**Credit line pricing under heterogeneous risk beliefs**

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**Abstract**

*We study a firm with multistage investment options and a bank providing a commitment for financing using a credit line. We consider possible differences in beliefs between equity holders and the bank about the risk of assets before agreeing on the provided credit line and show that unfavorable beliefs by the bank reduce credit line capacity and lead to underinvestment. Constraints on alternative sources of financing cause firms to rely more heavily on bank credit lines even when facing unfavorable beliefs by the bank about the risk of assets. Higher loan commitment fees charged by the bank on the unused portion of the credit line accelerate initial investment but may reduce follow-on investment and use of the credit line. Our analysis examines the optimal choice between accelerated versus sequential investment and provides predictions on the optimal credit line drawdown activity used to finance these investments.*

## 1. Introduction

Many small size firms or firms operating in underdeveloped financial markets face external financing constraints and rely heavily on bank financing. Indeed, Campello et al. (2011) show that firms that are private and below investment grade have generally higher lines of credit compared to public and investment-grade firms. Since credit lines are commitments on behalf of the bank to provide future financing, initial differences in opinion between equity and the bank about the risk of the assets may distort equity holders' policies relating to investment and default timing and have important effects on firm value, as well as on the value of the credit lines. Relying on bank financing may also exasperate agency conflicts between equity and debt holders over the optimal timing of investments (e.g., Leland, 1998 and Mauer and Sarkar, 2005). From the bank's perspective, loan commitment fees charged on the unused portion of the credit line may be used to affect credit line levels and drawdown activity. The purpose of this paper is to develop a continuous-time real options framework that incorporates these realistic features. Specifically, our model considers a firm having series of investment options and a credit line. The framework includes commitment fees on unused debt commitment, heterogeneous beliefs between debt and equity holders about the risk of the assets and external investment financing costs (resulting in financing constraints and stronger reliance on credit lines). We also analyse optimal sequential versus accelerated investment policies and implications on the optimal credit line drawdown activity of firms. We provide a number of new findings and predictions summarized below.

*Firstly*, consistently with the evidence in Thakor and Whited (2010) we show that the more unfavourable debtholders beliefs are (i.e., the higher their perceived volatility of assets), the lower the firm value and the larger the delay in investment. Thakor and Whited (2010) indeed show that firm vis-à-vis investors disagreement (and not asymmetric information) negatively affects investments and firm valuations. We further show that unfavourable debt holder beliefs reduce leverage and result in an increase in credit spreads. While direct empirical evidence on this result is not available, Jimenez et al. (2009) show that firms with prior history of defaults (most likely being the ones mostly facing unfavourable debt holder beliefs about future risks) use credit lines less. Acharya et al. (2014) also show evidence that higher risk of firm's cash flows results in lower use of credit lines. Our results suggest that unfavourable debt holder beliefs impose an indirect credit line financing constraint. An increase in credit spreads under unfavourable debt beliefs is also broadly in line with evidence in Campello et al. (2011) showing that deteriorated market conditions due to the crisis of 2008 have increased interest rate markups on credit lines. We show that agency costs of debt are lower when the firm faces more unfavourable beliefs. This result is consistent with other studies (e.g., Egami, 2009 and Charalambides and Koussis, 2018) and the notion that lower levels of leverage (which exist in our model due to unfavourable beliefs) are associated with lower levels of agency costs.

*Secondly*, we find that high external investment financing costs do not significantly alter investment policy and may actually lead to an acceleration of investment. This result is due to the fact that in our model the firm may still resort to the credit line for financing investment. Indeed, survey evidence in  Lins et al., 2010 show that lines of credit are used to finance investments while operational cash flows are used as a buffer for liquidity shortages. Our model does not consider a possible downward adjustment in credit line limits or credit line revocation (as suggested in Shockley and Thakor, 1997 and more recently by Acharya et al., 2014) since the focus is not on the use of a credit line to finance shortages in liquidity but on financing new investments. Our approach of allowing for bank financing using lines of credit when other financing alternatives may be limited is also supported by evidence in Campello et al. (2011) who show evidence that lines of credit were used to boost investment even amid the financial crisis of 2008. We show however that the inability of a firm to tab external markets for financing results in more bank debt used even when the firm faces unfavorable debt holder beliefs about the risks of assets which leads to an increase in leverage, earlier default following investment and an increase in credit spreads and the agency costs of debt.

*Thirdly*, we show that high loan commitment fees encourage equity holders to invest earlier in order to avoid incurring fees on unused debt commitment. However, when equity holders face unfavourable debt holder beliefs about the risk of assets, high commitment fees may result in a delay in initial and follow-on investment and a reduction of drawdown from the credit line. Our analysis also provides a formal pricing explanation for the observed inverse relationship between credit spreads and commitment fees (see

Shockley and Thakor, 1997). We further show that the expected level of total commitment fees received by the bank is higher when commitment fees are high even though the level of credit line is reduced. Higher commitment fees also result in higher agency costs of debt.

*Fourthly*, we provide implications for the optimal timing of sequential investment and the optimal drawdown of the credit line. We show that sequential investment is optimal compared to an accelerated one when follow-on investment options are out-of-the-money (less profitable ex-ante). Within our sequential setup we show that for very unfavourable debt holder beliefs about the risk of assets the firm delays investments and reduces the use of the credit line. In this case the credit line is only used to finance early stage investments while follow-on investments are mostly self-financed. Just like in the single stage framework, the sequential setup confirms that the availability of the credit line alleviates external financing constraints allowing investment policy in different stages to remain substantially unchanged or even to be triggered earlier for unfavourable debt holder beliefs. These results further support Sufi (2009) and Nikolov et al. (2018) suggesting that the credit line is an important source of financing alleviating financing constraints. Within a sequential setting high commitment fees accelerate initial investment but lead to substantial delays of follow-on investment when the firm faces unfavourable beliefs.

The paper is organized as follows. Section 2 provides a brief review of other related literature. Section 3 describes our theoretical framework. Section 4 provides sensitivity results and model predictions. The last section concludes.

## 2. Related literature

### 2.1. Firms' use of credit lines

The importance of credit lines is highlighted in Ergungor (2001), Berg et al. (2016) and Chava and Jarrow (2008) who show that more than 80% of all commercial and industrial lending is in the form of commitments. Holmstrom and Tirole (1998) theoretically justify the existence of loan commitments suggesting that they provide an insurance against liquidity shortages. Furthermore, Shockley and Thakor (1997) explain how the different types of loan commitment fees act as mechanisms for differentiating the risk type of borrowers under asymmetric information. Indeed, some evidence suggests the use of loan commitments in the provision of liquidity insurance. For example, Campello et al. (2011) find that during the crisis firms burn their credit lines and cash to fund operations (bypassing attractive investment opportunities). However, the liquidity insurance theoretical explanation of credit lines is contradicted by evidence that credit lines may be revoked by the bank when actually in most need by firms. Sufi (2009) shows that a credit line is a viable liquidity insurance only for firms that maintain high cash flows to avoid banks' covenant violations. In response to this contradiction, Acharya et al. (2014) theoretically extend the framework of Holmstrom and Tirole (1998) and also show empirically that firms with high liquidity risk may use cash instead of the credit line to obtain liquidity insurance.

Importantly, besides providing liquidity insurance, empirical evidence suggests that credit lines may be used to finance investment activities. Indeed, Lins et al. (2010) provide survey evidence based on CFOs responses finding that credit lines are used to finance investments whereas firms use operational cash as a liquidity buffer. Furthermore, across the different countries used in the survey firms have greater use of lines of credit when external credit markets are poorly developed. Lins et al. (2010) not only show that lines of credit are the dominant source of liquidity for companies around the world (comprising about 15% of assets) but also that they are primarily used to exploit investment opportunities in good times. Nini et al. (2009) show evidence that private credit agreements are indeed used to finance capital expenditure and when these agreements have restrictions on capital expenditures they demonstrate that they can have important effects on firm's investment policies and values. Sufi (2009) shows that lack of access to a line of credit is a more statistically powerful measure of financial constraints than traditional measures used in the literature. In sum, the evidence implies that in the absence of alternative financing sources a credit line may remain an important financing choice for firms to finance their investment activities. Thus, similarly to Nikolov et al. (2018) in this paper we consider credit line financing availability for investments even when external financing sources become scarce or unavailable.

### 2.2. Pricing of credit lines

Pricing models of credit lines loan commitments that accommodate realistic features such as investment options and default risk are scarce. Thakor et al. (1981) provides an early attempt to value loan commitments as European put options and Mauer and Sarkar (2005) analyzes loan commitments in the context of financing a single-stage investment option. Egami (2009) extends this framework to include the risk of default prior to exercising an expansion option, as well as, time-to-build. Sarkar and Zhang (2016) studies loan commitments but focuses on performance-sensitive debt. Our model contributes to the literature by including heterogeneous beliefs between debt and equity holders about the volatility of assets, commitment fees for the unused part of the debt commitment and external financing costs. To our knowledge we are also the first to study a multistage model with partial drawdowns. Our study revisits the sequential vis-à-vis single-stage accelerated implementation of investments (see e.g., Kort et. al., 2010) within an optimal capital structure framework. We provide a direct measure of loan commitment fees in order to contribute to studies that empirically attempt to estimate the total costs of loan financing including the fees for embedded options (see Berg et al., 2016). Our analysis of drawdown intensity of credit lines is also useful in empirical studies aiming to link the drawdown activity along the life of bank-borrower relationship (e.g., see Jimenez et. al., 2009).

### 2.3. Heterogenous beliefs

Our modeling of heterogeneous beliefs is conceptually related to Jung and Subramanian (2013), Yang (2013) and Bayar et al. (2015) who analyze the implications of heterogeneous beliefs on firms' capital structure decisions. Our approach, however, aims in retaining a tractable continuous-time framework found in related contingent claim literature (e.g., Leland, 1994, Hackbarth and Mauer, 2012) by focusing on credit lines. Other related studies include Thakor and Whited (2010) and Dittmar and Thakor (2007), however their focus is on manager-shareholder disagreement.[1] Furthermore, compared to these studies our modeling of differences in beliefs (similarly to Yang, 2013) is more directly determined by prior disagreement about the volatility of the underlying value

[1] In Dittmar and Thakor, 2007 managers and shareholders draw from different but potentially correlated priors and the measure of disagreement is determined by the correlation of priors. It should be noted that our work is also different from papers analyzing asymmetric information and signaling as an approach to reveal information. This is because asymmetric information models assume an insider holds superior information to outsider investors and analyze whether good type firms can separate from bad firms in equilibrium. For some interesting recent contributions see Morellec and Shürhoff (2011), Fulghieri et al. (2015) and Strebulaev et al. (2016).

driver. Hackbarth (2008) (see also Hackbarth, 2009) studies managerial traits and their impact on capital structure and agency costs using a contingent claims model. His framework explores managerial optimism while our model (similarly to Yang, 2013, Dittmar and Thakor, 2007 and Thakor and Whited, 2010) does not consider which group of investors holds "correct" beliefs. Allen and Gale (1999) (see also Giat et al., 2009) explain that differences in opinion is particularly important for the financing of innovations, which, due to the absence of previous information and high risks may lead to divergent opinions about their prospects.

## 3. Theoretical framework

### 3.1. Assumptions and basic claims

We assume that the value of firm's unlevered assets $V$ of a completed project follows the stochastic process:

$$\frac{dV}{V} = \mu dt + \sigma_k dZ \qquad k = E, D \tag{1}$$

where $\mu$ is the real drift (expected rate of change or capital gains) of the assets, $\sigma_k$ is the volatility and $dZ$ is a standard Weiner process. Due to heterogeneous beliefs, the volatility perceived by debt holders ($\sigma_D$) may be different than that of equity holders ($\sigma_E$). Furthermore, the firm obtains constant cash flows $\delta V dt$ per interval $dt$ once investment is initiated.

We assume that the firm has no assets in place at t = 0 and holds investment options which cost $X_i$ per investment stage $i$. In the more general setup we allow for two investment stages, $i = 0,1$ which implies three operation phases denoted by *j*:

- Operation phase *j* = 0: between time zero and the time where initial investment stage 0 is triggered.
- Operation phase *j* = 1: between investment stage 0 and investment stage 1.
- Operation phase *j* = 2: following investment stage 1.

A bank provides a credit line (loan commitment) to the firm[2] with terms defined at $t$ = 0. The bank rationally prices each drawdown of the credit line with the first drawdown amounting to $D_0^1(V_I^0)$ obtained at investment stage 0 which is triggered at $V_I^0$ and is payable from operational phase 1 onwards and the second amounting to $D_1^2(V_I^1)$ anticipated at the optimal time of investment 1 which is triggered at $V_I^1$ and payable in operational phase 2. Thus, the total commitment agreed at $t$ = 0 is $K = D_0^1(V_I^0) + D_1^2(V_I^1)$. Agency conflicts arise because the bank commits on the debt financing terms today allowing the firm to borrow on a future date. Thus, as in Mauer and Sarkar (2005), once the loan commitment is in place equity holders have an incentive to select the investment timing by optimizing equity instead of overall firm (equity plus debt) value. We extend Mauer and Sarkar (2005) in a multistage setting and include proportional costs $c$ for unused loan commitment (see Berg et al., 2016) and external financing investment costs $\varphi$ which are proportional to the initial and subsequent stage investment financing deficits $(X_0 - D_0^1(V_I^0))$ and $(X_1 - D_1^2(V_I^1))$ respectively. We also study the effect of heterogeneous beliefs between equity and debt holders.

