The Impact of Optimism on the Order and Timing of Entry under Duopolistic Competition

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Abstract

We incorporate optimism about future growth prospects into a real-options duopolistic setting and show that an optimistic firm will enter earlier than an otherwise identical non-optimistic competitor but at a higher threshold than if neither firm were optimistic. More generally, optimism can change entry order: a more optimistic firm may enter first even if its competitor has higher monopoly and duopoly revenues and lower costs (as long as the difference is not too great). Optimism can thus compensate for competitive advantage in costs and revenues in determining entry order. Furthermore, in contrast to the impact of optimism in non-competitive settings where investment thresholds decrease with optimism, an optimistic firm's entry thresholds as a leader in duopolistic competition when it needs to enter pre-emptively *increase* with the firm's optimism.

Keywords: Optimism, entry order, duopolistic competition, real options,

JEL Classification Numbers: G31, G32, G40

1 Introduction

Empirical studies have consistently found that entrepreneurs are more optimistic than the general population (Puri and Robinson (2013b), Puri and Robinson (2013a), Cooper et al. (1988), Landier and Thesmar (2009))and that they were so even before they became self-employed (Dawson and Henley (2012)), consistent with the idea that optimism is an inherent trait. Existing studies of the impact of decision-makers' inherent optimism on investment decisions such as Hackbarth (2009) show optimism decreases investment thresholds, bringing forward investment, because of the greater perceived present value of future cash flows arising from the project. In this paper we use real options analysis to investigate the impact of entrepreneurial inherent optimism on the entry order and investment thresholds of firms in a duopoly setting.

We start by considering otherwise identical firms (with the same entry costs and revenues after entry, both as temporary monopolists and as duopolists in the long run) and show that if one firm is run by an optimistic entrepreneur and the other by a non-optimisitc (realistic) entrepreneur, then the optimist will enter first and thus enjoy the temporarily higher monopolistic profits. Moreover, if he enters in order to pre-empt the realist (which will be the case if the entrepreneur's optimism level is not too great), the optimist will enter at a higher threshold than an equivalent firm run by a non-optimistic entrepreneur, in stark contrast to the results in a non-competitive setting.

We then generalise to consider competition between two asymmetric firms which may have different levels of profitability, both as monopolist and duopolist, and different investment costs for entering the market initially. Here we show that an entrepreneur's optimism can move him earlier in the entry order. As shown by Kong and Kwok (2007), in the absence of optimism, which firm enters first depends on the firms' relative advantages in both monopoly and duopoly settings. Considering two firms, A and B, if one firm (B) dominates in both duopoly and monopoly market entry, this firm will enter first; however a sufficiently strong advantage as a duopolist can outweigh a disadvantage as a monopolist, and vice versa.

We show that optimism can change the entry order when the relative competitive strengths of the two firms are similar (*i.e.* the thresholds at which each firm is willing to pre-empt their competitor in order to enter the market as leader are close). If the weaker firm (A) is run by an optimistic entrepreneur, this optimism can compensate for the firm's competitive disadvantage in costs and revenues. The firm run by a realist (B) can be stronger as both a monopolist and in duopoly and yet will still not enter the market first.

The reason for this is that Firm A's optimism discourages pre-emption by its realistic rival. Firm A's optimism inflates its estimates of future value; this is why it enters at a lower threshold in the absence of competitive pressure (Hackbarth (2009)). In a duopoly setting, this result remains true when the optimist is the follower: the more optimistic an entrepreneur is about future revenue growth, the earlier it enters as second mover. However, this earlier entry by the optimistic entrepreneur (A) reduces the time over which a realistic first entrant would enjoy higher monopoly profits and thus decreases the realistic competitor (B)'s value of entering the market as leader. The threshold at which the realist B will be willing to pre-empt and enter as first mover thus *increases* with Firm A's optimism (Firm A's optimism does not affect its competitor's value as a follower). On the other hand, a greater level of optimism reduces the optimist's pre-emption threshold. Thus even if the realistic firm has a competitive advantage in costs and revenues (so if A and B were both realistic then B's pre-emptive threshold would be lower than A's pre-emptive threshold), since A's optimism decreases A's and increases B's thresholds, if B's original competitive advantage was not too great, optimism can reverse the exercise order: an optimistic A may be willing and able to pre-empt B and enter the market first. Moreover, as long as B's pre-emption threat is sufficiently strong,[1] the optimistic A will only enter the market once demand exceeds B's pre-emption threshold. In this case, increases in A's optimism delay A's pre-emptive entry, since B's pre-emption threshold increases with A's optimism.

Our work extends the real options literature. A number of papers (e.g. Dixit and

[1] If B's pre-emption threat is sufficiently small that A can enter at its perceived optimal leader threshold, then further increases in optimism will decrease A's entry threshold as leader. This is discussed in greater detail in Section 2.

Pindyck (1994)) have investigated strategic interactions between firms in duopoly allowing for asymmmetry in investment costs (Pawlina and Kort (2006)) and most recently also asymmetry in profitability as both monopolist and duopolist (Kong and Kwok (2007)). Kong and Kwok (2007) find the entry order is affected by firms' relative advantages in both monopoly and duopoly. If one firm dominates both as first entrant and in duopoly, they will enter the market first. More generally there is a trade-off: relative strength as first entrant can offset weakness in the duopoly market to 'win' the pre-emption game and enter first. We introduce another type of potential asymmetry between rival entrepreneurial firms: differences in the entrepreneurs' perceived growth rates, or optimism levels, and show optimism acts as a source of pre-emptive advantage which can offset competitive weaknesses to secure entry as leader, particularly for firms which have a relative advantage in duopoly (but not as monopolist) but where product market competition reduces overall profits significantly in duopoly, and when volatility is low.

Whilst the assertion of managerial overconfidence and/or optimism has been investigated empirically in the Finance literature (see e.g. Malmendier and Tate (2005)), there are nevertheless relatively few papers which consider the implications of these biases on investment decisions under uncertainty. Hackbarth (2009) shows optimism decreases exercise thresholds for investment options and goes on to consider impacts on firm leverage (see also Hackbarth (2008), Kamoto (2014)). More recently Smit and Matawlie (2017) model the effects of optimism about growth rates on the choice between direct acquisition and via a toehold strategy. They argue that optimistic bidders are more likely to pursue direct acquisitions rather than toehold acquisition strategies and find supporting empirical evidence. Their rationale builds on Hackbarth (2009)'s result that investment thresholds decrease as a decision-maker's optimism increases. However, none of these papers consider the impact of optimism on strategic incentives in a competitive setting, where we find the opposite comparative static result can hold.

A number of studies have suggested reasons why inherent or dispositional optimism or overconfidence may provide advantages in competitive settings. Goel and Thakor (2008) argue that overconfidence can increase the likelyhood of promotion to CEO. Kyle and Wang

(1997) show that overconfidence induces traders to trade more agressively (or equivalently have higher capacity in a Cournot duopoly model), which causes realistic rivals to trade less (select lower quantities) (See also Odean (1998)) In contrast, our model does not rely on differences in the choice of capacity or size by optimists. Instead our results show that, even in the absence of such capacity impacts, differences in optimists' timing of entry can discourage pre-emption by rivals. Adding in these features would only strengthen our results, and we leave this for future work.

