies the conditions above is left for future research.

4. Results

For the baseline firm, we set both the expected operating cash flow $\mu$ and its volatility $\sigma$ to 0.18. We choose to set $\mu = \sigma$ as this entails that the probability of a negative cash flow occurring is approximately $\Phi(-1) \approx 16\%$, in consistency with observed data.\(^{11}\) We set the return of the risk-free asset to $r = 4\%$ and the carry cost of cash to $\lambda = 2\%$. The operating assets’ value loss upon managerial dismissal, i.e., the managerial entrenchment parameter, $\gamma$, is set to 6.5%.\(^{12}\) To separate the effect of managerial entrenchment from tunneling, we initially set $\theta = 0$. Lastly, we set $\phi = 0.01$ and $p = 1.03$, resulting in financing costs of 11.4% of the gross equity issuance for the baseline case.

The solution based on these parameter values results in a cash ratio of 18.8% at payout $(\bar{C}/V(\bar{C}))$ and

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\(^{11}\)Indicatively, the frequency of negative cash flows per firm with at least 10 years of data in Compustat is once every 6.7 years. By setting $\mu = \sigma = 0.18$, we diverge to some extent from numerical values of $\mu = 0.18$ and $\sigma = 0.09$ used in previous contributions (e.g., Bolton et al., 2011; Décamps et al., 2011), as the resulting $\mu/\sigma$ ratio implies a very modest, in comparison, probability of a negative cash flow (just 2.3% per year). Our baseline values have been chosen purely on realism and expositional purposes; setting $\sigma = 0.09$ has no qualitative impact on our results.

\(^{12}\)In a related setup, Lambrecht and Myers (2008) set costs of collective action for firms with strong investor rights to 3% of net operating assets, and those for weak investor rights to 10% respectively. We use the midpoint for our baseline case.
2.5% upon refinancing ($\bar{C}/V(\bar{C})$) for the firm under the control of the entrenched manager.\textsuperscript{13} These ratios are 6.4% and 3.1% respectively for the benchmark agency-free firm.

Prior to analyzing effects of all relevant parameters on cash management policies and values, we first highlight the implications of introducing a self-interested manager to our setup.

4.1. Policy and value implications of managerial entrenchment

Proposition 1 describes a non-monotonic relation between the marginal value of cash and levels of cash: the marginal value of cash decreases with cash for its low levels, but increases with cash when its balance is relatively high. Panel (a) of Figure 1 plots the marginal value of cash for the firm run by the self-interested manager (solid line) and the respective function for an otherwise identical agency-free firm (dashed line). The frictionless marginal value of cash ($V^E_C = 1$) is depicted by the dotted line. The U-shape of the marginal value of cash is a result of the suboptimal payout decision caused by the introduction of the self-interested manager and the ensuing second-best equity issuance decision by shareholders.

[Please insert Figure 1 about here]

For the ease of analysis, we decompose the value of corporate cash into three components: its face value, its precautionary benefit, and its opportunity cost; and define the net marginal value of cash as the difference between the precautionary benefit and the opportunity cost of holding an incremental unit of cash in the firm rather than outside of it.\textsuperscript{14} The precautionary benefit of one dollar of cash reflects the contribution of this dollar to the decrease in the probability of the firm incurring external financing costs; while the opportunity cost reflects the loss in value that is due to the slower rate at which cash effectively accumulates inside the firm rather than outside of it ($r - \lambda - \theta$ rather than $r$).

As discussed above, the wedge between the value of the firm and that of the outside option, created by managerial entrenchment, allows the manager to delay payouts and, most importantly, to accumulate liquid funds uncontestedly. Although delaying payout increases both the precautionary benefit and the opportunity cost of holding cash, the effect of the latter prevails. This is to be expected since the policy under examination constitutes a deviation from the optimal payout level where, under the optimal payout policy, they would equate. As a consequence, the total value of cash drops, and so does firm value.

\textsuperscript{13}Or, equivalently, gross equity proceeds ($p(\bar{C} + \phi)/V(0)$) of 2.9%.

\textsuperscript{14}In other words, the net marginal value of cash is the marginal value of cash net of its face value ($V_C - 1$).
Next, we separate the effect that the introduction of the self-interested manager has on the two components of the net marginal value of cash. Higher payout thresholds lead to higher marginal precautionary benefits as the extension of the cash reserves’ upper limit lends more scope to every dollar of cash for avoiding issuance costs. For instance, the marginal precautionary benefit of a dollar of cash at $C^L$ for the agency-free firm is zero. At the same time, the same dollar has a positive precautionary value in a firm run by the self-interested manager as its addition to cash reserves decreases further the probability of incurring the costs of external financing. It also holds that setting a higher payout threshold entails primarily a higher marginal opportunity cost as every dollar of cash in corporate reserves contributes less to increasing the probability of hitting the payout trigger. Taking the same limiting example as before, the marginal opportunity cost of a dollar of cash received at $C^L$ for the agency-free firm is zero and, considering the marginal precautionary benefit of zero, the market value of this dollar of cash is equal to its face value ($V_c = 1$). Instead, the same dollar of cash, i.e., one received at the same level of corporate cash reserves (recall that $C^L < \bar{C}$), has a positive opportunity cost in the firm run by the self-interested manager as it yields a lower return than the prevailing return in the market. Even after adding up the marginal precautionary benefit, deviating from the optimal payout policy means that the marginal cost exceeds the marginal benefit, resulting thus in a negative net (i.e., below the par value) marginal value of cash.

Multiple results ensue from this drop in the marginal value of cash. First, the marginal value of cash drops below its face value ($V_c < 1$) for a substantial range of cash levels. This means that in this range one dollar of cash is more valuable in the pockets of shareholders rather than in corporate reserves. Note that our framework allows for a marginal value of cash lower than one despite the absence of a fixed cost of distribution, which would otherwise be needed in an agency-free cash model to generate a similar feature. This value shortfall is very much in line with empirical evidence of cash market values being estimated at levels lower than one dollar (Faulkender and Wang, 2006; Pinkowitz, Stulz, and Williamson, 2006; Dittmar and Mahrt-Smith, 2007; Pinkowitz and Williamson, 2007). Panel (b) of Figure 1 highlights that this effect may be so substantial that the total value of cash inside the firm could drop below its face value ($V(c) - V(0) < c$).

