Bid price under uncertainty: the case of Palmeirópolis Auction

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Abstract

Mineral rights auctions bids are subject to price and geological uncertainty and are affected by managerial flexibility clauses. We analyze the case of a mineral auction in Brazil of a deposit of copper, zinc, lead, silver, gold and cadmium in an area known as Palmeirópolis Polymineral Complex under the real options approach. The model considers both the price and the exploratory-technical uncertainty. The auction rules allow for the firm to abandon the project at different stages of the process, as long as certain prior requirements of capital investment or payments have been met. Considering all options that concession agreements offers, the project value increases by $10.33 million, or 26.85%.

1 Introduction

In many countries, rights of exploration of natural resources such as oil, gas and minerals are typically auctioned by the governing authority to the highest bidder. These auctions can take many forms, and the rules may include contractual clauses and conditions that attempt to insure that bidders are serious in their intent, that they put forward their best offer, to reduce technical uncertainty and others.

Many of these clauses allow the bidder to abandon the project up to a period of time during the process, in case the preliminary survey results indicate that the exploration potential of the field turns out to be below expectations. In addition, they may require that the bidder pay additional amounts to progress to the next stages or others. On the other hand, the mere existence of such clauses impacts the value of the concession and thus, that of the bid, as they affect the risk and return of the project. This information could be useful so that for the governing body can adequately price the bid, and to assist the potential concessionaire in determining its preferred bid price. Given that, these clauses have option-like characteristics, as they provide the bidders an opportunity to decide whether to continue or to abandon at each of the stages of the project, they must be valued using option-pricing methods.

In this article we develop a such as the real options model to determine the value that such clauses add to a mineral rights contract, using the case of the auction of the Palmeirópolis Polymineral Complex in the State of Tocantins , Brazil, in 2019 as a numerical example. Our results indicate that

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these contractual flexibilities significantly increase the value of the project in the case of the Palmeirópolis complex.

This paper is organized as follows. After this introduction we present a brief literature review, followed by a numerical example. In section 4 we present the real options model and results, and in section 5 we conclude.

2 Literature Review

The most common method for asset valuation is the Discounted Cash Flow (DCF), which considers future expected cash flow discounted by a risk-adjusted discount rate. In investments with high uncertainty, such as mineral resources, extreme events are more likely than low uncertainty projects, and average-based methods may distort the valuation. In addition, DCF does not capture the value of any embedded flexibilities the project may have. Real Options Analysis (ROA), which derived from the seminal work on financial options of Black and Scholes (1973) and Merton (1973), takes into account the fact that managerial flexibility has option-like characteristics, which can be valued using option pricing methods. Authors such as Dixit and Pindyck (1994), Trigeorgis (1995) and Copeland and Antikarov (2001) subsequently contributed to the consolidation of the research in the field.

The first ROA application was in natural resources, which the owner had the rights, i.e, an option to develop given the expected price of the commodity (Tourinho, 1979). Brennan and Schwartz (1985) analyzed the case of investments in natural resources and showed that the real options approach could be used both for valuation and decision making purposes. They apply real option analysis to value managerial flexibility in for copper mines considering price, cost and reserves as the three main sources of uncertainty and concludes that differences in assumptions may lead to large differences in valuation results.

Cortazar and Casassus (1998) analyze the value of the option to expand production capacity of a copper mine under the real options approach and shows that a significant fraction of investment value derives from the flexibility to delay the investment. Cortazar, Schwartz, and Casassus (2003) consider a natural resource exploration investments when there is price and geological–technical uncertainty and flexibility to defer investment and once developed, to suspend and resume operations at any time depending on uncertain future cash flow expectations. Results show that a significant portion of the project value is due to the operational flexibility available to the managers. Miranda, Brandão, and Lazo Lazo (2017) used a discrete real options model to determine the value of embedded options in a junior silver mine in Peru and show that these managerial flexibilities have significant impact on the value of the mine. Guj and Chandra (2019) compared several techniques to obtain the value a mining project and conclude that the use of the volatility of the individual uncertainties provide a more accurate real option values as shown by Copeland and Antikarov (2001) and Brandão, Dyer, and Hahn (2005).

The stochastic modeling of commodity prices has been widely discussed in the literature. Brennan and Schwartz (1985) adopted the Geometric Brownian Motion (GBM), which is a simple and tractable model. The binomial lattice model of Cox, Ross, and Rubinstein (1979) is based on the GBM. However, the equilibrium of economic prices is not easily justified with this model as is generally assumed that commodity prices follow a mean reverting process. Schwartz (1997) proposed one-factor models, while Schwartz and Smith (2000) developed more detailed two factor models. Nonetheless, while adding more factors usually provides more adherence to the historical series, parameter determination and the ROA model become significant more complex.

Research that unites technical and price uncertainty are important in the exploratory mineral phase, which follows several paths. Value of Information (VOI) is one way of dealing with this issue, since the accumulated knowledge reduces the uncertainty of a projects (Dias, 2004; Grenadier, 2015). Another way is to propose a particular stochastic model for the deposit, which interacts with the project's value under the stochastic price model (Cortazar et al., 2003). At the end, it is important to design a model consistent with the industry's reality, which is something difficult to capture. What is known, and as shown in (Rudolph & Goulding, 2017), is that the error of the mineral deposit declaration decreases over time, being a common principle between the two research streams.

3 Numerical Example

The Palmeirópolis Polymineral Complex is a Volcanic Proterozoic Deposit (1,200 Millions years) with occurrence of copper, zinc, lead, silver, gold and cadmium (Araujo, Fawcett, & Scott, 1995), with high potential in copper extraction Fig.(1). This has relation with the formation events of the Brasilia Belt and the magmatic's arches existing till these events (Juliani, Monteiro, & Fernandes, 2016).

The Brazilian Geological Service (CPRM) is a government agency responsible for the dissemination of geological knowledge, research and the granting of mineral exploration rights in Brazil. Since the 1970s, CPRM also holds the rights to 330 mining areas divided into 30 blocks (CPRM, 2019a). These rights include the southern area of the state of Tocantins, in the city of Palmeirópolis, located 500 kilometers north of Brasilia. This area was subject to two research surveys, one in the 70s and another in the 80s. Given that it is an area with a strong economic potential, in 2019 the Brazilian government decided to auction off the exploration rights of this field.

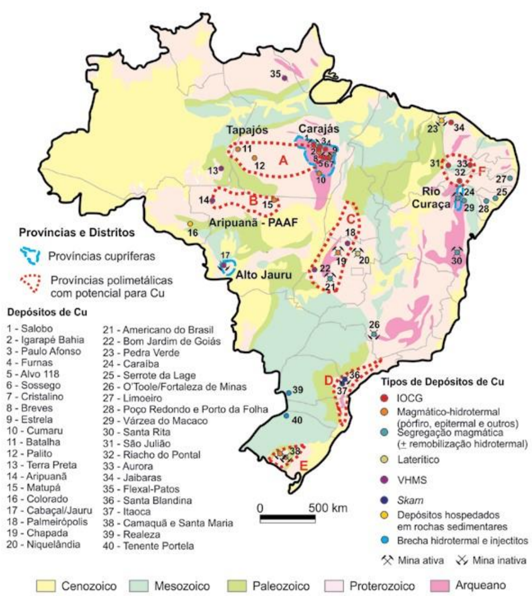

Figure 1: Copper deposits in Brazil - #18 - Palmeirópolis Complex- Juliani et al. (2016)

The auction was structured in three phases Fig.(2) (CPRM, 2019b), in a model that is similar to concession regimes of oil exploration areas. Upon payment of the entry bonus ($450,000), the winner of the auction was to be granted access to audit CPRM´s database information for a period of six months. Once this period was over the winner had the option to continue to the second phase, which required mandatory exploratory investments of $6 million in drilling costs over two years. This stage involved two exploratory sub stages: Exploration-1 and Exploration-2. In Exploration-1 stage, which would last six months, the winner was required to invest at least 40 % of the total exploration budget for this state, while in the 18 month Exploration-2 stage the firm was required to invest the remaining 60 % of the budget. Following that, the winner had the option to abandon the project with no penalty. Otherwise, a payment of $ 1.8 million had to be made, followed by another payment of $2.3 million at the end of six months, which would allow the firm to receive the exclusive rights to explore until the exhaustion of ore deposit. At the completion of each stage the investor has the right to abandon the project.

