Effects of Creative Destruction on the Size and Timing of an Investment*

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Abstract

In the current innovation-driven economy project lives are short due to the economic phenomenon *creative destruction*. This paper investigates its implications for optimal firm investments. We distinguish between evolutionary and revolutionary innovation, where in the first (second) case the lifetime of the option to invest is (in)finite. We find that if the firm is a monopolist, a reduced length of the project life does not affect the size of the investment but the firm waits longer for better market conditions before it invests. If, in addition, the option to invest could also expire in finite time, the firm invests earlier and less. Besides a monopoly setting we also investigate a duopoly. An entry deterring incumbent invests in the same way as the monopolist except that the incumbent will invest earlier in the revolutionary scenario where the investment option never expires.

When, initially, firms are both potential entrants, the project life being finite reduces the incentive to preempt the investment of the opponent. Finally, we show that considerable value losses will be achieved when the project life being finite is mistakenly not taken into account in taking the investment decision. This value loss is enlarged by the preemption effect just mentioned.

Keywords: investment analysis, finite project life, capacity choice, real options, uncertainty

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1 Introduction

In today's innovation driven economy (Christensen et al. (1998), McGrath (2019), Geelen et al. (2019)) creative destruction considerably reduces product lives (Jovanovic and Tse (2010)).[1] A prominent example is the mobile phone industry where the innovative success of Apple and Samsung caused the end of the economic lifetime of Nokia phones. For this reason it is more and more important to take account of the finiteness of product lifetime in capital investment decisions of firms. The present article focuses on this topic within a monopoly as well as a duopoly framework. In particular, we consider a scenario where the occurrence of one event can end market activities related to the current product. An obvious example is thus the arrival of a drastic innovation that makes selling this product obsolete.[2]

The article considers capital investment opportunities of firms, where investing implies that the particular firm acquires a production plant. The firms have to decide when and how much to invest. Taking into account the uncertain economic environment, we deal with a real option problem. The real options literature took off with the seminal works of Dixit and Pindyck (1994) and Trigeorgis (1996), mainly considering the optimal timing of investment where the project life is infinite. Contributions where also the investment size is determined are, e.g., Capozza and Li (1994), Dangl (1999), and Bar-Ilan and Strange (1999). Bensoussan and Chevalier-Roignant (2019) allow for sequential capacity decisions. A finite project life, but then for investment projects with exogenously given size, is considered in Gryglewicz et al. (2008).[3]

This paper also considers finite project life but then with the investment size being endogenous. Since Schumpeter we know that technological change is the driving force of "creative destruction". The latter term stands for the decay of long-standing products followed by more innovative, disruptive ones. The present paper shows that ignoring the possibility that products get outdated, could lead to suboptimal investment decisions resulting in considerable value losses. These losses are especially significant in highly competitive markets. There it could happen that, in the race to be the first investor with the reward of achieving a time period with monopoly profits, a firm runs the risk of investing far too much at a too early stage.

Creative destruction is the arrival of a more innovative, disruptive product, which makes the current product obsolete. Our theory states that it is important to distinguish between two

[1] He and Matvos (2016) focus on the role of debt in the process of liquidating firms that have become inefficient due to technological innovation shocks.

[2] Bollen (1999) builds a discrete-value lattice to analyze a framework in which the firm faces a product lifecycle where demand first grows and then decreases. In the resulting complex setup the use of numerical methods is required to solve the model. Bollen (1999) concludes that contraction (expansion) options are under (over) valued in the standard real options literature that ignores the declining demand phase.

[3] Investment decisions with finite-lived collars are considered in Adkins et al. (2019). However, in that paper the project as such has infinite length.

scenarios. First, a firm's investment in a plant to produce the current product, could at the same time be a significant boost of the development process ultimately leading to the launch of the disruptive product. This situation is most likely the case when the firm's current product arose from a *revolutionary* innovation (Stibel, 2011). Revolutionary innovations are innovations of products that no one else has thought of before. Here we think for example of the first automobile, the phonograph, and the first smartphone. In such a situation the investment option life is long where in our theoretical model we take the extreme case of an *infinite investment option life*, thereby implicitly assuming that the development of the disruptive product replacing the current one is starting only after the firm's investment in the current product.

Second, this development process could already have started before the firm has undertaken the investment, resulting in a *finitely lived investment option*. Here we think of *evolutionary* innovation (Stibel, 2011), like better engines for cars, better sound systems for the phonograph, or the next generation smartphones. The point is that at the time a new innovation has arrived, one is already working on the next innovation. The exercise time of this investment option hardly affects the process of evolutionary innovation.

Competition is of prime importance for the profitability of today's investment projects. For investment projects of exogenously given size, frameworks including competition have elaborately been taken into account within the theory of real options, as emphasized in survey contributions by Grenadier (2000), Chevalier-Roignant et al. (2011), and Azevedo and Paxson (2014). Hellmann and Thijssen (2018) extend this literature by allowing for ambiguity. Competition is for sure an important element when determining the size of an investment project. Focusing on flexibility, Anupindi and Jiang (2008) present duopoly models where firms decide on capacity under demand uncertainty, but they do not consider investment timing. Recently, a research line started in which competing firms have to simultaneously determine timing and size of their investments (see Huisman and Kort (2015), Sarkar (2020) and also the survey article by Huberts et al. (2015)).[4] The present paper belongs to this strand of research, where our extension lies in the consideration of finite time projects. To our knowledge, the combination of competing firms that can decide about their investment size and finite time projects has not been studied yet within one model.

A paper that comes close to our work is Chevalier-Roignant et al. (2019). They focus on an oligopolistic industry, where firms develop product innovations that arrive according to a Poisson process with an endogenously determined Poisson parameter. This so-called development stage is not modeled by us. Instead, we let the firms decide about the timing and the size of the investment, which is not considered in Chevalier-Roignant et al. (2019).

[4]These works form a theoretical basis to underpin the empirical analysis of entry deterrence in the casino industry by Cookson (2018).

We study a framework where future demand is subject to stochastic shocks, which admit a geometric Brownian motion process. Firms are not active initially but have an opportunity to enter the market. They have to determine the optimal time to do so and at the same time decide about the size of the production capacity. The product market is considered to be homogeneous and firms produce up to capacity. The characteristics we focus on are that, first, the project has a finite life. At some unknown point in the future an event takes place due to which the firms have to stop selling the product. Such an event could be, for instance, the launch of an innovative product by an outside firm that destroys demand of the current product. The moment this occurs is uncertain and cannot be influenced by the firms under consideration. We assume that its timing satisfies a Poisson process. Second, we make a distinction between revolutionary and evolutionary innovations. In the first case the option to invest has an infinite lifetime, whereas in the case of evolutionary innovation the lifetime of the option to invest is finite. Then the option expires at a point in time determined by the same Poisson process. This can be motivated by the fact that in our example the launch of an innovative product does not only end the life of the project after market entry, but also before the investment takes place it destroys the value of the option to invest in a plant producing that particular product.

We start off with a monopoly framework in which the project has a finite life and, in addition, the life of the option to invest is also finite, reflecting evolutionary innovation. This will lead to an earlier investment because as long as the firm does not invest, it runs the risk of not being able to invest at all due to the possibility that the option will vanish. The firm speeding up investment implies that it will invest at the moment that the output price is smaller. For this reason the firm decides to invest less when the option could expire in finite time. If the investment option exists forever as long as the firm has not exercised it, as is the case for revolutionary innovation , we find that the optimal size of the investment is not affected by the probability that the project will end in finite time. However, at the same time the investment threshold goes up, implying that it is optimal for the firm to invest later.

When we turn to a duopoly framework, we first distinguish between the first investor (or leader), who becomes the incumbent upon investing, and the second investor (or follower), who becomes the potential entrant once the other firm has invested. We first again consider the scenario where the investment option life is finite for both, i.e., evolutionary innovation. Hence, we impose that when the unforeseen event takes place, for both firms this implies that the life of the project as well as the investment option life comes to an end. Second, we analyze the situation in which the investment option life is infinite for the incumbent, which resembles revolutionary innovation. For the potential entrant we impose that, as long as it has not invested yet while the incumbent has already done so, the life of its investment option ends at the moment that the project of the incumbent comes to an end. The motivation is that when this event takes place due to which selling this product stops, also the option to invest becomes worthless.

As soon as the entrant will invest, the first investor is already in and has taken its decisions, namely when to invest and how much. Therefore, the entrant in fact faces the same decision as the monopolist in the case of a finite project and option life, while taking into account that the market size has reduced because the incumbent has already taken part of it. So in such a situation, as for the monopolist it also holds for the entrant that when the probability that the project and the option life stops increases, the entrant will invest earlier and less.

For the incumbent it also holds that it invests earlier and less. As in the monopoly case without entry threat, the incumbent wants to preempt the event that the investment option will expire. For the entrant, the fact that the incumbent invests less implies that for a given quantity the output price is larger, which makes investing more attractive. As a result, the entrant will invest earlier and more compared to the situation where the incumbent's option life had infinite length.

When analyzing such a situation, we get that, when the investment option life is infinite, the project life being finite induces the incumbent to invest the same amount but later compared to a situation with the project life being infinite. In addition, the leader invests earlier than the monopolist without entry threat, facing a project with the same length.

If in the duopoly investment game firm roles are *endogenous*, both firms are entitled to invest first. We know already from Fudenberg and Tirole (1985) that there will be a preemption equilibrium with an early investment of the first investor. Since the investment payoff is lower if the project life is shorter, firms are less inclined to become the first investor. Therefore, a finite project life mitigates the preemption effect. Finally, we check what will happen if, in taking their investment decisions, firms make the mistake to ignore that project lives are finite. We show that the reduction in value is huge, and even loss-making strategies can be expected in case of sufficiently short project lives. This value loss will be considerably enlarged by the preemption effect just mentioned.

The paper is organized as follows. Section 2 presents the model setting, whereas the monopoly problem is studied in Section 3. In Section 4 we extend the analysis towards a duopoly. The duopoly investment game with endogenous firm roles, and the value losses resulting from mistakenly not taking into account that the project life is finite, are analyzed in Section 5 and Section 6 concludes this paper.

2 Model setting

Consider a homogeneous product market where the output price is given by

$$P(t) = X(t) f(Q(t)), \tag{1}$$

in which $Q(t)$ is the market quantity and $f'(Q(t)) < 0$. Future demand is uncertain, which is modeled by letting $X(t)$ follow the geometric Brownian motion process

$$dX(t) = \mu X(t)dt + \sigma X_(t)dz(t), \tag{2}$$

where μ is a parameter reflecting the trend, σ is the uncertainty parameter, and $dz(t)$ is the increment of a Wiener process. As such the inverse demand function contains multiplicative demand shocks (Anupindi and Jiang (2008)).

There are two firms having an option to invest in production capacity denoted by K_L for the first investor and K_F for the second investor. The subscript L stands for the first investor being the leader in the investment game, and the subscript F indicates that the second investor is the follower. Both K_L and K_F are endogenous, i.e., both the leader and the follower can decide on their investment size. Firms produce up to capacity, implying that $Q(t) = K_L$ after the first investor has invested, and $Q(t) = K_L + K_F$ after the second investor also has done so.