Equity holders have the option to default at an optimal default trigger $V_B^i$ prior to exercising their investment option at optimal trigger $V_I^i$, where $i$ = 0,1 denotes the investment stage. Thus, $V_B^0$ defines default triggered between t = 0 and initial investment 0 and $V_B^1$ denotes the default trigger between investment stage 0 and 1. In the multistage framework the firm decides the optimal timing of the initial investment 0 and upon investment obtains the value of assets $V_I^0$. In the second stage, investment is triggered at $V_I^1$ and the firm expands the value of assets $V$ at a cost $X_1$ and exchanges existing assets with their expanded version $e_G V_I^1$ where $e_G > 1$. After the final investment we assume a Leland (1994) framework which assumes no more investment stages are available and that equity holders choose only an optimal timing of default at an optimal $V_B$.

[2] Our paper does not focus on the reasons why firms choose to use loan commitments instead of other forms of financing. Some explanations in the literature are based on asymmetric information (see e.g., Holmstrom and Tirole, 1998) or the benefits of relationship banking (e.g., Boot, 2000). Our analysis assumes that a line of credit remains a possible source of financing available for firms even when they have limited access to other forms of financing (e.g, access to markets).

In the single stage (accelerated) investment framework there are only two operation phases (before and following the single investment stage). Within the single stage investment framework we assume the firm obtains $e_G V_I^0$ at the investment trigger $V_I^0$ with an investment cost *X*. The single stage framework provides also the solution to the case of accelerating investment in the sequential-multistage framework when $X = X_0 + X_1$. Thus, while in a sequential setting the firm stages investment by first paying $X_0$ obtaining $V_I^0$ and then additionally investing $X_1$ at $V_I^1$ and exchanging $V_I^1$ for an expanded version $e_G V_I^1$, in the single-stage accelerated model the firm pays all costs $X = X_0 + X_1$ at once and obtains the expanded assets $e_G$ immediately. In the limit, optimal solutions within the sequential framework where $V_I^1 \to V_I^0$ would imply that the sequential model collapses to a single stage accelerated investment model.

Let $H(V)$ denote the value of a contingent claim on the value of the project $V$. Depending on specific claim holders' beliefs, we follow standard arguments in the real options pricing literature (see for example, Dixit and Pindyck, 1994) to show that the contingent claim satisfies the following differential equation:

$$\frac{1}{2} H_{VV} \sigma_k^2 V^2 + (r - \delta) V H_V - rH = 0 \qquad k = E, D \tag{2}$$

A constant continuously compounded risk-free rate *r* is used as the discount rate under risk neutrality. Unlike standard settings, the above differential equation depends on which group of investors (equity or debt holders) beliefs about volatility risk is used which results in alternative *perceived* values for different claims. Similarly to Dittmar and Thakor (2007) and Yang (2013) our model does not consider which group of investors has the correct estimates. We assume that each group estimates is common knowledge (i.e., investors share information about their estimates when negotiating the credit line). Therefore, each claim holder uses their own estimates to value their claims accounting for the effect of the other group beliefs. Our assumption, as pointed in Dittmar and Thakor (2007) rests on economic theory which does not restrict the existence of heterogenous prior beliefs (see Dittmar and Thakor, 2007, p. 5). In fact, as pointed out by Dittmar and Thakor (2007) and Allen and Gale (1999) such contexts are particularly relevant when information signals are interpreted differently, such as in R&D investments (see also Giat et al. 2009) or when there is lack of reliable data for beliefs to be updated. Lack of reliable-verifiable data may be particularly relevant for small or private firms which indeed rely heavily on bank financing.[3]

The general solution of the above claim $H(V)$ can be expressed as a linear combination of two independent solutions of the form $AV^{\beta}$ as follows (see Dixit and Pindyck, 1994 p.142):

$$H(V) = A_1^H V^{\beta_1^k} + A_2^H V^{\beta_2^k} \qquad k = E, D \tag{3}$$

Parameters $A_1^H$ and $A_2^H$ are constants to be determined by relevant boundary conditions alongside particular solutions depending on the contingent claim (equity, debt or firm value). Solutions for $\beta_1^k$ and $\beta_2^k$ are obtained by trying $AV^{\beta^k}$ in the differential equation (2) which results in the following fundamental quadratic equation:

$$Q = \frac{1}{2}\sigma_k^2 \beta^k \left(\beta^k - 1\right) + (r-\delta)\beta^k - r = 0 \qquad k = E, D \tag{4}$$

The two roots of the quadratic (depending on whether equity and debt holders' beliefs are used) are:

$$\beta_1^k = \frac{1}{2} - \frac{(r-\delta)}{\sigma_k^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma_k^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_k^2}} > 1 \tag{5}$$

$$\beta_2^k = \frac{1}{2} - \frac{(r-\delta)}{\sigma_k^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma_k^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_k^2}} < 0$$

where $k = E, D$.

For the solution of different claim values we start from the final stage and work backwards. The final stage solutions for equity and debt are shown in the next subsection. In order to obtain solutions before the final investment we follow Hirth and Uhrig-Homburg (2010) and Hackbarth and Mauer (2012) and define two basic claims. Firstly,

[3] A related issue is whether updates of beliefs lead to an eventual convergence of (posterior) beliefs. As pointed out in Dittmar and Thakor (2007) and Allen and Gale (1999) convergence of beliefs may not occur due to limited interaction times and/or lack of reliable data. This may be a reasonable assumption within our setting where equity and debt holders interact only a few times. We do not consider Bayesian updating of beliefs, however, earlier versions of the paper also explored the possibility of alignment of beliefs in our multistage framework.

conditional that the current project value $V$ is between a lower bound of $V_B^i$ and an upper threshold $V_I^i$, we define $J(V; V_B^i, V_I^i, k)$ to be the value of the basic claim that pays 1 when $V$ reaches $V_I^i$ and becomes worthless at $V_B^i$. This basic claim involves no intermediate payments and since it is a contingent claim on $V$ it satisfies the ordinary differential equation (2). Note that this claim depends on equity holders ($k = E$) or debt holders ($k = D$) beliefs about volatility.

Using the general solution in equation (3) for $J(V; V_B^i, V_I^i, k)$ subject to boundary conditions $J(V_B^i; V_B^i, V_I^i, k) = 0$ and $J(V_I^i; V_B^i, V_I^i, k) = 1$ results in the following solution:

$$J(V; V_B^i, V_I^i, k) = \frac{(V_B^i)^{\beta_2^k} V^{\beta_1^k} - (V_B^i)^{\beta_1^k} V^{\beta_2^k}}{(V_I^i)^{\beta_1^k} (V_B^i)^{\beta_2^k} - (V_I^i)^{\beta_2^k} (V_B^i)^{\beta_1^k}} \quad \text{for } V_B^i < V < V_I^i \tag{6}$$

Secondly, we define $L(V; V_B^i, V_I^i, k)$ to be the value of a basic claim that pays 1 when $V$ reaches $V_B^i$ and becomes worthless at $V_I^i$. This claim satisfies the ordinary differential equation (2) subject to boundary conditions $L(V_B^i; V_B^i, V_I^i, k) = 1$ and $L(V_I^i; V_B^i, V_I^i, k) = 0$ which results in the following solution:

$$L(V; V_B^i, V_I^i, k) = \frac{(V_I^i)^{\beta_1^k} V^{\beta_2^k} - (V_I^i)^{\beta_2^k} V^{\beta_1^k}}{(V_I^i)^{\beta_1^k} (V_B^i)^{\beta_2^k} - (V_I^i)^{\beta_2^k} (V_B^i)^{\beta_1^k}}, \quad \text{for } V_B^i < V < V_I^i \tag{7}$$

The derivation of solutions for $J(V; V_B^i, V_I^i, k)$ and $L(V; V_B^i, V_I^i, k)$ is shown in Appendix A. Following Hackbarth and Mauer (2012) we also provide the probability of default $\Pi_L(V; V_B^i, V_I^i, k)$ and the probability of investment $\Pi_J(V; V_B^i, V_I^i, k)$ at each stage:

$$\Pi_L(V; V_B^i, V_I^i, k) = \frac{(V_I^i)^{2\lambda_k / \sigma_k^2} - (V)^{2\lambda_k / \sigma_k^2}}{(V_I^i)^{2\lambda_k / \sigma_k^2} - (V_B^i)^{2\lambda_k / \sigma_k^2}} \tag{8}$$

$$\Pi_J(V; V_B^i, V_I^i, k) = 1 - \Pi_L(V; V_B^i, V_I^i, k)$$

where $\lambda_k = -(\mu - \frac{\sigma_k^2}{2})$. In many corporate planning applications or when estimating probabilities using real data the real drift $\mu$ is commonly used in equation (8). Under risk-neutrality we replace with $\mu = (r - \delta)$.

### 3.2. The value of equity and debt in the final operation phase

Due to the exercise of the growth option the asset value in the last operation phase becomes $V' = e_G V$. Using Ito's Lemma it is easy to see that $V'$ follows a geometric Brownian motion like Eq.(1). Thus, following investment, equity value $E(V')$ satisfies the following partial differential equation (see equation (2)):

$$\frac{1}{2}E_{V'V'}\sigma_E^2 V'^2 + (r - \delta)V'E_{V'} - rE + \delta V' - (1-\tau)R = 0 \tag{9}$$

Equity holders during this last operating phase obtain cash inflows $\delta V'$ and pay the tax deductible at a corporate tax rate $\tau$ coupon $R$ to debt holders. With two investment stages and two drawdown of loan commitment $R = R_0 + R_1$, else when there is only a single-stage with one drawdown $R = R_0$.

The solution for $E^j(V')$ where $j$ denotes the operation phase is of the following form:

$$E^j(V') = V' - (1-\tau)\frac{R}{r} + A_1^E V'^{\beta_1^E} + A_2^E V'^{\beta_2^E} \tag{10}$$

where $\beta_i^E, i = 1{,}2$ are defined in equation (5) above and the constants $A_i^E, i = 1{,}2$ are determined by applying the following boundary and smooth-pasting conditions:

$$E^j(V_B') = 0 \tag{11}$$

$$\frac{\partial E^j}{\partial V'}\Big|_{V'=V_B'} = 0 \tag{12}$$

Applying condition (11) results in $A_1^E = 0$ and $A_2^E = \left((1-\tau)\frac{R}{r} - V_B'\right)\left(\frac{1}{V_B'}\right)^{\beta_2^E}$. Using these results and applying the smooth-pasting condition in equation (12) results in the optimal default trigger in the last operational phase[4]:

$$V_B = \frac{-\beta_2^E(1-\tau)}{1-\beta_2^E}\frac{R}{e_G r} \tag{13}$$

Thus, the value of equity following the last operation phase $j$ is given by:

[4] Note that the bankruptcy trigger is defined in terms of $V$. The actual default trigger is $e_G V_B$.

$$E^j(V) = e_G V - (1-\tau)\frac{R}{r} + \left((1-\tau)\frac{R}{r} - e_G V_B\right)\left(\frac{V}{V_B}\right)^{\beta_2^E} \tag{14}$$

It is understood in the solution above that the last operational phase for the sequential-multistage model is $j$ = 2 while for the single stage $j$ = 1.

The value of each drawdown (debt) in the two investment stages $i = 0,1$ in the last operational phase $j$ denoted by $D_i^j(V')$ satisfies the differential equation (2) and includes the flow of coupon $R_i$ received each period:

$$\frac{1}{2}D^j_{i\,V'V'}\sigma_D^2 V'^2 + (r-\delta)V' D^j_{i\,V'} - rD_i^j + R_i = 0 \tag{15}$$

The general solution for each credit line drawdown value is of the following form:

$$D_i^j(V) = \frac{R_i}{r} + A_1^D V'^{\beta_1^D} + A_2^D V'^{\beta_2^D} \tag{16}$$

The credit line drawdown value satisfies the following boundary conditions:

$$\lim_{V'\to\infty} D_i^j(V') = \frac{R_i}{r} \tag{17}$$

$$D_i^j(V'_B) = (1-b)\psi_i V'_B \tag{18}$$

where $\psi_i = \frac{R_i}{R}$ corresponds to the fraction of assets allocated to each drawdown in the event of default.[5] Within a single stage framework with one drawdown only we set $\psi_0 = 1$ in equation (18).

Applying equation (17) to the general solution of equation (16) implies that $A_1^D = 0$. From equation (18) we also obtain that $A_2^D = \left((1-b)V_B - \frac{R}{r}\right)\left(\frac{1}{V_B}\right)^{\beta_2^D}$. Replacing these results into the general solution we obtain the value of each debt drawdown $i$ in the final operation phase $j$:

[5] This is similar to the pari-passu rule used in Hackbarth and Mauer (2012) to assign equal footing to value of assets at default to different lenders. In our case, we have a single lender and so it claims 100% of the net of bankruptcy cost asset value, however, each drawdown can be thought to claim part of that value depending on its coupon level.

$$D_i^j(V) = \frac{R_i}{r} + \left((1-b)\psi_i(e_G V_B) - \frac{R_i}{r}\right)\left(\frac{V}{V_B}\right)^{\beta_2^D} \quad (19)$$

With single investment stage $j = 1$ and we have only one drawdown so $i = 0$ and $\psi_0 = 1$. For the sequential framework $j = 2$ and there are two drawdowns at each investment stage $i = 0, 1$ and thus also $\psi_i = \frac{R_i}{R}$ .We note that debt holders use their perceived volatility which affects the probability of bankruptcy through the auxiliary parameter $\beta_2^D$, as well as, equity holders beliefs regarding the determination of optimal default trigger $V_B$ (see equation 13).

