Petroleum Reserves: Traditional, Probabilistic and with Abandonment Real Options

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By: Marco Antonio Guimarães Dias[a] & Roberto Evelim Penha Borges[b]. Version: 2019-06-19

Abstract

This paper discusses the estimative of oil reserves volume, by comparing the traditional approach based on the discounted cash flow (DCF), the probabilistic DCF approach, and the real options approach. In all cases, we consider both the residual value and the abandonment cost. The traditional DCF uses *expected* cash flow. The probabilistic approach explicit the uncertainty/scenarios, but using the DCF rule in each scenario. The real option approach is also probabilistic, but considering the value of waiting for better condition before exercising the abandonment option. We use the binomial method considering the oil price uncertainty for both probabilistic and option approaches to evaluate both the monetary and the volume values of the reserve. The option approach gives the higher economic value for the oilfield, followed by the probabilistic approach. We found that, ex-ante, the reserve volume is usually optimistic with traditional DCF than with the options approach, except for mature fields. If we consider the options approach the most rigorous, it means that in most cases the reported reserves are optimistic. However, *ex-post*, the use of real option rule will result in higher recoverable volume (reserves) than the DCF rule because the options approach considers the value of waiting for a better oil price before abandoning, whereas the DCF only consider the expected cash flow. For the volume calculus with abandonment option, the paper introduces the technique of pruned Pascal's triangle to access the probability of continuing to produce in any node from the binomial tree. The conclusion is that oil companies using deterministic DCF are *ex-ante* optimistic regarding oil reserve volume, but ex-post abandon too early the oilfields when using the DCF rule.

Keywords: petroleum reserves, real options, abandonment option, petroleum exploration & production, probabilistic reserves, reserves with option to abandon, pruned Pascal's triangle.

(a) Lecturer, Dep. of Electrical Engineering, PUC-Rio. Support Decision Methods Research Group. Applied Computational Intelligence Laboratory. E-mail: marcoagd@ele.puc-rio.br
(b) Doctor by the Departamento de Engenharia de Computação e Automação, Universidade Federal do Rio Grande do Norte; Reservoir Manager, UO-RNCE/Petrobras. E-mail: roberto.borges@petrobras.com.br

1) Introduction and the Literature on Reserves

According to the Society of Petroleum Engineers (SPE, 2007), petroleum resources are the estimated quantities of hydrocarbons naturally occurring on or within the Earth's crust. When these resources are "*anticipated to be commercially recoverable by application of development projects to known accumulations from a given date forward under defined conditions*", they can be called reserves[1].

In short, petroleum reserve is the economically recoverable volume of hydrocarbons for a given development investment. International efforts to standardize the definitions of petroleum resources and reserves and how they are estimated began in the 1930s[2]. In 1997, in an industry standardization effort, the Society of Petroleum Engineers (SPE) and the World Petroleum Council (WPC) jointly approved the "Petroleum Reserves Definitions" including suggestions for *probabilistic reserves* estimation procedures. In 2007, SPE and WPC, together with the American Association of Petroleum Geologists (AAPG), the Society of Petroleum Evaluation Engineers (SPEE) and Society of Exploration Geophysicists (SEG), released their Petroleum Resources Management System (PRMS) document (SPE, 2007), which consolidates, builds on, and replaces guidance previously published.

The PRMS prescribes a consistent approach to estimating petroleum quantities, evaluating development projects, and presenting results within a comprehensive classification framework. The definitions and the related classification system provide a measure of comparability between projects, groups/portfolio of projects, and total reserves from different petroleum companies, with recommendations to forecast production profiles and recoveries. Therefore, by following PRMS, companies can reduce the subjective nature of resources estimation. However, this is not enough or the best oil companies can do, as will be discussed here.

Companies often estimate reserves using more than one criterion. Stock exchange regulators, like SEC (Security Exchange Commission) in USA, have their own (evolving) rules for oil and gas reporting disclosures. Their goal is to allow the best possible comparison between listed companies; thus, their premises are pre-defined and must be followed with little space for particular assumptions. For example, SEC determines the oil price and the discount rate every company should use in economic assessment. Olsen, Lee and Blasingame (2011) talk about *reserves overbooking* (balance sheet reporting optimistic reserves) and show some refinements made by SEC to avoid this problem.

[1] Discovered and recoverable, but noneconomic petroleum volumes, are named *contingent resources*; whereas undiscovered volumes from exploratory prospects, are named *prospective resources* (SPE, 2007; SPE, 2011).
[2] But earlier articles on the subject had been published, such as Pack (1917), a geologist from a US agency.

National regulatory authorities typically require companies to report reserves as well. Although many times the regulation is based in some international guideline, like PRMS, their concern is generally more related to the country petroleum supply. Hence, they usually do not restrict the estimation to contractual limits, as opposed to SEC. In order to increase the reliability about the reported reserves, companies can also hire an independent certification service. A reserves certification is an estimation made by a third-party company, usually a renowned consultancy firm, based on data provided by the hirer. For internal critical analysis of reserves, companies frequently use a different set of assumptions. The estimation of resources involves the interpretation of volumes and values that have an inherent degree of uncertainty. Therefore, a sensibility analysis is often used, and the results are restricted to internal scrutiny in most cases. In order to support corporate planning (e.g., for reserves replacement), companies may use particular premises (like discount rate), scenarios (for example, for oil prices) and more robust methods (like probabilistic approach) to assess the economically recoverable petroleum volume. In this way, some companies use internally and sometimes report more than one reserve number on the same oilfield[3] ("SPE reserve", "SEC reserve", "reserve with corporate premises", "probabilistic reserve"), reflecting different assumptions or methodologies for assessing the economically recoverable volume. In addition to different geological interpretations, there are market issues with great impact on reserve like the future oil price, the future daily rig rate, and others, whose estimates vary between companies.

Reserves are classified as *proved*, *probable* and *possible*, depending on the degree of geological uncertainty (volume existence and reservoir productivity)[4]. Companies can reduce these uncertainties before development with investment in information (more appraisal wells, long-term production test, pilot systems, etc.). However, the market uncertainty remains the same. A "proved" reserve may become non-commercial if the oil price falls, a "contingent resource" (noneconomic volume) may become a reserve with improved market condition (e.g., if the oil price rises)[5]. The impact of market conditions on the reserve volume has not been studied with due rigor and depth in the literature that this problem requires. The main goal of this paper is to address this important topic using the modern approach to investment under uncertainty, namely the real options theory, including a comparison with the traditional discounted cash flow (DCF) approach and with the probabilistic DCF approach.

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[3] Example: http://www.investidorpetrobras.com.br/en/press-releases/proved-reserves-petrobras-2018
[4] See the Petroleum Resources Classification Framework from PRMS (2007, item 1.1).
[5] E.g., see http://crudeoilpeak.info/oil-reserves-and-resources-as-function-of-oil-price (in particular the figures 12-14).

Dermirmen (2007) presents a good overview on the reserve estimation methods used in the industry, including definitions on different kinds of reserves (e.g., proved, probable and possible) and discusses some issues related to a stochastic approach to reserve volume estimation. He highlights the necessity of reliable reserves report by the oil companies: "*Confidence in reserves disclosures became a public issue, and there were calls from investors and lending institutions for more-reliable reserves estimates*". As in Dermirmen (2007), here "field reserves estimates" are expectation values, not proved or proved + probable volumes. He defends the probabilistic approach "… *generally should be preferred over a deterministic approach*", but focusing on geological uncertainties. He did not analyze the effect of the oil price uncertainty on the reserve volume estimation as done here. In order to take into account the economic/market effect on the reserve volume, instead of explicitly modeling the economic (e.g., oil price) uncertainty, he uses a controversial approach much used in the industry: the truncation of the reserve volume distribution at a minimum economical volume ("economic cutoff"). Of course, the minimum economically recoverable volume depends on the evolving market condition especially the uncertain future oil prices and the specific oilfield operational costs, as considered here. There are some additional problems to work with truncated distributions, e.g., the truncated distribution expected value is higher than the expected value without truncation, which can generate an optimistic information, being necessary to multiply by an economic factor to correct. While Dermirmen (2007) uses the *probabilistic reserves* expression looking only geologic/reservoir uncertainties, here this term is used focusing oil price uncertainty.

Some oil companies report the reserves life indicator named Reserve Life Index (RLI), a reserve-to-production ratio that gives an idea about the years of production with the current developed reserves. RLI indicates how long reserves will last at the current production rate with no additions to reserves. If B is the reserve volume (number of barrels) and Q the current annual production, RLI = B/Q, which is typically measured in years. If B and/or Q change with oil price, RLI changes as well. This index can even increase when oil prices fall depending on the operational cost (Opex - operational expenditure): if the variable cost is high, but the fixed cost is relatively low as in mature onshore oilfields, can be optimal to stop temporally some wells (instead of permanent abandonment), reducing Q and hence rising RLI. With (permanent) abandonment of some wells, may occur the opposite, reducing more B than Q, decreasing the RLI. So, this index is not much appropriated to perform empirical analysis on the oil prices impact on reserve.

Apergis, Ewing & Payne (2016) applied time-series methods to assess the impact of oil prices on the reserve life. With onshore oilfields data from the state of Texas over the years 1977–2013, they found significative evidence of oil price influence on the RLI and "*that the reserve life index and real oil prices are cointegrated, thus exhibiting a long run equilibrium relationship*". Their results show asymmetric effects, with higher impact from positive oil price shocks. But this specific result may be biased by the onshore data. It would be interesting to do a similar study for offshore oilfields, which have a much higher fixed Opex, so that the definitive abandonment is more frequent than the temporary stopping option when compared with the onshore oilfields used in their study.

After this introduction, the paper is organized as follow. Item 2 performs a review of literature about the project abandonment decisions. Item 3 discusses the DCF and options approaches using simple examples and anticipating some intriguing results that DCF reports optimistic reserves, but options (and probabilistic) rules ex-post results in higher ultimate recovery volumes. Item 4 details the options approach using binomial method, including the new feature of pruned Pascal triangle to calculate the production probability (hence, the expected reserve) under options rule. Item 5 presents a case study comparing the reserves volume and its economic value for the three decision rules (DCF, probabilistic and options). Item 6 presents the concluding remarks and suggestions for future research. Item 7 list the bibliographical references and item 8 provides a short glossary of terms.