For both firms the investment cost is sunk and proportional to the acquired capacity. If the capacity size is K, investment costs are equal to δK, with δ being the unit investment cost. The firms are risk neutral and discount with a fixed rate r. As usual (see, e.g., Dixit and Pindyck (1994)), we impose that $r > \mu$.

For both firms it holds that their project has a finite life. To do so we introduce a parameter $\lambda > 0$, and assume that during the next time instant dt the probability that the project stops is λdt. This is different from, e.g., Huisman and Kort (2015) in which investment project lives are infinite.

Concerning the option to invest we distinguish between scenarios where innovation is revolutionary and thus the investment option exists forever, and where innovation is evolutionary and thus for both firms the investment option ceases to exist over the next time instant dt with probability λdt. This is the same parameter λ as the one that governs the project life. The idea is that with probability λdt some exogenous event happens that makes the current market obsolete. Here we can think of a product innovation resulting in a new product that takes all demand away from the current product. This ends the project life when the firm has already invested, and makes the option to invest in this project worthless in case the firm has not invested yet. Following this interpretation the parameter λ can be interpreted as the "creative destruction parameter". When innovation is evolutionary, demand follows the process

$$dX_E(t) = \mu X_E(t)dt + \sigma X_E(t)dz(t) - X_E(t)dq(t), \tag{3}$$

where

$$dq(t) = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1-\lambda dt \end{cases}, \tag{4}$$

while revolutionary innovation implies the process to follow

$$dX_R(t) = \begin{cases} \mu X_R(t)dt + \sigma X_R(t)dz(t) & \text{if } T < T^* \\ \mu X_R(t)dt + \sigma X_R(t)dz(t) - X_R(t)dq(t) & \text{if } T \geq T^* \end{cases}, \tag{5}$$

where T^* is the first passage time related to the investment threshold X^*:

$$T^* = \min_T \left\{ X(t) \geq X^* \right\}. \tag{6}$$

As we already stated, we assume that firms always produce up to capacity, i.e., the market quantity $Q(t)$ is equal to the total available capacity acquired by the firms that have invested by time t. Alternatively, volume flexibility can be assumed where either capacity can be left idle (e.g. Van Mieghem (1998)) or the firm can produce above capacity level against extra costs (e.g. Besanko et al. (2010)). Departing from our production up-to-capacity assumption the analysis is considerably simplified, especially in a duopoly framework. On the other hand, based on the analysis developed in, e.g., Hagspiel et al. (2016), we expect it will not change the qualitative aspects of our results when we relax this constraint. In reality it both can happen that firms always produce up to capacity, due to e.g. fixed costs associated to labor, commitment to suppliers, and production ramp-up (Goyal and Netessine (2007)), or that they can leave some capacity idle in case of a downturn. Typical examples where firms produce up to capacity are the steal industry and the semiconductor industry. When demand falls, a semiconductor fabrication plant keeps producing at its maximum. Running costs are low, because these plants are highly automated with few staff. Therefore, a policy of producing up to capacity makes sense but also results in decreasing prices. This happened through most of the year 2019.[5]

[5]The Economist, Jan 16th 2020 edition.

3 Monopoly

Let us first consider the evolutionary setting, i.e. that the "creative destruction" parameter λ relates to the project life and to the investment option. The idea is that with probability λdt an outside product innovation makes the current product obsolete. This ends not only the project life when the firm has already invested, but also makes the option to invest in this project worthless in case the firm has not invested yet describing an evolutionary innovation. The firm has to determine the optimal time to invest and the optimal size of the investment. In other words, a profit-maximizing firm faces the following maximization problem:

$$\max_{T\geq 0, K\geq 0}\left(V\left(X(T),K\right)-e^{-rT}\delta K\right), \tag{7}$$

in which

$$V(X(T),K)=E\left(\int_T^{\infty}e^{-rt}X(t)f\left(K\right)K dt \mid X\left(0\right)=X\right), \tag{8}$$

where T is the time of the investment, and K is the capacity level that the firm acquires at time T. The expectation sign is there because future cash flows are uncertain. This is due to the fact that the output price $X(t)f(K)$ depends on the geometric Brownian motion process $X(t)$, and that with probability λdt the project will stop during the next time instant dt.

As explained in, e.g., Dixit and Pindyck (1994), we treat this problem as an optimal stopping problem. To do so we distinguish between a continuation region, where X, and thus also the output price $Xf(K)$, is too low for an investment to be optimal, and a stopping region where X is large enough for the firm to invest. In between the two regions we have the boundary X^* being the threshold value triggering investment. At the moment of the investment the firm has to determine the investment level K^*. To determine X^* and K^* we use the following conditions. First, for a given X we maximize the stopping value with respect to K. Then, to determine X^* we value match and smooth paste the value functions of the stopping and the continuation region at the “free boundary” X^*.

3.1 Evolutionary innovation

Let us first consider the case of evolutionary innovation. Then it holds that in both the continuation and the stopping region we have to take into account that the investment option as well as the project can become worthless with probability λdt, we denote this by a subscript E referring to evolutionary innovation. For a more elaborate mathematical derivation we refer to the proof of Proposition 1. The proofs of all propositions and corollaries can be found in the appendix.

Proposition 1. *In case of evolutionary innovation, the optimal investment threshold satisfies*

$$X_E^*(K) = \frac{\beta_\lambda}{\beta_\lambda - 1} \frac{\delta\left(r+\lambda-\mu\right)}{f(K)}, \tag{9}$$

and the corresponding capacity level is given by

$$\beta_\lambda \frac{K_E^* f'\left(K_E^*\right)}{f\left(K_E^*\right)} + 1 = 0, \tag{10}$$

where β_λ is the positive root (larger than one) of the quadratic equation

$$\frac{1}{2}\sigma^2\beta_\lambda^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta_\lambda - (r+\lambda) = 0. \tag{11}$$

An explicit expression of the capacity size can be obtained once we specify $f(Q(t))$. In the duopoly model that we analyze in the next section, we take inverse demand to be linear in the quantity:

$$P(t) = X(t)\left(1-\eta Q(t)\right), \tag{12}$$

which, due to the fact that the firm produces up to capacity, implies that after investment of the monopolist the output price is given by

$$P(t) = X(t)\left(1-\eta K\right). \tag{13}$$

For the linear inverse demand case Proposition 1 translates into the result presented in the following corollary.

Corollary 1. *If the inverse demand function is given by (12), the optimal investment threshold satisfies*

$$X_E^* = \frac{\beta_\lambda + 1}{\beta_\lambda - 1}\delta\left(r+\lambda-\mu\right), \tag{14}$$

and the corresponding capacity level is given by

$$K_E^* = \frac{1}{\eta\left(\beta_\lambda + 1\right)}. \tag{15}$$

Following Dixit and Pindyck (1994, p. 200) we can view finite project lives as infinite project lives with increased discount rates, i.e., "*Formally, we can regard the project as infinite-lived, but augment the rate at which future profits are discounted by adding the Poisson death parameter, so that the discount rate increases from r to $r+\lambda$*". In this way the results of Huisman and Kort (2015) directly translate to the optimal capacity and threshold from Corollary 1.

3.2 Revolutionary innovation

We now consider the revolutionary scenario where the project life is finite while the option to invest remains existent as long as the firm has not invested yet. To analyze this problem we again distinguish between the stopping and the continuation region. In the stopping region nothing has changed. However, in the continuation region we have to take into account that the investment option has an infinite life. Analogous to Proposition 1 we get the following result.

Proposition 2. *In case of revolutionary innovation, the optimal investment threshold equals*

$$X_R^*(K) = \frac{\beta}{\beta - 1}\frac{\delta\left(r + \lambda - \mu\right)}{f(K)}, \tag{16}$$

whereas the investment size implicitly satisfies

$$\beta\frac{K_R^* f'\left(K_R^*\right)}{f\left(K_R^*\right)} + 1 = 0, \tag{17}$$

where β is the positive root of the quadratic equation

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0. \tag{18}$$

An important conclusion we can draw from the expression for the investment size (17) is that capacity size K_R^* does not depend on λ. Hence, this holds despite of the fact that at each time instant the project ends with probability λdt. Still, the creative destruction parameter λ has an effect on the investment decision, because expression (16) learns that the investment threshold goes up with λ, which implies that the monopolist will invest later, provided that the initial level of the process X, which is X_0, falls below the threshold level $X_R^*(K)$. This means that the reduced project length causes that the firm invests later in the same capacity size.

For the linear inverse demand case, Proposition 2 translates to

$$X_R^*(K) = \frac{\beta}{\beta - 1}\frac{\delta\left(r + \lambda - \mu\right)}{1 - \eta K}, \tag{19}$$

and

$$K_R^* = \frac{1}{\eta\left(\beta + 1\right)}. \tag{20}$$

This straightforwardly leads to the result specified in the next corollary.

Corollary 2. *If the inverse demand function is given by (12), the optimal investment threshold* *satisfies*

$$X_R^* = \frac{\beta+1}{\beta-1}\delta\left(r+\lambda-\mu\right), \tag{21}$$

and the corresponding capacity level is given by

$$K_R^* = \frac{1}{\eta\left(\beta+1\right)}. \tag{22}$$

From the two quadratic polynomials (11) and (18) it is obtained that

$$\beta_\lambda > \beta. \tag{23}$$

Comparing, for the linear inverse demand function, the investment decision for evolutionary innovation, (14)-(15), with revolutionary innovation, (21)-(22), we conclude that for the finite time option the investment threshold is lower, $X_E^* < X_R^*$, indicating an earlier investment, and that the size of the investment is smaller, $K_E^* < K_R^*$. The reason is that the monopolist wants to preempt the event that the option to invest is not available anymore. Investing earlier implies that at the moment of the investment the output price is lower, given the capacity size. Therefore, the firm will spend less on investing so that it acquires a smaller capacity.

Note that the optimal decisions under an infinite option life do not correspond to the work of Huisman and Kort (2015) with a higher perceived discount rate. Still it is true that the perceived discount rate in the stopping region equals $r+\lambda$. However, since the option life is infinite, in the continuation region the λ does not play a role so that we still have r as the discount rate there.

4 Duopoly

We consider a scenario with two firms competing for a market share. Both have to decide on when to invest, i.e., when to enter the market, and on how much to invest, which determines the size of the production capacity. Throughout this section we impose that the inverse demand function is the one of expression (12).

One firm is assigned to be the leader, which has the right to invest first. The other firm is the follower, which has the choice to invest at the same time as the leader or to invest later. In a situation where the leader has invested and the follower still waits, we in fact have an incumbent-entrant situation. For this reason we sometimes call the leader the incumbent and the follower the (potential) entrant.

Hence, what we consider here is a situation of exogenous firm roles in the sense that beforehand we assign the leader role to one firm and the follower role to the other firm. The next section analyzes a duopoly scenario with endogenous firm roles. This means that both firms are able to invest first, giving rise to so-called preemption equilibria.

