Strategic Capacity Investment under Uncertainty with Volume Flexibility

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Abstract

This article considers investment decisions in an uncertain and competitive framework, where the firm(s) can adjust the output quantities according to the market demand fluctuations. In particular, 3 cases will be studied in this paper (we currently only finished two). The first case is a duopoly model with exogenous firm roles, where a first investor, the leader, always producing up to full capacity (dedicated) and a second investor, the follower, being able to adjust output levels within constraint of the installed capacity (flexible). Both firms need to decide on the investment timing and investment capacity size. The main findings are as follows. Compared to a situation where the follower always produces up to full capacity, the leader has a larger incentive to accommodate a flexible follower. This is because the leader also benefits from the follower's volume flexibility. Due to the first mover advantage, the leader's value is higher than the follower's value, despite the follower's technological advantage in flexibility. The second case is where the leader can adjust the output quantity while the follower cannot. The third case is where both the leader and the follower can adjust their output quantities.

Keywords: Investment under Uncertainty, Volume Flexibility, Entry Deterrence/Accommodation, Capacity Choice, Duopoly

JEL classification: E22, C73, D81

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The authors would like to thank Dolf Talman, Nick Huberts, Maria Lavrutich, Verena Hagspiel, Cláudia Nunes, Bert Willems, and participants of the 1st Young Academics on Real Options Workshop in Leipzig (March 2017) for their valuable comments. Xingang Wen gratefully acknowledges support from the German Research Foundation (DFG) via CRC 1283.

1 Introduction

Uncertainty is a main characteristic of the business environment nowadays. The technology advancement has shortened product life cycles, increased product variety, and indulged more demanding consumers. This contributes to the uncertainty in consumer demand and poses challenges on the manufacturing firms. The ability to produce to the least cost is no longer enough. The capability to absorb demand fluctuations has become an important competitive issue. Flexibility is considered an adaptive response to the environmental uncertainty (Gupta and Goyal, 1989). Browne et al. (1984) have defined eight different flexibility types, among which, the volume flexibility is described as "the ability to operate an FMS (Flexible Manufacturing Systems) profitably at different production volumes." Later on, Sethi and Sethi (1990) further describe volume flexibility as "the ability to be operated profitably at different overall output levels." According to Beach et al. (2000), utilizing flexibility presents performance-related benefits. Numerous studies have supported the importance of volume flexibility (Jack and Raturi, 2002). For instance, Goyal and Netessine (2011) show that volume flexibility may help the firm combat the product demand uncertainty. In a monopolistic market, Hagspiel et al. (2016) and Wen et al. (2017) analyze the volume flexibility's influences on a monopolistic investor's investment decision and show that it increases the value of the investment. In a competitive setting, an important question for the investors would be how the volume flexibility influences investment decisions and the investors' strategic interactions.

This article considers volume flexibility in a homogenous good market. Demand is linear and subject to stochastic shocks, which follow a geometric Brownian motion process. There are two firms that decide on entering the market by investing in a production plant. More specifically, they have to decide about the investment timing and the investment capacity. As an extension to Huisman and Kort (2015), where the strategic interaction of two dedicated firms is studied, we plan to study several cases in order to gain insight about the volume flexibility. These cases, together with Huisman and Kort (2015) are summarized in the following table 1.

	Dedicated Leader	Flexible Leader
Dedicated Follower	Huisman and Kort (2015)	Case 2
Flexible Follower	Case 1	Case 3

Table 1: Overview of the cases for exogenous firm roles

- Case 1: exogenous duopoly with volume-dedicated leader and volume-flexible follower

Case 1 is such that, the first investor, the leader, has dedicated technology. The follower, i.e., the firm that invests secondly, has volume flexibility. The leader always produces up to capacity and has a first mover advantage. The follower can adjust output levels according to market demand. A surprising outcome is that, because the market price is affected by the follower's flexible output, the leader benefits from the follower's flexibility when market demand is low. This is because the follower reduces the output quantity in such a case. The analysis in this case starts with a market where no firms are active. Then two intervals on market demand are identified for the leader, with one interval where it is optimal to deter the entry of the flexible follower and the other one where it is optimal to accommodate the entry. We find that compared to a dedicated follower, the leader is less likely to deter a flexible follower. This is because when there is demand uncertainty, both the leader and the flexible follower tend to wait for more information about the future market and invest later. For the entry deterrence strategy, the leader has an incentive to overinvest to deter the entry of the follower[1]. Being dedicated and unable to the instant market demand, the leader is more vulnerable to the negative demand shocks. For the follower, the volume flexibility yields higher values and thus motivates to invest earlier compared with a dedicated follower. This results in a shorter monopoly period for the leader and diminishes the attractiveness of entry deterrence compared to the case where the follower is dedicated. Furthermore, compared to a dedicated follower, it is more likely for the leader to accommodate a flexible follower. For the accommodation strategy, the two firms invest at the same time, so the incentive to overinvest in order to deter the follower's entry disappears. The market price reacts to the follower's output adjustment, and this diminishes the leader's vulnerability to demand uncertainty. The incentive to overinvest in order to reduce the capacity size of the flexible follower and to benefit from the follower's output adjustment is still strong. This makes accommodation of the flexible follower more attractive to the leader.

We also find that in a fast growing market, the flexible follower produces below capacity right after investment. While in a slowly growing or shrinking market, the flexible follower produces up to capacity right after investment. In the intermediate case, the flexible follower produces up to capacity right after investment when uncertainty is low and below capacity when uncertainty is high. These findings are the same as that for the flexible monopolist by Wen et al. (2017). The strategic interactions between the leader and the flexible follower do not influence these results. Moreover, there is "free riding"on the follower's flexibility since the volume flexibility affects market prices, and thus enlarges the profitability of the leader. So, the flexible follower cannot fully capture the innovative benefits from the technology advancement. However, this does not diminish the follower's incentive to invest in the volume flexibility technology, because it still generates a larger value for the follower regardless the leader chooses and entry deterrence or entry accommodation strategy.

- Case 2: exogenous duopoly with volume-flexible leader and volume-dedicated follower

- Case 3: exogenous duopoly with volume-flexible leader and volume-flexible follower

[1]Overinvesting refers to that a firm invests more capacity as the first investor than when investing simultaneously with the other firm at a predetermined point of time.

2 Dedicated leader and flexible follower

2.1 Model Setup

Consider a framework where two firms can invest in production capacity to enter a market or serve a particular demand. Of the two firms, the follower (second investor) has volume flexibility technology and adjust output levels up to the installed capacity after the investment. The leader (first investor) has no such technology and can only produce at full capacity level. Denote by $K_D \geq 0$ and $K_F \geq 0$ the capacity of the dedicated leader and the flexible follower, respectively. For both firms, the unit cost for capacity investment is $\delta > 0$ and the unit cost for production is $c > 0$. The price at time $t \geq 0$ is $p(t)$, given by the inverse demand function

$$p(t) = X(t)\left[1 - \gamma\left(q_D(t) + q_F(t)\right)\right],$$

where $\gamma > 0$ is a constant, $q_D(t)$, equal to K_D, and $q_F(t)$, no larger than K_F, denote the production output for the dedicated and flexible firm at time t, respectively, and the uncertainty in demand, $\{X(t)|t \geq 0\}$, follows a geometric Brownian Motion (GBM) process

$$\mathrm{d}X(t) = \alpha X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}W_t,$$

in which $X(0) > 0$, α is the trend parameter, $\sigma > 0$ is the volatility parameter, and dW_t is the increment of a Wiener process. The inverse linear demand function has among others been adopted by Pindyck (1988) and Huisman and Kort (2015). Both firms are risk neutral and have a discount rate of r, which is assumed to be larger than α, the trend of GBM $X(t)$. This is to prevent that it is optimal for the firms to always delay the investment (see Dixit and Pindyck, 1994). From now on I drop the argument of time whenever there can be no misunderstanding.

2.2 Flexible Follower's Optimal Investment Decision

The leader is assumed to be already in the market when the flexible follower makes investment decisions. Given $X(t) = X$ and the leader's investment capacity K_D, denote $\pi_F(X, K_D, K_F)$ as the profit for the flexible follower after investing in capacity K_F. The follower is flexible and can adjust its output quantity between 0 and the invested capacity K_F. The output maximizes the follower's profit flow, which is equal to

$$\pi_F(X, K_D, K_F) = \max_{0 \leq q_F \leq K_F} \left\{X\left[1 - \gamma\left(K_D + q_F\right)\right] - c\right\} q_F.$$

Given $0 \leq K_D < 1/\gamma$, the optimal output level for the follower is

$$q_F(X, K_D, K_F) = \begin{cases} 0 & 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{X-c}{2\gamma X} - \frac{K_D}{2} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ K_F & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \tag{1}$$

The corresponding profit flow is given by

$$\pi_F(X, K_D, K_F) = \begin{cases} 0 & 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{(X-c-\gamma X K_D)^2}{4\gamma X} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ (X - c - \gamma X K_D) K_F - K_F^2 \gamma X & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \tag{2}$$

The flexible follower's investment decision is solved as an optimal stopping problem and can be formalized as

$$\sup_{T\geq 0, K_F \geq 0} E\left[\int_T^\infty \pi_F(X(t), K_D, K_F)\exp(-rt)\mathrm{d}t - \delta K_F \exp(-rT)\middle| X(0)\right],$$

conditional on the available information at time 0, where T is the time when the flexible follower invests, and K_F is the acquired capacity at time T. Denote by $V_F(X, K_D, K_F)$ the value for the flexible follower, and it satisfies the Bellman equation

$$rV_F = \pi_F + \frac{1}{\mathrm{d}t} E[\mathrm{d}V_F]. \tag{3}$$

Applying Ito's Lemma, substituting and rewriting lead to the following differential equation (see also, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_F(X, K_D, K_F)}{\partial X^2} + \alpha X \frac{\partial V_F(X, K_D, K_F)}{\partial X} - rV_F(X, K_D, K_F) + \pi_F(X, K_D, K_F) = 0. \tag{4}$$

Substituting (2) into (4) and employing value matching and smooth pasting for $X = c/(1-\gamma K_D)$ and $X = c/(1 - \gamma K_D - 2\gamma K_F)$ yield the follower's value after investment as given by

$$V_F(X, K_D, K_F) = \begin{cases} L(K_D, K_F) X^{\beta_1} & 0 < X < \frac{c}{1-\gamma K_D}, \\ M_1(K_D, K_F) X^{\beta_1} + M_2(K_D) X^{\beta_2} & \\ \quad + \frac{(1-\gamma K_D)^2 X}{4\gamma(r-\alpha)} - \frac{c(1-\gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r+\alpha-\sigma^2)} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ N(K_D, K_F) X^{\beta_2} - \frac{cK_F}{r} + \frac{XK_F(1-\gamma K_D - \gamma K_F)}{r-\alpha} & X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \end{cases} \tag{5}$$

in which

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{6}$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < -1. \tag{7}$$

The expressions of $L(K_D, K_F)$, $M_1(K_D, K_F)$, $M_2(K_D)$, $N(K_D, K_F)$ can be found in Appendix A.1. If $K_D = 0$, the model reduces to the monopoly case.

The follower does not produce right after the investment for $0 < X < c/(1-\gamma K_D)$. Thus, $L(K_D, K_F)X^{\beta_1}$ is positive and represents the option value to start producing in the future as soon as X reaches $c/(1-\gamma K_D)$. $M_1(K_D, K_F)X^{\beta_1}$ is negative and corrects for the fact that if X reaches $c/(1 - \gamma K_D - 2\gamma K_F)$, the follower's output will be constrained by the installed capacity level. $M_2(K_D)X^{\beta_2}$ has both a negative and a positive effect. The negative effect corrects for the positive quadratic form of cash flows even when X drops below $c/(1-\gamma K_D)$. The positive effect comes from the option that the follower would temporarily suspend production for a too small market demand. When $\sigma^2 < r+\alpha$, the negative effect dominates the positive effect, and if $\sigma^2 > r+\alpha$ the positive effect dominates[2]. $N(K_D, K_F)X^{\beta_2}$ is positive and describes the option value that if demand decreases, i.e., X drops below $c/(1-\gamma K_D - 2\gamma K_D)$, the follower produces below full capacity. The optimal investment decision is found in two steps. First, given K_D and the level of X, the optimal value of K_F is found by maximizing $V_F(X, K_D, K_F) - \delta K_F$, which yields $K_F(X, K_D)$. Second, the optimal investment threshold $X_F^*(K_D)$ for the follower can be derived. The two steps are summarized in the following proposition, where

$$F(\beta) \quad = \quad \frac{2\beta}{r} - \frac{\beta-1}{r-\alpha} - \frac{\beta+1}{r+\alpha-\sigma^2}. \tag{8}$$

and $\bar{\sigma}$ is such that

$$\bar{\sigma}^2 = \frac{-2\left(\Lambda-\alpha^2\right)(2r-\alpha) + 4\sqrt{r\Lambda\left(\Lambda-\alpha^2\right)(r-\alpha)}}{\Lambda-(2r-\alpha)^2}, \tag{9}$$

with $\Lambda = \left(\frac{2\delta r(r-\alpha)-\alpha c}{c}\right)^2$. $\bar{\sigma} > 0$ is a value of the drift parameter that determines if the follower produces below or up to capacity right after investment. $\bar{\sigma}$ is only defined for $r - c/\delta < \alpha \le \delta r^2/(c+\delta r)$.

Proposition 1 *Given that the dedicated firm has already invested capacity $K_D \in [0, 1/\gamma)$, there are two possibilities for the follower's investment decisions:*

1. *Suppose $\alpha > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \alpha \le \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$. The follower produces below capacity right after investment. For any $X \ge c/(1-\gamma K_D)$, the optimal capacity $K_F(X, K_D)$ that maximizes $V(X, K_D, K_F) - \delta K_F$ is given by*

$$K_F\left(X, K_D\right) = \frac{1}{2\gamma}\left(1-\gamma K_D - \frac{c}{X}\left[\frac{2\delta\left(\beta_1-\beta_2\right)}{c\left(1+\beta_1\right)F\left(\beta_2\right)}\right]^{\frac{1}{\beta_1}}\right), \tag{10}$$

and the optimal investment threshold $X_F^*(K_D)$ satisfies

$$\frac{c(1-\gamma K_D)F(\beta_1)}{4\gamma\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2} + \frac{1}{4\gamma}\left[\frac{\beta_1-1}{\beta_1}\frac{X(1-\gamma K_D)^2}{r-\alpha} - \frac{2c(1-\gamma K_D)}{r} + \frac{\beta_1+1}{\beta_1}\frac{c^2}{X(r+\alpha-\sigma^2)}\right] - \delta K_F(X, K_D) = 0. \tag{11}$$

If $X(0) < X_F^*(K_D)$, then the optimal capacity of the follower is $K_F^*(K_D) = K_F\left(X_F^*(K_D), K_D\right)$. If $X(0) \ge X_F^*(K_D)$, then the follower invests at $t=0$ with capacity $K_F^*(K_D) = K_F(X(0), K_D)$.

[2]Compared to Hagspiel et al. (2016), the dominance of positive and negative effect can be determined in this paper. This is probably due to the fact that I adopt a multiplicative inverse demand structure, and they study an additive inverse demand function.

2. *Suppose $\alpha \le r - c/\delta$, or both $r - c/\delta < \alpha \le \delta r^2/(c+\delta r)$ and $\sigma \le \bar{\sigma}$. Then the follower produces up to capacity right after investment. For any $X \ge c/(1-\gamma K_D)$, the optimal capacity $K_F(X, K_D)$ satisfies*

$$\frac{c\,(1+\beta_2)\,F\,(\beta_1)}{2\,(\beta_1-\beta_2)}\left(\frac{X(1-2\gamma K_F-\gamma K_D)}{c}\right)^{\beta_2}+\frac{X(1-2\gamma K_F-\gamma K_D)}{r-\alpha}-\frac{c}{r}-\delta=0, \tag{12}$$

and the optimal investment threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned}&\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\frac{X}{c}\right)^{\beta_2}\left((1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right)\\&\quad+\frac{(\beta_1-1)X}{\beta_1}\,\frac{K_F-\gamma K_D K_F-\gamma K_F^2}{r-\alpha}-\frac{cK_F}{r}-\delta K_F=0,\end{aligned} \tag{13}$$

with $K_F = K_F(X, K_D)$. If $X(0) < X_F^*(K_D)$, then the optimal capacity of the follower is $K_F^*(K_D) = K_F(X_F^*(K_D), K_D)$. If $X(0) \ge X_F^*(K_D)$, then the follower invests at $t = 0$ with capacity $K_F^*(K_D) = K_F(X(0), K_D)$.

From Proposition 1, the influence of the leader's investment capacity on the follower's investment decision is concluded in Corollary 1. Their proof can be found in Appendix A.2 and A.3.

Corollary 1 *The dedicated leader's capacity level K_D influences the follower's investment decision such that if the leader invests more, then the follower invests later and invests less.*

This result is intuitive because the leader always produces up to capacity after investment, and the more the leader invests, the smaller market share is left for the flexible follower. When deciding on the capacity, the follower takes the future market demand into consideration. Thus, a smaller market share decreases the follower's investment capacity. Moreover, given the current market demand level, the market price decreases if the leader invests more. This would lower the follower's potential profits and delay the follower's entry because the follower prefers to wait for a higher market price.

2.3 Dedicated Leader's Optimal Investment Decision

The leader also takes the follower's decisions into consideration when deciding on the market entry. Suppose the leader invests at t with capacity size K_D and $X(t) = X$. Corollary 1 shows that the leader's capacity influences the follower investment timing. Assume there exists a capacity size for the leader, $\hat{K}_D(X)$, such that the follower's optimal threshold satisfies $X_F^*(\hat{K}_D) = X$, and when the follower produces below capacity right after investment, $\hat{K}_D(X)$ can be derived from (11) as to satisfy

$$\begin{aligned}&\frac{c(1-\gamma K_D)F(\beta_1)}{2\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2}+\frac{\beta_1-1}{2\beta_1}\frac{X(1-\gamma K_D)^2}{r-\alpha}-\frac{c(1-\gamma K_D)}{r}+\frac{\beta_1+1}{2\beta_1}\frac{c^2}{X(r+\alpha-\sigma^2)}\\&\qquad-\delta(1-\gamma K_D)+\frac{c\delta}{X}\left(\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}}=0.\end{aligned} \tag{14}$$

From Corollary 1 it can be concluded that if $K_D \le \hat{K}_D(X)$, then $X \ge X_F^*(K_D)$, implying that the follower invests at the same time with the leader. If $K_D > \hat{K}_D(X)$, then $X < X_F^*(K_D)$, implying that the follower invests later than the leader. The former corresponds to the leader's entry accommodation strategy and the latter corresponds to the entry deterrence strategy, as described by Huisman and Kort (2015). In the following analysis, the leader's entry accommodation and entry deterrence strategy are characterized as the local optimum for the leader's value maximization problem given by

$$\sup_{K_D \geq 0} E\Bigg[\int_0^T (K_D(1-\gamma K_D)X(t) - c_D K_D)\, e^{-rt}\mathrm{d}t \\ + \int_T^\infty (K_D(1-\gamma K_D - \gamma q_F(X,K_D,K_F))X(t) - c_D K_D)\, e^{-rt}\mathrm{d}t - \delta_D K_D \Bigg| X(0) = X\Bigg],$$

where T is the moment that the flexible follower invests. Note that $T > 0$ under the entry deterrence strategy and $T = 0$ under the entry accommodation strategy.