2) Review of Literature on Abandonment Decisions

This review of the literature on abandonment decisions will also serve to discuss some concepts used in the traditional, probabilistic and options approaches that we are going to analyze in the other items.

Robichek & Van Horne (1967) is a known early research in project abandonment decisions. They used Monte Carlo (MC) simulation to address the uncertain cash flows and for incorporating the effects of abandonment for the investment decision. In their conclusions, they correctly argue "*since having the option to abandon never decreases project value, the typical consequences of ignoring the option would be to underestimate the value of a project*". Although they used the term "option to abandon", they did not use the modern options theory (that only appeared in the next decade) and their approach is more related with the probabilistic method (with MC). Robichek & Van Horne used the following decision rule: in each scenario, the abandonment is optimal if the present value of the *expected* future cash flows is lower than the abandonment value (salvage value[6]) of the project. This is, in essence, the DCF approach. However, they did not consider that future abandonment might be more valuable than current abandonment. Dyl & Long (1969) criticized this rule suggesting a rule that consider all possible future abandonment opportunities[7]. Here we consider this issue by working backwards in a dynamic programming fashion. In addition, instead of the expected future cash flows, real options theory considers all future scenarios starting from each scenario. Robichek & Van Horne also realized that the abandonment option reduces the risk: "*much of the downside risk can be eliminated if the Project is abandoned when events turn unfavorable*". But they didn't know how this issue changes the discount rate: "*The determination of this effect is extremely complex*". The option's discount rate problem was solved few years later, with the modern option pricing theory.

Bonini (1977) is other known earlier reference on abandonment decisions under uncertainty in capital projects. Similar to Robichek & Van Horne, "*project would be abandoned in any year if estimated future cash flows (appropriately discounted) did not exceed the current abandonment value*", argues Bonini. But in constrast with Robichek & Van Horne, Bonini considered that the future abandonment might be better than current abandonment. In order to do this, his analysis was performed backwards with discrete-time dynamic programming. However, Bonino used the firm's cost of capital as discount rate. He did not use the real options theory, which had not yet been born at the time (the first real option mathematical model was Tourinho, 1979). The risk-adjusted discount rate for projects with options is different from the case without options: the abandonment option reduces the project risk; it is like a partial insurance. In the section 3 we will also use a discrete-time dynamic programming, but in options' style, with risk-penalized probabilities that allows to discount with risk-free interest rate.

The real options literature, 25 years after the first textbooks of Dixit & Pindyck (1994) and Trigeorgis (1996), it is now well established in academy and with growing use in firms. Tourinho (1979) was the earliest real option model in petroleum. Dias (2004) gives an overview of petroleum real options models, with focus in exploration and production (E&P). Early real options articles on abandonment decisions include entry-exit models (the firm has the option to reenter after an exit) and models using the analogy with the (financial) put option. The entry-exit real options models include Brennan & Schwartz (1985) and Dixit (1989), but they are not well suitable for oilfield abandonment because the

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[6] The revenue obtained by selling the project assets when abandoning. Here is used the term salvage (or residual) value for all dates, typically decreasing with the time, but Robichek & Van Horne use the term *salvage value* only in the last date to abandon the project. Before, they use the term *abandonment value*.

[7] Later, Robichek & Van Horne (1969) recognized this problem with their original rule.

option to redevelop (reentry) a depleted and abandoned oilfield is negligible and almost never occurs. Early real options papers on abandonment option includes Kensinger (1980) and Myers & Majd (1983; 1990). Both explore the analogy abandon option with (financial) put option. Kensinger considered stochastic both, the project value and the salvage value following correlated stochastic processes. It is similar to the option to exchange one risk asset (project) by other risk asset (the alternative use of the assets in place, the salvage value). Kensinger used the geometric Brownian motion (GBM), the most used stochastic process in economics and finance, but with negative drift for the salvage value. However, he used European option type, assuming the abandonment decision will be made only on a specific date (but this is not realistic in most cases). Myers & Majd also used the GBM with negative drift, but to model the uncertainty on project value V and considering the more realistic American option type (which can be exercised in any date, in contrast with European option). They also consider (in appendix) the case of stochastic salvage value (also a GBM with negative drift), correlated with V. Here we will discuss the analogy, but also the differences between the (financial) American put option and the (real) option to abandon.

In terms of empirical evidence, Berger, Ofek & Swary (1996) found "*strong support for the predictions of abandonment option theory*" using 1,043 US accounting data from the years 1984-93. Moel & Tufano (2002) presents a similar conclusion with data from 285 North American gold mines. This does not mean that these executives know and use the modern theory of real options. It can be said that executives' intuition that DCF would lead to premature abandonment causes them to claim "strategic reasons" to consider waiting for better conditions before abandoning projects irreversibly. This intuition also leads executives to postpone projects with NPV > 0, if that NPV is small and if the uncertainty is high. This behavior is consistent with real options theory but not with DCF, which recommends doing all projects with NPV > 0 and does not consider the value of the wait.

Olsen & Stensland (1988) is closer to the specific *petroleum* literature on abandonment options. They used two GBM, one for the commodity price P(t) and the other for the production rate q(t). The GBM for q(t) has negative drift, implying that the expected production rate follows an exponential decline. In contrast with this article, they assumed that the option to abandon is perpetual and did not consider abandonment costs (environmental recovery) and salvage values. They use risk-free discount rate, but without changing the probability measure as here, finding analytical solution for the optimal stopping problem. However, their conclusion is similar to the one of this paper in our *ex-post* view: "*Thus, uncertainty will tend to prolong the extraction period compared to the deterministic case.*"

Begg, Bratvold & Campbell (2004) analyze the abandonment option for an oilfield using mean-reversion to model the oil price uncertainty, but their focus is the analysis of different abandonment decision rules as candidates to maximize the oilfield value, not the effect on reserve volume as in this present work. Jafarizadeh & Bratvold (2012) discussed the option to abandon an oilfield, but more as an illustration of two-factor stochastic process for oil prices, combining mean-reversion with GBM.

More recent real options models for oilfield abandonment include Guedes & Santos (2016), Abadie & Chamorro (2017), Borges et al. (2018) and Borges et al. (2019). Guedes & Santos analyze the sequential E&P options: exploration options, appraisal options, investment/scaling options and abandonment options. They found the abandon as the most valuable option, but they consider the abandon in all E&P phases, including failure of exploration/appraisal related with geological uncertainties (existence, quality and volume). Here we are concerned with the abandonment *after* the oilfield development and both the option value (oilfield value considering the option to abandon) and the oilfield volume that results from the optimal management of the abandonment option. Abadie & Chamorro discuss options to delay and to abandon with focus in short-lived oilfields (tight oil production) using more sophisticated stochastic process for oil price (mean reversion and stochastic volatility). Borges et al. (2018) and Borges et al. (2019) use the fuzzy real options approach to evaluate an oilfield abandonment option, correcting a methodology used in this literature (*fuzzy pay-off method*) with a new approach. However, none of these articles discusses the effect of abandonment option on the reserve volume as here.

3) Discounted Cash Flow versus Explicit Uncertainty/Options Approaches

The traditional DCF abandonment rule is "... *a project should be abandoned at that point in time when its abandonment value exceeds the net-present value of subsequent expected future cash flows discounted at the cost-of-capital rate*" (Robichek and Van Horne, 1967). In simpler and more precise terms, the correct DCF abandonment rule is <u>to abandon the project on the date that maximizes the NPV</u> (net present value) considering only expected values. The NPV includes all the abandonment costs and salvage values as well as the expected cash flows. Note that the rule is not abandon at the first negative expected cash flow[8]. It may be optimal to produce even with negative cash flow, because postponing the abandonment has the additional benefit of *postponing the abandon cost*. The

[8] However, the first author experience when working in an oil company showed that some reserve auditors insisted on considering abandonment in the first negative cash flow, whereas others auditors accepted the maximum NPV rule, even producing with negative cash flows. The former case, with suboptimal decision, results in lower reserve reporting, and we name this naïve suboptimal rule as "wrong DCF" to distingue from the (right) DCF.

benefit of the delayed cost increases with the discount rate and may be more than sufficient to offset the negative cash flow. For example, postponing one year a cost of \$ 100 million, which is applied on the capital market at an interest rate of $r = 4\%$ p.a., means \$ 4 million annual benefit, which offsets a negative cash flow up to - \$ 4 million. To formalize, let A be the abandonment cost and SV the salvage value, $CF(\Delta t)$ the production cash flow in the next time interval Δt and r the interest rate. Even if $CF(\Delta t) < 0$, it is optimal to keep producing if the financial gain from postponing the cost A net of the benefit SV is sufficient to offset the negative cash flow. In short, keep producing if:

$$CF(\Delta t) + \{(A - SV)\,[(1 + r)^{\Delta t} - 1]\} \geq 0 \qquad (1)$$

Otherwise, it is optimal (by the DCF rule) to abandon the field. In case of abandonment, the firm receives the abandonment payoff net of taxes:

$$\text{Abandonment Payoff}(t) = [- A + SV(t)]\,(1 - T_c) \qquad (2)$$

Where T_c is the corporate tax rate. This tax does not change the rule from eq. 1 (multiplication by a positive number, $1 - T_c$, does not change the inequality). The salvage value typically decreases with the time (economic depreciation) until a scrap value. The cash flows from the production also tend to decline with the time because the declining production of the oil/gas fields.

The simple numerical example below illustrates both the traditional DCF rule and how the decision changes if we use different scenarios and consider the options.

Consider a mature oilfield that, by the first time (at $t = 0$), is faced with a negative expected cash flow if the production continues from $t = 0$ to $t = 1$. Consider only two possible production periods, so that the abandonment is obligatory at $t = 2$ (end of the concession or end of equipment life). Consider the discount rate $r = 4\%$ p.a., the abandonment cost A = \$ 100 million, the corporate tax rate $T_c = 34$ %, and the remaining data in the Table 1 below (in million \$).