### 3.3. The sequential model

From a methodological perspective this section extends Hackbarth and Mauer (2012) with the addition of another stage of investment and default decisions in the presence of heterogeneous beliefs.[6] We start from the final stage (see previous section) and then move backwards. Thus, the value of equity $E^2(V)$ following the second investment evaluated at $V$ was given in equation (14): $E^2(V) = e_G V - (1-\tau)\frac{R}{r} + \left((1-\tau)\frac{R}{r} - e_G V_B\right)\left(\frac{V}{V_B}\right)^{\beta_2^E}$ with $R = R_0 + R_1$ , $V_B = \frac{-\beta_2^E(1-\tau)}{(1-\beta_2^E)}\frac{R}{e_G r}$ for $V > V_B$. Note that following the last investment stage there are no more debt commitment fees to be incurred by equity holders since the full amount of the loan commitment has already been drawn.

The value of second (final) drawdown initiated in investment stage 1 with payments in operation period 2, denoted by $D_1^2$ and of the first investment stage drawdown in period 2 (denoted by $D_0^2$) have been derived in equation (19) and are as follows: $D_i^2(V) = \frac{R_i}{r} + \left((1-b)\psi_i(e_G V_B) - \frac{R_i}{r}\right)\left(\frac{V}{V_B}\right)^{\beta_2^D}$ , $V > V_B$ where $\psi_i = \frac{R_i}{R}$ , $i = 0{,}1$ corresponds to the fraction of assets that goes to each drawdown in the event of default.

[6] Our model focuses on multiple drawdowns of a credit line from a single borrower whereas Hackbarth and Mauer (2012) focus on multiple borrowers and priority rules among debt issuers. Hackbarth and Sun (2015) provide a multistage sequential framework for investment and debt financing, however, their framework does not focus on credit lines.

Moving one step back at the initial investment stage triggered at $V_I^0$ and using the basic claims derived in section 3.1, the value of equity in operation stage 1 denoted by $E^1(V)$ is then calculated as follows:

$$E^1(V) = V - \frac{R_0(1-\tau)}{r} - \frac{cD_1^2(V_I^1)}{r} + \left(\frac{cD_1^2(V_I^1)}{r} + \frac{R_0(1-\tau)}{r} - V_B^1\right) L(V; V_B^1, V_I^1, k = E) + \left(E^2(V_I^1) + D_1^2(V_I^1) - X_1 - \varphi\left(X_1 - D_1^2(V_I^1)\right)1_{X_1 > D_1^2} + \frac{cD_1^2(V_I^1)}{r} + \frac{R_0(1-\tau)}{r} - V_I^1\right) J(V; V_B^1, V_I^1, k = E) \quad (20)$$

where $V_B^1 < V < V_I^1$.

Equity value in operation stage 1 has an intuitive interpretation.[7] The first three terms capture the value of assets net of after tax coupons and commitment fees (on the yet to be drawn second stage drawdown). The subsequent term adjusts previous mentioned values of assets, after tax coupon and commitment fees in the event of default while the third term captures the anticipated additional values received, paid or given up in the event of exercise of the follow-on investment option at $V_I^1$. Note that at investment $V_I^1$ the value of assets is replaced by a scaled version equal to $e_G V_I^1$. To see this note that at investment $V_I^1$ we have that $J(V_I^1; V_B^1, V_I^1, E) = 1$ and $L(V_I^1; V_B^1, V_I^1, E) = 0$, so $E^1(V_I^1) = \left(E^2(V_I^1) + D_1^2(V_I^1) - X_1 - \varphi\left(X_1 - D_1^2(V_I^1)\right)1_{X_1 > D_1^2}\right)$. Also note that equation (20) incorporates proportional external financing costs φ incurred for financing investments from alternative (to credit line) sources.

Using the basic claims of Section 3.1, in operation stage 1 the value of the initial investment stage (0) drawdown within operational phase 1, $D_0^1(V)$ is as follows:

$$D_0^1(V) = \frac{R_0}{r} + \left((1-b)V_B^1 - \frac{R_0}{r}\right) L(V; V_B^1, V_I^1, k = D) + \left(D_0^1(V_I^1) - \frac{R_0}{r}\right) J(V; V_B^1, V_I^1, k = D) \quad (21)$$

for $V_B^1 < V < V_I^1$.

The value of the first drawdown involves the present value of perpetual coupon payments (first term), an adjustment in value in the event of default within operations' phase 1

[7] Instead of using the basic claims, one can derive equation (20) by applying differential equation (2) for equity with cash inflows $\delta V - R_0(1-\tau) - cD_1^2(V_I^1)$ per period and boundary conditions $E^1(V_B^1) = 0$ and $E^1(V_I^1) = \left(E^2(V_I^1) + D_1^2(V_I^1) - X_1 - \varphi\left(X_1 - D_1^2(V_I^1)\right)1_{X_1 > D_1^2} + \frac{cD_1^2(V_I^1)}{r} + \frac{R_0(1-\tau)}{r} - V_I^1\right)$. Similar solutions by applying boundary conditions can be used to derive other contingent claims described in this section.

(second term) and the anticipated value of the first drawdown expected to be received in the event that the follow-on investment option is exercised. The value of commitment fees $T^1(V)$ for operation stage 1 can be calculated as follows:

$$T^1(V) = \frac{cD_1^2(V_I^1)}{r} - \frac{cD_1^2(V_I^1)}{r} L(V; V_B^1, V_I^1, k) - \frac{cD_1^2(V_I^1)}{r} J(V; V_B^1, V_I^1, k) \tag{22}$$

For the subsequent optimization of investment and default triggers by equity holders we evaluate equation (22) using equity holders beliefs, hence $k = E$ is used above.

Moving one more step backwards we derive values at $t = 0$. Using the basic claims values firm value $F(V)$ in stage 0 (received by equity holders thus equivalent to equity value), is the following:

$$F(V) = -\frac{cK}{r} + \frac{cK}{r} L(V; V_B^0, V_I^0, k = E) + \Big(E^1(V_I^0) + D_0^1(V_I^0) - X_0 - \varphi\big(X_0 - D_0^1(V_I^0)\big)1_{X_0 > D_0^1} + \frac{cK}{r}\Big) J(V; V_B^0, V_I^0, k = E) \tag{23}$$

where $K = D_0^1(V_I^0) + D_1^2(V_I^1)$ is the total value of the loan commitment and $V_B^0 < V < V_I^0$.

Finally, the value of total commitment fees at t = 0 is given by:

$$T(V) = \frac{cK}{r} - \frac{cK}{r} L(V; V_B^0, V_I^0, k) + \Big(T^1(V_I^0) - \frac{cK}{r}\Big) J(V; V_B^0, V_I^0, k) \tag{24}$$

The optimization conditions for solving for the optimal boundaries $V_I^0, V_B^0, V_I^1, V_B^1$ are the following:

$$\frac{\partial F}{\partial V}\Big|_{V=V_I^0} = \frac{\partial E^1}{\partial V}\Big|_{V=V_I^0} \tag{25a}$$

$$\frac{\partial F}{\partial V}\Big|_{V=V_B^0} = 0 \tag{25b}$$

$$\frac{\partial E^1}{\partial V}\Big|_{V=V_I^1} = \frac{\partial E^2}{\partial V}\Big|_{V=V_I^1} \tag{25c}$$

$$\frac{\partial E^1}{\partial V}\Big|_{V=V_B^1} = 0 \tag{25d}$$

For brevity we do not show here the system of non-linear equations implied by the smooth-pasting conditions and we explain the derivation of these equations in the Appendix B. In the above optimization we take into account that equity holders after agreeing on loan commitment act opportunistically by maximizing equity instead of overall firm (equity plus debt) values. This corresponds to the second-best solutions which extends Mauer and Sarkar, (2005) single stage model analysis of loan commitments. Debt holders internalize this risk in the valuation of the loan commitment since the thresholds are considered in debt valuation equations. For comparison, we also calculate the first-best optimization where we set commitment fees c = 0 and replace equations (25a) and (25c) with the following optimization conditions[8]:

$$\frac{\partial F}{\partial V}|_{V=V_I^0} = \frac{\partial E^1}{\partial V}|_{V=V_I^0} + \frac{\partial D_0^1}{\partial V}|_{V=V_I^0} \tag{26a}$$

$$\frac{\partial F^1}{\partial V}|_{V=V_I^1} = \frac{\partial E^2}{\partial V}|_{V=V_I^1} + \frac{\partial D_0^2}{\partial V}|_{V=V_I^1} + \frac{\partial D_1^2}{\partial V}|_{V=V_I^1} \tag{26b}$$

Note that $F^1(V) = E^1(V) + D_0^1(V)$. The above optimization conditions correspond to the optimization under a scenario of no agency conflicts where one maximizes total firm value (both equity and debt).

For the optimization of drawdown payments and hence the capital structure, we create two fine grids for $R_0$ and $R_1$ and select the combination that maximizes initial firm value (23) (which is the value obtained by equity holders obtain at t = 0). We use this approach both when we consider the second-best optimization for investment where coupon optimization is used with conditions (25a)-(25d) and for the first-best approach where we replace (25a) and (25c) for (26a) and (26c). For the agency costs calculations we use the following:

$$AC = \frac{F^1(V) - (F^2(V) + T(V))}{(F^2(V) + T(V))} \tag{27}$$

In the above equation "1" denotes first-best and "2" denotes second-best. To provide a better comparison between first-best and second-best solutions and the corresponding effect of agency conflicts we add back the costs paid on commitment fees under the second-

[8] We have also calculated the case where equity holders maximize equity value plus only the new financing received at each round of financing without pre-commitment. This implies replacing conditions (25a) and (25c) with $\frac{\partial F}{\partial V}|_{V=V_I^0} = \frac{\partial E^1}{\partial V}|_{V=V_I^0} + \frac{\partial D_0^1}{\partial V}|_{V=V_I^0}$ and $\frac{\partial E^1}{\partial V}|_{V=V_I^1} = \frac{\partial E^2}{\partial V}|_{V=V_I^1} + \frac{\partial D_1^2}{\partial V}|_{V=V_I^1}$ respectively. These conditions were applied in Hackbarth and Mauer (2012) and were coined "second-best" solution. When these conditions are applied they provide (as expected) firm values that are in between the first-best and the second-best cases that we consider in this paper. Our conditions (25a)-(25b) are more appropriate for our context since credit line commitment financing terms are defined at *t* = 0.

best solution (which do not exist for the first-best solution). We also calculate the leverage ratio at each investment stage 1 (final stage) and 0 (initial stage) as follows:

$$Lev_1 = \frac{K}{E^2(V)+K} \text{ where } K = D_0^1(V_I^0) + D_1^2(V_I^1) \tag{28a}$$

$$Lev_0 = \frac{D_0^1(V_I^0)}{E^1(V_I^0)+D_0^1(V_I^0)} \tag{28b}$$

We also investigate drawdown activity by calculating the fraction of the total commitment used in the initial drawdown as $D_0^1(V_I^0)/K$. Finally, the credit spreads for each drawdown at each investment stage are calculated as follows:

$$CS_1 = \frac{R_1}{D_1^2(V_I^1)} - r \tag{29a}$$

$$CS_0 = \frac{R_0}{D_0^1(V_I^0)} - r \tag{29b}$$

**3.4. The single-stage (accelerated) investment model**

For the single stage model (with one drawdown) and using the basic claims defined in the section 3.1 we define firm value at $t = 0$ as follows:

$$F(V) = -\frac{cK}{r} + \frac{cK}{r}L(V; V_B^0, V_I^0, E) + \left(E^1(V_I^0) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right)J(V; V_B^0, V_I^0, E) \tag{30}$$

where $X$ is the single stage cost for investment. In equation (30), $E^1(V_I^0)$ is obtained from equation (14) and the loan commitment $K = D_0^1(V)$ from equation (19) (with $\psi_0 = 1$).

The optimal second-best investment $V_I^0$ and default trigger $V_B^0$ are found by solving a system of two equations which result from applying the following smooth-pasting conditions[9]:

$$\frac{\partial F}{\partial V}\Big|_{V=V_I^0} = \frac{\partial E^1}{\partial V}\Big|_{V=V_I^0} \tag{31a}$$

$$\frac{\partial F}{\partial V}\Big|_{V=V_B^0} = 0 \tag{31b}$$

For first-best optimization we replace (31a) with :

[9] See the Appendix B for the derivation of the non-linear equations implied by the smooth-pasting conditions.

$$\frac{\partial F}{\partial V}|_{V=V_I^0} = \frac{\partial E^1}{\partial V}|_{V=V_I^0} + \frac{\partial D_0^1}{\partial V}|_{V=V_I^0} \tag{32}$$

where for the derivative of $D_0^1(V)$ we use equation (19) with $\psi_0 = 1$ (since there is only single drawdown). We obtain the optimal drawdown payment $R_0$ that maximizes firm value in equation (30) using a dense $R_0$ grid search. We also calculate the total expected loan commitment fees as follows:

$$T(V) = \frac{cK}{r} - \frac{cK}{r} L(V; V_B^0, V_I^0, k) - \frac{cK}{r} J(V; V_B^0, V_I^0, k) \tag{33}$$

For the calculation of agency costs we use equation (27) as was shown in the sequential model. For the leverage ratio at the single investment stage we use equation (28b) with $K = D_0^1(V_I^0)$ (single drawdown) and for credit spreads equation (29b) as defined in the sequential model above. As pointed earlier, we use the single-stage model to study accelerated investment when the firm pays all costs $X = X_0 + X_1$ at once an optimal $V_I^0$ and obtains the expanded assets $e_G V_I^0$ immediately.