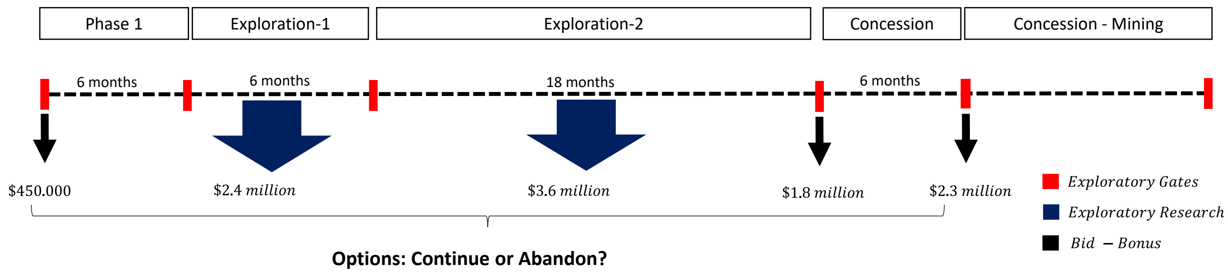

Figure 2: Palmeirópolis's auction

The exploratory contract model offers several options for postponing the investment or even leaving the project. Not considering these options, ie, applying the Discounted Cash Flow (DCF) method, is not seeking the best for the investor and disregarding existing values. For this reason, the Real Options approach is the method for this kind of situation. Moreover, not considering technical uncertainty in a coherent model, it is to disregard the great function of this phase, which seeks to mitigate the doubts about deposits for companies to invest. Thus, it was considered for each step, a model of technical uncertainty, with the principle of variance's reduction throughout the deposits' research.

The Palmeirópolis mineral complex has occurrence of copper, zinc, lead, silver, gold and cadmium. Nevertheless, the gross information in CPRM's reports are related with zinc, copper and lead. Even in the review analysis for this auction, the consulting firm Saga modeled a base scenario considering zinc and copper production. This paper propose modeling also those two variables, but in the dimension of uncertainty of prices and ore's grade.

3.1 Price Analysis

The zinc historical time series was obtained in quandl website, between the years 2004 and 2019. It is refer to zinc monthly spot price, 98% high grade pure in US$ per metric tonne Fig.(3-a). The copper prices also was downloaded in quandl and the analysis was done in the same interval. It is a London Metal Exchange (LME) spot monthly price, with CIF European ports and grade A cathode, listed in US$ per metric of ton Fig.(3-b).

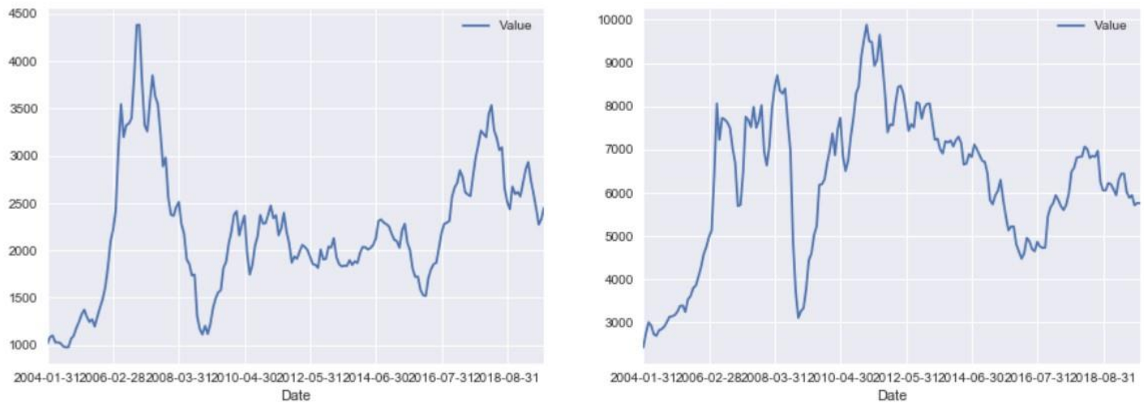

Figure 3: Historical prices a) Zinc and b) Copper

The Augmented Dickey-Fuller test was applied in both data. We reject the null-hypothesis (H_0) with α = 5% of normality of first difference prices ($p\text{-}value_{zinc}$= 0.0061 and $p\text{-}value_{copper}$= 0.0468). This means that we reject a Geometric Brownian Motion for both series in this interval. For that reason, we adopted for the price's model (S) the Geometric Mean Reverse stochastic process of Schwartz (1997), as we can see in Eq.(1).

$$dX = \kappa(\alpha - X)dt + \sigma dz \quad (1)$$

where $X = ln(S)$

Bastian-Pinto, Brandão, and Ozório (2016) present a solution for the expected value and the simulation of Schwartz (1997) stochastic process, considering $\alpha = ln(\overline{S})$. Next step is to extract the parameters using the model in the time series range, as we expose in the Table (1).

Asset	κ	α	σ	Equilibrium Price ($/ton)
Zinc	0.428	7.846	24.36%	2470.58
Copper	0.566	7.846	23.98%	6787.15

Table 1: Zinc and Copper - Parameters of the Schwartz (1997) Model

The following Eq.(2 and 3) present the expected value and the variance of the process . To consider the expected value of S_t, it is necessary to apply log-normal propriety ($[S_t] = e^{\mathbb{E}[X_t]+0.5*Var[X_t]}$). The simulation is obtained with Eq.(4) (Bastian-Pinto et al., 2016).

$$\mathbb{E}[X_t] = ln(S_{t_0})e^{-\kappa(t-t_0)} + (\alpha - \frac{\sigma^2}{2\kappa})(1 - e^{-\kappa(t-t_0)}) \quad (2)$$

$$Var[X_t] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(t-t_0)}) \quad (3)$$

$$S_{t+1} = e^{ln(S_t)e^{-\kappa\Delta t} + [\alpha - \frac{\sigma^2}{2\kappa}](1-e^{-\kappa\Delta t}) + \sigma\sqrt{\frac{1-e^{-2\kappa\Delta t}}{2\kappa}}N(0,1)} \quad (4)$$

The result below is fifty simulations in ten years period Fig.(4). Both simulations started at the price of last data (2019/11/26). Even with the half life ($-ln0.5/\kappa$) of Zinc is longer (one year and a half) than the copper (one year and three months), the starting zinc's price ($2451.65) is closed to the Equilibrium's price (Table 1), so the appearance of average long-term is a line. In the case of copper price's simulation, the effect of convergence to equilibrium's price is more clear.

