Project Financing vs. Corporate Integration: Decision-Making under Operational and Financial Risks

Maximilian Schreiter[a,*]

[a]HHL Leipzig Graduate School of Management, Jahnallee 59, 04109 Leipzig, Germany

Abstract

The article addresses the choice between setting up a project entity and integrating the project into a corporation from the perspective of a dynamic corporate finance model in the absence of operational synergies. The analysis builds upon a classic continuous-time framework with tax benefits of debt, costs of default and, more innovatively, a benefit or cost of abandonment derived for two projects with stochastic, correlated revenues. By implementing the model with a simulation-based approach, I show that foremost high correlations, high volatilities and high portions of fixed costs as well as heterogenous volatilities, bankruptcy costs and operational cost structures stimulate independent project structures. Further, a more difficult or costly access to external funds raises the benefits of merging activities in a combined firm.

Keywords: Project Financing, Financial Synergies, Default Risk, Abandonment Options, Optimal Capital Structure, Real Options
JEL classification: G12, G31, G32, G33

* Corresponding author. Email: maximilian.schreiter@hhl.de.

January 15, 2020

1. Introduction

Investors and firms are confronted with numerous choices when designing and adjusting their financial structures. The dimension mostly discussed in practice and regularly researched since the famous paper by Modigliani and Miller (1958) is the optimal mix of equity and debt. Further important choices to be made concern the type of debt securities, bank loans versus market bonds, the maturity of debt or the redemption schedules of debt, to name just a few. Beyond those dimensions, investors or firms can also choose to either finance a new project within the existing corporation (corporate finance) or to set up a separate entity, e.g., special purpose entity (SPE), which operates and finances the project legally independent with injected, nonrecourse capital (project finance - PF).[1] Picking between these two options is more than an exotic niche challenge as we observe a total debt volume of USD 328 billion in project loans and bonds for 2018 (Project Finance International, 2019), which compares to a total of approx. USD 1,300 billion of debt raised by corporates other than financial institutions (Societe Generale, 2018). The resulting share of 25 percent may even increase to more than 50 percent when investments greater USD 500 million are considered (Etsy and Christov, 2002).[2]

Thus, the question that arises is: What determines the choice between setting up a project entity and integrating the project into the corporation? This paper tackles the choice from a pure structured finance perspective by applying a stochastic model in the spirit of Goldstein, Ju, and Leland (2001) where benefits and drawbacks of bundling versus non-bundling of projects are compared under operational and financial risks. I intentionally exclude any operational synergies from the analysis[3] and focus on financial synergies only in order to identify motives for PF.[4]

Admittedly, the underlying question is not novel and has been raised by researchers before. Lewellen (1971) identifies a coinsurance effect of merging activities, i.e., a combined firm reduces the risk of default, is able to impose more debt and, thus, generates higher tax savings. A whole series of literature confirms Lewellen's coinsurance effect and rather examines the distribution of value created by combining firms between debtholders and equity investors (see, e.g., Higgins and Schall, 1975; Kim and McConnell, 1977; Scott, 1977; Shastri, 1990). None of these papers consider the risk of default explicitly and instead provide examples with strictly positive cash flow projections. However, Scott (1977) and Sarig (1985) are first in suggesting that the possibility of negative cash flows may generate a negative merger effect.

Leland (2007) formalizes this issue as "LL effect" - A value destruction resulting from the loss of separate limited liabilities. He presents a formal two-period model with explicit capital structure optimizations for both choices, the sum of the two separate projects and the combined firm. His model illustrates a trade-off between the two choices and provides implications regarding structural preferences under different setups. According to Leland (2007), the net effect of combining activities is more likely to be positive for low correlations, low volatilities and rather similar volatilities. Moreover, similar and rather high costs of default also impact financial synergies positively. Consequently, separate project structures are favourable whenever volatilities or default costs differ significantly. By testing various parameter settings, he is also able to estimate the relative size of financial synergies, often a low one digit percentage of the project value, and by doing so he identifies clear rationales for empirical observations. Most importantly, he shows that for relatively large, high-risk projects the benefits of separate financing

1 An overview of more formal text book definitions is provided by Etsy (2014).

2 PF is not a novel phenomenon but roots back to the Middle Ages (see, e.g., Kensinger and Martin, 1988). Modern PF emerged in the 1980s with the primary goal of financing large US energy projects Müllner (2017) and grew significantly between 1994 (USD 17.7 billion in project loans and bonds) and 2008 (USD 262.5 billion) (Etsy, 2014). After the financial crisis, the sector recovered to pre-crisis level in 2013.

3 The sum of the projects' operational cash flows equals the operational cash flow of the combined firm (see, e.g., Leland, 2007; Banal-Estanol et al., 2013, for equivalent setups).

4 Flannery et al. (1993), John and John (1991), John (1993) and Chemmanur and John (1996) analyze how merging activities alter operational cash flows, match with managerial abilities and create control issues.

can exceed ten percent - a strong argument for the frequent observation of PF in such settings.

The paper of Banal-Estanol, Ottaviani, and Winton (2013) presents some insightful extensions to Leland (2007). In accordance with Leland (2007), they develop a two-period (one time-step) model but consider pure debt investments only. Default emerges whenever the return at maturity is not sufficient to cover the investment outlay. By implementing pure debt investments, Banal-Estanol et al. (2013) reveal setups where separate financing is advantageous although the leverage itself is similar. Additionally, they show that the tradeoff of coinsurance and risk-contamination, a term they define for adverse effects of merging activities, persists for empirically observed return distributions aside the normal distribution assumed by Leland (2007). Based upon this analysis, they uncover that increasing average returns favor the bundling of projects while an increase in the negative skewness of returns provokes the opposite. Beyond that, results of Leland (2007) concerning the impact of correlation, volatility and default costs are widely confirmed.

The model developed in this article extends the work of Leland (2007) and Banal-Estanol et al. (2013). I switch from a two-period (one time-step) model to infinite time. Thus, I can study the dynamics of the two correlated projects in a more general way, and can consider more realistic default conditions over time. In particular, I allow for default triggered by illiquidity (or by the breach of a covenant) and by over-indebtedness in a constant barrier setting. Both triggers are common in capital structure literature (see Strebulaev and Whited, 2011, for a detailed overview and discussion), but have not been applied to the topic of financial synergies. The first of the two triggers is referred to as "exogenous" because it cannot be set freely by investors, while the other is "endogenous" as equity investors can determine at what (optimal) point they file for bankruptcy. Exogenous default implies that no additional capital can be induced although it might be economically meaningful, while endogenous default sets the assumption that investors have "deep pockets" and can inject capital as long as it maximizes their economic value. Leland (2007) and Banal-Estanol et al. (2013) implement exogenous default triggers only. Furthermore, my stochastic underlying follows a geometric Brownian motion (gBm) but is not a version of firm or project value (see, e.g., the classic capital structure model of Leland, 1994). Instead, I assume revenues to follow a gBm resulting in stochastic EBITs, equivalently to Goldstein et al. (2001). Motives to favor revenues over EBITs when creating a stochastic structural model lie in the richer set of potential analyses, namely the reflection of cost structures and abandonment options, and in the more consistent fit with empirical data.[5]

Implementing the extended model allows to draw numerous more general conclusions: First, with unlimited capital access there is no positive net effect of bundling regarding the option to abandon. As long as investors can inject capital without additional agency costs, no financial argument for merging activities exists under full-equity financing. Second, turning towards partial-debt financing the net effects of bundling regarding the net benefits of debt become positive for both settings of capital access with decreasing correlation. Third, the magnitude of net effects of bundling is higher under fully restricted capital access. These results indicate that merging activities in a combined firm is more beneficial if the access to external funds is difficult. In consequence, private firms and investors operating in less developed capital markets may rather tend to bundle activities in conglomerate-like structures while it is expected that we find rather separated project structures with public firms and in very well developed capital markets.

Regarding specific parameter settings, the analysis reveals that heterogeneity with respect to cost structures, growth, risk and bankruptcy costs lower the financial net effects (net benefits of debt) of bundling irrespective of the underlying capital access regime. While Leland (2007) does not consider cost structures and growth, my results for risk and bankruptcy costs are in accordance to his work and would economically point towards mergers of rather similar partners and towards the use of corporate

[5] See Section 2 for a more comprehensive discussion.

financing for projects similar to the existing business.

Moreover, I find with respect to the level of parameter values that high volatility of cash flows, and a high portion of fixed costs within the cost structure support a project structure with independent legal entities as the likelihood of risk contamination increases in such settings. In the exogenous model, I identify a similar relationship for the revenue's expected drift rate, and for bankruptcy costs. However, the impact of these two flips in the endogenous model as equity investors alter their abandonment and bankruptcy triggers accordingly. Economically, I provide strong indication for the separation of highly volatile activities (similar to Leland (2007) and Banal-Estanol et al. (2013)) but also for separating projects with a jointly high level of fixed costs.

While not a core topic of this article, looking at separate firms or projects and including the option (endogenous model) or obligation (exogenous model) of abandonment triggers an additional insight: For all cost structures with a high fixed costs proportion, abandonment is a highly relevant value component and shall not be suppressed, neither in dynamic models of capital structure nor in corporate valuation in general.

Beyond the literature strand presented so far, there exists a literature strand on the advantages of PF due to reduced agency conflicts. Pinto and Alves (2016) and Müllner (2017) provide comprehensive summaries of the benefits found. In short, the following additional findings can be concluded: (i) PF reduces costly agency conflicts among capital providers as all of them enter the investment at the same point in time (Brealey et al., 1996; Etsy, 2004). (ii) Companies with high leverage avoid the debt overhang problem by preventing opportunity costs of underinvestment (Fabozzi et al., 2006; Gatti, 2012). (iii) Due to its rather isolated nature, PF is less affected by asymmetric information and by such reduces the problem of underinvestment (see, e.g., Nevitt and Fabozzi, 2000). This article does not treat the issue of agency conflicts explicitly. However, by developing an exogenous model, where no additional capital can be injected, and an endogenous model, where unlimited funds can be added as long as it is value-generating, I consider the two extreme cases of agency conflicts implicitly. In the endogenous model, we face a complete market with symmetric information where raising money is always possible at a net present value of zero. In the exogenous model, the firm or investors in the firm cannot raise additional capital but need(s) to rely on internal cash flows only - the most extreme scenario that can emerge from asymmetric information and the resulting agency conflicts.

The remainder of the article is organized as follows: Section 2 introduces the basic setup including revenue and expense structure of firm and project as well as the underlying stochastic properties. In Section 3, I derive the managerial choice to abandon either of the two separate activities irrespective of financing while Section 4 presents the consequences of debt issuance reflecting tax shield and bankruptcy costs for firm and project separately. The simulation approach required to implement the model for bundled activities is developed in Section 5. In Section 6, I compare the outcomes of bundling and separating activities based on a numerical application of the model. Findings are summarized in formal conjectures before Section 7 concludes the article.

2. Basic setup

I commence by following the assumptions of a standard contingent claims framework in the spirit of Goldstein et al. (2001), where the capital market is arbitrage-free and complete with respect to all N firms operating in the economy. Each individual firm operates like an EBIT-generating machine, where its value stems from its entitlement to an uncertain stream of cash flows generated by this machine.[6]

[6] There are no assets in place, and the corporation has no excess cash at hand.

Key Assumption 2.1 (EBIT decomposition)**.** *Instead of considering the EBIT $\tilde{X}$ as the stochastic underlying, I decompose $\tilde{X}$ to*

$$\tilde{X} = \tilde{R}(1 - \gamma) - F, \tag{2.1}$$

with revenue $\tilde{R}$ representing the stochastic underlying, γ being a constant variable costs portion of $\tilde{R}$, and F reflecting constant fixed costs.

The benefits of this extension are threefold: First, such a setup allows to analyze the impact of cost structures, i.e., fixed versus variable expenses, on the capital structure in general and on the choice of PF versus merging activities in particular. Second, the existence of fixed expenses results in the creation of an abandonment option or obligation, adding a further realistic managerial choice to the model. That is, it might be optimal to abandon operations even under full equity-finance. Third, the stochastic underlying $\tilde{R}$ is (almost) always positive and normally distributed log-changes of revenues can be regularly observed empirically while log-changes of EBITs are usually not normally distributed (see, e.g., Kutzker and Schreiter, 2019).[7]

Key Assumption 2.2 (Additional project)**.** *Each individual firm is endowed with monopoly access to an additional infinitely lived project. The project represents another EBIT-generating machine with features congruent to the ones of the basic firm. Investors of the firm have the choice to either bundle the basic firm, indexed with $i = 1$, and the additional project, $i = 2$, in one legal entity, subscripted with f, or separate them in to independent legal entities with same ownership. For the latter, I use the subscript $\sum$ to indicate that I refer to the sum of the two legally independent units.*

Adding a second stochastic underlying allows me to investigate the key question of this paper: What determines the choice setting up a project entity, with project financing (PF) and integrating the project into the corporation, with corporate financing (CF)?

Please note that I will occasionally refer to the basic firm and the additional project as simply two separate projects in order to provide a more concise description.

With respect to the nature of uncertainty my assumptions are very standard (see, e.g., Goldstein et al., 2001; Strebulaev, 2007; Hackbarth and Mauer, 2012). Let $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\geq 0})$ be a filtered probability space, where Ω is the set of all possible events θ such that the uncertain revenues $\tilde{R}_{1,t}, \tilde{R}_{2,t} \in (0, \infty)$, $(\mathcal{F}_t)_{t\geq 0}$ is the filtration produced by the augmented σ-algebra of $(\tilde{R}_{1,t}, \tilde{R}_{2,t})_{t\geq\infty}$ and $\mathbb{P}$ is a bivariate probability measure. For any real-world probability $\mathbb{P}$ there exists a corresponding risk-neutral probability $\mathbb{Q}$. As long as investors have power utility functions, physical drifts $\mu_{\mathbb{P}i}$ can be adjusted towards risk neutral drifts μ_i by applying the relationship[8]

$$\mu_i = \mu_{\mathbb{P}i} - \lambda_i, i = 1, 2, \tag{2.2}$$

where $\lambda_i = r_{iA} - r$ is the firm specific risk premium with r_A representing the cost of capital corresponding to the underlying's risk under all equity financing. r is the (assumed) constant risk free rate of interest.

Further assume that the risk neutral revenue dynamics are governed by two correlated geometric Brownian motions with constant volatilities σ_i, such that the dynamics of the projects' revenues are given

---

7 The vast majority of capital structure models is build upon the assumption that the stochastic underlying follows a geometric Brownian motion and is, thus, log-normally distributed. Log-normally distributed variables are strictly positive, a property the EBIT is missing in reality.

8 See Goldstein et al. (2001) for an elegant proof of the relationship.

by

$$\frac{d\tilde{R}_i}{\tilde{R}_i} = \mu_i dt + \sigma_i dW_{i,t}, i = 1, 2, \tag{2.3}$$

where $W_{i,t}$ are Wiener processes under the risk neutral measure $\mathbb{Q}$. The correlation ρ of $W_{1,t}$ and $W_{2,t}$ is assumed to be constant over time,

$$\rho dt = (dW_{1,t}, dW_{2,t}). \tag{2.4}$$

In line with Modigliani and Miller (1958), future cash flows are invariant to financial policy. Further, assume a constant and deterministic corporate tax rate τ resulting in an unlevered free cash flow of

$$F\tilde{C}F_{i,t} = \left(\tilde{R}_{i,t}(1-\gamma_i) - F_i\right)(1-\tau), i = 1, 2. \tag{2.5}$$

For simplicity but without loss of generality, I do not consider other cash-relevant items like capital expenditures or change in net working capital.

If both EBIT-generating machines are combined, the merged revenue stream satisfies

$$\tilde{R}_{f,t} = \tilde{R}_{\sum,t} = \tilde{R}_{1,t} + \tilde{R}_{2,t}, \tag{2.6}$$

irrespective of the legal setup, i.e., one combined firm or two independent projects. The same holds true for the free cash flow with $F\tilde{C}F_{f,t} = F\tilde{C}F_{\sum,t} = F\tilde{C}F_{1,t} + F\tilde{C}F_{2,t}$.