Key to our results is the premise that optimism is a trait inherent to an individual which thus cannot be changed, rather than a choice variable. Sharot and Dolan (1998) suggests a mechanistic explanation for the persistence of unrealistic optimism. They found evidence supporting the existence of systematic selective updating of beliefs whereby negative information was less likely to be incorporated, leading to overoptimistic expectations relating to future events. Furthermore, the magnitude of the difference in coding of positive and negative updates as measured by activity in different parts of the brain was larger for individuals exhibiting greater optimism. Dispositional optimism in young adults has also been found to be predicted by heredity (explaining 25% (Plomin and McClearn (1992))), early childhood reported temperament (Heinonen and Keltikangas-Jarvinen (2005)) and family socio-economic status (Heinonen and Keltikangas-Jarvinen (2006)).

The inherent nature of optimism is also supported by de Meza and Arabsheibani (2019) who found financial optimism to be positively correlated with both divorce and smoking, suggesting the existence of a psychological trait. Furthermore Kaniel and Robinson (2010) and Matthews and Kuller (2004) amongst others have found persistence, i.e. high correlation of measured optimism levels over time. Dawson and Henley (2012) found individuals who become self-employed were more optimistic before switching, consistent with an optimistic disposition pre-dating the decision to become self-employed, although being self-employed also increased measured optimism. The correlation between optimism and self-employment is well documented (Puri and Robinson (2007, 2013), Cooper et al. (1988), Landier and Thesmar (2009)) and extends to events outside the entrepreneur's locus of control. Koudstaal and Sloof (2015) find entrepreneurs are more optimistic (have both greater "dispositional optimism" and "more optimistic attributional style when bad events occur") than managers. (Bengtsson and Ekeblom (2014)) find Swedish entrepreneurs have more optimistic (but also more accurate) beliefs than the general population about future economic conditions. We assume optimistic entrepreneurs have higher expectations about future growth rates of demand for their firm's products.

Details of the model and solution are found in Section 2, Section 3 presents the results and Section 4 concludes.

2 Model

2.1 General Setting

Consider two entrepreneurs A and B who have the same investment opportunity to set up a new project and will compete in the same market. The subscripts for all the variables in this paper denote entrepreneurs. Once the entrepreneur pays the lump-sum cost I_i with $i \in \{A, B\}$, he will receive instantaneous revenue cash flows $\pi(X_t) = DX_t$ each period indefinitely. Here D is deterministic and can take one of four different values $D \in \{D_A^m, D_B^m, D_A^d, D_B^d\}$. The superscript 'm' denotes monopolistic profit and 'd' denotes duopolistic profit. The entrepreneur who enters first is the leader and enjoys monopoly profits until the other entrepreneur (i.e. follower) enters. After both entrepreneurs have entered the market, each receives a lower duopolistic profit which may differ between the entrepreneurs. Market demand is represented by X_t which follows the stochastic process:

$$dX_t = \nu X_t dt + \eta X_t dB_t \tag{1}$$

where ν and η are constants corresponding to drift rate and volatility respectively. The two entrepreneurs may have different subjective beliefs about the future growth rate of market demand, ν, and thus denote each entrepreneur's subjective belief by ν_A and ν_B. The opportunity cost of forgone cash flows δ, defined by $\delta = r - \nu$ may thus also differ between the entrepreneurs ($\delta_A \neq \delta_B$). If the drift rate and opportunity cost of foregone cash flows under non-optimistic beliefs are ν_0, $\delta_0 = r - \nu_0$ respectively and the entrepreneur's optimism level is denoted by α, entrepreneur A's belief in the growth rate is $\nu_A = \nu_0 + \alpha_A$ and thus the corresponding opportunity cost of foregone cash flows $\delta_A = \delta_0 - \alpha_A$.[2]

Both entrepreneurs choose their leader and follower thresholds optimally, i.e. to maximise their expected perceived value given their competitors' best responses. We solve the resulting strategic game backwards in time, by firstly finding each entrepreneur's follower threshold (which takes account of his own subjective beliefs, i.e. optimism), secondly finding each entrepreneur's value of entering as leader, and hence the pre-emptive threshold above which he is willing to pre-empt his competitor in order to enter the market as first mover. Note each entrepreneur's pre-emptive threshold depends not only on their own optimism, but also on the subjective beliefs of their competitor, since the latter affects the threshold at which they lose their monopoly advantage. We also find each entrepreneur's optimal leader's threshold in the absence of competitive pre-emption pressure. Finally we compare both entrepreneur's pre-emption and optimal leader thresholds to determine which firm enters first (the one with the lower pre-emptive threshold) and at what level of market demand, which depends on the relationship between the leader's optimal threshold and their rival's pre-emptive threshold.

2.2 Follower's value function and investment threshold

First suppose that one entrepreneur (leader) has already invested. The other entrepreneur (follower) only has to consider the optimal time to enter the market as part of a duopoly. Since the follower is last to invest, the decision of the follower is equivalent to a stand-alone investment option problem.

We denote the follower value $V_A^F(x)$ as entrepreneur A's perceived value and apply the standard dynamic programming method in continuous time to obtain the Hamilton–Jacobi–Bellman

[2]Note that δ_A represents an opportunity cost of delaying the investment so that $\delta_A > 0$ (see Dixit and Pindyck [1994, p.149]). This puts a natural upper bound on the optimism levels considered in this paper, i.e. $\alpha_A < \delta_0$ and $\alpha_B < \delta_0$.

(HJB) equation which is given by

$$rV_A^F dt = \max_t \left\{ D_A^d x dt + \mathbb{E}(dV_A^F) \right\} \tag{1}$$

Applying Itô's lemma, we have

$$rV_A^F(x) = D_A^d x + (r + \alpha_A - \delta_0) x \frac{\partial V_A^F(x)}{\partial x} + \frac{1}{2}\eta^2 x^2 \frac{\partial^2 V_A^F(x)}{\partial x^2} \tag{2}$$

with boundary conditions

$$V_A^F(0) = 0 \tag{3}$$

$$V_A^F(\bar{x}_A^F) = \frac{D_A^d \bar{x}_A^F}{\delta_0 - \alpha_A} - I_A \tag{4}$$

$$\left.\frac{\partial V_A^F(x)}{\partial x}\right|_{x=\bar{x}_A^F} = \frac{D_A^d}{\delta_0 - \alpha_A} \tag{5}$$

The first two conditions are value-matching conditions and the last one is the smooth-pasting condition. Solving (3)) - (5) yields the follower's value function

$$V_A^F(x) = \begin{cases} \left(\dfrac{D_A^d \bar{x}_A^F}{\delta_0 - \alpha_A} - I_A \right) \left(\dfrac{x}{\bar{x}_A^F} \right)^{\beta_A} & , x < \bar{x}_A^F \\ \\ \dfrac{D_A^d x}{\delta_0 - \alpha_A} - I_A & , x \geq \bar{x}_A^F \end{cases} \tag{6}$$

and optimal follower's threshold $\bar{x}_A^F$ which is given by

$$\bar{x}_A^F = \frac{\beta_A}{\beta_A - 1} \frac{(\delta_0 - \alpha_A) I_A}{D_A^d} \tag{7}$$

where β_A equals

$$\beta_A = \frac{1}{2} - \frac{r + \alpha_A - \delta_0}{\eta^2} + \sqrt{\left(\frac{r + \alpha_A - \delta_0}{\eta^2} - \frac{1}{2} \right)^2 + \frac{2r}{\eta^2}} > 1 \tag{8}$$

We can follow the same steps to obtain the follower's value function of entrepreneur B and also his optimal follower threshold. Note each entrepreneur's value functions before and after investment as follower depend on their own subjective beliefs but are independent of their rival's level of optimism.