The leader's investment value is generated by the leader's profit flow. Before the follower's entry, the leader is the only producer in the market. After the follower's entry, both firms are active in the market. The follower might not produce, produce below, and produces up to capacity after investment. Thus there are three cases for the leader's profit flow. For the given GBM level X and the leader's capacity size K_D, the leader's profit flow $\pi_D(X,K_D)$ is given by

$$\pi_D(X,K_D) = \begin{cases} K_D(1-\gamma K_D)X - c_D K_D & \text{if } 0 < X < \frac{c}{1-\gamma K_D}, \\ \frac{K_D}{2}(X - \gamma X K_D + c - 2c_D) & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ X K_D[1-\gamma(K_D + K_F^*(K_D))] - c_D K_D & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases}$$

Applying Ito's Lemma, substituting and rewriting leads to the following differential equation (see, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_D(X,K_D)}{\partial X^2} + \alpha X \frac{\partial V_D(X,K_D)}{\partial X} - rV_D(X,K_D) + \pi_D(X,K_D) = 0.$$

Substituting π_D into this differential equation and employing value matching and smooth pasting at $X = c/(1-\gamma K_D)$ and $X = c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$ give the value of the leader after the follower's investment as

$$V_D(X,K_D) = \begin{cases} \mathcal{L}(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{c_D K_D}{r} & \text{if } 0 \leq X < \frac{c}{1-\gamma K_D}, \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} & \\ \quad + \frac{X K_D(1-\gamma K_D)}{2(r-\alpha)} + \frac{(c-2c_D)K_D}{2r} & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) > \frac{X-c}{2\gamma X} - \frac{K_D}{2}, \\ \mathcal{N}(K_D)X^{\beta_2} - \frac{c_D K_D}{r} & \\ \quad + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X & \text{if } X \geq \frac{c}{1-\gamma K_D} \text{ and } K_F^*(K_D) \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}. \end{cases} \tag{15}$$

The derivation and expressions of $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, $\mathcal{N}(K_D)$, and their signs can be found in Appendix A.4. For $0 \leq X < c/(1-\gamma K_D)$, the demand is so low that the follower's production is temporarily suspended. However, the dedicated leader still produces at full capacity. In the leader's value function, $\mathcal{L}(K_D)X^{\beta_1}$ measures the decrease in the leader's value when the follower resumes production in the future. This happens as soon as X becomes larger than $c/(1-\gamma K_D)$. For $X \geq c/(1-\gamma K_D)$ and $K_F^*(K_D) > (X-c)/(2\gamma X) - K_D/2$, i.e., $c/(1-\gamma K_D) \leq X < c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, the follower produces below capacity right after investment. $\mathcal{M}_1(K_D)X^{\beta_1}$ corrects for the fact that if X reaches $c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, then the production of the follower is constrained by the installed capacity, hence the value of the leader increases. The term $\mathcal{M}_2(K_D)X^{\beta_2}$ denotes the decrease in the leader's option value, due to the fact that when X falls below $c/(1-\gamma K_D)$, the market demand becomes so small that the follower suspends production, whereas the leader still produces at full capacity, which results in negative profit. For $X \geq c/(1-\gamma K_D)$ and $K_F^*(K_D) \leq (X-c)/(2\gamma X) - K_D/2$, i.e., $X \geq c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, the follower produces up to capacity right after investment. The term $\mathcal{N}(K_D)X^{\beta_2}$ corrects for the fact that when X drops below $c/(1-\gamma K_D - 2\gamma K_F^*(K_D))$, the follower produces below capacity, and the value of the leader would increase.

The leader's strategies are analyzed for two cases, i.e., the follower produces below and up to capacity right after investment. This is because according to Wen et al. (2017), the flexible firm always produces right after investment. Before the follower invests, the leader's value function consists of two parts with one part from the monopolistic profit flow, and the other part correcting for the fact that the leader loses its monopoly privilege when the follower invests. Given that the leader invests at X, let the leader's value before the follower's entry be

$$V_D(X, K_D) = \mathcal{B}(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{c_D K_D}{r},$$

where $\mathcal{B}(K_D)$ has different expressions and will be derived for the two cases[3]. The leader's value function after the follower's investment is shown in (15). Then in every case both the entry deterrence and the entry accommodation strategy are analyzed.

- The flexible follower produces below capacity right after investment when $\alpha > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$.

Given that the leader invests at X, the value function before and after the follower's entry is as follows

$$V_D(X, K_D) = \begin{cases} \mathcal{B}_1(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{c_D K_D}{r} & X < X_F^*(K_D), \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X + \frac{(c-2c_D)K_D}{2r} & X \geq X_F^*(K_D), \end{cases} \tag{16}$$

with

$$\mathcal{B}_1(K_D) = \mathcal{M}_1(K_D) + \mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D) - \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D) + \frac{cK_D}{2r}X_F^{*-\beta_1}(K_D), \tag{17}$$

[3] $\mathcal{B}(K_D)$ and $\mathcal{L}(K_D)$ are different. According to Dixit and Pindyck (1994), the fundamental component in the leader's value function, i.e., $\frac{K_D(1-\gamma K_D)}{r}X - \frac{K_D}{r}$, is generated by the profit flows. $\mathcal{L}(K_D)X^{\beta_1}$ describes the deviation of $V_D(X, K_D)$ from the fundamental component due to the possibility that X will move across the boundary $\frac{c}{1-\gamma K_D}$. $\mathcal{B}(K_D)X^{\beta_1}$ describes the deviation of $V_D(X, K_D)$ from the fundamental component due to the possibility that X will move across the follower's optimal investment threshold X_F^*.

according to value matching condition at $X_F^*(K_D)$, which is defined by (11). Intuitively, $\mathcal{B}_1(K_D)$ is negative (see Appendix A.5). It corrects for the fact that when $X(t)$ reaches $X_F^*(K_D)$, the follower enters the market, putting an end to the leader's monopolistic privilege. The leader's entry deterrence and accommodation strategies, when the follower produces below capacity right after investment, are described in the following proposition (see also Appendix A.5 for the proof).

Proposition 2 *Suppose $\alpha > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \alpha \le \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$.*

(a) Entry Deterrence Strategy

The entry deterrence strategy will be considered whenever $X \in (X_1^{det}, X_2^{det})$, where X_1^{det} satisfies

$$\left(\frac{X^{det}}{X_F^*(0)}\right)^{\beta_1}\left[-\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)+\frac{c^{1-\beta_2}X_F^{*\beta_2}(0)}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)-\frac{X_F^*(0)}{2(r-\alpha)}+\frac{c}{2r}\right]$$
$$+\frac{X^{det}}{r-\alpha}-\frac{c_D}{r}-\delta_D=0, \quad (18)$$

where $X_F^*(0)$ can be derived from (10) *and* (11) *given that $K_D = 0$, and X_2^{det} together with $K_D^{det}(X_2^{det})$ satisfy* (14) *and*

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_1(K_D)(X^{det})^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X^{det}-\frac{c_D}{r}-\delta_D=0. \quad (19)$$

The optimal investment threshold X_D^{det} and investment capacity K_D^{det} are

$$\begin{aligned} X_D^{det} &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c_D}{r}+\delta_D\right), \\ K_D^{det} &\equiv K_D^{det}(X_D^{det}) = \frac{1}{(\beta_1+1)\gamma}, \end{aligned}$$

when $X < X_D^{det}$ and $X_D^{det} \in [X_1^{det}, X_2^{det}]$. If $X_D^{det} \le X \le X_2^{det}$, in order to implement the entry deterrence strategy, the leader invests immediately at X with capacity $K_D^{det}(X)$ that satisfies (19)*. Then the value of the entry deterrence strategy is*

$$V_D^{det}(X)=\mathcal{B}_1(K_D^{det}(X))X^{\beta_1}+\frac{K_D^{det}(X)\left(1-\gamma K_D^{det}(X)\right)}{r-\alpha}X-\frac{c_D K_D^{det}(X)}{r}-\delta_D K_D^{det}(X). \quad (20)$$

(b) Entry Accommodation Strategy

The entry accommodation strategy will be considered if $X \ge X_1^{acc}$, where X_1^{acc} and the corresponding $K_D^{acc}(X_1^{acc})$ satisfy (14) *and*

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)(X^{acc})^{\beta_1}+\frac{1-\gamma K_D-\beta_2\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)(X^{acc})^{\beta_2}$$
$$+\frac{1-2\gamma K_D}{2(r-\alpha)}X^{acc}+\frac{c-2c_D}{2r}-\delta_D=0. \quad (21)$$

The optimal investment threshold X_D^{acc} satisfies

$$\frac{c}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{\beta_1 X^{acc}}{c(\beta_1+1)}\right)^{\beta_2}+\frac{(\beta_1-1)X^{acc}}{2(r-\alpha)(\beta_1+1)}+\frac{c-2c_D}{2r}-\delta_D=0, \quad (22)$$

when $X < X_D^{acc}$ *and* $X_D^{acc} \geq X_1^{acc}$. *The optimal investment capacity for the entry accommodation strategy is*

$$K_D^{acc} \equiv K_D^{acc}(X_D^{acc}) = \frac{1}{(\beta_1+1)\gamma}.$$

If $X \geq X_D^{acc}$, *in order to implement the entry accommodation strategy, the leader invests immediately at* X *with capacity* $K_D^{acc}(X)$ *that satisfies* (21). *The value of the entry accommodation strategy is*

$$V_D^{acc}(X) = \mathcal{M}_1(K_D^{acc}(X))X^{\beta_1} + \mathcal{M}_2(K_D^{acc}(X))X^{\beta_2} + \frac{K_D^{acc}(X)\,(1-\gamma K_D^{acc}(X))}{2(r-\alpha)}X + \frac{(c-2c_D)K_D^{acc}(X)}{2r} - \delta_D K_D^{acc}(X). \tag{23}$$

- The flexible follower produces up to capacity right after the investment when $\alpha \leq r - c/\delta$, or both $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ and $\sigma \leq \bar{\sigma}$.

Similar to where the follower produces below capacity right after investment, given that the leader invests at X, the value function before and after the follower's entry can be written as

$$V_D(X, K_D) = \begin{cases} \mathcal{B}_2(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{c_D K_D}{r} & X < X_F^*(K_D), \\ \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X - \frac{c_D K_D}{r} & X \geq X_F^*(K_D), \end{cases} \tag{24}$$

with

$$\mathcal{B}_2(K_D) = \mathcal{N}(K_D)X_F^{*\beta_2-\beta_1}(K_D) - \frac{\gamma K_D K_F^*(K_D)}{r-\alpha}X_F^{*1-\beta_1}(K_D), \tag{25}$$

according to the value matching condition at the flexible follower's investment threshold $X_F^*(K_D)$, which is defined by (13).

Similar as $\mathcal{B}_1(K_D)$, $\mathcal{B}_2(K_D)$ corrects for the fact that when the follower enters the market, i.e. X reaches $X_F^*(K_D)$, it would put an end to the leader's monopoly privilege. Thus, $\mathcal{B}_2(K_D)$ is negative, shown in Appendix A.6. Because $X_F^*(K_D)$ increases with K_D according to Corollary 1, it is possible for the dedicated leader to delay the entry of flexible follower through the entry deterrence strategy by investing $K_D^{det}(X) > \hat{K}_D(X)$. Otherwise, the two firms invest at the same time, implying the leader applies the entry accommodation strategy by investing $K_D^{acc} \leq \hat{K}_D(X)$. This critical size for the leader's capacity, $\hat{K}_D(X)$, can be derived from (13) with the follower's optimal investment capacity $K_F^*(X) \equiv K_F^*(K_D(X))$ satisfying (12).

The leader's investment decision under entry deterrence and accommodation strategies, when the follower produces up to capacity right after investment, are summarized in the following proposition with the proof in Appendix A.6.

Proposition 3 *Suppose* $\alpha \leq r - c/\delta$, *or both* $r - c/\delta < \alpha \leq \delta r^2/(c+\delta r)$ *and* $\sigma \leq \bar{\sigma}$.

(a) Entry Deterrence Strategy

The entry deterrence strategy is possible if $X \in \left(X_1^{det}, X_2^{\det}\right)$. X_1^{det} *satisfies*

$$\frac{c}{2(\beta_1-\beta_2)}\left(\frac{X^{det}}{X_F^*(0)}\right)^{\beta_1}\left(\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left[\left(\frac{X_F^*(0)}{c}\right)^{\beta_2}-\left(\frac{X_F^*(0)\left(1-2\gamma K_F^*(0)\right)}{c}\right)^{\beta_2}\right]\right.$$
$$\left.-\frac{\beta_1-\beta_2}{r-\alpha}\frac{2\gamma X_F^*(0)K_F^*(0)}{c}\right)+\frac{X^{det}}{r-\alpha}-\frac{c_D}{r}-\delta_D=0, \qquad (26)$$

where $K_F^*(0)$ *and* $X_F^*(0)$ *can be derived from* (12) *and* (13) *given that* $K_D=0$. X_2^{det}, $K_D^{det}(X_2^{det})$ *and* $K_F^*(X_2^{det})$ *satisfy* (12), (13), *and*

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_2(K_D)(X^{det})^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X^{det}-\frac{c_D}{r}-\delta_D=0. \qquad (27)$$

The optimal investment threshold X_D^{det} *and the corresponding optimal capacity* K_D^{det} *are equal to*

$$\begin{aligned} X_D^{det} &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c_D}{r}+\delta_D\right), \\ K_D^{det} &\equiv K_D^{det}(X_D^{det})=\frac{1}{(\beta_1+1)\gamma}, \end{aligned}$$

if $X<X_D^{det}$ *and* $X_D^{det}\in[X_1^{det},X_2^{det}]$. *If* $X_D^{det}\le X<X_2^{det}$, *in order to implement the entry deterrence strategy, the leader invests immediately at* X *with capacity* $K_D^{det}(X)$ *that satisfies* (27). *The value of the entry deterrence strategy is*

$$V_D^{det}(X)=\mathcal{B}_2(K_D^{det}(X))X^{\beta_1}+\frac{K_D^{det}(X)\left(1-\gamma K_D^{det}(X)\right)}{r-\alpha}X-\frac{c_DK_D^{det}(X)}{r}-\delta_DK_D^{det}(X). \qquad (28)$$

(b) Entry Accommodation Strategy

The entry accommodation strategy is possible if $X>X_1^{acc}$. X_1^{acc}, $K_D^{acc}(X_1^{acc})$, *and* $K_F^*(X_1^{acc})$ *satisfy* (12), (13), *and*

$$\frac{(1-\gamma K_D-\beta_2\gamma K_D)(X^{acc})^{\beta_2}}{K_D(1-\gamma K_D)}\mathcal{N}(K_D)+\frac{X^{acc}(1-\gamma K_D-\gamma K_F^*(K_D))(1-2\gamma K_D)}{(r-\alpha)(1-\gamma K_D)}-\frac{c_D}{r}-\delta_D=0. \qquad (29)$$

The optimal investment threshold X_D^{acc} *satisfies*

$$\frac{c(X^{acc})^{\beta_2}}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\left(\frac{1-\gamma K_D^{acc}}{c}\right)^{\beta_2}-\left(\frac{1-\gamma K_D^{acc}-2\gamma K_F^*(K_D^{acc})}{c}\right)^{\beta_2}\right)$$
$$+\frac{(\beta_1-1)X^{acc}}{\beta_1(r-\alpha)}\left(1-\gamma K_D^{acc}-\gamma K_F^*(K_D^{acc})\right)-\frac{c_D}{r}-\delta_D=0, \qquad (30)$$

if $X<X_D^{acc}$ *and* $X_D^{acc}\ge X_1^{acc}$. *The optimal investment capacity for the entry accommodation strategy is*

$$K_D^{acc}\equiv K_D^{acc}(X_D^{acc})=\frac{1}{(\beta_1+1)\gamma}.$$

If $X\ge X_D^{acc}$, *in order to implement the entry accommodation strategy, the leader invests immediately at* X *and the corresponding capacity* $K_D^{acc}(X)$ *satisfies* (29). *The value of the entry accommodation strategy* *is*

$$V_D^{acc}(X) = \mathcal{N}(K_D^{acc}(X))X^{\beta_2} + \frac{K_D^{acc}(X)\,(1-\gamma K_D - \gamma K_F^*(K_D^{acc}(X)))}{r-\alpha}X - \frac{c_D K_D^{acc}(X)}{r} - \delta_D K_D^{acc}(X). \tag{31}$$

A numerical example is provided to illustrate the possibility for the entry deterrence and accommodation strategies in Figure 1 and 2. Note that in this example the follower produces below capacity right after investment. Similar analysis can be conducted for the follower producing up to capacity right after investment.

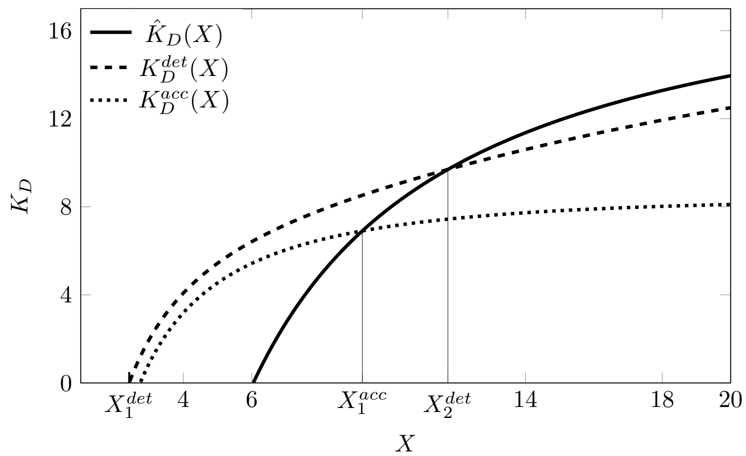


Figure 1: Illustration of $\hat{K}_D(X)$, $K_D^{det}(X)$, and $K_D^{acc}(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 1 illustrates the capacity levels $\hat{K}_D$, K_D^{det}, and K_D^{acc} as functions of X. For the given parameter values, the leader implements the deterrence strategy for $X \in [X_1^{det}, X_2^{det}]$, and the accommodation strategy for $X \geq X_1^{acc}$. When both strategies are implementable, the leader chooses the strategy that generates higher values. More specifically, for the given parameter values in Figure 1, $X_1^{det} = 2.42$, $X_2^{det} = 11.73$. The optimal threshold for the entry deterrence strategy is $X_D^{det} = 6.30$. Suppose the current level of geometric Brownian motion is X. If $X < 6.30$, to delay the entry of the flexible follower, the leader waits until X reaches 6.30. For any X between 6.30 and 11.73, the leader needs to invest immediately to delay the flexible follower. For $X > 11.73$, the entry deterrence strategy is not possible because the market demand is large enough for both firms to be active. Moreover, $X_D^{acc} = 8.50 < X_1^{acc} = 9.23$, which makes X_D^{acc} have no meaning for the leader in this numerical example. This is because X has to reach X_1^{acc} to make the follower invest at the same time as the leader.

Figure 2 shows the value of the entry deterrence strategy V_D^{det} and accommodation strategy V_D^{acc} as functions of X, for the case that the flexible follower produces below capacity right after investment. Note that $V_D^{det} = V_D^{acc}$ at $X = \hat{X}$. For $X_1^{det} < X < \hat{X}$, the deterrence strategy is chosen and the leader invests at $X_D^{det} = 6.30$ with capacity $K_D(X_D^{det}) = 6.67$. For $X \geq \hat{X}$, the leader implements the accommodation

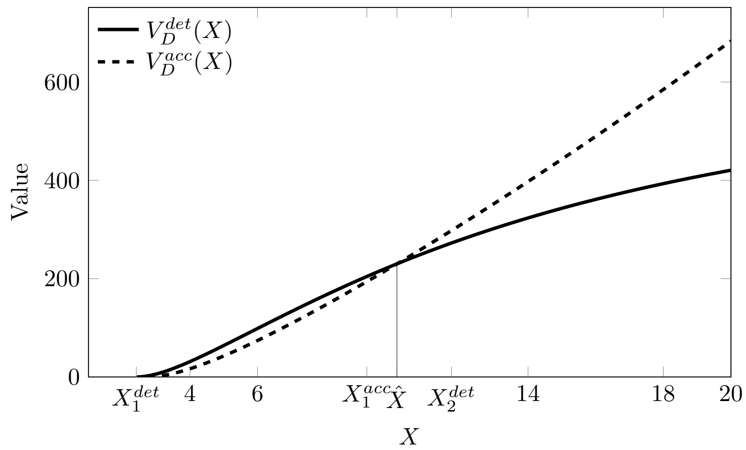


Figure 2: Illustration of $V_D^{det}(X)$ and $V_D^{acc}(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

strategy. Given that $\hat{X} > X_1^{acc}$, the leader invests immediately with capacity level $K_D^{acc}(X)$ if $X \geq \hat{X}$.

It can be concluded from Proposition 2 and 3 that the accommodation strategy is not possible if $X < X_1^{acc}$, and the deterrence strategy is not possible if $X > X_2^{det}$. When $X_1^{acc} < X < X_2^{det}$, the strategy that gives higher value will be chosen. Huisman and Kort (2015) have shown analytically that $X_1^{acc} < X_2^{det}$ when there is no volume flexibility. Figure 3 checks numerically whether this still holds for a flexible follower. Departing from the default parameter values $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, and $\gamma = 0.05$, when changing σ, α, r, c, δ and γ, X_2^{det} is always larger than X_1^{acc}. Thus, it can be assumed that $X_2^{det} > X_1^{acc}$ also holds for a flexible follower[4]. However, different from Huisman and Kort (2015), where $X_D^{acc} < X_1^{acc}$ always holds, the numerical analysis in Figure 3 shows that for significantly small α or δ, $X_D^{acc} > X_1^{acc}$. Note that X_D^{acc} implies that the market demand should be large enough to accommodate both firms. When α is small or negative, and the follower produces up to full capacity right after investment, a larger market demand is required to accommodate two firms. This leads to $X_D^{acc} > X_1^{acc}$. When δ is small, i.e., investing is less costly, both firms are encouraged to install larger capacities and a larger X_D^{acc} results. The above analysis is summarized in the following proposition.

Proposition 4 *Denote $\hat{X}$ as*

$$\hat{X} = \min\{X | X_1^{acc} < X < X_2^{det} \text{ and } V_D^{acc}(X) = V_D^{det}(X)\}.$$

[4]Given that $X_2^{det} > X_1^{acc}$, there is no boundary solution when analyzing the entry deterrence and entry accommodation strategies, which are two local optimum for the leader's investment problem.