Unfortunately, the dynamics of $\tilde{R}_{f,t}$ are not described by a diffusion process with constant instantaneous mean and standard deviation. By the Îto-Doeblin theorem, we find

$$\frac{d\tilde{R}_f}{\tilde{R}_f} = \left(\frac{\tilde{R}_1}{\tilde{R}_f}\right)\mu_1 dt + \left(\frac{\tilde{R}_2}{\tilde{R}_f}\right)\mu_2 dt + \left(\frac{\tilde{R}_1}{\tilde{R}_f}\right)\sigma_1 dW_{1,t} + \left(\frac{\tilde{R}_2}{\tilde{R}_f}\right)\sigma_2 dW_{2,t}, \tag{2.7}$$

where the weights $\tilde{R}_i/\tilde{R}_f$ determining mean $\tilde{\mu}_f$ and standard deviation $\tilde{\sigma}_f$ of the combined revenue are stochastic, and thus, $\tilde{\mu}_f$ and $\tilde{\sigma}_f$ are stochastic themselves.

Hence, I will not be able to extend closed form valuation formulas of the separates from subsequent chapters to the combined firm setting. However, by applying numerical methods, I am going to draw important economic implications in the form of conjectures.

3. Abandonment choices and the unlevered values of separate projects

Due to the existence of fixed costs F_i, the firm considered here faces the risk of operational losses for both of its EBIT-generating machines. Depending on whether owners of the firm are willing or able to inject fresh capital in case of operational losses, I postulate two versions of my model captured in the Key Assumptions 3.1 and 3.2.

Key Assumption 3.1 (Endogenous Abandonment Option)**.** *If the firm's equity investors are not capital-constrained*[9]*, the firm will hold the option to irreversibly abandon one or both of its projects at any future time. They choose the (endogenous) abandonment triggers B_i^* such that the operating values of the*

9 In their standard work on capital structure models, Strebulaev and Whited (2011) illustrate this condition by referring to investors with "deep pockets". Such investors are able to inject fresh money into the firm as long as it maximizes the value of their investment, i.e., the firm does not face the risk of illiquidity.

separate projects $V_{i,0}^{U}(\tilde{R}_{i,t})$ are maximized,

$$\max_{B_i \leq R_{i,0}} V_{i,0}^{U}(\tilde{R}_{i,t}). \tag{3.1}$$

Key Assumption 3.2 (Exogenous Abandonment Obligation)**.** *In sharp contrast, investors and the firm itself can be fully capital-constrained, i.e., they are not able to provide additional capital if operational losses occur. In that case, the (exogenous) abandonment triggers B_i^{ex} are reached when revenues are not sufficient to cover costs, or in continuous time right in the moment when revenues equal costs. Thus,*

$$\begin{aligned} 0 &= \left(B_i^{ex}(1-\gamma_i) - F_i\right)(1-\tau) \\ B_i^{ex} &= \frac{F_i}{1-\gamma_i}. \end{aligned} \tag{3.2}$$

For simplicity but without loss of generality, I assume that the net value of abandonment (salvage value minus closing costs) equals zero under both regimes.

While Key Assumptions 3.1 and 3.2 are regularly not featured in capital structure models (see, e.g., Goldstein et al., 2001; Hackbarth et al., 2007; Strebulaev, 2007; Hackbarth and Mauer, 2012), they are not novel. Wong (2009) examines the interplay of an (endogenous) abandonment option and fixed costs with results comparable to the subsequent ones. In contrast to my model, he adds the dimension of investment timing but does not consider multiple projects or financing choices.

I integrate the abandonment option (or obligation) into the valuation framework of $V_{i,0}^{U}(\tilde{R}_{i,t})$ as follows: Let B_i, with $B_i < R_{i,0}$, be the (endogenous or exogenous) abandonment trigger hit from above by the revenue process $\tilde{R}_{i,t}$ at the stochastic stopping time $T^{Bi} = \inf\{t \geq 0 : \tilde{R}_{i,t} = B_i\}$. $V_{i,0}^{U}(\tilde{R}_i)$ is therefore given by

$$\begin{aligned} V_{i,0}^{U}(\tilde{R}_{i,t}) =& E^{\mathbb{Q}}\left[\int_0^{T^{Bi}} e^{-rt}\left(\tilde{R}_{i,t}(1-\gamma_i) - F_i\right)(1-\tau)dt\right] \\ =& E^{\mathbb{Q}}\left[\int_0^{\infty} e^{-rt}\left(\tilde{R}_{i,t}(1-\gamma_i) - F_i\right)(1-\tau)dt\right. \\ &\left. - \int_{T^{Bi}}^{\infty} e^{-r(t-T^{Bi})}\left(\tilde{R}_{i,t}(1-\gamma_i) - F_i\right)(1-\tau)dt\right], \end{aligned} \tag{3.3}$$

where $E^{\mathbb{Q}}$ represents the certainty equivalent of the expected value.

Based on the strong Markov property of Brownian motions, I transform Eq. (3.3) towards

$$\begin{aligned} V_{i,0}^{U}(\tilde{R}_{i,t}) =& \frac{R_{i,0}(1-\gamma_i)(1-\tau)}{r-\mu_i} - \frac{F_i(1-\tau)}{r} \\ & - E^{\mathbb{Q}}\left[e^{-rT_{Bi}}\right] E^{\mathbb{Q}}\left[\int_0^{\infty} e^{-rt}\left(\tilde{R}_{i,t}(1-\gamma_i) - F_i\right)(1-\tau)dt\right]. \end{aligned} \tag{3.4}$$

The term $E^{\mathbb{Q}}\left[e^{-rT_{Bi}}\right]$ is similar to a perpetual, down-and-in, cash-at-hit-or-nothing, single barrier option with payoff 1 in case $\tilde{R}_{i,t}$ hits B_i and zero otherwise (see, e.g., Couch et al., 2012). There exists a well-known analytic solution to such an option given by[10]

$$P_{Bi} = E^{\mathbb{Q}}\left[e^{-rT_{Bi}}\right] = \left(\frac{B_i}{R_{i,0}}\right)^{y_i}, \tag{3.5}$$

with

$$y_i = \frac{1}{\sigma_i^2}\left[(\mu_i - \frac{\sigma_i^2}{2}) + \sqrt{(\mu_i - \frac{\sigma_i^2}{2})^2 + 2r\sigma_i^2}\right]. \tag{3.6}$$

Subsequently, I refer to P_{Bi} as contingent present value factor of abandonment. Substituting Eq. (3.5) into Eq. (3.4) and rearranging yields

$$V_{i,0}^{U}(\tilde{R}_{i,t}) = \frac{R_{i,0}(1-\gamma_i)(1-\tau)}{r-\mu_i} - \frac{F_i(1-\tau)}{r} + \left(\frac{F_i(1-\tau)}{r} - \frac{B_i(1-\gamma_i)(1-\tau)}{r-\mu_i}\right)P_{Bi}. \tag{3.7}$$

The term

$$V_{i,0}^{A}(\tilde{R}_{i,t}) = \left(\frac{F_i(1-\tau)}{r} - \frac{B_i(1-\gamma_i)(1-\tau)}{r-\mu_i}\right)P_{Bi} \tag{3.8}$$

represents the value of the abandonment option (or obligation). While the abandonment trigger under full capital constraint, $B_i = B_i^{ex}$, has been determined in Eq. (3.2), the derivation of the abandonment trigger without capital constraint (see Extension 3.1), $B_i = B_i^*$, is subject to the optimization problem $\max_{B_i \leq R_{i,0}} V_{i,0}^{U}(\tilde{R}_{i,t})$. Solving the problem's first order condition, $\delta V_{i,0}^{U}(\tilde{R}_{i,t})/\delta B_i = 0$, results in the optimal boundary of abandonment

$$B_i^* = \frac{y_i}{1+y_i}\frac{r-\mu_o}{r}\frac{F_i}{1-\gamma_i}. \tag{3.9}$$

I provide the comprehensive derivation of this solution in Appendix A. My results are congruent to Wong (2009).

Based on Eq. (3.2) and (3.9), B_i^{ex} and B_i^* are known and I can present the values of the abandonment options (or obligations) under both capital access regimes,

$$V_{i,0}^{A,ex}(\tilde{R}_{i,t}) = F_i(1-\tau)\left(\frac{1}{r}\frac{1}{r-\mu_i}\right)\left(\frac{F_i}{R_{0i}(1-\gamma_i)}\right)^{y_i} \tag{3.10}$$

$$V_{i,0}^{A,*}(\tilde{R}_{i,t}) = \frac{F_i(1-\tau)}{r}\frac{1}{1+y_i}\left(\frac{y_i}{1+y_i}\frac{r-\mu_i}{r}\frac{F_i}{R_{0i}(1-\gamma_i)}\right)^{y_i}, \tag{3.11}$$

respectively. With $r > 0$, it is evident from Eq. (3.6) that $y > 0$. Combined with the condition $r > \mu$, the following proposition can be invoked:

Proposition 3.1 (Value of abandonment option with unlimited capital access)**.** *If there is no capital restriction and fixed costs are strictly positive, $F_i > 0$, the value of the abandonment option $V_{i,0}^{A,*}(\tilde{R_{i,t}})$ will be strictly positive, too, and increasing with $F_i/(1-\gamma_i)$, the fixed costs' portion of total costs. The optimal boundary of abandonment B_i^* lies below the operating costs ($\gamma_i\tilde{R_{i,t}}$ and F_i) implying that investors need access to additional funds in order to materialize the maximum unlevered value $V_{i,0}^{U}(\tilde{R_{i,t}}, B_i^*)$. In the*

[10] For a comprehensive derivation see, e.g., Dixit and Pindyck (1994, pp. 315-316).

absence of fixed costs, $F_i = 0$, the value of the abandonment option is zero.

Proof. See Appendix B. □

If the access to capital is restricted, abandonment is not an option but rather an obligation and its value either positive or negative depending on the growth prospects of firm or project.

Proposition 3.2 (Value of abandonment option under full capital restriction)**.** *The value of the abandonment obligation under full capital restriction is positive, $V_{i,0}^{A,ex}(\tilde{R}_{i,t}) > 0$, if $\mu_i < 0$ and $F_i > 0$, and it is negative, $V_{i,0}^{A,ex}(\tilde{R}_{i,t}) < 0$, if $\mu_i > 0$ and $F_i > 0$. For $F_i = 0$, the value of the abandonment option is zero.*

Proof. See Appendix C. □

My results confirm once more that valuing firms or projects without reflecting the flexible choices available to investors, in particular the option to abandon, may lead to significant mispricing. Specifically, investment sectors with a high share of fixed expenses, e.g., semiconductor industry, wind parks or oil exploration, contain a material built-in option value of abandonment. Moreover, securing access to additional funds with agency costs as low as possible in order to exercise the abandonment option at the optimal point in time is value-enhancing - a strong argument for the fact that public and/or well-governed firms regularly carry higher valuation multiples than others.

The subsequent chapter extends the model towards the effects of financing decisions on the setup developed.

4. Financing choices and the levered values of separate projects

The firm considered follows a financing policy equivalent to Leland (1994). It issues console bonds promising an infinite stream of (continuous) coupons C_i as long as the firm or the project remains solvent. The credit risk-adjusted coupon payments are chosen such that the levered value of the firm $V_{i,0}^{L}(\tilde{R}_{i,t})$ is maximized. I extend the basic framework of Leland as follows:

Key Assumption 4.1 (Project Financing vs. Corporate Financing)**.** *The firm chooses to either issue one console bond with C_f which is served by the combined cash flows of the two projects (*Corporate finance*) or issue two console bonds with C_i which are served separately by the distinct cash flows of the projects (*Project finance*). The two bonds under project financing are non-recoursing as the basic firm and the project operate as legally independent entities.*

In either case, issuing bonds generates a continuous stream of tax shields, τC_f or $\sum_{i=1}^{2} \tau C_i$, as coupons reduce the firm's tax burden. In contrast to this benefit of debt, issuing bonds also creates the risk of bankruptcy with certain costs attached. In particular, a fraction α_f or α_i, with $\alpha_{f/i} \in [0, 1]$, of the then prevailing unlevered value of firm or project is lost in case of bankruptcy.

There are two classic triggers of bankruptcy which are both reflected in my model. First, bankruptcy of the firm or the project will be triggered if the respective stochastic process(es) $\tilde{R}_{f/it}$ hit(s) a certain boundary $H^*_{f/i}$ which is endogenously chosen in order to maximize the equity value. This is the case of unsecured debt where equity investors are once more assumed to be able to settle operational losses and coupon obligations with external funds ("deep pocket" assumption). Proposition 4.1 provides a closed form solution for the optimal boundary H^*_i. Due to the stochastic properties of $\tilde{R}_{f,t}$, there is no closed form solution to H^*_f.

Second, I consider the case of secured debt where bankruptcy will occur if stochastic EBITs $\tilde{X}_{f/i,t} = \tilde{R}_{f/i,t}(1 - \gamma_{f/i}) - F_{f/i}$ are not sufficient to comply with an interest coverage covenant $\delta_{f/i}C_{f/i}$ such that the exogenous bankruptcy boundary $H^{ex}_{f/i}$ to our stochastic process $\tilde{R}_{f/i,t}$ takes the form of

$$H^{ex}_{f/i} = \frac{\delta_{f/i} C_{f/i} + F_{f/i}}{1 - \gamma_{f/i}}. \tag{4.1}$$

$\delta_{f/i}$ represents an interest coverage ratio. If $\delta_{f/i} = (1 - \tau)$, $H^{ex}_{f/i}$ is set to illiquidity and implies that equity investors have no frictionless access to external funds[11] Irrespective of the chosen (endogenous or exogenous) bankruptcy trigger: When $\tilde{R}_{f/i,t}$ hits $H_{f/i}$, debtholders take control of the firm or the project leaving them with an asset value of $(1 - \alpha_{f/i})V^H_{f/i}$, where $V^H_{f/i}$ is the unlevered value of firm or project at the time of hitting.

Note that all subsequent derivations exclusively concern the projects separately (subindex i). While all general model mechanics also work for the combined firm (subindex f), it is not possible to develop a framework of closed form solutions for f as we are confronted with stochastic $\tilde{\mu}_f$ and $\tilde{\sigma}_f$.

In accordance to the reflection of the abandonment option in section 3, I integrate the stochastic hitting of the bankruptcy trigger H_i into the model by constructing a perpetual, down-and-in, cash-at-hit-or-nothing, single barrier option with payoff 1 in case $\tilde{R}_{i,t}$ hits H_i from above and zero otherwise. Let $T^{Hi} = \inf\{t \geq 0 : \tilde{R}_{i,t} = H_i\}$ be the stochastic stopping time with $H_i < R_{i,0}$. The price of the option described is therefore given by

$$P_{Hi} = E^{\mathbb{Q}}\left[e^{-rT_{Hi}}\right] = \left(\frac{H_i}{R_{i,0}}\right)^{y_i}, \tag{4.2}$$

where y_i corresponds to Eq. (3.6).