In Table 1, the cash flows, salvage values and abandonment costs are before income tax and in present values at each date t, whereas the NPV numbers are after the income tax (multiplied by 0.66 to consider the 34% of corporate tax, see below for NPV calculus details). NPV numbers are all in present value at $t = 0$, but we denote τ the date that the project is abandoned in the NPV calculus. The last line shows the production from date t to date $t + 1$ (at $t = 2$ the abandonment is obligatory).

Table 1 – Simple DCF Example (in million US$)

	t = 0	t = 1	t = 2
Expected Net Cash Flow (CF) producing one year more	– 1	– 5	-
Salvage Value (SV)	4	2	0
- Abandonment Cost (A)	- 100	- 100	- 100
NPV(τ) for each date of abandonment (after tax) at t = 0	– 63.36	– 62.85	– 64.85
Annual Production from t to t + 1 (million barrels)	10	9	–

The expected cash flows are net of operational costs and royalties. If produce, a negative CF(t) is incurred and a new decision at t + 1 is considered. If abandon at t = 0, the negative expected cash flow CF(t) for the next year (- 1 in present value at t = 0) is avoided, but pays the abandonment cost A and receives the salvage value SV(t). If produce from t = 0 to t = 1, the firm get a negative expected operating cash flow, but we postpone the abandonment cost (although also postpone the SV). If we produce from t = 0 to t = 2, we get two negative expected cash flows, but we benefit from delaying two periods the abandonment cost. What is the abandonment date that maximizes the NPV? See below that it is not the date t = 0, even avoiding the negative expected cash flow. The NPV(τ) (in present value at t = 0) for each possible abandonment date τ are:

at $\tau = 0$ (immediate): $NPV(\tau = 0) = (-100 + 4) * (1 - 0.34) = -63.36$;

at $\tau = 1$: $NPV(\tau = 1) = \{-1 + [(-100 + 2)/(1 + 0.04)]\} * (1 - 0.34) = -62.85$; and

at $\tau = 2$: $NPV(\tau = 2) = \{-1 + [-5/(1 + 0.04)] + [-100/(1 + 0.04)^2]\} * (1 - 0.34) = -64.85$.

So, in this example, the abandonment date that maximizes the NPV is t = 1: it is optimal to produce one year even with negative expected cash flow, because the benefit to postpone the cost of abandon A is higher and compensate this negative operational result. It is not optimal to delay more, because the second year expected cash flow is too negative (– 5) compared with the benefit to postpone one year more the abandonment cost (= r A = 0.04 x 100 = 4) and the SV drops to zero. We can also use the rule from eq.(1) to see that is optimal to produce at t = 0 ($-1 + (100 - 4)(1.04 - 1) = 2.84 > 0$) but not at t = 1 ($-5 + (100 - 2)(1.04 - 1) = -1.08 < 0$). Note that with a higher discount rate (e.g., 6%), could be better to produce two years ($-5 + (100 - 2)(1.06 - 1) = 0.88 > 0$).

Now, consider the same example, but splitting E[CF(1)] into two scenarios: instead of considering only the expected cash flow at t = 1, consider that the expected cash flow E[CF(1)] = – 5 is due to the weighting of two scenarios with 50% chances each: the scenarios $CF(1)^+ = 5$ (oil price rises) and $CF(1)^- = -15$ (oil price drops). Figure 1 shows this case with explicit uncertainty (right decision tree) and the previous case using only expected value as done by DCF (left decision tree).

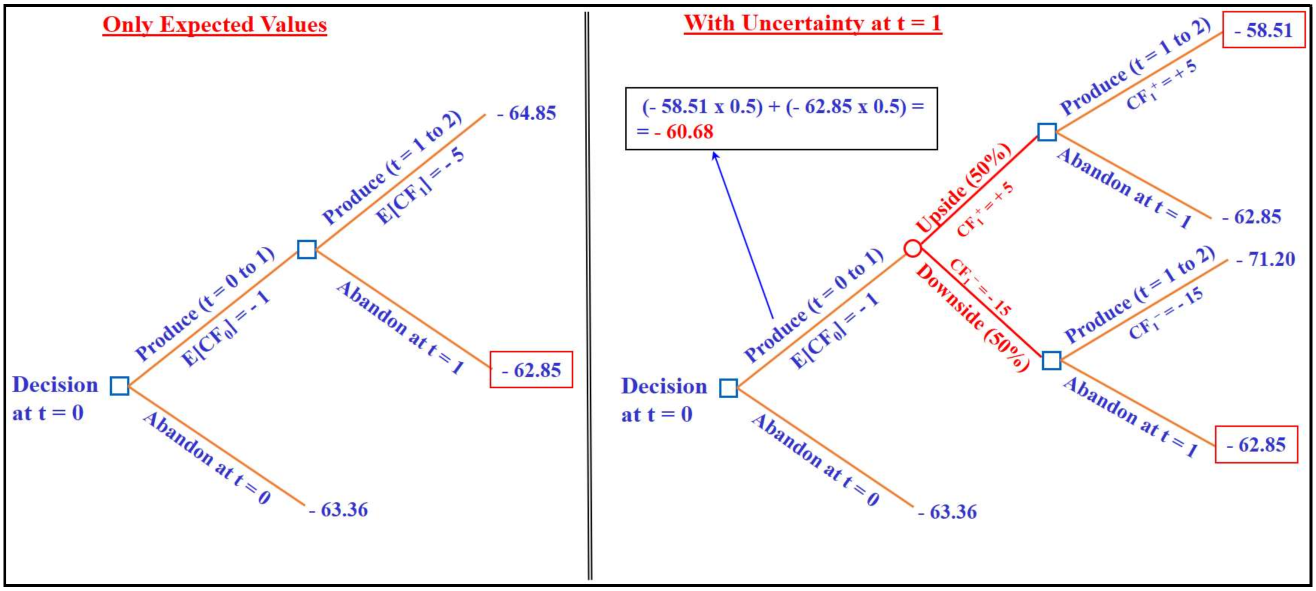

Figure 1 - Example with r = 4% p.a. Left: Only Expected Values; Right: With Uncertainty

In the case with uncertainty, there are two scenarios for NPV($\tau = 2$) that we call $NPV^+(\tau = 2)$ and $NPV^-(\tau = 2)$:

$NPV^+(\tau = 2) = \{-1 + [+5/(1 + 0.04)] + [-100/(1 + 0.04)^2]\} * (1 - 0.34) = -58.51.$

$NPV^-(\tau = 2) = \{-1 + [-15/(1 + 0.04)] + [-100/(1 + 0.04)^2]\} * (1 - 0.34) = -71.20.$

The NPV producing from t = 0 to t = 1 and producing from t = 1 to t = 2 in the upside scenario and abandon at t = 1 in the downside scenario, indicated in Figure 1 (left), is – 60.68 > – 62.85 > – 63.36.

So, instead of producing only from t = 0 to t = 1 (abandonment at t = 1), the optimal rule is: produce the first period; after that, in t = 1 if the market improves, produce also in the second period and abandon at t = 2, whereas if the market is worse at t = 1, stop production and abandon at t = 1. With these scenarios, there is 50% chance of producing more than in the case of traditional DCF rule that stops production with 100% chances at t = 1. So, in this case the expected reserve volume is higher, when considering scenarios/uncertainty, than with DCF rule (which sees only expected cash flows).

Now, consider the same data, but with a much higher discount rate of r = 8% per period. Figure 2 presents this example for two subcases (only expected values and considering the uncertainty). For the DCF rule (using expected cash flows, left decision tree in Figure 1), the reader can check that now it is optimal to produce two periods, because the abandonment date that maximizes the NPV is t = 2: NPV(τ = 2) = – 60.30 > NPV(τ = 1) = – 60.55 > NPV(τ = 0) = – 63.36. However, if we expand the expected cash flow into two scenarios (right decision tree in Figure 2), as done before (previous case with r = 4%), we get $NPV^+(\tau = 2)$ = – 54.19 and $NPV^-(\tau = 2)$ = – 66.41. In this case, the optimal rule is to produce two periods only if at t = 1 the revealed market scenario is favorable ("upside"). If the scenario at t = 1 is "downside" (50% chances), the optimal is to produce only one period (t = 0 to 1) and abandon at t = 1.

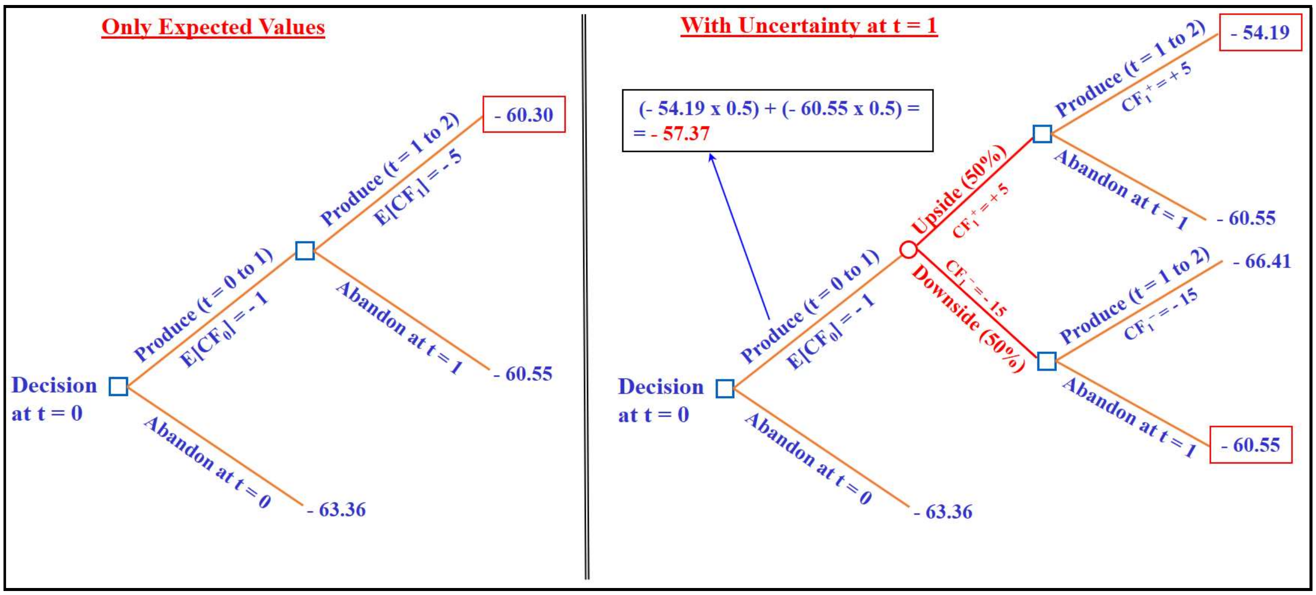

Figure 2 – Example with r = 8% p.a. Left: Only Expected Values; Right: With Uncertainty

Figure 2 shows an inverted case (compared with r = 4% from Figure 1): the DCF rule looking only expected values, produces until t = 2, whereas with explicit uncertainty at t = 1, there are only 50% chances to produce until t = 2. So, in the uncertainty case, considering the option to produce and abandon in each scenario, points to lower reserve volume than the DCF approach.