Second, the drop in the value of cash results in fewer net proceeds from equity issuances when the firm seeks financing, i.e. $\tilde{C} < \tilde{C}^L$. As cash under the control of an entrenched manager becomes less valuable for investors, they are willing to inject a smaller amount of funds compared to an otherwise identical agency-free firm.

Third, the marginal value of cash is U-shaped with respect to cash levels. As cash reserves accumulate and approach the payout trigger, both the marginal precautionary benefit and the marginal opportunity cost of a dollar of cash shrink and eventually disappear, leading to the marginal cash value converging to its face value. Given that the marginal value of a dollar of cash is valued at less than one dollar (see above) for high
levels of cash, this implies that above a certain level of cash reserves $C^*$ and closer to the payout threshold, the marginal value increases towards its face value. More precisely, the precautionary benefit of an additional dollar of cash decreases at a slower rate than its opportunity cost for high levels of cash, resulting in this dollar being valued more than the previous one added in the cash account. This feature differs from the monotonically decreasing marginal value of cash in previous agency-free contributions (Bolton et al., 2011; Décamp et al., 2011) and yields a novel empirical implication indicating a scope for polynomial tests when it comes to cash holding valuation.

Finally, it is worth noticing that the marginal value of cash increases for very low levels of cash with the introduction of the self-interested manager. This effect is due to the lumpy resetting of the cash balance upon hitting the refinancing barrier ($C^* = 0$).\textsuperscript{15} There are two complementing reasons behind this result: a drop in the opportunity cost close to refinancing due solely to the increase of the payout threshold and a further drop in the opportunity cost due to the simultaneous adjustment of the refinancing target. To highlight the first one, consider a hypothetical situation where the resetting target would not be affected by an increase in the payout threshold. Given that the marginal precautionary benefit would remain virtually unaffected for low levels of cash, an increase in the net marginal value of cash would have to be a result of a decrease in the marginal opportunity cost. Focusing only on the latter, i.e. costs of external financing aside, hitting the refinancing barrier leads to a jump in the cash reserves from zero to the resetting target; an increase in the payout threshold sets the unaltered resetting target further from the former. This means that hitting zero (losing one dollar) is less “valuable” for the same (issuance) cost, i.e. one dollar of cash received close to zero is less “costly” than before; hence the increase in the net marginal value. This effect is complemented by the ensuing decrease in the resetting target as a response to the suboptimally high payout threshold. Lower net equity issuances decrease the marginal precautionary benefit (as shareholders inject less cash into the firm, making refinancing less costly for them), but further decrease the marginal opportunity cost, i.e. they exacerbate the effect described above. Similar to the primary effect on moderate and high levels of cash, the change in the marginal opportunity cost dominates the marginal precautionary benefit. This feature of our model indicates that the effect on the market value of cash of introducing the self-interested manager is not uniform across all levels of cash. Most notably, low cash balances in firms suffering from higher managerial entrenchment may be valued higher than similar reserves in firms with less entrenched managers.

4.2. Comparative statics of cash management policies

Having discussed the overall impact of our framework, we now turn to studying the separate effects of the model’s parameters on cash policies and values. We extend the results to testable implications by expressing

\textsuperscript{15}The reader can check that in a model where restocking is incremental, the marginal value of cash decreases throughout the entire range of cash levels.
the same features as a ratio of firm value, a proxy of which (cash over book value) is typically used in related empirical research.

4.2.1. Managerial entrenchment

Panel (a) of Figure 2 illustrates further the effect of managerial entrenchment on cash policies. As the value of the shareholder’s outside option decreases (an increase in $\gamma$), the manager becomes more irreplaceable and can extract more rents from her decision-making position over time, e.g. through an overall overinvestment in negative NPV projects (empire-building). In our framework, this translates into a delayed dividend payout (higher $\bar{C}$), which leads to mostly lower marginal values of cash, i.e. the value of an additional dollar held inside the firm drops. This drop in value results in fewer proceeds from equity issuances (lower $\tilde{C}$). Panel (a) of Figure 2 also highlights that as entrenchment approaches to zero, the entrenchment-prone firm’s cash policies converge to the cash policies of the agency-free firm (i.e. the shareholder’s outside option). As a matter of fact, the model of Décamps et al. (2011) is nested in our framework, of which it represents a special case where agency problems are eliminated, i.e. both managerial entrenchment ($\gamma$) and tunneling ($\theta$) are set to zero.

The graph illustrates that an increase in the cost of managerial replacement results in an increase in the wedge between refinancing ($\bar{C}$) and payout ($\tilde{C}$) levels, and hence a longer expected time until payout after an equity issuance has taken place. This divergence of the two threshold levels resembles (qualitatively) the effect that an increase in the proportional cost of issuance ($p - 1$) would have on cash policy, as depicted in Panel (b) of Figure 2. Due to this feature, cash management models use proportional cost of issuance parameters as a reduced-form proxy of adverse selection problems to replicate the time lag between financing and payout observed in real data. In fact, setting the proportional cost of equity issuance to zero in an agency-free model would result in shareholders replenishing cash reserves fully (up to the payout threshold), which would imply a high probability of payout ($= 1 - \Phi \left(-\frac{\mu}{\sigma}\right)$) in the next instance. This result is highlighted by the convergence of the thresholds for the benchmark firm (dashed lines) at $p = 1$ in Panel (b) of Figure 2.

The absence of a managerial entrenchment parameter, and of its feature of generating a wedge between refinancing targets and payout thresholds, from an empirical estimation would make the proportional cost of equity issuance a dual-purpose tool. This naturally would lead to upwardly biased model estimates and render them difficult to interpret. An interesting feature of our framework is that, by incorporating a more direct measure of (perceived) agency problems, it alleviates the extra burden from the proportional cost
parameter, allowing it to reflect purely the marginal cost incurred from issuing one extra dollar of equity. As shown in Panel (b) of Figure 2, our framework is able to accommodate even the inexistence of a proportional cost of equity issuance while still maintaining a substantial time lag between financing and payout (solid lines).