With Eq. (4.2) at hand, closed form expressions for the value of tax shield,

$$V^{TS}_{i,0}(\tilde{R}_{i,t}) = \frac{\tau C_i}{r} - P_{Hi}\frac{\tau C_i}{r} = (1 - P_{Hi})\frac{\tau C_i}{r} \tag{4.3}$$

and for the value of bankruptcy costs,

$$V^{BC}_{i,0}(\tilde{R}_{i,t}) = \alpha_i V^H_i P_{Hi} \tag{4.4}$$

can be formed irrespective of the type (endogenous or exogenous) of bankruptcy trigger. Derivations of Eq. (4.3) and (4.4) follow Leland (1994). The unlevered value of basic firm or project V^H_i at hitting is defined like V^U_i in Eq. (3.7), except that the starting point of the stochastic process is not $R_{i,0}$ but H_i. Hence,

$$V^H_i = \frac{H_i(1-\gamma_i)(1-\tau)}{r-\mu_i} - \frac{F_i(1-\tau)}{r} + \left(\frac{F_i(1-\tau)}{r} - \frac{B_i(1-\gamma_i)(1-\tau)}{r-\mu_i}\right)\left(\frac{B_i}{H_i}\right)^{y_i}, \tag{4.5}$$

with $B_i < H_i$. I determine the net benefit of debt as

$$V^{NB}_{i,0}(\tilde{R}_{i,t}) = V^{TS}_{i,0}(\tilde{R}_{i,t}) - V^{BC}_{i,0}(\tilde{R}_{i,t}). \tag{4.6}$$

In combination, net benefits of debt $V^{NB}_{i,0}(\tilde{R}_{i,t})$ and unlevered value of firm or project $V^U_{i,0}(\tilde{R}_{i,t})$ form the

---

[11] In practice, we almost certainly observe interest coverage covenants of $\delta_{f/i} > 1$ indicating that debtholders face some sort of agency problem, e.g. asymmetric information, which they intend to hedge against. Stulz and Johnson (1985) present detailed work on the economic motivation of secured debt.

levered value of firm,

$$V^L_{i,0}(\tilde{R}_{i,t}) = V^U_{i,0}(\tilde{R}_{i,t}) + V^{NB}_{i,0}(\tilde{R}_{i,t}). \tag{4.7}$$

Substituting Eq. (4.2)-(4.6) into Eq. (4.7) yields a closed form solution to the levered value of firm or project being a function of bankruptcy trigger H_i such that

$$\begin{aligned} V^L_{i,0}(\tilde{R}_{i,t}) = V^U_{i,0}(\tilde{R}_{i,t}) &+ \left(1 - \left(\frac{H_i}{R_{i,0}}\right)^{y_i}\right)\frac{\tau C_i}{r} - \alpha_i \left(\frac{H_i}{R_{i,0}}\right)^{y_i}\left(\frac{H_i(1-\gamma_i)(1-\tau)}{r-\mu_i} - \frac{F_i(1-\tau)}{r}\right. \\ &\left. + \left(\frac{F_i(1-\tau)}{r} - \frac{B_i(1-\gamma_i)(1-\tau)}{r-\mu_i}\right)\left(\frac{B_i}{H_i}\right)^{y_i}\right). \end{aligned} \tag{4.8}$$

Eq. (4.8) determines the total value for both, equity investors and debtholders. To split this value, I consider the claim of debtholders separately. As they receive C_i as long as project i has not filed for bankruptcy ($t < T^{Hi}$) and $(1-\alpha_i)V^H_i$ in the moment bankruptcy is triggered ($t = T^{Hi}$), I obtain

$$\begin{aligned} D_{i,0}(\tilde{R}_{i,t}) &= (1 - P_{Hi})\,\frac{C_i}{r} && +(1-\alpha_i)V^H_i P_{Hi} \\ &= \left(1 - \left(\frac{H_i}{R_{i,0}}\right)^{y_i}\right)\frac{C_i}{r} && +(1-\alpha_i)\left(\frac{H_i}{R_{i,0}}\right)^{y_i}\left(\frac{H_i(1-\gamma_i)(1-\tau)}{r-\mu_i} - \frac{F_i(1-\tau)}{r}\right. \\ & && \left. + \left(\frac{F_i(1-\tau)}{r} - \frac{B_i(1-\gamma_i)(1-\tau)}{r-\mu_i}\right)\left(\frac{B_i}{H_i}\right)^{y_i}\right). \end{aligned} \tag{4.9}$$

Deducting $D_{i,0}(\tilde{R}_{i,t})$ from $V^L_{i,0}(\tilde{R}_{i,t})$ results in the value of equity $E_{i,0}(\tilde{R}_{i,t})$ given by

$$\begin{aligned} E_{i,0}(\tilde{R}_{i,t}) =& V^U_{i,0}(\tilde{R}_{i,t}) - (1-\tau)\frac{C_i}{r} - P_{Hi}\left(V^H_i - (1-\tau)\frac{C_i}{r}\right) \\ =& V^U_{i,0}(\tilde{R}_{i,t}) - (1-\tau)\frac{C_i}{r} - \left(\frac{H_i}{R_{i,0}}\right)^{y_i}\left(\frac{H_i(1-\gamma_i)(1-\tau)}{r-\mu_i} - \frac{F_i(1-\tau)}{r}\right. \\ &\left. + \left(\frac{F_i(1-\tau)}{r} - \frac{B_i(1-\gamma_i)(1-\tau)}{r-\mu_i}\right)\left(\frac{B_i}{H_i}\right)^{y_i} - (1-\tau)\frac{C_i}{r}\right). \end{aligned} \tag{4.10}$$

With Eq. (4.8) and (4.10), it is possible to frame and solve the optimization problem generating the endogenous, optimal bankruptcy trigger H^*_i. The following proposition presents the result.

Proposition 4.1 (Optimal bankruptcy trigger)**.** *In the absence of any constraints on access to capital and with unsecured debt, equity investors are free to determine the bankruptcy trigger H^*_i such that their equity value is maximized,*

$$\max_{H^*_i \le R_{i,0}} E_{i,0}(\tilde{R}_{i,t}). \tag{4.11}$$

Solving the first-order condition for the optimization problem yields the optimal bankruptcy trigger[12]

$$H^*_i = \frac{y_i}{1+y_i}\frac{r-\mu_o}{r}\frac{C_i + F_i}{1-\gamma_i}. \tag{4.12}$$

Comparing Eq. (4.12) *and Eq.* (3.9)*, it is evident that, with $C_i > 0$, $H^*_i > B^*_i$.*

[12] It is trivial to show that the second-order condition is satisfied.

Proof. Set the first derivative of $E_{i,0}(\tilde{R}_{i,t})$ with respect to H_i^* equal to zero,

$$\frac{\delta E_{i,0}(\tilde{R}_{i,t})}{\delta H_i^*} = \left(\frac{H_i^*}{R_{i,0}}\right)^{y_i} \left(\frac{(1-\tau)C_i y_i}{H_i^* r} - \frac{(1-\gamma_i)(1-\tau)(1+y_i)}{r-\mu_i} + \frac{F_i(1-\tau)y_i}{H_i^* r}\right) \stackrel{!}{=} 0. \tag{4.13}$$

Rearranging for H_i^* results in Eq. (4.12). □

In case of an endogenous bankruptcy trigger H_i^*, the optimal level of debt, i.e. the optimal coupon C_i^*, is determined by maximizing the levered firm value $V_{i,0}^L(\tilde{R}_{i,t})$ under the condition that Eq. (4.12) holds. With an exogenous bankruptcy trigger H_i^{ex}, the optimization problem alters slightly as maximizing $V_{i,0}^L(\tilde{R}_{i,t})$ is now subject to Eq. (4.1). I capture the solution to both optimization problems in the subsequent propositions.

Proposition 4.2 (Optimal Coupon Level with Unsecured Debt and Endogenous Bankruptcy)**.** *The optimal coupon level C_i^* set to maximize the levered firm value $V_{i,0}^L(\tilde{R}_{i,t})$, subject to an endogenously chosen bankruptcy trigger H_i^* maximizing the investors' equity value $E_{i,0}(\tilde{R}_{i,t})$, can be evaluated numerically by applying the expression*

$$C_i^* = \frac{r}{r-\mu_i}\frac{1+y_i}{y_i}\left(1+\frac{y_i}{C_i^*+F_i}\left(C_i^*\left(1+\alpha_i\frac{1-\tau}{\tau}\right)+2F_i\alpha_i\frac{1-\tau}{\tau}\right)\right)^{-\frac{1}{y_i}} R_{i,0}(1-\gamma_i) - F_i, \tag{4.14}$$

where C_i^* remains on the right-hand side preventing an analytic solution to the problem. However, for the special case of fixed costs equal to zero, Eq. (4.14) *collapses to*

$$C_{i,F=0}^* = \frac{r}{r-\mu_i}\frac{1+y_i}{y_i}\left(1+\frac{y_i}{+}\alpha_i y_i\frac{1-\tau}{\tau}\right)^{-\frac{1}{y_i}} R_{i,0}(1-\gamma_i), \tag{4.15}$$

an analytic solution congruent to the results obtained by Leland (1994) and Goldstein et al. (2001).

Proof. See Appendix D. □

Proposition 4.3 (Optimal Coupon Level with Secured Debt and Exogenous Bankruptcy)**.** *With debt secured via an interest coverage ratio of δ (implying bankruptcy at H_i^{ex} and exogenous abandonment via illiquidity at B_i^{ex}, the optimal coupon level C_i^{ex} alters to*

$$C_i^{ex} = \frac{1}{\delta}\left[\left(1+\frac{y_i\delta\tau}{\delta C_i^{ex}+F_i}(\tau C_i^{ex}-\alpha_i F_i(1-\tau)) + \frac{r}{r-\mu_i}\frac{1-\tau}{\tau}\alpha_i\delta(1+y_i)\right)^{-\frac{1}{y_i}} R_{i,0}(1-\gamma_i) - F_i\right], \tag{4.16}$$

which constitutes a closed form solution requiring numerical evaluation as C_i^{ex} remains on the right-hand side. In line with Proposition 4.2 let $F_i = 0$, Eq. (4.16) *collapses to*

$$C_{i,F=0}^{ex} = \frac{1}{\delta}\left[\left((1+y_i)(1+\frac{r}{r-\mu_i}\frac{1-\tau}{\tau}\alpha_i\delta)\right)^{-\frac{1}{y_i}} R_{i,0}(1-\gamma_i)\right], \tag{4.17}$$

which can be solved analytically.

Proof. See Appendix E. □

5. Simulation-based implementation of the model for the combined firm

As combining both projects, $i = 1, 2$, in a single legal entity, subindexed by f, triggers a diffusion process with stochastic (path-dependent) drift $\tilde{\mu}_f$ and volatility $\tilde{\sigma}_f$ (see Eq. (2.7)), I generate a discrete-time approximation of the model and solve it via Monte Carlo simulation.

The discrete version of single risk neutral revenue dynamics is represented by

$$\tilde{R}_{i,t+\Delta t} = \tilde{R}_{i,t} e^{(\mu_i - \frac{1}{2}\sigma_i^2)\Delta t + \sigma_i \sqrt{\Delta t} z_i} \tag{5.1}$$

where $\Delta t \in \,]\,0, \infty\,[$ is a discrete time step and z_i are standard correlated normal variates.

To achieve correlated revenue paths, I apply the Cholesky decomposition. In a first step, I generate two uncorrelated normal variates z_1 and z_3. While z_1 can be used instantly to generate the revenue dynamics $\tilde{R}_{1,t+\Delta t}$ of the basic firm, z_3 is required as a support variate to create the normal variate z_2 which is the normal variate of the project correlated with z_1. According to the Cholesky decomposition, it follows

$$z_2 = \rho z_1 + (1 - \rho)^2 z_3. \tag{5.2}$$

With z_1 and z_2 at hand, both correlated revenue streams can be simulated pathwise over time. Robust simulation results require a high number of simulated paths in order to map desired distributions correctly. By testing simulation results for the two single EBIT-generating machines based on different parameter settings against the analytic solutions derived in Sections 3 and 4, I find $N = 100,000$ paths to be sufficient for an error tolerance of less than one percent. Regarding the model with exogenous triggers, I increase the number of paths even to $N = 200,000$ as there are no optimization algorithms needed for abandonment and bankruptcy triggers. For approximating an infinite lifetime, I model three hundred years ($T = 300$) as the discounted value contribution of cash flows after such a long time span converges to zero. The length of a time step is set to one year ($\Delta t = 1$), although this choice partially contradicts the propositions of continuous abandonment and bankruptcy triggers. However, shorter time steps exponentially increase the required calculation resources without impacting the general relationship between summed values of two independent entities and the values of the combined firm.

Fig. 1 depicts the average developments of drift rate μ and volatility σ over time for the two separate projects and for the combined firm.

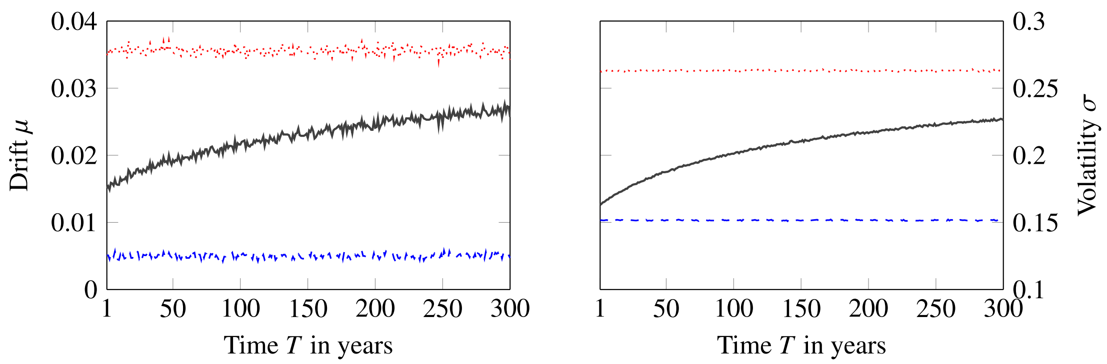

Figure 1: Drift rate μ and volatility σ over time. The figure provides the average developments of drift rate μ (left graph) and volatility σ (right graph) over time for the separate projects (blue, dashed for basic firm, and red, dotted for project) and for the combined firm (dark grey, solid). Results are based on the following set of parameters: $R_{1,0} = 100, R_{2,0} = 50, \mu_1 = 0.005, \mu_2 = 0.035, \sigma_1 = 0.15, \sigma_2 = 0.25, N = 200,000$.

The graphs illustrate the need for a simulation-based numerical solution as we face time-varying and path-dependent drift rates and volatilities for the combined firm.

Income before interests and taxes (EBIT X) and free cash flow to firm (FCF) of the combined firm are still given by Eq. (2.1) and (2.5). FCFs of each path are discounted to $t = 0$ and summed up to a present value. The average of all present values gives the expected value of the unlevered firm without option or obligation to abandon,

$$\begin{aligned} V^{U,NA}_{i/f,0}(\tilde{R}_{i/f,t}) &= V^{U}_{i/f,0}(\tilde{R}_{i/f,t}) - V^{A,*/ex}_{i/f,0}(\tilde{R}_{i/f,t}) \\ &= \frac{\sum_{n=1}^{N} \sum_{t=1}^{T} FCF_{n,i/f,t} e^{-rt}}{N}. \end{aligned} \tag{5.3}$$

Subsequently, an abandonment trigger $B^{*/ex}_{f/i}$ is set. Whenever a revenue path falls below $B^{*/ex}_{f/i}$, abandonment is triggered and all future FCF are set to zero. Adding this indicator condition to Eq. (5.3) results in

$$V^{U}_{i/f,0}(\tilde{R}_{i/f,t}) = \frac{\sum_{n=1}^{N} \sum_{t=1}^{T} FCF_{n,i/f,t} e^{-rt} \mathbb{1}_{\{\tilde{R}_{n,i/f,t} > B^{*/ex}_{f/i}\}}}{N}. \tag{5.4}$$

For the exogenous model with full capital restriction, the trigger $B^{ex}_{f/i}$ is defined similar as in Eq. (3.2). $B^{*}_{f/i}$, the abandonment trigger of the endogenous model needs to be identified via iteration in order to satisfy Eq. (3.1).

Note that the abandonment trigger of the combined firm in the endogenous model, B^{*}_{f}, only reflects an approximation as path-dependent revenue dynamics would also require path-dependent triggers to maximize $V^{U}_{f,0}(\tilde{R}_{f,t})$. Since no numerical procedures are known to generate such a setting, I choose the approximation described above.

Deducting Eq. (5.3) from Eq. (5.4) yields the value of the abandonment option (or obligation),

$$V^{A}_{i/f,0}(\tilde{R}_{i/f,t}) = V^{U}_{i/f,0}(\tilde{R}_{i/f,t}) - V^{U,NA}_{i/f,0}(\tilde{R}_{i/f,t}). \tag{5.5}$$

Turning to the effects of debt financing in the simulation-based approach, I proceed equivalently. First, I define the cash flows of benefits from tax savings ($\tau C_{i/f,t}$) and of costs from bankruptcy ($\alpha_{i/f} V^{U}_{n,i/f,t}$). Subsequently, I model the bankruptcy condition, $\tilde{R}_{n,i/f,t} \leq H^{*/ex}_{f/i}$ where tax savings cease to exist and bankruptcy costs emerge. Eq. (4.1) provides the trigger $H^{ex}_{f/i}$ for the exogenous model, while $H^{*}_{f/i}$, the endogenous trigger, is determined via iteration to satisfy Eq. (4.12). For the combined firm, H^{*}_{f} again only represents an approximation as path-dependent triggers would be required to maximize the levered value $V^{L}_{f,0}(\tilde{R}_{f,t})$. Thus, in accordance with the endogenous version of $V^{U}_{f,0}(\tilde{R}_{f,t})$, we may face a slight underestimation.