## 4. Numerical results and discussion

### 4.1. Single-stage (accelerated) investment model

For our sensitivity results in this section we use the base case parameter values of Leland (1994) with an additional assumption of a positive opportunity cost $\delta = 6\%$. Other parameters values are as follows: value of assets $V = 100$, risk-free rate $r = 0.06$, investment cost $X = 100$, $e_G = 1$, bankruptcy costs $b = 0.5$ and tax rate $\tau = 0.35$. Initially we assume away the presence of equity financing costs ($\varphi = 0$) and loan commitment fee ($c = 0\%$) in order to focus on the impact of heterogeneous beliefs. For the symmetric beliefs case we use a volatility $\sigma_E = \sigma_D = 0.25$. For the sensitivity analysis we fix the estimates for equity holders and vary debt holders' beliefs. Therefore, for $\sigma_D < 0.25$ equity holders face favorable beliefs and by increasing $\sigma_D$ we study more unfavorable debt holders beliefs. We provide sensitivity results in Table 1 where the case of symmetric beliefs is highlighted in bold. Given our initial assumption of equity financing costs ($\varphi = 0$) and loan commitment fee ($c = 0\%$) the symmetric case corresponds to the Mauer and Sarkar (2005) model (where it should be noted however we model unlevered assets instead of prices as the underlying source of uncertainty). With $c = 0\%$ assumed initially there is no default risk prior to investment and we thus we do not report $V_B^0$ in the table (since we obtain $V_B^0 \to 0$ in all sensitivity results). Based on our extensive sensitivity analysis in Table 1 we summarize our first result.

[Insert Table 1]

**Prediction 1:** *The effect of heterogeneous beliefs between equity and debt holders regarding the volatility of assets.* [10]*(Single stage model.)*

In the absence of external financing costs and commitment fees, more unfavourable debt-holders beliefs (higher $\sigma_D$) result in:

a) Lower firm value, debt and leverage ratios
b) A higher investment trigger (i.e., there is a delay in investment)
c) A lower default trigger (i.e., there is a delay in bankruptcy after investment) for unfavourable debt holder beliefs
d) Higher credit spreads
e) Lower agency costs

Prediction 1 (a) shows that unfavourable debt holders beliefs create an indirect debt capacity constraint causing a reduction in leverage. This is broadly in line with low leverage due to financing constraints and frictions reported in Devos et al. (2012). Our results are also consistent with the theoretical result presented in Jung and Subramanian (2013) regarding the reduction in the use of long-term debt when outside investors have unfavourable beliefs. Yang (2013) offers an alternative explanation where optimistic managers may prefer equity relative to debt (which they consider undervalued) thus driving down leverage ratios. It is important to note that similar insights but for different reasons can be obtained within an asymmetric information setting. Within an asymmetric information setting good firms face costlier financing in their effort to separate from bad quality firms (e.g., see Morellec and Shürhoff, 2011). Thus, predictions like the negative impact of differences in beliefs on firm values (Prediction 1 (a)) and investment (Prediction

[10] The predictions are based on second-best solutions. The directional effects are similar for the first-best case except that we observe that credit spreads follow an inverse U-shape with respect to debt holders beliefs.

1(b)) can also be derived within an asymmetric information context (see for example Myers and Majluf, 1984 and Claus, 2011) which makes empirical testing difficult. However, Thakor and Whited (2010) provide supporting evidence that firm versus investor disagreement and *not* asymmetric information reduces valuations and investment.[11] Unfavourable beliefs increase credit spreads (Prediction 1 (d)) which is broadly consistent with evidence in Campello et al. (2011) showing an increase in interest markups on credit lines during the crisis (where a deterioration of debt holders beliefs about the firms' risks may have risen). Finally, it is interesting to note that unfavourable debt holder beliefs may lead to a negative relationship between leverage and credit spreads (see Prediction1 (a) and (d)) challenging traditional views (e.g., ratio analysis and Z-scores) which would associate higher leverage ratios with higher default risk and credit spreads.

Prediction 1 (e) shows that the more unfavourable debtholders beliefs become (higher $\sigma_D$ ) the lower the total agency costs of debt. This result is related to Egami (2009) (see also Charalambides and Koussis, 2018) who find that agency costs are lower when leverage ratios are low. Our analysis provides further insights related to heterogeneous beliefs. We show that when debt holder beliefs are favourable, agency costs increase since equity holders deviate from first-best policies in order to exploit financial benefits using higher leverage (see optimal policies in Table 1). On the other hand, when faced with unfavourable debt holders beliefs, equity holders anticipate little financial benefits arising from debt. In this case, optimizing the timing of investment and default leads to policies that are more aligned with the corresponding first-best solutions. Interestingly, agency costs tend to almost zero when equity holders face highly unfavourable beliefs by debt holders.

We next focus on the effect of external financing costs.[12] Figure 1 explores the impact of external financing costs on firm value, investment and default, debt, leverage ratios and credit spreads by focusing on second-best (equity optimized) solutions. The last panel in the figure explores the impact of external financing costs on the agency costs of

[11] Ascioglu et al. (2008) shows empirically that firms facing higher information asymmetry may lead to lower valuations but not lower investment. Empirically distinguishing between heterogeneous beliefs and asymmetric information proxies is indeed a challenging task (see discussion in Thakor and Whited, 2010). Thus further research may be needed to distinguish more clearly between information-based theories.

[12] Other papers consider the effect of (exogenous) debt financing constraints in a contingent claim framework and on investment timing (see Shibata and Nishihara, 2012 and Koussis and Martzoukos, 2012). In this paper we focus more broadly on constraints on any external source of financing allowing the firm in this case to only have access to the credit line.

debt. The figure panels explore the case of zero equity financing costs (solid line) compared with a high equity financing costs (dotted line). For the high equity financing costs we use $\varphi = 1$ to proxy for the case of limited access to external markets.

[Insert Figure 1 here]

Our results show that when debt holders beliefs are lower or at par with those of equity holders regarding volatility, external financing constraints has no effect on firm value, the investment and default policy, debt, leverage ratios and credit spreads. This result is expected since for this range of debt holders beliefs equity holders fully finance investment with the credit line (credit line debt financing in this range of beliefs often exceeds the investment cost level of 100). However, with unfavorable debt holder beliefs, equity holders would partly finance investments with alternative sources of financing if they did not face any external financing constraints. To see this, observe that when equity holder are unconstrained ($\varphi = 0$) and face unfavourable beliefs ($\sigma$ of debt higher than 0.25) their optimal level of credit line financing (see panel 4) would drop below the level of investment (which is at the level of 100). This implies that they would optimally finance part of the investment with other external sources. Therefore, for unfavorable debt holders beliefs the constraint arising due to external financing costs becomes binding. In this case our results show that when faced with external financing costs (high $\varphi$) equity holders resort to just enough financing from the credit line to cover the level of investment. In the constrained region, firm value drops more significantly under second-best compared to the first-best case which results in an increase in agency costs (as shown in panel 7). We summarize the following results regarding the effect of external financing costs[13]:

**Prediction 2:** *The effect of external financing costs (Single-stage model)*

External financing constraints (high external financing costs) result in:

a) Lower firm value when debt holders have unfavorable beliefs (otherwise, for favorable beliefs firm value remains unchanged because firms use the credit line for financing)

[13] The summary result provides predictions under a second-best solution. Similar directional effects are observed for first-best solutions (not shown for brevity) except that we observe a delay in investment when debt holders have unfavorable beliefs under the first-best solutions.

b) An investment trigger which is not significantly different compared to the case with no external financing costs for favorable beliefs but is triggered earlier for more unfavorable beliefs
c) An increase in the default trigger after investment (i.e, default triggered earlier); this becomes more pronounced when debt holders beliefs become more unfavorable
d) Credit line (debt) levels which remain close to the level of investment when debt holders have unfavorable beliefs resulting in an increase in leverage ratios
e) An increase in credit spreads which is more pronounced when debt holders have unfavorable beliefs
f) An increase in agency costs in the region where debt holders have more unfavorable beliefs

Hirth and Homburg (2010(b)) show that firms delay investment when facing external financing costs and have low internal liquidity. On the other hand, our results show that investment policy is not substantially changed in the presence of external financing constraints and may actually triggered earlier for unfavorable debt holder beliefs (Prediction 2 (b)). The difference in the obtained result is due to the fact that in our context equity holders retain access to a bank credit line (even if that becomes expensive when debt holders have unfavorable beliefs). Thus, in our case the credit line can be used to alleviate external financing constraints. Yang (2013) shows that high equity financing costs force firms to resort to higher debt levels thus driving leverage ratios to higher levels (consistent with Prediction 2 (d)). This is also consistent with evidence in Campello et al. (2011) showing that firms that are private or have below-investment grade ratings (i.e., those firms that within our setting would have high external financing costs) have higher lines-of-credit to assets ratios (compared to public and investment-grade firms). In Hirth and Homburg (2010(b)) model, higher levels of liquidity provide an alternative financing source which reduces investment distortions and the level of agency costs. Although our model does not analyze liquidity choice, our insights concerning Prediction 2(f) are similar: lower levels of external financing constraints (i.e., higher availability of an alternative financing source) reduces deviations from first-best investment policies and the agency costs of debt. Our results further suggest that external financing constraints have a significant impact on agency costs only when combined with significantly unfavorable debt holder beliefs about the risk of the firm.

Our subsequent analysis focuses on the impact of loan commitment fees. Berg et al. (2016) analyze the importance of these fees in loan pricing by showing that they reflect options embedded in these contracts. Figure 2 compares the solutions between commitment fees of c = 0.1% (solid line) with c = 0.5% (dotted line). A debt commitment fee of 0.5% is in line with the median debt commitment fee reported in Berg et al. (2016).

[Insert Figure 2 here]

The effect of loan commitment fees is summarized in Prediction 3 below.

**Prediction 3:** *The effect of loan commitment fees (Single-stage model)*

Higher loan commitment fees result in:

a) Lower firm value.
b) Earlier investment and higher probability of investment unless debt holders beliefs are highly unfavourable (in which case there is a delay in investment).
c) Earlier default prior to investment and a delay in default following investment.
d) Lower levels of credit line (debt) and leverage ratios; the reduction is more significant the more unfavourable debt holders beliefs become.
e) Lower credit spreads; the reduction is more pronounced the more unfavourable debt holders beliefs become.
f) An increase in total expected cost of commitment fees which follows an inverse U-shape with respect to debt holders beliefs.
g) Higher agency costs of debt.

Focusing on the case of at par beliefs ($\sigma_E = \sigma_D$) (see Figure 2) shows that our model suggests an inverse relationship between loan commitment fees and credit spreads (supporting the evidence in Shockley and Thakor, 1997). Our result provides the pricing mechanism for this result: an increase in loan commitment fees decreases the level of the loan commitment resulting in a reduction in leverage and adjustments in firms optimal policies that lead to lower risk of default and hence also lower credit spreads.

In Prediction 3 (g) we also observe that higher commitment fees distort a firm's optimal policies and result in higher agency costs irrespective of debt holders' beliefs. Agency costs are significant: with loan commitment fees c = 0.5% which is the median charge found in Berg et al. (2016) agency costs range between 4% for unfavourable debt holders' beliefs to more than 31% for favourable debt holders' beliefs. Thus while higher commitment fees create incentives for accelerated investment they also create distortions in investment policy and higher levels of agency costs. This provides a plausible explanation why banks did not increase fees too much during the crisis (Campello et al. 2011).

## 4.2. Sequential investment and use of the credit line

### 4.2.1. Sequential vs single stage (accelerated) investment

Our first step in the analysis of the sequential model is to investigate the conditions where a sequential strategy is preferred over a single-stage (accelerated) investment. Kort et al. (2010) focus on the effect of uncertainty on the choice between accelerated vs sequential investment with no debt financing and by incorporating economies of scale when the firm accelerates investment. We revisit the choice of sequential vs accelerated investment in our more general setting with credit line debt financing. However, we choose to allow for equal footing between accelerated and sequential strategies, i.e., we do not assume economies of scale for accelerated investment in order to investigate the "pure" underlying forces underlying the choice between the two strategies. As we see next the crucial factor affecting the attractiveness of each strategy is the "moneyness" of the follow-on investment options which is defined as follows: when the expansion factor of second stage is high (low) or when the second stage investment cost is low (high) then follow-on second stage option to invest is considered more in-the-money (out-of-the-money). We show below that a sequential investment is preferred over an accelerated one when the second-stage investment is relatively more out-of-the-money (and vice versa).