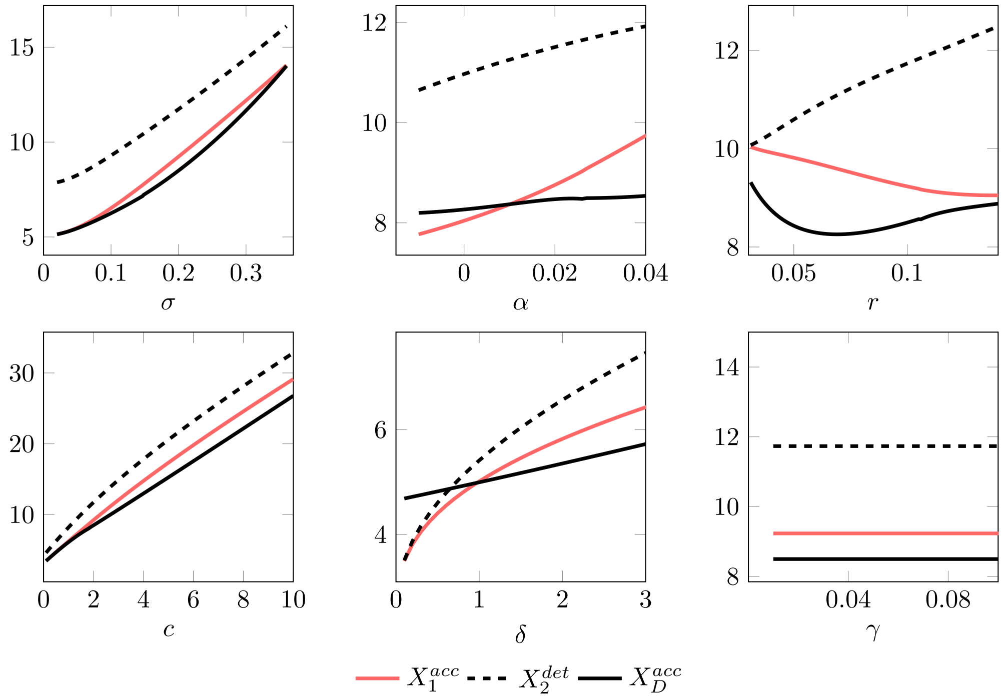


Figure 3: Illustration of X_1^{acc}, X_2^{det}, and X_D^{acc}. Default parameter values are $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

Let $X(t) = X$, the optimal investment capacity for the leader is

$$K_D^*(X) = \begin{cases} K_D^{det}(X_D^{det}) & \textit{if } \ 0 \le X < X_D^{det}, \\ K_D^{det}(X) & \textit{if } \ X_D^{det} \le X < \hat{X}, \\ K_D^{acc}(X_D^{acc}) \ \textit{or} \ K_D^{det}(\hat{X}) & \textit{if } \ \hat{X} \le X < X_D^{acc}, \\ K_D^{acc}(X) & \textit{if } \ X \ge \max\{\hat{X}, X_D^{acc}\}. \end{cases} \tag{32}$$

The optimal investment threshold for the leader is

$$X_D^* = \begin{cases} X_D^{det} & \textit{if } \ 0 \le X < X_D^{det}, \\ X & \textit{if } \ X_D^{det} \le X < \hat{X}, \\ X_D^{acc} \ \textit{or} \ \hat{X} & \textit{if } \ \hat{X} \le X < X_D^{acc}, \\ X & \textit{if } \ X \ge \max\{\hat{X}, X_D^{acc}\}. \end{cases} \tag{33}$$

The leader's and the follower's optimal investment capacities, $K_D^*(X)$ and $K_F^*(X)$, are demonstrated in Figure 4 when the follower produces below capacity right after investment. For given parameter values $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, and $\delta = 0.5$, then $X_D^{det} = 4.3050$ and $\hat{X} = 4.4779$. If $X < X_D^{det}$, the leader waits until X reaches X_D^{det} to implement the entry deterrence strategy. If $X_D^{det} \le X < \hat{X}$, the entry deterrence strategy is implemented immediately at X. When $X \ge \hat{X}$, the leader chooses entry accommodation strategy because it yields higher value. Different from Huisman and Kort (2015) that $X_D^{acc} < \hat{X}$, we have in this numerical example that $X_1^{acc} = 4.4072 < \hat{X} < X_D^{acc} = 4.8238$. For $\hat{X} \leq X < X_D^{acc}$, the leader waits until X reaches X_D^{acc}, i.e., the leader is holding an option to invest in the accommodation strategy. This is shown in Figure 4 as the void area for the interval $\hat{X} \leq X < X_D^{acc}$.

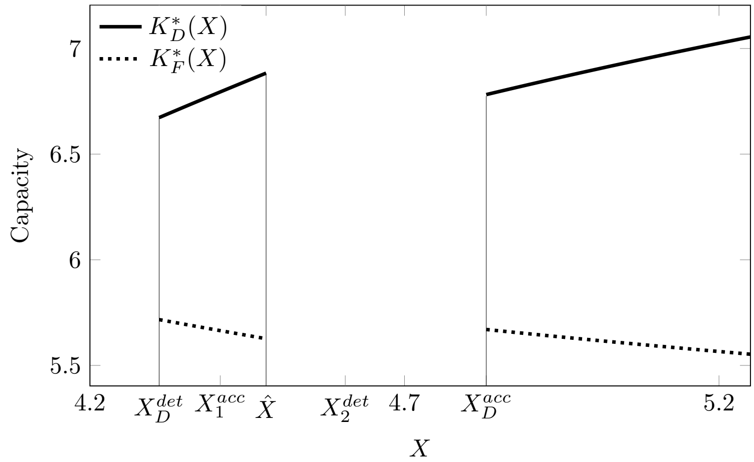


Figure 4: Illustration of $K_D^*(X)$ and $K_F^*(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\sigma = 0.2$, $\gamma = 0.05$, $c = 2$, $\delta = 0.5$.

Figure 5 demonstrates the values of the leader and the follower as functions of X when the follower produces below capacity right after investment. If $X < X_D^{det}$, the leader waits to invest with the entry deterrence strategy capacity. The follower is also waiting to invest, and expects the leader to invest at X_D^{det} with capacity $K_D^{det}(X_D^{det})$. If $X_D^{det} \leq X < \hat{X}$, the leader invests immediately at level X with deterrence capacity $K_D^{det}(X)$. When $\hat{X} \leq X < X_D^{acc}$, the leader implements entry accommodation strategy and waits to invest at X_D^{acc} with capacity $K_D^{acc}(X_D^{acc})$. The follower invests at the same time but with capacity $K_F^*(K_D^{acc}(X_D^{acc}))$. Because of the switch from the entry deterrence to accommodation, the leader's value function has a kink and the follower's value function is shown to jump at $\hat{X}$. When $X \geq X_D^{acc}$, the leader invests immediately with the entry accommodation strategy capacity $K_D^{acc}(X)$. The follower also invests at the same time as the leader but with capacity $K_F^*(K_D^{acc}(X))$.

2.4 Influence of Flexibility

In order to analyze the influence of the follower's volume flexibility, the optimal investment decisions without flexibility are derived in Appendix B. By comparing the leader's investment decisions with a flexible and with a dedicated follower, we get the following proposition.

Proposition 5 *Volume flexibility does not influence the leader's investment decisions under entry deterrence strategy. Moreover, it also does not influence the leader's optimal capacity under entry accommodation strategy.*

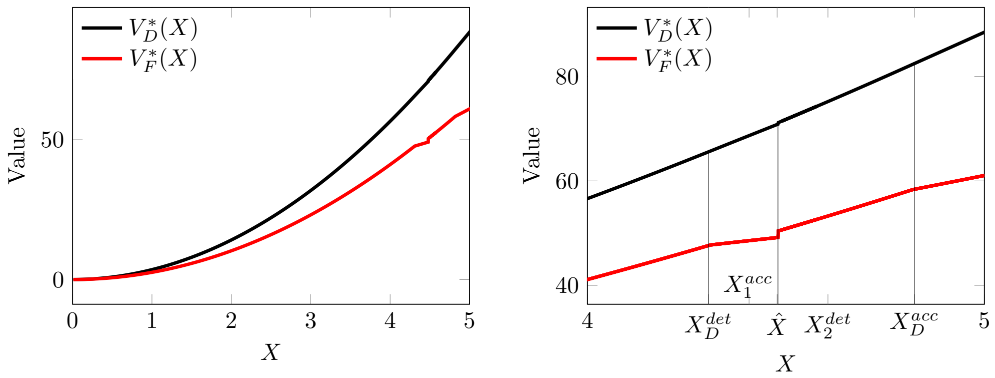


Figure 5: Illustration of $V_D^*(X)$ and $V_F^*(X)$ when the flexible follower produces below capacity right after investment. Parameter values are $\alpha = 0.03$, $\sigma = 0.2$, $r = 0.1$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

In this section, numerical analysis is carried out to investigate how flexibility influences the leader's and the follower's investment decisions. More specifically, we consider the possibility of each strategy by comparing X_1^{det}, X_2^{det}, and $\hat{X}$, with and without the follower's volume flexibility. The analysis of $\hat{X}$ is because that the leader only switches to accommodation strategy when $X \geq \hat{X}$. We analyze how flexibility influences the leader's optimal capacity and option values at $\hat{X}$. Moreover, this section also considers the follower's optimal investment decisions, with and without volume flexibility, under the leader's deterrence and accommodation strategies. The influence of flexibility on the follower's values at the moment of investment is also analyzed.

2.4.1 Flexibility Influences Leader

This subsection analyzes numerically the dedicated leader's investment strategies. For the given parameter values in Figure 6, it is demonstrated in the left penal that $X_1^{acc} > X_D^{acc}$ when the follower is flexible, which makes the optimal threshold X_D^{acc} have no meaning as when the follower is not flexible by Huisman and Kort (2015). From Proposition 4, the leader invests at $\hat{X}$ if $\hat{X} \geq X_D^{acc}$, because accommodation strategy generates higher value. In the left penal, it is also shown that $\hat{X} > X_1^{acc}$, implying that it is possible to implement at $\hat{X}$ the accommodation strategy. So we further analyze the influence of follower's flexibility on $\hat{X}$. $\hat{X}$ when follower is flexible is smaller than when follower is not flexible, implying that the leader switches to accommodation strategy earlier and the accommodation strategy is more likely, see Figure 7 for more.

Moreover, $\hat{X}$ increases with σ as shown in left penal of Figure 6. This means that the leader switches to accommodation strategy later in a more volatile market. The intuition is that both the leader and the follower invest more in case of upward demand shocks when there is more uncertainty, shown in the right panel. Furthermore, the right panel also shows that when switching to accommodation strategy, the leader invests less if the follower is flexible. This will be explained further in 2.4.2.

The follower's flexibility influences the possibility for the leader to implement two strategies. The analysis is carried out by considering the interval $[X_1^{det}, X_2^{det}]$, where the entry deterrence strategy is possible, and

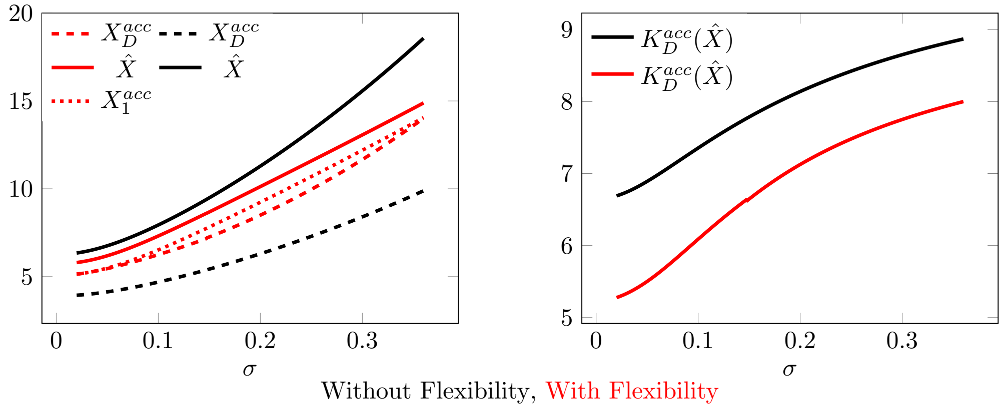


Figure 6: Illustration of X_D^{acc}, $\hat{X}$, and $K_D^{acc}(\hat{X})$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

$X \geq \hat{X}$, where the accommodation strategy is considered. Figure 7 demonstrates that, the interval to implement deterrence strategy shrinks and the interval to implement accommodation strategy enlarges when the follower is flexible. The changes in the intervals reflect the tendency for the leader to implement the corresponding strategy. It holds that for the given parameters, the leader tends to delay the flexible follower's entry less and is more likely to implement the accommodation strategy.

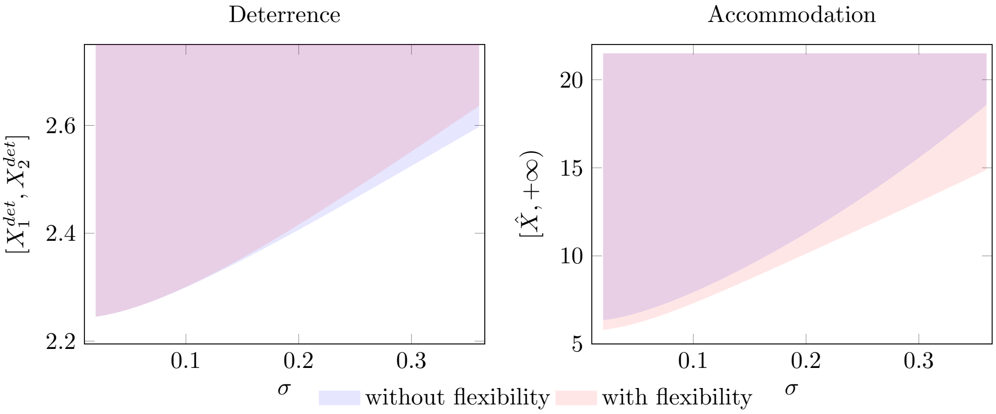


Figure 7: Illustration of X_1^{det}, X_2^{det}, and $\hat{X}$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

The leader's tendency to implement different strategies depends on how the follower's flexibility influences its value. Figure 8 illustrates the leader's value under the entry deterrence strategy at threshold X_D^{det} and accommodation strategy at $\hat{X}$, with and without flexibility. For the entry deterrence strategy, it is shown that the leader's value at X_D^{det} under the follow's flexibility is no larger than that without flexibility. This is because the flexibility does not change the leader's investment threshold and capacity, but it makes the follower enter the market earlier, see Figure 8, implying an earlier end to the leader's monopoly privilege.

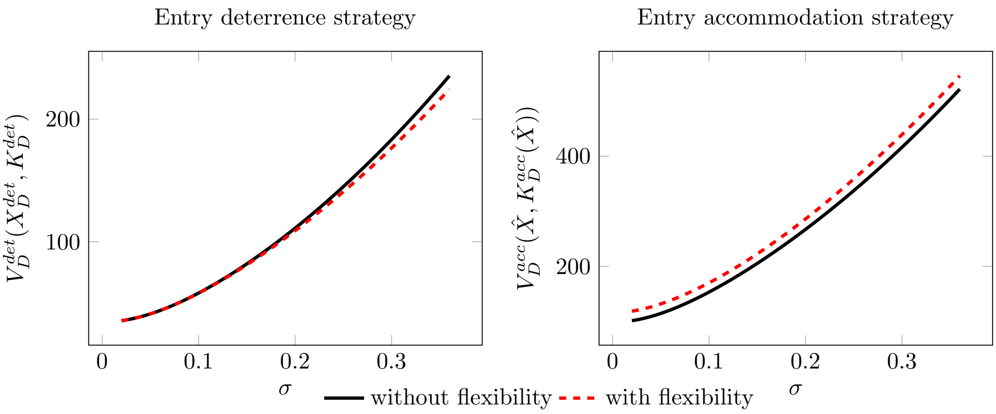


Figure 8: Illustration of $V_D^{det}(X_D^{det}, K_D^{det})$ when investing at the optimal threshold X_D^{det}, and $V_D^{acc}(\hat{X}, K_D^{acc}(\hat{X}))$ when investing at level $\hat{X}$ with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Under the accommodation strategy, the leader invests earlier and less when the follower is flexible. However, as shown in Figure 8, the leader's values at the moment of investment is larger than that without flexibility[5]. This implies that when implementing the accommodation strategy, the leader also benefits from the follower's flexibility. However, if the leader deters follower's entry, then the follower's flexibility decreases its value.

2.4.2 Flexibility Influences Follower

In this subsection, we analyze how the volume flexibility influences the follower's investment threshold, capacity, and value under different leader strategies.

When the leader implements entry deterrence strategy, the flexible follower invests earlier with more capacity and has higher value, as shown in Figure 9. Given that the follower can adjust output levels to the market demand, and prefers to invest more in case the market demand increases in the future. Intuitively the firm would invest later so that the market demand is higher to compensate for the larger investment costs. However, as shown in Figure 9, this is not the case because of another effect that the technological advantage yields higher values for the follower (right panel) and motivates the follower to invest earlier. For the given parameter values, It is apparently that the latter effect dominates. Besides, the difference between with and without flexibility increases with σ for the follower. This is because for smaller σ, market uncertainty is low and the flexible follower produces up to capacity right after investment, so the differences in $X_F^*(K_D^{det})$ and $K_F^*(K_D^{det})$ are relatively small. However, with more market uncertainty, i.e., larger σ, the flexible follower produces below capacity right after investment and puts more capacity on hold for future positive demand

[5]Note that $\hat{X}$ is different, depending on whether the follower's flexible. When comparing investment values at different $\hat{X}$s, the comparison is made at a predetermined point of time, for instance, at $\hat{X}$ where the follower is flexible. The discount factor for the leader's value when the follower is flexible is equal to $\left(\hat{X}_{Li}/\hat{X}_{Lf}\right)^{\beta_1}$, where $\hat{X}_{Lf}$ stands for $\hat{X}$ in the flexible follower situation, and $\hat{X}_{Li}$ stands for $\hat{X}$ in the inflexible follower situation.

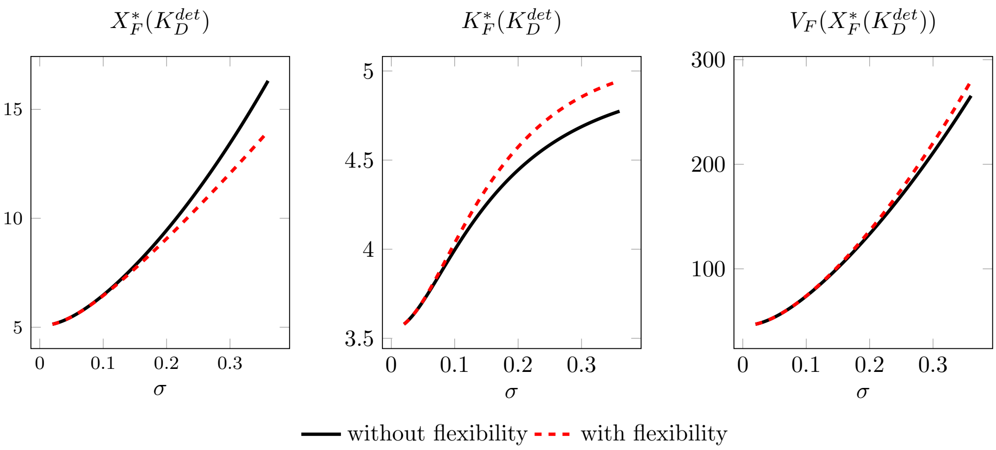


Figure 9: Illustration of $X_F^*(K_D^{det})$, $K_F^*(K_D^{det})$, and $V_F(X_F^*(K_D^{det}))$ under entry deterrence strategy with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

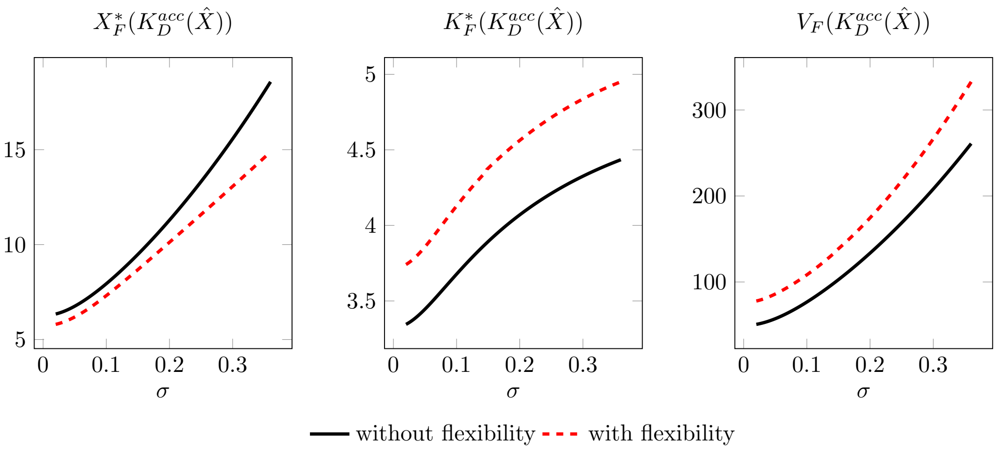


Figure 10: Illustration of $X_F^*(K_D^{acc}(\hat{X}))$, $K_F^*(K_D^{acc}(\hat{X}))$, and $V_F(K_D^{acc}(\hat{X}))$ under the entry accommodation strategy with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

shocks, so the differences are relatively large.