Table 2 shows the remaining reserves of this field for the DCF rule (see only expected values) and for the options case, with bold highlighting for the rule that predict the higher remaining reserve volume. It uses the last line of the Table 1, which shows the potential production from t to t + 1 (10 million barrels and 9 million barrels in the first and second period, respectively) for the cases discussed above.

Table 2 - Remaining Reserve Volume (million barrels) for Different Rules and Discount Rates

Discount Rate	DCF (only expected values)	Uncertainty/Options
4% p.a.	10	**14.5** (= 10 + [9 x 50%])
8% p.a.	**19**	14.5

So, the rule (DCF or options) with higher reserve volume depends on parameters like the discount rate. Other parameter changes can invert the rule with higher reserve. For example, reconsider the case of r = 8%, but with E[CF(1)] = - 5.5 (instead - 5) and splitting the - 5.5 into two scenarios (50% chances each) of $CF(1)^{+} = + 4.75$ and $CF(1)^{-} = - 15.75$. The reader can verify that DCF rule now produces only in the first year (10 million bbl), whereas the options rule does not chance (produces 14.5 million bbl). So, a more negative E[CF(1)] points higher reserve volume to options rule.

Table 3 presents the NPV for each rule for these two cases of discount rate. Note that even when the DCF reports higher reserve volume, the options rule in both cases of r presents the higher NPV (with bold highlighting). In fact, this is not a particular result: the option rule always points a value equal or higher than the value obtained with the DCF rule.

Table 3 – NPV (in millions $) for Different Rules and Discount Rates

Discount Rate	DCF (only expected values)	Uncertainty/Options
4% p.a.	- 62.85	**- 60.68**
8% p.a.	- 60.30	**- 57.37**

These simple numerical examples show that traditional DCF rule may report higher or lower reserve when compared with the probabilistic approach. The probabilistic approach has some similarities with the options approach (see below the differences) so that the reserve volume may be higher or lower than DCF approach. Questions: overall, is the reserve higher or lower with options? Are the oil companies reporting optimistic, realistic or pessimistic reserves when using DCF? Is there a more important effect on reserve? Yes, as we see below.

In the beginning of the oilfield productive life, the expected revenue is much higher than the Opex because the expected net operational cash flows must be sufficiently positive to pay the investment (if not, the development project has negative NPV and the project is not approved). So, the first effect (DCF indicating higher reserves than probabilistic or option approaches) occurs in the beginning of the oilfield life: DCF indicating production “with certainty”, whereas probabilistic and options indicating some probability of abandonment after some few years. When the oilfield becomes more “geriatric”, with negative cash flows, the second effect occurs: probabilistic and options approach may recommend continuing production in some scenarios, whereas DCF may recommend immediate abandonment by looking only expected values.

In the first producing years the E[CF] is very positive and the DCF points 0% chance of abandonment (100% chance of producing), whereas in options/probabilistic approach explicit the uncertainty so that even with E[CF(t)] > 0, some scenarios can be so negative that is optimal the abandonment. So, while DCF points producing with 100% of probability, the options approach points producing with less than 100% probability (positive probability of abandonment). In the later years, the opposite occurs: with E[CF(t)] sufficiently negative, the DCF points 0% chances of producing and 100% of abandonment, whereas the options/probabilistic approach identify favorable scenarios in which is still optimal to continue the production, so that points a positive probability of producing.

Overall, in the case of new oil/gas fields, what is the most important effect for reserves reporting? In quantitative terms, due to the production decline of this non-renewable commodity, the production in the *first years has a much higher contribution to reserve volume* than the last years of production. So, when the firm is reporting for the first time the project reserves, we can say that typically DCF points optimistic reserves when compared with options approach (and probabilistic approach). However, for mature fields, the opposite can occur (options reporting higher reserves than DCF).

Figure 3, from a base case that is detailed in the item 5, illustrates this point. Note that from the second year the expected production with options begins to be lower than that of DCF. This is because there is a probability of low oil prices occurring, which results in a positive probability of abandonment. The DCF approach does not “see” these abandonment scenarios. Can a large price change occur in two years? Recall that in June 2014 the average Brent’s oil price was near US$112/bbl, whereas in June 2016 this average price was around US$ 50/bbl and even lower some months before: in January 2016, the Brent was around US$ 32/bbl. In the first six months of 2019, the Brent oil price oscillated in the range 54-75 US$/bbl.

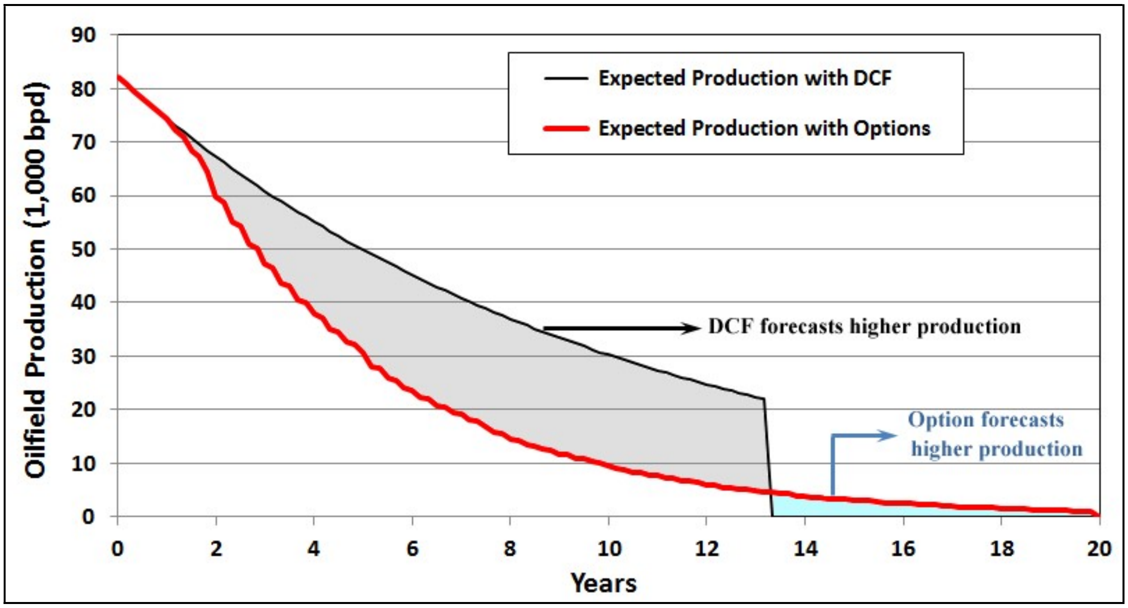

Figure 3 – DCF x Options Production (Reserve) Expectations

The largest shaded area (gray) in Figure 3 shows the DCF *additional* production forecast in relation to the options' forecast, whereas the smaller shaded area (blue) shows the opposite case. In this example, the cumulative economical production (reserves) forecast for DCF was 222.77 million bbl, whereas for option was 149.60 million bbl. If the reserves estimate from options perspective is more realistic, the DCF was very optimistic, reporting a reserve number 48.9% higher! Therefore, the difference is very relevant and the reserves reported by oil companies can be too optimistic.

Here we focus on reserve estimate when making (or just after) the development investment, because this is the most important epoch for investors, when large investment are performed and new reserves additions are reported. We call "ex-ante forecast" the reserve report in this period.

Someone could collect empirical data comparing the original reserves reports with ultimate reserves from abandoned oilfields, which could indicate the opposite: the original reserve reports indicated a lower reserve than fulfilled. However, in the vast majority of cases, these oilfields had *additional investments* (like infill drilling, enhanced oil recovery with fluids injection, etc.) As pointed out in SPE (2007), "*Reserves must further satisfy four criteria: they must be discovered, recoverable, commercial, and* <u>*remaining (as of the evaluation date) based on the development project(s) applied*</u>" (emphasis added). So, the reserves report does not consider future investment projects.

The Figure 3 shows that if we are reporting reserves between years 12 and 13 (reserves from mature fields instead new fields), the opposite situation occurs. When the average net operating cash flows becomes sufficiently negative, if the decision maker is using options theory, the oilfield life is extended (compared with DCF) because options consider scenarios where market conditions improve, the "good news principle" (Dixit & Pindyck, 1994, p.41 n.9). So, while ex-ante DCF reports higher reserves, ex-post the options rule produce more time than the DCF rule.

Hence, in practice, options recommendation results in ultimate higher reserves, even ex-ante reporting lower reserves!

The probabilistic DCF considered in this paper uses the DCF rule (e.g., the criterion in eq. 1), but considering many scenarios (not only the expected value). The probabilistic DCF does not consider the value of waiting for better scenario (as the options' rule). But, because consider many scenarios, the probabilistic DCF generally presents intermediate results between traditional DCF and real options. Typically, the value (NPV) obtained with probabilistic DCF is higher than with traditional DCF, but lower than the option value, as we see in the example presented in item 5.

4) The Option to Abandon Approach to Reserve Forecast

In this item, we detail the options model using the binomial tree method. The main option methodologies are the discrete-time binomial tree, the differential equation and the simulation (Monte Carlo) approaches. We choose the binomial approach because it is more intuitive and thus has more chances of this reserves estimative methodology to be used by managers of oil companies.