Figure 3 presents sensitivity results with respect to the expansion factor $e_G$ which varies the moneyness of the second stage investment option. Other parameters are as follows: value of unlevered assets $V$ = 100, risk-free rate r = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E = \sigma_D = 0.25$, investment costs $X_0$ =100, $X_1$ = 100, bankruptcy costs b = 0.5 and tax rate $\tau = 0.35$, equity financing costs $\varphi = 0$ and loan commitment fees c = 0.5%. For the single stage (accelerated) we set X = 200 (=$X_0 + X_1$ ). Figure 3 panel (1) shows firm values under an accelerated vs sequential investment and panel (2) shows the investment thresholds for the first stage (0) and second stage (1) investment of the sequential model compared with the single investment trigger (0) for the accelerated investment model.

[Insert Figure 3 here]

In Figure 3, panel (1) we observe that when the second stage option is less in the money (low expansion factor $e_G$) then the sequential investment strategy dominates. However, as the option to invest in the second stage becomes more attractive (for higher $e_G$) the sequential strategy converges to a single-stage investment, i.e., the firm follows an accelerated investment. Figure 2, Panel (2), confirms this insight. The figure shows that when the second stage option to expand is out-of-the money the distance between the investment thresholds of the first and the second investment threshold of the sequential model is high (there is greater delay of second stage relative to first stage and hence investment is staged). However, as the second stage option to invest becomes more attractive, the firm's second stage investment threshold is closer to the first, until eventually the two stages collapse into the single-stage threshold. We have verified that the above results indeed depend on second stage option moneyness (relative magnitude of $e_G$ and the investment cost $X_2$). For example when the second stage investment becomes more expensive (e.g. $X_2 = 200$) then the sequential strategy is preferred for a wider range of low expansion factor levels. We summarize the following result.

**Prediction 4:** *Sequential vs single stage (accelerated) investment*

For low enough follow-on investment option moneyness (low $e_G$ or high second stage cost) a sequential strategy is preferred over a single-stage (accelerated) investment. When follow-on investment becomes more attractive (increase moneyness) the firm implements both stages simultaneously leading to accelerated investment.

Following the above result and in order to retain the sequential setup in the subsequent sensitivity analysis we use a relatively out-of-the money second stage with $X_2 = 100$ and $e_G = 1.8$ as a base case. All other parameters remain the same as before. Note that we also use a level of loan commitment fees c = 0.5% (median level used in Berg et al., 2016) in order to allow the study of the level of total commitment fees and default risk prior to investment.

Figure 4 and Figure 5 show sensitivity results with respect to the effect of external financing costs in the presence of heterogeneous beliefs in the sequential model. Figure 4 focuses on the effect on firm value, investment and default thresholds, probability of investment and default and the effect on total expected value of loan commitment fees. Figure 5 focuses on the effect of external financing costs on initial and follow-on (second stage) investment's leverage ratios, the total level of loan commitment, the optimal drawdown at each stage, the credit spreads at investment stage and the agency costs of debt.

[Insert Figure 4 & 5 here]

We first discuss the effect of heterogeneous beliefs for the base case of the sequential model with no external financing costs (focusing only on the solid lines in Figures 4 and 5). We observe that most of the implications we have summarized for the single stage model (see Prediction 1) also hold for the sequential model. Specifically, we observe that more unfavourable debt-holders beliefs (higher $\sigma_D$) result in lower firm values, lower credit line levels and leverage ratios in the different investment stages, higher investment triggers (resulting in a delay in investment in both investment stages), lower default triggers at each operational phase when debt beliefs about risk are highly unfavourable (resulting in a delay in bankruptcy) and lower agency costs of debt. We are not able to confirm the same increasing pattern for credit spreads for more unfavourable beliefs in the sequential model where instead we observe that credit spreads may actually decrease for very unfavourable beliefs.[14]

Our sequential model analysis provides some additional new insights. We find that for very unfavourable debt holder beliefs the firm delays investment (both initial and follow-on) and utilizes the credit line for financing only for early stage investments. Follow-on investments are substantially delayed at very high asset value levels and the firm does not drawdown from the credit line. Instead the firm self-finances follow-on investments. To verify this one notices in Figure 4, panel (4) that investment in the second stage is delayed substantially, that the probability of second stage investment substantially

[14] Since in this section we have now assumed positive commitment fees levels which have somewhat reduced leverage and credit spreads this may have created some distortion on the "pure" effect of heterogeneous beliefs (see Figure 2 panel (9)).

drops for more unfavourable beliefs (Fig.4, panel (8)) and that in Figure 5 panel (4) for very unfavourable beliefs the firm draws down the credit line only for financing initial investment. We summarize the following main results:

**Prediction 5:** *The effect of heterogeneous beliefs between equity and debt holders regarding the volatility of assets (Sequential model).*

More unfavourable debt-holders beliefs (higher $\sigma_D$) result in:

a) Lower firm value.
b) Lower credit line levels (debt) and leverage ratios at different investment stages and low drawdown of credit line for follow-up investments for very unfavourable beliefs.
c) Higher investment triggers for each investment stage (i.e., delay in investment in different stages) and a reduction in probability of investment for unfavourable debt beliefs.
d) Lower default triggers before and following investment stages in the region of very unfavourable debt holder beliefs.
e) Lower credit spreads only in the region of very unfavourable debt holder beliefs.
f) Lower agency costs.

A further results from the sequential model shows that total commitment fees follow an inverse U-shape as a function of debt holder volatility beliefs (Fig.4, panel (9)).

We next move to the discussion of the impact of external financing costs. We summarize these results as follows (which can be easily verified by resorting to the results of Figure 4 and 5):

**Prediction 6:** *The effect of external financing costs (Sequential model).*

External financing constraints (high external financing costs) result in:

a) A reduction in firm value for unfavorable debt holder beliefs (otherwise firm value may remain unchanged since firms use credit line for financing investments).
b) An increase in leverage ratios at each stage of investment with a more substantial increase in leverage in the region where debt holder beliefs are unfavorable.
c) Drawdown of credit line which is used to fully finance investment levels and thus a total commitment level in-line with total investment levels.

d) Investment triggers which are not significantly different compared to the case with no external financing costs for favorable beliefs but triggered earlier for more unfavorable beliefs. An increase in the probability of investment in each stage which is more pronounced for follow-up investments.
e) An increase in default triggers before and after investment resulting in earlier default; this becomes pronounced when debt holders beliefs become more unfavorable.
f) An increase in credit spreads which is more pronounced when debt holders have unfavorable beliefs.
g) An increase in agency costs which more pronounced in the region where debt holders have more unfavorable beliefs

The results confirm and extend the insights obtained from the single-stage model within a sequential setting. Importantly, our results show that the availability of the credit line allows investment policy not to be substantially changed in the presence of external financing constraints and actually to be triggered earlier for unfavourable debt holder beliefs (Prediction 6 (d)). This result supports Sufi (2009) and Nikolov et al. (2018) suggesting that the credit line is an important source of financing alleviating financing constraints. Our results are also supported by Lins et al. (2010) and other recent literature showing evidence on the use of credit lines to finance investment and Campello et al. (2011) showing that firms with less access to alternative public sources of external financing costs exhibit greater use of credit lines compared to firms with better access to public markets for financing.

Figure 6 and 7 analyze the impact of loan commitment fees. We summarize these results as follows:

[Insert Figure 6 & 7 here]

**Prediction 7:** *The effect of loan commitment fees (Sequential stage model)*

Higher loan commitment fees result in:

a) Lower firm value.

b) Earlier investment and higher probability of initial stage investment; a delay in follow-on investment with a reduction of probability of investment for unfavourable debt holder beliefs.
c) Earlier default prior to investment and a delay in default following investment.
d) Lower total commitment (debt) and leverage ratios; the reduction is more significant for follow-on investment and for the region of more unfavourable debt holders beliefs.
e) Lower credit spreads for more unfavourable debt holders beliefs.
f) An increase in total expected cost of commitment fees which follows an inverse U-shape with respect to debt holders beliefs.
g) A higher level of agency costs of debt.

These results corroborate with those of the single stage model (see Prediction 3) and also provide with some additional insights for sequential decisions as follows. First, we find that in a sequential setting high commitment fees only accelerate initial investment (just like in the single-stage model), however, they may lead to a delay of follow-on investment when the firm faces unfavorable beliefs. These results suggest that when banks charge substantial commitment fees and share a pessimistic view about firm's risk with the firm then the firm's optimal reaction will to only use the credit line for early-on investments and to substantial postpone and self-finance follow-on investments. Agency costs due to the distortion in firm's policies remain significant as in the single-stage. Furthermore the inverse relationship between commitment fees and credit spreads for at par beliefs ($\sigma_E = \sigma_D$) (in support of evidence in Shockley and Thakor, 1997) remains within the sequential setup.

## 5. Conclusions

In this paper we have developed a framework with heterogeneous beliefs between equity and debt holders under a loan commitment credit line agreement. We study the impact of heterogeneous beliefs on firm value, optimal capital structure, investment and default timing, credit spreads and the level of agency costs. Our analysis shows that unfavourable beliefs by debt holders reduce firm value, optimal leverage and result in delayed investment and an increase in credit spreads. With external financing costs, equity holders resort to debt financing even when faced with debt holders' unfavourable beliefs which results in an increase in leverage and credit spreads. We show that higher loan commitment fees result in earlier initial investment, however they may lead to a delay in follow-on investments when the firm faces unfavourable beliefs by debt holders about the risk of the assets. Furthermore, under higher loan commitment fees we find that credit line levels and leverage ratios are lower and that this reduction is more significant for more unfavourable debt holder beliefs. We show that agency costs of debt are lower when equity holders face unfavourable beliefs, higher external financing costs and higher loan commitment fees.

Our framework also provided implications on the choice between accelerated versus sequential investments and the drawdown activity of the credit line in relation to debt holder beliefs about the risk of the assets, external financing constraints and loan commitment fees. We confirm several results from the empirical literature on credit lines and provide further empirical predictions on this growing and important literature.

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## Appendix A

In this appendix we show the derivation of the value of the basic claims $J(V;V_B^i,V_I^i,k)$ and $L(V;V_B^i,V_I^i,k)$ shown in equations (6) and (7) respectively.

First, to derive the value of $J(V;V_B^i,V_I^i,k)$ we use the general solution in equation (3) and apply the two boundary conditions as follows:

$$J(V_B^i;V_B^i,V_I^i,k) = A_1(V_B^i)^{\beta_1^k} + A_2(V_B^i)^{\beta_2^k} = 0 \tag{A1}$$

$$J(V_I^i;V_B^i,V_I^i,k) = A_1(V_I^i)^{\beta_1^k} + A_2(V_I^i)^{\beta_2^k} = 1 \tag{A2}$$

Solving the system of equations in (A1) and (A2) determines the value of the constants $A_1$ and $A_2$ and results in the solution for $J(V;V_B^i,V_I^i,k)$ shown in equation (6).

Similarly, for $L(V;V_B^i,V_I^i,k)$ we apply the general solution in equation (3) alongside the two boundary conditions as follows:

$$L(V_B^i;V_B^i,V_I^i,k) = A_1(V_B^i)^{\beta_1^k} + A_2(V_B^i)^{\beta_2^k} = 1 \tag{A3}$$

$$L(V_I^i;V_B^i,V_I^i,k) = A_1(V_I^i)^{\beta_1^k} + A_2(V_I^i)^{\beta_2^k} = 0 \tag{A4}$$

Solving the system of equations in (A3) and (A4) determines the value of the constants $A_1$ and $A_2$ and results in the solution for $L(V;V_B^i,V_I^i,k)$ shown in equation (7).

**Appendix B**

*Multistage model*

In this section of the Appendix we characterize explicitly the smooth-pasting equations for the multistage model. First, for (25a) and (25b) we need:

$$\frac{\partial F}{\partial V}|_{V=y} = \frac{cK}{r}\frac{\partial L(V;V_B^0,V_I^0,E)}{\partial V}|_{V=y} + \left(E^1(V_I^0) + D_0^1(V_I^0) - X_0 - \varphi\big(X_0 - D_0^1(V_I^0)\big)1_{X_0>D_0^1} + \frac{cK}{r}\right)\frac{\partial L(V;V_B^0,V_I^0,E)}{\partial V}|_{V=y} \quad \text{(A5)}$$

evaluated either at $y = V_I^0$ (needed for 25a) or $y = V_B^0$ (needed for 25b).

For (25a), (25c) and (25d) we also need the following:

$$\frac{\partial E^1}{\partial V}|_{V=y} = 1 + \left(\frac{cD_1^2(V_I^1)}{r} + \frac{R_0(1-\tau)}{r} - V_B^1\right)\frac{\partial L(V;V_B^1,V_I^1,E)}{\partial V}|_{V=y} + \left(E^2(V_I^1) + D_1^2(V_I^1) - X_1 - \varphi\big(X_1 - D_1^2(V_I^1)\big)1_{X_1>D_1^2} + \frac{cD_1^2(V_I^1)}{r} + \frac{R_0(1-\tau)}{r} - V_I^1\right)\frac{\partial J(V;V_B^1,V_I^1,E)}{\partial V}|_{V=y}$$

(A6)

which is evaluated for $y = V_I^0, V_I^1, V_B^1$ (for (25a), (25c) and (25d) respectively).