4.2.2. Cost-of-carry and managerial tunneling

A slower cash accumulation, due to either market imperfections ($\lambda$) or agency problems ($\theta$), results unarguably in lower firm values since a larger proportion of liquid assets is lost per unit of time. As a dollar of cash becomes more valuable outside the firm for high levels of cash, an earlier payout is optimal. This result is depicted by the drop in the payout threshold of the agency-free firm $C^L$ (dotted line) in Panel (a) of Figure 3. The graph also illustrates that the payout threshold drops for the firm run by the self-interested manager too (solid and dashed lines for the effect of a change in $\theta$ and $\lambda$ respectively) upon the deceleration of the cash reserve build-up, regardless of the source. What differs between the impact of the two cash depleting factors is the rate of change in the payout threshold, but also the direction of their respective effect on the resetting target.

[Please insert Figure 3 about here]

Looking into the effect on the payout threshold, one can notice that an increase in the common cost-of-carry element $\lambda$ leads to a decrease in value of both the professionally-managed firm and the shareholders' outside option. However, because of managerial entrenchment, the former is expected to hold a higher quantity of cash and hence loses more value than the latter. As a result, the wedge between the two narrows and the manager is forced to pay out earlier in order to maintain her position. The decrease in the payout threshold is more pronounced in the case of the tunneling parameter ($\theta$) as the drop in firm value is not mitigated by a respective drop in the value of the shareholders' outside option. Indicatively, an increase in $\lambda$ from 2% (baseline value) to 4% leads to a drop in the payout threshold from 18.8% to 13.4%, while an equivalent increase in $\theta$ from 0 to 2% shrinks the payout threshold to 11.2%.

Turning to the refinancing target, Panel (b) of Figure 3 shows that a deceleration in the cash accumulation results in lower cash injections from shareholders into the benchmark agency-free firm (dotted line). A higher proportion of cash being wasted when held into corporate reserves, makes shareholders reluctant to invest as much into the firm’s activities. Although this effect is mitigated by the ensuing earlier payouts as discussed above, it is not strong enough to revert the overall impact on the size of equity proceeds when the deceleration is due to the common cost-of-carry element, also in the case of the entrenchment-prone firm (dashed line).
However, the latter effect is more pronounced when the deceleration is due to the manager-specific tunneling parameter $\theta$, where the stronger effect on the payout threshold results in an increase in the amount of equity proceeds collected upon refinancing.

4.2.3. Productivity and economy-wide parameters

Turning to the remaining parameters of our framework, their effects on the corporate liquidity policy and the ensuing predictions are in line with those of previous cash management models (e.g. Décamps et al. 2011). The payout threshold is decreasing in the firm’s ability to generate cash flows ($\mu$) and the marginal cost of equity issuance ($p$), but increasing in the volatility of cash flows ($\sigma$), the fixed cost of refinancing ($\phi$), and the risk-free rate ($r$). With the exception of the diverging effect of $p$ discussed in Section 4.2.1, the change in the equity issuance target ratio following the change in one of the parameters above qualitatively parallels the effect on the payout threshold.

4.3. The value of cash

Changes in parameter values obviously impact the marginal value of cash. Although not explicitly modeled in this paper, the value of cash serves as a good indicator of the willingness of shareholders, *ceteris paribus*, to finance corporate activities (e.g. undertaking an investment or repaying some outstanding debt) using internally generated funds. To this end, we let each parameter vary from their base case value and plot the difference in the marginal value functions over the support of the inaction region. We then try to isolate the main source(s) for the observed change.

4.3.1. Managerial entrenchment

As described in Section 4.1, the introduction of managerial entrenchment affects the marginal value of cash positively for low cash levels and negatively for high cash levels. Figure 4 plots the difference in the marginal value of cash function (solid line) caused by an increase in $\gamma$ from 0.04 to 0.08 and depicts a similar qualitative effect. As the cost of dismissing the manager does not affect the cash accumulation process, the only impact on the value of cash stems from the deferral of payouts (i.e. higher $\overline{C}$) and the consequent drop in the amount of funds raised (i.e. higher $\tilde{C}$) at refinancing. We isolate the separate effects of these two changes on the precautionary marginal value ($PV_c$) and the “relative” marginal value ($RV_c$), and compare them to the combined effect on the total marginal value of cash.$^{16}$ We find that the effects of the drop in

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$^{16}$To better illustrate this distinction, think of the value to shareholders as the sum of their obligations (outflows) due to refinancing, and their rights (inflows) as sole claimants of the firm’s payouts. The value gained from the decrease in their expected outflows by each additional dollar of cash reserves is defined as the precautionary marginal value ($PV_c$) and is equal to the first derivative of the expected refinancing costs with respect to the level of cash. Given $\overline{C}$ and $\tilde{C}$, the function satisfies...
The value gained from the increase in expected inflows by each additional dollar of cash reserves is defined as the “relative” marginal value ($RV_c$) and is equal to the first derivative of the expected payouts with respect to the level of cash. In fact, this is equal to the face value of one unit of cash net of the opportunity cost of holding this unit of cash inside the firm rather than the alternative of investing it at the rate of $r$ outside of it. Given $\tilde{C}$ and $\bar{C}$, the function satisfies the following system of equations:

$$r \ PV_c(c) = \left[ \mu + (r - \lambda) \right] PV_c(c) + \frac{\sigma^2}{2} PV_{cc}(c)$$

$$PV_c(\bar{C}) = 0$$

$$PV_c(0) = PV(\tilde{C}) - p(\tilde{C} + \phi)$$

The sum of $PV_c$ and $RV_c$ equals the marginal value of cash, $V_c$. 