We model the uncertainty of oil prices along the time with the most popular stochastic process, the geometric Brownian motion (GBM), used by Black (1976) to model commodities and in pioneering real option models in petroleum extraction (Tourinho, 1979; and Paddock et al, 1988). GBM is much simpler than other models: less parameters to estimate, it is easier to interpret and American options are homogeneous in the underlying asset and in the exercise price[9]. Econometric tests like unit root test does not reject the GBM when using 30-40 years of data (Dixit & Pindyck, 1994, p. 78; Dias, 2015, p. 128). Although other stochastic processes can be used, such us mean-reversion with jumps models, the main conclusions of this paper should not change if other processes are used. As pointed out by Pindyck (1999, section 6), "*...generally the size of the error is small, at least relative to errors that would be tolerable in real options applications*.", and mention one paper that found errors of 5% when comparing mean-reversion with GBM, "*which would be significant for a financial option. In case of a capital investment decision, ... an error of this size is unlikely to be important*.". Pindyck

[9] So that we can calculate the option F by unity of investment (F/I) or per barrel of reserve (F/B in $/bbl) and then multiply by the investment I or by the reserve volume B to get the option value. Mean-reversion models do not share this homogeneity (see the textbook of Dias, 2015, chapter 23; and a theorem in Merton, 1973, p.149).

(2001, p. 26) reaffirms: "*... the GBM assumption is unlikely to lead to large errors in the optimal investment rule.*". It is left for a future work to analyze the impact on reserve volume using other stochastic processes. The authors think that variations in the choice of the stochastic process for the oil prices produce a second order effect on the reserve volume estimate.

The GBM stochastic differential equation for the oil price P is given below, where α is the drift rate, σ is the oil price volatility, and $dz = N(0, 1) \sqrt{dt}$ is the Brownian/Wiener increment, with N(0, 1) being the standard normal density.

$$dP = \alpha P \, dt + \sigma P \, dz \tag{3}$$

Because the options discount rate is a complex problem, we use a *certainty equivalent* approach that penalizes the probability in order to use the risk-free discount rate. The penalized probability is named *risk-neutral probability measure*. Risk-neutral probability is only a smart math trick to use the risk-free discount rate to calculate the present value of options[10]. The risk-neutral GBM stochastic differential equation (see, e.g., McDonald, 2006, eq. 20.26) for the oil price P is given by one of the two equivalent equations below:

$$dP^Q = (\alpha - \pi) P \, dt + \sigma P \, dz^Q \tag{4a}$$

$$dP^Q = (r - \delta) P \, dt + \sigma P \, dz^Q \tag{4b}$$

Where π is the oil price risk-premium (in % p.a.), r is the risk-free interest rate and δ is the oil price convenience yield[11]. The superscript Q denotes risk-neutral probability measure ("Q-measure") in the random terms dP and dz. Equation (4a) shows that the drift α is penalized by a risk premium π in the GBM risk neutral version. In the penalized risk neutral world, we use the risk free discount rate r.

In the binomial method, after one period, the price P can go up to $P^+ = u P$ with risk-neutral probability q or can go down to $P^- = d P$ with probability $1 - q$, where u is the upside factor and d is the downside factor. The Cox & Ross & Rubinstein (1979) binomial equations for the GBM, but considering the dividend yield (see, e.g., Back, 2005, p.93; or Dias, 2014, p.231-232), are given by:

$$u = \text{Exp.}[\sigma \sqrt{\Delta t}\,] \tag{5}$$

$$d = 1/u \tag{6}$$

[10] See any textbook of financial options (e.g., McDonald, 2006) or real options (e.g., Trigeorgis, 1996).

[11] Convenience yield is the benefit on holding inventories of a commodity. It is like a non-cash dividend that justifies the investment in storage facilities. It can be measured in futures markets using the relation $F(t) = P \exp(r - \delta)\, t$, where F(t) is the price of a futures contract to delivery in t years; P is the spot price; r is the risk-free rate. See details in derivatives textbooks such as McDonald (2006), Geman (2005), or Dias (2015).

$$q = \frac{e^{(r-\delta)\Delta t} - d}{u - d} \quad (7)$$

While for option valuation purposes we must use risk-neutral probabilities (because we need to calculate the option present value with the discount rate r), to calculate the expected reserve volume with options and probabilistic approaches we must use real (or true or statistical) probabilities. For the reserves volume expectation is necessary to work with *true* probabilities. In the binomial, we call the up and down risk-neutral probabilities as "q" and "1 – q". For the true probability we use "p" and "'1 – p" for the up and down scenarios, respectively. The true probability of up movement in the binomial is given by:

$$p = \frac{e^{(\mu-\delta)\Delta t} - d}{u - d} \quad (8)$$

Where μ is the total rate of return of the oil price (or the risk-adjusted discount rate), which can be estimate using the CAPM. Here, all rates (r, δ and μ) are in continuous time (logarithm returns)[12]. The difference between the last two equations is only the replacement of the risk-free interest rate r by the risk-adjusted rate μ.

In order to calculate the probability of occurrence of each node of the tree, it is necessary to calculate the number of paths that reach a certain node of the tree. Pascal's triangle[13] is a triangular array of the binomial coefficients very known in mathematics. In the binomial tree context, it is used to calculate the number of paths to reach a specific node in the tree. The triangle is built as follow: start with "1" at first node (here node x(0, 0)) and the other nodes follow a rule of adding the two numbers from the previous adjacent nodes to get the node number. Figure 4 below illustrates the Pascal's triangle.

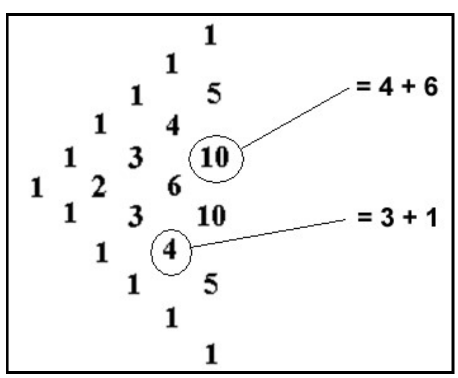

Figure 4 – Pascal's Triangle

[12] If r_d is the discrete-time interest rate and r_c the continuous-time interest rate, we can get r_c using $r_c = Ln(1 + r_d)$.
[13] It is known as the Yanghui triangle in China.

Here it will be used to estimate the probability to reach each binomial tree node in order to estimate the expected reserve volume considering only nodes with production. For the case without earlier abandonment, the figure below shows the binomial tree and the Pascal's triangle, which gives the number of paths to reach a node. For example, in t = 3 we have four nodes (oil price scenarios), the nodes (from bottom to up) x(3, 0), x(3, 1), x(3, 2) and x(3, 3). The extreme scenarios have only one path to reach each one: (down, down, down) for x(3, 0) and (up, up, up) for x(3, 3). Hence, the probabilities to reach the nodes x(3, 0) and x(3, 3) are p^3 and $(1 - p)^3$, respectively. But the interior nodes we have more than one path to reach. For example, the node x(3, 2) (in t = 3, the second node from the top) has three paths: (up, up, down), (up, down, up) and (down, up, up). In all the three cases, we have two "ups" and one "down", so the probability for each path is $p^2 (1 - p)$. As we have three paths, the probability to reach the node x(3, 2) is $3 p^2 (1 - p)$, as indicated in the figure below. The Pascal´s triangle gives the number of paths reaching each node (right bottom in the figure).

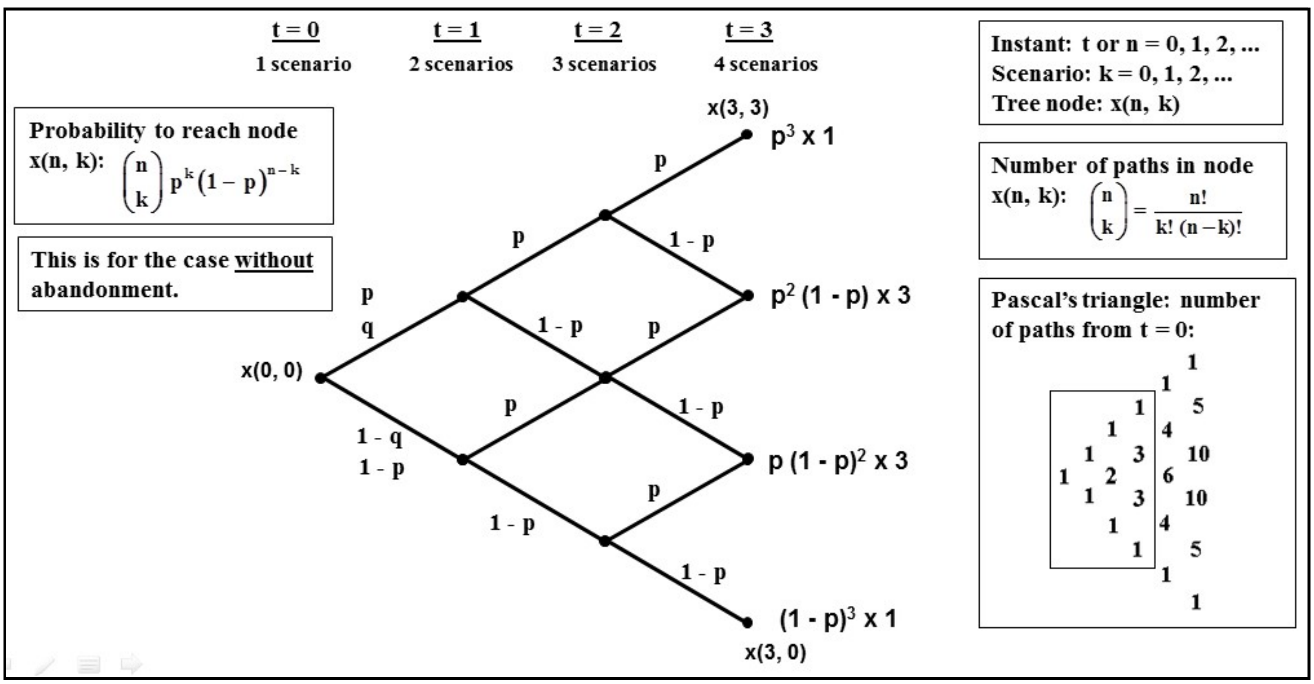

Figure 5 – Binomial Tree and the Node Probabilities without Abandonment Option

So, the number of paths reaching a generic node x(n, k) is given by the binomial coefficient:

$$\text{Number of Paths at } x(n, k) = \binom{n}{k} = \frac{n!}{k!\,(n - k)!} \tag{9}$$

Because each path at node x(n, k) has probability $p^k (1 - p)^{n-k}$ (k "ups" and n – k "downs"), the probability to reach the generic node x(n, k) is:

$$\text{Probability to reach } x(n, k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{10}$$

This is known, but it does not apply directly to our case, because we can abandon in one node so that all paths reaching this abandoned node are "killed" and the paths do not continue in terms of oil production. Example: in the Figure 4, if we exercise the option to abandon in the node x(1, 0), all paths from this node will not exist anymore for production purposes (dotted lines in the figure). The nodes x(2, 0) and x(3, 0) will never be reached and the nodes x(2, 1), x(3, 1) and x(3, 2) will be reached by fewer paths when compared with the case without earlier abandonment. How to calculate the nodes probability in this case?