In the above (A5-A6) expressions we have:

$$\frac{\partial L(V;V_B^i,V_I^i,k)}{\partial V}|_{V=y} = \frac{\frac{1}{y}\left(\beta_2(V_I^i)^{\beta_1^k}(y)^{\beta_2^k} - \beta_1(V_I^i)^{\beta_2^k}(y)^{\beta_1^k}\right)}{\left[(V_I^i)^{\beta_1^k}(V_B^i)^{\beta_2^k} - (V_I^i)^{\beta_2^k}(V_B^0)^{\beta_1^k}\right]} \quad \text{(A7)}$$

$$\frac{\partial J(V;V_B^i,V_I^i,k)}{\partial V}|_{V=y} = \frac{\frac{1}{y}\left(\beta_1(V_B^0)^{\beta_2^k}(y)^{\beta_1^k} - \beta_2(V_B^0)^{\beta_1^k}(y)^{\beta_2^k}\right)}{\left[(V_I^0)^{\beta_1^k}(V_B^0)^{\beta_2^k} - (V_I^0)^{\beta_2^k}(V_B^0)^{\beta_1^k}\right]} \quad \text{(A8)}$$

For (25c) we obtain $\frac{\partial E^2}{\partial V}|_{V=V_I^1}$ as follows:

$$\frac{\partial E^2}{\partial V}|_{V=V_I^1} = e_G + \beta_2^E\left((1-\tau)\frac{R}{r} - e_G V_B\right)\left(\frac{V_I^1}{V_B}\right)^{\beta_2^E}\left(\frac{1}{V_I^1}\right) \quad \text{(A9)}$$

For first-best solutions (see equations (26a) and (26b)) we need $D_i^j(V)$ given by equation (19) in the main text. These leads to the following:

$$\frac{\partial D_i^j}{\partial V}|_{V=V_I^i} = \beta_2^D\big((1-b)\psi_i(e_G V_B) - \frac{R_i}{r}\big)\left(\frac{V_I^i}{V_B}\right)^{\beta_2^D}\left(\frac{1}{V_I^i}\right) \tag{A10}$$

(A10) can be used to evaluate $\frac{\partial D_0^1}{\partial V}|_{V=V_I^0}$ and $\frac{\partial D_1^2}{\partial V}|_{V=V_I^1}$ needed for expressions (26a)-(26b).

*Single stage model*

For the single stage model we need the following:

$$\frac{\partial F}{\partial V}|_{V=y} = \frac{cK}{r}\frac{\partial L(V;V_B^0,V_I^0,E)}{\partial V}|_{V=y} + \Big(E^1(V_I^0) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\Big)\frac{\partial J(V;V_B^0,V_I^0,E)}{\partial V}|_{V=y} \tag{A11}$$

The above is evaluated at $y = V_I^0$ for equation (31a) and at $y = V_B^0$ for equation (31b). The terms $\frac{\partial L(.)}{\partial V}|_{V=y}$ and $\frac{\partial J(.)}{\partial V}|_{V=y}$ can be obtained from (A7) and (A8) respectively.

For equation (31a) one also needs:

$$\frac{\partial E^1}{\partial V}|_{V=V_I^0} = e_G + \beta_2^E\left((1-\tau)\frac{R}{r} - e_G V_B\right)\left(\frac{V_I^0}{V_B}\right)^{\beta_2^E}\left(\frac{1}{V_I^0}\right) \tag{A12}$$

For first-best solutions of equation (32) one needs $D_0^1(V)$ given by equation (19) in the main text with $\psi_0 = 1$. This leads to the following:

$$\frac{\partial D_0^1}{\partial V}|_{V=V_I^0} = \beta_2^D\big((1-b)(e_G V_B) - \frac{R_0}{r}\big)\left(\frac{V_I^0}{V_B}\right)^{\beta_2^D}\left(\frac{1}{V_I^0}\right) \tag{A13}$$

We also note that for the first best solution we set commitment fees c equal to zero.

## Table 1: Single stage model: The effect of heterogeneous beliefs between debt and equity holders with respect to volatility (σ)

### A. First-best

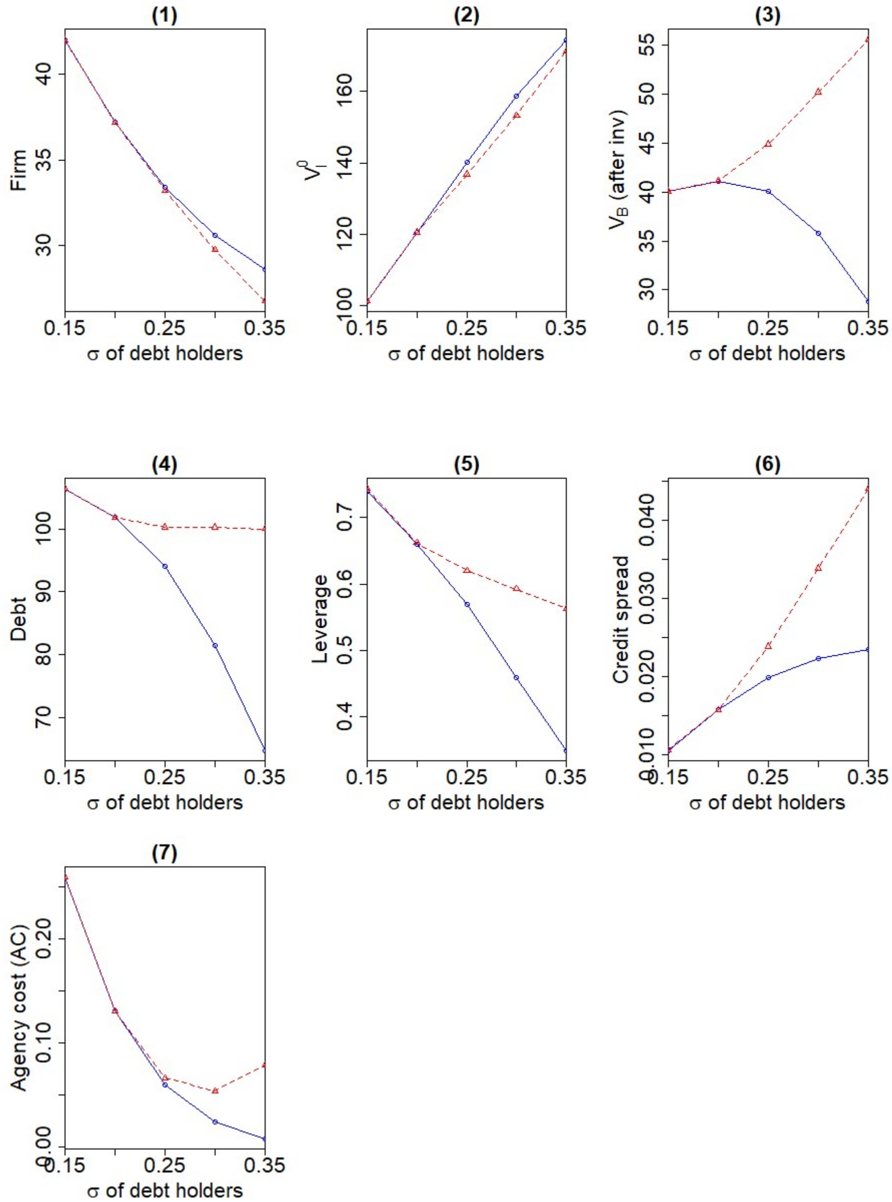
| Volatility | Firm value | Inv. Trigger ($V_I^0$) | Bankruptcy after inv. ($V_B$) | Optimal Capital Structure at Investment Trigger $V_I^0$: Equity | K=Debt | Leverage | Credit line payment ($R_0$) | Credit Spread | Agency (AC) |
|---|---|---|---|---|---|---|---|---|---|
| $\sigma_D$ =0.15 | 52.88 | 140.08 | 71.60 | 33.20 | 169.63 | 0.84 | 13.4 | 0.0190 | 0.258 |
| $\sigma_D$ =0.20 | 42.03 | 157.43 | 66.79 | 51.80 | 151.09 | 0.74 | 12.5 | 0.0227 | 0.130 |
| **$\sigma_D$ =0.25 = $\sigma_E$** | **35.42** | **171.42** | **57.71** | **74.97** | **127.62** | **0.63** | **10.8** | **0.0246** | **0.060** |
| $\sigma_D$ =0.30 | 31.34 | 182.43 | 45.95 | 101.59 | 101.04 | 0.50 | 8.6 | 0.0251 | 0.024 |
| $\sigma_D$ =0.35 | 28.87 | 190.32 | 33.66 | 128.47 | 74.30 | 0.37 | 6.3 | 0.0248 | 0.008 |

### B. Second-best:

| Volatility | Firm value | Inv. Trigger ($V_I^0$) | Bankruptcy after inv. ($V_B$) | Optimal Capital Structure at Investment Trigger $V_I^0$: Equity | K=Debt | Leverage | Credit line payment ($R_0$) | Credit Spread |
|---|---|---|---|---|---|---|---|---|
| $\sigma_D$ =0.15 | 42.02 | 101.30 | 40.08 | 36.74 | 121.06 | 0.74 | 7.5 | 0.0105 |
| $\sigma_D$ =0.20 | 37.18 | 120.57 | 41.14 | 51.99 | 101.71 | 0.66 | 7.7 | 0.0157 |
| **$\sigma_D$ =0.25 = $\sigma_E$** | **33.41** | **140.12** | **40.08** | **71.05** | **93.65** | **0.57** | **7.5** | **0.0198** |
| $\sigma_D$ =0.30 | 30.60 | 158.81 | 35.80 | 94.85 | 81.21 | 0.46 | 6.7 | 0.0223 |
| $\sigma_D$ =0.35 | 28.63 | 174.53 | 28.86 | 121.17 | 64.83 | 0.35 | 5.4 | 0.0234 |

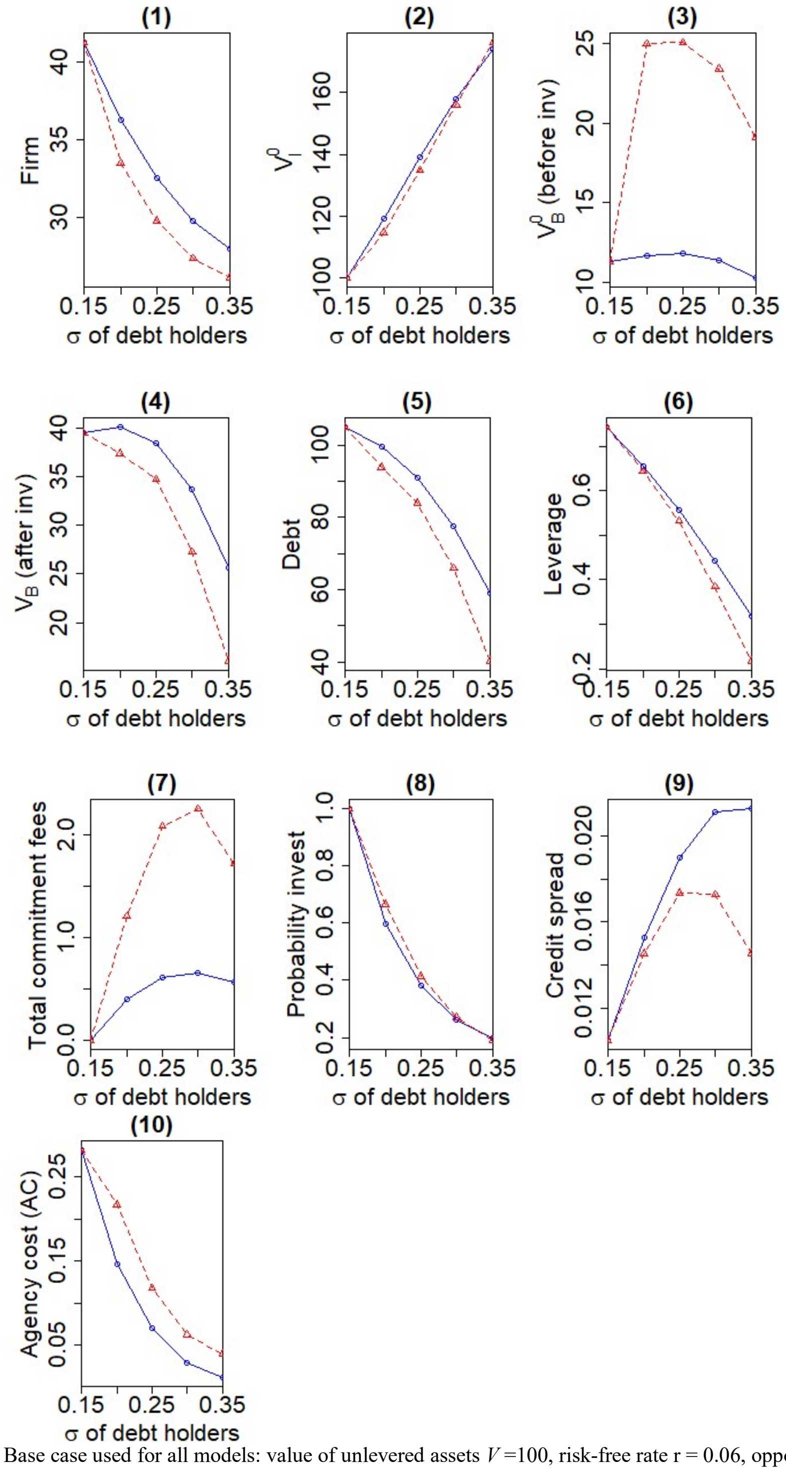
Base case parameters: value of unlevered assets $V$ = 100, risk-free rate $r$ = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E$ = 0.25, investment cost $X$ = 100, $e_G = 1$, bankruptcy costs $b$ = 0.5, tax rate $\tau$ = 0.35,equity financing costs φ = 0 and loan commitment fees c = 0. Sensitivity analysis is with respect to debt holders perceived estimate of volatility $\sigma_D$. The single-stage model used in the sensitivity results is described in Section 3.4. Firm value is derived in equation (27), the investment trigger $V_I^0$ and bankruptcy trigger $V_B^0$ before investment for second-best are derived using system of equations (31a) and (31b) and for first-best (31a) is replaced with (32a). Due to c = 0 there is no default risk prior to investment and hence $V_B^0 \to 0$ in our solutions in all cases (thus not reported). $V_B$ denotes the bankruptcy trigger following investment (see equation (13). The value of equity at the investment trigger $V_I^0$ is derived using equation (14) and total loan commitment (debt) $K$ using equation (19) for $D_0^1(V)$ with $\psi_0 = 1$. Leverage is the ratio of debt ($K$) over equity plus debt. Optimal capital structure is obtained using a dense grid search over the credit line payment $R_0$ with increments 0.1. Credit spread is derived by dividing optimal $R_0$ with $D_0^1(V)$ and subtracting the risk-free rate. Agency costs shows the percentage differences between the first-best and second-best firm solutions calculated as in equation (27). A summary of the main results of the Table is provided in Prediction 1 of the main text which shows that in the absence of external financing costs and commitment fees more unfavorable debt-holders beliefs (higher $\sigma_D$) result in: lower firm value, debt and leverage ratios, a higher investment trigger (i.e., there is a delay in investment), a lower default trigger (i.e., there is a delay in bankruptcy after investment) for unfavorable debt holder beliefs, higher credit spreads. These predictions are based on second-best solution while the first-best differs only in that there is an inverse U-shape with respect to credit spreads. The model also shows that more unfavorable debt-holders beliefs (higher $\sigma_D$) result in and lower agency costs.