We first treat the scenario where both the project life is finite and the investment option has a finite life, followed by an analysis of the case where the investment option for the leader is infinite.

4.1 Evolutionary innovation

We consider an evolutionary situation in which the product market has a finite life also before the current product is launched. Hence, already when the current product is not produced yet, product innovation carried out by outside firms can wipe away demand for this product. The implication is that not only after investment the project ends with probability λdt, but also before the investment the option to invest will vanish with probability λdt. This holds for both the leader and the follower.

We first consider the follower's decision in a situation that the leader already has invested and acquired a capacity level K_L. After determining the follower's optimal investment decision we turn to the leader's problem. We derive the leader's optimal investment decision, taking into account how the follower will react. The next proposition specifies the optimal investment decision of the follower.

Proposition 3. *The value function of the follower at the moment of investment is equal to*

$$V_F(X, K_L, K_F) = \frac{XK_F\left(1-\eta\left(K_L+K_F\right)\right)}{r+\lambda-\mu}. \tag{24}$$

Given the capacity level K_L of the leader, the optimal investment threshold of the follower, in case of evolutionary innovation, is given by

$$X^*_{F,E}(K_L) = \frac{\beta_\lambda+1}{\beta_\lambda-1}\frac{\delta\left(r+\lambda-\mu\right)}{1-\eta K_L}, \tag{25}$$

whereas the capacity level equals

$$K^*_{F,E}(K_L) = \frac{1-\eta K_L}{\eta\left(\beta_\lambda+1\right)}. \tag{26}$$

The decision of the follower is qualitatively similar as the one of the monopolist when the option life is finite. The only change is that due to the investment of the leader, the reservation price has reduced by a factor ηK_L. This explains that, apart from this factor ηK_L, the investment decision is the same as the monopolist's investment decision expressed in (14)-(15). For the leader the follower's investment decision (25)-(26) provides important information in the sense that increasing its capacity K_L not only reduces the follower's capacity but also lets the follower invest later, so that the leader enjoys a longer period during which it is the monopolist in the market. Hence, as already concluded by Huisman and Kort (2015), the leader has two reasons to overinvest.

Turning to the leader's problem we first have to remark that, based on the fact that the follower can invest at the same time or later than the leader, we have to distinguish between an *entry deterrence* and an *entry accommodation* strategy. With an entry deterrence strategy the leader invests so much that it will generate a monopoly period for itself due to the fact that the follower will invest later. From (25) we obtain that the acquired capacity level should then be such that

$$K_L > \hat{K}(X) = \frac{1}{\eta}\left(1 - \frac{\beta_\lambda + 1}{\beta_\lambda - 1}\frac{\delta\left(r + \lambda - \mu\right)}{X}\right). \tag{27}$$

If the capacity size does not satisfy this constraint, the follower will invest at the same time as the leader and then we are in the entry accommodation scenario.

Under the entry deterrence strategy the leader value in the stopping region is given by

$$V_L^{\det}(X, K_L) - \delta K_L = \frac{K_L\left(1 - \eta K_L\right)}{r + \lambda - \mu}X - \delta K_L - \left(\frac{X}{X^*_{F,E}(K_L)}\right)^{\beta_\lambda}\frac{K_L \eta K^*_{F,E}(K_L)}{r + \lambda - \mu}X^*_{F,E}(K_L). \tag{28}$$

The first two terms represent the leader value if the leader were a monopolist until the end of the project. The last term is a negative correction for the fact that the follower will enter as soon as X reaches the follower threshold $X^*_{F,E}$. Then the output price will be reduced by an amount $\eta K^*_{F,E} X^*_{F,E}$, leading to an instantaneous revenue reduction of $K_L \eta K^*_{F,E} X^*_{F,E}$. This negative correction needs to be properly discounted, because it follows from the entry of the follower taking place at a later point in time. In fact, this discounting is achieved by the term $\left(\frac{X}{X^*_{F,E}}\right)^{\beta_\lambda}$ denoting the stochastic discount factor, that is, it holds that $\left(\frac{X}{X^*_{F,E}}\right)^{\beta_\lambda} = E\left(e^{-(r+\lambda)(t-T_F)}\right)$, where T_F is the (stochastic) entry time of the follower, taking place as soon as the stochastic process X reaches the follower threshold $X^*_{F,E}$ for the first time. Comparing this to the analogous term in expression (29) of Huisman and Kort (2015), we see that in their paper "our" β_λ is replaced by β. This implies that, because $\beta_\lambda > \beta$ (see (23)) and $X < X^*_{F,E}$ (which holds in the region considered here), the negative correction due to the follower's entry is smaller in our case. The finiteness of the project works here: it may be possible that the project is stopped already before the follower enters. This mitigates the strategic effect in the sense that the negative correction of the leader value due to the possible follower entry is lower.

Application of the value matching and the smooth pasting conditions, and maximizing (28) with respect to K_L, where we have to take into account that $X^*_{F,E}$ and $K^*_{F,E}$ depend on K_L as obtained from (25)-(26), gives the following result.

Proposition 4. *The leader will consider the entry deterrence strategy whenever X is in the interval $\left(X_1^{\text{det}}, X_2^{\text{det}}\right)$, where X_1^{det} is implicitly given by*

$$\frac{X_1^{\text{det}}}{r+\lambda-\mu} - \delta + \left(\frac{X_1^{\text{det}}}{X_{F,E}^{*}(0)}\right)^{\beta_\lambda} \frac{\delta}{\beta_\lambda - 1} = 0, \tag{29}$$

and

$$X_2^{\text{det}} = \frac{\beta_\lambda + 1}{\beta_\lambda - 1} 2\delta \left(r+\lambda-\mu\right). \tag{30}$$

The leader's entry deterrence investment threshold, in case of evolutionary innovation, equals

$$X_{L,E}^{\text{det}}(K_L) = \frac{\beta_\lambda}{\beta_\lambda - 1} \frac{\delta\left(r+\lambda-\mu\right)}{1-\eta K_L}, \tag{31}$$

and the investment size $K_{L,E}^{\text{det}}(X_L)$ is implicitly determined by

$$\frac{1-2\eta K_{L,E}^{\text{det}}}{r+\lambda-\mu} X_L - \delta + \left(\frac{X_L}{X_{F,E}^{*}(K_{L,E}^{\text{det}})}\right)^{\beta_\lambda} \frac{K_{L,E}^{\text{det}} \eta K_{F,E}^{*}(K_{L,E}^{\text{det}}) X_{F,E}^{*}(K_{L,E}^{\text{det}})}{r+\lambda-\mu} \frac{1-\left(\beta_\lambda+1\right)\eta K_{L,E}^{\text{det}}}{K_{L,E}^{\text{det}}\left(1-\eta K_{L,E}^{\text{det}}\right)} = 0. \tag{32}$$

Expression (31) is an important one, because it helps to understand the various effects of the creative destruction parameter λ. If we take into account that λ also influences $K_{L,E}^{\text{det}}$, as becomes apparent from expression (32), three effects can be distinguished, namely an *NPV effect* via λ itself, a *quantity effect via* $K_{L,E}^{\text{det}}$, and a *strategic* effect via β_λ. The *NPV effect* takes into account that the project is expected to last shorter if λ increases, implying a smaller period during which revenues are earned. This makes the project less attractive, implying that the investment threshold $X_{L,E}^{\text{det}}$ will increase.

The *quantity effect* results from the fact that we can obtain from (32) that $K_{L,E}^{\text{det}}$ decreases in λ, having the obvious interpretation that the leader will invest less in the project when it has a shorter expected duration. A smaller investment means that the firm has a lower cash outflow, implying that revenues need not be that high to make an investment profitable. Hence, due to the quantity effect $X_{L,E}^{\text{det}}$ will decrease.

The *strategic effect* is the effect coming from the fact that finite project duration results in a positive probability that the project has already stopped before the follower will enter. This mitigates the effect of competition on the leader's investment decision, and thus the negative effect of future follower entry on the expected net present value of the investment. For this reason investing becomes more attractive so that the strategic effect reduces the leader threshold $X_{L,E}^{\text{det}}$.

In total, the leader threshold $X_{L,E}^{\text{det}}$ is positively influenced by the *NPV effect,* and negatively by the *quantity* and the *strategic effect*. In principle the two expressions (31)-(32) can be used to solve for the threshold X_L^{det} and the investment size $K_{L,E}^{\mathrm{det}}$. The next proposition presents the outcome.

Proposition 5. *Under an entry deterrence strategy the leader's investment threshold is, in case of evolutionary innovation, given by*

$$X_{L,E}^{\mathrm{det}} = \frac{\beta_\lambda + 1}{\beta_\lambda - 1}\delta\left(r + \lambda - \mu\right), \tag{33}$$

and the corresponding capacity size is equal to

$$K_{L,E}^{\mathrm{det}} = \frac{1}{\eta\left(\beta_\lambda + 1\right)}. \tag{34}$$

Similar as in the monopoly setting with finite option life, also here expressions (33) and (34) are analogous to the comparable results in Huisman and Kort (2015), but then with augmented discount rate $r+\lambda$. To analyze the effects of the creative destruction parameter λ, note that, since β_λ is increasing in λ, the effect of λ on $K_{L,E}^{\mathrm{det}}$ is negative. The effect of λ on $X_{L,E}^{\mathrm{det}}$ can be determined by the sign of the derivative of $X_{L,E}^{\mathrm{det}}$ as given in the following corollary.

Corollary 3. *The derivative of* (33) *with respect to* λ *is*

$$\frac{\partial X_{L,E}^{\mathrm{det}}}{\partial \lambda} = \frac{\delta\beta_\lambda\left(\sigma^2\left(\beta_\lambda^2 - 1\right) + 2\left(r + \lambda - \mu\right)\right)}{\left(\beta_\lambda - 1\right)\left(2\left(r + \lambda\right) + \sigma^2\beta_\lambda^2\right)} > 0. \tag{35}$$

Corollary 3 implies that taking into account creative destruction, delays market entry of the leader. In other words, the *NPV effect* dominates the *quantity* and the *strategic effect*.

We refrain from analyzing the entry accommodation strategy as it is similar to the strategy in Huisman and Kort (2015) where r is replaced by $r+\lambda$.

4.2 Revolutionary innovation

Now we consider a revolutionary situation in which the product market has a finite life only from the moment the leader has invested. This situation could arise when the launch of the current product triggers outside firms to start developing a new project that makes the current product obsolete. The probability that these firms obtain a breakthrough in their innovation process is assumed to be equal to λdt. The implication is that the lifetime of the option to invest is infinite for the leader, but after the leader has invested, due to creative destruction the project will end with probability λdt. For the follower this implies that once the leader has invested, the lifetime of the follower's investment option becomes finite. This is because, once the outside firms accomplish the breakthrough that makes the current product obsolete, the opportunity to invest in the current product market also becomes worthless. Hence both the follower's project and the follower's investment option life is finite. Since also in the previous section there was a finite option life for the follower, the follower's investment decision is exactly the same as in Proposition 3. Hence, $X_{F,R}^{*}(K_L) = X_{F,E}^{*}(K_L)$ and $K_{F,R}^{*}(K_L) = K_{F,E}^{*}(K_L)$. Of course, K_L can be different, which we will find out below, so that the follower could still invest at a different time in a different capacity size.