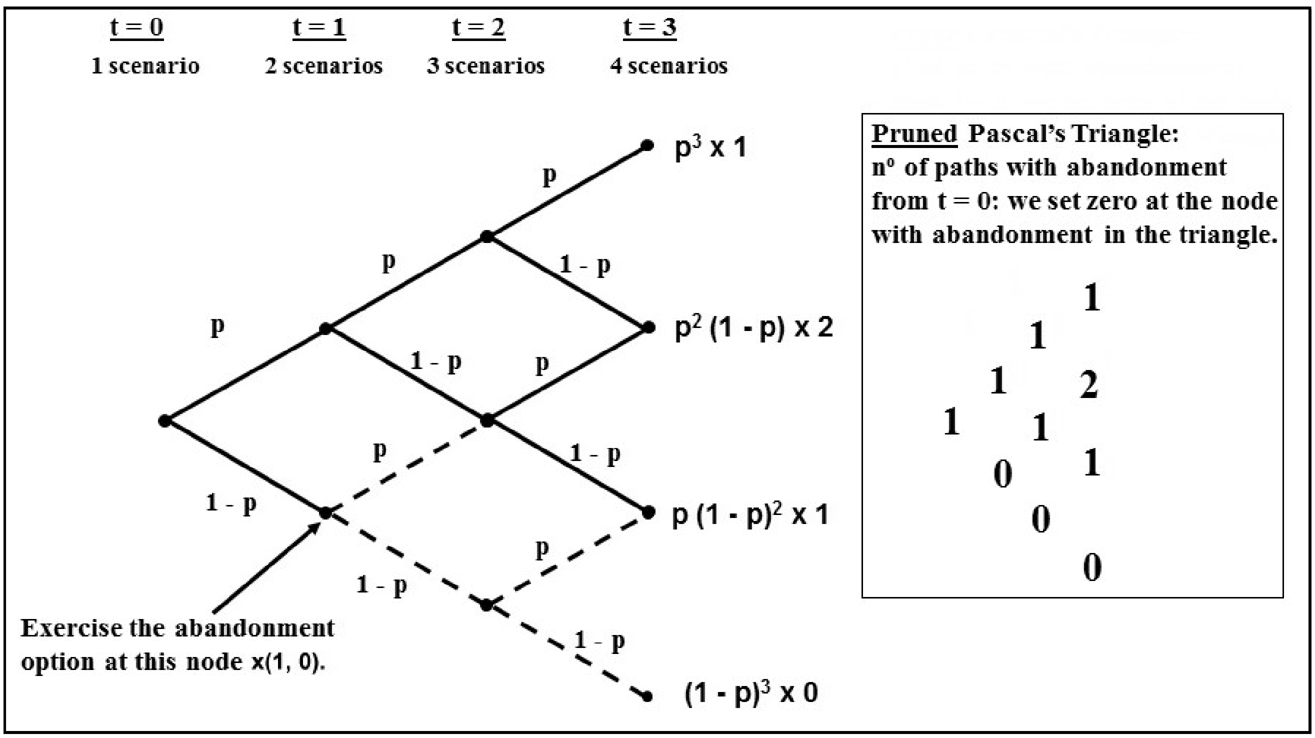

Figure 6 – Binomial Tree with Abandonment Option: Pruned Pascal's Triangle

The probability for one *path* doesn't change, $p^k (1 - p)^{n-k}$, but the number of active nodes change and so the number of paths reaching a specific node. This paper proposes a very simple recursive approach, the Pruned Pascal's Triangle. In this approach, we first solve the option-pricing problem, so that we know the nodes with abandonment. With this information, we construct the pruned triangle: set zero for each node in which is optimal the abandon. After that, use the Pascal's triangle rule (sum the numbers of the adjacent previous two nodes) as shown in Figure 6. Example, with zero at x(1, 0), the number of paths reaching x(2, 1) now is only one (1 + 0) instead two (1 + 1) and the number of paths reaching x(3, 2) is now only two (1 + 1 instead of 1 + 2), etc. By using this pruned triangle rule, we map all the number of paths in each node of the tree considering the abandonment, denoted by $N(n, k)_{p\,a}$. The probability of reach the node x(n, k) with production, considering the abandonment option, is:

$$\text{Probability to reach x(n, k) with option} = N(n, k)_{pa}\ p^{k}\ (1 - p)^{n-k} \tag{11}$$

So, for each instant n we can estimate the probability of producing at this date. Without previous or immediate abandon, the probability of producing is 100%[14]. However, with abandonment, this probability is less than 100% and this issue must be considered in expected reserve volume calculus. Figure 7 illustrates this issue from one numerical case (the same case used in Figure 3).

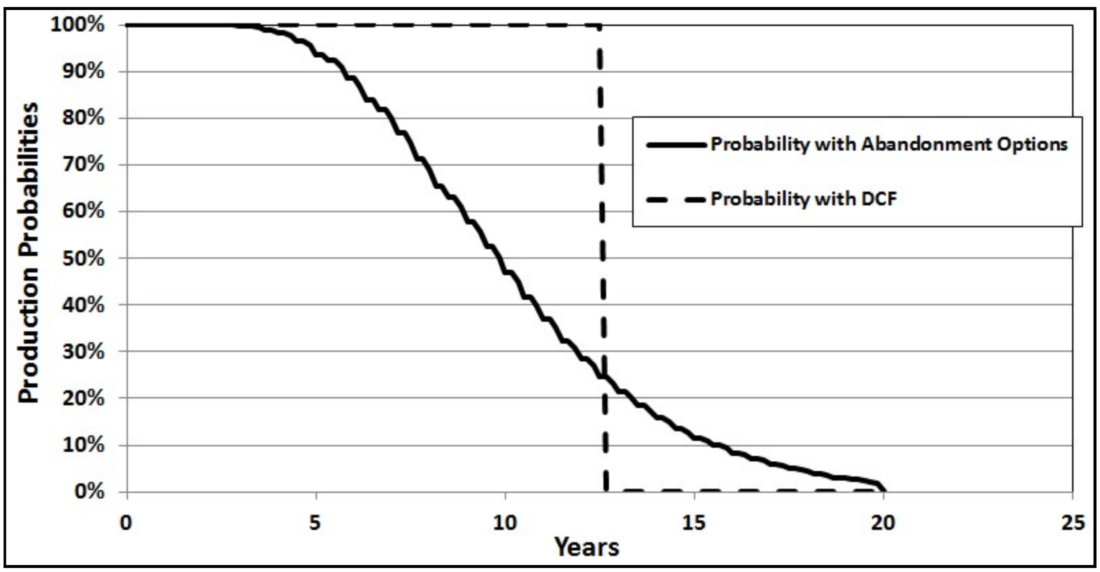

Figure 7 – Oil Production Probabilities: Options x DCF Approaches

5) Case Studies and Numerical Comparison

In this section, we present some numerical simulations in order to compare the three approaches to estimate the reserve volume (traditional DCF, probabilistic, and option) in terms of reserves volumes and the oilfield values. Let us consider a base-case with round numbers. Later, we present a sensitivity analysis for critical parameters like the oil price volatility.

[14] Without abandonment, the summation in k (for all k in a given date n) for the previous equation is 100%.

Let Q(t) be the oil production flow rate curve (in million barrels/year), Q(0) the initial production and λ the exponential decline rate factor. In any instant t, the production rate is:

$$Q(t) = Q(0)\, e^{-\lambda t} \tag{12}$$

In the binomial tree, we use time-step Δt between the decision points. The operational cash flow in this interval Δt depends on the production Q, the oil price P, the variable operational cost C (fixed), the fixed operational cost CF, the royalty rate[15] ROY and income tax rate T_c. It is given by:

$$CF(t) = \{Q(t)\, [\{P(t)\, (1 - ROY)\} - C] - CF\}\, \Delta t\, (1 - Tc) \tag{13}$$

Table 4 presents the economic/financial parameters used in the base-case.

Table 4 – Economic/Financial Parameters Used in the Base-Case

Input/Parameter	Value
Oil price at t = 0: P(0)	US$ 50/bbl
Oil price volatility: σ	30% p.a.
Risk-free interest rate: r	4% p.a.
Oil convenience yield: δ	6% p.a.
Risk-adjusted discount rate: μ	6% p.a.
Corporate income-tax rate: T_c	34%
Royalty rate: ROY	10%

The current oil price is based in the price seen in May 2017. The oil price volatility is a rounded value from a 30-years oil price time-series estimative. The risk-free interest rate is based in Treasury bonds from a long-term perspective. The oil price convenience yield is measured in futures markets using WTI contract for delivery in 18 months. The risk-adjusted discount rate considers a risk-premium of 2% p.a. over the risk-free rate. A sensitivity analysis for σ, r, and μ will be presented. The corporate income tax and royalty rates match the Brazilian economy conditions.

Table 5 presents the specific oilfield parameters used in the base-case.

[15] A compensation for the depletion of nonrenewable resources. It is a kind of tax on gross revenue.