**Figure 1. The effect of external financing costs: single-stage model**



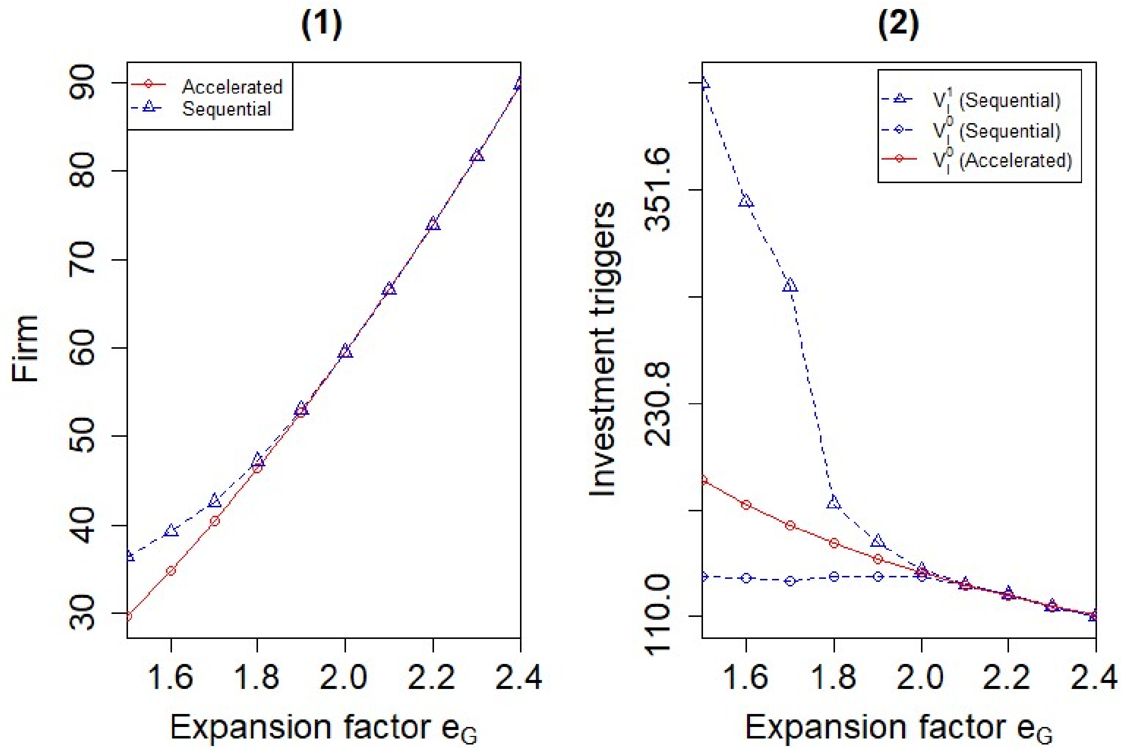

Parameters used: value of unlevered assets $V$ =100, risk-free rate $r$ = 0.06, opportunity cost $\delta$= 0.06, volatility $\sigma_E$ = 0.25, investment cost $X$ = 100, $e_G = 1$, bankruptcy costs $b$ = 0.5 and tax rate $\tau$ = 0.35, loan commitment fees $c$ = 0. Equity financing costs $\varphi$ = 0 (solid line) or $\varphi$ = 1 for constrained (dotted line). Sensitivity analysis is with respect to debt holders perceived estimate of volatility $\sigma_D$. Due to c = 0 there is no default risk prior to investment and hence $V_B^0 \rightarrow 0$ in our solutions in all cases. The single-stage model of section 3.4. is used for the calculations. All variable definitions are provided also in Table 1. The figure results are summarized in Prediction 2 of the main text and are as follows. External financing constraints (high external financing costs) result in lower firm value when debt holders have unfavorable beliefs (otherwise it may remain unaltered because firms use the credit line for financing), an investment trigger which is not significantly different compared to the case with no external financing costs for favorable beliefs but is earlier for more unfavorable beliefs, an increase in the default trigger after investment (i.e, default triggered earlier) which becomes more pronounced when debt holders beliefs become more unfavorable, credit line (debt) levels which remain close to the level of investment when debt holders have unfavorable beliefs resulting in an increase in leverage ratios, an increase in credit spreads which is more pronounced when debt holders have unfavorable beliefs and an increase in agency costs in the region where debt holders have more unfavorable beliefs.

**Figure 2. The effect of loan commitment fees: single-stage model**



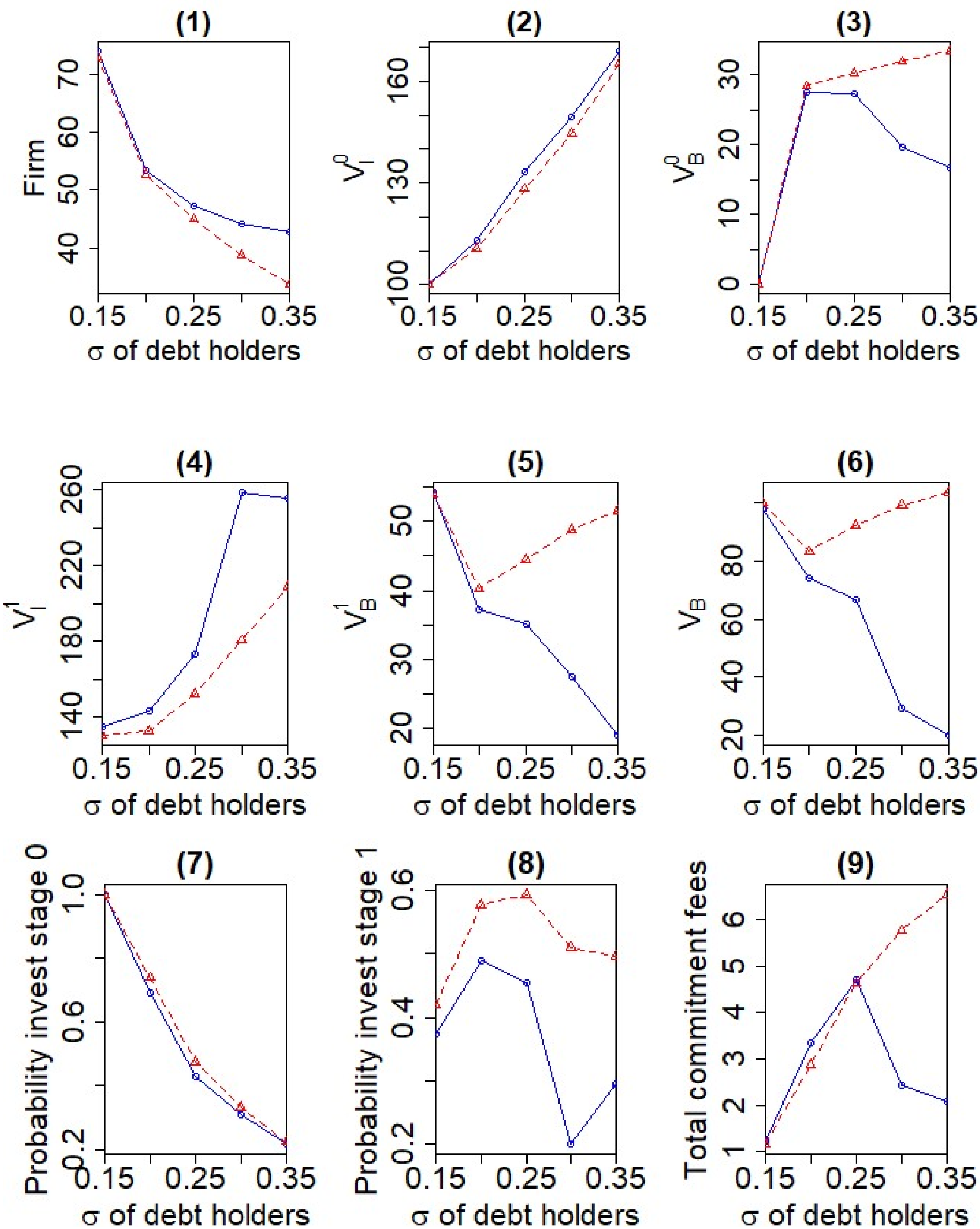

Base case used for all models: value of unlevered assets $V$ =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility $\sigma_E$ = 0.25, investment cost $X$ = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0. Loan commitment fees c = 0.1% (solid line) or c = 0.5% (dotted line). Sensitivity analysis is with respect to debt holders perceived estimate of volatility $\sigma_D$. The single-stage model of section 3.4. is used for the calculations. All variable definitions are provided also in Table 1. The figure results are summarized in Prediction 3 of the main text and are as follows. Higher loan commitment fees result in lower firm value, earlier investment and higher probability of investment unless debt holders beliefs are highly unfavorable (in which case there is a delay in investment), earlier default prior to investment and a delay in default following investment, lower levels of credit line (debt) and leverage ratios; the reduction is more significant the more unfavorable debt holders beliefs become, lower credit spreads; the reduction is more pronounced for more unfavorable debt holders beliefs, total expected cost of commitment fees which follow an inverse U-shape.

**Figure 3. Accelerated vs Sequential investment**



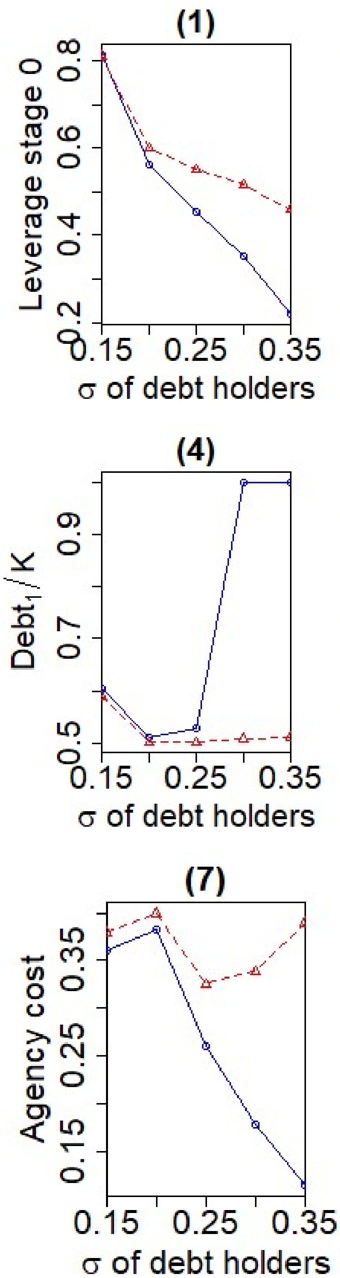

Base case used for sequential model: value of unlevered assets $V$ =100, risk-free rate $r$ = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E$ = 0.25, investment costs $X_0 = X_1$ =100, varying expansion factor $e_G$ , bankruptcy costs $b$ = 0.5 and tax rate $\tau$ = 0.35, equity financing costs $\varphi$ = 0, loan commitment fees $c$ = 0.5% . For the single stage model we use $X= X_0 + X_1$ = 200. Panel (a) shows firm values under a single-stage versus a sequential investment, panel (b) shows the investment thresholds for first stage (0) and second stage (1) for the sequential model versus the threshold for the single state (accelerated) model. The sequential model is described in section 3.3. and the single-stage model is described in section 3.4. The results are summarized in Prediction 4 of the main text and are as follows. For low enough follow-on investment option moneyness (low $e_G$) a sequential strategy is preferred over a single-stage (accelerated) investment. When follow-on investment becomes more attractive (increase in $e_G$) the firm implements both stages simultaneously leading to accelerated investment.

