Trade credit contracts: Design and regulation

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Abstract

This paper provides a theoretical analysis of trade credit within a real options framework. We show that under trade credit the buyer delays the decision to stop, getting closer to the supply chain optimal stopping threshold. Therefore, trade credit serves as a coordination device. Moreover, we show that the supplier can optimally choose to offer trade credit for free, since this will guarantee her business for a longer period of time. Optimal trade credit design is analyzed for an integrated supply chain (cooperative solution) and for external procurement (Nash bargaining and Stackelberg solutions). When regulation imposes a limit on trade credit maturity, we show that the two parties, buyer and supplier, might have difficulties in undoing regulation, despite the complementarity between price discount and trade credit. The model's predictions are in line with recent empirical evidence on the effects of regulation in the retail and trucking industry.

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1. Introduction

Trade credit as a source of financing has been generally neglected by the literature (as opposed to bank or bond financing), although it is the most important source of financing not only for small and medium enterprises, but also for large ones. Cuñat (2007) provides evidence that trade credit accounts for roughly one-fourth of the total assets of a representative firm and abouth half of the short-term debt in two different samples of medium-sized UK firms and small US firms. Similarly, Yang and Birge (2018) claim that as of June 2016, accounts payable (the amount of trade credit owed by buyers to suppliers) were 3.3 times as large as bank loans on the aggregated balance sheet of non-financial US businesses. Moreover, for large public retailers in North America, accounts payable represent roughly one third of their liabilities. In this paper, we contribute to this literature by providing a first theoretical analysis of trade credit within a real options framework, which allows us to shed light on important issues related to the design of trade credit terms, and the effects of regulation. Furthermore, we contribute to the long-standing debate over whether trade credit is an expensive form of financing, or on the contrary a low-cost financing source (Yang and Birge, 2018).

On the one hand, the popular view in the initial literature on trade credit was that trade credit was an expensive form of finance (Cuñat, 2007; Petersen and Rajan, 1997).[1] Researchers trying to address the so-called trade credit puzzle attempted to answer the questions "why does trade credit appear to be so expensive?" and "why is trade financed by suppliers instead of banks" (Cuñat, 2007, p. 492). Several explanations have been offered in the literature: trade credit is used as a price discrimination device (Brennan et al., 1988), trade credit can serve as a warranty for product quality (Long et al., 1994), trade credit can be used to signal information to banks in the context of asymmetric information in times of credit rationing (Biais and Gollier, 1997), or trade credit can be justified for buyer-specific products (Cuñat, 2007). Moreover, another common assumption is that a large manufacturer finances a relatively small and young financially constrained retailer

[1] Cuñat (2007) offers the example of a two-part trade credit contract called "2-10 net 30", where if customers pay within 10 days they qualify for a 2% discount, otherwise they can pay up to 30 days after delivery. This discount on early payment implies receiving credit at 2% for 20 days, which is equivalent to a 44% annual interest rate.

(Petersen and Rajan, 1997).

On the other hand, recent empirical evidence seems to challenge these two assumptions. First, Giannetti et al. (2011) find that a majority of firms in their sample receive trade credit at low cost, only a minority of firms reporting that their main supplier offers early payment discounts. In fact, the median firm in their sample receives trade credit at zero cost. Moreover, Yang and Birge (2018) find that the most common trade credit terms in their sample are "net terms", which are interest-free loans from suppliers to buyers.[2] Secondly, there is recent evidence that many small suppliers finance large financially unconstrained buyers (Fabbri and Klapper, 2016; Klapper et al., 2012; Murfin and Njoroge, 2015).[3]. The question that naturally arises is why do small financially constrained suppliers offer trade credit for free to large unconstrained buyers. Why don't they just offer a price discount? As pointed out by Giannetti et al. (2011), a big challenge for future research is to answer this question. A possible explanation suggested in the empirical papers of Klapper et al. (2012) and Fabbri and Klapper (2016) is that they offer trade credit as a competitive gesture, in order to attract the buyer. Fabbri and Klapper (2016) show that suppliers are more likely to offer trade credit and better credit terms to powerful and important customers. A price reduction is observable by competitors and can trigger a price war, detrimental for the whole industry, while trade credit is less aggressive and more flexible. They also show evidence of complementarity between these two instruments, price discount and trade credit: firms that lowered prices relative to the previous year are more likely to offer trade credit.

Anecdotal evidence from the retail and automotive industry also seems to support this new perspective on trade credit from small suppliers to large buyers and seems to suggest that large buyers abuse their bargaining power in order to extract margins from small suppliers.[4] This has drawn the attention

[2]For example, under the "net 30" term, buyers need to pay supplier within 30 days of delivery.