In case of an entry deterrence strategy the leader invests as in the following proposition.

Proposition 6. *The leader will consider the entry deterrence strategy whenever X is in the interval $\left(X_1^{\text{det}}, X_2^{\text{det}}\right)$, where X_1^{det} is implicitly given by (29) and X_2^{det} is given by (30). The leader's entry deterrence investment size, in case of revolutionary innovation, equals*

$$K_{L,R}^{\text{det}} = \frac{1}{\eta\left(\beta+1\right)}, \tag{36}$$

and the investment threshold $X_{L,R}^{\text{det}}$ is implicitly determined by

$$\left(X_{L,R}^{\text{det}}\right)^{\beta_\lambda}\frac{\left(\beta_\lambda-\beta\right)\delta}{\beta\left(\beta_\lambda-1\right)}\left(\frac{\beta\left(\beta_\lambda-1\right)}{\left(1+\beta\right)\left(1+\beta_\lambda\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda}+X_{L,R}^{\text{det}}\frac{\left(\beta-1\right)}{\left(r+\lambda-\mu\right)\left(\beta+1\right)}-\delta=0. \tag{37}$$

Concerning the entry deterrence investment strategy we have an explicit expression for the investment size, whereas the investment threshold is implicitly determined. As in the monopoly case with infinite option life (Corollary 2), also here the investment size (36) is not influenced by the creative destruction parameter λ. The capacity $K_{L,R}^{\text{det}}$ is in fact the Stackelberg leader quantity level, because due to the production-up-to-capacity assumption, the leader, or incumbent, is committed to produce the quantity $K_{L,R}^{\text{det}}$ at any time after the investment. The follower, or (future) entrant, will adjust its capacity level accordingly (see (26)).

Although we do not know $X_{L,R}^{\text{det}}$ explicitly, from the implicit expression (37) we can derive that the threshold is between certain bounds, as the following corollary shows.

Corollary 4. *Under an entry deterrence strategy, in case of revolutionary innovation, the leader's investment threshold $X_{L,R}^{\text{det}}$, which is implicitly defined by (37), is in between the following values*

$$X_{L,E}^{\text{det}} = \frac{\beta_\lambda+1}{\beta_\lambda-1}\delta\left(r+\lambda-\mu\right) < X_{L,R}^{\text{det}} < \frac{\beta+1}{\beta-1}\delta\left(r+\lambda-\mu\right) = X_R^{*}. \tag{38}$$

From the upper bound of $X_{L,R}^{\text{det}}$ in (38), and expression (21) we obtain that the leader invests earlier than the monopolist without entry threat. The lower bound of $X_{L,R}^{\text{det}}$ in (38), which coincides with the evolutionary threshold $X_{L,E}^{\text{det}}$, shows that the leader will invest later if the investment option will not expire in finite time. It makes sense that, as in the monopoly case, if the investment option will expire in finite time, this will accelerate the investment decision of the leader.

From the results of the last proposition and corollary and Proposition 5, we obtain that, when the leader's investment option could expire in finite time, he invests earlier and therefore less. Note that for the follower this is a good situation, since it holds that for any follower quantity the output price will be higher. Therefore, the follower will respond by investing earlier and more compared to the situation where the investment option of the leader has infinite life.

Turning to the *entry accommodation* strategy, it holds that the investment capacity size $K_L^{\mathrm{acc}}(X)$ is such that $K_L^{\mathrm{acc}}(X) \leq \hat{K}(X)$ where $\hat{K}(X)$ is given by (27). We then are in a situation that the leader invests at the same time as the follower. However, despite the fact that leader and follower invest at the same time, it is still the case that the leader acts first, where the follower has the choice to invest later or to follow suit, where it chooses for the latter option in the entry accommodation case. As a result we obtain the Stackelberg equilibrium in investment quantities. In this situation, one reason for overinvestment, namely that the follower will invest later if the leader quantity is higher, does not exist. However, we still have the mechanism that a larger value of K_L will result in a smaller capacity of the follower, as confirmed by expression (26). The following proposition contains the leader's entry accommodation strategy.

Proposition 7. *For the accommodation strategy, the value function of the leader is given by*

$$V_L^{acc}(X, K_L) = \frac{K_L\left(1 - \eta\left(K_L + K_F^*(X, K_L)\right)\right)}{r + \lambda - \mu} X. \tag{39}$$

The leader will consider the entry accommodation strategy, in case of revolutionary innovation, whenever $X \geq \max\left(X_1^{\mathrm{acc}}, X_{L,R}^{\mathrm{acc}}\right)$*, where*

$$X_1^{\mathrm{acc}} = \frac{3 + \beta_\lambda}{\beta_\lambda - 1} \delta\left(r + \lambda - \mu\right), \tag{40}$$

$$X_{L,R}^{\mathrm{acc}} = \frac{1 + \beta}{\beta - 1} \delta\left(r + \lambda - \mu\right), \tag{41}$$

and the corresponding capacity size is equal to

$$K_L^{\mathrm{acc}}(X) = \frac{1}{2\eta}\left(1 - \frac{\delta\left(r + \lambda - \mu\right)}{X}\right). \tag{42}$$

The optimal accommodation threshold from the perspective of the leader is $X_{L,R}^{\mathrm{acc}}$, whereas X_1^{acc} is the minimum level of X for which the follower is willing to invest at the same time as the leader. The conclusion is that only when $X \geq \max\left(X_1^{\mathrm{acc}}, X_{L,R}^{\mathrm{acc}}\right)$, both firms are willing to invest at the same time.[6]

If the probability that the project will end goes up, the effect on X_1^{acc} is positive because $\frac{\partial X_1^{\text{acc}}}{\partial \lambda} > 0$ (see Appendix A.8 where $a = 3$). A shorter expected project life reduces the investment's net present value (NPV), due to which the follower wants to invest later. However, the follower also has an incentive to invest earlier because with probability λdt it can happen that the option disappears within the next time period of length dt. Apparently, the NPV effect dominates here. Furthermore, $X_{L,R}^{\text{acc}}$ increases with λ, because due to the NPV effect simultaneously investing with the follower is less profitable. Further, we see that an increase of λ will result in the leader to invest less because of the obvious reason that the project is expected to last during a shorter period of time.

5 Duopoly with Symmetric Firm with Endogenous Firm Roles

Since the option life for the follower is always finite, we already concluded in the previous section that the strategies of the follower are independent of the scenario involving evolutionary or revolutionary innovations. It also holds that the optimal capacity size of the leader $K_L^{\text{det}}(X)$ follows from the first order condition of the leader value in the stopping region, which thus does not depend on the option value. Moreover, in the resulting preemption equilibrium, which is explained later, the timing also only depends on the leader value in the stopping region. Therefore, we can conclude that the resulting equilibrium outcome is the same for revolutionary and evolutionary innovations. Hence, in this section we can drop the subscripts E and R, so that

$$
\begin{aligned}
X_F^*(K_L) &= X_{F,R}^*(K_L) = X_{F,E}^*(K_L),\\
K_F^*(K_L) &= K_{F,R}^*(K_L) = K_{F,E}^*(K_L),\\
K_L^{\text{acc}}(X) &= K_{L,E}^{\text{acc}}(X),\\
K_L^{\text{det}}(X) &= K_{L,E}^{\text{det}}(X).
\end{aligned}
$$

We analyze a situation where the firms are completely symmetric, which implies that both firms are entitled to invest first. This means that it is not known beforehand which firm will be the leader or the follower. Hence, firm roles are endogenous in the duopoly investment game. To determine the equilibrium for such a scenario we follow the approach of, originally, Fudenberg and Tirole (1985) (see also Thijssen et al. (2012) and Riedel and Steg (2017)), which starts out with developing two different curves, as depicted in Figure 1. Note that we actually see four curves there, but this is because we depict two different situations: finite and infinite project life.

[6] Note that in case the project has infinite life, i.e. $\lambda = 0$, it always holds that $X_{L,R}^{\text{acc}} < X_1^{\text{acc}}$, implying that for $X > X_1^{\text{acc}}$ the leader accommodates entry in the sense that the leader and the follower will invest at the same time.

For the infinite variant we refer to Huisman and Kort (2015), which can be obtained by setting $\lambda = 0$. We denote the latter strategies by small letters x and k rather than capitals. For each situation a leader and a follower curve is drawn. The leader curve connects points representing the leader value that results from investing immediately, taking into account that the follower invests at the corresponding follower threshold (see Proposition 3). Note that for small values of the geometric Brownian motion process, i.e. $X < X_1^{\text{det}}$, the leader value equals zero, which is because the output price is too low for the leader to profitably invest in a capacity level larger than zero.

The follower curve connects follower values resulting from investing at the follower threshold. This explains why the follower curve is situated above the leader curve for small values of X. In between the values X_1^{det} and $\hat{X}$ the leader applies the entry deterrence strategy, whereas for $X > \hat{X}$ entry accommodation is applied (see also Huisman and Kort (2015)). Thus $\hat{X}$ is defined by

$$\hat{X} = \min\left\{X \in \left(X_1^{\text{acc}}, X_2^{\text{det}}\right) \middle| V_L^{\text{acc}}\left(X, K_L^{\text{acc}}(X)\right) - \delta K_L^{\text{acc}}(X) = V_L^{\text{det}}\left(X, K_L^{\text{det}}(X)\right) - \delta K_L^{\text{det}}(X)\right\}.$$

Note that the follower curve jumps upwards for $X = \hat{X}$. This is because, if the leader moves from an entry deterrence to an entry accommodation strategy, the leader's investment size jumps down, because one reason for overinvestment, namely that by investing more the follower invests later, disappears.