Table 5 – Specific Oilfield Parameters in the Base-Case

Input/Parameter	Value
Initial production: Q(0)	30 million bbl/year
Production decline rate: λ	10% p.a.
Variable Operational Cost: C	20 US$/bbl
Fixed Operational Cost: CF	US$ 200 million/year
Maximum Production Life: T	20 years
Salvage Value at t = 0: SV(0)	US$ 800 million
Salvage Value at t = T: SV(T)	0
Abandonment Cost: A	US$ 300 million
Potential Reserves (without early abandon): B_P	261,57 million bbl

The initial production corresponds to 82,192 barrels per day. The decline rate is typical of many oilfields. The operational cost inputs are drawn from an offshore oilfield with rounded numbers. The maximum production life can be either because of the equipment life limit or by legal restriction (in Brazil, the maximum production time of oil concessions is 27 years). The salvage value at t = 0 considers a floating production unit (like a FPSO) that can be used in other oilfield, but with remaining life of 20 years, so that the salvage/residual value after 20 years is negligible. The abandonment cost considers the abandon of 12 producing and injection subsea wells. The *potential* reserve is the cumulative production considering the maximum production life (without early abandonment), which is assumed 20 years of production in this numerical example.

We build this binomial model with an Excel spreadsheet, which has several binomial trees: real option trees (with risk-neutral probability), probabilistic DCF tree, and true probability trees for the expected volume calculus. We present the results from the base case in Table 6:

Table 6 – Base Case Results

	Oilfield Value (million $)	Expected Reserve (million bbl)	Expected Reserve Life (years)
Without Early Abandon	837.81	261.57	20
DCF	1,196.86	222.77	13.33
Probabilistic DCF	1,935.93	112.65	5.93
Options Approach	2,048.53	149.60	8.24

The table above presents the case "without early abandon" for comparison purposes (shows the potential reserve if the oilfield produces all the 20 years). As expected, the options approach has the highest monetary value because the options rule maximizes the oilfield value. The DCF forecast exhibits the highest reserve volume, 48.9 % higher than the one reported by the options approach (but with much lower oilfield value, only 58.4% of the options case). The probabilistic DCF reports an even lower reserve volume and its rule results in lower oilfield value than the options. However, the probabilistic approach generates a much higher oilfield value than the traditional DCF and a little lower value than the option approach.

We calculate also the optimal oil price to abandon along the oilfield life. This optimal abandonment price is named the option exercise threshold, denoted by P*(t). In the binomial the initial price P(0) can be above (keep producing) or below (abandonment is optimal). If it is above, we rerun the binomial tree with a lower P(0) until we get P*(0) where we are indifferent between producing and abandoning. This is a disadvantage of the binomial approach when compared with the method of differential equation solved by finite differences (using a grid P x t, we run only once the finite differences software to obtain the threshold curve).

For the base case we generate the following thresholds chart, which shows both the real option and the probabilistic DCF optimal abandonment oil prices.

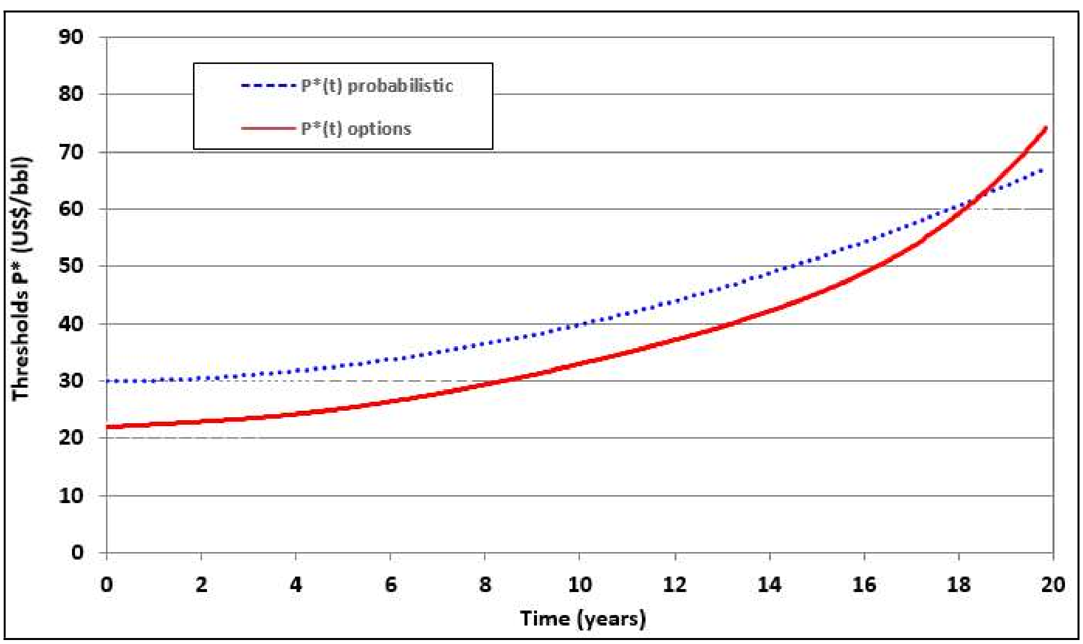

Figure 8 – Oil Price Thresholds for the Base Case

The options threshold curve is under the probabilistic threshold curve for almost all the oilfield life, except near the expiration. But, if we consider a more "pure" case, with no savage value (only the effect of waiting for a better oil price), the options curve will always be below the probabilistic curve. The Figure 7 shows the base case, except that the savage value is zero.

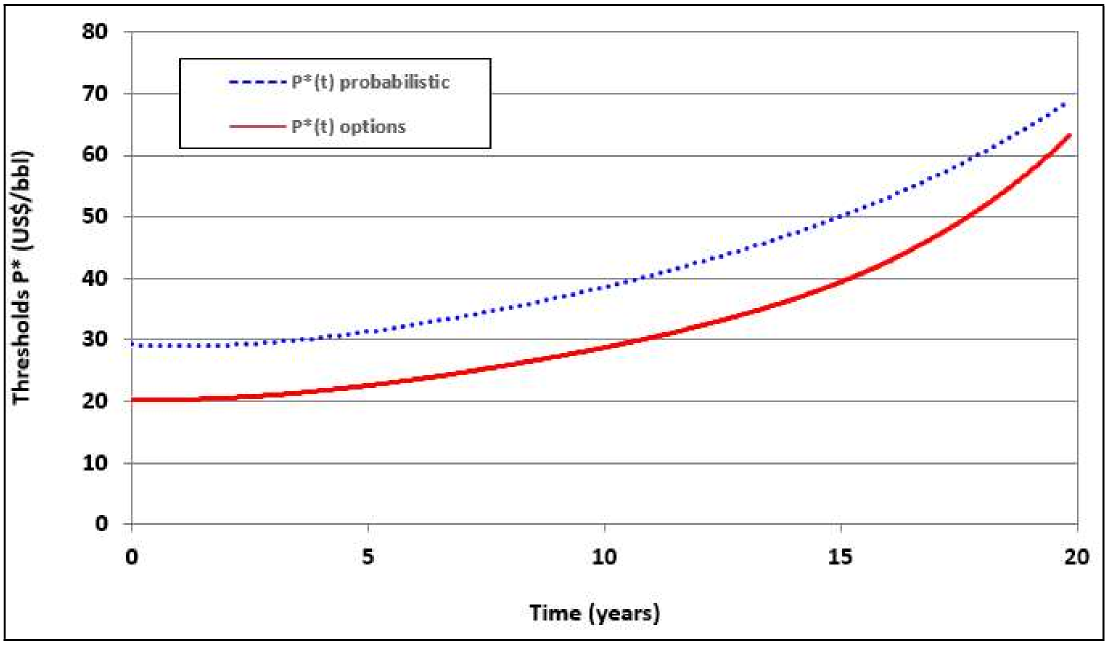

Figure 9 - Oil Price Thresholds for the Base Case without Savage Value

We perform also a sensitivity analysis for the oil price volatility for several model results (reserve volume, oilfield value and average time to abandon) in the following charts:

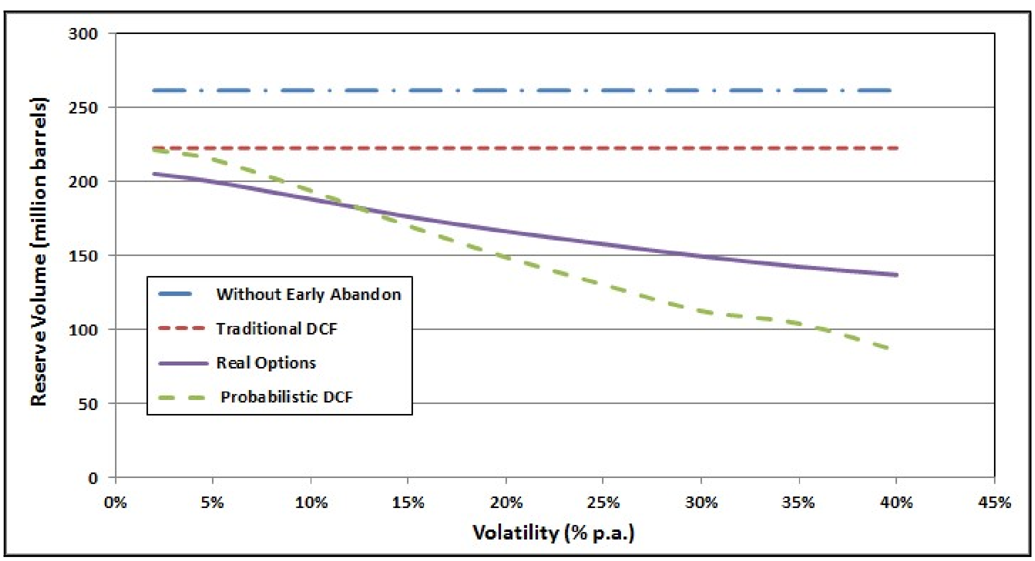

Figure 10 – Effect of Volatility in the Reserve Volume

Higher oil price volatility means that the oil price trajectory reach more extremes values (both higher and lower oil prices). The options approach (and the probabilistic one) “see” more scenarios where is optimal the abandonment, whereas DCF looks only the expected oil price curve that remains the same. This issue decreases the reserve volume for the options approach because increases the abandonment probability. However, as showed in the Figure 11 below, the oilfield value is higher with higher oil price volatility in the options approach due to very high prices scenarios with high volatility (whereas the option to abandon limit the losses in the very low oil prices scenarios).