The dedicated leader switches from deterrence to accommodation strategy at $\hat{X}$. Note that for the accommodation strategy, the follower invests at the same time as the leader, thus $X_F^*(K_D^{acc}(\hat{X}))$ in Figure 10 is the same as $\hat{X}$ in Figure 6. Figure 10 shows that under leader's accommodation strategy, the flexible follower also invests earlier and more, and has higher value than an inflexible follower. The reason is similar as that in the deterrence strategy. Given that two firms invest at the same time, the leader also invests earlier than that when the follower is dedicated. For the leader, investing earlier implies smaller investment capacity because the leader is dedicated.

2.4.3 First Mover Advantage v.s. Technological Advantage

This subsection investigates whether the follower's technological advantage in volume flexibility can overcome the leader's first mover advantage.

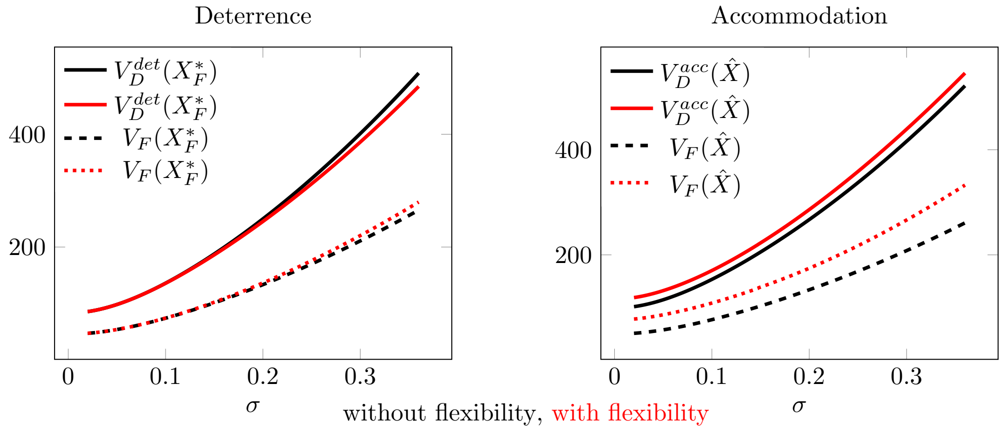


Figure 11: Comparison of $V_D^{det}(X_F^*, K_D^{det}(X_F^*))$ and $V_F(X_F^*, K_D^{det}(X_F^*))$ under the entry deterrence strategy, $V_D^{acc}(\hat{X}, K_D^{acc}(\hat{X}))$ and $V_F(K_D^{acc}(\hat{X}))$ under the entry accommodation strategy, with and without flexibility. Parameter values are $r = 0.1$, $\alpha = 0.03$, $\gamma = 0.05$, $c = 2$, $\delta = 10$.

Figure 11 compares the leader and the follower's values for the entry deterrence and accommodation strategies, with and without flexibility. The leader always has higher values than the follower, implying the first mover advantage cannot be leapfrogged by the volume flexibility advantage. Gal-Or (1985) has shown with symmetric players that the leader has larger profits compared to the follower if the follower's reaction function is downward-sloping. In my model with asymmetric firms and continuous time setting, the result is similar in that the optimal follower's optimal capacity decreases with the leader's installed capacity. Another possible reason is that the leader benefits from the follower's volume flexibility without sharing costs for these benefits.

2.5 Conclusion

This section introduces volume flexibility into the strategic capacity investment problem under uncertainty. In the duopoly framework, the follower has technological advantage over the leader in that the follower can adjust output quantity within the constraint of installed production capacity, and the leader always produces up to capacity. When making decisions about investment timing and investment capacity, the leader not only takes into account the incentives to preempt, but also the influence of the follower's volume flexibility on the market price. This is because the flexible follower competes against the dedicated leader on one hand, and on the other hand makes the market price fluctuate less when there is demand volatility. We show that compared to a dedicated follower, the dedicated leader is more likely to accommodate the entry of the flexible follower. This is due to that the entry deterrence strategy decreases the leader's value when the follower is flexible, and the accommodation strategy increases the leader's value. The leader does not like to deter because volume flexibility makes the follower to enter the market earlier and thus shortens the leader's monopoly period. Whereas when implementing the accommodation strategy, two firms enter the market later than that under the deterrence strategy, so the market demand is larger. In a way, the leader benefits more from the less fluctuating market prices due to follower's volume flexibility.

3 Preemption Analysis between the Flexible Firm and the Dedicated Firm [6]

Duopoly model with two firms, both firms need to decide about the investment timing and capacity. Exogenous firm roles. The leader is volume flexible in the way that it can adjust output within the constraint of invested capacity. The follower is dedicated in the way that it always produces up to full capacity after investment. This section is to help find out whether the leader prefers to be dedicated or flexible when the follower is dedicated, also to get insight about the leader's technology choice before investment.

3.1 Model

Multiplicative market inverse demand function is $P_t = X_t(1 - \gamma(Q_L + Q_F))$ with $\gamma \geq 0$, and Q_L, Q_F are the output quantities for the leader and the follower. X_t follows a geometric Brownian motion $\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}w_t$, where $\mathrm{d}w_t$ is the increment of a Wiener process. Given the leader is volume flexible and the follower is dedicated, it holds that $Q_L \leq K_L$, and $Q_F = K_F$, where K_L and K_F are their investment capacities. The unit production and investment cost for the follower are c and δ respectively. The unit production cost and investment cost for the leader are c_L and δ_L.

3.2 Follower's investment decision

Given that the leader is already in the market and producing Q_L, the follower's instantaneous profit flow after investment depends on whether the leader's production, which ranges in the interval $[0, K_L]$. Thus, the follower's instantaneous profit at the moment of investment X is

$$\pi_F(X, Q_L, K_F) = \Big(X(1 - \gamma(Q_L + K_F)) - c\Big)K_F, \quad 0 \leq Q_L \leq K_L$$

At the moment of market entry of the follower, i.e., at X, there are three cases/regions for the leader's output: no production ($Q_L = 0$), producing below capacity ($0 < Q_L < K_L$) and producing up to capacity ($Q_L = K_L$). Apparently, the leader's output would depend on the market demand and influence the value the follower. Note that if the leader is producing below capacity when the follower enters the market, the

[6] The notations in this section are independent from the previous section. This is because the draft is still about research work in progress.

leader produces an output that maximizes it's instantaneous profit, i.e.,

$$Q_L(X, K_F) = \frac{X(1-\gamma K_F) - c_L}{2\gamma X}.$$

Lemma 1 *The follower's value at the moment of investment is*

$$V_F(X, K_L, K_F)$$
$$= \begin{cases} \frac{X(1-\gamma K_F)K_F}{r-\mu} - \frac{cK_F}{r} - \delta K_F, & \text{if } X \leq \frac{c_L}{1-\gamma K_F}, \\ \frac{X(1-\gamma Q_L(X,K_F)-\gamma K_F)K_F}{r-\mu} - \frac{cK_F}{r} - \delta K_F, & \text{if } \frac{c_L}{1-\gamma K_F} < X \leq \frac{c_L}{1-\gamma K_F - 2\gamma K_L}, \\ \frac{X(1-\gamma K_L-\gamma K_F)K_F}{r-\mu} - \frac{cK_F}{r} - \delta K_F, & \text{if } X > \frac{c_L}{1-\gamma K_F-2\gamma K_L}. \end{cases} \quad (34)$$

Lemma 1 shows that the leader's volume flexibility does not have option values for the dedicated follower. This is due to the fact that the dedicated follower does not influence the boundaries of each region. The follower's investment capacity K_F depends on the leader's investment/output decisions, which makes the follower's value function continuous but not differentiable on the boundaries.

Suppose the value before investment is denoted as $A_F X^{\beta_1}$, then in every region, we can derive the follower's optimal investment decision summarized in the following proposition.

Theorem 1 *The follower's optimal investment decision for different regions, is given by*

$$\begin{cases} X_F^* = \frac{\beta_1+1}{\beta_1-1}(r-\mu)\left(\frac{c}{r}+\delta\right) \\ K_F^* = \frac{1}{(\beta_1+1)\gamma} \end{cases} \quad \text{if } X \leq \frac{c_L}{1-\gamma K_F},$$
$$\begin{cases} X_F^* = \frac{\beta_1+1}{\beta_1-1}\left(2(r-\mu)\left(\frac{c}{r}+\delta\right) - c_L\right) \\ K_F^* = \frac{1}{(\beta_1+1)\gamma} \end{cases} \quad \text{if } \frac{c_L}{1-\gamma K_F} < X \leq \frac{c_L}{1-\gamma K_F - 2\gamma K_L}, \quad (35)$$
$$\begin{cases} X_F^* = \frac{\beta_1+1}{(\beta_1-1)(1-\gamma K_L)}(r-\mu)\left(\frac{c}{r}+\delta\right) \\ K_F^* = \frac{1-\gamma K_L}{(\beta_1+1)\gamma} \end{cases} \quad \text{if } X > \frac{c_L}{1-\gamma K_F - 2\gamma K_L}.$$

Theorem 1 shows that if the market demand is small (region 1 and region 2) and the leader is not producing at full capacity, the follower invests with a monopolistic capacity size. Given that the follower's investment is not influenced by the leader and the exogenous firm roles, it can be concluded that they will either invest sequentially (referred as entry deterrence) such that the leader invests at $X < X_F^*$, or they invest at the same time at $X \geq X_F^*$ (referred as entry accommodation). If the market demand is large enough such that the leader is producing at full capacity, the follower's investment decision is influenced by the leader's investment capacity. It is possible that the leader can deter the follower's market entry. In the following we will analyze the leader's optimal investment decision and check the leader's deterrence strategy.

3.3 Leader's investment decision

Suppose the unit cost of the leader's investment is denoted by δ_L. We can derive the leader's value function when both firms are in the market. For the three regions where the leader does not produce, produces below capacity and produces up to capacity when both firms are active, the leader's profits flow is given by

$$\pi_L(X, K_L) = \begin{cases} 0 & \text{if } X < \frac{c_L}{1-\gamma K_F}, \\ \frac{1}{4\gamma}\left(X(1-\gamma K_F)^2 - 2(1-\gamma K_F)c_L + \frac{c_L^2}{X}\right) & \text{if } \frac{c_L}{1-\gamma K_F} \le X < \frac{c_L}{1-\gamma K_F - 2\gamma K_L}, \\ X(1-\gamma K_L - \gamma K_F)K_L - c_L K_L & \text{if } X \ge \frac{c_L}{1-\gamma K_F - 2\gamma K_L}. \end{cases} \tag{36}$$

Then the value function of the leader when both firms are active in the market takes the following form, where β_1 and β_2 are the positive and negative root for $\sigma^2\beta^2 + (2\alpha - \sigma^2)\beta - 2r = 0$.

$$V_L(X, K_L) = \begin{cases} LX^{\beta_1} & \text{if } X \le \frac{c_L}{1-\gamma K_F}, \\ M_1 X^{\beta_1} + M_2 X^{\beta_2} & \\ +\frac{1}{4\gamma}\left(\frac{X(1-\gamma K_F)^2}{r-\mu} - \frac{2(1-\gamma K_F)c_L}{r} + \frac{c_L^2}{X(r+\mu-\sigma^2)}\right) & \text{if } \frac{c_L}{1-\gamma K_F} < X \le \frac{c_L}{1-\gamma K_F - 2\gamma K_L}, \\ NX^{\beta_2} + \frac{X(1-\gamma K_L - \gamma K_F)K_L}{r-\mu} - \frac{c_L K_L}{r} & \text{if } X > \frac{c_L}{1-\gamma K_F - 2\gamma K_L}. \end{cases} \tag{37}$$

To get the value functions for the leader, we need to derive the expressions for L, M_1, M_2 and N. Intuitively, L is positive and represents the option value in case that the market demand increases, which happens once X becomes lager than $c_L/(1-\gamma K_F)$. M_1 is negative and corrects for the fact that when X becomes larger than $c_L/(1-\gamma K_F - 2\gamma K_L)$, then the leader's output is constrained by its investment capacity K_L. M_2 is negative and is a correction such that if X becomes smaller than $c_L/(1-\gamma K_F)$, then the leader would suspend production. N is positive and corrects for the fact that if the market demand decreases, i.e., X drops below $c_L/(1-\gamma K_F - 2\gamma K_L)$, then the flexible leader can produce below capacity.

Denote the boundary of the regions as $X_1 = c_L/(1-\gamma K_F)$ and $X_2 = c_L/(1-\gamma K_F - 2\gamma K_L)$. From the value matching and smooth pasting conditions at the boundaries, we have the following equations.

$$LX_1^{\beta_1} = M_1 X_1^{\beta_1} + M_2 X_1^{\beta_2} + \frac{1}{4\gamma}\left(\frac{X_1(1-\gamma K_F)^2}{r-\mu} - \frac{2(1-\gamma K_F)c_L}{r} + \frac{c_L^2}{X_1(r+\mu-\sigma^2)}\right), \tag{38}$$

$$L\beta_1 X_1^{\beta_1-1} = M_1\beta_1 X_1^{\beta_1-1} + M_2\beta_2 X_1^{\beta_2-1} + \frac{1}{4\gamma}\left(\frac{(1-\gamma K_F)^2}{r-\mu} - \frac{c_L^2}{X_1^2(r+\mu-\sigma^2)}\right), \tag{39}$$

$$\begin{aligned} M_1 X_2^{\beta_1} + M_2 X_2^{\beta_2} + \frac{1}{4\gamma}\left(\frac{X_2(1-\gamma K_F)^2}{r-\mu} - \frac{2(1-\gamma K_F)c_L}{r} + \frac{c_L^2}{X_2(r+\mu-\sigma^2)}\right) &= NX_2^{\beta_2} \\ &+\frac{X_2(1-\gamma K_L-\gamma K_F)K_L}{r-\mu} - \frac{c_L K_L}{r}, \end{aligned} \tag{40}$$

$$\begin{aligned} M_1\beta_1 X_2^{\beta_1-1} + M_2\beta_2 X_2^{\beta_2-1} + \frac{1}{4\gamma}\left(\frac{(1-\gamma K_F)^2}{r-\mu} - \frac{c_L^2}{X_2^2(r+\mu-\sigma^2)}\right) &= N\beta_2 X_2^{\beta_2-1} \\ &+\frac{(1-\gamma K_L-\gamma K_F)K_L}{r-\mu}. \end{aligned} \tag{41}$$

Then it can be derived that

$$M_2 = -\frac{X_1^{-\beta_2-1}}{\beta_1-\beta_2}\frac{c_L^2}{4\gamma}\left(\frac{\beta_1-1}{r-\mu} - \frac{2\beta_1}{r} + \frac{\beta_1+1}{r+\mu-\sigma^2}\right),$$

$$
\begin{aligned}
M_1 &= \frac{X_2^{-\beta_1-1}}{\beta_1-\beta_2}\frac{c_L^2}{4\gamma}\left(\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2}\right), \\
L &= \frac{c_L^2\left(X_2^{-\beta_1-1}-X_1^{-\beta_1-1}\right)}{4\gamma(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2}\right), \\
N &= \frac{c_L^2\left(X_2^{-\beta_2-1}-X_1^{-\beta_2-1}\right)}{4\gamma(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\mu}-\frac{2\beta_1}{r}+\frac{\beta_1+1}{r+\mu-\sigma^2}\right).
\end{aligned}
$$

From the case of dedicated leader and flexible follower (in my job market paper), it holds that

$$
\begin{aligned}
\frac{\beta_1-1}{r-\mu}-\frac{2\beta_1}{r}+\frac{\beta_1+1}{r+\mu-\sigma^2} &> 0, \\
\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2} &< 0.
\end{aligned}
$$

Thus, we can conclude $M_1 < 0$, $M_2 < 0$, $L > 0$ and $N > 0$. Their signs correspond to the intuition mentioned above. For every region, we can analyze the leader's strategies, i.e., deterrence and accommodation.

Given the exogenous firm roles, there are basically two situations for the follower's investment: investing at the same time as the leader, or investing later than the leader in the corresponding region. The following analysis and calculation in every region are based on the following two situations.

Region 1: The leader does not produce when the follower invests, i.e., $X \leq X_1$

- Entry Accommodation

Given that the leader and the follower enter the market at the same time, the value function of the leader is equal to

$$
V_L(X, K_L) = \begin{cases} A_L X^{\beta_1} & X < X_L^*, \\ L X^{\beta_1} & X \geq X_L^*. \end{cases}
$$

X_L^* is the investment threshold of the leader in this region. Because of the exogenous firm roles, we have that $X_L^* \geq \frac{\beta-1}{\beta+1}(r-\mu)\left(\frac{c}{r}+\delta\right)$. A_L and L have different implications. A_L represents the option value from investing, and L represents the option value that the firm starts producing when the market demand is large enough. The optimal investment capacity $K_L(X)$ maximizes $LX^{\beta_1} - \delta_L K_L$ for a given X. For a given capacity level K_L, the corresponding investment threshold $X(K_L)$, satisfies the value matching and smooth pasting conditions, i.e.,

$$
\begin{aligned}
A_L X(K_L)^{\beta_1} &= L X(K_L)^{\beta_1} - \delta_L K_L, \\
\beta_1 A_L X(K_L)^{\beta_1 - 1} &= \beta_1 L X(K_L)^{\beta_1-1}.
\end{aligned}
$$

The two conditions will not hold unless $K_L = 0$. So the leader does not investment in this region.

- Entry Deterrence

If the follower enters the market later than the leader, then the leader's value function, before and after the follower's investment at X_F^*, is denoted by

$$V_L(X, K_L) = \begin{cases} B_L X^{\beta_1} & X < X_F^*, \\ L X^{\beta_1} & X \geq X_F^*. \end{cases}$$

$B_L = L$ represents the option value that the leader resumes production in the future, though it is not producing at X_F^*. Moreover, the leader's value before and after investment is given by

$$V_L(X, K_L) = \begin{cases} A_L X^{\beta_1} & X < X_L^*, \\ B_L X^{\beta_1} & X \geq X_L^*. \end{cases}$$

From the analysis above, the leader will not invest in this region.

Region 2: The leader produces below capacity when the follower invests, i.e., $X_1 < X \leq X_2$

Given that the follower's investment threshold X_F^* in this region is not influenced by the leader's investment, there are two possibilities for the leader's investment: the leader invests no earlier than the follower (referred as accommodation), and the leader invests earlier than the follower (referred as deterrence). Between these two strategies, the leader would choose the strategy that generates larger value.

- Entry Deterrence

Because the leader's capacity decision does not influence the follower's investment decision, the entry deterrence (sequential investment) can only happen when the leader invests at $X < X_F^*$. The boundaries for this region, X_1 and X_2, are given by

$$\begin{aligned} X_1 &= \frac{c_L}{1 - \gamma K_F^*} = \frac{(\beta_1 + 1) c_L}{\beta_1}, \\ X_2(K_L) &= \frac{c_L}{1 - \gamma K_F^* - 2\gamma K_L} = \frac{(\beta_1 + 1) c_L}{\beta_1 - (\beta_1 + 1) 2\gamma K_L}. \end{aligned}$$

Before the follower enters the market, the leader is the monopolist and adjusts the output quantity Q_L according to the demand fluctuations. The leader's monopoly profit is equal to

$$\pi_L(X, Q_L) = \Big(X(1 - \gamma Q_L) - c_L \Big) Q_L.$$

Thus, the optimal output quantity for a given X is given by $Q_L(X) = \frac{X - c_L}{2\gamma X}$, and the leader's corresponding profit is

$$\pi_L(X) = \frac{X^2 - 2c_L X + c_L^2}{4\gamma X}.$$

Then we can write the leader's value after the follower enters the market as

$$V_L(X, K_L) = \begin{cases} B_1 X^{\beta_1} + \frac{1}{4\gamma}\left(\frac{X}{r-\mu} - \frac{2c_L}{r} + \frac{c_L^2}{X(r+\mu-\sigma^2)} \right) & X < X_F^*; \\ M_1(K_L) X^{\beta_1} + M_2 X^{\beta_2} + \frac{1}{4\gamma}\left(\frac{X}{r-\mu}\left(\frac{\beta_1}{\beta_1+1}\right)^2 - \frac{2c_L\beta_1}{r(\beta_1+1)} + \frac{c_L^2}{X(r+\mu-\sigma^2)} \right) & X \geq X_F^*. \end{cases}$$

From the value matching condition at X_F^*, we can derive that

$$B_1(K_L) \quad = \quad M_1(K_L) + M_2 X_F^{*\beta_2-\beta_1} - \frac{X_F^{*-\beta_1}}{4\gamma(\beta_1+1)}\left(\frac{X_F^*}{r-\mu}\frac{2\beta_1+1}{\beta_1+1} - \frac{2c_L}{r}\right).$$

Let the leader's value before and after investment be

$$V_L(X, K_L) \quad = \quad \begin{cases} AX^{\beta_1} & X < X_L, \\ B_1(K_L)X^{\beta_1} + \frac{1}{4\gamma}\left(\frac{X}{r-\mu} - \frac{2c_L}{r} + \frac{c_L^2}{X(r+\mu-\sigma^2)}\right) & X \geq X_L. \end{cases}$$

The optimal investment capacity $K(X)$ then satisfies the following implicit equation

$$\frac{\mathrm{d}M_1(K_L)}{\mathrm{d}K_L}X^{\beta_1} - \delta_L \quad = \quad -\frac{\beta_1+1}{\beta_1-\beta_2}\frac{c_L}{2}\left(\frac{\beta_2-1}{r-\mu} - \frac{2\beta_2}{r} + \frac{\beta_2+1}{r+\mu-\sigma^2}\right)\left(\frac{X}{X_2(K_L)}\right)^{\beta_1} - \delta_L = 0.$$

Thus, it holds that

$$-\frac{\beta_1+1}{\beta_1-\beta_2}\frac{c_L}{2}\left(\frac{\beta_2-1}{r-\mu} - \frac{2\beta_2}{r} + \frac{\beta_2+1}{r+\mu-\sigma^2}\right)\left(\frac{X}{X_2(K_L)}\right)^{\beta_1} = \delta_L.$$

For a given level of X, the corresponding investment capacity is

$$K_L(X) = \frac{1}{2\gamma}\left(\frac{\beta_1}{\beta_1+1} - \frac{c_L}{X}\left(-\frac{c_L(\beta_1+1)}{2\delta_L(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu} - \frac{2\beta_2}{r} + \frac{\beta_2+1}{r+\mu-\sigma^2}\right)\right)^{-1/\beta_1}\right). \tag{42}$$

From the value matching and smooth pasting conditions, it can be derived that for a given K_L, the investment threshold $X(K_L)$ can be derived from

$$\frac{1}{4\gamma}\left(\frac{X(\beta_1-1)}{r-\mu} - \frac{2\beta_1 c_L}{r} + \frac{c_L^2(\beta_1+1)}{X(r+\mu-\sigma^2)}\right) - \beta_1\delta_L K_L = 0. \tag{43}$$

Solving (42) and (43) yield the optimal investment entry deterrence decision. The deterrence is possible under the two conditions: $X_F^* > X_L^{det}$ and $X_F^* \in [X_1, X_2)$.