Having determined the curves representing the leader and the follower value, we can determine the equilibrium, i.e., the level of X at which the first investor will invest and its capacity size, which then, by Proposition 3, in turn determines the follower's investment decision. We do this for the situation where the market is small initially, i.e. $X_0 < X_P$, in which X_P is the point at which the leader curve intersects the follower curve from below, see Figure 1. The firms will refrain from investing immediately, because as long as the follower curve is situated above the leader curve, a better strategy is to wait and invest at the follower threshold. On the other hand, if the firms wait with investing until $X > X_P$, a first mover advantage arises, because the leader value exceeds the follower value. In such a situation it would be better for one firm to invest at a slightly lower value of X to *preempt* its competitor so that it obtains the leader value. Therefore, in equilibrium the first investor invests at X_P, in capacity $K_L^{\text{det}}(X_P)$ (see Proposition 4) and the second investor waits with investment until X reaches the corresponding follower threshold $X_F^*(K_L^{\text{det}}(X_P))$ and invests $K_F^*(K_L^{\text{det}}(X_P))$. Because the preemption effect is so dominantly present, this equilibrium is called a preemption equilibrium and X_P the preemption point determined by

$$V_L^{\text{det}}\left(X_P, K_L^{\text{det}}(X_P)\right) - \delta K_L^{\text{det}}(X_P) = F_F(X_P). \tag{43}$$

All small X-values x in Figure 1 represent the case of $\lambda = 0$, which corresponds to the infinite

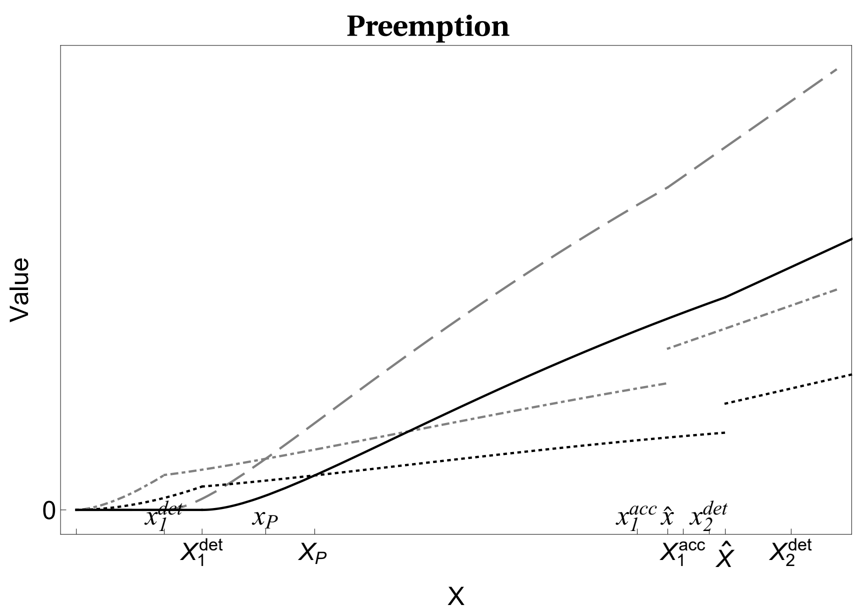


Figure 1: Optimal value functions for the leader and follower as a function of X. The parameter values are $r = 0.1; \mu = 0.06; \delta = 0.1; \eta = 0.05; \lambda = 0.02; \sigma = 0.1$. The gray lines represent the case of an infinite project length and the black lines represent the case of a finite project life. The black solid line and gray dashed represent the value function of the leader and the black dotted and gray dot-dashed line the value function of the follower.

project life variant. The preemption effect is less dominant if the project life is finite, which can be inferred from the fact that $X_P > x_P$. The finiteness of the project implies that the investment payoff is lower, and therefore firms are less inclined to be the first investor. So, like in the case of exogenous firm roles treated in Section 4, also here we get that the first investor will invest at a later point in time.

It is important to notice that both the leader and the follower curve, and thus also the corresponding preemption equilibrium, do not depend on whether the option to invest has a finite or infinite life for the first investor. The leader curve does not change because it is based on the leader investing immediately. The follower curve is based on the follower investing later or at the same time as the leader. This implies that at the moment of the investment the option life of the follower is finite in both situations so there is no change either. This confirms what we stated in the beginning of the section, namely that the equilibrium outcome does not depend on the innovation being revolutionary of evolutionary.

5.1 How important is it to realize project life is finite

Mainstream real options contributions consider the lifetime of an investment project to be infinite. However, in every day life we more and more realize that innovations are abound (McGrath (2019)) and product lifetime is mostly finite due to creative destruction. The analysis in this paper takes account of that and this section explores how important it is to explicitly model finite lifetime. To quantify, we derive the relative loss in value when the project life is finite, whereas firms ignore this feature in their investment decisions. To do so, we compare the value functions in two situations. First, we take the value functions of the monopolist, leader and follower based on the strategies earlier derived in this paper. Second, we consider value functions resulting from standard real options strategies, i.e. firms invest assuming that the project life is infinite, whereas in fact the project can end at each time with probability λdt. As a result, we define the relative loss as the value of the investor based on the optimal time and size decisions taking the finitiness of the project into account and discounted to some common value X_0, minus the value of the investor based on the suboptimal time and size decisions which would have been optimal if the project lives were infinite and discounted to the same X_0, divided by the optimal value. The stochastic discount factor is influenced by β_λ in case of evolutionary innovation and by β in case of revolutionary innovation. We denote the optimal investment decisions, i.e., the optimal threshold and capacity strategy dependent on λ, by capital letters and the suboptimal decisions which ignore the fact that the project is finite by small letters.

So far the value functions were defined without the direct inclusion of the costs, therefore we introduce $V^n = V - \delta K$ as the *net* value function. For the monopolist the net value is defined as

$$V_M^n(X,K) = \frac{K(1-\eta K)X}{r+\lambda-\mu} - \delta K. \tag{44}$$

And the loss of the monopolist in case of evolutionary innovation is then

$$\ell_M^{(E)}(\lambda) = \frac{\left(\frac{X_0}{X_E^*}\right)^{\beta_\lambda} V_M^n\left(X_E^*, K_E^*\right) - \left(\frac{X_0}{x^*}\right)^{\beta_\lambda} V_M^n(x^*, k^*)}{\left(\frac{X_0}{X_E^*}\right)^{\beta_\lambda} V_M^n\left(X_E^*, K_E^*\right)}, \tag{45}$$

and in case of revolutionary innovation the relative loss of the monopolist of ignoring the finiteness of the project is

$$\ell_M^{(R)}(\lambda) = \frac{\left(\frac{X_0}{X_R^*}\right)^{\beta} V_M^n\left(X_R^*, K_R^*\right) - \left(\frac{X_0}{x^*}\right)^{\beta} V_M^n(x^*, k^*)}{\left(\frac{X_0}{X_R^*}\right)^{\beta} V_M^n\left(X_R^*, K_R^*\right)}. \tag{46}$$

For the duopoly strategies we introduce the notation S_L and S_F as the optimal decision sets at the preemption point of the leader and follower, respectively,

$$S_L = \left\{X_P, X_F^*(K_L^{\det}(X_P)), K_F^*(K_L^{\det}(X_P)), K_L^{\det}(X_P)\right\} = \{X_P, S_F\}, \tag{47}$$

and s_L and s_F as the suboptimal equivalent under the assumption that projects have an infinite length

$$s_L = \left\{x_P, x_F^*(k_L^{\text{det}}(x_P)), k_F^*(k_L^{\text{det}}(x_P)), k_L^{\text{det}}(x_P)\right\} = \{x_P, s_F\}. \tag{48}$$

Recall that the decisions at the preemption point for evolutionary and revolutionary innovations coincide. The net value of the leader is defined as

$$V_L^n(X, X_F, K_F, K_L) = \frac{K_L(1-\eta K_L)}{r+\lambda-\mu}X - \left(\frac{X}{X_F}\right)^{\beta_\lambda}\frac{K_L\eta K_F}{r+\lambda-\mu}X_F - \delta K_L, \tag{49}$$

and the net value of the follower as

$$V_F^n(X_F, K_F, K_L) = \frac{X_F K_F(1-\eta(K_L+K_F))}{r+\lambda-\mu} - \delta K_F. \tag{50}$$

Now, the loss of the leader for evolutionary innovation is defined as

$$\ell_L^{(E)}(\lambda) = \frac{\left(\frac{X_0}{X_P}\right)^{\beta_\lambda} V_L^n(S_L) - \left(\frac{X_0}{x_P}\right)^{\beta_\lambda} V_L^n(s_L)}{\left(\frac{X_0}{X_P}\right)^{\beta_\lambda} V_L^n(S_L)}, \tag{51}$$

and for revolutionary innovation as

$$\ell_L^{(R)}(\lambda) = \frac{\left(\frac{X_0}{X_P}\right)^{\beta} V_L^n(S_L) - \left(\frac{X_0}{x_P}\right)^{\beta} V_L^n(s_L)}{\left(\frac{X_0}{X_P}\right)^{\beta} V_L^n(S_L)}. \tag{52}$$

The losses of the follower are in case of evolutionary and revolutionary innovations

$$\ell_F^{(E)}(\lambda) = \frac{\left(\frac{X_0}{X_P}\right)^{\beta_\lambda} V_F^n(S_F) - \left(\frac{X_0}{x_P}\right)^{\beta_\lambda} V_F^n(s_F)}{\left(\frac{X_0}{X_P}\right)^{\beta_\lambda} V_F^n(S_F)}, \tag{53}$$

and

$$\ell_F^{(R)}(\lambda) = \frac{\left(\frac{X_0}{X_P}\right)^{\beta} V_F^n(S_F) - \left(\frac{X_0}{x_P}\right)^{\beta} V_F^n(s_F)}{\left(\frac{X_0}{X_P}\right)^{\beta} V_F^n(S_F)}, \tag{54}$$

respectively.

Note that the initial value of $X_0 \in (0, X_P)$, $X_0 \in (0, X_E^*)$ or $X_0 \in (0, X_R^*)$ to which the values are discounted, is irrelevant as it drops out in the definition of the loss function. Since discounting takes place in the continuation region, the stochastic discount factor is influenced by β_λ for evolutionary innovation and by β for revolutionary innovation. As we saw earlier, the strategies of the leader and follower at the preemption point are independent of the type of innovation. For the monopolist, the threshold and size are different for evolutionary and revolutionary innovations.

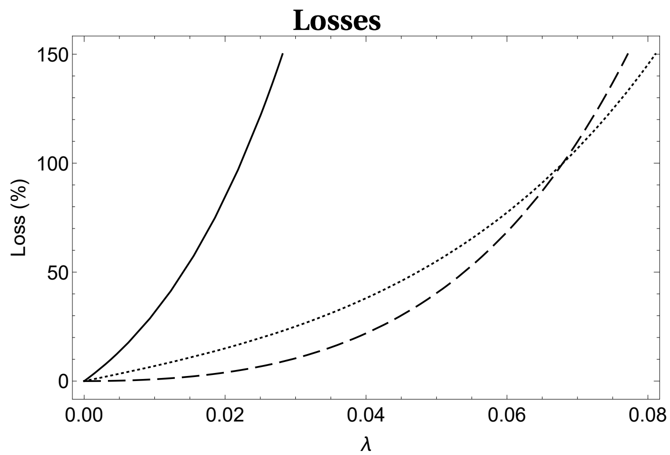
The loss of not taking the finiteness of the project into account is shown in Figure 2. Figure 2a shows the losses in case of evolutionary innovation and Figure 2b in case of revolutionary innovation. We conclude that, if the expected lifetime is shorter than 15 or 45 years (note that expected lifetime equals $1/\lambda$), the resulting suboptimal decisions lead to a loss that is more than 100% for both the follower and monopolist, and the leader respectively.