From the procedure stated above, it follows that the value of tax shields in the simulation-based approach is defined by

$$V^{TS}_{i/f,0}(\tilde{R}_{i/f,t}) = \frac{\sum_{n=1}^{N} \sum_{t=1}^{T} \tau C_{i/f,t} e^{-rt} \mathbb{1}_{\{\tilde{R}_{n,i/f,t} > H^{*/ex}_{f/i}\}}}{N}, \tag{5.6}$$

and that the value of bankruptcy costs can be expressed by

$$V^{BC}_{i/f,0}(\tilde{R}_{i/f,t}) = \frac{\sum_{n=1}^{N} \sum_{t=1}^{T} \alpha_{i/f} V^{U}_{n,i/f,t} e^{-rt} \mathbb{1}_{\{\tilde{R}_{n,i/f,t} \leq H^{*/ex}_{f/i}\}}}{N}. \tag{5.7}$$

Deducting Eq. (5.7) from Eq. (5.6) results in the value of net benefits of debt

$$V^{NB}_{i/f,0}(\tilde{R}_{i/f,t}) = V^{TS}_{i/f,0}(\tilde{R}_{i/f,t}) - V^{BC}_{i/f,0}(\tilde{R}_{i/f,t}). \tag{5.8}$$

To arrive at optimal coupon payments $C^{*/ex}_{i/f,0}$, another iteration is applied with target function $\max V^{NB}_{i/f,0}(\tilde{R}_{i/f,t})$ and subject to the underlying conditions for $H^{*/ex}_{f/i}$. Thus, I face a two-step iteration process in the endogenous model as for any tested coupon level $C^{*}_{i/f,0}$, the corresponding optimal bankruptcy trigger $H^{*}_{i/f}$ needs to be determined. Due to this increased complexity, I reduce the number of simulation paths to $N = 100,000$ as outlined before.

In the subsequent chapter, I apply the analytic and the simulation-based model to draw economic implications.

6. Combined firm versus separate entities: Application of the model and economic conclusions

Consider a firm running a basic business ($i = 1$) and having the opportunity to start a new project ($i = 2$). Both, the basic business and the new project, are EBIT-generating machines with all the features developed in this paper. There exists a base setup and numerous sensitivity setups for the two to examine the effects of differences and levels of key parameter values. Table 1 summarizes all settings.

All parameter setups of Tab. 1 are tested with varying correlations ($\rho \in (-1, 1)$) of $\tilde{R}_{i,t}$. Moreover, I generate sensitivity analyses concerning three ρ-levels ($\rho = 0.0, 0.4, 0.8$) for varying parameters of the additional project ($i = 2$) only. The settings developed are investigated for both scenarios of capital access, the fully restricted version where only internal funds are available (exogenous model) and the unlimited access version where capital is available as long as net present values of investing are greater or equal to zero (endogenous model).

Investors in the current firm are free to choose the legal setup, i.e., founding a new independent entity for the new project or integrating the new project in the existing entity. Keeping both separate allows investors to set individual capital structures and, in the endogenous model, individual abandonment and bankruptcy triggers. Combining both projects may create a diversification effect, i.e., financial synergies, that reduce the expected value of asymmetric bankruptcy costs and increase the expected value of asymmetric tax shields.

I compare the net benefits of the chosen legal setup twofold. I start by examining the effect regarding operating values with the metric

$$\begin{aligned}\mathit{\Delta} V^{A} &= \frac{V^{A}_{f,0}(\tilde{R}_{f,t}) - (\sum_{i=1}^{2} V^{A}_{i,0}(\tilde{R}_{i,t}))}{V^{U}_{2}} \\ &= \frac{V^{A}_{f,0}(\tilde{R}_{f,t}) - V^{A}_{\Sigma,0}(\tilde{R}_{\Sigma,t})}{V^{U}_{2}}, \end{aligned} \tag{6.1}$$

where setting the absolute net effect $V^{A}_{f,0}(\tilde{R}_{f,t}) - V^{A}_{\Sigma,0}(\tilde{R}_{\Sigma,t})$ in relation to the operating value of the additional project V^{U}_{2} allows for an estimation of the effect's overall relevance.

Congruently, I define a relative $\mathit{\Delta}$-measure for the net benefits of debt such that

$$\begin{aligned}\mathit{\Delta} V^{NB} &= \frac{V^{NB}_{f,0}(\tilde{R}_{f,t}) - (\sum_{i=1}^{2} V^{NB}_{i,0}(\tilde{R}_{i,t}))}{V^{U}_{2}} \\ &= \frac{V^{NB}_{f,0}(\tilde{R}_{f,t}) - V^{NB}_{\Sigma,0}(\tilde{R}_{\Sigma,t})}{V^{U}_{2}}. \end{aligned} \tag{6.2}$$

Again, I relate the absolute net effect $V^{NB}_{f,0}(\tilde{R}_{f,t}) - V^{NB}_{\Sigma,0}(\tilde{R}_{\Sigma,t})$ to the operating value of the additional project V^{U}_{2} as this enables me to directly add and compare the two measures.

Table 1: Parameter setups. The table contains all setups tested by the model. The critical parameters μ, σ, γ, F, α and δ are varied with respect to the difference between the basic business ($i = 1$) and the new project ($i = 2$), as well as with respect to the general level. Parameter values that differ from the base case are highlighted.

Setting		Component	R_0	μ	σ	γ	F	α	δ	r	τ
Base		"Basic" firm	**100**	**0.005**	**0.15**	**0.3**	**30**	**0.2**	**0.7**	**0.03**	**0.3**
		Project	**50**	**0.01**	**0.15**	**0.4**	**10**	**0.2**	**0.7**	**0.03**	**0.3**
Drift rate	Difference 1	"Basic" firm	100	**0.005**	0.15	0.3	30	0.2	0.7	0.03	0.3
		Project	50	**-0.01**	0.15	0.4	10	0.2	0.7	0.03	0.3
	Difference 2	"Basic" firm	100	**0.005**	0.15	0.3	30	0.2	0.7	0.03	0.3
		Project	50	**0.00**	0.15	0.4	10	0.2	0.7	0.03	0.3
	Level 1	"Basic" firm	100	**-0.01**	0.15	0.3	30	0.2	0.7	0.03	0.3
		Project	50	**-0.01**	0.15	0.4	10	0.2	0.7	0.03	0.3
	Level 2	"Basic" firm	100	**0.01**	0.15	0.3	30	0.2	0.7	0.03	0.3
		Project	50	**0.01**	0.15	0.4	10	0.2	0.7	0.03	0.3
Volatility	Difference 1	"Basic" firm	100	0.005	**0.15**	0.3	30	0.2	0.7	0.03	0.3
		Project	50	0.01	**0.10**	0.4	10	0.2	0.7	0.03	0.3
	Difference 2	"Basic" firm	100	0.005	**0.15**	0.3	30	0.2	0.7	0.03	0.3
		Project	50	0.01	**0.20**	0.4	10	0.2	0.7	0.03	0.3
	Level 1	"Basic" firm	100	0.005	**0.10**	0.3	30	0.2	0.7	0.03	0.3
		Project	50	0.01	**0.10**	0.4	10	0.2	0.7	0.03	0.3
	Level 2	"Basic" firm	100	0.005	**0.20**	0.3	30	0.2	0.7	0.03	0.3
		Project	50	0.01	**0.20**	0.4	10	0.2	0.7	0.03	0.3
Cost structure	Difference 1	"Basic" firm	100	0.005	0.15	**0.3**	**30**	0.2	0.7	0.03	0.3
		Project	50	0.01	0.15	**0.6**	**0**	0.2	0.7	0.03	0.3
	Difference 2	"Basic" firm	100	0.005	0.15	**0.3**	**30**	0.2	0.7	0.03	0.3
		Project	50	0.01	0.15	**0.0**	**60**	0.2	0.7	0.03	0.3
	Level 1	"Basic" firm	100	0.005	0.15	**0.6**	**0**	0.2	0.7	0.03	0.3
		Project	50	0.01	0.15	**0.6**	**0**	0.2	0.7	0.03	0.3
	Level 2	"Basic" firm	100	0.005	0.15	**0.0**	**60**	0.2	0.7	0.03	0.3
		Project	50	0.01	0.15	**0.0**	**30**	0.2	0.7	0.03	0.3
Bankruptcy costs	Difference 1	"Basic" firm	100	0.005	0.15	0.3	30	**0.2**	0.7	0.03	0.3
		Project	50	0.01	0.15	0.4	10	**0.1**	0.7	0.03	0.3
	Difference 2	"Basic" firm	100	0.005	0.15	0.3	30	**0.2**	0.7	0.03	0.3
		Project	50	0.01	0.15	0.4	10	**0.6**	0.7	0.03	0.3
	Level 1	"Basic" firm	100	0.005	0.15	0.3	30	**0.1**	0.7	0.03	0.3
		Project	50	0.01	0.15	0.4	10	**0.1**	0.7	0.03	0.3
	Level 2	"Basic" firm	100	0.005	0.15	0.3	30	**0.4**	0.7	0.03	0.3
		Project	50	0.01	0.15	0.4	10	**0.4**	0.7	0.03	0.3
Coverage ratio	Difference 1	"Basic" firm	100	0.005	0.15	0.3	30	0.2	**0.7**	0.03	0.3
		Project	50	0.01	0.15	0.4	10	0.2	**0.5**	0.03	0.3
	Difference 2	"Basic" firm	100	0.005	0.15	0.3	30	0.2	**0.7**	0.03	0.3
		Project	50	0.01	0.15	0.4	10	0.2	**1.0**	0.03	0.3
	Level 1	"Basic" firm	100	0.005	0.15	0.3	30	0.2	**1.0**	0.03	0.3
		Project	50	0.01	0.15	0.4	10	0.2	**1.0**	0.03	0.3
	Level 2	"Basic" firm	100	0.005	0.15	0.3	30	0.2	**1.3**	0.03	0.3
		Project	50	0.01	0.15	0.4	10	0.2	**1.3**	0.03	0.3

6.1. *Analytic vs. simulation-based solution: A consideration of single EBIT-generating machines*

In this introductory analysis, I examine the two projects separately applying analytic solutions from sections 3 and 4. Moreover, I compare results to the simulation-based approach. Tab. 2 presents the outcome for the base setup.

Table 2: Separate analysis of basic firm and new project under base setup via analytic solution and simulation-based approach. The table displays, from the left to the right, values without abandonment option/obligation ($V_{i,0}^{U,NA}$), values of the abandonment option/obligation ($V_{i,0}^{A}$), operating values ($V_{i,0}^{U}$), values of net benefits of debt ($V_{i,0}^{NB}$), total levered values ($V_{i,0}^{L}$) and leverage ratios ($D_{i,0}/V_{i,0}^{L}$) for basic firm and new project based on exogenous and endogenous model. Analytic results are provided for all value components while simulation results are depicted and compared to the analytic ones for $V_{i,0}^{U}$, $V_{i,0}^{L}$ and $D_{i,0}/V_{i,0}^{L}$.

"Basic" firm												
	Analytic			**Simu.**	*Δ%*	**Analytic**		**Simu.**	*Δ%*	**Analytic**	**Simu.**	*Δ% − pts.*
	$V_{i,0}^{U,NA}$	$V_{i,0}^{U,A}$		$V_{i,0}^{U}$		$V_{i,0}^{NB}$		$V_{i,0}^{L}$			$D_{i,0}/V_{i,0}^{L}$	
Endo.	1260	33.6	1293.6	1282.4	-0.0086	239.2	1532.8	1521.9	-0.0071	0.7143	0.7279	0.0136
Exog.	1260	-43.5	1216.5	1221.6	0.0042	48.3	1264.8	1287.9	0.0183	0.3544	0.401	0.0467
Project												
	Analytic			**Simu.**	*Δ%*	**Analytic**		**Simu.**	*Δ%*	**Ana.**	**Simu.**	*Δ% − pts.*
	$V_{i,0}^{U,NA}$	$V_{i,0}^{U,A}$		$V_{i,0}^{U}$		$V_{i,0}^{NB}$		$V_{i,0}^{L}$			$D_{i,0}/V_{i,0}^{L}$	
Endo.	816.7	3.9	820.5	812.8	-0.0095	172.3	992.8	979	-0.0139	0.7627	0.7526	-0.0101
Exog.	816.7	-20.6	796.1	795.4	-0.0008	47.6	843.7	852.9	0.0109	0.4261	0.4616	0.0355

Most importantly, Tab. 2 confirms that the discrete-time approximation via simulation works well as operating values $V_{i,0}^{U}$ in the endogenous and in the exogenous model lie within one percent while levered values $V_{i,0}^{L}$ lie within two percent. Moreover, results illustrate the claims of propositions 3.1 and 3.2 from section 3 as $V_{i,0}^{A}$ is positive in the endogenous model and negative in the exogenous model if $\mu > 0$. Investors with "deep pockets" can expect higher operating values, are able to issue more debt to achieve their optimal capital structure and, thus, also face higher expected net benefits of debt. These results are robust throughout all parameter choices for the new project as Tab. 3 proves.

6.2. *Combined firm versus project approach: The impact on operating values*

Entering the analysis of potential benefits from bundling diverse projects in a combined firm versus a project approach, I start by considering value effects irrespective of financing and, thus, I focus on the operating value with built-in abandonment option or obligation. Fig. 2 provides the results of this analysis in the base setup for exogenous and endogenous abandonment triggers, i.e., full capital restriction and unlimited capital access.

While bundling projects in a combined firm does not provide any operating value-add under unlimited capital access (right graph), the effect of diversification may generate significant additional operating value if no external funding is available and correlation is sufficiently low (left graph).

Fig. 3-6 report the relative net effects in operating value ΔV^{A} as defined in Eq. (6.1) for all parameter setups and variations regarding cost structure, drift and volatility.[13] Based on the reported outcomes, a more general finding (see Conjecture 6.1) with respect to the operating net effect under unlimited capital access and more specific findings (see Conjecture 6.2) dealing with the impact of parameters under full capital restriction can be drawn.

[13] The operating value V^{U} is independent of bankruptcy costs α and coverage ratio δ. Thus, the setups regarding these two parameters have been omitted in Fig. 3 and 5.

Table 3: Comparative statics for value components of the basic firm. The table comprises operating values without abandonment option/obligation ($V_{1,0}^{U,NA}$), values of the abandonment option/obligation ($V_{1,0}^{A}$), operating values in total ($V_{1,0}^{U}$), values of net benefits of debt ($V_{1,0}^{NB}$), total levered values ($V_{1,0}^{L}$) and leverage ratios ($D_{1,0}/V_{1,0}^{L}$) for the endogenous model (left) and the exogenous model (right) resting upon the base case parameters of the basic firm with adjustments made to these parameters as outlined in the first column.