[3]Murfin and Njoroge (2015) document that the largest two decile firms by size are net receivers of trade credit in the retail industry.

[4]See John Plender, "The Wal-Martisation of almost everything", The Financial Times, 15 November 2004; John Plender, Martin Simons and Henry Tricks, "Cash benefit: how big supermarkets fund expansion by using suppliers as bankers", The Financial Times, 7 December 2005; "Competition policy in the EU: Big chains enjoy a buyer's market", The Economist, 15 February 2007; "FT interview transcript: Christine Lagarde", The Financial Times, 12 May 2008, among others.

of policy makers and regulators, who have tightened regulation by reducing the maturity of trade credit contracts, in an attempt to protect suppliers. Two such cases have been recently analyzed in the empirical papers of Breza and Liberman (2017) and Barrot (2016). Supermarkets in Chile saw their trade credit maturity reduced from 90 to 30 days, while French trucking firms were prevented from extending to their customers trading terms exceeding 30 days (representing a 15% reduction in payment terms).

In this paper, we contribute to this debate by providing a first theoretical analysis of this issue. Within a real options framework, we assume that the price at which the buyer sells the final product follows a geometric Brownian motion. The supplier sells the product to the buyer at the wholesale price and incurrs a production cost. The buyer can optimally choose the time at which to stop the business. We derive the buyer, supplier and supply chain values first under no trade credit, and second under trade credit. In the no trade credit case, the buyer stops too early compared to the supply chain optimal. In fact, the supply chain is only coordinated (and the value of the supply chain maximized) if the wholesale price is equal to the production cost. On the contrary, in the trade credit case, we show that the stopping threshold optimally chosen by the buyer is lower than the one without trade credit. The buyer delays the decision to stop when receiving trade credit, getting closer to the supply chain optimal. Thus, trade credit is effectively a coordination device. Moreover, although offering trade credit implies a cost of delay for the supplier (receiving the payment later), it also has the advantage of guaranteeing her business for a longer period of time. Given this trade-off, we show that as long as the trade credit maturity is not too high, the supplier can optimally choose to offer trade credit for free. Our model's implications are in line with recent empirical evidence suggesting trade credit is offered at a low cost (Giannetti et al., 2011).

We consider three different solutions in designing the optimal trade credit terms, i.e., the pair of optimal wholesale price (price charged by the supplier to the buyer) and optimal trade credit maturity. The first one is the cooperative solution corresponding to a vertically integrated supply chain. We show that there exist an infinite number of pairs (wholesale price, trade credit maturity) that maximize the value of the supply chain. While under no trade credit the supply chain is only coordinated when the wholesale price is equal to production cost, with trade credit it is possible to coordinate the supply chain for any wholesale price. The second solution we analyze is the Nash bargaining solution corresponding to external procurement. In this case, the outcome depends on the status quo of the two parties and their bargaining power. Even under this non-cooperative solution it is possible to achieve first best. The optimal pair of trade credit terms is one of the infinite number of pairs from the cooperative solution that splits the supply chain value according to the bargaing power of the parties. Third, we analyze the Stackelberg solution, with the leader being the buyer and the follower the supplier. This strategy characterizes well recent empirical and anecdotal evidence that large retailers finance themselves through trade credit from their smaller and weaker suppliers.

Finally, we study the impact of regulation, in particular, the effect of a limit on trade credit maturity imposed by regulators. We show that although regulation can be undone in case of an integrated supply chain, this is not the case for external procurement. Even though the two parties, buyer and supplier, can reduce the wholesale price as a complementary strategy to the imposed limitation in trade credit maturity, they cannot undo regulation. The decrease in the wholesale price cannot fully compensate for the loss in value caused by the decrease in trade credit maturity. One of the parties will be negatively affected by regulation. Therefore, the model predicts that following such a change in trade credit regulation will decrease and internal procurement will increase.

Our work makes important contributions to two strands of the literature. First, we extend current research on real options by applying a real options framework to trade credit. Although corporate debt has been extensively analyzed within a real options framework, to our knowledge this is the first paper to apply it to trade credit. Within the real options literature, the forward start model of Shackleton and Wojakowski (2007) has been recently employed to explain findings from different areas ranging from duopolies (Pereira and Rodrigues, 2014) to public-private partnerships (Adkins et al., 2018). However, a "forward stop" model, as the one we employ, similar to the deferred American put model of Gerber and Shiu (1993), has not been exploited in the literature. Second, we contribute to the operations management literature on supply chains. Different contract types have been shown to coordinate the supply chain, such as buy back contracts, revenue sharing contracts and quantity flexibility contracts (Cachon, 2003). Our cooperative solution can be seen as a new coordination mechanism for the supply chain, in the context of trade credit. Finally, our work on trade credit design is also relevant for practitioners who welcome normative insights.