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4.3.2. Cost-of-carry and managerial tunneling

Incorporating agency considerations unveils a dual effect of $\theta$ on the value of cash. In models considering only optimal liquidity policy, higher tunneling activity reduces the value of cash. This direct effect simulates the moral hazard problem in information asymmetry environments à la Jensen (1986). Nevertheless, in a model allowing for deviations from payout optimality, higher tunneling activity also decreases the marginal contribution of the manager to firm value, forcing earlier payouts. This indirect effect yields higher values of cash.

Before moving on to the breakdown of the aggregate marginal value effect of managerial tunneling, it is worthwhile to first look into the respective effect of the cost of carrying cash that is common to all firms ($\lambda$). Figure 5 depicts the change in the marginal values of cash for the agency-free firm ($\Delta_\lambda V_C^L$, dot-dashed line) and for the firm run by the self-interested manager ($\Delta_\lambda V_C$, solid line) after a 2% increase (from 0.02 to 0.04) in $\lambda$. As in traditional cash management models, an increase in the cost of carrying cash leads more commonly to lower marginal values of cash in the agency-free case. This is true for the largest portion of the
negatively skewed cash holdings distribution, but the opposite holds close to refinancing due to the lumpy restocking of liquid assets.

One would expect that an increase in the cost-of-carry would have a far more pronounced impact on marginal cash values for the firm run by the self-interested manager compared to the moderate effect depicted on the graph, since the payout in that case occurs at suboptimally high levels of cash. This would indeed be the case if the change did not alter the cash management of the firm; panels (a) and (b) of Figure 6 illustrate this direct effect for the entrenchment-prone and agency-free firm respectively. Yet, recall that, at the same time, an increase in $\lambda$ leads to a drop in the payout threshold as shareholders want to cash out earlier to compensate for the loss in the speed of accumulation inside the firm. This payout acceleration also affects the marginal cash value; panels (c) and (d) depict this indirect effect for both types of firms.

It is worth pointing out that in the case of the firm run by the self-interested manager (panels (a) and (c)) the indirect effect counteracts the direct effect, while this does not seem to be the case for the agency-free firm (panels (b) and (d)). In fact, the indirect effect is far more modest for the agency-free firm and the aggregate marginal value is mostly driven by the direct effect of the slower cash accumulation. Interestingly, the difference between the indirect effect of the two cases is not caused by a stronger effect on the relative value of cash (dotdashed lines), but rather by the virtual lack of an effect on the precautionary value of cash for the firm run by the self-interested manager. In the standard case of the agency-free firm, the condition determining the firm’s cash policy is the balance between benefits (avoid refinancing costs) and losses (cost of carrying) from an extra dollar of cash. Therefore, a small change in the underlying parameters can have a substantial impact on the precautionary value of cash. In our framework, however, the condition determining payout is the managerial replacement trigger which may be far from the levels where an additional dollar of cash would still make a sizeable impact on the precautionary aspect of cash value. In fact, this absence of an effect on the precautionary value of cash in the entrenchment-prone case explains both the modest downward parallel shift and the large increase in the marginal cash value for higher cash levels observed on Figure 5.
Figure 5 also plots the change in the marginal values of cash for the firm run by the self-interested manager after a similar 2% increase (from 0 to 0.02) in \( \theta \) (\( \Delta \theta V_C \), solid line). Given that the direct effect is the same as in the respective 2% increase in \( \lambda \), most of the difference has to come from the increase in the relative value of cash due to the drop of the payout threshold. Given that the change in the payout trigger is more sensitive to changes in \( \theta \) rather than \( \lambda \) as discussed above, the indirect effect is stronger and causes an increase in the marginal value of cash almost throughout the entire support of cash levels.

Combining the results above leads to novel implications for corporate governance policies. Tightening the monitoring of managerial actions (i.e. lowering \( \theta \)) would result in less corporate resources being wasted on unprofitable projects per dollar of cash. However, an extra dollar of cash may contribute less on average to bridging the gap until payout as the manager would still try to cash in as much of her comparative advantage by hoarding more resources within her reach. This latter effect dominates the former, leading to a decrease in the value of cash. At the same time, adding credibility to an irreversible replacement threat (i.e. decreasing \( \gamma \)) acts as a disciplining mechanism forcing the manager to self-restrict her tunneling activity\(^\text{17}\) through earlier payouts. Consequently, the relative contribution of an extra dollar of cash to the reduction of the time until payout, and hence its value, increases.

4.3.3. Productivity and economy-wide parameters

The impact of the remaining parameters included in the model is depicted in Figure 7. We observe that a change in the parameter value for all \( \mu, \sigma, r, \phi, p \) affects the marginal value of the firm run by the self-interested manager in a qualitative similar fashion as it does for the agency-free firm for low and moderate cash ratios. In particular, an increase in \( \mu \) results in higher marginal cash values for low cash ratios and lower for moderate cash ratios; an increase in \( \sigma \) results in lower values for a dollar of corporate reserves at low cash ratios and higher ones at moderate cash ratios; while increases in \( r, \phi, \) or \( p \) result in higher cash values for low and moderate cash ratios. Nevertheless, closer to payout, i.e. for high cash ratios, the effect of a change in parameter values on the marginal cash value of the entrenchment-prone firm is more prominently influenced by the impact on the value of shareholders’ option to replace incumbent management. In fact, changes that trigger earlier payouts (e.g. an increase in \( \mu \)) result in higher marginal values of cash close to the threshold, while changes that delay payouts (e.g. increases in \( \sigma, r, \phi, p \)) cause an opposite effect.