Under this entry deterrence strategy, it holds that $X < X_2$, thus $2\gamma K_L(X) > 1 - \gamma K_F^* - \frac{c_L}{X}$. It is equivalent to

$$-\frac{c_L(\beta_1+1)}{2\delta_L(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu} - \frac{2\beta_2}{r} + \frac{\beta_2+1}{r+\mu-\sigma^2}\right) > 1.$$

This induces the same definition of the Region 2 for deterrence strategy as the case for accommodation strategy, and also the same as the definition of same Region 2 in the monopoly model, where the flexible firm produces below capacity right after investment.

- Entry Accommodation

Under this strategy, both firms would enter the market at the same time. This only happens if the leader's investment threshold $X_L^{acc} \geq X_F^*$ in our exogenous firm-role setting, and the follower would invest with capacity

$$K_F(X) = \frac{1}{2\gamma}\left(1 - \frac{1}{X}\left(2(r-\mu)\left(\frac{c}{r}+\delta\right) - c_L\right)\right).$$

Note that if otherwise such that $X_L^{acc} < X_F^*$, then the follower would just invest at X_F^*, because the follower's investment decision X_F^* and K_F^* are not influenced by the leader. The leader's timing decision X_L^{acc} also influences the boundary of this region, i.e., $X_1(K_F(X))$ and $X_2(K_F(X), K_L)$. This is because the leader's accommodation strategy timing decision influences the follower's investment capacity $K_F(X_L^{acc})$. The follower's capacity $K_F(X_L^{acc})$, together with $K_L(X_L^{acc})$, influences the boundaries for the leader to suspend production X_1, to produce below and up to capacity X_2, and also $M_1(X, K_F, K_L)$ and $M_2(X, K_F)$.

With the entry accommodation strategy, the value function of the leader is equal to

$$V_L(X, K_L) = \begin{cases} B_L(K_L)X^{\beta_1} & X < X_L^{acc}(K_L), \\ M_1(X, K_L)X^{\beta_1} + M_2(X)X^{\beta_2} & X \geq X_L^{acc}(K_L). \\ +\frac{1}{4\gamma}\left(\frac{X(1-\gamma K_F(X))^2}{r-\mu} - \frac{2(1-\gamma K_F(X))c_L}{r} + \frac{c_L^2}{X(r+\mu-\sigma^2)}\right) & \end{cases}$$

From the follower's investment capacity $K_F(X)$, it can be derived that

$$\begin{aligned}
\frac{\mathrm{d}K_F(X)}{\mathrm{d}X} &= \frac{1}{2\gamma X^2}\left(2(r-\mu)\left(\frac{c}{r}+\delta\right) - c_L\right) = \frac{1-2\gamma K_F}{2\gamma X}, \\
\frac{\mathrm{d}X_1(X)}{\mathrm{d}X} &= \frac{\gamma c_L}{(1-\gamma K_F(X))^2}\frac{\mathrm{d}K_F}{\mathrm{d}X} = \frac{1-2\gamma K_F}{2(1-\gamma K_F)}\frac{X_1}{X}, \\
\frac{\partial X_2(X, K_L)}{\partial X} &= \frac{\gamma c_L}{(1-\gamma K_F(X) - 2\gamma K_L)^2}\frac{\mathrm{d}K_F}{\mathrm{d}X} = \frac{1-2\gamma K_F}{2(1-\gamma K_F - 2\gamma K_L)}\frac{X_2}{X}, \\
\frac{\partial M_1(X, K_L)}{\partial X} &= -\frac{(\beta_1+1)M_1}{X_2}\frac{\partial X_2(X, K_L)}{\partial X} = -\frac{(\beta_1+1)(1-2\gamma K_F)M_1}{2X(1-\gamma K_F - 2\gamma K_L)}, \\
\frac{\mathrm{d}M_2(X)}{\mathrm{d}X} &= -\frac{(\beta_2+1)M_2}{X_1}\frac{\mathrm{d}X_1(X)}{\mathrm{d}X} = -\frac{(\beta_2+1)(1-2\gamma K_F)M_2}{2X(1-\gamma K_F)}.
\end{aligned}$$

The value matching and smooth pasting conditions at $X_L^{acc}(K_L)$ yield the following equations:

$$\begin{aligned}
B_L(K_L)X^{\beta_1} =& M_1(K_L)X^{\beta_1} + M_2(K_L)X^{\beta_2} \\
&+ \frac{1}{4\gamma}\left(\frac{X(1-\gamma K_F(X))^2}{r-\mu} - \frac{2(1-\gamma K_F(X))c_L}{r} + \frac{c_L^2}{X(r+\mu-\sigma^2)}\right) - \delta_L K_L, \\
\beta_1 B_L(K_L)X^{\beta_1-1} =& \beta_1 M_1(K_L)X^{\beta_1-1} + \beta_2 M_2(K_L)X^{\beta_2-1} \\
&- \frac{(\beta_1+1)(1-2\gamma K_F)M_1}{2(1-\gamma K_F - 2\gamma K_L)}X^{\beta_1-1} - \frac{(\beta_2+1)(1-2\gamma K_F)M_2}{2(1-\gamma K_F)}X^{\beta_2-1} \\
&+ \frac{1}{4\gamma}\left(\frac{\gamma K_F(X)(1-\gamma K_F(X))}{r-\mu} + \frac{c_L(1-2\gamma K_F(X))}{rX} - \frac{c_L^2}{(r+\mu-\sigma^2)X^2}\right).
\end{aligned}$$

Thus, it can be derived that

$$\begin{aligned}
\beta_1 B_L(K_L)X^{\beta_1} =& \beta_1 M_1 X^{\beta_1} - \frac{(\beta_1+1)(1-2\gamma K_F)M_1}{2(1-\gamma K_F - 2\gamma K_L)}X^{\beta_1} + \frac{\beta_2 - 1 + 2\gamma K_F}{2(1-\gamma K_F)}M_2(K_L)X^{\beta_2} \\
&+ \frac{1}{4\gamma}\left(\frac{X\gamma K_F(X)(1-\gamma K_F(X))}{r-\mu} + \frac{c_L(1-2\gamma K_F(X))}{r} - \frac{c_L^2}{X(r+\mu-\sigma^2)}\right).
\end{aligned}$$

So $X_L^{acc}(K_L)$ satisfies the following implicit equation

$$\frac{(\beta_1+1)(1-2\gamma K_F)M_1}{2(1-\gamma K_F-2\gamma K_L)}X^{\beta_1}+\frac{2\beta_1-\beta_2+1-2(\beta_1+1)\gamma K_F}{2(1-\gamma K_F)}M_2X^{\beta_2}-\beta_1\delta_L K_L$$
$$+\frac{1}{4\gamma}\left(\frac{X(1-\gamma K_F(X))}{r-\mu}(\beta_1-(1+\beta_1)\gamma K_F(X))-\frac{c_L}{r}\big(2\beta_1+1-2(\beta_1+1)\gamma K_F(X)\big)+\frac{(\beta_1+1)c_L^2}{X(r+\mu-\sigma^2)}\right)=0. \tag{44}$$

e.g.,

$$\frac{(\beta_1+1)(1-2\gamma K_F)M_1}{2(1-\gamma K_F-2\gamma K_L)}X^{\beta_1}+\frac{2\beta_1-\beta_2+1-2(\beta_1+1)\gamma K_F}{2(1-\gamma K_F)}M_2X^{\beta_2}-\beta_1\delta_L K_L$$
$$+\frac{1}{4\gamma}\left[(\beta_1+1)\left(\frac{X(1-\gamma K_F(X))^2}{r-\mu}-\frac{2c_L(1-\gamma K_F(X))}{r}+\frac{c_L^2}{X(r+\mu-\sigma^2)}\right)-\frac{X(1-\gamma K_F(X))}{r-\mu}+\frac{c_L}{r}\right]=0$$

For a given level of X, the leader would invest with capacity size $K_L(X)$. The leader's investment size does not influence the follower's investment capacity, i.e., $\partial K_F/\partial K_L=0$. Thus, in this region, it can be derived that $\mathrm{d}X_1/\mathrm{d}K_L=0$ and $\mathrm{d}X_2/\mathrm{d}K_L=2\gamma X_2^2/c_L$. Furthermore, we can get

$$\begin{aligned}\frac{\mathrm{d}M_2(X)}{\mathrm{d}K_L}&=0,\\ \frac{\mathrm{d}M_1(X,K_L)}{\mathrm{d}K_L}&=-\frac{2\gamma(\beta_1+1)X_2}{c_L}M_1.\end{aligned}$$

So, $0\le K_L(X)\le\frac{\beta_1}{2\gamma(\beta_1+1)}$ satisfies the following equation

$$-\frac{(\beta_1+1)c_L}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2}\right)\left(\frac{X}{X_2(K_L)}\right)^{\beta_1}-\delta_L=0, \tag{45}$$

or

$$-\frac{2\gamma(\beta_1+1)X_2(K_L)}{c_L}M_1(X,K_L(X))X^{\beta_1}-\delta_L=0,$$

By substituting

$$X_2(X)=\frac{c_L}{1-\gamma K_F(X)-2\gamma K_L}=\frac{2c_L}{1+\frac{1}{X}\Big(2(r-\mu)\left(c/r+\delta\right)-c_L\Big)-4\gamma K_L}$$

into (45), we can derive that for a given X, the leader's investment capacity $K_L(X)$ is

$$K_L(X)=$$
$$\frac{1}{4\gamma X}\left(X+2(r-\mu)\left(\frac{c}{r}+\delta\right)-c_L-2c_L\left(-\frac{c_L(1+\beta_1)}{2\delta_L(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2}\right)\right)^{-1/\beta_1}\right) \tag{46}$$

In order for the accommodation strategy in this region to hold, we have $X\le X_2$, thus $2\gamma K_L\ge 1-\gamma K_F-\frac{c_L}{X}$. In other words,

$$-\frac{c_L(\beta_1+1)}{2\delta_L(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2}\right)\ge 1.$$

This is the same definition as in the monopoly model. So whether the firm produces up to or below capacity when the follower enters the market depends on the parameter values. If $\mu>\delta_L r^2/(c_L+\delta_L r)$, or both

$r - c_L/\delta_L < \mu \leq \delta_L r^2/(c_L + \delta_L r)$ and $\sigma > \bar{\sigma}$, then the leader produces below capacity when the follower enters. Because the transition between producing below and up to capacity is smooth, it can be concluded that if $\mu \leq r - c_L/\delta_L$, or both $r - c_L/\delta_L < \mu \leq \delta_L r^2/(c_L + \delta_L r)$ and $\sigma \leq \bar{\sigma}$, then the leader produces up to capacity when the follower enters the market. Moreover

$$\bar{\sigma}^2 = \frac{-2\left(\Lambda - \alpha^2\right)(2r - \alpha) - 4\sqrt{r\Lambda\left(\Lambda - \alpha^2\right)(r - \alpha)}}{\Lambda - (2r - \alpha)^2}, \tag{47}$$

with $\Lambda = \left(\frac{2\delta_L r(r-\alpha) - \alpha c_L}{c_L}\right)^2$. $\bar{\sigma}$ is only defined for $r - c_L/\delta_L < \mu \leq \delta_L r^2/(c_L + \delta_L r)$.

- Numerical

The leader's strategy depends on the parameters, like the leader's unit production cost c_L and unit investment δ_L for instance. If the leader's unit costs are too large, then the leader would find it difficult to invest early. This makes entry deterrence strategy less likely to happen. On the other hand, if the leader's unit costs are small, then the entry deterrence strategy is more likely. More specifically, when the follower invests at X_F^* with investment capacity K_L^*, and the leader produces below capacity when the follower invests, this would imply $X_1 < X_F^*$, i.e.,

$$c_L < \frac{2\beta_1}{2\beta_1 - 1}(r - \mu)\left(\frac{c}{r} + \delta\right).$$

There is also a lower bound for c_L. An extreme case is that $c_L = 0$, then we would have $X_1 = X_2 = 0$, and the region that the leader produces below capacity when the follower enters the market disappears. The fact is that the leader always produces up to capacity when $c_L = 0$. So c_L should be large enough that the equation $X_2 > X_1$ holds, i.e., it is possible that the leader produces below capacity. Furthermore, because c_L also influences the follower's investment threshold X_F^*, the smaller c_L, the larger X_F^*. Another case is that c_L cannot be so small such that $X_F^* > X_2$, which contradicts the fact that the follower invests when the leader is producing below capacity. Overall, the value of c_L should make it hold in this region that

$$X_1 < X_L^{det} < X_F^* < X_2.$$

For the parameter values we use in the Numerical section, c_L is in the approximate range of 1.8 and 3.27 to make the leader produces below capacity when the follower invests. Given the complexity of the implicit equation, we resort to numerical analysis, and start with the deterrence strategy first. Note that all the parameter values used for the figure are in the range that defines this region.

From the theoretical result of the entry deterrence strategy, by substituting (42) into (43), we get the conclusion that the entry deterrence investment threshold X_L^{*det} makes the following function $F(X)$ equal to 0.

$$F(X) = \frac{1}{4\gamma}\left(\frac{X(\beta_1 - 1)}{r - \mu} - \frac{2\beta_1 c_L}{r} + \frac{c_L^2(\beta_1 + 1)}{X(r + \mu - \sigma^2)}\right) - \frac{\beta_1 \delta_L}{2\gamma}\left(\frac{\beta_1}{\beta_1 + 1}\right.$$

$$-\frac{c_L}{X}\left(-\frac{c_L(\beta_1+1)}{2\delta_L(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu}-\frac{2\beta_2}{r}+\frac{\beta_2+1}{r+\mu-\sigma^2}\right)\right)^{-1/\beta_1}\Bigg). \tag{48}$$

However, $F(X) > 0$ for most of the parameter values. This can be shown in Figure 12. $F(X) > 0$ implies that there is no solution for X_L^{*det} and the entry deterrence is not possible in this region. The most interesting parameter is μ. As shown in Figure 12e, $F(X) = 0$has solutions for some values of μ. Specifically, if $\mu \geq 0.0375$, then there are solutions for $F(X) = 0$. We carry out further analysis. When there are two solutions, we checked and found that the smaller solution yields a result, where the upper bound for this region is $X_2 < X_F^*$. This contradicts the definition that the leader is producing below capacity when the follower enters the market. So we take the larger solution of the two and get the results as shown in Figure 13, and further compare the accommodation and deterrence strategy for these values of μ.

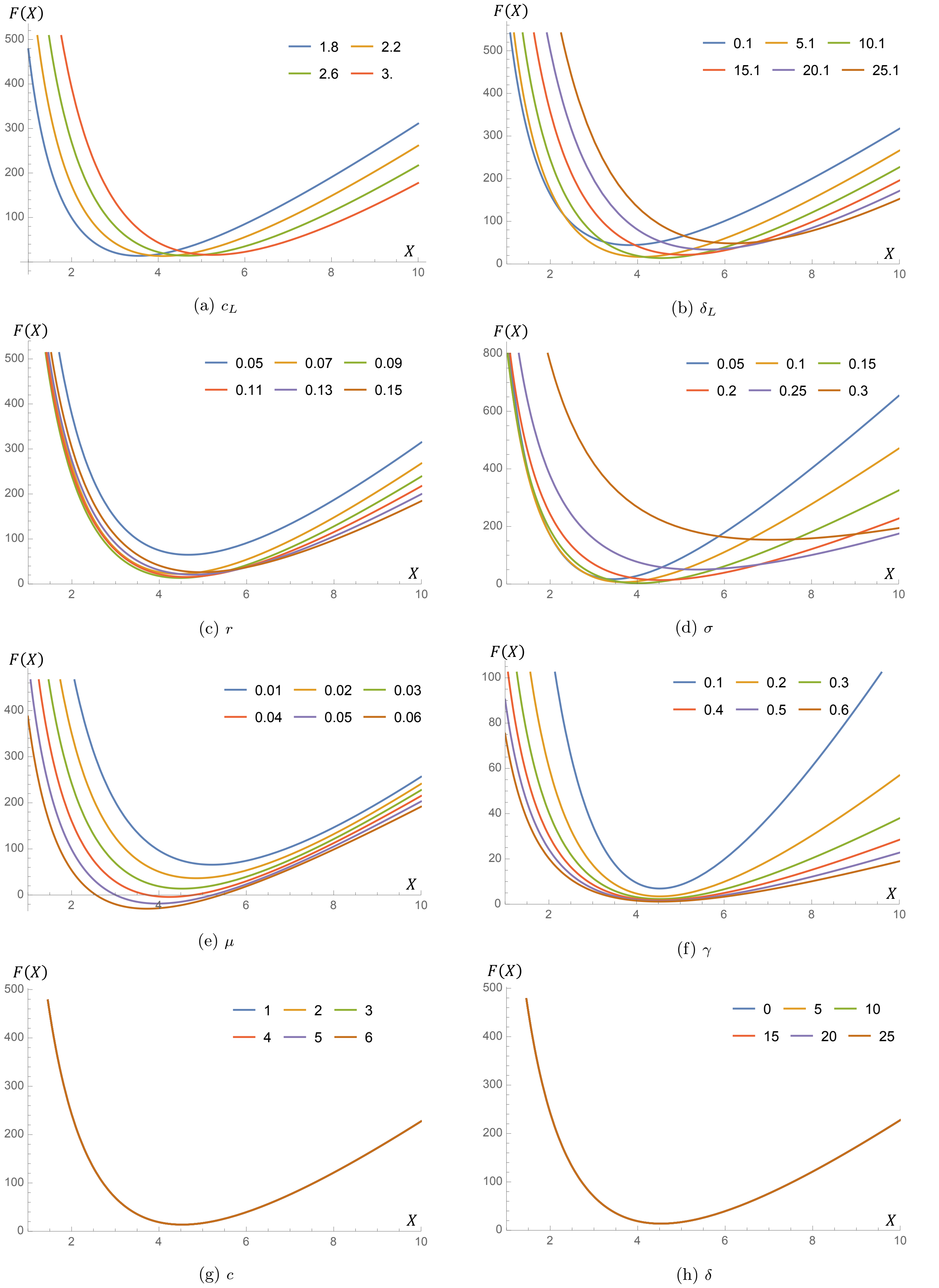
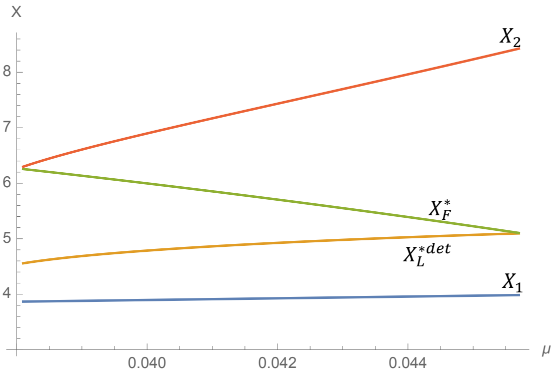
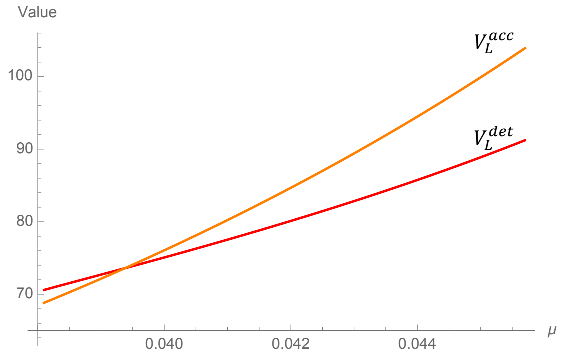


Figure 12: Illustration of the function $F(X)$ changing with parameter values. Default parameter values are $r = .1$, $\mu = .03$, $\sigma = .2$, $\gamma = .05$, $c = c_L = 2.5$, and $\delta = \delta_L = 10$.