The rest of the paper is organized as follows. Section 2 describes the financial setup, first under no trade credit and then under trade credit. Section 3 analyzes the benefits of offering trade credit for free. Section 4 presents the design of trade credit terms under different cooperative and non-cooperative solutions. In section 5 we analyze the social planner's perspective and discuss issues related to regulation. A numerical illustration and comparison of the model's implications with empirical evidence on trade credit is provided in section 6. Finally, section 7 concludes.

2. The framework

A firm, called the buyer, produces a final good using as an input another good that it buys from the supplier. The project under study consists in selling on the market a final good whose (retail) price is denoted by P. This price is random and and it is well described by

$$dP_t = \mu P_t dt + \sigma P_t dW_t$$

The wholesale price c that the buyer pays to the supplier is fixed, and the supplier faces a fixed production cost γ, with $c \geq \gamma$. For the time being we will consider c as given. Later on, in Section 4 the wholesale price will be determined either as a cooperative solution in the case of a vertically integrated supply chain, or as the result of a Nash bargaining non-cooperative game between the buyer and the supplier. The instantaneous net cash flow per unit of goods sold for the buyer is therefore $P_t - c$, while the one of the seller is $c - \gamma$. For simplicity and for the sake of analytical tractability, we consider a perpetual demand which is normalized to one.[5] Each firm is risk neutral, thus each firm maximizes expected profits. Let us denote by B the value of the project from the buyer's perspective and by S the value from the supplier's perspective. V stands for the value of a supply chain that comprises both the buyer and the supplier, so that $V = B + S$.

We will now analyze two cases. First, the simple case in which delivery and payment of the goods delivered by the supplier to the buyer coincide.

[5] The wholesale price contract has been extensively studied in the context of the newsvendor model in the operations management literature (see Lariviere and Porteus, 2001). The standard newsvendor model is used to determine the optimal inventory level in a context of fixed prices and stochastic demand. Many variations of this model exist, such as deterministic and price dependent demand as in Bresnahan and Reiss (1985), the newsvendor model with stochastic and price dependent retail demand or the single location base stock model with infinite horizon stochastic demand (Cachon, 2003).

Therefore, there is no delay in payment, and the only contractual parameter is the wholesale price, c. Second, we will consider the case in which payment occurs some time after the delivery of the goods, thus the supplier is offering trade credit to the buyer. In this case, the two contractual parameters are the wholesale price, c, and the trade credit maturity, Δ.

2.1. The no trade credit case

In case there is no trade credit, we assume that the buyer can decide to stop the project at any time. Of course he will undertake this decision so as to maximize his profit.[6] Let's denote by τ such an optimal time. Since the project has no special expiration, the optimal decision will occur as soon as the sale price P reaches some threshold K from above. Hence, the optimal time may be defined by $\tau = \inf\{t : P_t = K\}$. Then the buyer and supplier valuations are given by:

$$\begin{aligned} B^{NT} &= E_0\left(\int_0^\tau (P_s - c)e^{-rs}ds\right) \\ S^{NT} &= E_0\left(\int_0^\tau (c - \gamma)e^{-rs}ds\right) \end{aligned}$$

for which we can derive explicit formulae (where r denotes the risk-free interest rate, and for convergence $r > \mu$).

Proposition 1: *The buyer's valuation of the project, given a stopping threshold K, is*

$$B^{NT}(c;K) = \left[\frac{P_0}{r-\mu} - \frac{c}{r}\right] - \left[\frac{K}{r-\mu} - \frac{c}{r}\right]\left(\frac{P_0}{K}\right)^{-X} \tag{1}$$

The supplier's valuation of the project is

$$S^{NT}(c;K) = \left[\frac{c}{r} - \frac{\gamma}{r}\right] - \left[\frac{c}{r} - \frac{\gamma}{r}\right]\left(\frac{P_0}{K}\right)^{-X} \tag{2}$$

The supply chain's valuation of the project is

$$V^{NT}(c;K) = \left[\frac{P_0}{r-\mu} - \frac{\gamma}{r}\right] - \left[\frac{K}{r-\mu} - \frac{\gamma}{r}\right]\left(\frac{P_0}{K}\right)^{-X} \tag{3}$$

[6]Following Cachon (2003), we adopt the convention that the upstream firm (the supplier) is female, and the downstream firm (the buyer) is male.

where

$$X = \frac{(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}.$$

In case the buyer chooses to stop production optimally, the endogenous threshold is

$$K^{NT} = \frac{c\,(r-\mu)}{r}\frac{X}{X+1}$$

must be plugged in the above formulae.