\(^{17}\)In a way similar to a manager’s choice of debt issuance as a voluntary self-constraint in Zwiebel’s (1996) capital structure model.
We conclude this section with an observation regarding the different effect that an alteration in the payout threshold has on the marginal cash value of the two firms (agency-free and run by the self-interested manager) close to payout. In a firm with no agency considerations, delayed distributions to shareholders are associated with higher cash values close to the payout trigger as its increase is the result of a shareholder value maximization criterion. Namely, firms that do not pay out until larger cash reserves have been accumulated are the ones facing higher risk- and time-adjusted costs of refinancing. Technically, that is to say that an earlier payout will have a stronger (negative) effect on the precautionary value of cash than on the cost of carrying cash. On the contrary, in firms facing an agency problem of managerial discretion, a higher payout threshold entails lower marginal cash values. This is true in our model as the amount being tunneled to managers at the expense of shareholders is linearly increasing with the level of cash stock; meaning that a delay in paying distributions to shareholders results in higher costs of carrying cash. Although an increase in the payout threshold also strengthens the precautionary value of cash, the rise is more modest than in the agency-free case since payout occurs at suboptimally high cash levels; and, overall, the marginal value of cash drops.

5. Risk management

In this section, we conjecture how the different claimants (shareholders and the manager) would behave if faced with the opportunity to reduce or amplify the riskiness of the firm’s cash flows.

As pointed out in Proposition 1, the value to shareholders is concave when cash reserves are low (from 0 to $C^*$) and convex when cash reserves are high (from $C^*$ to $\overline{C}$). This implies that shareholders would prefer to adopt risk-decreasing strategies in low states and risk-increasing strategies in high states of the cash variable. Hence, if a frictionless futures contract whose price is a Brownian Motion partially positively correlated to the one driving the firm’s cash flows ($W$) were available, they would like the firm to enter a short position when $c \in (0,C^*)$ and a long position when $c \in (C^*,\overline{C})$. Intuitively, shareholders would like to reduce cash flow volatility when the likelihood of incurring issuance costs is higher and the cost of holding cash ($\theta C$) is low, i.e. close to refinancing. However, as cash increases and the probability of running out of cash decreases, the cost from the manager’s tunneling activities grows and considerably delays payout. Hence, at high levels of cash, it may be more profitable for shareholders to increase cash flow volatility: a positive outcome would lead to earlier payout, while a negative outcome would be mitigated by a reduction of the cash return shortfall from tunneling.

Nonetheless, this strategy would be implemented only if the control rights of the firm’s risk management policy lay with shareholders. If, however, the latter were to be delegated to the firm’s management, the risk
strategy of the firm could differ. In order to determine the strategy that the manager would choose, one needs to characterize her value function, which we denote by $M(c)$. It satisfies the following equation:

$$M(C_t) = \theta C_t \Delta t + e^{-r \Delta t} \mathbb{E}_t [M(C_{t+\Delta t})],$$  

which leads to the ODE:

$$\frac{1}{2} \sigma^2 M_{cc} + [\mu + (r - \lambda - \theta) c] M_c - r M + \theta c = 0. \quad (26)$$

ODE (26) is subject to two conditions stemming from our setup. The first one reflects that incremental payout occurs at $\bar{C}$; at this point, adding a unit of cash returns no additional value to the manager as the entire unit is paid out as dividend to shareholders. Hence,

$$M_c(\bar{C}) = 0 \quad (27)$$

The second condition relates to refinancing. As soon as the firm runs out of cash, the reserves are replenished up to $\hat{C}$ and the manager’s value function satisfies

$$M(0) = M(\hat{C}) \quad (28)$$

Combining these two conditions with the ODE (26) yields the following proposition.

**Proposition 2.** For a firm facing equity issuance costs and run by a sufficiently entrenched self-serving manager, it holds that

1. The value to the firm’s manager, $M(c)$, is U-shaped with respect to cash stock $c$, i.e. decreasing in $[0, \hat{C})$ and increasing in $(\hat{C}, \bar{C})$.
2. $M(c)$ is convex for $c \in [0, C^M)$ and concave for $c \in (C^M, \bar{C}]$, where $\hat{C} < C^M < \bar{C}$.

Proposition 2 reveals the risk preferences of the manager with respect to the level of cash holdings. For low levels of cash, the manager would benefit from a (temporary) increase in the volatility of cash flows as hitting the downward bound of cash reserves results in a costless (from her perspective) refinancing and, hence, an upward jump in her payoffs. For high levels of cash, the manager’s return from tunneling activities approaches its upward bound (achieved at the payout threshold) and reduces her willingness to increase the probability of low states reoccurring. Hence, in the presence of the same frictionless futures contract as above, the manager would like the firm to hold a long position in $(0, C^M)$ and a short position in $(C^M, \bar{C})$. Hence, in case the risk management strategy is delegated to the firm’s manager, she would be more likely to engage in risk-increasing activities when cash reserves are low and undertake risk-mitigating actions when these are high.
Figure 8 summarizes the shareholder-manager conflict on the corporate risk management strategy arising from the analysis above. Shareholders are willing to decrease the firm’s risk-taking when cash holdings lie in \((0, C^*)\) and increase it for cash levels in \((C^*, C)\); while managers prefer to increase the firm’s cash flow risk when cash holdings lie in \((0, C^M)\) and conversely for cash levels in \((C^M, C)\). Hence, there are conflicting preferences among the two parties towards the boundaries of the no-payout region and different risk management strategies could be adopted depending on the allocation of control rights.

[Please insert Figure 8 about here]

In the case where the riskiness of cash flows lies within managerial control, our framework reflects two noted agency conflicts with respect to risk taking. At the lower end of the cash support, the restricted opportunities for tunneling and the lumpy refinancing may lead an entrenched manager to increase the firm’s cash flow riskiness at the detriment of shareholders’ preferences. Outside our model, a propensity to increase cash flow risk would weaken the relation between risk and required return in the decision-making process of a firm and resemble the biased undertaking of negative-NPV projects, as attributed to managerial hubris (Roll, 1986). At the higher end of the support, the increased perks and the upper bound of cash reserves being set at the payout threshold may induce the manager to take risk-reducing actions although shareholders would be better off with higher cash flow volatility. In a real-life setting, this reluctance for risk-taking could be manifested as managerial slack hindering firm value maximization, in line with Hicks’ (1935) “quiet life”.\(^{18}\)

Interestingly, conflicts regarding the risk management strategy of the firm do not occur throughout the full range of possible cash levels. In fact, this is the case towards the center of the no-payout region where an alignment of the risk preferences of both parties could occur, i.e. if \(C^* = C^M\). For instance, a positive (negative) difference of the expression \((C^* - C^M)\) would mean that a lower (higher) exposure to cash flow risk may be benefiting both parties; in the case where \(C^* = C^M\), there is disagreement on the risk management policies across the entire range \([0, C]\).