		Endogenous model					**Exogenous model**				
	$V_{1,0}^{U,NA}$	$V_{1,0}^{U,A}$	$V_{1,0}^{U}$	$V_{1,0}^{NB}$	$V_{1,0}^{L}$	$D_{1,0}/V_{1,0}^{L}$	$V_{1,0}^{U,A}$	$V_{1,0}^{U}$	$V_{1,0}^{NB}$	$V_{1,0}^{L}$	$D_{1,0}/V_{1,0}^{L}$
μ_1											
-0.01	525	108	633	109	742	0.697	79	604	46	650	0.507
0	933	55	988	177	1166	0.707	0	933	49	982	0.414
0.01	1750	17	1767	339	2106	0.723	-92	1658	45	1703	0.284
σ_1											
0.1	1260	7	1267	266	1533	0.746	-18	1242	108	1351	0.491
0.15	1260	34	1294	239	1533	0.714	-44	1216	48	1265	0.354
0.2	1260	74	1334	228	1561	0.694	-65	1195	20	1215	0.242
γ_1, F_1											
0.6, 0	1120	0	1120	215	1335	0.720	0	1120	139	1259	0.533
0.3, 30	1260	34	1294	239	1533	0.714	-44	1216	48	1265	0.354
0, 60	1400	107	1507	276	1782	0.712	-138	1262	16	1277	0.224
α_1											
0.1	1260	34	1294	262	1555	0.756	-44	1216	72	1288	0.455
0.2	1260	34	1294	239	1533	0.714	-44	1216	48	1265	0.354
0.4	1260	34	1294	205	1498	0.644	-44	1216	20	1237	0.207
δ_1											
0.7	1260	34	1294	239	1533	0.714	-44	1216	48	1265	0.354
1	1260	34	1294	239	1533	0.714	-44	1216	24	1240	0.235
1.3	1260	34	1294	239	1533	0.714	-44	1216	12	1229	0.162

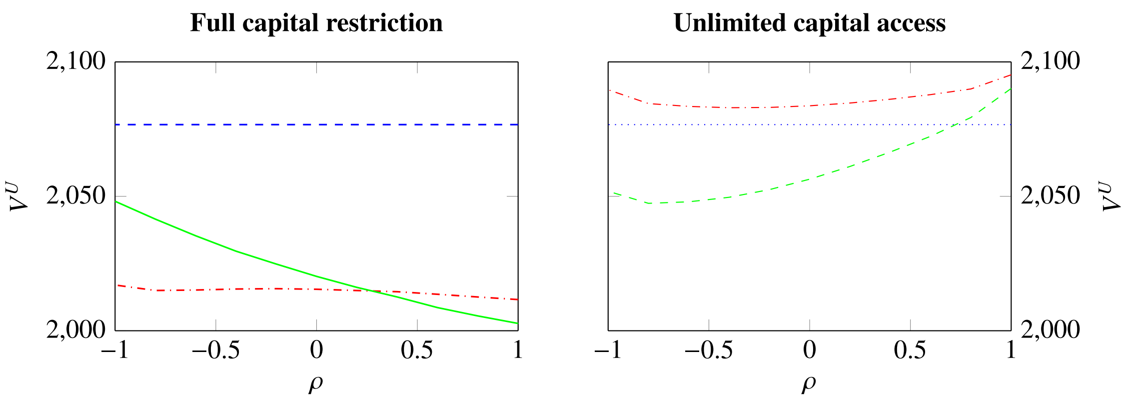

Figure 2: Comparison of operating values with abandonment option/ obligation for the combined firm vs. the sum of separate projects. This figure depicts the sum of the operating values of basic firm and project without abandonment option ($V^{U,NA}$, blue, dotted line), the sum of the operating values of the two kept in separate entities with separate abandonment option (V_{Σ}^{U}, red, dashdotted line), and the operating value of the combined firm where both are bundled in a legal entity (V_f^U, green, dashed line) with one joint abandonment option under full capital restriction (left graph) and under unlimited capital access (right graph). Results rest upon the base set of parameters.

Conjecture 6.1 (Operating net effects of project bundling in a combined firm under unlimited capital access)**.** *With unlimited capital access, bundling projects where revenues $\tilde{R}_{i,t}$ interact with correlation $\rho < 1$ is never beneficial with respect to the operating value in comparison to separating the basic firm and the project in two independent legal entities. It follows that*

$$V^{U}_{f,0}(\tilde{R}_{f,t}) \leq V^{U}_{\sum,0}(\tilde{R}_{i,t}) \tag{6.3}$$

As the operating values without abandonment option are equal, I eliminate the value component $V^{U,NA}(\tilde{R}_{f,t})$ from both sides of (6.3) *and arrive at*

$$V^{A}_{f,0}(\tilde{R}_{f,t}) \leq \sum_{i=1}^{2} V^{A}_{i,0}(\tilde{R}_{i,t}) = V^{A}_{\sum,0}(\tilde{R}_{i,t}). \tag{6.4}$$

The flexibility to abandon the basic firm and the project separately is more beneficial than the effect of diversification, if external funds can be accessed without restrictions until the value-maximizing trigger B^*_i is hit.

Please note: While bundling is not beneficial with respect to operating values under unlimited capital access, it does not result in adverse outcomes in practice as the combined firm still has the flexibility to abandon one of its projects if the individual trigger B^*_i is hit.

Conjecture 6.2 (Operating net effects of project bundling in a combined firm under full capital restriction)**.** *With no external funds available, the effect of bundling, which increases with decreasing correlation ρ, may surpass the benefit of holding separate abandonment options for basic firm and project. Whether bundling in a combined entity creates positive operating net effects depends upon the specific parameter settings as depicted in Fig. 5 and 6. The following conclusions regarding parameter settings can be made:*

1. *A higher portion of fixed costs F within the cost structure of the two projects has a positive impact on the operating net effect of bundling ΔV^A (see settings "Difference 2" and "Level 2" vs. "Difference 1" and "Level 1" in Fig. 5).*

2. *A higher heterogeneity of cost structures of the projects has a positive impact on the operating net effect of bundling ΔV^A (see upper-left graph of Fig. 6).*

3. *An increasing overall drift rate level μ results in higher net effects of bundling ΔV^A (see settings "Level 1" vs. "Level 2" in Fig. 5).*

4. *A jointly negative drift rate μ of the combined firm leads to negative net effects of bundling ΔV^A (see settings "Level 1" vs. "Level 2" in Fig. 5).*

5. *A higher heterogeneity of drift rates μ_i has a negative impact on the operating net effects of bundling ΔV^A (see upper-right graph of Fig. 6).*

6. *An increasing overall volatility level σ results in lower net effects of bundling ΔV^A (see settings "Level 1" vs. "Level 2" in Fig. 5).*

7. *A higher heterogeneity of volatilities σ_i has a positive impact on the operating net effects of bundling ΔV^A (see lower-left graph of Fig. 6).*

The results of this chapter provide important economic implications, although the effects of capital structure choices have not been included into the analysis, yet. Initially, if there is a complete capital market with symmetric distribution of information, diversifying a firm's revenue base will not deliver

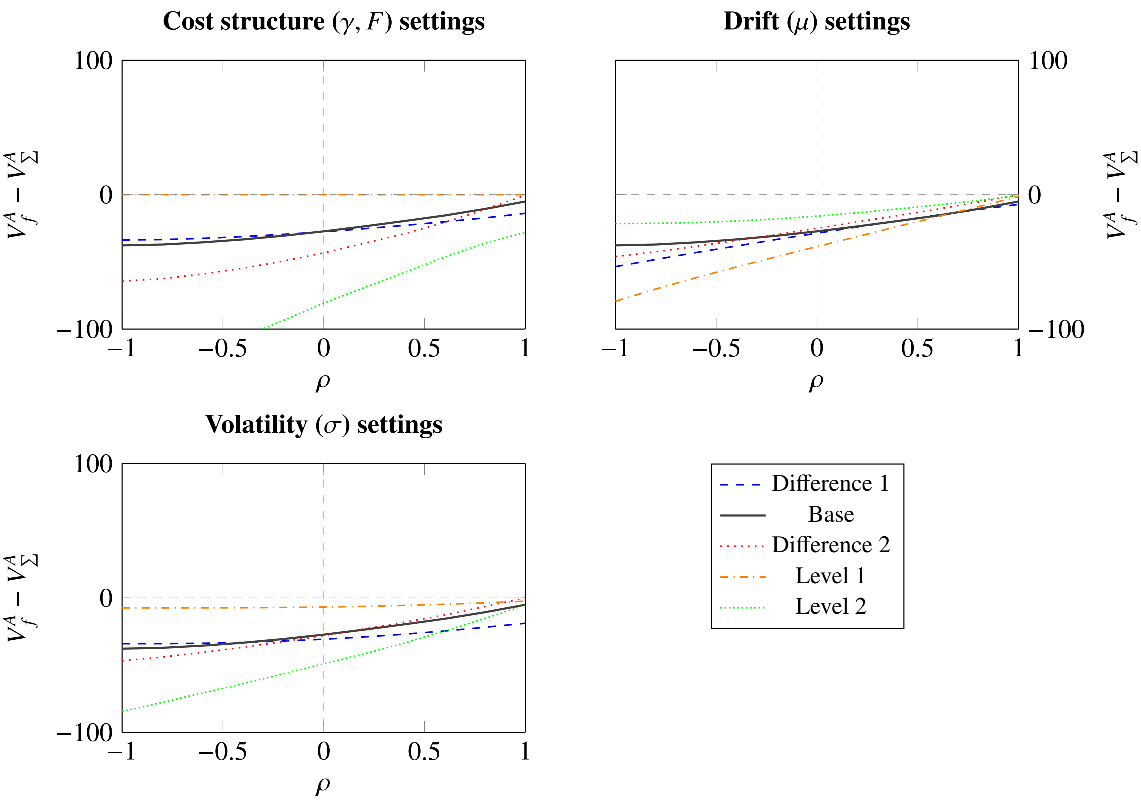

Figure 3: Operating net effect of bundling (ΔV^A) with unlimited capital access for different parameter settings. All graphs compare the outcomes of ΔV^A for correlations $\rho \in (-1, 1)$. The upper-left graph depicts all settings with varying cost structure (γ, F), the upper-right one all settings with varying drift rate μ and the lower-left one all settings with varying volatility σ. The settings examined for each parameter are "Difference 1" (blue, dashed), "Base" (black, solid), "Difference 2" (red, dotted), "Level 1 (orange, dashdotted) and "Level 2" (green, densely dotted).

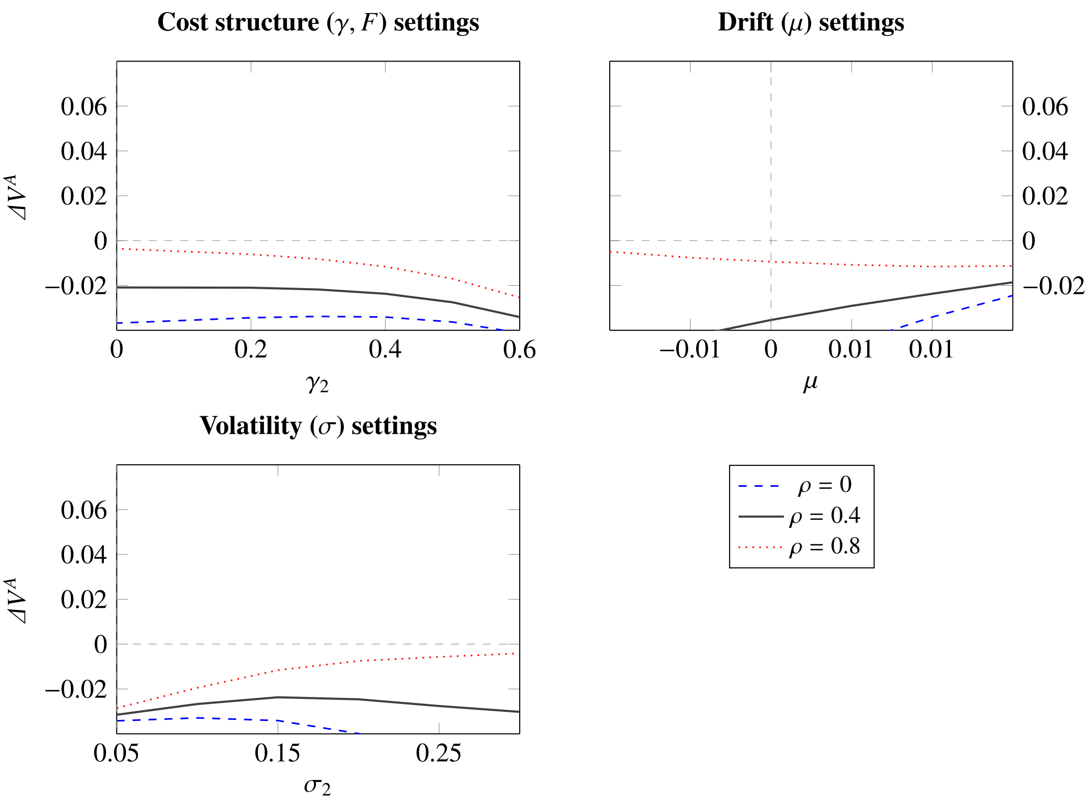

Figure 4: Operating net effect of bundling (ΔV^A) with unlimited capital access for varying parameters of project $i = 2$. All graphs compare the outcomes of ΔV^A for varying parameters of project $i = 2$ under three correlation setups: $\rho = 0.0$ (blue, dashed), $\rho = 0.4$ (black, solid) and $\rho = 0.8$ (red, dotted). The upper-left graph depicts varying cost structures (γ_2, F_2), the upper-right one varying drift rates μ_2 and the lower-left one varying volatilities σ_2. Concerning cost structures, F_2 is adjusted by $\Delta\gamma_2 R_{2,0}$ for any change (Δ) of γ_2. All parameters not varied in the respective analysis are defined according to the base set.

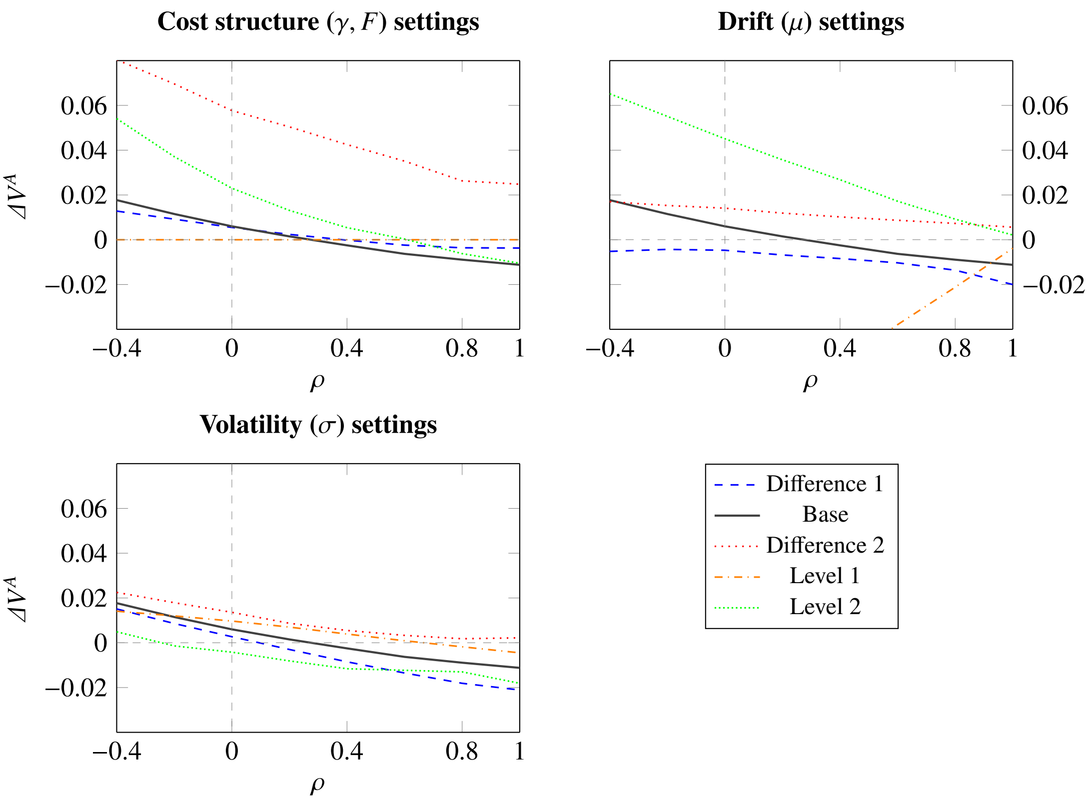

Figure 5: Operating net effect of bundling ($\varDelta V^A$) under full capital restriction for different parameter settings. All graphs compare the outcomes of $\varDelta V^A$ for correlations $\rho \in (-1, 1)$. The upper-left graph depicts all settings with varying cost structure (γ, F), the upper-right one all settings with varying drift rate μ and the lower-left one all settings with varying volatility σ. The settings examined for each parameter are "Difference 1" (blue, dashed), "Base" (black, solid), "Difference 2" (red, dotted), "Level 1 (orange, dashdotted) and "Level 2" (green, densely dotted).