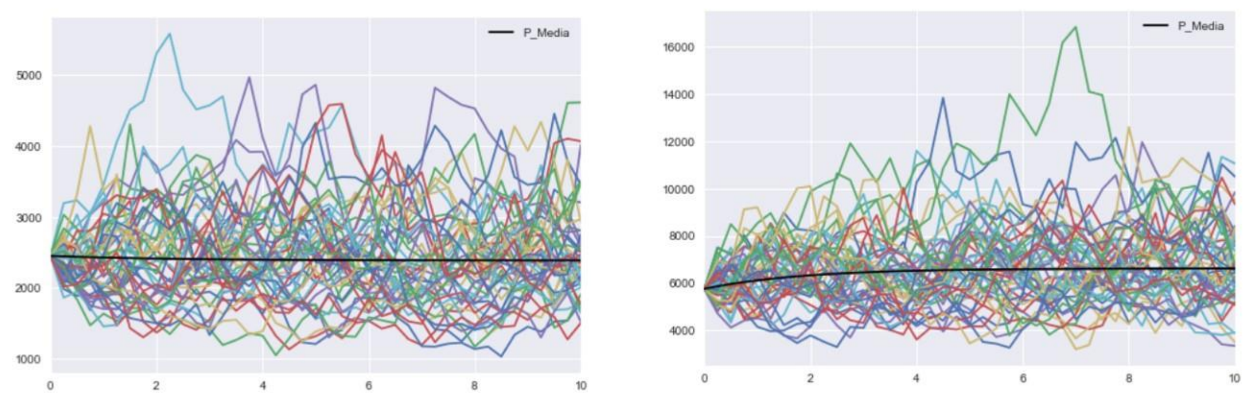

Figure 4: Simulation in 10 years - a) Zinc and b) Copper

3.2 Geological Uncertainty

In 70's and 80's, CPRM did two considerable exploratory campaigns with drills, soil geochemistry and geophysics indirect methods to better understand the volcanic ore deposit. Palmeirópolis complex has number of prospects inside the concession area, but in the portfolio reassessment for this bid, the consulting SAGA with the technical experts in CPRM agreed that still need more hard information about the deposit to declare as mineral reserve. They consider three prospects with the best potential economic value: C1, C3 and C4. In addition, those areas have more drills and analysis than others do, so the qualification is more accurate.

The sample analysis was reorganized in the reassessment report's Saga-CPRM (Table 2). We note that all histograms of ore's grade were a log-normal like distribution.

Measures	C1		C3		C4	
	Zn(%)	Cu(%)	Zn(%)	Cu(%)	Zn(%)	Cu(%)
Samples	481	479	242	242	266	266
Minimum	0.01	0.01	0.07	0.07	0.07	0.006
Maximum	25.6	9.07	25.6	8.32	23.6	1.6
Average	3.87	0.99	6.79	1.71	1.51	0.15
Standard deviation	5.61	1.34	6.64	1.67	2.40	0.19
Coefficient of variation	1.44	1.35	0.98	0.97	1.59	1.25

Table 2: Geochemistry of samples in C1, C3 and C4 deposits - Saga and CPRM docs

Saga consulting modeled the three deposit and concluded that the Palmeirópolis complex has 6.54 Mt of ore. Using a Sub-level Open Stoping method of production, they project to extract around 4.7 Mt of ore. The average of each mineralization and the correlation with zinc and cooper's elements can been seen in the Table (3).

Model	C1			C3			C4		
	Zn(%)	Cu(%)	$\rho Zn,Cu$	Zn(%)	Cu(%)	$\rho Zn,Cu$	Zn(%)	Cu(%)	$\rho Zn,Cu$
Average	5.85	1.59	0.74	6.19	1.55	0.97	1.49	0.14	0.46

Table 3: Geological Model C1, C3 and C4 deposits - Saga and CPRM docs

As the preference for production is to begin with the highest zinc concentration, Saga modeled the plant with the sequence of C3, C1 and C4 deposits. Our geological uncertainty was extract from theses information (tables 2 and 3). As we have the average of the model for each deposit, we can estimate from the coefficient of variation of Table (2) the variance of log-normal distribution deposit. For each mineralization we can obtain the normal parameters (μ and σ) that drives the log-normal distribution. For uncertainty model, it is possible to simulate just one random variable (multivariate normal distribution), as the principle that there is a correlation of occurrence of both chemicals elements.

The randomized concentration is controlled by the maximum of sample percentage of each chemical element, as you can see in Table (2). That control avoids deposits around 100% of ore. Besides that, Saga offer the production of each level of underground mining. With this information, it is necessary to adjust the percentage of each level in relation of the role randomized deposit percentage.

During exploration phase, the uncertainty of ore concentration drop down (Rudolph & Goulding, 2017). This effect is associated with reveled information through direct and indirect methods, like, for example, drilling and electromagnetic very low frequency. Since the contract requires a mineral survey, we propose a change of variance of initial model along the exploration, without changing the average of the estimation. We design a reduction in 40% in the variance at the first phase of exploration and twice of 40% in the second phase, accumulating 78.4% in reduction of variance at the last phase.

3.3 Production and costs

Saga consulting estimated ten year to produce 4.7 Mt volume of rock, which has a rump up of processing between years one and three, flattening in next year in 500 tones by year. The fourth year of production, it will be a mix of the last level of C3 mineralization with the first level of C1. That also happen in the seventh year, mixing the end of C1 mines and the beginning of C4.

The Fig.(5) shows zinc's concentration by year of production. The plot expose the mean and standard deviation of thousands of simulation. The gray line is the Saga's model. It is possible to note a similar values in the mean (square and diamond shapes) to the Saga's model. However, it is slightly different in the fourth year. Indeed, this effect is the independent sort of C3 and C1 percentage, much more than the result of maximum value control of log-normal distribution.

Saga reports that the mineralization C1 and C3 have geological correlation, both are secondary process, unlike the deposit C4, which has original mineralization, with greater dispersion of cooper and zinc. Indeed, adopting the correlation of deposit would turn the model more realistic, but also more difficult to solve. Besides these facts, in the image it is clear the reducing of the uncertainty by standard deviation (bars blue and rose) in each explorations' phases and each year of production.

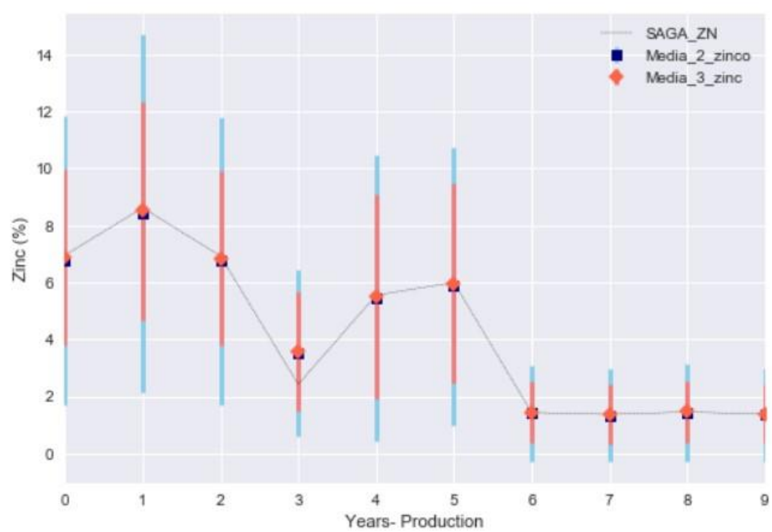

Figure 5: Simulation- Zinc's percentage versus years of production

Saga estimated the OPEX by volume of production, not by concentration mineral, corresponding, at the end, of 21% of CAPEX. Therefore, in that way, we can oscillate the concentration without affect in the costs or investments (Appendix A).