In Figure 9, we plot the switching points in the risk preferences, i.e. the inflection points of the respective value functions, of shareholders \((C^*)\) and management \((C^M)\) against the parameters of main interest, \(\gamma\) and \(\theta\). Given that shareholders would prefer to reduce cash flow risk for cash levels lower than \(C^*\) and increase risk otherwise, and that management would gain from reducing cash flow risk for cash levels above \(C^M\) and

\(^{18}\)Using Bertrand and Mullainathan’s (2003) interpretation of managers avoiding “difficult decisions and costly efforts” associated to the opening and closure of plants, the manager’s aversion for risk at high levels of cash capacity would push wider the thresholds triggering these decisions.
increase risk otherwise, the shaded area in both sides of the panel indicates the range of cash levels for which both parties would benefit from the lower cash flow riskiness. To illustrate the portion of the stationary distribution of cash as a proxy of the likelihood of the risk preferences of shareholders and manager being aligned, both thresholds are scaled by the payout threshold $C$.

It is worthwhile to note that the resulting predictions vary depending on the allocation of the control rights. If the risk management decision is made by shareholders (the manager), the propensity for reducing cash flow risk decreases (increases) with managerial entrenchment, but increases (decreases) with tunneling. Looking at the particular case where the two parties have to collectively agree on the risk management strategy, panel (a) shows that, as $\gamma$ increases, the two thresholds not only approach zero, but also converge, i.e. the portion of the cash distribution in which risk-reducing actions are undertaken is narrower. Conversely, panel (b) reveals that both thresholds approach the payout threshold as $\theta$ increases, but at different rates such that the range of values for which hedging would occur is wider.

[Please insert Figure 9 about here]

The results of the analysis above are best summarized in the form of empirically testable implications. First, *all else equal*, different risk management activities can occur throughout the range of a firm’s cash levels. In the case where the risk management policy needs to be collectively decided between shareholders and the manager, risk management activities may have a hump-shaped relation to cash reserves; that is, the firm engages in risk-reducing strategies for moderate levels of cash but not in the proximity of refinancing- or payout-triggering levels. This prediction deviates from the traditional view of substitutability between liquidity and risk management (e.g. Bolton et al., 2011) as in our model these policies result from an interplay between the manager’s and shareholders’ preferences. Contrary to the common assumption of managerial risk-aversion (Stulz, 1984; Smith and Stulz, 1985), the manager in our model is entrenched enough to be willing to engage in risk-increasing activities at low values of cash. However, since such a policy would not be ratified by shareholders, this risk-shifting propensity will not materialize leading to the hump-shaped relation described above.

The model also yields predictions regarding the relation between risk management policies and agency considerations. In particular, risk-reducing activities are increasing in tunneling, or more generally, and consistently with previous literature (e.g. DeMarzo and Duffie, 1991; Breeden and Viswanathan, 1998;

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19Note that although the distribution of cash is not uniform, it is negatively skewed for all parameter values examined in this study with the mode being at the truncation (payout) threshold. Hence, the probability density function increases with the level of cash.
DaDalt, Gay, and Nam, 2002), hedging is increasing in information asymmetry. Recall that the marginal value of a dollar of cash decreases with the effectiveness of monitoring. This reflects the fact that holding cash becomes more costly for shareholders, which naturally reduces their incentives for risk management. Furthermore, the model predicts that firms operated by more entrenched managers will engage less often in risk-reducing activities. Similar to the argument above, the associated drop in the marginal value of cash increases the risk tolerance of shareholders and reduces their propensity to hedge.

Regarding the empirical testing of the above predictions, one needs to notice that two manifestations of poor (good) corporate governance, more (less) tunneling and high (low) managerial entrenchment, have conflicting effects on the firm’s propensity for risk-reducing actions. Hence, outcomes of empirical tests involving aggregate measures of corporate governance can very well be mixed. In this light, it comes as little surprise that empirical evidence on the effects of corporate governance on hedging is inconclusive.20 Moreover, these two aspects of corporate governance can be significantly correlated to each other and both are shown here to affect risk preferences; hence, unless both are adequately controlled for in an empirical specification, results are prone to omitted variable biases. Finally, note that an omission of either tunneling and managerial entrenchment would violate the ceteris paribus clause of the U-shaped relation between risk-taking and cash levels. Given that these are driving both cash levels and risk preferences, such an omission could yield a biased empirical result of complementarity between liquidity and risk management policies.

6. Conclusions

We develop a cash management model in which two governance-related imperfections, ineffectiveness of monitoring and managerial entrenchment, distort a firm’s payout and refinancing decisions. We demonstrate that sufficient levels of managerial entrenchment generate a wide range of cash ratios and a non-monotonic cash value function with respect to cash levels. We further use the model to show that the expression of the shareholder-manager conflict with respect to corporate risk-taking is a function of the levels of liquid assets the firm holds.

In the presented model, both managerial entrenchment and the effectiveness of monitoring managerial actions, conflicting facets of corporate governance, lead to later payouts to shareholders and hence higher corporate cash reserves. In addition, they both reduce the marginal value of cash; indicating that restraining the manager’s leeway does not necessarily increase the contribution of a dollar of cash to firm value. Turning to the attitude towards risk, a managerial asset substitution problem may give rise to hubris at low cash levels, while the desire of managers for cash reserves to remain high is in line with the hypothesis of “quiet

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20See Aretz and Bartram (2010) for a comprehensive review of the empirical literature.
"life". Finally, the model generates an intersection area where both shareholders and managers are better off reducing cash flow risk, which hints to an untested hump-shaped relation between corporate cash levels and hedging.
A. Appendix

A.1. Lemma 1

Lemma 1. For any smooth function $f(x)$ satisfying the differential equation

$$
\frac{\sigma^2}{2} f''(x) + [\mu + (r - \lambda - \theta) x] f'(x) - r f(x) + \delta x = 0 \tag{A.1}
$$

its second derivative $f''(x)$ has at most one root in $\mathbb{R}$.