In Table 1 we show the strategies, net firm values and losses for three values of λ coinciding with an infinite expected lifetime, and an expected lifetime of 50 years and 10 years corresponding to the values of λ being equal to $\{0.1, 0.02, 0\}$. Table 1a presents the moment of entry of the leader – the preemption point – X_P, how long the follower subsequently waits until entering the market, $X_F^*(K_L^{\text{det}}(X_P))$, and for both the associated capacity, $K_L^{\text{det}}(X_P)$ and $K_F^*(K_L^{\text{det}}(X_P)$. When $\lambda = 0$ we are in the scenario in which projects have infinite lives, $S_L = s_L$, resulting in early market entries and large capacities. If those strategies are still implemented although in reality projects are finite, the associated percentage losses are provided by $\ell_L^{(E)}(\lambda), \ell_F^{(E)}(\lambda)$ for evolutionary innovation and $\ell_L^{(R)}(\lambda), \ell_F^{(R)}(\lambda)$ for revolutionary innovation.

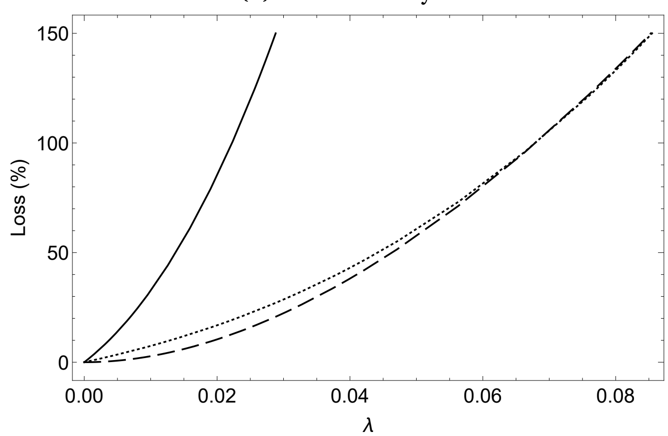
The relative loss of the leader is bigger than the loss of the follower, because, when thinking that project life is infinite ($\lambda = 0$) while it is in fact not ($\lambda = 0.1$ or $\lambda = 0.02$), it is investing far too much (5.53 rather than 3.48 or 4.90) at a too early stage (0.0105 rather than 0.0228 or 0.0132). The follower also enters the market too early and invests too much compared to the optimal strategies, but the relative loss of the follower is not so high as that of the leader. The reason is that in infinite time the leader invests a lot, implying that, since capacities are strategic substitutes, the follower is already a cautious investor even when it thinks that time is infinite while it is not.

The strategic effect causing the leader's relative loss to be much higher is due to the preemption effect, resulting in a strategy where the leader has to overinvest at an early point in time. Introducing this finite project life makes that the leader will invest at a considerable later time in a significantly smaller capacity size. Both of these effects result in a huge loss if the fact that project life is finite is not taken into account when the leader decides about its investment strategy. However, even for the follower it holds that the infinite strategy is *loss-making* when the expected lifetime in reality is 10 years, which is the case when the net value is negative.

The monopolist's relative loss is comparable to that of the follower which is depicted in Table 1b. Here we see that for a given λ, the monopolist will enter the market sooner and invest



(a) Evolutionary



(b) Revolutionary

Figure 2: Loss due to ignoring finiteness of project for the leader, follower and monopolist as a function of λ. The parameter values are $r = 0.1; \mu = 0.06; \delta = 0.1; \eta = 0.05; \sigma = 0.1$. The solid line represents the case of leader $\ell_L^{(i)}(\lambda)$, the dotted line represents the loss function of the follower $\ell_F^{(i)}(\lambda)$, and the dashed line represents the monopolist value reduction $\ell_M^{(i)}(\lambda)$. The upper figure shows these losses for $i = E$, the evolutionary scenario, and the lower figure for $i = R$, the revolutionary scenario.

less when the option can vanish. The earlier market entrance is incentivised by the fact that the option can disappear, which on its turn causes the firm to invest less to still ensure a profitable project at the lower demand and price level.

We can conclude that it is of high importance to incorporate the finiteness of the lifetime of projects when determining the optimal entry and capacity strategies under uncertainty.

Figure 3 depicts the leader and follower curve for two different values of σ. An increase in uncertainty results in the follower investing later. This is because more uncertainty increases the value of waiting with investment (see, e.g., Dixit and Pindyck (1994)). For the leader the same holds but on the other hand the investment project has become more attractive for the leader: the follower investing later means that the leader's monopoly period lasts longer. As a

Strategies

λ	X_P	$K_L^{\text{det}}(X_P)$	$V_L^n(S_L)$	$V_L^n(s_L)$	$\ell_L^{(E)}(\lambda)$	$\ell_L^{(R)}(\lambda)$
0.1	0.0228	3.48	0.0658	-0.276	3983.1%	1531.5%
0.02	0.0132	4.90	0.169	0.0174	84.2%	85.2%
0	0.0105	5.53	0.2511	0.2511	0%	0%
λ	$X_F^*(K_L^{\text{det}}(X_P))$	$K_F^*(K_L^{\text{det}}(X_P))$	$V_F^n(S_F)$	$V_F^n(s_F)$	$\ell_F^{(E)}(\lambda)$	$\ell_F^{(R)}(\lambda)$
0.1	0.0350	4.26	0.226	-0.128	261.3 %	200.8 %
0.02	0.0263	5.27	0.609	0.447	15.0 %	16.8 %
0	0.0243	5.59	0.950	0.950	0%	0%

(a) Duopoly

λ	X_E^*	K_E^*	$V_M^n(X_E^*, K_E^*)$	$V_M^n(x^*, k^*)$	$\ell_M^{(E)}(\lambda)$
0.1	0.0289	5.15	0.274	-0.177	369.7%
0.02	0.0199	6.98	0.806	0.617	3.94%
0	0.0176	7.73	1.312	1.312	0%
λ	X_R^*	K_R^*	$V_M^n(X_R^*, K_R^*)$	$V_M^n(x^*, k^*)$	$\ell_M^{(R)}(\lambda)$
0.1	0.0616	7.73	1.312	-0.177	198.6%
0.02	0.0264	7.73	1.312	0.617	10.41%
0	0.0176	7.73	1.312	1.312	0%

(b) Monopoly

Table 1: For three different λs, the preemption point, the threshold of the follower and the capacity decisions of the leader of follower are shown in the upper panel. The lower panel shows the optimal threshold and capacity of the monopolist as given by Corollary 1 and 2. The net value function V_i^n is shown when the investors take the finiteness of the projects into account, and also the suboptimal values are shown when the investors make their decisions based on the assumption that the projects have infinite length. The associated losses are provided for evolutionary and revolutionary innovations. The parameters are again $r = 0.1; \mu = 0.06; \delta = 0.1; \eta = 0.05; \sigma = 0.1$.

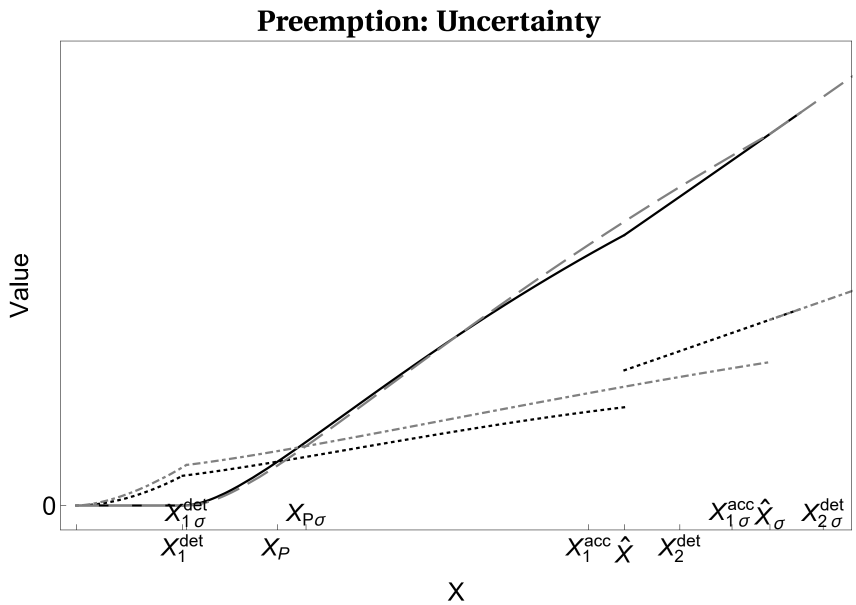


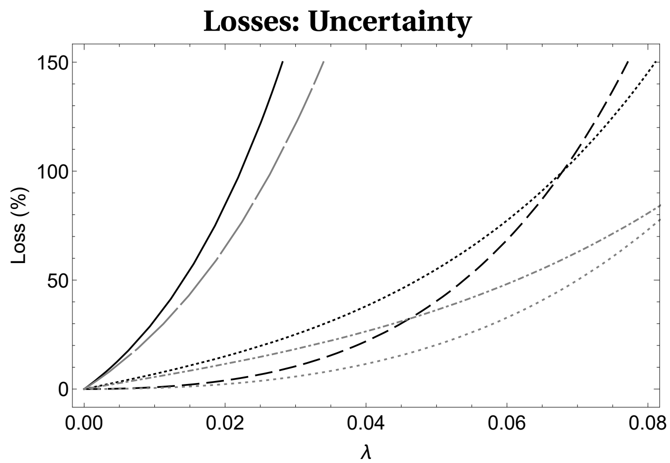
Figure 3: Optimal value functions for the leader and follower where the gray lines represent the case when $\sigma = 0.2$ and the black lines when $\sigma = 0.1$. The black solid line and gray dashed line represent the value function of the leader and the black dotted and gray dot-dashed line the value function of the follower.

result, the preemption point is delayed but only a little. The subscript σ indicates the higher $\sigma = 0.2$ compared to the assumption of $\sigma = 0.1$ that we used for the other numerical examples. Similarly, in Figure 4 we depict the losses when uncertainty is increased. The figure shows that, if firms invest as if the project life is infinite while it is not, the losses become smaller for both firms, as well as for the monopolist, when uncertainty goes up. Presumably this is because under increased uncertainty, firms not only invest later but also more (Dixit (1993)), and therefore investment sizes get closer to the level corresponding to the infinite project life case.

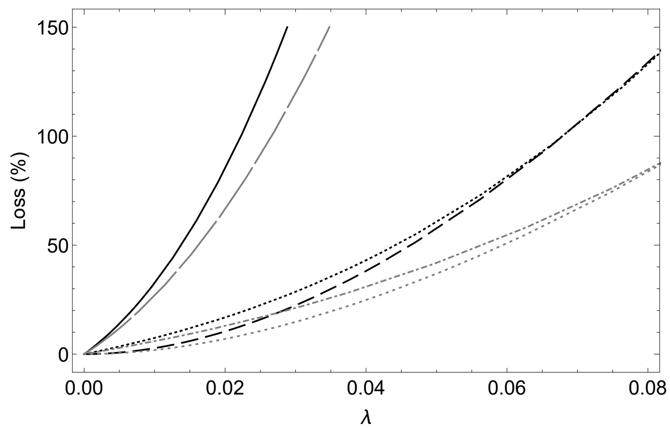
6 Conclusions

An often overlooked characteristic in economic research is the finiteness of investment projects. Due to technological progress drastic innovations appear that make current products obsolete. We show that having a finite life of the project delays the investment in a monopoly market while the investment size remains the same. If we take into account the possibility of future entry of a competitor, we see a similar picture.