3.4 Investment and discounting rate

Saga consulting estimated $ 70 million of CAPEX. They collected market prices of Mining Equipment, considering this amount is equal to 25% of total of investment. Part of this expenditure is to maintenance of mine processing (Appendix B), as the underground mine, plant and infrastructure.

The exploration investment (bidding and research) is out of this amount. For the discounting rate, Saga use 10%. Maybe this is a low discount rate for a project like that. The risk-adjusted discount rate can be determined from CAPM model, as the Eq.(5):

$$\mu = r_f + \beta(E[R_m] - r_f) + r_p \quad (5)$$

On the Damoradam website, we obtained the country premium risk ($r_p = 4.17\%$) and beta for the mining sector (β = 1.42). On quandl-FRED we obtained 10-Year Treasury Constant Maturity Rate and calculated the mean of the year 2019 ($r_f = 2.17\%$). For expected return market, we use S&P Metal and Mining Index (XME index), downloaded on yahoo finance. The time series is in the Fig.(6) and we calculated an average of annual return since 2015 ($E[R_m]$ = 8.41%). The result of the expected return (μ) is 15.21%.

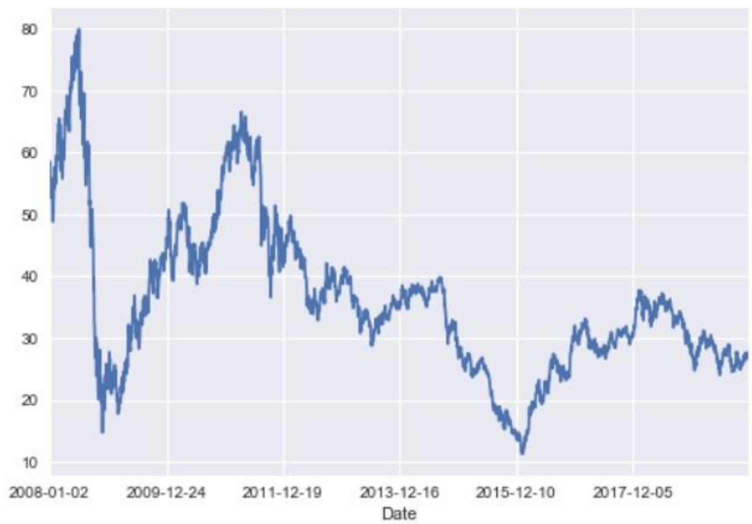

Figure 6: XME: yahoo Finance

3.5 Cash Flow and firm's initial value

Comparing to Saga's cash flow model, we turn the exploration phase exclusive to research and drilling (Appendix A), without the underground cave, plant and infrastructure investments. That is similar to reality of industry, which postpone the maximum as possible the sunk cost. It is a natural strategy to use all options during the exploration to reduce the uncertainty, related with geology and market. That is the reason why our model has one more year before the exploitation: six months to analyze CPRM information, two year of exploration and six months to conclude the transfer of concession. Besides that, for Saga, the price is constant along the production. For base Cash Flow we used the average of Mean Reverse Motion of Schwartz (1997), as we can see in the Eq.(2 and 3) and not included the investment.

Using a CAPM's results (μ = 15.21%) as the discounting rate, the VP_{0base} is \$ 123 million and at the auction's moment (*bid*- VP_{-3base}) is \$ 80 million. For the investment, using the same discounting rate, the I_0 is \$ 54.78 million. Then, the Discounted Cash Flow (DCF_{-3}), i.e bypass the price and technical uncertainties, and also without exploration cost, has the net of \$ 44 million at the auction's moment.

3.6 Market risk premium estimation

The project's volatility depends in which phase the investor is. The method adopted is the one proposed by Brandão, Dyer, and Hahn (2012). But before estimate the volatility of the project, it is necessary to calculate the risk premium, using the simulation to check. For the risk market premium, the geological uncertainty is irrelevant and the correlation with price uncertainty is zero (Cortazar et al., 2003). Therefore, the Monte Carlo simulation has only price's uncertainty. With CAPM model Eq.(5), the VP_0 average is equal to \$123 million, like the determinism model Fig.(7). The minimum is \$59 million and the maximum is \$229 million. Consequently, without the geological uncertainty and exploration's cost, the probability than Present Value (Operation Cash Flow) be negative is 0% and 0.01% to be less than the investment (\$ 54 million). Furthermore, is possible tocheck CAPM model, with distribution of the return ($\nu = ln(\widetilde{VP}_1/VP_0)$). The result is 14.48% ($=\overline{\nu} + \sigma_\nu^2/2$), similar to CAPM ($\mu$).

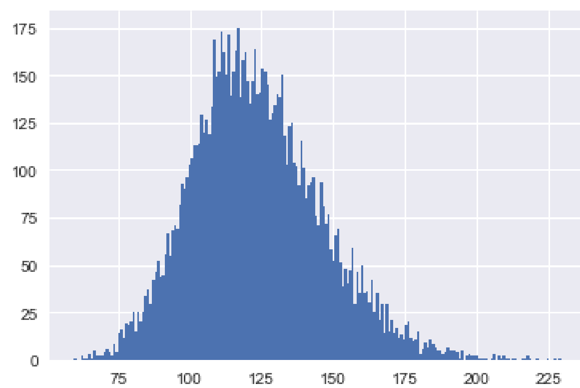

Figure 7: Monte Carlo Simulation- Price Uncertainty - $\widetilde{VP}_0$

To find the risk premium of each commodity (λ_{zn} and λ_{cu}), we apply the Freitas and Brandão (2010) methodology. It is consist of minimum difference of two deterministic (base case) cash flow, one with discount by CAPM model (μ) and other by risk-free rate ($rp + rf$). Nevertheless, our model has two outputs (λ_{zn} and λ_{cu}), having the possibility to hit a local minimum. To avoid that, it is necessary assume that the relations of the returns of the commodities is the same than the risk-premium of each commodity. A plot of returns per month between 2004 and 2009 shows that the correlation is 0.72($\rho_{zn,cu}$) and the returns of zinc correspond to 76% of returns of cooper ($return_{zn} = 0.76 return_{cu}$). With this control, the discount in $\alpha_{zn,cu}$ for minimize the function are 0.168 and 0.167. To find the premium is necessary to multiply by $\kappa_{zn,cu}$ (Schwartz & Smith, 2000), arriving to λ_{zn}= 7.20% and λ_{cu}= 9.46%. Assuming that all premium risk are from these uncertainties (λ_{CAPM} = 8.87%), the portfolio would have 26.17% (w_{zn}) of λ_{zn} and 73.83%(w_{cu}) of λ_{cu} To double check the estimation, we run the model with α^* and risk-free premium ($rf + rp$), resulting the average simulation (=\$ 122.5 million) around the simulation with CAPM (=\$ 123 million).

3.7 Project volatility during exploration phase

The first phase (analyze of CPRM data) has a higher geological uncertainty because it is reapplies the variance of the samples' rocks and also is driving by prices stochastic processes Fig.(8). This result looks realistic, which is the phase with higher risk for the investor. On the other hand, is which the investor pay less for continue playing the game. Comparing with a model without a technical risk (section 3.6), now the probability for the present value to be less than 0 is 10.95% and to be less than investment (\$ 54 million) is 35.53%.