Proposition 1: Optimism accelerates the investment as a follower, i.e. the optimal follower's threshold $\bar{x}_A^F$ is a strictly decreasing function of the optimism level α_A.

Proof. See Appendix 1.

Proposition 1 states that an optimistic entrepreneur tends to enter the market earlier than an equivalent rational one (i.e. with identical monopoly and duopoly profitabilities and identical costs). Furthermore, the more optimistic the entrepreneur is, the earlier their entry as second mover. This arises because the optimistic entrepreneur overestimates the growth trend (drift rate) of future profits. A higher expected growth rate induces entrepreneurs to exercise investment options earlier whilst higher volatility causes them to delay: the optimal threshold is the result of a trade-off between them. Optimism plays a role in "increasing" the optimistic entrepreneur's anticipated growth rate, thereby accelerating investment for the follower problem which is actually same as a stand-alone investment option.

2.3 Leader's value function and investment threshold

We now consider each entrepreneur's optimisation problem for entering as a leader, taking account of their rival's behaviour as follower (i.e. his competitor's optimal follower threshold). Intuitively, the leader's value is defined as the value to the entrepreneur if he was the leader/monopolist already, taking account of the follower's later entry, after which his profits will fall to the duopoly level. We can also use Bellman's equation to derive the leader's value function, e.g. for entrepreneur A

$$rV_A^L dt = \max_t \left\{ D_A^m x dt + \mathbb{E}(dV_A^L) \right\} \tag{1}$$

Applying Itô's lemma, then we have

$$rV_A^L(x) = D_A^m x + (r + \alpha_A - \delta_0) x \frac{\partial V_A^L(x)}{\partial x} + \frac{1}{2}\eta^2 x^2 \frac{\partial^2 V_A^L(x)}{\partial x^2} \tag{2}$$

with boundary conditions

$$V_A^L(0) = -I_A \tag{3}$$

$$V_A^L(\bar{x}_B^F) = \frac{D_A^d \bar{x}_B^F}{\delta_0 - \alpha_A} - I_A \tag{4}$$

Hence, the leader's value evaluated at the moment of investing equals

$$V_A^L(x) = \begin{cases} \dfrac{D_A^d - D_A^m}{\delta_0 - \alpha_A}\bar{x}_B^F\left(\dfrac{x}{\bar{x}_B^F}\right)^{\beta_A} + \dfrac{D_A^m}{\delta_0 - \alpha_A}x - I_A & , x < \bar{x}_B^F \\ \\ \dfrac{D_A^d x}{\delta_0 - \alpha_A} - I_A & , x \geq \bar{x}_B^F \end{cases} \tag{5}$$

Note that $V_A^L(x)$ is only the leader's value function after the leader enters the market. The leader's value function before entry, $V_A^{LB}(x)$, satisfies the following equation,

$$rV_A^{LB}dt = \max_t \left\{0 \cdot dt + \mathbb{E}(dV_A^{LB})\right\} \tag{6}$$

or equivalently

$$rV_A^{LB}(x) = (r + \alpha_A - \delta_0)x\frac{\partial V_A^{LB}(x)}{\partial x} + \frac{1}{2}\eta^2 x^2 \frac{\partial^2 V_A^{LB}(x)}{\partial x^2} \tag{7}$$

with boundary conditions

$$V_A^{LB}(0) = 0 \tag{8}$$

$$V_A^{LB}(\bar{x}_A^E) = V_A^L(\bar{x}_A^E) \tag{9}$$

where $\bar{x}_A^E$ is the actual entry point of entrepreneur A as the leader. Solving the above equations gives

$$V_A^{LB}(x) = \frac{D_A^d - D_A^m}{\delta_0 - \alpha_A}\bar{x}_B^F\left(\frac{x}{\bar{x}_B^F}\right)^{\beta_A} + \left(\frac{D_A^m}{\delta_0 - \alpha_A}\bar{x}_A^E - I_A\right)\left(\frac{x}{\bar{x}_A^E}\right)^{\beta_A} \tag{10}$$

If entrepreneur A can enter at his optimal leader threshold $\bar{x}_A^L$, i.e. $\bar{x}_A^E = \bar{x}_A^L$, which satisfies the smooth-pasting condition

$$\left.\frac{\partial V_A^{LB}(x)}{\partial x}\right|_{x=\bar{x}_A^L} = \left.\frac{\partial V_A^L(x)}{\partial x}\right|_{x=\bar{x}_A^L} \tag{11}$$

after taking derivatives of equation (46) and (52) and substituting $x = \bar{x}_A^L$, we have

$$\frac{D_A^d - D_A^m}{\delta_0 - \alpha_A}\beta_A\left(\frac{\bar{x}_A^L}{\bar{x}_B^F}\right)^{\beta_A - 1} + \left(\frac{D_A^m}{\delta_0 - \alpha_A}\bar{x}_A^L - I_A\right)\frac{\beta_A}{\bar{x}_A^L} = \frac{D_A^d - D_A^m}{\delta_0 - \alpha_A}\beta_A\left(\frac{\bar{x}_A^L}{\bar{x}_B^F}\right)^{\beta_A - 1} + \frac{D_A^m}{\delta_0 - \alpha_A} \tag{12}$$

Thus the optimal leader's threshold $\bar{x}_A^L$ is given by

$$\bar{x}_A^L = \frac{\beta_A}{\beta_A - 1}\frac{(\delta_0 - \alpha_A)I_A}{D_A^m} \tag{13}$$

We can see that the optimal leader's threshold $\bar{x}_A^L$ has the same form as the optimal investment threshold of the pure monopoly case where there is no competition and the deterministic part of payoff in period is D_A^m. However, the value function of pure monopoly case is different from $V_A^{LB}(x)$ here even though they have the same optimal threshold.

As mentioned before, the optimal leader's threshold $\bar{x}_A^L$ is not always achievable because of competition. Both entrepreneurs are willing to give up some option value as long as they can capture an additional monopolistic profit by pre-empting their rivals. An entrepreneur will have an incentive to pre-empt as soon as their leader's value exceeds follower's value. At $x = 0$, we know that the leader's value (after entry) is negative since the payoff is too low to cover the sunk cost whereas the option value of being the follower is zero . As x increases, the leader's value is growing faster than the follower's value because we assume the certain part of monopolistic profit is always greater than that of duopolistic profit. So we define each entrepreneur's pre-emptive threshold (e.g. $\bar{x}_A^P$ for A's pre-emptive threshold) as the lowest value of x for which an entrepreneur's follower's value equals their leader's value as the *pre-emptive threshold*. This solves

$$\bar{x}_A^P = \inf\{x : V_A^L(x) = V_A^F(x)\} \tag{14}$$

or

$$\bar{x}_A^P = \inf\left\{x : \frac{D_A^d - D_A^m}{\delta_0 - \alpha_A}\bar{x}_B^F\left(\frac{x}{\bar{x}_B^F}\right)^{\beta_A} + \frac{D_A^m}{\delta_0 - \alpha_A}x - I_A = \left(\frac{D_A^d \bar{x}_A^F}{\delta_0 - \alpha_A} - I_A\right)\left(\frac{x}{\bar{x}_A^F}\right)^{\beta_A}\right\} \tag{15}$$

Entrepreneur B's pre-emptive threshold $\bar{x}_B^P$ can be calculated similarly. If $\bar{x}_A^P$ and $\bar{x}_B^P$ both exist, then the entrepreneur with the lower pre-emptive threshold will be the leader. For example, if $\bar{x}_A^P < \bar{x}_B^P$, entrepreneur A will be first to enter the market at $\bar{x}_A^E = \min\{\bar{x}_A^L, \bar{x}_B^P\}$. If$\bar{x}_A^E = \bar{x}_B^P$, this signifies *pre-emptive investment*.