**Figure 4. The effect of external financing costs: sequential model**



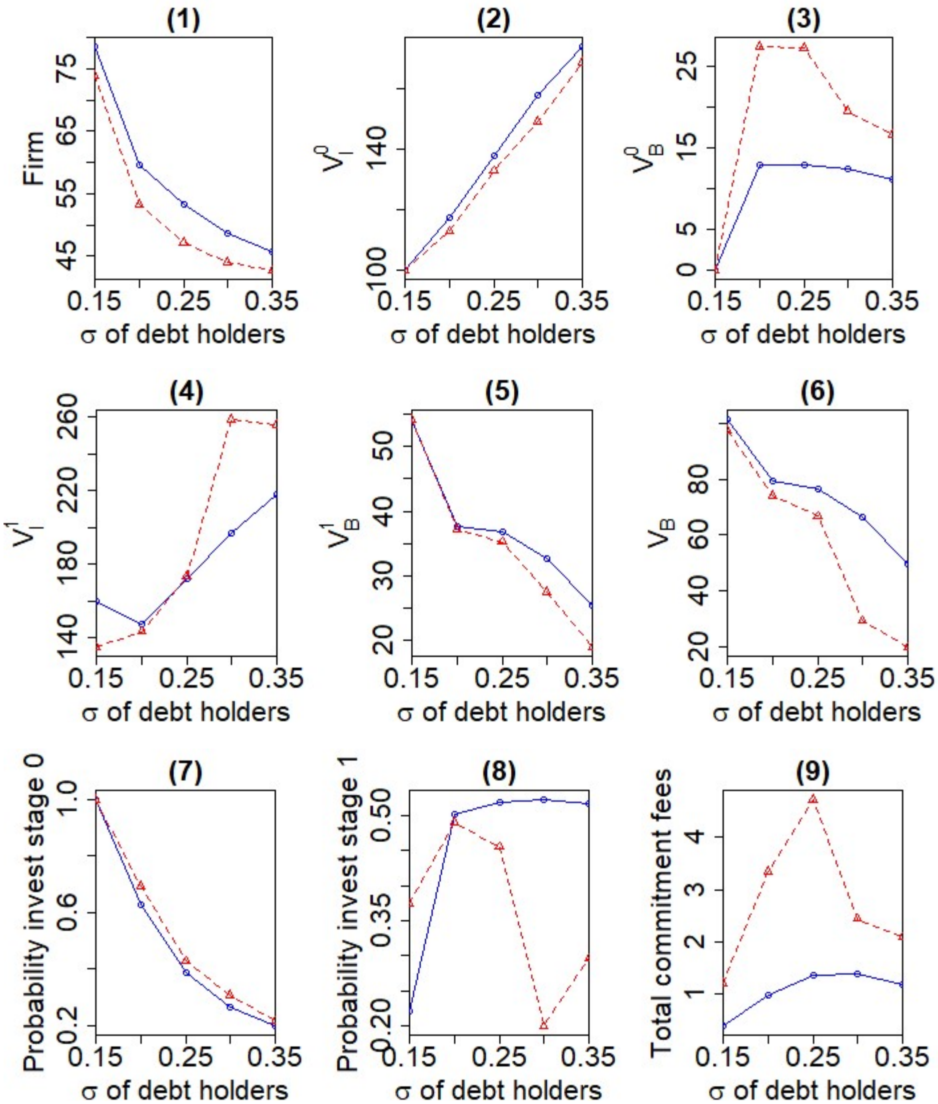

Base case parameters: unlevered assets $V$=100, risk-free rate $r$ = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E$= 0.25, investment costs $X_0 = X_I$ =100, $e_G$=1.8 , bankruptcy costs $b$ = 0.5 and tax rate $\tau$ = 0.35, loan commitment fees $c$ = 0.5% , equity financing costs φ = 0 (solid line), equity financing costs φ = 1 (dotted line). The sequential model is described in section 3.3. "Firm" denotes firm value derived in equation (23) using an optimal capital structure of varying initial investment stage credit line payments ($R_0$) and follow-on investment stage credit line payments ($R_1$) with increments of 0.1. $V_I^i$ $i = 0,1$ denote the investment trigger in investment stage $i$ and $V_B^i$, i=0,1 the bankruptcy triggers in operation phase 0 (before first investment) and 1 (after first investment and before second investment). $V_B$ denotes the bankruptcy trigger in the last operation phase. The probability of investment is calculated in equation (8) and total commitment fees in equation (24). Figure 4 & 5 results are summarized in Prediction 5 and 6 of the main text.

## Figure 5. The effect of external financing costs on debt financing and agency costs of debt: sequential model



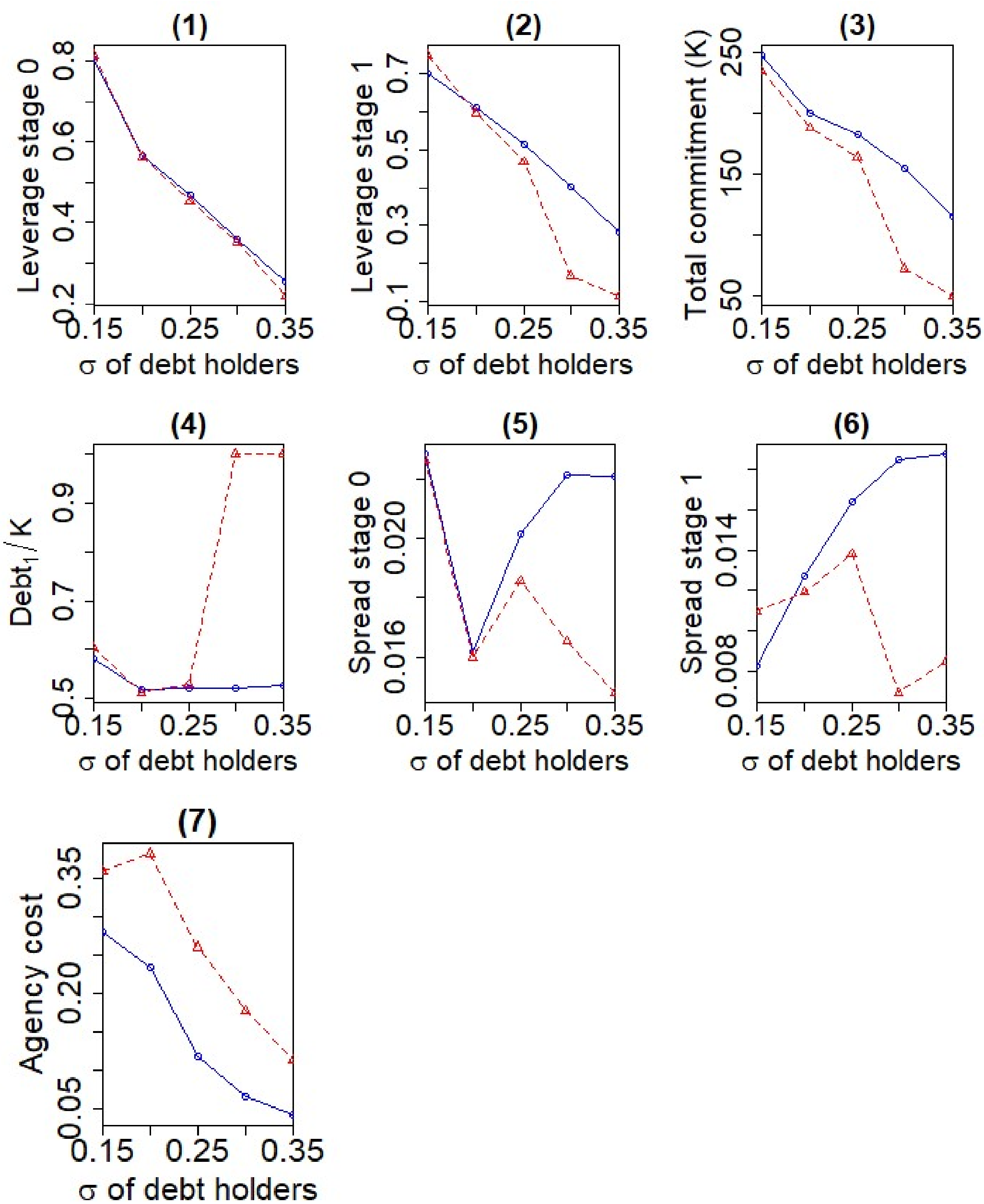

Base case parameters: unlevered assets $V$=100, risk-free rate $r$ = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E$ = 0.25, investment costs $X_0 = X_I$ =100, $e_G$=1.8 , bankruptcy costs $b$ = 0.5 and tax rate $\tau$ = 0.35, loan commitment fees $c$ = 0.5% , equity financing costs φ = 0 (solid line), equity financing costs φ = 1 (dotted line). The sequential model is described in section 3.3. Leverage (stage $i = 0$) denotes drawdown (debt) value $D_0^1(V_I^0)$ (see equation 21) divided by equity value $E^1(V_I^0)$ plus $D_0^1(V_I^0)$ and of stage 1 it is the sum of $D_0^2(V_I^1)$ and $D_1^2(V_I^1)$ (see equation 19) divided by equity $E^2(V_I^1)$ (equation (14) plus total debt ($D_0^2(V_I^1)$ +$D_1^2(V_I^1)$ ). Leverage ratios are shown for an optimal capital structure of varying initial investment stage credit line payments ($R_0$) and follow-on investment stage credit line payments $R_1$ with increments of 0.1. Total commitment $K$ is the sum $D_0^1(V_I^0)$ and $D_1^2(V_I^1)$. $\text{Debt}_1/K$ is the fraction of the total commitment corresponding to the first drawdown $D_0^1(V_I^0)$/K. Spread of stage 0 is $\frac{R_0}{D_0^1(V_I^0)} - r$ and spread of stage 1 is $\frac{R_1}{D_1^2(V_I^1)} - r$. Agency costs are calculated as in equation (27). Figure 4 & 5 results are summarized in Prediction 5 and 6 of the main text.

## Figure 6. The effect of loan commitment fees: sequential model




Base case used for sequential model (see section 4.2): value of unlevered assets $V$ =100, risk-free rate $r$ = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E$ = 0.25, investment costs $X_0 = X_I$ =100, $e_G$=1.8 , bankruptcy costs $b$ = 0.5 and tax rate $\tau$ = 0.35, equity financing costs φ = 0, loan commitment fees $c$ = 0.1% (solid line) and c = 0.5% (dotted line). "Firm" denotes firm value derived in equation (23) using an optimal capital structure of varying initial investment stage credit line payments ($R_0$) and follow-on investment stage credit line payments ($R_1$) with increments of 0.1. $V_I^i$ $i = 0,1$ denote the investment trigger in investment stage $i$ and $V_B^i$, i=0,1 the bankruptcy triggers in operation phase 0 (before first investment) and 1 (after first investment and before second investment). $V_B$ denotes the bankruptcy trigger in the last operation phase. The probability of investment is calculated in equation (8) and total commitment fees in equation (24). Figure 6 & 7 results are summarized in Prediction 7 of the main text.

**Figure 7. The effect of loan commitment fees on debt financing and agency costs of debt: sequential model**




Base case used for sequential model (see section 4.2): value of unlevered assets $V$ =100, risk-free rate $r$ = 0.06, opportunity cost $\delta$ = 0.06, volatility $\sigma_E$ = 0.25, investment costs $X_0 = X_I$ =100, $e_G$=1.8 , bankruptcy costs $b$ = 0.5 and tax rate $\tau$ = 0.35, equity financing costs φ = 0, loan commitment fees $c$ = 0.1% (solid line) and c = 0.5% (dotted line). Leverage (stage $i = 0$) denotes drawdown (debt) value $D_0^1(V_I^0)$ (see equation 21) divided by equity value $E^1(V_I^0)$ plus $D_0^1(V_I^0)$ and of stage 1 it is the sum of $D_0^2(V_I^1)$ and $D_1^2(V_I^1)$ (see equation 19) divided by equity $E^2(V_I^1)$ (equation (14) plus total debt ($D_0^2(V_I^1)$ +$D_1^2(V_I^1)$ ). Leverage ratios are shown for an optimal capital structure of varying initial investment stage credit line payments $(R_0)$ and follow-on investment stage credit line payments $R_1$ with increments of 0.1. Total commitment $K$ is the sum $D_0^1(V_I^0)$ and $D_1^2(V_I^1)$. Debt$_1$/$K$ is the fraction of the total commitment corresponding to the first drawdown $D_0^1(V_I^0)$/K. Spread of stage 0 is $\frac{R_0}{D_0^1(V_I^0)} - r$ and spread of stage 1 is $\frac{R_1}{D_1^2(V_I^1)} - r$. Agency costs are calculated as in equation (27). Figure 6 & 7 results are summarized in Prediction 7 of the main text.