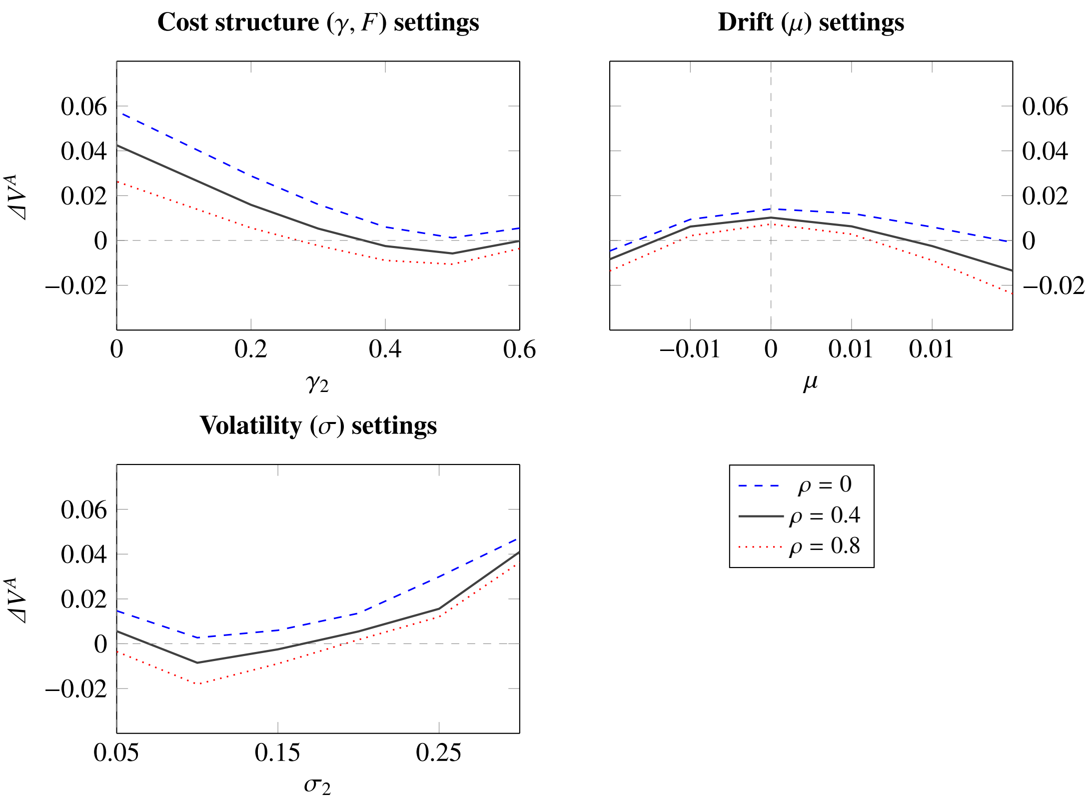

Figure 6: Operating net effect of bundling (ΔV^A) under full capital restriction for varying parameters of project $i = 2$. All graphs compare the outcomes of ΔV^A for varying parameters of project $i = 2$ under three correlation setups: $\rho = 0.0$ (blue, dashed), $\rho = 0.4$ (black, solid) and $\rho = 0.8$ (red, dotted). The upper-left graph depicts varying cost structures (γ_2, F_2), the upper-right one varying drift rates μ_2 and the lower-left one varying volatilities σ_2. Concerning cost structures, F_2 is adjusted by $\Delta\gamma_2 R_{2,0}$ for any change (Δ) of γ_2. All parameters not varied in the respective analysis are defined according to the base set.

any value-add. However, under full capital restrictions, i.e., there is neither access to external funds nor to internal excess cash, bundling diversified ($\rho < 1$) activities in a firm regularly provides value which increases with decreasing ρ. In practice, most firms face a mix of the two extreme scenarios where, for instance, the access to fresh capital is possible but costly due to agency costs generated by information asymmetries. In the light of these insights, it becomes evident that structural models like this are able to offer a novel perspective on the long lasting discussion on the rationale of building conglomerates. Investors and firms operating in less developed capital markets shall tend more towards building conglomerates than public firms operating in very well developed capital markets.

The insights of this analysis appear to be in particular important for firms and projects with highly positive growth assumptions[14], high portions of fixed costs, high cash flow volatilities and, in general, for firms operating very heterogenous projects or segments.

6.3. Combined firm versus project approach: The impact on net benefits of debt

Turning towards the impact of financing on the decision to bundle projects in a combined firm, I start by examining the generated values of tax shields and values of bankruptcy costs in the endogenous model and in the exogenous one. Fig. 7 compares the two models for the base setting.

The first observation from Fig. 7 is concerned with the net benefits of debt where I identify a significant value-add effect of bundling for correlations $\rho < 1$ in the base setting for both models. As expected, the total net benefit of debt and also the optimal leverage ratio is higher on an absolute level in the endogenous model, irrespective of whether bundling takes place. However, the effect of bundling versus non-bundling is more severe in the exogenous model, where the net benefit of debt of the combined firm exceeds the sum of the two separate entities by 287 percent for $\rho = -1$ (47 percent in the endogenous model) and by 51 percent for $\rho = 0$ (8 percent in the endogenous model). Over all correlations, $\rho \in (-1, 1)$, I find a premium of 65 percent in the exogenous model when examining the base setting.

Testing all parameter settings (see Fig. 8-11) illustrates one more general, and well-expected, insight: Keeping the basic firm and the new project separate tends to be value-destroying for lower degrees of correlation and can become, depending on the specific parameter setup, value-enhancing for higher degrees of correlation. Beyond that, I again postulate specific Conjectures regarding the impact of parameters for both scenarios of capital access and begin with the endogenous model:

Conjecture 6.3 (Financial net effects of project bundling in a combined firm under unlimited capital access)**.** *With unlimited capital access, bundling projects, where revenues $\tilde{R}_{i,t}$ interact with correlation $\rho < 1$, creates a positive financial net effect which decreases with increasing ρ. Whether non-bundling is more beneficial due to the flexibility gained (possibility of choosing capital structures individually) depends upon the specific parameter setting as depicted in Fig. 8 and 9. The following conclusions regarding parameter settings can be made:*

1. *A higher portion of fixed costs F within the cost structure of the two projects has a negative impact on the financial net effect of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 8).*
2. *An increasing overall drift rate level μ results in higher financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 8).*
3. *A higher heterogeneity of drift rates μ_i has a negative impact on the financial net effects of bundling ΔV^{NB} (see upper-right graph of Fig. 9).*

[14] Results are obviously also significant for firms and projects with negative growth assumptions as bundling leads to value destruction.

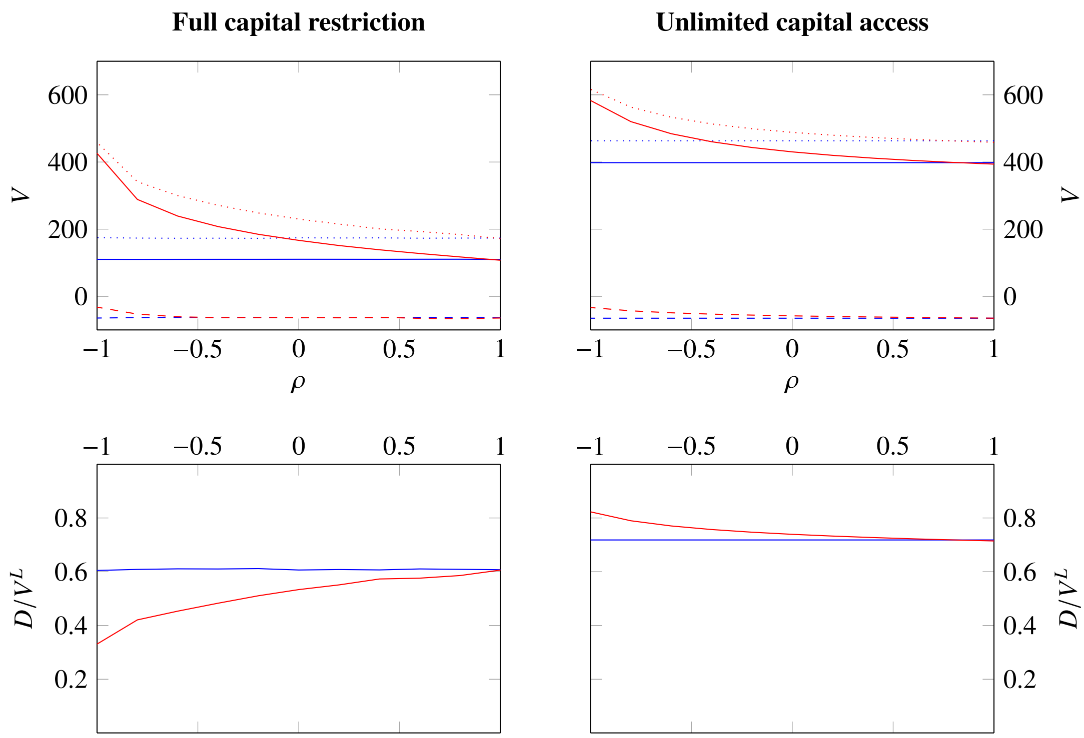

Figure 7: Comparison of value and leverage effects of capital structure optimization for the combined firm vs. the sum of the two separate entities. The graphs on the left hand side show results under full capital restriction while graphs on the right hand side depict outcomes for unlimited capital access. In all graphs, the blue lines represent the sum of the two separate entities and the red ones describe the combined firm, both always along correlations ranging from $\rho \in (-1, 1)$. The upper graphs depict the values of net benefits of debt, V_f^{NB} and V_Σ^{NB} (solid lines), the values of tax shields, V_f^{TB} and V_Σ^{TB} (dotted lines), and the values of bankruptcy costs, V_f^{BC} and V_Σ^{BC} (dashed lines). The lower graphs compare the leverage ratios, D_f/V_f^L and D_Σ/V_Σ^L. Results rest upon the base set of parameters.

4. *An increasing overall volatility level σ results in lower financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 8).*

5. *A higher heterogeneity of volatilities σ_i has a negative impact on the financial net effects of bundling ΔV^{NB} (see mid-left graph of Fig. 9).*

6. *Increasing overall bankruptcy costs α result in higher financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 8).*

7. *A higher heterogeneity of bankruptcy costs α_i has a negative impact on the financial net effects of bundling ΔV^{NB} (see mid-left graph of Fig. 9).*

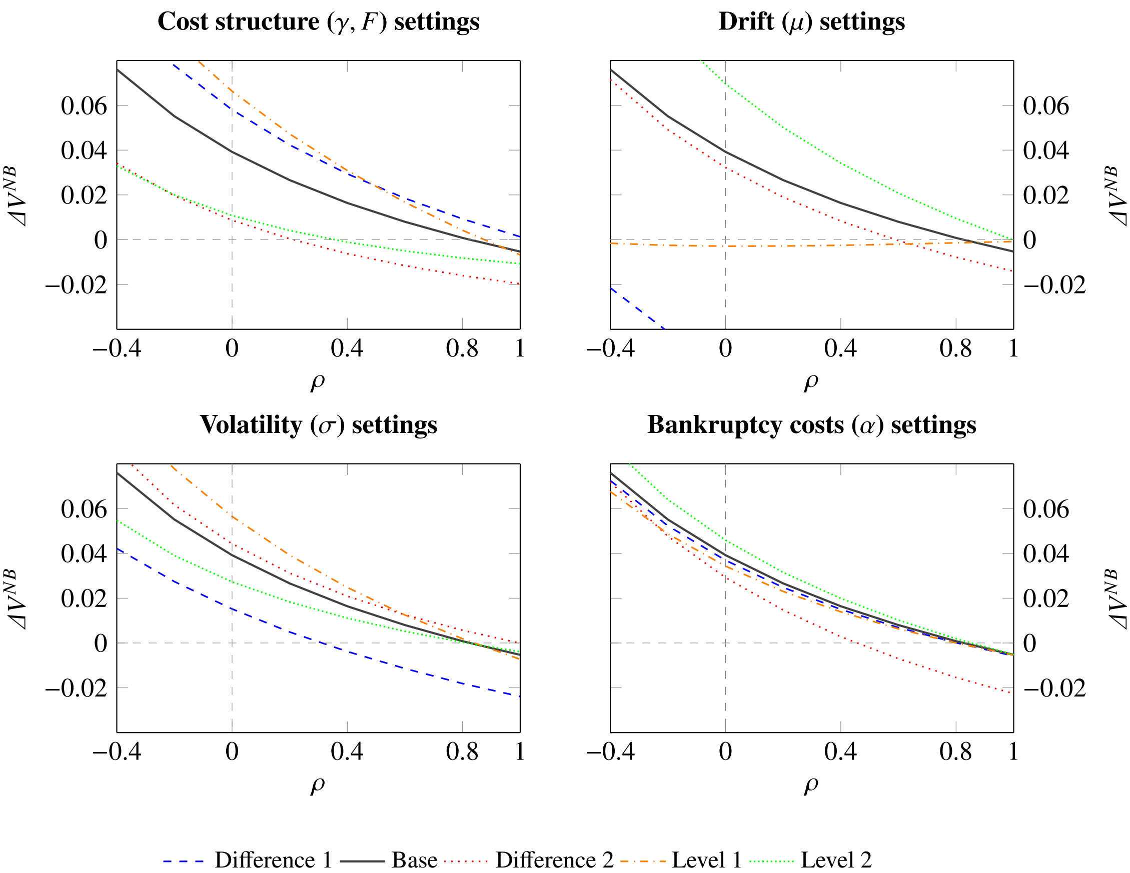

Figure 8: Financial net effect of bundling (ΔV^{NB}) with unlimited capital access for different parameter settings. All graphs compare the outcomes of ΔV^{NB} for correlations $\rho \in (-1, 1)$. The upper-left graph depicts all settings with varying cost structure (γ, F), the upper-right one all settings with varying drift rate μ, the lower-left one all settings with varying volatility σ and the lower-right one all settings with varying bankruptcy costs α. The settings examined for each parameter are "Difference 1" (blue, dashed), "Base" (black, solid), "Difference 2" (red, dotted), "Level 1 (orange, dashdotted) and "Level 2" (green, densely dotted).