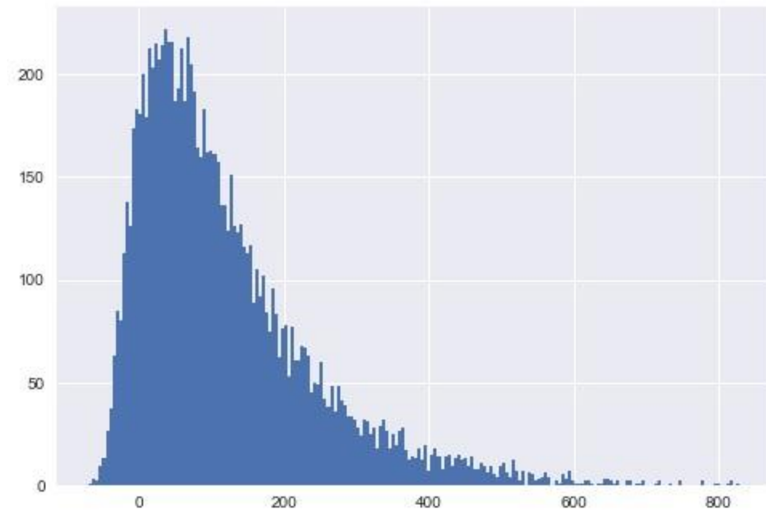

Figure 8: Monte Carlo Simulation- Phase 1 - $\widetilde{VP}_0$

The return can be calculated with the log of simulation in date 1 ($\widetilde{VP}_1$) over Initial Value (VP_{0base}). It turns nonexistent values because the simulation returns negative results. Ignoring those non values, arrives a spread returns and asymmetric histogram Fig.(9), with 114.31% of standard deviation (σ).

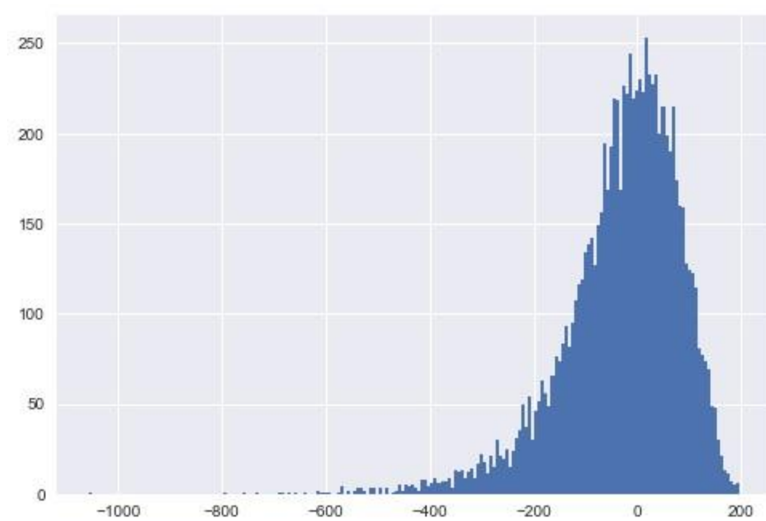

Figure 9: Monte Carlo Simulation- Phase 1 - $\widetilde{Return}_0$

The initial exploration phase (Exploration-1) begins six month after the bid, thus the prices should be an expected value until this phase Eq.(2 and 3) and after a stochastic process simulation Eq.(4). The variance of ore's grade is going to reduce 40% in this Monte Carlo simulation. This happens because the owner invests in knowledge doing research and drilling, but also, together with second Exploration phase, are the most expensive for the investor. The return is present in Fig.(10), with project's volatility reduce for 102.61%.

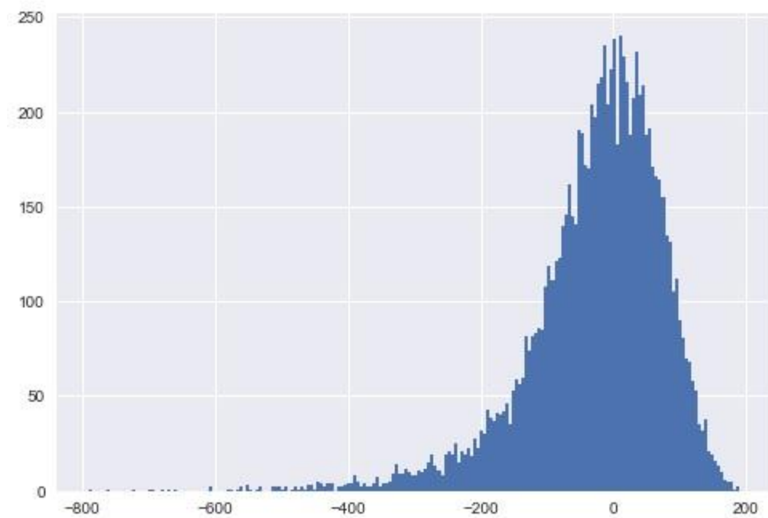

Figure 10: Monte Carlo Simulation - Exploration 1 - $\widetilde{Return}_0$

The third phase (Exploration-2) starts one year later the bid, so until this date needs to project the expected price of copper and zinc and for the following years, a stochastic process. The variance of ore's grade reduce twice of 40% over the last variance, with a total reduction of 78.4% of the first model. The return can be seen in the Fig.(11). Comparing with the previous returns, it turns the shape of distribution less asymmetric. The volatility of the project reduce to 67.72% in the Exploration-2.

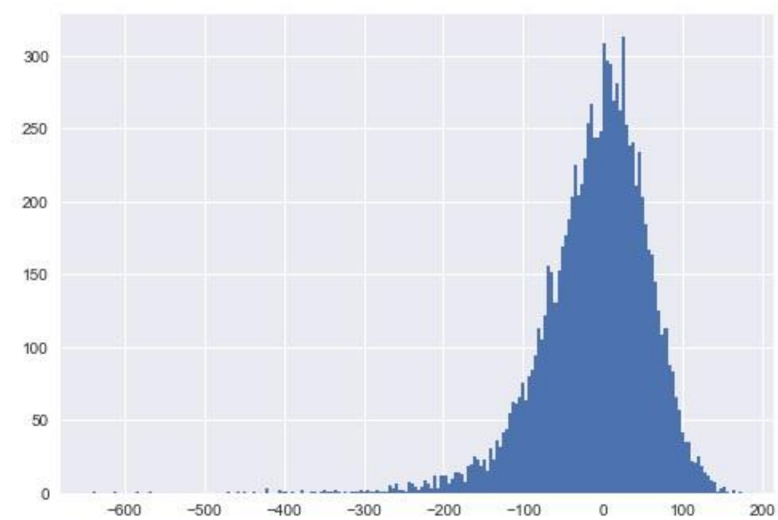

Figure 11: Monte Carlo Simulation - Exploration 2 - $\widetilde{Return}_0$

The last phase before the deposit's exploitation, the investor has no or less doubts about geology and the deposits, which the project's volatility is drive only by the prices changes. For that, we assume that the ore's grade initial expectation was correct. As you can see in the Fig.(12), the returns has a normal shape distribution, without the asymmetric negative tail in the previous phases. The volatility in the Cession phase drops to 10.84%.