Interesting is the case of evolutionary innovation, where besides a finite project life, also the investment option will expire in finite time. We show that then there are multiple contradictory effects on the investment timing of the incumbent, such as an NPV effect, which delays the firm's investment, a quantity effect, and a strategic effect. The latter two accelerate the firm's



(a) Evolutionary



(b) Revolutionary

Figure 4: Loss due to ignoring finiteness of project for the leader, follower and monopolist as a function of λ for two different values of σ. The parameter values are $r = 0.1; \mu = 0.06; \delta = 0.1; \eta = 0.05$ and $\sigma = 0.1$ for the black lines and $\sigma = 0.2$ for the gray lines. The black solid and gray long-dashed lines show $\ell_L^{(i)}(\lambda)$, the black dotted and gray dash-dotted lines show $\ell_F^{(i)}(\lambda)$, and the black dashed and gray dotted lines show $\ell_M^{(i)}(\lambda)$ in the upper figure for $i = E$ and in the lower figure for $i = R$.

investment, but still the total effect is such that also here investment is delayed. In general it holds that, compared to the scenario of revolutionary innovation where the project life is finite and the option life is infinite, the firm will invest earlier and less.

In the duopoly game with endogenous firm roles there is an incentive to become the first investor, and a preemption equilibrium results. We show that this preemption effect is mitigated by the project life being finite. However, when firms mistakenly ignore the finiteness of the project when taking their investment decisions, the preemption effect considerably enlarges the already large value loss.

Future research could point to different aspects. First, the present paper considers that the lifetime of a product can end due to the occurrence of more innovative products on the market.

If such an innovation is drastic, this event essentially stops all activities related to the current product not only of the focal firm, but also of its competitors. As an alternative it would be interesting to consider the finiteness of the economic lifetime of production factors. In such a case, the end of a project owned by one firm would not mean the end of the project of other firms too, instead these could still be continued. We intend to analyze such a scenario with independent events triggering the termination of projects in our future research. Second, a scenario could be considered where the firms themselves control the innovation activities with the implication that the innovation speed λdt, and thus the probability that the lifetime of the current product will end, will depend on the level of R&D activities (see also Martzoukos and Zacharias (2013)). Third, effects of ambiguity could be studied, in the sense that firms are ambiguous about the true values of the drift and the uncertainty parameters governing the demand system (see, e.g., Sarkar (2020)).

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A Appendix

This appendix contains the proofs of the propositions.

A.1 Proof of Proposition 1

Consider first the continuation region and denote by $F(X)$ the value of the option to invest. In the continuation region we have to take into account that the investment option can become worthless with probability λdt. Application of Ito's lemma gives

$$E\left(dF_E(X)\right) = \left(\mu X F_E'(X) + \frac{1}{2}\sigma^2 X^2 F_E''(X) - \lambda F_E(X)\right)dt. \tag{55}$$

Combining this with the Bellman equation results in the following differential equation that $F_E(X)$ has to satisfy:

$$(r+\lambda) F_E(X) = \frac{1}{2}\sigma^2 X^2 F_E''(X) + \mu X F_E'(X). \tag{56}$$

Solving the differential equation, while taking into account that $F_E(0) = 0$, gives

$$F_E(X) = A_E X^{\beta_\lambda}, \tag{57}$$

in which A_E is an unknown constant.

In the stopping region the value of the firm $V(X,K)$ satisfies the following differential equation:

$$rV(X,K) = Kf(K)X + \frac{1}{2}\sigma^2 V''(X,K) + \mu V'(X,K) - \lambda V(X,K), \tag{58}$$

where the last term on the right-hand side results from the fact that with probability λdt the project ends and thus its value jumps down from $V(X,K)$ to zero. Taking into account that $V(0,K) = 0$ and that we abstract away from speculative bubbles, solving the differential equation gives

$$V(X,K) = \frac{Kf(K)X}{r+\lambda-\mu}. \tag{59}$$

So, it is clearly seen that the larger the probability the project ends, the lower the value of the firm after investment, as reflected by the presence of λ in the denominator.

Maximizing value function $V(X,K) - \delta K$ with respect to K gives the optimal capacity size K^* for every given level of X:

$$\frac{\left(f(K) + Kf'(K)\right)X}{r+\lambda-\mu} - \delta = 0. \tag{60}$$

Standard real options analysis, shows that the value of the option to invest, denoted by F is equal to (57). To determine the optimal trigger X^*, we employ the value matching and smooth pasting conditions:

$$F_E(X^*) = V(X^*, K) - \delta K, \tag{61}$$

$$F_E'(X)_{X=X^*} = V'(X, K)_{X=X^*}. \tag{62}$$

Substituting (57) and (59) into (61) and (62) leads to $V'(X,K)X = (V(X,K) - \delta K)\beta_\lambda$. Solving for X_E^* gives the first result. If we plug (9) into the first-order condition with respect to K as given by (60), then we get the implicit function (10).

A.2 Proof of Corollary 1

If the inverse demand is linear in the quantity as given by (12) then $f(K) = 1 - \eta K$. Substituting this into the implicit function of Proposition 1 and solving for K gives both results.

A.3 Proof of Proposition 2

Now that the option has an infinite life, expression (55) has to be replaced by

$$E(dF_R(X)) = \left(\mu X F_R'(X) + \frac{1}{2}\sigma^2 X^2 F_R''(X)\right) dt. \tag{63}$$

Inserting this in the Bellman equation results in the differential equation

$$rF_R(X) = \frac{1}{2}\sigma^2 X^2 F_R''(X) + \mu X F_R'(X), \tag{64}$$

and the solution

$$F_R(X) = A_R X^\beta, \tag{65}$$

Similar as in the proof of Proposition 1, the first-order condition with respect to K remains (60). The value matching and smooth pasting conditions now lead to $V'(X,K)X = (V(X,K) - \delta K)\beta$. Solving for X_R^* gives the results.

A.4 Proof of Corollary 2

If the inverse demand is linear in the quantity as given by (12) then $f(K) = 1 - \eta K$. Substituting this into (17) and solving for K gives both results.

A.5 Proof of Proposition 3

Maximizing $V_F(X, K_L, K_F) - \delta K_F$, where $V_F(X, K_L, K_F)$ is given by (24), with respect to K_F gives the optimal capacity size of the follower, given the level X and the capacity size of the leader K_L:

$$K_F^*(X, K_L) = \frac{1}{2\eta}\left(1 - \eta K_L - \frac{\delta(r + \lambda - \mu)}{X}\right). \tag{66}$$

Before the follower has invested, thus when $X < X_F^*(K_L)$, the firm holds an option to invest. The option value is

$$F_F(X) = A_F X^{\beta_\lambda}. \tag{67}$$

Solving the corresponding value matching and smooth pasting conditions gives

$$X_F^*(K_L, K_F) = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{\delta(r + \lambda - \mu)}{1 - \eta(K_L + K_F)}. \tag{68}$$

After solving the system of equations we obtain (25) and (26).

A.6 Proof of Proposition 4

The value function of the leader at the moment of investment for the deterrence strategy is given by (28). Substituting (25) and (26) into this equation results in

$$V_L^{\det}(X, K_L) - \delta K_L = \frac{K_L(1 - \eta K_L)}{r + \lambda - \mu}X - \delta K_L - \left(\frac{X(\beta_\lambda - 1)(1 - \eta K_L)}{(\beta_\lambda + 1)\delta(r + \lambda - \mu)}\right)^{\beta_\lambda}\frac{K_L\delta}{\beta_\lambda - 1}.$$

Maximizing with respect to K_L gives the following first-order condition:

$$\begin{aligned}\phi(X, K_L) = &\frac{(1 - 2\eta K_L)}{r + \lambda - \mu}X - \delta\\ &- \left(\frac{X(\beta_\lambda - 1)(1 - \eta K_L)}{(\beta_\lambda + 1)\delta(r + \lambda - \mu)}\right)^{\beta_\lambda}\frac{(1 - (\beta_\lambda + 1)\eta K_L)\delta}{(\beta_\lambda - 1)(1 - \eta K_L)} = 0.\end{aligned} \tag{69}$$

Solving (69) gives $K_L^{\det}(X)$ as given by (32). Setting $K_L = 0$ in equation (69) gives equation $X_1^{\det}$. Furthermore, the leader cannot use the deterrence strategy anymore if we have that $X_F^*\left(K_L^{\det}(X)\right) \leq X$. Let us define $X_2^{\det}$ as

$$X_F^*\left(K_L^{\det}(X_2^{\det})\right) = X_2^{\det}. \tag{70}$$

To determine $X_2^{\det}$ we substitute equation (25) for X into (69) which solves for K_L

$$K_L = \frac{1}{2\eta}. \tag{71}$$

Substituting this into (25) gives

$$X_2^{\text{det}} = \frac{\beta_\lambda + 1}{\beta_\lambda - 1} 2\delta \left(r + \lambda - \mu\right). \tag{72}$$

Before the leader has invested, thus when $X < X_L^{\text{det}}$, the firm holds an option to invest. The option value is

$$F_{L,E}^{\text{det}}(X) = A_{L,E}^{\text{det}} X^{\beta_\lambda}, \tag{73}$$

when the option life of the investment is of finite length. The value matching and smooth pasting conditions to determine $X_{L,E}^{\text{det}}$ lead together to the condition $V'(X,K)X - (V(X,K) - \delta K)\beta_\lambda = 0$. Define

$$\begin{aligned}
\varphi(X, K_L) &= V'(X, K_L)X - (V(X, K_L) - \delta K_L)\beta_\lambda, \\
&= \left(\frac{K_L\left(1 - \eta K_L\right)}{r + \lambda - \mu} X - \beta_\lambda \left(\frac{X\left(\beta_\lambda - 1\right)\left(1 - \eta K_L\right)}{\left(\beta_\lambda + 1\right)\delta\left(r + \lambda - \mu\right)}\right)^{\beta_\lambda} \frac{K_L \delta}{\beta_\lambda - 1}\right) - \\
&\quad \beta_\lambda \left(\frac{K_L\left(1 - \eta K_L\right)}{r + \lambda - \mu} X - \delta K_L - \left(\frac{X\left(\beta_\lambda - 1\right)\left(1 - \eta K_L\right)}{\left(\beta_\lambda + 1\right)\delta\left(r + \lambda - \mu\right)}\right)^{\beta_\lambda} \frac{K_L \delta}{\beta_\lambda - 1}\right), \\
&= \frac{K_L\left(1 - \eta K_L\right)}{r + \lambda - \mu} X\left(1 - \beta_\lambda\right) + \beta_\lambda \delta K_L, \\
&= 0,
\end{aligned} \tag{74}$$

which solves for (31).

A.7 Proof of Proposition 5

Substituting (31) into (69) and solving for K gives (33). The corresponding threshold $X_{L,E}^{\text{det}}$ can be calculated by substituting the optimal quantity into (31).