Besides the already postulated general findings regarding the financial net effects in the exogenous model, Conjecture 6.4 summarizes specific insights with respect to the model parameters:

Conjecture 6.4 (Financial net effects of project bundling in a combined firm under full capital restriction)**.** *With no external funds available, bundling projects, where revenues $\tilde{R}_{i,t}$ interact with correlation $\rho < 1$,*

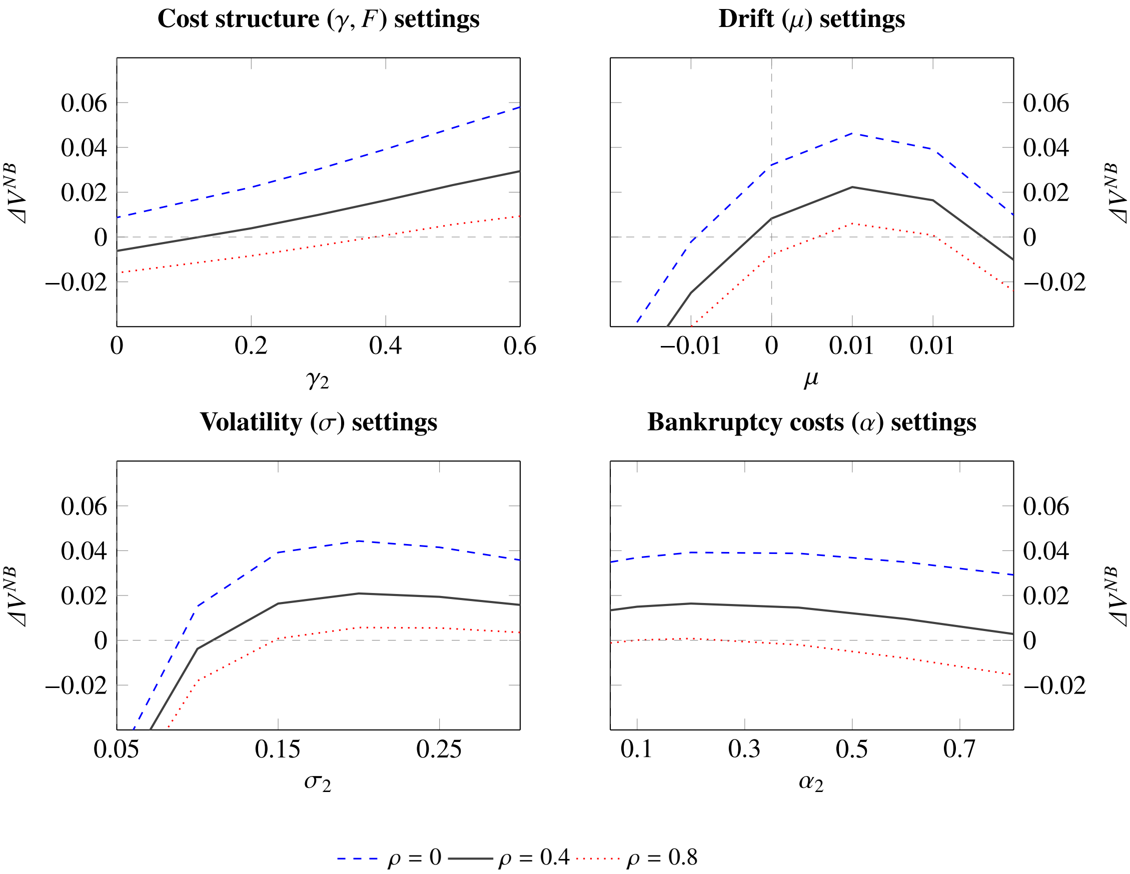

Figure 9: Financial net effect of bundling (ΔV^{NB}) with unlimited capital access for varying parameters of project $i = 2$. All graphs compare the outcomes of ΔV^{NB} for varying parameters of project $i = 2$ under three correlation setups: $\rho = 0.0$ (blue, dashed), $\rho = 0.4$ (black, solid) and $\rho = 0.8$ (red, dotted). The upper-left graph depicts varying cost structures (γ_2, F_2), the upper-right one varying drift rates μ_2, the lower-left one varying volatilities σ_2 and the lower-right one varying bankruptcy costs α_2. Concerning cost structures, F_2 is adjusted by $\Delta\gamma_2 R_{2,0}$ for any change (Δ) of γ_2. All parameters not varied in the respective analysis are defined according to the base set.

creates a positive financial net effect which decreases with increasing ρ. Whether non-bundling is more beneficial due to the flexibility gained (possibility of choosing capital structures individually) depends upon the specific parameter settings as depicted in Fig. 10 and 11. The following conclusions regarding parameter settings can be made:

1. *A higher portion of fixed costs F within the cost structure of the two projects has a negative impact on the financial net effect of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 10).*
2. *A higher heterogeneity of cost structures of the projects has a negative impact on the financial net effect of bundling ΔV^{NB} (see upper-left graph of Fig. 11).*
3. *An increasing overall drift rate level μ results in lower financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 10).*
4. *A higher heterogeneity of drift rates μ_i has a positive impact on the financial net effects of bundling ΔV^{NB} (see upper-right graph of Fig. 11).*
5. *An increasing overall volatility level σ results in lower financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 10).*
6. *A higher heterogeneity of volatilities σ_i has a negative impact on the financial net effects of bundling ΔV^{NB} (see mid-left graph of Fig. 11)*
7. *Increasing overall bankruptcy costs α result in lower financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 10).*
8. *A higher heterogeneity of bankruptcy costs α_i has a negative impact on the financial net effects of bundling ΔV^{NB} (see mid-right graph of Fig. 11).*
9. *Increasing overall coverage ratios δ result in lower financial net effects of bundling ΔV^{NB} (see settings "Level 1" vs. "Level 2" in Fig. 10).*
10. *A higher heterogeneity of coverage ratios δ_i has a negative impact on the financial net effects of bundling ΔV^{NB} (see lower-left graph of Fig. 11).*

Overall, the net effects of bundling have a higher magnitude with no external funds available in comparison to the model with unlimited capital access.

Again, the analysis points towards an important economic implication as merging activities in a combined firm is more relevant if the access to external funds is more difficult. In general, separating projects or activities in independent legal entities will be rather beneficial, if projects are very heterogenous regarding their cost structures, cash flow volatilities as well as bankruptcy costs, and if the correlation of the projects' revenues is rather high. Moreover, the analysis reveals that risky firms or projects, i.e., firms with highly volatile activities, shall lean towards separation. This result is congruent with the analysis of operating net effects and in accordance with the findings of Leland (2007) and Banal-Estanol et al. (2013).

7. Conclusion

This article shades further light on an investor's or firm's choice between setting up a legally independent entity with nonrecourse financing to run a new project and integrating the new project in an existing firm with shared financing. The analysis is based on a stochastic model incorporating options (or obligations) to abandon operations (under full-equity financing) and to file for bankruptcy (under partial-debt

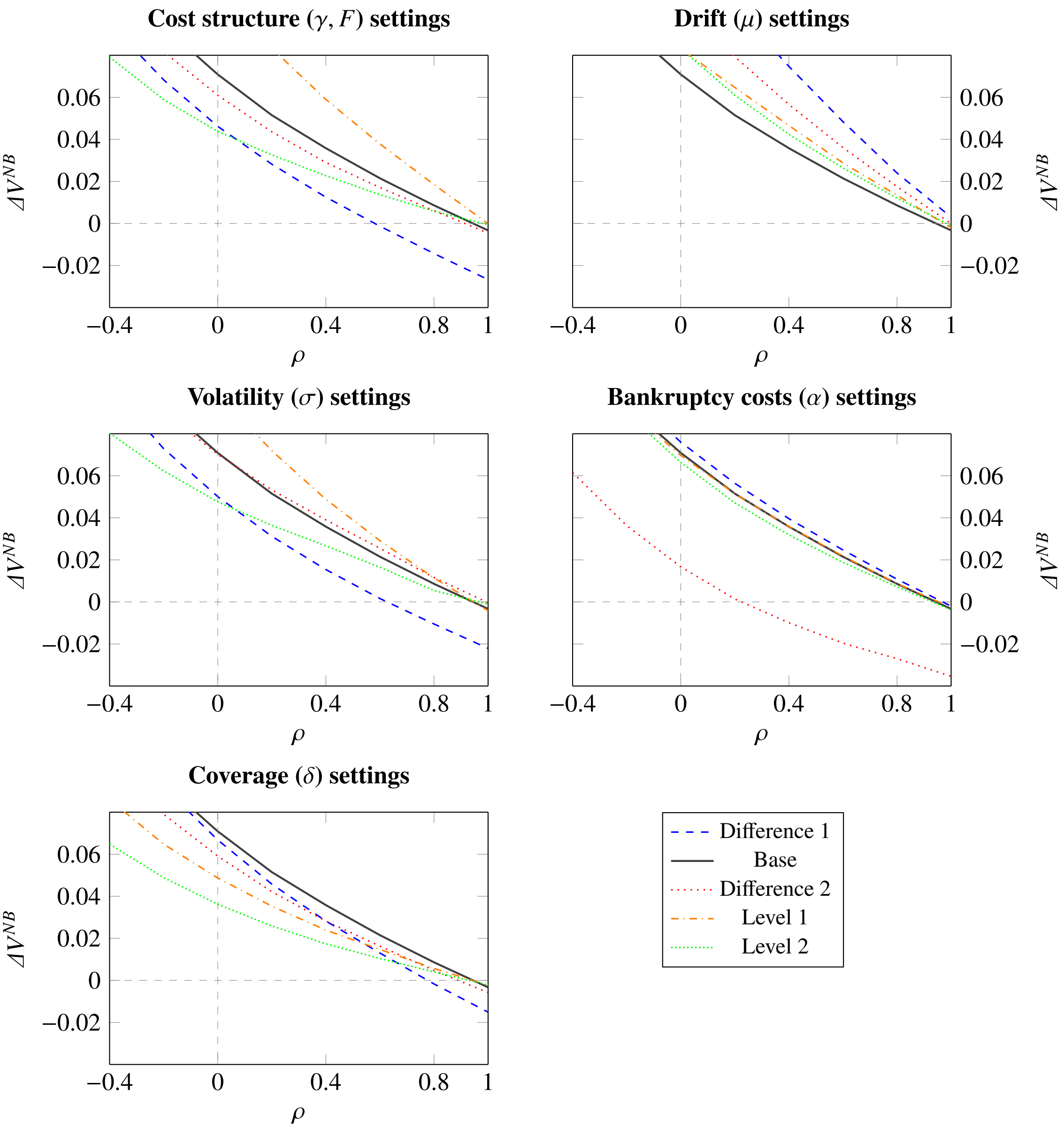

Figure 10: Financial net effect of bundling ($\varDelta V^{NB}$) under full capital restriction for different parameter settings. All graphs compare the outcomes of $\varDelta V^{NB}$ for correlations $\rho \in (-1, 1)$. The upper-left graph depicts all settings with varying cost structure (γ, F), the upper-right one all settings with varying drift rate μ, the mid-left one all settings with varying volatility σ, the mid-right one all settings with varying bankruptcy costs α and the lower-left one all settings with varying coverage ratio δ. The settings examined for each parameter are "Difference 1" (blue, dashed), "Base" (black, solid), "Difference 2" (red, dotted), "Level 1 (orange, dashdotted) and "Level 2" (green, densely dotted).

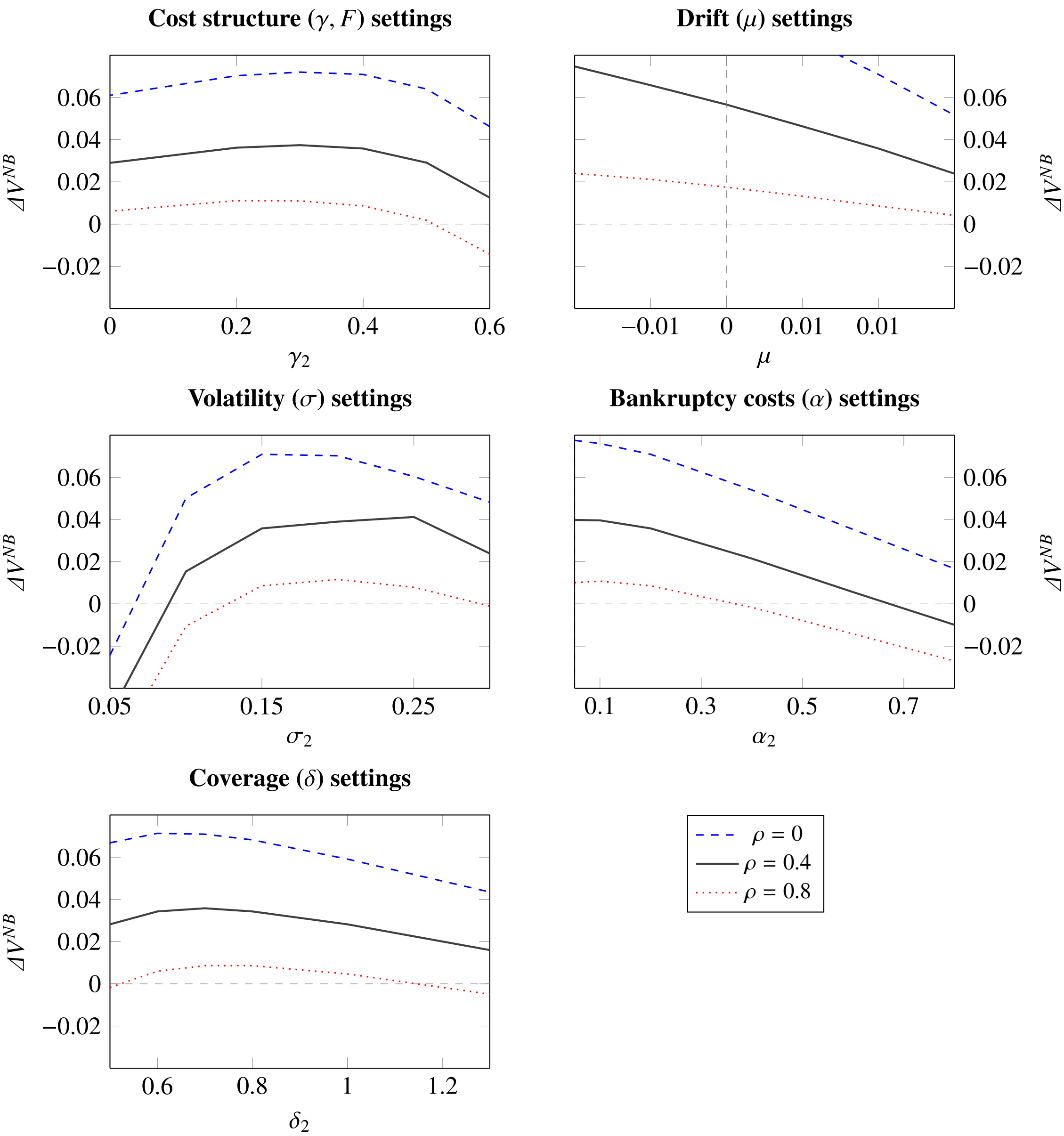

Figure 11: Financial net effect of bundling (ΔV^{NB}) under full capital restriction for varying parameters of project $i = 2$. All graphs compare the outcomes of ΔV^{NB} for varying parameters of project $i = 2$ under three correlation setups: $\rho = 0.0$ (blue, dashed), $\rho = 0.4$ (black, solid) and $\rho = 0.8$ (red, dotted). The upper-left graph depicts varying cost structures (γ_2, F_2), the upper-right one varying drift rates μ_2, the mid-left one varying volatilities σ_2, the mid-right one varying bankruptcy costs α_2 and the lower-left one varying coverage ratios δ_2. Concerning cost structures, F_2 is adjusted by $\Delta\gamma_2 R_{2,0}$ for any change (Δ) of γ_2. All parameters not varied in the respective analysis are defined according to the base set.

financing). Operational synergies and the explicit reflection of agency conflicts are excluded to isolate the pure benefits and drawbacks stemming from operational and financial risks. By examining two regimes of capital access, unlimited access versus internal capital only, implicit conclusions regarding agency costs can be drawn.

In the sections 3 and 4, I derive closed-form solutions for separate projects regarding the corporate valuation framework outlined in section 2. Although this step is only a necessary prerequisite for the model comparing the choice described above, I identify important economic implications from it: Irrespective of chosen capital structures, the option or obligation to abandon is a relevant valuation component in all setups with fixed costs. It is always positive with unlimited capital access and under full capital restriction if the revenue's drift rate is negative, while it turns negative under full capital restriction with positive drift rates. Not reflecting on abandonment leads to misrepresentations of corporate values, in particular for areas with high fixed costs.

As the translation of the developed corporate valuation framework towards the combined firm requires numerical methods, section 5 present a simulation-based approach to the problem while findings are discussed in section 6. First, the analysis reveals that with unlimited capital access, there is no operating value-add, i.e., no positive abandonment option value, from merging activities. In other words, under full-equity finance and with investors that can unrestrictedly add further funds, merging activities does not create any value in the absence of operational synergies. Second, with only internal funds available positive effects of bundling projects are regularly generated for the abandonment option value (operating net effect) and for the net benefits of debt (financial net effect) with decreasing correlation. Financial net effects of bundling turn also positive with unlimited capital access and decreasing correlation. However, the magnitude of effects is higher under full capital restriction. These results clearly indicate that merging activities in a combined firm is more beneficial if the access to external funds is difficult. Thus, it can be expected that conglomerate-like structures are more prevalent for private firms and in less developed capital markets, while public firms operating in very well developed capital markets may tend towards separation.