(a) Deterrence Capacity



(b) Value Comparison

Figure 13: Illustration of possible deterrence strategy for the flexible leader. Default parameter values are $r = .1$, $\sigma = .2$, $\gamma = .05$, $c = c_L = 2.5$, and $\delta = \delta_L = 10$.

Figure 13a shows that the entry deterrence by the flexible leader is possible for $\mu \in [0.0381, 0.0457]$. In Figure 13b, the values of the deterrence and accommodation strategy are compared. It shows that only for $\mu \in [0.0381, 0.0393]$, the deterrence can generate a larger value than the accommodation strategy. For this range of parameter μ, we further study whether the dedicated firm can preempt the flexible firm. That is, we calculate the preemption point of the dedicated (non-flexible) firm and the flexible firm, shown in Figure 14.

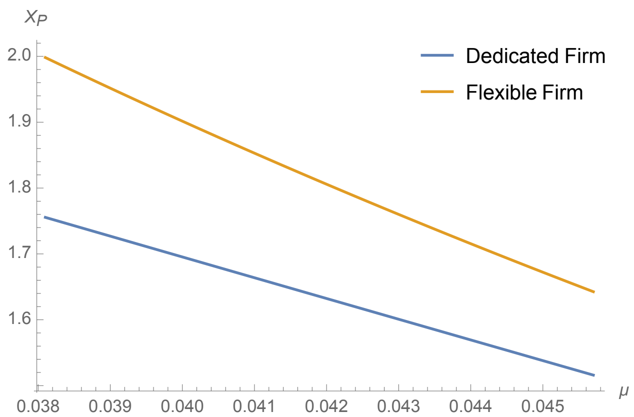


Figure 14: Illustration of the dedicated and flexible firms' preemption as functions of μ. Default parameter values are $r = .1$, $\sigma = .2$, $\gamma = .05$, $c = c_L = 2.5$, and $\delta = \delta_L = 10$.

Figure 14 shows that the preemption points of the dedicated firm is always smaller than the flexible firm. This implies that the dedicated firm always preempts the flexible firm.

4 Flexible leader and flexible follower

5 Conclusion

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Appendix

A Flexible Follower

A.1 Expression of $L_1(K_D, K_F), M_1(K_D, K_F), M_2(K_D)$, and $N(K_D, K_F)$

In the follower's value function $V_F(X, K_D, K_F)$, the lengthy expression for L_1, M_1, M_2 and N_2 are as follows,

$$
\begin{aligned}
L(K_D, K_F) &= \frac{c^2 F(\beta_2)}{4\gamma(\beta_1-\beta_2)}\left(\left(\frac{1-\gamma K_D}{c}\right)^{\beta_1+1} - \left(\frac{1-2\gamma K_F-\gamma K_D}{c}\right)^{\beta_1+1}\right),\\
M_1(K_D, K_F) &= -\frac{c^2 F(\beta_2)}{4\gamma(\beta_1-\beta_2)}\left(\frac{1-2\gamma K_F-\gamma K_D}{c}\right)^{\beta_1+1},\\
M_2(K_D) &= \frac{c^2 F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\frac{1-\gamma K_D}{c}\right)^{\beta_2+1},\\
N(K_D, K_F) &= \frac{c^2 F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\left(\frac{1-\gamma K_D}{c}\right)^{\beta_2+1} - \left(\frac{1-\gamma K_D-2\gamma K_F}{c}\right)^{\beta_2+1}\right).
\end{aligned}
$$

In order to get more insight of the value function, I analyze the signs for these four expressions. Given that $r > \alpha$, it holds that $\beta_1 > 1$ and $F(\beta_2) > 0$. From Wen et al. (2017), it also holds that $\beta_2 < -1$, and $F(\beta_1) < 0$ when $\sigma^2 < r+\alpha$; $-1 < \beta_2 < 0$, and $F(\beta_1) > 0$ when $\sigma^2 > r+\alpha$. Thus, it can be concluded that $L(K_D, K_F) > 0$, $M_1(K_D, K_F) < 0$, and $N(K_D, K_F) > 0$. If $\sigma^2 < r+\alpha$, then $M_2(K_D) < 0$, and if $\sigma^2 > r+\alpha$, then $M_2(K_D) > 0$.

A.2 Proof of Proposition 1

The optimal investment capacity $K_F(X, K_D)$ of the follower maximizes $V_F(X, K_D, K_F) - \delta K_F$. The analysis is carried out for three different regions.

- Region 1: $0 < X < c/(1-\gamma K_D)$.

Given the expression of L_1, the first order condition of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F gives

$$
\frac{c(1+\beta_1)F(\beta_2)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\gamma K_F-\gamma K_D)}{c}\right)^{\beta_1} - \delta = 0. \tag{A.1}
$$

Thus,

$$
K_F(X, K_D) = \frac{1}{2\gamma}\left(1-\gamma K_D - \frac{c}{X}\left[\frac{2\delta(\beta_1-\beta_2)}{cF(\beta_2)(1+\beta_1)}\right]^{\frac{1}{\beta_1}}\right). \tag{A.2}
$$

- Region 2: $X \geq c/(1-\gamma K_D)$ and $K_F > \frac{X-c}{2\gamma X} - \frac{K_D}{2}$.

Given the expression of M_1 and M_2, taking the first order condition of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F yields

$$
K_F(X, K_D) = \frac{1}{2\gamma}\left(1-\gamma K_D - \frac{c}{X}\left[\frac{2\delta(\beta_1-\beta_2)}{cF(\beta_2)(1+\beta_1)}\right]^{\frac{1}{\beta_1}}\right). \tag{A.3}
$$

- Region 3: $X \geq c/(1-\gamma K_D)$ and $K_F \leq \frac{X-c}{2\gamma X} - \frac{K_D}{2}$.

Given the expression for N_2, the first order condition of $V_F(X, K_D, K_F) - \delta K_F$ with respect to K_F yields that $K_F(X, K_D)$ must satisfy

$$\frac{c(1+\beta_2)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\gamma K_F-\gamma K_D)}{c}\right)^{\beta_2} + \frac{X(1-2\gamma K_F-\gamma K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0. \tag{A.4}$$

The optimal investment threshold $X_F^*(K_D)$ in each region can be derived by the value matching and smooth pasting conditions at $X_F^*(K_D)$:

$$\begin{cases} AX_F^{*\beta_1}(K_D) & = V_F\left(X_F^*(K_D), K_D, K_F\left(X_F^*(K_D), K_D\right)\right) - \delta K_F\left(X_F^*(K_D), K_D\right), \\ \beta_1 AX_F^{*\beta_1-1}(K_D) & = \frac{\mathrm{d}}{\mathrm{d}X}\left[V_F\left(X_F^*(K_D), K_D, K_F\left(X_F^*(K_D), K_D\right)\right) - \delta K_F\left(X_F^*(K_D), K_D\right)\right]. \end{cases}$$

Thus, $X_F^*(K_D)$ satisfies the following implicit equation

$$\begin{aligned} & V_F\left(X_F, K_D, K_F\left(X_F, K_D\right)\right) - \delta K_F\left(X_F, K_D\right) \\ = \; & \frac{X_F(K_D)}{\beta_1}\frac{\mathrm{d}\left[V_F\left(X_F, K_D, K_F\left(X_F, K_D\right)\right) - \delta K_F\left(X_F, K_D\right)\right]}{\mathrm{d}X}. \end{aligned} \tag{A.5}$$

- Region 1

The implicit equation (A.5) implies that

$$\delta K_F = 0. \tag{A.6}$$

- Region 2

The optimal threshold $X_F^*(K_D)$ satisfies

$$\begin{aligned} & -\frac{F(\beta_2)c^{1-\beta_1}(1-2\gamma K_F-\gamma K_D)^{1+\beta_1}X^{\beta_1}}{4\gamma(\beta_1-\beta_2)} + \frac{c^{1-\beta_2}(1-\gamma K_D)^{1+\beta_2}}{4\gamma(\beta_1-\beta_2)}F(\beta_1)X^{\beta_2} + \frac{(1-\gamma K_D)^2 X}{4\gamma(r-\alpha)} \\ & \qquad - \frac{c(1-\gamma K_D)}{2\gamma r} + \frac{c^2}{4\gamma X(r+\alpha-\sigma^2)} - \delta K_F \\ = \; & \frac{X}{\beta_1}\left[\frac{\beta_2 F(\beta_1)c^{1-\beta_2}(1-\gamma K_D)^{1+\beta_2}X^{\beta_2-1}}{4\gamma(\beta_1-\beta_2)} + \frac{(1-\gamma K_D)^2}{4\gamma(r-\alpha)} - \frac{c^2}{4\gamma X^2(r+\alpha-\sigma^2)}\right] \\ & \qquad - \frac{F(\beta_2)c^{1-\beta_1}(1-\gamma K_D-2\gamma K_F)^{1+\beta_1}X^{\beta_1}}{4\gamma(\beta_1-\beta_2)}, \end{aligned}$$

which is equivalent to

$$\begin{aligned} & \frac{c(1-\gamma K_D)F(\beta_1)}{4\gamma\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2} + \frac{1}{4\gamma}\left[\frac{\beta_1-1}{\beta_1}\frac{X(1-\gamma K_D)^2}{r-\alpha} - \frac{2c(1-\gamma K_D)}{r}\right. \\ & \qquad \left. + \frac{\beta_1+1}{\beta_1}\frac{c^2}{X(r+\alpha-\sigma^2)}\right] - \delta K_F = 0. \end{aligned} \tag{A.7}$$

- Region 3

The optimal investment threshold $X_F^*(K_D)$ satisfies

$$\frac{c\left[(1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right]F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\frac{X}{c}\right)^{\beta_2}+K_F\left(\frac{(1-\gamma K_D-\gamma K_F)X}{r-\alpha}-\frac{c}{r}-\delta\right)$$
$$=\frac{\beta_2}{\beta_1}\frac{c\left[(1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right]F(\beta_1)}{4\gamma(\beta_1-\beta_2)}\left(\frac{X}{c}\right)^{\beta_2}+\frac{K_F}{\beta_1}\frac{X(1-\gamma K_D-\gamma K_F)}{r-\alpha}.$$

Rearranging terms yields

$$\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\frac{X}{c}\right)^{\beta_2}\left[(1-\gamma K_D)^{1+\beta_2}-(1-2\gamma K_F-\gamma K_D)^{1+\beta_2}\right]$$
$$+\frac{(\beta_1-1)K_F}{\beta_1}\frac{X(1-\gamma K_D-\gamma K_F)}{r-\alpha}-\frac{cK_F}{r}-\delta K_F=0. \tag{A.8}$$

Note that in the monopoly case by Wen et al. (2017), whether the flexible firm produces up to capacity depends on the economic setting. Similarly as for the follower in the duopoly situation, if the firm produces below capacity right after investment, then $q_F(X,K_D,K_F(X,K_D))<K_F(X,K_D)$, i.e.,

$$\frac{1}{2\gamma}\left(1-\gamma K_D-\frac{c}{X}\left[\frac{2\delta\left(\beta_1-\beta_2\right)}{cF\left(\beta_2\right)\left(1+\beta_1\right)}\right]^{\frac{1}{\beta_1}}\right)>\frac{X(1-\gamma K_D)-c}{2\gamma X}.$$

It is equivalent to

$$2\delta(\beta_1-\beta_2)<cF(\beta_2)(1+\beta_1), \tag{A.9}$$

which is the same as in the monopoly case. Furthermore, it can be deduced that

$$2\delta(\beta_1-\beta_2)\geq cF(\beta_2)(1+\beta_1) \tag{A.10}$$

defines Region 3, where the firm produces up to capacity right after investment. The definitions of Region 2, equation (A.9), and Region 3, equation (A.10), for the flexible follower firm are the same as that for the flexible monopoly firm in Wen et al. (2017).

A.3 Proof of Corollary 1

- Region 2

Derive $\mathrm{d}X_F^*(K_D)/\mathrm{d}K_D$ and check whether the leader's installed capacity level would delay the flexible follower's investment. Dividing (11) by $(1-\gamma K_D)$ yields that

$$\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\frac{X(1-\gamma K_D)}{c}\right)^{\beta_2}-\frac{\delta}{2\gamma}\left(1-\frac{c}{X(1-\gamma K_D)}\left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right]^{\frac{1}{\beta_1}}\right)$$
$$+\frac{1}{4\gamma}\left[\frac{\beta_1-1}{\beta_1}\frac{X(1-\gamma K_D)}{r-\alpha}-\frac{2c}{r}+\frac{\beta_1+1}{\beta_1}\frac{c^2}{r+\alpha-\sigma^2}\frac{1}{X(1-\gamma K_D)}\right]=0. \tag{A.11}$$

Comparing (A.11) with the implicit equation that determines the optimal investment threshold in the corresponding monopoly model, I find that $X(1-\gamma K_D)$ replaces X^* in the corresponding monopoly case. Apparently, $(1-\gamma K_D)X_F^{K_D}$ is a constant that solves (A.11). Thus, it can be concluded

$$\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D}=\frac{\gamma X_F^*(K_D)}{1-\gamma K_D}>0, \tag{A.12}$$

implying that investing in more capacity by the dedicated leader would delay the investment of the flexible follower. According to (10), taking the derivative of $K_F^*(K_D)$ with respect to K_D, it follows that

$$\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D} \quad = \quad -\frac{\gamma K_F^*(K_D)}{1-\gamma K_D} \leq 0. \tag{A.13}$$

This implies that an increase in the inflexible leader's investment capacity decreases the flexible follower's optimal capacity to invest with.

- Region 3

The investment timing $X_F^*(K_D)$ and investment capacity $K_F^*(K_D)$ are determined by (12) and (13) when the follower produces up to capacity right after the investment. Rewriting these two equations yields

$$\frac{F(\beta_1)c^{1-\beta_2}(1+\beta_2)H^{\beta_2}(K_D)}{2(\beta_1-\beta_2)} + \frac{H(K_D)}{r-\alpha} - \frac{c}{r} - \delta = 0, \tag{A.14}$$

and

$$\frac{c^{1-\beta_2}F(\beta_1)\left[W^{1+\beta_2}(K_D) - H^{1+\beta_2}(K_D)\right]}{4\gamma\beta_1 X_F^*(K_D)} - \left(\frac{c}{r}+\delta\right)K_F^*(K_D) + \frac{(\beta_1-1)\left(W^2(K_D)-H^2(K_D)\right)}{4\gamma\beta_1(r-\alpha)X_F^*(K_D)} = 0, \tag{A.15}$$

respectively, where

$$\begin{aligned} W(K_D) &= X_F^*(K_D)(1-\gamma K_D), \\ H(K_D) &= X_F^*(K_D)(1-\gamma K_D - 2\gamma K_F^*(K_D)). \end{aligned}$$

Note that $H(K_D)$ is a constant and solves (A.14). From $\mathrm{d}H(K_D)/\mathrm{d}K_D = 0$, it follows that

$$\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} = \frac{\gamma X_F^*(K_D)}{1-\gamma K_D - 2\gamma K_F^*(K_D)}\left(2\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D}+1\right). \tag{A.16}$$

$W(K_D)$ solves equation (A.15). Taking the derivative of (A.15) with respect to K_D yields

$$\left(\frac{(1+\beta_2)c^{1-\beta_2}F(\beta_1)W^{\beta_2}(K_D)}{2\beta_1} + \frac{(\beta_1-1)W(K_D)}{\beta_1(r-\alpha)} - \left(\frac{c}{r}+\delta\right)\right)\frac{\gamma K_F^*(K_D) + (1-\gamma K_D)\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D}}{1-\gamma K_D - 2\gamma K_F^*(K_D)} = 0,$$

implying,

$$\frac{c(1+\beta_2)F(\beta_1)}{2\beta_1}\left(\frac{W(K_D)}{c}\right)^{\beta_2} + \frac{(\beta_1-1)W(K_D)}{\beta_1(r-\alpha)} = \frac{c}{r}+\delta. \tag{A.17}$$

(A.17) implies that $W(K_D)$ is also a constant and satisfies

$$\frac{\mathrm{d}W(K_D)}{\mathrm{d}K_D} = -\gamma X_F^*(K_D) + (1-\gamma K_D)\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} = 0.$$

It can be further derived that

$$\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} = \frac{\gamma X_F^*(K_D)}{1-\gamma K_D} > 0. \tag{A.18}$$

Moreover, from (A.16) and (A.18), it follows that

$$\frac{\mathrm{d}K_F^*(K_D)}{\mathrm{d}K_D} = -\frac{\gamma K_F^*(K_D)}{1-\gamma K_D} < 0. \tag{A.19}$$

Thus, for the case that the flexible follower produces up to capacity right after the investment, the dedicated leader can delay and decrease the investment of the follower by investing in a larger capacity.

A.4 Expressions of $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, $\mathcal{N}(K_D)$

Employing value matching and smooth pasting at $X_1 = c/(1-\gamma K_D)$ and $X_2 = c/\left(1-\gamma K_D - 2\gamma K_F^*(K_D)\right)$, then for a given K_D $(0 \le K_D < 1/\gamma)$, it can be derived that

$$\mathcal{M}_2(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{c}{1-\gamma K_D}\right)^{-\beta_2}, \tag{A.20}$$

$$\mathcal{M}_1(K_D) = -\frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left(\frac{c}{1-\gamma K_D-2\gamma K_F^*(K_D)}\right)^{-\beta_1}, \tag{A.21}$$

$$\mathcal{L}(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left[\left(\frac{c}{1-\gamma K_D}\right)^{-\beta_1}-\left(\frac{c}{1-\gamma K_D-2\gamma K_F^*(K_D)}\right)^{-\beta_1}\right], \tag{A.22}$$

$$\mathcal{N}(K_D) = \frac{cK_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left[\left(\frac{c}{1-\gamma K_D}\right)^{-\beta_2}-\left(\frac{c}{1-\gamma K_D-2\gamma K_F^*(K_D)}\right)^{-\beta_2}\right] \tag{A.23}$$

In order to check the signs for $\mathcal{L}(K_D)$, $\mathcal{M}_1(K_D)$, $\mathcal{M}_2(K_D)$, and $\mathcal{N}(K_D)$, first analyze the signs of $(\beta-1)/(r-\alpha)-\beta/r = \frac{\alpha\beta-r}{r(r-\alpha)}$ for $\beta=\beta_1$ and $\beta=\beta_2$.

If $\alpha \ge 0$, then $\alpha\beta_2 - r < 0$ because $\beta_2 < 0$. If $\alpha < 0$, then $\alpha\beta_2 - r = \alpha\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha}-\sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}\right)$, with $\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha} > 0$. From $\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha}\right)^2-\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2-\frac{2r}{\sigma^2} = -\frac{r}{\alpha}+\frac{r^2}{\alpha^2} > 0$, we get $\alpha\beta_2 - r < 0$. So, $\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r} < 0$.

If $\alpha \le 0$, then $\alpha\beta_1 - r < 0$. If $\alpha > 0$, then $\alpha\beta_1 - r = \alpha\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha}+\sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}\right)$, with $\frac{1}{2}-\frac{\alpha}{\sigma^2}-\frac{r}{\alpha} < 0$, because $r > \alpha$. From $\left(\frac{r}{\alpha}+\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2-\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2-\frac{2r}{\sigma^2} = \frac{r^2}{\alpha^2}-\frac{r}{\alpha} > 0$, it holds that $\alpha\beta_1 - r < 0$. So, $\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r} < 0$.

Thus, it can be concluded that when $0 \le K_D < 1/\gamma$, then $\mathcal{L}(K_D) < 0$, $\mathcal{M}_1(K_D) > 0$, $\mathcal{M}_2(K_D) < 0$, $\mathcal{N}(K_D) > 0$.