A.8 Proof of Corollary 3

Let

$$X = \frac{\beta_\lambda + a}{\beta_\lambda - 1} \delta \left(r + \lambda - \mu\right), \tag{75}$$

where $\beta_\lambda > 1$ is the positive root that solves (11) and is given by

$$\beta_\lambda = \frac{-(\mu - \frac{\sigma^2}{2}) + \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + \lambda)}}{\sigma^2}. \tag{76}$$

The derivative is

$$\frac{\partial X}{\partial \lambda} = -\frac{1+a}{(\beta_\lambda - 1)^2}\frac{\partial \beta_\lambda}{\partial \lambda}\delta(r+\lambda-\mu) + \frac{\beta_\lambda + a}{\beta_\lambda - 1}\delta, \tag{77}$$

where

$$\frac{\partial \beta_\lambda}{\partial \lambda} = \frac{1}{\sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda)}} > 0. \tag{78}$$

By rewriting (11) and (76), we can also express the derivative of β_λ with respect to λ as

$$\frac{\partial \beta_\lambda}{\partial \lambda} = \frac{\beta_\lambda}{r+\lambda+\frac{1}{2}\sigma^2\beta_\lambda^2}. \tag{79}$$

Plugging (79) into (77) leads to

$$\frac{\partial X}{\partial \lambda} = \frac{\delta}{(\beta_\lambda - 1)^2\left(2(r+\lambda)+\sigma^2\beta_\lambda^2\right)}\Upsilon, \tag{80}$$

where

$$\begin{aligned}\Upsilon = &-2a(r+\lambda) - 4\left(r+\lambda-\frac{1}{2}\mu(1+a)\right)\beta_\lambda + \\ &\left(2(r+\lambda) - a\sigma^2\right)\beta_\lambda^2 - \sigma^2(1-a)\beta_\lambda^3 + \sigma^2\beta_\lambda^4.\end{aligned} \tag{81}$$

We have to show that this derivative is positive. Since $\beta_\lambda > 1, \delta > 0$ and $r > \mu$ we thus concentrate on Υ. By applying twice the equality of (11) and rewriting, we obtain the following equalities that simplify the derivative to (82),

$$\begin{aligned}\Upsilon = &\overbrace{-2a(r+\lambda)}^{\text{use (11)}} - 4\left(r+\lambda-\frac{1}{2}\mu(1+a)\right)\beta_\lambda + \\ &\left(2(r+\lambda)-a\sigma^2\right)\beta_\lambda^2 - \sigma^2(1-a)\beta_\lambda^3 + \sigma^2\beta_\lambda^4, \\ &= \beta_\lambda\left(\sigma^2\beta_\lambda^2(\beta_\lambda-1) + a\sigma^2(\beta_\lambda-1)^2 - 2\left(\overbrace{(r+\lambda)}^{\text{use (11)}} - \mu\right) + 2(r+\lambda)(\beta_\lambda-1)\right), \\ &= \beta_\lambda(\beta_\lambda-1)\left(\sigma^2\beta_\lambda^2 + a\sigma^2(\beta_\lambda-1) - 2\left(\mu+\frac{1}{2}\sigma^2\beta_\lambda\right) + 2(r+\lambda)\right), \\ &= \beta_\lambda(\beta_\lambda-1)\left(\sigma^2(\beta_\lambda+a)(\beta_\lambda-1) + 2(r+\lambda-\mu)\right).\end{aligned}$$

Hence,

$$\frac{\partial X}{\partial \lambda} = \frac{\delta\beta_\lambda(\beta_\lambda-1)\left(\sigma^2(\beta_\lambda+a)(\beta_\lambda-1)+2(r+\lambda-\mu)\right)}{(\beta_\lambda-1)^2\left(2(r+\lambda)+\sigma^2\beta_\lambda^2\right)}. \tag{82}$$

Since $r > \mu$ it follows that $r + \lambda - > \geq 0$ and because $\beta_\lambda > 1$ it follows that the derivative is positive for $a > -1$. Hence it holds for both $X_{L,E}^{\mathrm{det}}$ when $a = 1$ and X_1^{acc} when $a = 3$.

A.9 Proof of Proposition 6

This proof is similar to the proof of Proposition 4, though here the option life is finite. The derivations are identical up to (72). Before the leader has invested, thus when $X < X_L^{\mathrm{det}}$, the firm holds an option to invest. The option value is

$$F_{L,R}^{\mathrm{det}}(X) = A_{L,R}^{\mathrm{det}} X^{\beta}, \tag{83}$$

when the option life of the investment is of infinite length. The value matching and smooth pasting conditions to determine $X_{L,R}^{\mathrm{det}}$ lead together to the condition $V'(X,K)X - (V(X,K) - \delta K)\beta = 0$. Define

$$\begin{aligned}
\varphi(X, K_L) &= V'(X, K_L)X - (V(X, K_L) - \delta K_L)\beta, \\
&= \left(\frac{K_L(1-\eta K_L)}{r+\lambda-\mu} X - \beta_\lambda \left(\frac{X(\beta_\lambda - 1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda} \frac{K_L\delta}{\beta_\lambda - 1}\right) - \\
&\beta\left(\frac{K_L(1-\eta K_L)}{r+\lambda-\mu} X - \delta K_L - \left(\frac{X(\beta_\lambda - 1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda} \frac{K_L\delta}{\beta_\lambda - 1}\right), \\
&= \frac{K_L(1-\eta K_L)}{r+\lambda-\mu} X(1-\beta) \\
&- (\beta_\lambda - \beta)\left(\frac{X(\beta_\lambda - 1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda} \frac{K_L\delta}{\beta_\lambda - 1} + \beta\delta K_L, \\
&= 0.
\end{aligned} \tag{84}$$

We now solve (69) and (84) simultaneously by the fact that $\phi(X, K_L) = 0$ and $\varphi(X, K_L) = 0$. Let $Z = \left(\frac{(\beta_\lambda-1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}$ then

$$\begin{aligned}
\varphi(X, K_L) &= -(\beta_\lambda - \beta) Z \frac{K_L\delta}{\beta_\lambda - 1} X^{\beta_\lambda} + \frac{K_L(1-\eta K_L)}{r+\lambda-\mu}(1-\beta)X + \beta\delta K_L = 0, \\
\phi(X, K_L) &= -Z\frac{(1-(\beta_\lambda+1)\eta K_L)\delta}{(\beta_\lambda-1)(1-\eta K_L)} X^{\beta_\lambda} + \frac{(1-2\eta K_L)}{r+\lambda-\mu} X - \delta = 0.
\end{aligned} \tag{85}$$

These two equations are equal to each other when the capacity is equal to

$$K_{L,R}^{\mathrm{det}} = \frac{1}{\eta(\beta+1)}. \tag{86}$$

Plugging this into $\phi(X, K_{L,R}^{\mathrm{det}}) = 0$ and $\varphi(X, K_{L,R}^{\mathrm{det}}) = 0$ leads to the implicit function for $X = X_{L,R}^{\mathrm{det}}$ solving (37).

A.10 Proof of Corollary 4

Define (37) as $\Phi(X)$

$$\begin{aligned}\Phi(X) = X^{\beta_\lambda}\frac{(\beta_\lambda-\beta)\delta}{\beta(\beta_\lambda-1)}\left(\frac{\beta(\beta_\lambda-1)}{(1+\beta)(1+\beta_\lambda)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}\\ +X\frac{(\beta-1)}{(r+\lambda-\mu)(\beta+1)}-\delta.\end{aligned} \tag{87}$$

We show that the implicit function that determines the leader's threshold is in between duopoly threshold under a finite option and the monopoly threshold under an infinite option. Plugging (33) into $\Phi(X)$ leads to

$$\Phi\left(\frac{\beta_\lambda+1}{\beta_\lambda-1}\delta(r+\lambda-\mu)\right) = \frac{(\beta_\lambda-\beta)\delta\left((\beta+1)\left(\frac{\beta}{1+\beta}\right)^{\beta_\lambda}-2\beta\right)}{\beta(1+\beta)(\beta_\lambda-1)} < 0, \tag{88}$$

since $\beta_\lambda > \beta > 1$. While plugging (21) into $\Phi(X)$ leads to

$$\Phi\left(\frac{\beta+1}{\beta-1}\delta(r+\lambda-\mu)\right) = \frac{(\beta_\lambda-\beta)\delta}{\beta(\beta_\lambda-1)}\left(\frac{\beta(\beta_\lambda-1)}{(\beta-1)(\beta_\lambda+1)}\right)^{\beta_\lambda} > 0. \tag{89}$$

Moreover, for $X > 0$,

$$\begin{aligned}\frac{\partial\Phi(X)}{\partial X} =& \beta_\lambda X^{\beta_\lambda-1}\frac{(\beta_\lambda-\beta)\delta}{\beta(\beta_\lambda-1)}\left(\frac{\beta(\beta_\lambda-1)}{(1+\beta)(1+\beta_\lambda)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}\\ &+\frac{(\beta-1)}{(r+\lambda-\mu)(\beta+1)} > 0.\end{aligned} \tag{90}$$

Hence, the optimal threshold of the leader under an infinite option, which is implied by $\Phi(X) = 0$, is in between these two strategies as displayed in (38).

A.11 Proof of Proposition 7

Substituting (66) into (39) and maximizing $V_L^{\mathrm{acc}}(X, K_L) - \delta K_L$ with respect to K_L gives (42)

$$K_L^{\mathrm{acc}}(X) = \frac{1}{2\eta}\left(1-\frac{\delta(r+\lambda-\mu)}{X}\right). \tag{91}$$

The leader will only use its accommodation strategy if the optimal quantity $K_L^{\text{acc}}(X)$ leads to immediate investment of the follower. So it should hold that $X_F^*(K_L^{\text{acc}}(X)) \leq X$. We define X_1^{acc} as

$$X_{F,R}^*(K_L^{\text{acc}}(X_1^{\text{acc}})) = X_1^{\text{acc}}. \tag{92}$$

Substitution of (25), where we use that $X_{F,R}^*(K_L) = X_{F,E}^*(K_L)$, and (91) into (92) and rearranging gives

$$X_1^{\text{acc}} = \frac{\left(\beta_\lambda + 3\right)\delta}{\beta_\lambda - 1}\left(r + \lambda - \mu\right). \tag{93}$$

For the accommodation strategy, the value matching and smooth pasting conditions with an infinite option to invest leads to the condition $V'(X,K)X - (V(X,K) - \delta K)\beta = 0$ we obtain two roots of which $X = \delta(r + \lambda - \mu)$ is not a valid solution and thus we have that $X_{L,R}^{\text{acc}}$

$$X_{L,R}^{\text{acc}} = \frac{1+\beta}{\beta - 1}\delta\left(r + \lambda - \mu\right). \tag{94}$$

Thus the leader will consider the entry accommodation strategy whenever $X \geq \max\left(X_1^{\text{acc}}, X_{L,R}^{\text{acc}}\right)$.