Regarding the parameter settings of the existing firm and the additional project, I identify that heterogeneity with respect to cost structures, growth, risk and bankruptcy costs lowers the financial net effects of bundling irrespective of the underlying capital access regime. This finding would economically indicate a tendency towards mergers of rather similar partners and for corporate financing of projects similar to the existing business. My results are in accordance with Leland (2007) for risk and bankruptcy costs.[15] With respect to the level of parameters, the analysis reveals that high risk, i.e., high volatility of cash flows, and a high portion of fixed costs within the cost structure support a project structure with independent legal entities as the likelihood of risk contamination increases in such settings. The same holds true for growth, i.e., the revenue's expected drift rate, and bankruptcy costs, but only in the exogenous model as the abandonment and bankruptcy triggers are not endogenously influenced by the parameter setting. In the endogenous model, the effects stemming from higher growth and higher bankruptcy costs flip as the respective triggers are adjusted by the equity investors. The outcomes for growth, risk and bankruptcy costs in the exogenous model match with the findings of Banal-Estanol et al. (2013).[16]

With the propositions and conjectures generated, this article presents testable hypotheses for the empirical analysis of project financing, an area that is still not well developed according to Müllner (2017), and of conglomerate discounts or the optimal scope of the firm, a literature strand that is "rich but still inconclusive" (Leland, 2007, p. 769). Moreover, the model may provide a solid base for incorporating agency conflicts and by such creating a more realistic setup between the two extreme scenarios of capital access. Another avenue for further research might be to incorporate other important capital struc-

15 Cost structures and growth are not considered in his work.

16 The article does not provide cost structure analysis and does not include an endogenous version of default.

ture choices, e.g., maturities or seniorities, in order to understand how those interact with the underlying choice of this article.

References

References

Banal-Estanol, A., Ottaviani, M., Winton, A., 2013. The flip side of financial synergies: coinsurance versus risk contamination. The Review of Financial Studies 26 (12), 3142–3181.

Brealey, R. A., Cooper, I. A., Habib, M. A., 1996. Using project finance to fund infrastructure investments. Journal of Applied Corporate Finance 9 (3), 25–39.

Chemmanur, T., John, K., 1996. Optimal incorporation, structure of debt contracts, and limited-recourse project financing. Journal of Financial Intermediation 5, 372–408.

Couch, R., Dothan, M., Wu, W., 2012. Interest tax shields: A barrier options approach. Review of Quantitative Finance and Accounting 39, 123–146.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainity. Princeton University Press, Princeton, New Jersey.

Etsy, B. C., 2004. Why study large projects? an introduction to research on project finance. European Financial Management 10 (2), 213–224.

Etsy, B. C., 2014. An overview of project finance and infrastructure finance, 2014 update. Harvard Business CaseHBS Case No. 214083.

Etsy, B. C., Christov, I., 2002. An overview of the project finance market - 2002 update. Harvard Business School Technical Note No. 202-105.

Fabozzi, F. J., Davis, H. A., Choudry, M., 2006. Introduction to Structured Finance, 1st Edition. Wiley Finance.

Flannery, M., Houston, J., Venkataraman, S., 1993. Financing multiple investment projects. Financial Management 22, 161–172.

Gatti, S., 2012. Project Finance in Theory and Practice â€“ Designing, Structuring, and Financing Private and Public Projects, 2nd Edition. Academic Press Advanced Finance Series.

Goldstein, R., Ju, N., Leland, H. E., 2001. An EBIT-based model of dynamic capital structure. Journal of Business 74 (4), 483–512.

Hackbarth, D., Hennessy, C., Leland, H. E., 2007. Can the trade-off theory explain debt structure? The Review of Financial Studies 20 (5), 1389–1428.

Hackbarth, D., Mauer, D. C., 2012. Optimal priority structure, capital structure, and investment. The Review of Financial Studies 25, 747–796.

Higgins, R., Schall, L., 1975. Corporate bankruptcy and conglomerate merger. The Journal of Finance 30, 93–113.

John, T. A., 1993. Optimality of spinoffs and allocation of debt. Journal of Financial and Quantitative Analysis 28, 139–160.

John, T. A., John, K., 1991. Optimality of project financing: Theory and empirical implications in finance and accounting. Review of Quantitative Finance and Accounting 1 (1), 51–74.

Kensinger, J. W., Martin, J. D., 1988. Project finance: raising money the old-fashioned way. Journal of Applied Corporate Finance 1 (3), 69–81.

Kim, H. E., McConnell, J., 1977. Corporate mergers and the co-insurance of corporate debt. The Journal of Finance 32, 349–365.

Kutzker, T., Schreiter, M., 2019. The optimal capital structure under risks of illiquidity and over-indebtedness in a double barrier option framework, working Paper.

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. The Journal of Finance 49 (4), 1213–1252.

Leland, H. E., 2007. Financial synergies and the optimal scope of the firm: Implications for mergers, spinoffs, and off-balance sheet finance. The Journal of Finance 62 (2), 765–807.

Lewellen, W., 1971. A pure financial rationale for the conglomerate merger. The Journal of Finance 26, 521–537.

Modigliani, F., Miller, M. H., 1958. The cost of capital, corporation finance and the theory of investment. The American Economic Review 48 (3), 261–297.

Müllner, J., 2017. International project finance: review and implications for international finance and international business. Management Review Quarterly 67 (2), 97–133.

Nevitt, P. K., Fabozzi, F. J., 2000. Project financing, 7th Edition. Euromoney Books.

Pinto, J. M., Alves, P. P., 2016. The choice between project financing and corporate financing: Evidence from the corporate syndicated loan market, working Paper.

Project Finance International, 2019. Pfi financial league tables 2018. Tech. rep., Project Finance International.

Sarig, O., 1985. On mergers, divestments, and options: A note. Journal of Financial and Quantitative Analysis 20, 385–389.

Scott, J., 1977. On the theory of corporate mergers. The Journal of Finance 32, 1235–1250.

Shastri, K., 1990. The differential effects of mergers on corporate security values. Research in Finance 8, 179–201.

Societe Generale, 2018. Debt capital markets - 2018 review and 2019 forecast. Tech. rep., Societe Generale.

Strebulaev, I. A., 2007. Do tests of capital structure theory mean what they say? The Journal of Finance 62 (4), 1747–1787.

Strebulaev, I. A., Whited, T. M., 2011. Dynamic models and structural estimation in corporate finance. Foundations and Trends in Finance 6 (1-2), 1–163.

Stulz, R. M., Johnson, H., 1985. An analysis of secured debt. Journal of FInancial Economics 14, 501–521.

Wong, K. P., 2009. The effects of abandonment options on operating leverage and investment timing. International Review of Economics & Finance 18 (1), 162–171.

A. Appendix: Derivation of the optimal boundary of abandonment B_i^*

In order to find the boundary of abandonment B_i^* that satisfies $\max_{B_i \leq R_{i,0}} V_{i,0}^U(\tilde{R}_{i,t})$, the first order condition

$$\frac{\delta V_{i,0}^U(\tilde{R}_{i,t})}{\delta B_i} = 0 \tag{A.1}$$

needs to be solved. Based on $V_{i,0}^U(\tilde{R}_{i,t})$ as defined in Eq. (3.7) and P_{Bi} from Eq. (3.5), I arrive at

$$\begin{aligned} 0 =& y_i \frac{F_i(1-\tau)}{r} \left(\frac{B_i}{R_{i,0}}\right)^{y_i} \frac{1}{B_i} - y_i \frac{(1-\gamma_i)(1-\tau)}{r-\mu_i} \left(\frac{B_i}{R_{i,0}}\right)^{y_i} \\ &- \frac{(1-\gamma_i)(1-\tau)}{r-\mu_i} \left(\frac{B_i}{R_{i,0}}\right)^{y_i}. \end{aligned} \tag{A.2}$$

Rearranging Eq. (A.2) for B_i yields

$$B_i^* = \frac{y_i}{1+y_i} \frac{r-\mu_o}{r} \frac{F_i}{1-\gamma_i}, \tag{A.3}$$

where B_i^* represents the optimal level of B_i.

B. Appendix: Proof of proposition 3.1

As the model's stochastic revenues $\tilde{R}_i$ bear idiosyncratic risk, we know from Eq. (2.2) that $r > \mu_i$. As long as $r > 0$ holds, it follows from Eq. (3.6) that $y_i > 0$.

Thus, all single terms of Eq. (3.11) are positive with $F_i > 0$:

$$V_{i,0}^{A,*}(\tilde{R}_{i,t}) = \frac{F_i(1-\tau)}{r} \frac{1}{1+y_i} \left(\frac{y_i}{1+y_i} \frac{r-\mu_i}{r} \frac{F_i}{R_{0i}(1-\gamma_i)}\right)^{y_i} > 0. \tag{B.1}$$

C. Appendix: Proof of proposition 3.2

With fixed costs $\mu_i < 0$ and $F_i > 0$, the inequality $V_{i,0}^{A,ex}(\tilde{R}_{i,t}) > 0$ collapses to

$$\begin{aligned} &V_{i,0}^{A,ex}(\tilde{R}_{i,t}) > 0 \\ &\left(\frac{1}{r} \frac{1}{r-\mu_i}\right) > 0. \end{aligned} \tag{C.1}$$

As long as $r > 0$ holds and with $r > \mu_i$ (see Eq. (2.2)), Ineq. (C.1) holds with $\mu_i < 0$ and flips with $\mu_i > 0$.

D. Appendix: Proof of proposition 4.2

The optimal coupon level C_i^* is set to maximize the levered firm value $V_{i,0}^L(\tilde{R}_{i,t})$, subject to the endogenously chosen bankruptcy trigger H_i^* as derived in Proposition 4.1, Eq. (4.12).

Substituting H_i^* and the optimal abandonment trigger B_i^* from Eq. (3.9) into $V_{i,0}^L(\tilde{R}_{i,t}$ as defined under Eq. (4.8) yields the following optimization problem:

$$V_{i,0}^L(\tilde{R}_{i,t}, C_i) = V_{i,0}^U(\tilde{R}_{i,t}) + \frac{\tau C_i}{r} - \left(\frac{(C_i + F_i)(r - \mu_i)y_i}{R_{i,0}(1 - \gamma_i)r(1 + y_i)}\right)^{y_i} \left[\frac{\tau C_i}{r} - \alpha_i \left(\frac{(C_i + F_i)(1 - \tau)}{r}\right.\right.$$
$$\left.\left.\frac{y_i}{1 + y_i} - \frac{F(1 - \tau)}{r}\left(1 - \frac{1}{1 + y_i}\left(\frac{F_i}{C_i + F_i}\right)^{y_i}\right)\right)\right] \rightarrow \max. \tag{D.1}$$

By forming the first derivative of $V_{i,0}^L(\tilde{R}_{i,t}, C_i)$ subject to C_i), setting it equal to zero and rearranging terms, I obtain

$$\begin{aligned}
\frac{dV_{i,0}^L}{dC_i} &\stackrel{!}{=} 0 \\
\frac{\tau}{r} &\stackrel{!}{=} \left(\frac{(C_i^* + F_i)(r - \mu_i)y_i}{R_{i,0}(1 - \gamma_i)r(1 + y_i)}\right)^{y_i} \\
&\left[\frac{\tau}{r}\left(1 + y\frac{C_i^*}{C_i^* + F_i}\right) + \frac{\alpha_i(1 - \tau)y_i}{r(1 + y_i)}\left(1 + y_i - \frac{F_i(1 + y_i)}{C_i^* + F_i}\right)\right],
\end{aligned} \tag{D.2}$$

where C_i^* represents the optimal coupon level.

Given that $r > 0$ and $\mu_i < r$, y_i has to be strictly positive. Hence, by applying the power of $1/y_i$ to both sides of Eq. (D.2) and rearranging for C_i^*, I arrive at the optimal coupon level

$$C_i^* = \frac{r}{r - \mu_i}\frac{1 + y_i}{y_i}\left(1 + \frac{y_i}{C_i^* + F_i}\left(C_i^*\left(1 + \alpha_i\frac{1 - \tau}{\tau}\right) + 2F_i\alpha_i\frac{1 - \tau}{\tau}\right)\right)^{-\frac{1}{y_i}} R_{0,1}(1 - \gamma_i) - F_i, \tag{D.3}$$

where C_i^* remains on the right-hand side preventing an analytic solution to the problem but allowing for numerical solution. However, for the special case of fixed costs equal to zero, Eq. (D.3) collapses to

$$C_{i,F=0}^* = \frac{r}{r - \mu_i}\frac{1 + y_i}{y_i}\left(1 + \frac{y_i}{+}\alpha_i y_i\frac{1 - \tau}{\tau}\right)^{-\frac{1}{y_i}} R_{0,1}(1 - \gamma_i), \tag{D.4}$$

which can be solved analytically.

E. Appendix: Proof of proposition 4.3

The optimal coupon level C_i^{ex} with debt secured via an interest coverage ratio is set to maximize the levered firm value $V_{i,0}^L(\tilde{R}_{i,t})$. However, in this setup there is no side constraint with respect to the bankruptcy trigger as this is exogenously given by H_i^{ex} as defined in Eq. (4.1).

Substituting H_i^{ex} and the exogenous abandonment trigger B_i^{ex} from Eq. (3.2) into $V_{i,0}^L(\tilde{R}_{i,t}$ as defined under Eq. (4.8) yields the following optimization problem:

$$V_{i,0}^L(\tilde{R}_{i,t}, C_i) = V_{i,0}^U(\tilde{R}_{i,t}) + \frac{\tau C_i}{r} - \left(\frac{\delta_i C_i + F_i}{R_{i,0}(1 - \gamma_i)}\right)^{y_i}\left[\frac{\tau C_i}{r} - \alpha_i\left(\frac{(\delta_i C_i + F_i)(1 - \tau)}{r - \mu_i}\right.\right.$$
$$\left.\left. - \frac{F_i(1 - \tau)}{r}\left(1 - \frac{1}{1 + y_i}\left(\frac{y_i F_i(r - \mu_i)}{(1 + y_i)(\delta_i C_i + F_i)r}\right)^{y_i}\right)\right)\right] \rightarrow \max. \tag{E.1}$$

By forming the first derivative of $V_{i,0}^L(\tilde{R}_{i,t}, C_i)$ subject to C_i), setting it equal to zero and rearranging terms, I obtain

$$
\begin{aligned}
\frac{dV^L_{i,0}}{dC_i} &\overset{!}{=} 0 \\
\frac{\tau}{r} &\overset{!}{=} \left(\frac{\delta_i C^{ex}_i + F_i}{R_{i,0}(1-\gamma_i)}\right)^{y_i} \\
&\left[\frac{\tau}{r} + \frac{y_i \delta_i}{r(\delta_i C^{ex}_i + F_i)}\left(\tau C^{ex}_i - \alpha_i F_i (1-\tau)\right) + \frac{\alpha_i \delta_i (1-\tau)}{r-\mu_i}(1+y_1)\right],
\end{aligned}
\tag{E.2}
$$

where C^{ex}_i represents the optimal coupon level with debt secured via an interest coverage ratio.

Given that $r > 0$ and $\mu_i < r$, y_i has to be strictly positive. Hence, by applying the power of $1/y_i$ to both sides of Eq. (E.2) and rearranging for C^{ex}_i, I arrive at the optimal coupon level

$$
C^{ex}_i = \frac{1}{\delta}\left[\left(1 + \frac{y_i \delta \tau}{\delta C^{ex}_i + F_i}(\tau C^{ex}_i - \alpha_i F_i(1-\tau)) + \frac{r}{r-\mu_i}\frac{1-\tau}{\tau}\alpha_i \delta (1+y_i)\right)^{-\frac{1}{y_i}} R_{i,0}(1-\gamma_i) - F_i\right], \tag{E.3}
$$

where C^{ex}_i remains on the right-hand side preventing an analytic solution to the problem but allowing for numerical solution. However, for the special case of fixed costs equal to zero, Eq. (E.3) collapses to

$$
C^{ex}_{i,F=0} = \frac{1}{\delta}\left[\left((1+y_i)(1 + \frac{r}{r-\mu_i}\frac{1-\tau}{\tau}\alpha_i \delta)\right)^{-\frac{1}{y_i}} R_{i,0}(1-\gamma_i)\right], \tag{E.4}
$$

which can be solved analytically.