In this case, A invests just before their competitor's pre-emption threshold in order to obtain the leader value. Differentiating $V_A^{LB}(x)$ with respect to the entry threshold $\bar{x}_A^E$:

$$\begin{aligned}
\frac{\partial V_A^{LB}(x)}{\partial \bar{x}_A^E} &= -\beta_A\left(\frac{D^m}{\delta_0 - \alpha_A} - \frac{I_A}{\bar{x}_A^E}\right)\left(\frac{x}{\bar{x}_A^E}\right)^{\beta_A} + \frac{D^m}{\delta_0 - \alpha_A}\left(\frac{x}{\bar{x}_A^E}\right)^{\beta_A} \\
&= \left((1 - \beta_A)\frac{D^m}{\delta_0 - \alpha_A} + \beta_A\frac{I_A}{\bar{x}_A^E}\right)\left(\frac{x}{\bar{x}_A^E}\right)^{\beta_A} > 0 \text{ if } \bar{x}_A^E < \bar{x}_A^L
\end{aligned}$$

shows that as long as $\bar{x}_A^E < \bar{x}_A^L$, the leader's value function increases with $\bar{x}_A^E$, i.e. the leader prefers to enter as late as possible in order to capture as much of the option value of waiting as possible. However, if they delay investment so $x > \bar{x}_B^P$, B will pre-empt, so $\bar{x}_A^E \leq \bar{x}_B^P$ is a binding constraint for A to enter the market first.

We have

Proposition 2: entrepreneur B's pre-emptive threshold $\bar{x}_B^P$ increases with entrepreneur A's optimism level α_A if $\bar{x}_B^P$ exists.

Proof. See Appendix 1.

Proposition 2 implies that, as long as the (more) optimistic entrepreneur enters in order to pre-empt his realistic (less optimistic) rival (i.e. at B's pre-emptive threshold), then the threshold at which the optimistic entrepreneur enters as leader increases as his level of optimism increases. This is in contrast to the standard results (e.g. Hackbarth (2009)) that investment thresholds decrease as optimism increases. The reason is that A's threshold is determined by his rival ($\bar{x}_A^E = \bar{x}_B^P$) and increased optimism decreases A's follower's threshold which reduces B's leader value. This in turn increases B's pre-emptive threshold and hence A's entry threshold as leader as long as B's pre-emptive threat is real (i.e. $\bar{x}_B^P < \bar{x}_A^L$).

Alternatively, if if $\bar{x}_A^L < \bar{x}_B^P$, or if only one pre-emptive threshold exists (e.g. $\bar{x}_A^P$), entrepreneur B will never have an incentive to pre-empt A, and so entrepreneur A becomes the dominant leader. This leads to *sequential investment*. In this case, the first entrant, e.g. A, can enter at his own optimal leader threshold (i.e. $\bar{x}_A^E = \bar{x}_A^L$).

3 Results

Our goal is to illustrate the effect of entrepreneurial optimism in a general setting for two firms with asymmetric costs of entry, monopolistic and duopolistic profitability, and run by entrepreneurs with different levels of optimism. We start by considering two special cases:

first recapping the case of duopoly where the firms have asymmetric costs and revenues but entrepreneurs are realistic (not optimistic). Then we consider the case where the two firms have identical costs and revenues but the entrepreneurs have different levels of optimism. Finally we generalise to incorporate asymmetry in both cash flows and subjective beliefs (optimism levels).

3.1 Asymmetric costs and revenues with symmetric entrepreneurial beliefs

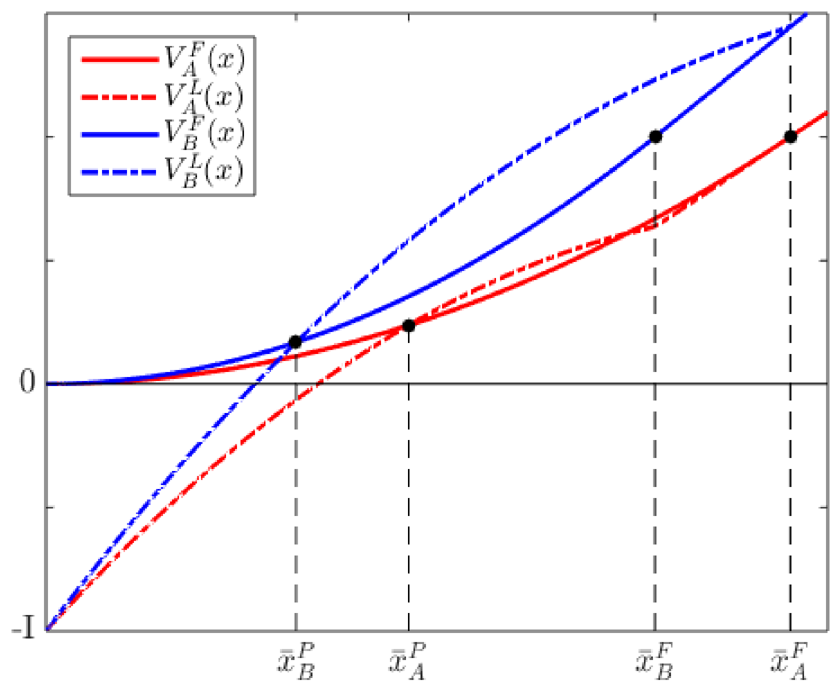


Figure 1: Leader's Value and Follower's Value for Two Asymmetric entrepreneurs. Parameter values: $r = 0.05$, $\nu = 0.02$, $\eta = 0.1$, $D_A^m = 1.5$, $D_A^d = 0.9$, $D_B^m = 1.8$, $D_B^d = 1.1$ and $I_A = I_B = 100$.

If the two entrepreneurs have different deterministic profitabilities and costs, this leads to different pre-emptive thresholds, as in Kong and Kwok (2007). The entrepreneur with the lower pre-emptive threshold will be the leader but will enter at the point just before his competitor's pre-emptive threshold. In addition, one of the pre-emptive thresholds may not exist, in which case only one entrepreneur has an incentive to be the leader (*sequential investment*). The entrepreneur who has a pre-emptive threshold will become the leader and can enter at his own optimal leader threshold without being concerned about his rival's pre-emption. Before moving on to our main results, we first explain how two pre-emptive thresholds would change or even disappear because of asymmetry.

We can interpret the pre-emptive threshold as the point above which the entrepreneur begins to have an incentive to be the leader. When x is below the pre-emptive threshold, it is not worthwhile to enter the market, which means the value of waiting outweighs the possible monopolistic profit gained by being the leader (i.e. follower's value exceeds leader's value). A relatively higher monopolistic profitability (i.e. higher D^m) directly motivates the entrepreneur to be the leader. Given the nature of the strategic game, one entrepreneur's pre-emptive threshold is also affected by the time until his rival will enter, when profits will decrease (which occurs at his rival's optimal follower threshold). How much an entrepreneur can earn as a monopolist in each time period is determined by his own firm characteristics, however it is the competitor who determines how long an entrepreneur remains as monopolist.