Proof: Differentiating (A.1) with respect to $x$ yields

$$
\frac{1}{2} \sigma^2 f'''(x) + [\mu + (r - \lambda - \theta) x] f''(x) - (\lambda + \theta) f'(x) + \delta = 0 \tag{A.2}
$$

Assume $f''$ has at least one root, and let the first one being denoted by $x^*$. Following (A.2), it holds

$$
\frac{1}{2} \sigma^2 f'''(x^*) = (\lambda + \theta) f'(x^*) \implies \text{sgn}[f'''(x^*)] = \text{sgn}[f'(x^*) - \frac{\delta}{\lambda + \theta}] \tag{A.3}
$$

Hence,

- If $f'(x^*) > \frac{\delta}{\lambda + \theta}$, then $f_x$ has a minimum at $x^*$, and
- If $f'(x^*) < \frac{\delta}{\lambda + \theta}$, then $f_x$ has a maximum at $x^*$.

Suppose that $f'(x^*) > \frac{\delta}{\lambda + \theta}$ and thus $f'''(x^*) > 0$. Denoting by $x^{*2}$ the next root of $f''$, it follows that $f''(x) > 0$ in $(x^*, x^{*2})$, and hence $f'(x^{*2}) > f'(x^*) > \frac{\delta}{\lambda + \theta}$. For $x^{*2}$ to be the next root of $f''$ though, $f''(x^{*2}) \leq 0$ has to hold. $f'''(x^{*2}) \leq 0 < f''(x^{*2})$ contradicts (A.3). The proof for $f'(x^*) < 0$ is similar and therefore omitted.

A.2. Proof of Proposition 1

Applying Lemma 1 to the differential equation (16), for $\delta = 0$, it follows that $V_{CC}^{MC}$ has at most a single root in $\mathbb{R}$.

Corollary: If $V_{CC}^{MC}$ has a root in $\mathbb{R}$, denoted by $C^*$, this would be a global minimum point of $V_c^{MC}$.

Proof: If $V_c^{MC}(C^*) < 0$, then following Lemma 1, $V_{CC}^{MC}(C^*) < 0$ and $C^*$ would be a maximum point of $V_c^{MC}$. Hence $V_c^{MC}(c) \leq V_c^{MC}(C^*) < 0$ for every $c \neq C^*$, which contradicts conditions (19) and (21).

Corollary: $V(c)$ is an increasing function of $c$ in $[0, \overline{C}]$.

Proof: Since $V_c^{MC}(c) \geq V_c^{MC}(C^*) > 0$, $V^{MC}(c)$ is an increasing function of $c$ in $\mathbb{R}$. Given that $V^L(c)$ is also an increasing function of $c$, so is $V(c)$ as defined in (??).

Next, we show that the change in the concavity in $V(c)$ happens in $(0, \overline{C})$, i.e. $C^* \in (0, \overline{C})$.

Lemma 2. $V_c^{MC}(0) > 1$ and $V_{CC}^{MC}(0) < 0$. 

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Proof: Since $V^M_c(\overline{C}) \geq V^M_c(\overline{C}) > 0$ and because of Lemma 1, $V^M_c$ is either decreasing throughout $[0, \overline{C}]$ or is U-shaped in $[0, \overline{C}]$, depending on whether the root of $V^M_{cc}$ is located in $(0, \overline{C})$. In any case, since $\bar{C} > 0$, it follows that $V^M_c(0) > 1$.

Substituting $c = 0$ in the differential equation (16), obtains

$$V^M(0) = \frac{\mu}{r} V^M_c(0) + \frac{\sigma^2}{2r} V^M_{cc}(0) > \frac{\mu}{r} + \frac{\sigma^2}{2r} V^M_c(0) \tag{A.4}$$

where the inequality is due to $V^M_c(0) > 1$. Given that the value of a firm with zero cash reserves in a frictionless world would be equal to $V^{FB}(o) = \frac{\mu}{r}$ (see Décamps et al., 2011), it follows that $V^M_{cc}(0) < 0$.

Lemma 3. $V^M_{cc}(\overline{C}) > 0$

Proof: Given (??) and 17, it follows that for a small $\epsilon > 0$

$$V^M(\overline{C} - \epsilon) > L(\overline{C} - \epsilon) \iff \frac{V^M(\overline{C}) - V^M(\overline{C} - \epsilon)}{\epsilon} < \frac{L(\overline{C}) - L(\overline{C} - \epsilon)}{\epsilon} \tag{A.5}$$

Taking the limit of $\epsilon$ on both sides of the inequality obtains $V^M_c(\overline{C} - \epsilon) < 1$. Given that $V^M_c(\overline{C}) = 1$ proves that $V^M_{cc}(\overline{C}) > 0$.

Summarizing Lemmata 2 and 3, we can write $V^M_c(0) < 0 < V^M_{cc}(\overline{C})$. Hence $C^* \in (0, \overline{C})$, $V^M_c(c)$ is U-shaped in $(0, \overline{C})$. Furthermore, $V(c)$ is concave in $[0, C^*)$ and convex in $(C^*, \overline{C})$.

A.3. Proof of Proposition 2

Applying Lemma 1 to the differential equation (26), for $\delta = \theta$, it follows that $M_{cc}$ has at most a single root in $\mathbb{R}$.

Lemma 4. The single root of $M_{cc}$ is located in $(0, \overline{C})$.

Proof: From (28) it follows that there is a $\hat{C} \in (0, \overline{C})$ where $M_c(\hat{C}) = 0$. Given that $M_c(\hat{C}) = M_c(\overline{C})$, there is a $C^M \in (\hat{C}, \overline{C})$ where $M_{cc}(C^M) = 0$.