A.5 Proof of Proposition 2

A.5.1 Negative $\mathcal{B}_1(K_D)$

Before the derivation of the dedicated leader's optimal investment capacity in the entry deterrence and accommodation strategies, first check the sign of $\mathcal{B}_1(K_D)$.

$$\begin{aligned}\mathcal{B}_1(K_D) &= \mathcal{M}_1(K_D)+\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D)-\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D)+\frac{cK_D}{2r}X_F^{*-\beta_1}(K_D)\\ &= \frac{cK_D}{2X_F^{*\beta_1}(K_D)}\left[-\frac{1}{\beta_1-\beta_2}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left(\frac{X_F^*(K_D)}{X_2(K_D)}\right)^{\beta_1}-\frac{1}{r-\alpha}\frac{X_F^*(K_D)}{X_1(K_D)}+\frac{1}{r}\right.\\ &\qquad\left.+\frac{1}{\beta_1-\beta_2}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{X_F^*(K_D)}{X_1(K_D)}\right)^{\beta_2}\right].\end{aligned}$$

For $X_F^*(K_D)/X_i(K_D)$ with $i=\{1,2\}$, it holds that

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{X_F^*(K_D)}{X_i(K_D)} = \frac{1}{X_i^2(K_D)}\left(\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}-\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}\right)=0.$$

This implies that $X_F^*(K_D)/X_i(K_D)$ is a constant and does not change with K_D. So I can set $K_D = 0$, then

$$\frac{X_F^*(K_D)}{X_1(K_D)} = \frac{X_F^*(0)}{c},$$

and

$$\frac{X_F^*(K_D)}{X_2(K_D)} = \frac{X_F^*(0)}{c}(1 - 2\gamma K_F^*(0)) = \left(\frac{2\delta(\beta_1 - \beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}}.$$

Equation (11) is the corresponding implicit equation to determine X^* in the monopoly case:

$$F(\beta_1)\left(\frac{X^*}{c}\right)^{\beta_2} + \frac{\beta_1 - 1}{r-\alpha}\frac{X^*}{c} - \frac{2\beta_1}{r} + \frac{\beta_1 + 1}{r+\alpha-\sigma^2}\frac{c}{X^*} - \frac{2\beta_1\delta}{c}\left(1 - \frac{c}{X^*}\left[\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right]^{\frac{1}{\beta_1}}\right) = 0. \quad \text{(A.24)}$$

Rewrite such that

$$\begin{aligned} \mathcal{B}_1(K_D) &= \frac{cK_D}{2(\beta_1-\beta_2)X_F^{*\beta_1}(K_D)}\left[-\left(\frac{\beta_2 - 1}{r-\alpha} - \frac{\beta_2}{r}\right)\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right. \\ &\quad \left. + \left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right)\left(\frac{X^*}{c}\right)^{\beta_2} - (\beta_1 - \beta_2)\left(\frac{1}{r-\alpha}\frac{X^*}{c} - \frac{1}{r}\right)\right] \\ &= \frac{cK_D}{2X_F^{*\beta_1}(K_D)}\mathcal{F}(X^*), \end{aligned}$$

where X^* satisfies (A.24). Next, I show numerically that $\mathcal{F}(X^*)$ is negative. The demonstration is shown in Figure A.1. Note that γ does not influence $\mathcal{F}(X^*)$, so the numerical analysis is just about the influence of α, σ, r, c, and δ. redo the pics with new parameter values The default parameter values are $\alpha = 0.05$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$. Some combination of parameter values does not make the flexible follower produce below capacity right after investment. After ruling out these combinations, $\mathcal{F}(X^*)$ changing with parameters is illustrated in Figure A.1. The numerical analysis confirms the conjecture that $\mathcal{B}_1(K_D)$ is negative when the flexible follower produces below capacity right after investment. In the following analysis, I take $\mathcal{B}_1(K_D)$ as negative.

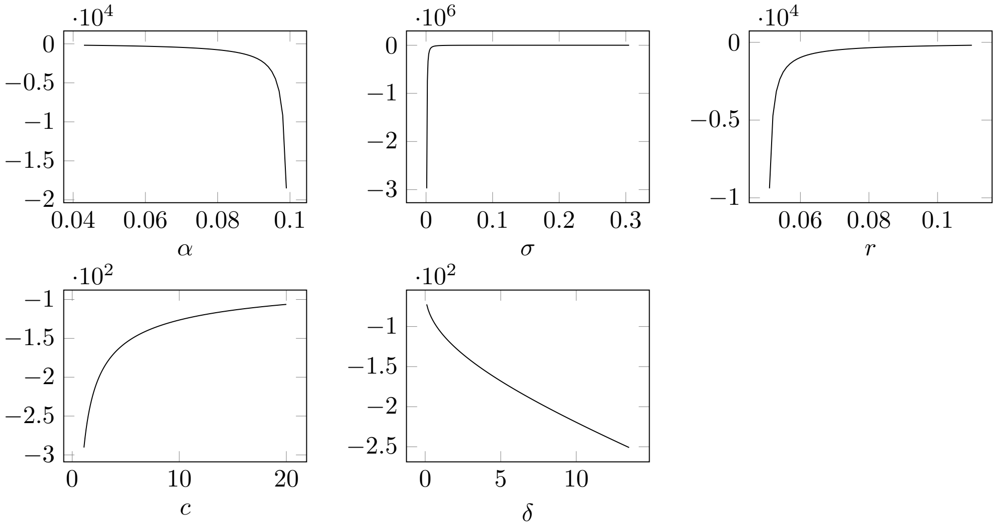

Figure A.1: Illustration of negative $\mathcal{F}(X^*)$ changing with α, σ, r, c, and δ. Default parameter values are $\alpha = 0.05$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$.

A.5.2 Proof of Proposition 2

In order to get the optimal investment decisions for the dedicated leader, I first calculate the first derivative of $\mathcal{B}_1(K_D)$ with respect of K_D. First, $\mathcal{M}_1(K_D)$ can be rewritten as

$$\begin{aligned}\mathcal{M}_1(K_D) &= -\frac{c^{1-\beta_1}K_D}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)\left[\frac{c}{X_F^*(K_D)}\left(\frac{2\delta(\beta_1-\beta_2)}{c(1+\beta_1)F(\beta_2)}\right)^{\frac{1}{\beta_1}}\right]^{\beta_1}\\ &= -\frac{K_D}{X_F^{*\beta_1}(K_D)}\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right).\end{aligned}$$

With $\mathrm{d}K_F^*(K_D)/\mathrm{d}K_D$ and $\mathrm{d}X_F^*(K_D)/\mathrm{d}K_D$ given by (A.12) and (A.13), it can be calculated that

$$\frac{\mathrm{d}\mathcal{M}_1(K_D)}{\mathrm{d}K_D} = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D).$$

Furthermore, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D) = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)X_F^{*\beta_2-\beta_1}(K_D).$$

Note also

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X_F^{*1-\beta_1}(K_D) = \frac{(1-\gamma K_D-\beta_1\gamma K_D)X_F^{*1-\beta_1}(K_D)}{2(r-\alpha)}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{cK_D}{2r}X_F^{*-\beta_1}(K_D) = \frac{cX_F^{*-\beta_1}(K_D)(1-\gamma K_D-\beta_1\gamma K_D)}{2r(1-\gamma K_D)},$$

then according to (17), it can be derived that

$$\frac{\mathrm{d}\mathcal{B}_1(K_D)}{\mathrm{d}K_D} = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_1(K_D).$$

Next, I analyze the entry deterrence and accommodation strategies for the dedicated leader, which include the optimal investment capacities and optimal investment thresholds.

1. Entry Deterrence Strategy

   The investment capacity $K_D^{det}(X)$ for a given level of X satisfies

$$\frac{\partial V_D(X,K_D)-\delta K_D}{\partial K_D} = \frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{B}_1(K_D)X^{\beta_1}+\frac{1-2\gamma K_D}{r-\alpha}X-\frac{c_D}{r}-\delta_D=0. \qquad \text{(A.25)}$$

   The entry deterrence strategy cannot happen when $K_D^{det}(X)<\hat{K}_D(X)$, which yields $X>X_2^{det}$ with X_2^{det} and $K_D^{det}(X_2^{det})$ satisfying (14) and (A.25). This is because the demand is high enough for the follower to invest immediately to enter the market. The entry deterrence strategy also does not happen when $K_D^{det}(X)<0$, yielding $X<X_1^{det}$ with X_1^{det} satisfying

$$\left[-\frac{\delta}{(1+\beta_1)F(\beta_2)}\left(\frac{\beta_2-1}{r-\alpha}-\frac{\beta_2}{r}\right)+\frac{c^{1-\beta_2}X_F^{*\beta_2}(0)}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\right.$$

$$-\frac{X_F^*(0)}{2(r-\alpha)}+\frac{c}{2r}\Big]\left(\frac{X_1^{det}}{X_F^*(0)}\right)^{\beta_1}+\frac{X_1^{det}}{r-\alpha}-\frac{c_D}{r}-\delta_D=0, \tag{A.26}$$

where $X_F^*(0)$ can be derived from (10) and (11) given that $K_D=0$. Thus, the entry deterrence strategy is only possible when $X\in(X_1^{det},X_2^{det})$. Suppose the investment threshold of the dedicated leader is $X^{det}(K_D)$ if the follower invests with capacity K_D in the entry deterrence strategy. The leader's value function before and after the investment is as follows

$$V_D(X,K_D)=\begin{cases}\mathcal{A}(K_D)X^{\beta_1} & X<X^{det}(K_D),\\ \mathcal{B}_1(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{c_DK_D}{r} & X^{det}(K_D)\le X<X_F^*(K_D),\\ \mathcal{M}_1(K_D)X^{\beta_1}+\mathcal{M}_2(K_D)X^{\beta_2} & \\ +\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X+\frac{(c-2c_D)K_D}{2r} & X\ge X_F^*(K_D).\end{cases} \tag{A.27}$$

The value matching and smooth pasting conditions to determine $X^{det}(K_D)$ are

$$\mathcal{A}(K_D)X^{\beta_1}=\mathcal{B}_1(K_D)X^{\beta_1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}X-\frac{c_DK_D}{r}-\delta_DK_D,$$
$$\beta_1\mathcal{A}(K_D)X^{\beta_1-1}=\beta_1\mathcal{B}(K_D)X^{\beta_1-1}+\frac{K_D(1-\gamma K_D)}{r-\alpha}.$$

Thus, the threshold of the entry deterrence strategy $X^{det}(K_D)$ is

$$X^{det}(K_D)=\frac{\beta_1}{\beta_1-1}\frac{r-\alpha}{1-\gamma K_D}\left(\frac{c_D}{r}+\delta_D\right). \tag{A.28}$$

Substituting $X^{det}(K_D)$ into (A.25), the optimal investment capacity K_D^{det} and investment threshold $X^{det}(K_D^{det})$ can be derived as

$$\begin{aligned}K_D^{det} &\equiv K_D^{det}(X^{det}(K_D^{det}))=\frac{1}{(\beta_1+1)\gamma},\\ X^{det}(K_D^{det}) &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c_D}{r}+\delta_D\right).\end{aligned}$$

2. Entry Accommodation Strategy

Note that from (A.20), we can get

$$\frac{\partial\mathcal{M}_2(K_D)}{\partial K_D}=\frac{1-\gamma K_D-\beta_2\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D).$$

The optimal capacity $K_D^{acc}(X)$ satisfies the following implicit equation

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)X^{\beta_1}+\frac{1-\gamma K_D-\beta_2\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_2(K_D)X^{\beta_2}+\frac{1-2\gamma K_D}{2(r-\alpha)}X+\frac{c-2c_D}{2r}-\delta_D=0. \tag{A.29}$$

The entry accommodation strategy only happens when $X\ge X_F^*(K_D)$, implying that the market demand is large enough to allow both the dedicated leader and the flexible follower to invest at the same time. Let X_1^{acc} be such that $X_1^{acc}=X_F^*(K_D^{acc}(X_1^{acc}))$, then X_1^{acc} and the corresponding $K_D^{acc}(X_1^{acc})$

satisfy (A.29) and (14). Suppose the dedicated leader invests at $X^{acc}(K_D)$ when the capacity level is K_D in the entry accommodation strategy, then the leader's value function before and after investment is

$$V_D(X,K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{acc}(K_D), \\ \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} & \\ +\frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X + \frac{c-2c_DK_D}{2r} & X \geq X_F^*(K_D) \geq X^{acc}(K_D). \end{cases} \quad \text{(A.30)}$$

The value matching and smooth pasting conditions to determine $X^{acc}(K_D)$ are

$$\mathcal{A}(K_D)X^{\beta_1} = \mathcal{M}_1(K_D)X^{\beta_1} + \mathcal{M}_2(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}X + \frac{(c-2c_D)K_D}{2r} - \delta_D K_D,$$

$$\beta_1\mathcal{A}(K_D)X^{\beta_1-1} = \beta_1\mathcal{M}_1(K_D)X^{\beta_1-1} + \beta_2\mathcal{M}_2(K_D)X^{\beta_2-1} + \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}.$$

Thus, the investment capacity $K_D^{acc}(X^{acc})$ and investment threshold $X^{acc}(K_D^{acc})$ satisfy equation (A.29) and

$$(\beta_1-\beta_2)\mathcal{M}_2(K_D)X^{\beta_2} + \frac{(\beta_1-1)K_D(1-\gamma K_D)}{2(r-\alpha)}X + \frac{(c-2c_D)\beta_1 K_D}{2r} - \beta_1\delta_D K_D = 0. \quad \text{(A.31)}$$

Rewrite these two equations, then $K_D^{acc}(X^{acc})$ and $X^{acc}(K_D^{acc})$ satisfy

$$\frac{1-\gamma K_D-\beta_1\gamma K_D}{K_D(1-\gamma K_D)}\mathcal{M}_1(K_D)X^{\beta_2} + \frac{1-\gamma K_D-\beta_2\gamma K_D}{1-\gamma K_D}\frac{c}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{X}{X_1}\right)^{\beta_2} \\ +\frac{1-2\gamma K_D}{2(r-\alpha)}X + \frac{c-2c_D}{2r} - \delta_D = 0,$$

and

$$\frac{c}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\frac{X}{X_1}\right)^{\beta_2} + \frac{(\beta_1-1)(1-\gamma K_D)}{2\beta_1(r-\alpha)}X + \frac{c-2c_D}{2r} - \delta_D = 0.$$

Solving these two equations yields

$$K_D^{acc} \equiv K_D^{acc}(X^{acc}(K_D^{acc})) = \frac{1}{(\beta_1+1)\gamma}.$$

A.6 Proof of Proposition 3

A.6.1 Negative $\mathcal{B}_2(K_D)$

When the flexible follower produces up to capacity right after investment, then

$$\begin{aligned} \mathcal{B}_2(K_D) &= \mathcal{N}(K_D)X_F^{*\beta_2-\beta_1}(K_D) - \frac{\gamma K_D K_F^*(K_D)}{r-\alpha}X_F^{*1-\beta_1}(K_D) \\ &= \frac{cK_D X_F^{*-\beta_1}(K_D)}{2(\beta_1-\beta_2)}\left[\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(\left(\frac{X_F^*(K_D)}{X_1(K_D)}\right)^{\beta_2} - \left(\frac{X_F^*(K_D)}{X_2(K_D)}\right)^{\beta_2}\right)\right. \\ &\qquad \left. +\frac{\beta_1-\beta_2}{r-\alpha}\left(\frac{X_F^*(K_D)}{X_2(K_D)} - \frac{X_F^*(K_D)}{X_1(K_D)}\right)\right]. \end{aligned}$$

Note that

$$\begin{aligned}
\frac{\mathrm{d}X_F^*(K_D)}{\mathrm{d}K_D} &= \frac{\gamma X_F^*(K_D)}{1-\gamma K_D}, \\
\frac{\mathrm{d}X_i(K_D)}{\mathrm{d}K_D} &= \frac{\gamma X_i(K_D)}{1-\gamma K_D},\ i \in \{1,2\}.
\end{aligned}$$

Thus for the terms $X_F^*(K_D)/X_i(K_D)$ with $i \in \{1,2\}$ in $\mathcal{B}_2(K_D)$,

$$\frac{\mathrm{d}}{\mathrm{d}K_D}\frac{X_F^*(K_D)}{X_i(K_D)} = \frac{1}{X_i^2(K_D)}\left(\frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D} - \frac{\gamma X_F^*(K_D)X_i(K_D)}{1-\gamma K_D}\right) = 0.$$

Similar to the case that flexible follower produces below capacity right after investment, $X_F^*(K_D)/X_1(K_D)$ and $X_F^*(K_D)/X_2(K_D)$ are constants and do not change with K_D. Thus

$$\begin{aligned}
\frac{X_F^*(K_D)}{X_1(K_D)} &= \frac{X_F^*(0)}{c}, \\
\frac{X_F^*(K_D)}{X_2(K_D)} &= \frac{X_F^*(0)}{X_2(0)}.
\end{aligned}$$

Let $X_F^*(0) = X^*$ and $X_2(0) = 1 - 2\gamma K^*$, with X^* as the optimal investment threshold and K^* as the optimal capacity in the monopoly case where the firm produces up to capacity right after investment. I rewrite $\mathcal{B}_2(K_D)$ as

$$\mathcal{B}_2(K_D) = \frac{cK_D X_F^{*-\beta_1}(K_D)}{2(\beta_1-\beta_2)}\mathcal{G}(X^*, K^*),$$

where X^* and K^* satisfy

$$\frac{c(1+\beta_2)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{(1-2\gamma K^*)X^*}{c}\right)^{\beta_2} + \frac{c}{r-\alpha}\frac{(1-2\gamma K^*)X^*}{c} - \frac{c}{r} - \delta = 0$$

and

$$\frac{cF(\beta_1)}{4\gamma\beta_1}\left(\left(\frac{X^*}{c}\right)^{\beta_2} - (1-2\gamma K^*)\left(\frac{(1-2\gamma K^*)X^*}{c}\right)^{\beta_2}\right) + \frac{\beta_1-1}{\beta_1}\frac{(1-\gamma K^*)X^*K^*}{r-\alpha} - \frac{cK^*}{r} - \delta K^* = 0.$$

$\mathcal{B}_2(K_D)$ is intuitively negative. However, it is too complicated to show this analytically. So I try to show it is negative numerically to verify the conjecture. Figure A.2 demonstrates $\mathcal{G}(X^*, K^*)$ changing with parameters. The default parameter values are given as $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$, and $\gamma = 0.05$. Some combination of parameter values does not define the case that the follower produces up to capacity right after investment. After ruling out such combinations, the negative $\mathcal{G}(X^*, K^*)$ is illustrated in Figure A.2. This confirms the conjecture that $\mathcal{B}_2(K_D)$ is negative. So in the following analysis, I assume negative $\mathcal{B}_2(K_D)$.

A.6.2 Proof of Proposition 3

I start with the derivative of $\mathcal{B}_2(K_D)$ with respect to K_D, where $K_F^*(K_D)$ and $X_F^*(K_D)$ are defined by (12) and (13), and $\mathrm{d}K_F^*(K_D)/\mathrm{d}K_D$ and $\mathrm{d}X_F^*(K_D)/\mathrm{d}K_D$ are defined by (A.18) and (A.19). It holds that

$$\frac{\mathrm{d}\mathcal{N}(K_D)}{\mathrm{d}K_D} = \frac{\mathcal{N}(K_D)(1-\gamma K_D - \beta_2\gamma K_D)}{K_D(1-\gamma K_D)},$$

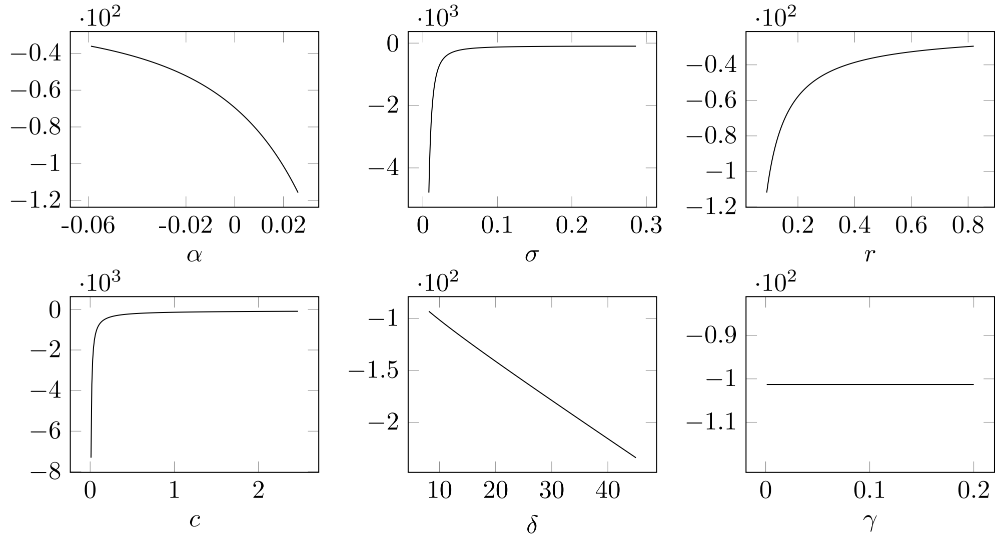

Figure A.2: Illustration of negative $\mathcal{G}(X^*, K^*)$ changing with α, σ, r, c, δ, and γ. Default parameter values are $\alpha = 0.02$, $r = 0.1$, $\sigma = 0.2$, $c = 2$, $\delta = 10$, $\gamma = 0.05$.

$$\frac{\mathrm{d}\mathcal{N}(K_D) X_F^{*\beta_2-\beta_1}}{\mathrm{d}K_D} = \frac{(1-\gamma K_D - \beta_1\gamma K_D)\mathcal{N}(K_D)}{K_D(1-\gamma K_D)} X_F^{*\beta_2-\beta_1},$$

and

$$\frac{\mathrm{d}}{\mathrm{d}K_D} K_D K_F^* X_F^{*1-\beta_1} = \frac{1-\gamma K_D - \beta_1\gamma K_D}{K_D(1-\gamma K_D)} K_D K_F^* X_F^{*1-\beta_1}.$$