To better understand the underlying mechanism, we give a particular example of asymmetric entrepreneurs as shown in Figure 2. Without loss of generality, we assume the same costs ($I_A = I_B$) but different D^m and D^d for the two entrepreneurs. Entrepreneur B has a higher monopolistic profit ($D^m_B > D^m_A$), which directly increases B's leader value and thus decreases $\bar{x}^P_B$. On the other hand, although the fact that B's duopolistic profit (D^d_B) is also higher than his rival's (D^d_A) increases B's follower value, it also significantly weakens his rival's incentive to pre-empt and decreases his rival's leader value because $\bar{x}^F_B$ is relatively smaller. If we further increase D^d_B and thus decrease $\bar{x}^F_B$, entrepreneur A's pre-emptive threshold is likely to disappear when his leader's value could never exceed his follower's value. Then entrepreneur B will become the dominant leader who is able to retain all of his option value as the leader.

In conclusion, relatively higher monopoly and duopoly profits (D^m and D^d) both contribute to enhancing the possibility of being leader, but whereas D^m strengthens the entrepreneur's incentive to pre-empt directly, D^d's primary impact is by weakening his rival's incentive to pre-empt. We will see that the impact of optimism will work through similar mechanisms, since optimism increases the perceived values of cash flows in both monopoly and duopoly.

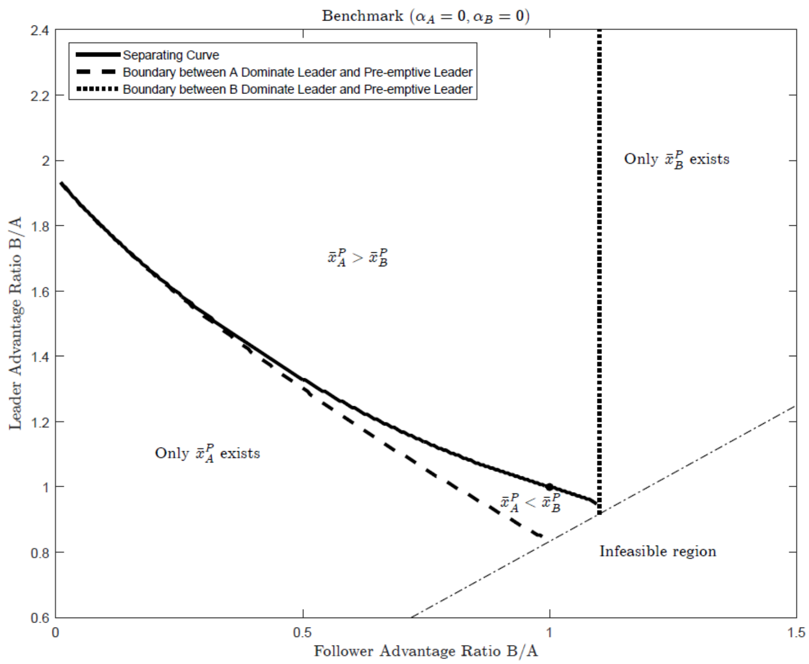
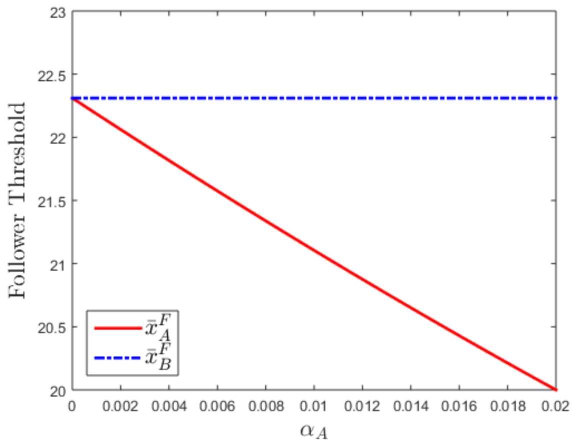


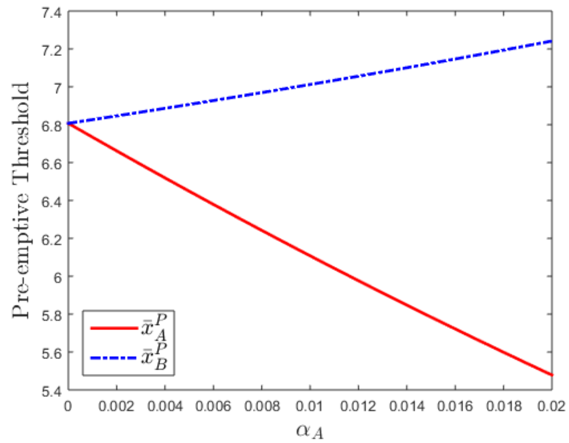
Figure 2: The Relative Magnitude of the Pre-emptive Thresholds $\bar{x}_A^P$ and $\bar{x}_B^P$. "Follower Advantage Ratio B/A" (FAR) equals $\frac{I_A D_B^d}{I_B D_A^d}$ and "Leader Advantage Ratio B/A" (LAR) equals $\frac{I_A D_B^m}{I_B D_A^m}$. Parameter values: $r = 0.05$, $\nu = 0.02$, $\eta = 0.3$ and $D_A^m / D_A^d = 1.2$.

In Figure 1, we only show one possible combination of asymmetric payoffs and costs. As for more asymmetric cases, we first define two important terms FAR and LAR as B's relative follower advantage to A (i.e. $\frac{I_A D_B^d}{I_B D_A^d}$) and B's relative leader advantage to A (i.e. $\frac{I_A D_B^m}{I_B D_A^m}$) respectively. Then we consider using FAR-LAR plane to identify different regions, namely $\{A$ is dominant leader$\}$, $\{A$ is pre-emptive leader$\}$, $\{B$ is pre-emptive leader$\}$ and $\{B$ is dominant leader$\}$, which correspond to $\{$Only $\bar{x}_A^P$ exists$\}$, $\{\bar{x}_A^P < \bar{x}_B^P\}$, $\{\bar{x}_B^P < \bar{x}_A^P\}$ and $\{$Only $\bar{x}_B^P$ exists$\}$ respectively.

Figure 2 shows the location of these regions for a fixed value of D_A^m / D_A^d (i.e. fixed first mover advantage for A), as in Kong and Kwok (2007). B becomes leader when it has a competitive advantage as a monopolist (high LAR) and as a duopolist (high FAR) whereas



(a) Effect on Follower Thresholds



(b) Effect on Pre-emptive Thresholds

Figure 3: The Effect of A's Optimism on Two Identical entrepreneurs' Thresholds. Parameter values: $r = 0.05$, $\delta_0 = 0.03$ and $\eta = 0.3$,$\alpha_B = 0$, $D_A^m = D_B^m = 2$ and $D_A^d = D_B^d = 1$.

A becomes leader when B's relative advantage as a monopolist (LAR) or as a duopolist (FAR) is low. The separating curve (bold solid curve) represents the situation when two entrepreneurs have the same pre-emptive thresholds even though their costs and payoffs are different. Intuitively, the separating curve is downward-sloping since one entrepreneur cannot dominate the other in terms of both follower's advantage and leader's advantage if they both end up with identical pre-emptive thresholds: as FAR decreases, LAR has to increase to compensate. An increase in B's competitive advantage over A as a leader (increase in LAR) decreases B's pre-emptive threshold and hence increases the range of parameter values, including FAR, for which B becomes leader.

3.2 Symmetric costs and revenues with asymmetric entrepreneurial beliefs

We now consider two entrepreneurs running identical firms, i.e. net income in monopoly and duopoly are identical as are the initial entry costs. However, entrepreneur A is optimistic whereas entrepreneur B is realistic. Figure 3 shows how their follower thresholds and more importantly pre-emptive thresholds change as A's optimism increases. In terms of the optimal follower threshold, an entrepreneur's optimism can only affect his own threshold, so we observe that $\bar{x}_B^F$ remains the same no matter how optimistic A is. However, A's follower threshold $\bar{x}_A^F$ decreases as we increase entrepreneur A's optimism level, consistent with Proposition 1.