Lemma 5. $M_{cc}(\overline{C}) < 0$

Proof: Substituting $c = \overline{C}$ into the differential equation (26) yields

$$M(\overline{C}) = \frac{\theta \overline{C}}{r} + \frac{\sigma^2}{2r} M_{cc}(\overline{C}) \tag{A.6}$$

Given that the first term $\left(\frac{\theta \overline{C}}{r}\right)$ represents the wealth to the manager if cash reserves remained indefinitely at $c = \overline{C}$, it follows that $M_{cc}(\overline{C}) < 0$.

Combining Lemmata 4 and 5, we deduce that $M_{cc} > 0$ in $[0, C^M)$ and $M_{cc} > 0$ in $(C^M, \overline{C}]$. Summarizing, the value to the firm’s manager, $M(c)$ is

- decreasing and convex in $[0, \hat{C})$.  

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• increasing and convex in $\left(\widehat{C}, C^M\right)$, and

• increasing and concave in $\left(C^M, \overline{C}\right)$
References


Figure 1: Marginal and total value of cash.
Figure 1 plots the marginal and total values of cash for the firm run by the self-interested manager (solid line) and the agency-free firm (dashed line), across levels of cash reserves $C$ within the inaction region for each firm. Panel (a) plots the marginal values of cash. The dotted line at $V_C = 1$ represents the marginal value of cash to shareholders in a frictionless setup. Panel (b) plots the difference between the value of cash inside the firm, $V(C) - V(0)$, and its value outside the firm, $C$. In both panels, the vertical gridlines represent the payout thresholds for the entrenchment-prone firm ($\overline{C}$) and the agency-free firm ($\overline{C}_L$), delineating the respective inaction regions.
Figure 2: Effect of entrenchment and proportional issuance costs on cash policies.

The figure plots the payout threshold (thick lines) and the refinancing target (thin lines) for the entrenchment-prone firm (solid lines) and the benchmark agency-free firm (dashed lines). In both panels, the payout thresholds ($C^\gamma$, $C^L$) and the refinancing targets ($\tilde{C}$, $\tilde{C}^L$) are expressed as ratios of firm value. Panel (a) plots the payout threshold and the refinancing target with respect to values of the entrenchment parameter ($\gamma$). Panel (b) plots the payout threshold and the refinancing target with respect to values of the proportional issuance cost parameter ($p$).
Figure 3: Effect of cost-of-carry and tunneling on cash policies.
The figure plots the payout threshold and the refinancing target for the entrenchment-prone firm (solid and dashed lines) and the agency-free firm (dotted lines) over values of the cost-of-carry ($\lambda$) and tunneling ($\theta$) parameters. In both panels, the payout thresholds ($C$, $\bar{C}$) and the refinancing targets ($\bar{C}_L$, $\dot{C}_L$) are expressed as ratios of firm value. Panel (a) plots the payout threshold with respect to $\lambda$ (dashed and dotted lines) and $\theta$ (solid line). Panel (b) plots the refinancing target with respect to $\lambda$ (dashed and dotted lines) and $\theta$ (solid line).
Figure 4: Effect of entrenchment on the marginal value of cash.
The graph plots the difference in the marginal value of cash (solid line) following an increase in $\gamma$ from 0.04 to 0.08. The graph also plots the difference in the marginal relative value of cash ($RV_C$, dashed line) following a change in the payout threshold similar to the one caused by the change in $\gamma$ above.

Figure 5: Effect of cost-of-carry ($\lambda$) and tunneling ($\theta$) on the marginal value of cash.
The graph plots the change in the marginal value of cash for the firm run by the self-interested manager (solid and dashed lines) and the agency-free firm (dot-dashed firm) caused by a change in $\lambda$ and $\theta$. The dashed line represents the change in the marginal value of cash of the entrenchment-prone firm following an increase in $\theta$ from 0 to 0.02, the solid line represents the change in the marginal value of cash of the entrenchment-prone firm following an increase in $\lambda$ from 0.02 to 0.04, and the dot-dashed line represents the change in the marginal value of cash of the agency-free firm following an identical increase (from 0.02 to 0.04) in $\lambda$. 
Figure 6: Decomposition of effect of cost-of-carry ($\lambda$) on marginal value of cash.
The graph plots the difference in the marginal value of cash (solid line), the marginal precautionary value of cash (dashed line), and the marginal relative value of cash (dot-dashed line) for the entrenchment-prone firm (Panels (a) and (c)) and the agency-free firm (Panels (b) and (d)) following a change in $\lambda$. Panels (a) and (b) plot the partial effect of a change in $\lambda$ from 0.02 to 0.04, while Panels (c) and (d) plot the partial effect of a change in the payout threshold ($\bar{C}$) similar to the one that ensues a change in $\lambda$ from 0.02 to 0.04.
Figure 7: Effect of productivity and economy-wide parameters on the marginal value of cash. The figure plots the difference in the marginal value of cash for the entrenchment-prone firm (solid line) and the entrenchment-free firm (dashed line) following a change in $\mu$ from 0.15 to 0.20 (panel (a)), in $\sigma$ from 0.15 to 0.20 (panel (b)), in $r$ from 0.04 to 0.06 (panel (c)), in $\phi$ from 0.01 to 0.02 (panel (d)), and $p$ from 1.02 to 1.04 (panel (e)).
Figure 8: Convexity of shareholders’ and manager’s value functions. The graph plots the value to shareholders net of the face value of cash (solid line) and the value to managers (dashed line) over the level of cash reserves. The plot identifies the levels of cash at which the convexities of shareholder value ($C^*$) and manager value ($C^M$) change. The shaded area represents the cash level interval for which both functions are concave.
Figure 9: Effect of entrenchment and tunneling on the collective risk-reducing propensity.
The figure plots the inflection points of the shareholder value function ($C^*$, blue line) and the manager value function ($C^M$, red line) over $\gamma$ (Panel (a)) and $\theta$ (Panel (b)). In both panels, the inflection points are expressed as ratios of the payout threshold ($C$). The shaded area represents the cash level interval for which both functions are concave.