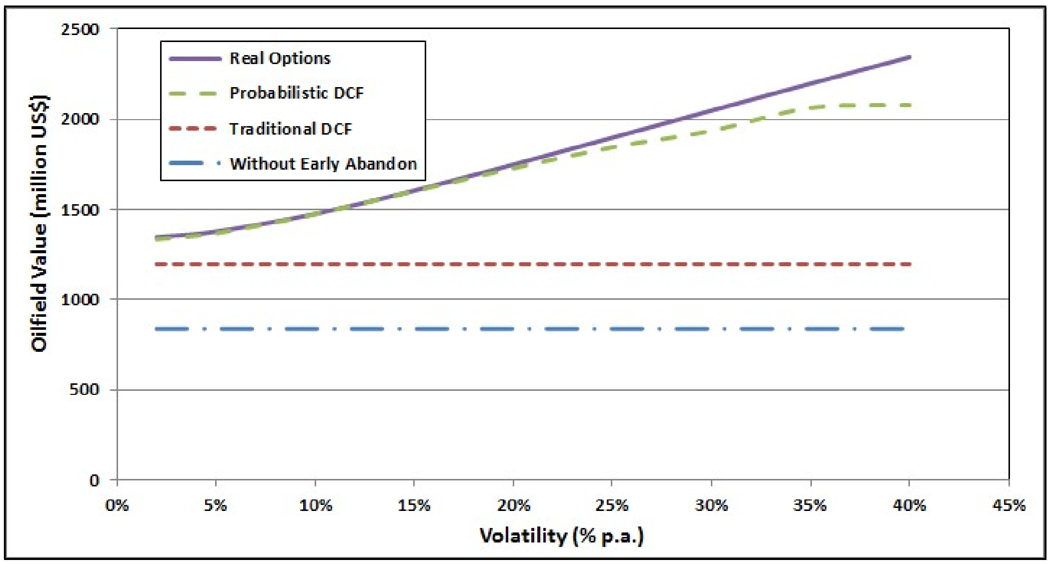

Figure 11 – Effect of Volatility in the Oilfield Value

Figure 12 below shows the expected oilfield life (or average time to abandon) for different abandonment rules. For each date t we consider the abandonment probability and so the expected time to abandon.

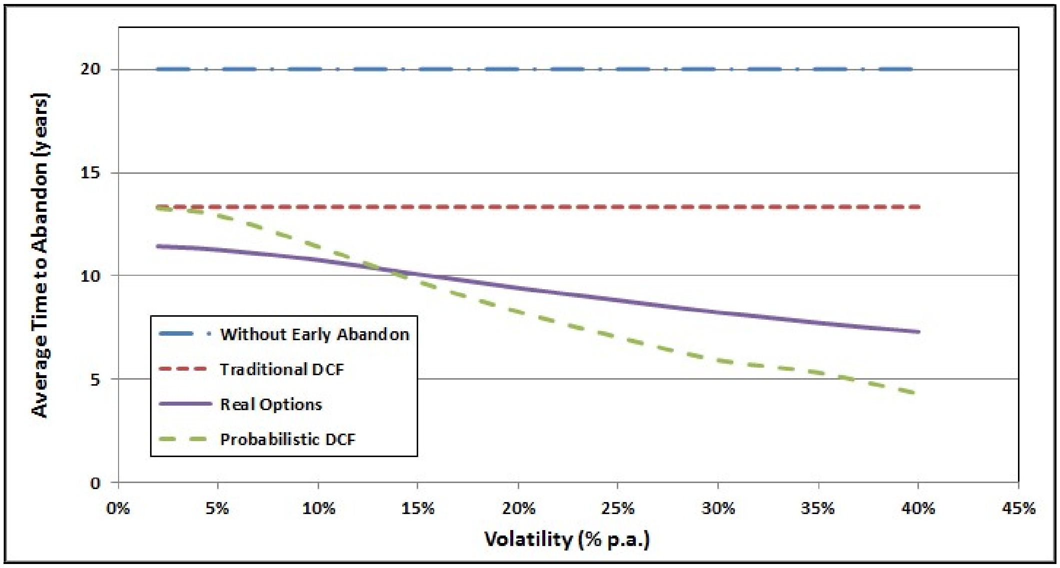

Figure 12 – Effect of Volatility in the Oilfield Expected Life

We can split the expected reserve volume in scenarios. The Figures 13 and 14 show the histograms of oilfield reserve volume for the probabilistic and options cases.

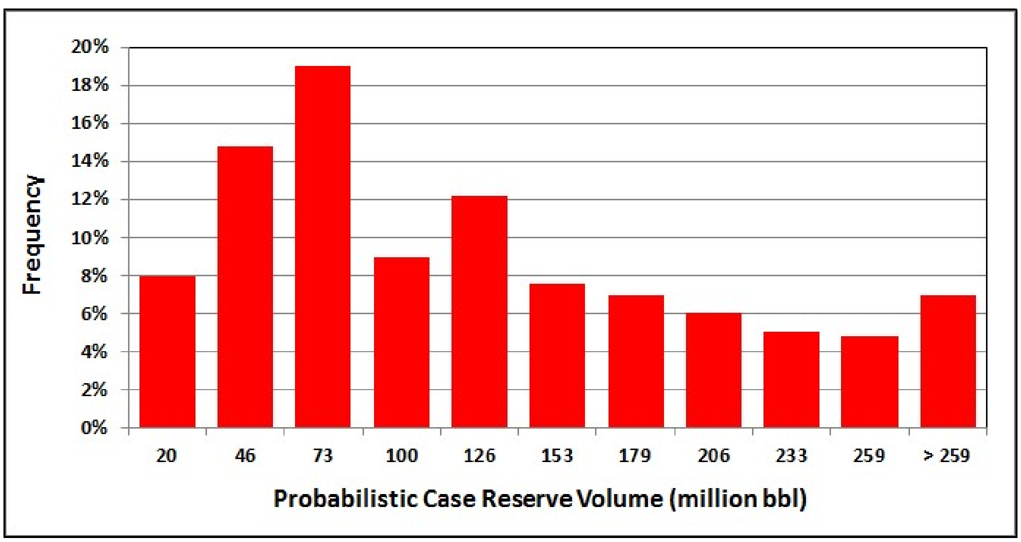

Figure 13 – Histogram of the Reserve Volume for the Probabilistic Case

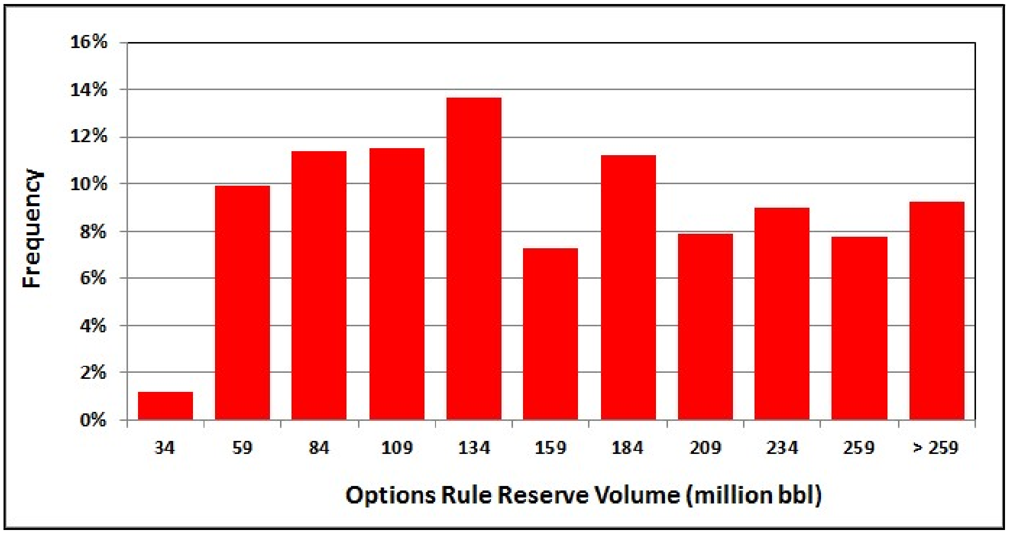

Figure 14 – Histogram of the Reserve Volume for the Options Case

We also use Monte Carlo simulation combined with binomial tree to address the effect of reservoir and geologic uncertainties on reserves, and in preliminary simulations we found that these technical uncertainties reduce even more the *expected* reserve, although the effect seems ex-ante small. This issue is left to a future research.

6) Concluding Remarks and Suggestions to Future Research

The paper model, using realistic case studies, shows that oil companies using deterministic DCF are *ex-ante* optimistic regarding oil reserve volume in new oil/gas fields, but *ex-post* abandon too early the oilfields when using the DCF rule. Therefore, it is recommended that oil-sector managers and regulators consider reporting also a more conservative reserve volume using the real options approach in order to provide additional and better information for more sophisticate investors. For mature fields, the opposite can occur: in many scenarios, options approach points a positive probability of producing, whereas DCF points 0% chance of keeping the production alive, as showed in Figure 7 after the year 13 of production.

We used the GBM for the oil prices because it is the most popular stochastic process and is sufficiently simple for the practitioners to be motivated in the use of option-pricing techniques for the reserves volume calculation. However, it can be easily extended to other stochastic processes like mean-reversion or mean-reversion with jumps (as in Dias & Rocha, 1998) or two-factor model (as in Jafarizadeh & Bratvold, 2012). Another extension could be an in depth analyses of the combined effect of oil price and geological uncertainties. These issues are left for a future research.

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8) Glossary

Abandonment Cost: it is the cost of removal of the installed platform/equipment and environmental recovery. In petroleum/gas fields, it includes the closure cost of each well. In offshore fields with subsea wells, the well's closure cost is by far the main item of abandonment cost.

Petroleum Reserves: the economically recoverable volume of hydrocarbons in a field with a given development project.

Salvage value: or residual value, is the value that can be obtained by selling the assets from the abandoned project (installations/equipment, platform, flowlines, etc.). It is value of the best alternative use of these assets. It is typically a time decreasing value and in many cases it is only a scrap value. Some authors use the term abandonment value.