In Figure 3(b), we note that two pre-emptive thresholds diverge correspondingly. To be specific, the pre-emptive threshold for entrepreneur B increases with A's optimism while the pre-emptive threshold for entrepreneur A follows a downward trend. Since the entrepreneur who has the smaller pre-emptive threshold will be the leader, this shows that optimism increases the likelihood of being the leader. In this case of symmetric firms, if both firms are run by "realistic" entrepreneurs, their pre-emptive thresholds would co-incide. Using a mixed strategy as in Thijssen and Kort (2012) would result in equal probabilities of either firm becoming the leader at the pre-emptive threshold. A's optimism breaks this symmetry: the more optimistic entrepreneur now enters the market first with probability 1.

Figure 3 also shows that the threshold at which the more optimistic entrepreneur will enter the market is higher than if he were rational even though his own pre-emptive threshold becomes smaller. This is because the winner of the pre-emption game will wait until just before his rival's pre-emptive threshold (as long as this is below his optimal leader's threshold). In the absence of competition, entrepreneurs would like to wait until the optimal leader's threshold before investing as this maximises the option value. The threat of pre-emption by his competitor limits how long the eventual leader can wait before investing. However the optimistic entrepreneur can wait longer than his realistic counterpart because his rival's pre-emptive threshold is higher.

The impact of changes in optimism levels on an optimistic entrepreneur's pre-emptive entry thresholds thus relies on the impact the entrepreneur's optimism has on his rival's incentives to pre-empt. By lowering the threshold at which he will enter as second mover, an increase in optimism reduces the rival's benefit of entering as leader by reducing the expected duration of her initial monopoly.

To see the underlying intuition why the two pre-emptive thresholds diverge, recall the pre-emptive threshold is defined as the first intersection of the entrepreneur's follower value and leader value. The pre-emptive threshold increases with the follower value whereas it decreases with the leader value.

Increasing A's optimism has no effect on B's value as a follower, since A has already entered the market. However, the impact A's optimism has in lowering its follower threshold has a negative effect on B's leader value, because it reduces the length of time B expects to earn the higher monopoly profits. Since B has less incentive to enter as leader, its pre-emptive threshold increases.

On the other hand, A's optimism about future product market growth increases the value he places on entering the market, both as a leader and as a follower. The increase in the leader value acts to decrease A's pre-emption threshold whereas the increase in the follower value acts in the opposite direction. In numerical simulations we always find that the impact on the leader's value dominates so pre-emptive thresholds for an optimistic entrepreneur decrease with their own optimism.

3.3 Asymmetric costs and revenues with asymmetric entrepreneurial beliefs

We now consider the general case when both firm characteristics (revenues and costs) and entrepreneurial beliefs (optimism levels) can differ.

3.3.1 Effect of optimism on entry order

To illustrate the main effect of optimism for asymmetric entrepreneurs, we plot the separating curve and boundaries between equilibria as first shown in Figure 3 again but increase entrepreneur A's optimism level. By keeping all the other parameters the same, we can compare how these curves will shift and how the regions will change as one entrepreneur (A) becomes more optimistic.

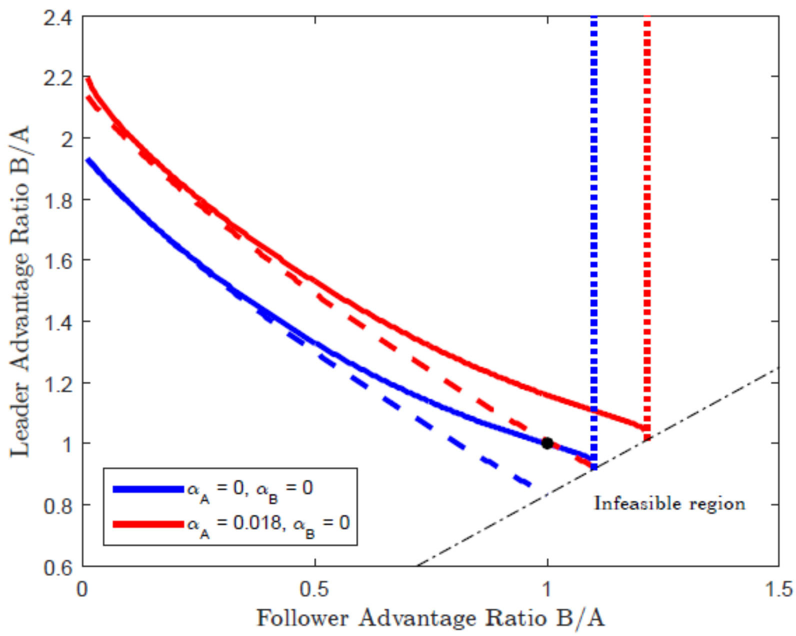
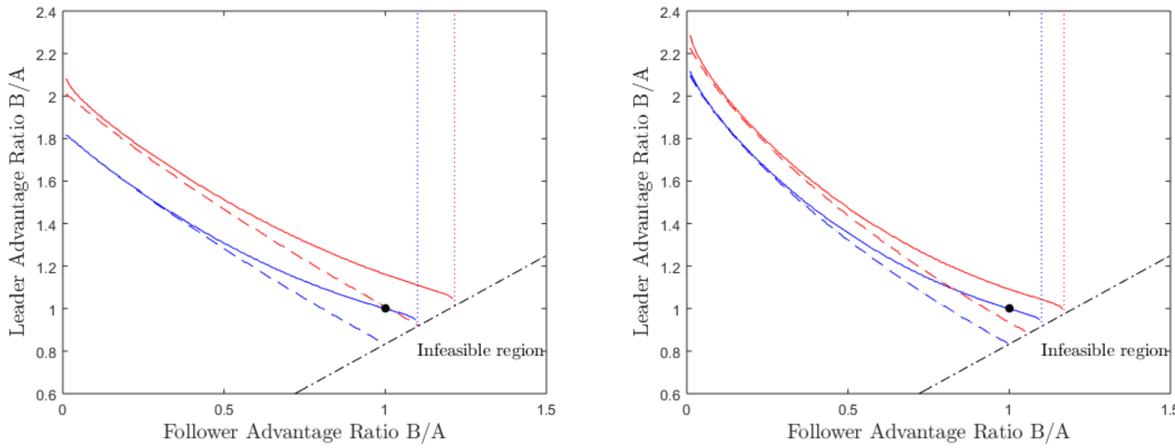


Figure 4: (Baseline) The Effect of A's Optimism in terms of Roles and Equilibria. Blue curves represent boundaries when both entrepreneurs are rational, while red curves represent boundaries when A becomes more optimistic. Parameter values: $r = 0.05$, $\delta_0 = 0.03$, $\eta = 0.3$ and $D_A^m/D_A^d = 1.2$.

Figure 4 shows that A's optimism increases the range of product market characteristics for which A enters as leader (the separating curves shift up and to the right). In particular there is a region (between the bold solid separating curves) where the entry order changes. In this region, when both entrepreneurs are realistic B has a pre-emptive advantage and thus enters first; however when A is optimistic B's pre-emptive threshold decreases whilst A's increases sufficiently that the order of the two entrepreneurs' pre-emptive thresholds switches. Effectively A's optimism provides an additional source of pre-emptive advantage. This implies a firm run by a realistic entrepreneur (B) can have a competitive advantage in both duopoly and monopoly markets (FAR and LAR ¿ 1) yet nevertheless her optimistic rival A will enter first. A's optimism reduces B's incentives to pre-empt, by reducing the expected duration of the period over which she can earn monopolistic profits. In turn this is due to A's earlier entry as follower due to his inherent optimism.