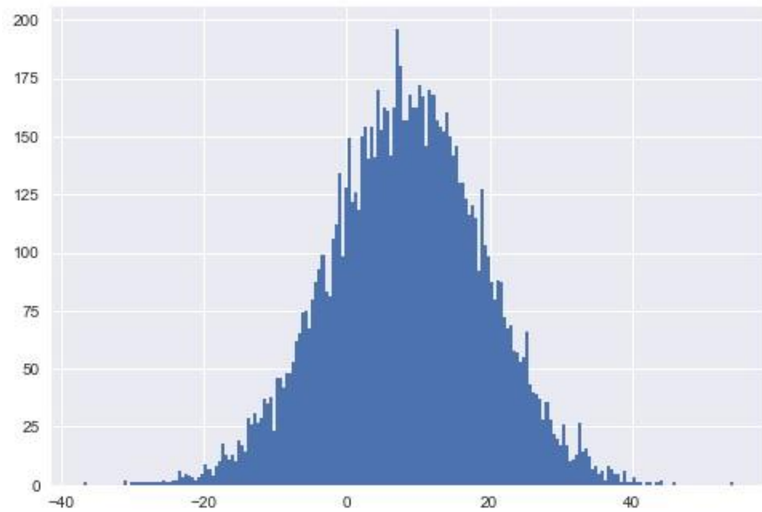

Figure 12: Monte Carlo Simulation - Cession Phase - $\widetilde{Return}_0$

The Table (4) summarizes the Monte Carlo simulations. Note that σ decreases throughout the exploratory phases. It is also noted that the value of μ without exploratory risk (Cession phase) is close to the risk-free rate, as we assume a risk-neutral process. It is not the same because there is a difference in time, and we apply expected price value until the cession phase, getting a different starting point than the simulation of section (3.6).

Before Exploitation	$\alpha(\%)$	$\sigma(\%)$	$\mu(\%)$
Phase-1	-30.43	114.31	34.91
Exploration-1	-26.29	102.61	26.35
Exploration-2	-9.62	67.72	13.30
Cession	8.31	10.84	8.90

Table 4: Returns in each exploration phase

4 Auction: Explorations with options

The phases described in Fig.(2) indicate several possibilities for investor abandonment during the exploration, i.e it is possible to model with an American's put option. One way to look at this problem is to consider that at each stage (or gate) the investor has the right to buy an investment opportunity to continue at the game (buy n call options in chain). However, it is not any time that the investor can buy theses tickets, only at the moment of changing phases. With that point of view, it turns the problem much easier, converting to a sequence of European call's options.

4.1 Underlying asset

Option modeling was done by the binomial method of (Cox et al., 1979). The software to model the binomial tree is DPL^{TM}. The discount rate is 15.21%, the free risk rate is 6.34%, the Project at auction moment ($VP_{-3}base$) is \$ 80 million and the Investment at the end of exploration (I_0) is \$54 million. The up factor is calculated as ($u = e^{\sigma\, sqrt(\Delta t)}$) and the down factor is ($d = \frac{1}{u}$). Risk neutral probability is given by the following equation:

$$p = \frac{(1+r)^{\Delta t} - d}{u - d}$$

For each phase, there is a volatility, as well as an up and down measure and a risk neutral probability. The Δt is the same (=0.5 year) for all probabilistic state. The underlying asset is the project value (=\$80 million), without options and sunk costs Fig.(13-a and b).

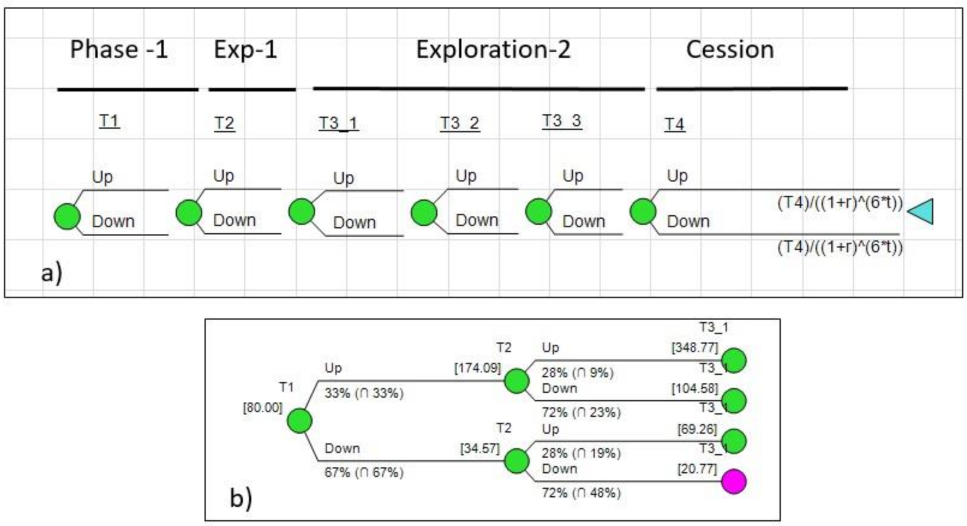

Figure 13: Underlying Asset: a) model and b) result

If we consider the investment in the mine exploitation and no options Fig. (14-a), the value is going to be the same as the determinism model (DFC_{-3}=\$44 million) Fig.(14-b).

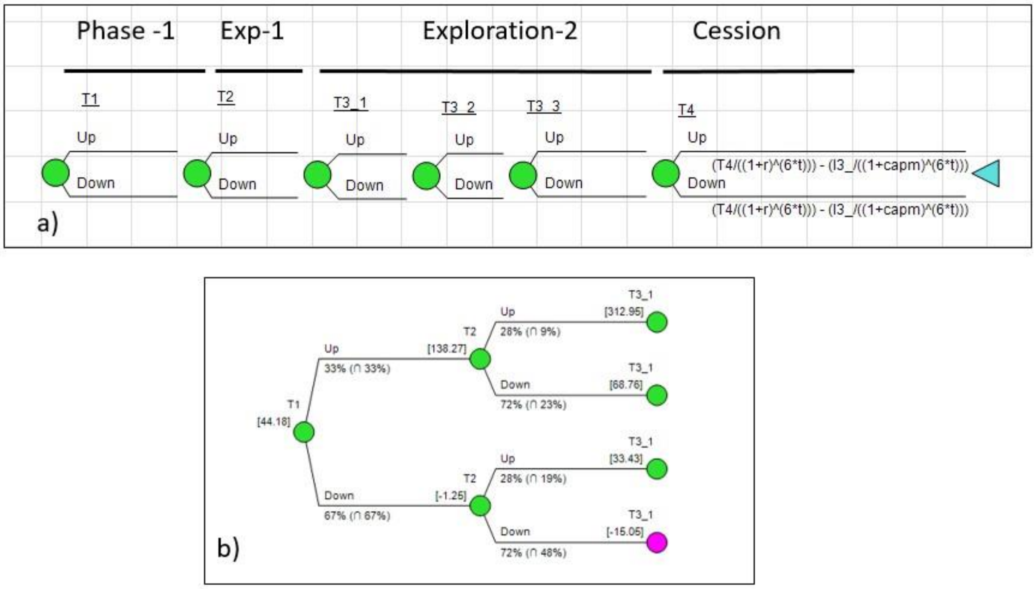

Figure 14: Underlying Asset with exploitation investment: a) model and b) result

Adding the explorations investments in each phase ($2.4 and 3.6 million) Fig.(15-a), we arrive in a net of $38.47 million Fig.(15-b). That situation is the reference for obtain the ticket's value offered by the auctioneer, ie the right but not the obligation to invest in Palmeirópolis' deposit.