Thus,

$$\frac{\mathrm{d}\mathcal{B}_2(K_D)}{\mathrm{d}K_D} = \frac{1-\gamma K_D - \beta_1\gamma K_D}{K_D(1-\gamma K_D)} \mathcal{B}_2(K_D).$$

1. Entry Deterrence Strategy

   The optimal capacity by the dedicated leader, $K_D^{det}(X)$, satisfies the first order condition

$$\begin{aligned} \frac{\partial V_D(X, K_D) - \delta K_D}{\partial K_D} &= \frac{\mathrm{d}\mathcal{B}_2(K_D)}{\mathrm{d}K_D} X^{\beta_1} + \frac{1-2\gamma K_D}{r-\alpha} X - \frac{c_D}{r} - \delta_D \\ &= \frac{1-\gamma K_D - \beta_1\gamma K_D}{K_D(1-\gamma K_D)} \mathcal{B}_2(K_D) X^{\beta_1} + \frac{1-2\gamma K_D}{r-\alpha} X - \frac{c_D}{r} - \delta_D = 0 \quad \text{(A.32)} \end{aligned}$$

   The entry deterrence strategy cannot happen if $K_D^{det}(X) < \hat{K}_D(X)$. If the dedicated leader invests at X, then the deterrence strategy is only possible when $X < X_2^{det}$. X_2^{det}, $K_D^{det}(X_2^{det})$, and $K_F^*(K_D^{det})$ satisfy (12), (13), and (A.32), with $X_F^*(K_D^{det}) = X_2^{det}$. Similar to the case that the flexible follower produces below capacity right after investment, the deterrence strategy is not possible if $K_D^{det} < 0$, which results that $X > X_1^{det}$ with X_1^{det} satisfying

$$\frac{c}{2(\beta_1-\beta_2)} \left(\frac{X_1^{det}}{X_F^*(0)}\right)^{\beta_1} \left(\left(\frac{\beta_1-1}{r-\alpha} - \frac{\beta_1}{r}\right) \left[\left(\frac{X_F^*(0)}{c}\right)^{\beta_2} - \left(\frac{X_F^*(0)\,(1-2\gamma K_F^*(0))}{c}\right)^{\beta_2} \right] - \frac{\beta_1-\beta_2}{r-\alpha} \frac{2\gamma X_F^*(0) K_F^*(0)}{c} \right) + \frac{X_1^{det}}{r-\alpha} - \frac{c_D}{r} - \delta_D = 0, \quad \text{(A.33)}$$

where $K_F^*(0)$ and $X_F^*(0)$ satisfy (12) and (13). Thus, the entry deterrence strategy is only possible if $X \in \left(X_1^{det}, X_2^{det}\right)$. If the leader applies the entry deterrence strategy and invests at $X^{det}(K_D)$ with capacity level K_D, then the value function before and after investment is

$$V_D(X,K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{det}(K_D), \\ \mathcal{B}_2(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{c_D K_D}{r} & X^{det}(K_D) \le X < X_F^*(K_D), \\ \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X - \frac{c_D K_D}{r} & X \ge X_F^*(K_D). \end{cases} \tag{A.34}$$

For a given capacity level K_D, from value matching and smooth pasting at $X^{det}(K_D)$, $X^{det}(K_D)$ must satisfy

$$\begin{aligned} \mathcal{A}(K_D)X^{\beta_1} &= \mathcal{B}_2(K_D)X^{\beta_1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}X - \frac{c_D K_D}{r} - \delta_D K_D, \\ \beta_1\mathcal{A}(K_D)X^{\beta_1-1} &= \beta_1\mathcal{B}_2(K_D)X^{\beta_1-1} + \frac{K_D(1-\gamma K_D)}{r-\alpha}. \end{aligned}$$

It can be derived that

$$X^{det}(K_D) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c_D}{r} + \delta_D\right). \tag{A.35}$$

K_D^{det} and $X^{det}(K_D^{det})$ satisfy (A.32), thus the optimal investment capacity K_D^{det} and investment threshold $X^{det}(K_D^{det})$ are

$$\begin{aligned} K_D^{det} &\equiv K_D^{det}(X^{det}(K_D^{det})) = \frac{1}{(\beta_1+1)\gamma}, \\ X^{det}(K_D^{det}) &= \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left(\frac{c_D}{r} + \delta_D\right). \end{aligned}$$

2. Entry Accommodation Strategy

   The investment capacity by the dedicated leader $K_D^{acc}(X)$ for a given level of X satisfies the first order condition

$$\begin{aligned} \frac{\partial V_D(X,K_D) - \delta K_D}{\partial K_D} &= \frac{\mathrm{d}\mathcal{N}(K_D)}{\mathrm{d}K_D}X^{\beta_2} + \frac{X(1-\gamma K_D - \gamma K_F^*(K_D))(1-2\gamma K_D)}{(r-\alpha)(1-\gamma K_D)} - \frac{c_D}{r} - \delta_D \\ &= \frac{(1-\gamma K_D - \beta_2\gamma K_D)\mathcal{N}(K_D)}{K_D(1-\gamma K_D)}X^{\beta_2} + \frac{X(1-\gamma K_D - \gamma K_F^*(K_D))(1-2\gamma K_D)}{(r-\alpha)(1-\gamma K_D)} \\ &\quad -\frac{c_D}{r} - \delta_D = 0. \end{aligned} \tag{A.36}$$

   The entry accommodation strategy only happens when the market has grown large enough to hold the two firms, i.e., $X \ge X_F^*(K_D)$. Define $X_1^{acc} = X_F^*\left(K_D^{acc}(X_1^{acc})\right)$, then X_1^{acc}, $K_D^{acc}(X_1^{acc})$, and $K_F^*(K_D^{acc})$ satisfy (12), (13), and (A.36). Suppose the dedicated leader uses the entry accommodation strategy and invests at $X^{acc}(K_D)$ with capacity K_D, then the leader's value function is

$$V_D(X,K_D) = \begin{cases} \mathcal{A}(K_D)X^{\beta_1} & X < X^{acc}(K_D), \\ \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X - \frac{c_D K_D}{r} & X \ge X_F^*(K_D) \ge X^{acc}(K_D). \end{cases} \tag{A.37}$$

From value matching and smooth pasting, I get that the investment threshold $X^{acc}(K_D)$ satisfies

$$\begin{aligned}\mathcal{A}(K_D)X^{\beta_1} &= \mathcal{N}(K_D)X^{\beta_2} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}X - \frac{c_D K_D}{r} - \delta_D K_D, \\ \beta_1\mathcal{A}(K_D)X^{\beta_1-1} &= \beta_2\mathcal{N}(K_D)X^{\beta_2-1} + \frac{K_D(1-\gamma K_D - \gamma K_F^*(K_D))}{r-\alpha}.\end{aligned}$$

Thus, it holds that $X^{acc}(K_D)$ must satisfy

$$\frac{\beta_1-\beta_2}{\beta_1}\mathcal{N}(K_D)X^{\beta_2} + \frac{\beta_1-1}{\beta_1(r-\alpha)}XK_D\left(1-\gamma K_D-\gamma K_F^*(K_D)\right) - \frac{c_D K_D}{r} - \delta_D K_D = 0. \tag{A.38}$$

Rewrite (A.36) and (A.38), then $X^{acc}(K_D^{acc})$ and K_D^{acc} satisfy

$$\begin{aligned}&\frac{1-\gamma K_D-\beta_2\gamma K_D}{1-\gamma K_D}\frac{cX^{\beta_2}}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(X_1^{-\beta_2}-X_2^{-\beta_2}\right) \\ &\qquad+\frac{X(1-\gamma K_D-\gamma K_F^*(K_D))}{r-\alpha}\frac{1-2\gamma K_D}{1-\gamma K_D}-\frac{c_D}{r}-\delta_D=0,\end{aligned}$$

and

$$\frac{cX^{\beta_2}}{2\beta_1}\left(\frac{\beta_1-1}{r-\alpha}-\frac{\beta_1}{r}\right)\left(X_1^{-\beta_2}-X_2^{-\beta_2}\right)+\frac{X(1-\gamma K_D-\gamma F^*(K_D))}{r-\alpha}\frac{\beta_1-1}{\beta_1}-\frac{c_D}{r}-\delta_D=0.$$

From

$$\frac{1-\gamma K_D-\beta_2\gamma K_D}{(\beta_1-\beta_2)(1-\gamma K_D)}=\frac{1}{\beta_1},$$

and

$$\frac{1-2\gamma K_D}{1-\gamma K_D}=\frac{\beta_1-1}{\beta_1},$$

it follows that the optimal investment capacity is

$$K_D^{acc}\equiv K_D^{acc}(X^{acc}(K_D^{acc}))=\frac{1}{(\beta_1+1)\gamma}.$$

A.7 Proof of Proposition 4

Given in the text.

A.8 Proof of Proposition 5

When the follower is flexible, from Proposition 2 and Proposition 3, the leader's entry deterrence strategy is the same regardless of whether the follower produces below or up to capacity right after investment. When there is no flexibility, the leader's entry deterrence (and entry accommodation strategy) can be found in Appendix B. The leader's entry deterrence strategy are the same regardless of with or without the follower flexibility. From Proposition 2 and Proposition 3, it also holds that the leader's investment capacity under entry accommodation strategy is $K_D^{acc}=\frac{1}{(\beta+1)\gamma}$, regardless of whether the follower produces below or up to capacity right after investment. This capacity level is the same as that when there is no follower flexibility.

B No Flexibility

This section analyzes what the follower and leader's decisions are when there is no flexibility. It means that both firms would always produce up to full capacity. For the follower, given that the leader invests and always produces K_D and the follower invests and always produces K_F, the profit flow at time t equals

$$\pi_F(t) = \left(X(t)\left(1-\gamma\left(K_D+K_F\right)\right)-c\right)K_F.$$

Here, I do not allow production suspension. So for a low level X, i.e., $X\left(1-\gamma(K_D+K_F)\right) < c$, the firms may have negative profit flows. Given the initial geometric Brownian motion level X, the value of the follower is

$$\begin{aligned} V_F(X, K_D, K_F) &= E\left[\int_{t=0}^{\infty} K_F\left(X(t)\left(1-\gamma\left(K_D+K_F\right)\right)-c\right)\exp\left(-rt\right)dt \mid X(0)=X\right] \\ &= \frac{XK_F(1-\gamma(K_D+K_F))}{r-\alpha} - \frac{cK_F}{r}. \end{aligned}$$

The follower's investment capacity maximizes

$$\max_{K_F>0} V_F(X, K_D, K_F) - \delta K_F,$$

thus, given X and K_D,

$$K_F(X, K_D) = \frac{1}{2\gamma}\left(1-\gamma K_D - \frac{r-\alpha}{X}\left(\frac{c}{r}+\delta\right)\right). \tag{B.1}$$

Before the investment, the follower holds an option to invest. Suppose the option value is

$$V_F(X, K_D) = A_F(K_L)X^{\beta_1}.$$

According to value matching and smooth pasting, the investment threshold $X_F(K_D, K_F)$ when investing with K_F satisfies

$$\begin{aligned} A_F X_F^{\beta_1} &= \frac{X_F^* K_F(1-\gamma(K_D+K_F))}{r-\alpha} - \frac{cK_F}{r} - \delta K_F, \\ \beta_1 A_F X_F^{\beta_1-1} &= \frac{K_F(1-\gamma(K_D+K_F))}{r-\alpha}. \end{aligned}$$

Thus,

$$X_F(K_D, K_F) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D-\gamma K_F)}\left(\frac{c}{r}+\delta\right). \tag{B.2}$$

Combining (B.1) and (B.2), the follower's optimal investment capacity and threshold are

$$K_F^*(K_D) = \frac{1-\gamma K_D}{(1+\beta_1)\gamma}, \tag{B.3}$$

$$X_F^*(K_D) = \frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right). \tag{B.4}$$

If $X_F^*(K_D) \leq X(0)$, then the follower would invest immediately at $t=0$ with capacity $K_F^*(X(0), K_D)$.

For the leader, to deter or accommodate the entry of the follower would be dependent on the leader's critical capacity level

$$\hat{K}_D(X) = \frac{1}{\gamma}\left(1 - \frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)X}\left(\frac{c}{r}+\delta\right)\right). \tag{B.5}$$

Entry Deterrence Strategy If the leader invests a capacity larger than $\hat{K}_D(X)$, then the follower invests later. However, if the leader invests a capacity not larger than $\hat{K}_D(X)$, then the follower invests at the same time with the leader. Suppose the investment threshold is $X_D^{det}(K_D)$ when investing capacity K_D, then the leader's value under entry deterrence strategy is assumed to be

$$V_D(X, K_D) = \begin{cases} A_D(K_D)X^{\beta_1} & \text{if } X < X_D^{det}(K_D), \\ B_D(K_D)X^{\beta_1} + \frac{XK_D(1-\gamma K_D)}{r-\alpha} - \frac{c_D K_D}{r} & \text{if } X_D^{\mathrm{det}}(K_D) \leq X < X_F^*(K_D), \\ \frac{\beta_1 X K_D(1-\gamma K_D)}{(1+\beta_1)(1-\alpha)} - \frac{c_D K_D}{r} & \text{if } X \geq X_F^*(K_D). \end{cases}$$

By value matching at $X_F^*(K_D)$, I get

$$B_D(K_D)X_F^{*\beta_1} + \frac{X_F^* K_D(1-\gamma K_D)}{r-\alpha} = \frac{\beta_1 X_F^* K_D(1-\gamma K_D)}{(\beta_1+1)(r-\alpha)}.$$

Thus,

$$B_D(K_D) = -\frac{K_D(1-\gamma K_D)X_F^*}{(\beta_1+1)(r-\alpha)}X_F^{*-\beta_1} = -\frac{K_D}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\left(\frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right)\right)^{-\beta_1}.$$

Suppose the leader invests at X, then the investment capacity under the deterrence strategy, $K_D^{det}(X)$, satisfies

$$-\frac{1-(\beta_1+1)\gamma K_D}{(\beta_1-1)(1-\gamma K_D)}\left(\frac{c}{r}+\delta\right)\left(\frac{X(\beta_1-1)(1-\gamma K_D)}{(\beta_1+1)(r-\alpha)\left(\frac{c}{r}+\delta\right)}\right)^{\beta_1} + \frac{X(1-2\gamma K_D)}{r-\alpha} - \frac{c_D}{r} - \delta_D = 0. \tag{B.6}$$

The corresponding value for the leader's entry deterrence strategy is

$$\begin{aligned} V_D^{det}(X) \quad = \quad & -\frac{K_D^{det}(X)}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\left(\frac{X(\beta_1-1)(1-\gamma K_D^{det}(X))}{(\beta_1+1)(r-\alpha)\left(\frac{c}{r}+\delta\right)}\right)^{\beta_1} \\ & + \frac{XK_D^{det}(X)(1-\gamma K_D^{det}(X))}{r-\alpha} - \frac{c_D K_D^{det}(X)}{r} - \delta_D K_D^{det}(X). \end{aligned} \tag{B.7}$$

If X is sufficiently small, then the optimal investment threshold is

$$X_D^{det} \quad = \quad \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D^{det})}\left(\frac{c_D}{r}+\delta_D\right). \tag{B.8}$$

Substitute (B.8) into (B.6) gives

$$1-(\beta_1+1)\gamma K_D^{det} = \left(1-(\beta_1+1)\gamma K_D^{det}\right)\left(\frac{\beta_1}{\beta_1+1}\frac{c_D+r\delta_D}{c+r\delta}\right)^{\beta_1}.$$

Thus,

$$K_D^{det} \quad = \quad \frac{1}{(\beta_1+1)\gamma},$$

$$X_D^{det} \equiv X_D^{det}(K_D^{det}) = \frac{(\beta_1+1)(r-\alpha)}{\beta_1 - 1}\left(\frac{c_D}{r} + \delta_D\right).$$

The corresponding follower's investment decisions are

$$\begin{aligned} K_F^*(K_D^{det}) &= \frac{\beta_1}{(\beta_1+1)^2\gamma}, \\ X_F^*(K_D^{det}) &= \frac{(\beta_1+1)^2(r-\alpha)}{\beta_1(\beta_1-1)}\left(\frac{c}{r}+\delta\right). \end{aligned}$$

Moreover, the entry deterrence strategy can not happen for

$$0 \le \hat{K}_D(X) < K_D^{det},$$

i.e.,

$$X_1^{det} \le X \le X_2^{det},$$

where

$$X_2^{det} = \frac{(\beta_1+1)(r-\alpha)}{\beta_1-1}\left((\beta+1)\left(\frac{c}{r}+\delta\right) - (\beta_1-1)\left(\frac{c_D}{r}+\delta_D\right)\right)$$

and X_1^{det} satisfies

$$-\frac{1}{\beta_1-1}\left(\frac{c}{r}+\delta\right)\left(\frac{X(\beta_1-1)}{(\beta_1+1)(r-\alpha)\left(\frac{c}{r}+\delta\right)}\right)^{\beta_1} + \frac{X}{r-\alpha} - \frac{c_D}{r} - \delta_D = 0. \tag{B.9}$$

If $X_D^{det} \le X$, then the deterrence strategy is implemented immediately with capacity $K_D^{det}(X)$ satisfying (B.6).

Entry Accommodation Strategy Under the entry accommodation strategy, the follower invests at the same time as the leader. Suppose the investment threshold is $X_D^{acc}(K_D)$ when investing capacity K_D, then the leader's value under entry accommodation strategy is assumed to be

$$V_D(X,K_D) = \begin{cases} A_D(K_D)X^{\beta_1} & \text{if } X < X_D^{acc}(K_D), \\ \frac{XK_D(1-\gamma K_D)}{2(r-\alpha)} + \frac{K_D}{2}\left(\frac{c}{r}+\delta-2\left(\frac{c_D}{r}+\delta_D\right)\right) & \text{if } X \ge X_D^{acc}(K_D). \end{cases}$$

For a given level of X, the investment capacity under the entry accommodation strategy is

$$K_D^{acc}(X) = \frac{1}{2\gamma}\left(1 + \frac{r-\alpha}{X}\left(\frac{c}{r}+\delta-2\left(\frac{c_D}{r}+\delta_D\right)\right)\right).$$

The accommodation strategy can only be chosen when $K_D^{acc}(X) \le \hat{K}_D(X)$, which means that it is only possible when

$$X \ge X_1^{acc} = (r-\alpha)\left(\frac{3\beta_1+1}{\beta_1-1}\left(\frac{c}{r}+\delta\right) - 2\left(\frac{c_D}{r}+\delta_D\right)\right).$$

Moreover, the value matching and smoothing pasting conditions yield that for the given capacity K_D, the investment threshold $X_D^{acc}(K_D)$ satisfies

$$A_D(K_D)X^{\beta_1} = \frac{XK_D(1-\gamma K_D)}{2(r-\alpha)} + \frac{K_D}{2}\left(\frac{c}{r}+\delta-2\left(\frac{c_D}{r}+\delta_D\right)\right),$$

$$\beta_1 A_D(K_D)X^{\beta-1} \quad = \quad \frac{K_D(1-\gamma K_D)}{2(r-\alpha)}.$$

Thus, it holds that

$$X_D^{acc}(K_D) = \frac{\beta_1(r-\alpha)}{(\beta_1-1)(1-\gamma K_D)}\left(2\left(\frac{c_D}{r}+\delta_D\right)-\left(\frac{c}{r}+\delta\right)\right).$$

Then the optimal investment capacity K_D^{acc} and the optimal investment threshold X_D^{acc} are

$$\begin{aligned}
K_D^{acc} &= \frac{1}{(\beta_1+1)\gamma}, \\
X_D^{acc} &= \frac{(\beta_1+1)(r-\alpha)}{(\beta_1-1)}\left(2\left(\frac{c_D}{r}+\delta_D\right)-\left(\frac{c}{r}+\delta\right)\right).
\end{aligned}$$

If $X_D^{acc} \leq X$, then the leader invests immediately at X with capacity

$$K_D(X) = \frac{1}{2\gamma}\left(1+\frac{r-\alpha}{X}\left(\frac{c}{r}+\delta-2\left(\frac{c_D}{r}+\delta_D\right)\right)\right).$$

Note that $X_1^{acc} > X_D^{acc}$. This means that the leader implements the accommodation strategy only when X reaches X_1^{acc}. Then the leader invests at X_1^{acc} with capacity

$$K_D(X_1^{acc}) = \frac{2}{(\beta_1+3)\gamma}.$$

The leader's value at X_1^{acc} is

$$V_D(X_1^{acc}, K_D(X_1^{acc})) = \frac{2}{(\beta_1-1)(\beta_1+3)\gamma}\left(\frac{c}{r}+\delta\right).$$

The corresponding follower's investment decisions under the leader's accommodation strategy are

$$\begin{aligned}
K_F^*(K_1^{acc}) &= \frac{\beta_1+1}{(\beta_1+3)\gamma}, \\
X_F^*(K_1^{acc}) &= \frac{(\beta_1+3)(r-\alpha)}{\beta_1-1}\left(\frac{c}{r}+\delta\right).
\end{aligned}$$