Furthermore, in unreported simulations we find that, just as in the case of symmetric



(a) Low Volatility $\eta = 0.2$

(b) High Volatility $\eta = 0.6$

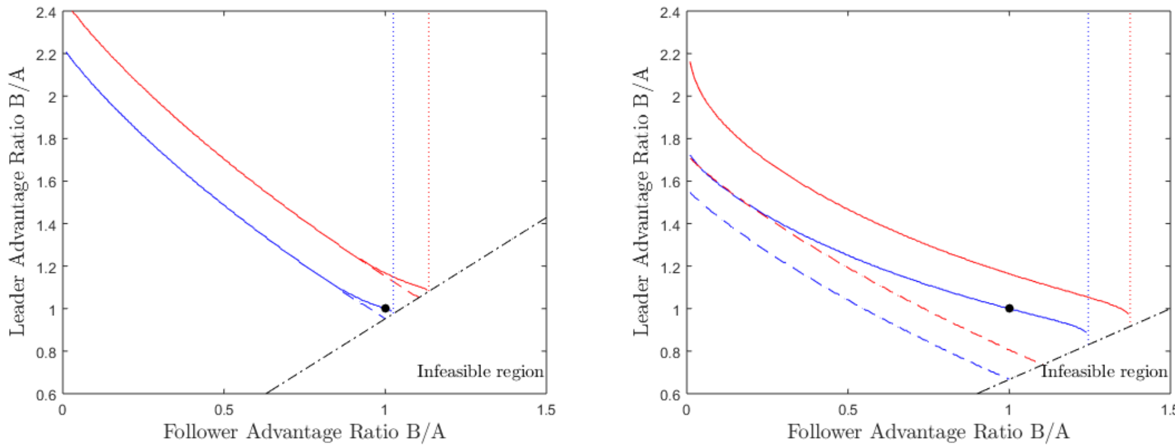
Figure 5: (Different Volatilities) The Effect of A's Optimism in terms of Roles and Equilibria. Parameter values: $r = 0.05$, $\delta_0 = 0.03$ and $D_A^m/D_A^d = 1.2$.

firms and consistent with Proposition 2, as long as the (more) optimistic entrepreneur enters in order to pre-empt his realistic (less optimistic) rival (i.e. at B's pre-emptive threshold), then the threshold at which the optimistic entrepreneur enters as leader increases as his level of optimism increases, consistent with Proposition 2. Whilst the follower's threshold decreases as optimism increases, as in prior literature which did not consider competition, pre-emption pressure reverses this effect when the optimistic entrepreneur enters pre-emptively as first mover.

3.3.2 Comparative statics

We now consider under which circumstances does optimism have the biggest effect on entry order. Specifically, we keep A's level of optimism fixed and investigate under which parameter combinations the region where a switch in the entry order occurs is largest.

We first investigate the effect of optimism for different levels of volatility. As shown in Figure 5, the entry order switching region between the red and blue curves is larger for low volatility. This is because investment timing involves a trade-off between the expected growth rate and uncertainty (volatility). If volatility increases, the desire to delay the investment



(a) Low FMA of A: $\frac{D_A^m}{D_A^d} = 1.05$ (b) High FMA of A: $\frac{D_A^m}{D_A^d} = 1.5$

Figure 6: (Different levels of A's first-mover advantage) The Effect of A's Optimism in terms of Roles and Equilibria. Parameter values: $r = 0.05$, $\delta_0 = 0.03$ and $\eta = 0.3$.

will be stronger. In this case, the same increment in the expected growth rate of market demand (i.e. same level of optimism) makes a smaller difference to the optimal investment timing.

We also investigate how the effect of optimism varies for different levels of first-mover advantage (FMA) of A (i.e. different values of $\text{FMA}_A = D_A^m/D_A^d$). (See Figure 6). A higher value of FMA_A means A's monopolistic profit is proportionately greater than A's duopolistic profit, or, in other words, A's profit decreases by a greater proportion on entry by his competitor. Figure 6 shows the impact of optimism for low and high levels of first mover advantage for A respectively.

It is clear that the region $\{A$ is pre-emptive leader$\}$ (i.e. the region between solid line and dashed line) expands as FMA_A increases. To understand the underlying reason, we note that $\bar{x}_A^P$, $\bar{x}_A^L$ and $\bar{x}_A^F$ would be very close together if FMA_A is sufficiently small[3]. However, as long as $\bar{x}_B^P$ exists, the inequality $\bar{x}_B^P < \bar{x}_A^F$ must hold: B's incentive to be the leader is only

[3]Imagine the extreme case when A's monopolistic profit equals its duopolistic profit (i.e. $D_A^m/D_A^d = 1$). Then it follows $\bar{x}_A^P = \bar{x}_A^L = \bar{x}_A^F$ since the monopolistic profit is the same as the duopolistic profit which means the decision of being the leader is the same as that of being the follower.

valid before A's entry. If $\bar{x}_A^P$ and $\bar{x}_A^L$ are only marginally smaller than $\bar{x}_A^F$, then $\bar{x}_B^P$ is less likely to be between $\bar{x}_A^P$ and $\bar{x}_A^F$. Thus, the case when $\bar{x}_B^P < \bar{x}_A^P$ is more likely to happen. On the other hand, $\bar{x}_A^P$ can be much smaller than $\bar{x}_A^F$ if $\text{FMA}_A = D_A^m/D_A^d$ is sufficiently large. The range of FAR and LAR for which $\bar{x}_B^P$ is located between $\bar{x}_A^P$ and $\bar{x}_A^F$ would be relatively larger. This is why the region $\{A \text{ is pre-emptive leader}\}$ expands as $\text{FMA}_A = D_A^m/D_A^d$ increases.

If we further compare the changes of the boundaries due to A's optimism in Figure 6 (a) and (b), the size of the entry order switching region is generally similar when B's leader advantage ratio and follower advantage ratio are both close to 1. However, the size of the region expands when A's first mover advantage is higher (Figure 6(b)) and when B has significant competitive advantage as a monopolist (LAR is high), but is at a significant disadvantage in the duopoly market (FAR is low). For these cases, optimism gives A a greater pre-emptive advantage, i.e. one which can compensate for a wider range of cashflow-based competitive advantages in determining entry order.

To understand why, note that for these firms B's first mover advantage ($\text{FMA}_B = \text{FMA}_A \times (\text{LAR}/\text{FAR})$), as well as A's first mover advantage is high: once both firms have entered the market, competition is fierce, significantly decreasing duopoly profits relative to either firm's monopoly profits. A's optimism affects both firms' pre-emptive thresholds, decreasing the threshold at which A is willing to pre-empt, whilst increasing B's by discouraging pre-emptive entry. The decrease in A's pre-emptive threshold $\bar{x}_A^P$ is magnified by its own high first mover advantage, because while both A's follower value and leader value are increased by optimism, the differential impact of optimism on the two values is greater when the ratio of monopoly to duopoly profits is higher. Higher own first mover advantage thus increases the impact of an entrepreneur's optimism on their own pre-emption incentives. On the other hand, the magnitude of the rival's first mover advantage affects the impact of optimism on the rival's pre-emptive thresholds. A's optimism decreases his follower thresholds and hence reduces the length of time B expects to enjoy monopoly profits were she to enter first. This decreases B's incentives to pre-empt. The larger the difference between B's monopoly and duopoly profits (the higher B's first mover advantage), the stronger the impact the reduction in duration has on B's leader value and thus the greater the increase in B's pre-emptive threshold.