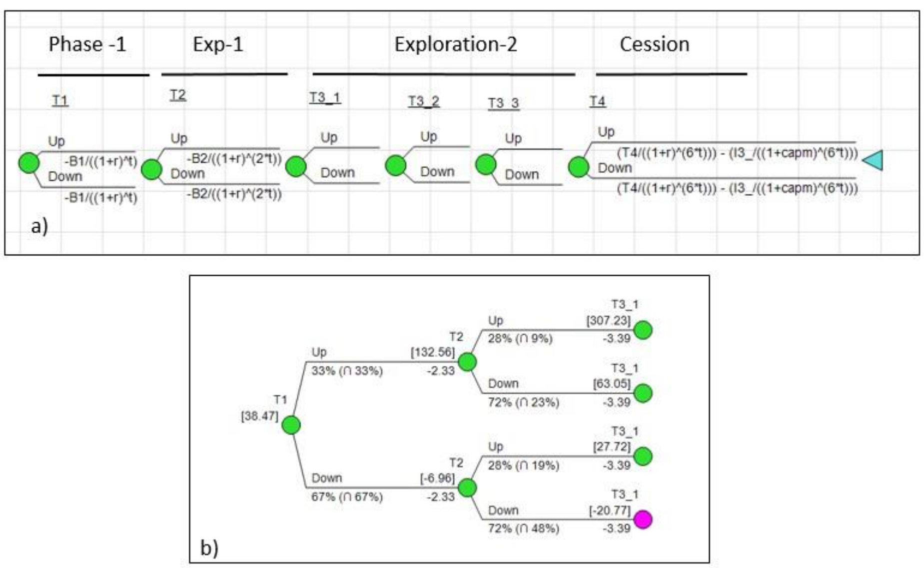

Figure 15: Underlying Asset with exploitation and exploration investments: a) model and b) result

4.2 Auction and options: Calls' Sequence

In this case, to model the right of explore and exploit the area can be done by series of call's options Fig.(16-a). At each gate, there is a decision to continue or leave. If continue, the bidder needs to pay the obligations. If it is advantage to leave, the investor has no cost, only at the last stage, which needs to pay the last part of the bonus bid. The value increase for the both situation without options, arriving in $46.86 million Fig.(16-b).

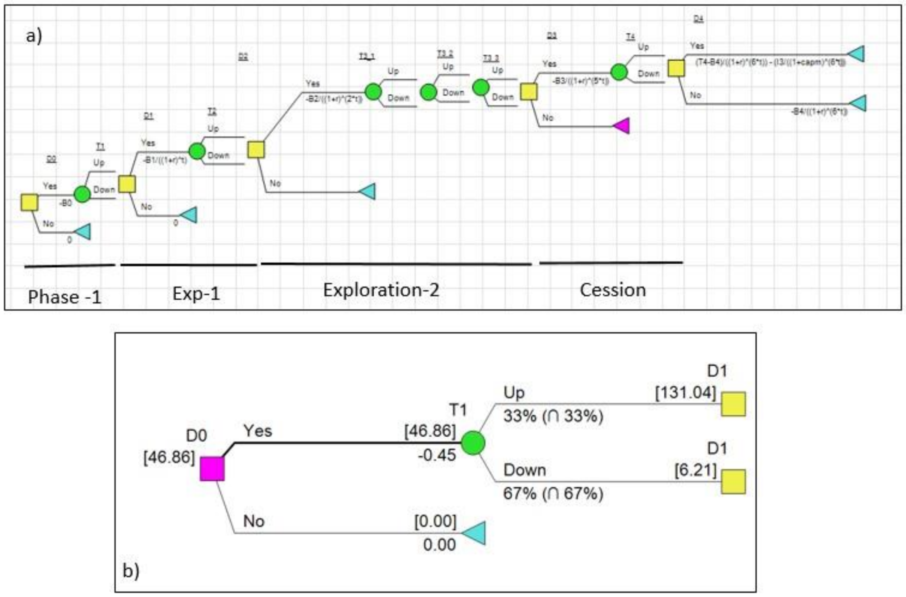

Figure 16: Project with Options: a) model and b) result

If we remove the bid bonus, the result is $48.8 million Fig.(17). With this final model, we can compare with the situation with the obligation to invest (=$ 38.47 million). That shows that the expected value of ticket offers by the auctioneer, in the present value, was $ 10.33 million (=48.80-38.47 million) and the auctioneer just grab $ 1.94 million of it (=48.80-46.86 million).

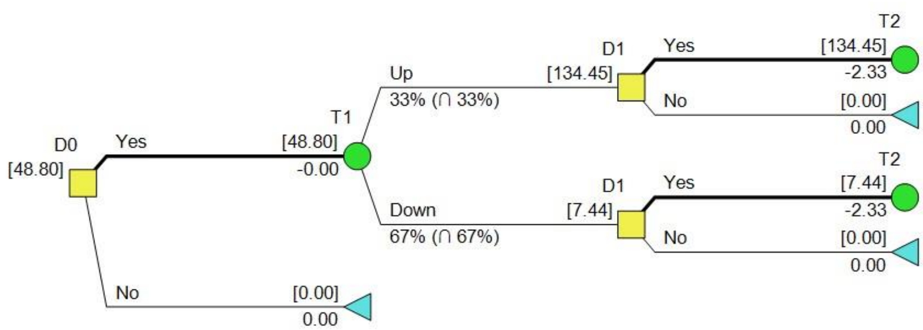

Figure 17: Project with Options less the bid bonus

This may seem unlikely as the total of bid value was $4.45 million and the premium for the auctioneer was $ 1.94 million. However, it is important to note that the investor can abandon at any gate, and we are working under the expectation of investing given the double uncertainty: price and geological. In fact, the value can vary from $0.45 to 4.5 million, depending on the volatility parameter.

4.3 Management decision: effect of technical uncertainty in exploratory projects

The study from the perspective of real options makes the manager reflect about uncertainty. Usually, the attention is over the market uncertainty. However, in this case technical uncertainties affect more the underlying asset than the commodity price. That can be noted by the variation effect over σ *in* Table (4). Moreover, the real options theory grabs the idea, which observing these uncertainties, the owner can changes actions during the business. Thus, the final value changes considerable, as noted in section (4.2).

Table (5) summarize the increase value that options analyses can present.

Asset	$ million	%
Base Case (no options)	38.47	-
With Options (+bid)	46.86	21.81
With Options (-bid)	48.80	26.85
Options	10.33	-
$\mathbb{E}$[Revenue] (auctioneer)	1.94	18.78

Table 5: Valuation summary ($ million)

5 Conclusions

Real Option Analysis offers powerful tools for analyzing investments, including auction situations. Technical uncertainty is usually considered in exploratory investments, but the change over time and the flexibility associated with these uncertainties are not so frequent. This work adds to the literature in the field by combining project volatility in the exploratory stages given the price and geological uncertainty and the consequences for the amount to be paid at the auction.

Firstly, cooper and zinc spot prices, between years 2004-2009 cannot be modeled by GBM. A MRM model is more compatible, widely used in commodity price, due to competition from suppliers and balance of supply and demand. Besides that, geological uncertainty changes throughout exploration. Therefore, it considered that the sample distribution of rocks ores' grade collected by CPRM represent an initial variation for the deposit models. The models considered that the variation would decrease over the course of exploration, but the average does not change. Considering these two uncertainties, it is noted that the project is significantly affected by the technical uncertainty and also that the volatility decreases throughout the exploratory stages.

Finally, considering that the auction is an option offered by the auctioneer, the project with obligation to explore and exploit has a value of $ 38.47 million and with the options rises to $ 48.80 million,i.e the right to explore worth $ 10.33 million or 26.85% of the base case. Given that the bonus is public information, the project with the obligations is $ 46.86 million, i.e the government only grabbed $1.94 million, or 18.78 % of the expected premium. The answer would be associated with few competitors in this auction.

Datasets and model

Data processing, cash flows, uncertainty simulation, parameters and volatility estimations were done with Python software. The Real Options model was solved with DPL^{TM} software.

References

Araujo, S. M., Fawcett, J. J., & Scott, S. D. (1995). Metamorphism of hydrothermally altered rocks in a volcanogenic massive sulfide deposit: the Palmeirópolis, Brazil, example. *Revista Brasileira de Geociencias*, *25*(3), 173-184.

Bastian-Pinto, C. d. L., Brandão, L. E. T., & Ozório, L. d. M. (2016). Valuing flexibility in electro-intensive industries: the case of an aluminum smelter. *Production, 26*(1), 28-38. doi: http://dx.doi.org/10.1590/0103-6513.174214

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy, 81*(3), 637-654. doi: https://doi.org/10.1086/260062

Brandão, L. E., Dyer, J. S., & Hahn, W. J. (2005). Using binomial decision trees to solve real-option valuation problems. *Decision Analysis, 2*(2), 69-88. doi: https://doi.org/10.1287/deca.1050.0040

Brandão, L. E., Dyer, J. S., & Hahn, W. J. (2012). Volatility estimation for stochastic project value models. *European Journal of Operational Research*, *220*(3), 642-648. doi: https://doi.org/10.1016/j.ejor.2012.01.059

Brennan, M. J., & Schwartz, E. S. (1985). Evaluating Natural Resource Investments. *Journal of Business*, *58*(2), 135-157.

Copeland, T., & Antikarov, V. (2001). *Real Options: A Practitioner's Guide*. Texere, New York.

Cortazar, G., & Casassus, J. (1998). Optimal timing of a mine expansion: implementing a real options model. *Quarterly Review of Economics and Finance*, *38*, 755-769. doi: https://doi.org/10.1016/S1062-9769(99)80100-8

Cortazar, G., Schwartz, E., & Casassus, J. (2003). Optimal exploration investments under price and geological-technical uncertainty: a real options model. In D. Paxson (Ed.), *Real R&D Options* (pp. 149–165). Burlington: Butterworth-Heinemann.

Cortazar, G., Schwartz, E. S., & Salinas, M. (1998). Evaluating Environmental Investments: A Real Options Approach. *Management Science*, *44*(8), 1059-1070. doi: 10.1287/mnsc.44.8.1059

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, *7*(3), 229-263. doi: https://doi.org/10.1016/0304-405X(79)90015-1

CPRM. (2019a). Ministro Bento Albuquerque anuncia edital de cessão de direitos minerários do Complexo Polimetálico de Palmeirópolis (TO). from https://www.cprm.gov.br/publique/Noticias/Ministro-Bento-Albuquerque-anuncia-edital-de-cessao-de-direitos-minerarios-do-Complexo-Polimetalico-de-Palmeiropolis-%28TO%29-5785.html

CPRM. (2019b). *Palmeiropolis Auction nº 001/2019-CPRM. Tender notice and data.* Retrieved from http://www.cprm.gov.br/publique/Acesso-a-Informacao/Leilao-Palmeiropolis%3A-Edital--n%BA-001%7C2019-CPRM-5567.html.

Dias, M. A. G. (2004). *Real options, learning measures, and bernoulli revelation processes.* Paper presented at the 8th Annaul International Conference on Real Options, Rio de Janeiro.

Dixit, A. K., & Pindyck, R. S. (1994). *Investment under Uncertainty*. Princeton: Princeton University Press.

Freitas, A., & Brandão, L. (2010). Real Options Valuation of e-learning Projects. *International Journal on E-Learning, 9*(3), 363-383.

Grenadier, S. R. (2015). Information Revelation Through Option Exercise. *The Review of Financial Studies, 12*(1), 95-129. doi: 10.1093/rfs/12.1.95

Guj, P., & Chandra, A. (2019). Comparing different real option valuation approaches as applied to a copper mine. *Resources Policy, 61*, 180-189. doi: https://doi.org/10.1016/j.resourpol.2019.01.020

Juliani, C., Monteiro, L., & Fernandes, C. M. D. (2016). Potencial mineral: cobre. *Recursos Minerais do Brasil: problemas e desafios*, 134-153.

Merton, R. C. (1973). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science, 4*(1), 141-183. doi: https://dx.doi.org/10.2307/3003143

Miranda, O., Brandão, L. E., & Lazo Lazo, J. (2017). A dynamic model for valuing flexible mining exploration projects under uncertainty. *Resources Policy, 52*, 393-404. doi: https://doi.org/10.1016/j.resourpol.2017.04.002

Rudolph, K. W., & Goulding, F. J. (2017). Benchmarking exploration predictions and performance using 20+ yr of drilling results: One company's experience. *AAPG Bulletin, 101*(2), 161-176. doi: 10.1306/06281616060

Schwartz, E., & Smith, J. E. (2000). Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science, 46*(7), 893-911. doi: 10.1287/mnsc.46.7.893.12034

Schwartz, E. S. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *Journal of Finance, 52*(3), 923-973. doi: 10.1111/j.1540-6261.1997.tb02721.x

Slade, M. E. (2001). Valuing managerial flexibility: An application of real-option theory to mining investments. *Journal of Environmental Economics and Management, 41*(2), 193-233. doi: 10.1006/jeem.2000.1139

Tourinho, O. A. F. (1979). *The valuation of reserves of natural resources: an option pricing approach.* (Doctoral PhD Dissertation), University of California, Berkeley.

Trigeorgis, L. (1990). A real options application in natural resource investments *Advances in futures and options research* (Vol. 4, pp. 153-164).

Trigeorgis, L. (1995). *Real options in capital investment : models, strategies, and applications*. Westport, Conn.: Praeger.

Appendix A

Year/Description	1	2	3	4	5	6	7	8	9	10
Zn Price($/ton)	2482.48	2479.82	2477.23	2475.18	2473.69	2472.65	2471.95	2471.48	2471.17	2470.97
Cu Price ($/ton)	6611.19	6688.14	6731.25	6755.53	6769.24	6777.00	6781.39	6783.89	6785.30	6786.10
Production (ton)	300,000	400,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
E[Zn]-grade (%)	6.95	8.61	6.93	2.42	5.55	6.00	1.43	1.37	1.45	1.38
E[Cu]-grade (%)	1.44	1.45	0.02	1.43	1.36	1.35	0.15	0.12	0.12	0.16
Operational Cost	12,690.91	16,921.21	21,151.51	21,151.51	21,151.51	21,151.51	21,151.51	21,151.51	21,151.51	21,151.51
Depreciation	4,727.27	4,727.27	4,727.27	4,727.27	4,727.27	4,727.27	4,727.27	4,727.27	4,727.27	4,727.27
Net Revenue	50,475.80	78,185.96	95,915.75	48,653.76	71,932.02	75,285.34	14,380.14	13,283.71	13,913.02	14,191.65
Tax	11,239.59	19,222.74	23,812.56	7,743.49	1,5658.10	1,6798.22	-3,909.54	-4,282.37	-4,068.36	-3,973.62
Operational Cash Flow	**25,535.78**	**40,478.28**	**49,033.35**	**18,785.68**	**33,683.76**	**35,829.89**	**-3,149.44**	**-3,851.15**	**-3,448.39**	**-3,270.06**

Table 5: Cash Flow - Thousand of dollars

Appendix B

Year	0	1	2	3	4	5	6	7	8	9	10
Investment	18,787.88	30,000.00	-	-	10,606.06	-	-	10,606.06	-	-	-

Table 6: Investment - Thousand of dollars