Hence when product market competition is fierce, i.e. overall duopoly profit levels are significantly lower than in monopoly, optimism has greater impact both in increasing an optimistic entrepreneur's incentives to pre-empt and in decreasing their realistic (less optimistic) competitor's pre-emption incentives. This implies that, as shown in Figure 6 (b), a given level of optimism changes the entry order for a wider range of product market characteristics when competition is fierce, allowing an entrepreneurial optimist who has a competitive advantage in duopoly but a competitive disadvantage as a monopolist to enter first when facing competitors with greater advantages as a monopolist (greater range of LARs) than an equivalent realist.

Optimism's pre-emptive advantage is thus greater for entrepreneurs with relative advantage in more competitive and less volatile markets.[4]

4 Conclusion

In this paper we investigate the impact of entrepreneurial optimism on the entry order and thresholds in a duopoly setting. We found entrepreneurial optimism could provide a pre-emptive advantage, allowing a more optimistic entrepreneur to enter the market first even if their rival has a competitive advantage in both monopoly and duopoly settings. Furthermore the range of competitive advantages that could be offset increased with the entrepreneur's optimism and was greater for entrepreneurs with relative duopolistic advantage in more competitive and less volatile markets. We also showed that an entrepreneur's investment threshold can *increase* with his optimism, if he is entering to pre-empt his rival: the opposite of the standard comparative static result.

[4]We also re-examine the effect for different levels of the opportunity cost of forgone cashflows, δ_0, but find little impact on the size of the entry order switching region.

Entrepreneurial optimism is a widespread trait, so it is important to understand its effects on decision-making under uncertainty more generally . Thus far only the impact on investment options have been considered in the literature. In future work it would be interesting to extend this to abandonment and other types of real options.

Appendix 1: Technical Proofs

Proof of Proposition 1

Given the functional form for β_A as in (34), we can rewrite

$$\frac{\beta_A}{\beta_A - 1} = \frac{1}{2}\frac{\eta^2}{(\delta_0 - \alpha_A)}\left[\sqrt{\left(\frac{r+\alpha_A-\delta_0}{\eta^2}-\frac{1}{2}\right)^2+\frac{2r}{\eta^2}}-\left(\frac{r+\alpha_A-\delta_0}{\eta^2}-\frac{1}{2}\right)+\frac{2r}{\eta^2}\right] \tag{A.1}$$

thus $\bar{x}_A^F$ equals

$$\bar{x}_A^F = \frac{\eta^2 I_A}{2D_A^d}\left[\sqrt{\left(\frac{r+\alpha_A-\delta_0}{\eta^2}-\frac{1}{2}\right)^2+\frac{2r}{\eta^2}}-\left(\frac{r+\alpha_A-\delta_0}{\eta^2}-\frac{1}{2}\right)+\frac{2r}{\eta^2}\right] \tag{A.2}$$

Take the first partial derivative w.r.t. α_A to give

$$\frac{\partial \bar{x}_A^F}{\partial \alpha_A} = \frac{I_A}{2D_A^d}\left[\frac{\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2}}{\sqrt{\left(\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2}\right)^2+\dfrac{2r}{\eta^2}}}-1\right] \tag{A.3}$$

If $\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2} < 0$, then $\dfrac{\partial \bar{x}_A^F}{\partial \alpha_A} < 0$. Otherwise, if $\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2} > 0$, then

$$\frac{\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2}}{\sqrt{\left(\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2}\right)^2+\dfrac{2r}{\eta^2}}} = \frac{\sqrt{\left(\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2}\right)^2}}{\sqrt{\left(\dfrac{r+\alpha_A-\delta_0}{\eta^2}-\dfrac{1}{2}\right)^2+\dfrac{2r}{\eta^2}}} < 1 \tag{A.4}$$

since $\dfrac{2r}{\eta^2} > 0$. Thus, we still have $\dfrac{\partial \bar{x}_A^F}{\partial \alpha_A} < 0$. Therefore $\bar{x}_A^F$ is strictly decreasing as α_A increases. ■

Proof of Proposition 2

Define $f(x, \alpha_A) = V_B^L(x, \alpha_A) - V_B^F(x)$ where only the leader's value of firm B is a function of α_A but the follower's value of firm B is unaffected by α_A. Given $\bar{x}_B^P$ does exist, $\bar{x}_B^P$ solves $f(\bar{x}_B^P, \alpha_A) = 0$, so we have

$$\frac{d\bar{x}_B^P}{d\alpha_A} = -\frac{\partial f(\bar{x}_B^P, \alpha_A)}{\partial \alpha_A}\bigg/\frac{\partial f(\bar{x}_B^P, \alpha_A)}{\partial x} \tag{A.5}$$

where $\dfrac{\partial f(\bar{x}_B^P, \alpha_A)}{\partial \alpha_A} = \dfrac{D_B^d - D_B^m}{\delta_0 - \alpha_B}(1-\beta_B)\left(\dfrac{\bar{x}_B^P}{\bar{x}_A^F}\right)^{\beta_B}\dfrac{d\bar{x}_A^F}{d\alpha_A} < 0$ since $\dfrac{D_B^d - D_B^m}{\delta_0 - \alpha_B} < 0$, $1-\beta_B < 0$, $\left(\dfrac{\bar{x}_B^P}{\bar{x}_A^F}\right)^{\beta_B} > 0$ and $\dfrac{d\bar{x}_A^F}{d\alpha_A} < 0$. Then consider

$$\frac{\partial f}{\partial x} = \beta_B\left[\frac{D_B^d - D_B^m}{\delta_0 - \alpha_B}\left(\frac{1}{\bar{x}_A^F}\right)^{\beta_B - 1} - \left(\frac{D_B^d \bar{x}_B^F}{\delta_0 - \alpha_B} - I_A\right)\left(\frac{1}{\bar{x}_B^F}\right)^{\beta_B}\right]x^{\beta_B - 1} + \frac{D_B^m}{\delta_0 - \alpha_B} \tag{A.6}$$

$$\frac{\partial^2 f}{\partial x^2} = \beta_B(\beta_B - 1)\left[\frac{D_B^d - D_B^m}{\delta_0 - \alpha_B}\left(\frac{1}{\bar{x}_A^F}\right)^{\beta_B - 1} - \frac{I}{\beta_B - 1}\left(\frac{1}{\bar{x}_B^F}\right)^{\beta_B}\right]x^{\beta_B - 2} < 0 \tag{A.7}$$

Thus, $f(x, \alpha_A)$ is a concave function of x. Let x_B^* denote the turning point of $f(x, \alpha_A)$. Recall the definition of $\bar{x}_B^P$, i.e. $\bar{x}_B^P = \inf\{x : f(x, \alpha_A) = 0\}$. If $\bar{x}_B^P$ does exist, we will always have $f(x_B^*, \alpha_A) > 0$ and $\dfrac{\partial f(\bar{x}_B^P, \alpha_A)}{\partial x} > 0$. Combining with $\dfrac{\partial f(\bar{x}_B^P, \alpha_A)}{\partial \alpha_A} < 0$, we prove that $\dfrac{d\bar{x}_B^P}{d\alpha_A} > 0$. ■

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