Proof: See Appendix.

This proposition is in line with standard results from the real option theory. It reminds us that the optimal threshold chosen by the buyer to stop production is lower than the Marshallian ones which would be either c or $\frac{c(r-\mu)}{r}$ depending on whether the stopping criteria is defined as the cash flow or the project value. Moreover, the stopping threshold is proportional to the cost c and it depends non-linearly on r, μ and σ.

Note that although the total supply chain value V^{NT} does not *directly* depend on the wholesale price c, it depends on it indirectly through the stopping threshold K^{NT}. In particular, we observe that when choosing the optimal stopping threshold the buyer does not take into account the supplier's profits. Therefore, the threshold chosen by the buyer does not maximize the total supply chain value. The optimal threshold that maximizes the total supply chain value is $K^{NT*} = \frac{\gamma(r-\mu)}{r}\frac{X}{X+1} = \frac{\gamma}{c}K^{NT}$. Since $c \geq \gamma$, we have that $K^{NT} \geq K^{NT*}$, that is, the buyer decides to stop production too early. One way to coordinate the supply chain, to make the buyer choose the stopping threshold that maximizes the supply chain value is to set the wholesale price equal to the production cost, i.e., $c = \gamma$. Then we have that $K^{NT} = K^{NT*}$, and the buyer captures all the profits, the supplier's valuation is equal to zero.[7] Vertical integration is one of the two most common models of supply chain strategies, along with the manufacturer Stackelberg strategy (see Baron et al., 2016). Different supply chain coordination mechanisms have been studied in the context of the newsvendor model: two-part tariff, buy back, quantity flexibility and revenue sharing. Cachon (2003) makes an

[7] At first sight, this "coordination" strategy seems odd because the coordination should discipline the buyer and the buyer is in a way compensated. But this extreme strategy corresponds to a vertical integration strategy where the buyer decides to face the production cost.

excellent survey of this literature. In the next subsection we propose a new coordination mechanism, trade credit.

2.2. Introducing trade credit

We now introduce trade credit. Let the maturity of the trade credit contract be denoted by Δ. Under such a contract, the buyer receives the goods from the supplier and starts production at time zero, however, he does not make any payments to the supplier until Δ. Therefore, the buyer receives the price of the final goods sold P starting at time zero, but only starts making a payment c to the supplier at Δ. Assuming that the buyer stops production whenever the price reaches a threshold K from above, we have the following two situations. If $P_\Delta > K$, the buyer will receive the price P until time $\tau > \Delta$ at which the threshold K is reached, and will make the payment c to the supplier until time $\tau + \Delta$. If $P_\Delta \leq K$, the production is stopped at time Δ, thus the buyer receives the price P until time Δ and makes the payment c until time 2Δ. The buyer's valuation is given by:

$$
\begin{aligned}
B^{TC} &= E_0\left(\int_0^\Delta P_s e^{-rs} ds\right) + E_0\left(1_{\{P_\Delta < K\}} \int_\Delta^{2\Delta} -ce^{-rs} ds\right) \\
&+ E_0\left[1_{\{P_\Delta > K\}}\left(\int_\Delta^\tau P_s e^{-rs} ds + \int_\Delta^{\tau+\Delta} -ce^{-rs} ds\right)\right]
\end{aligned}
$$

Rearranging we have:

$$
B^{TC} = E_0\left(\int_0^\Delta P_s e^{-rs} ds\right) - \int_\Delta^{2\Delta} ce^{-rs} ds + E_0\left[1_{\{P_\Delta > K\}}\left(\int_\Delta^\tau P_s e^{-rs} ds + \int_{2\Delta}^{2\Delta+(\tau-\Delta)} -ce^{-rs} ds\right)\right]
$$

Similarly, the supplier's valuation is given by:

$$
S^{TC} = \int_\Delta^{2\Delta} (c-\gamma) e^{-rs} ds + E_0\left[1_{\{P_\Delta > K\}}\left(\int_{2\Delta}^{2\Delta+(\tau-\Delta)} (c-\gamma) e^{-rs} ds\right)\right]
$$

The following proposition provides explicit formulae for the buyer and supplier valuations in the trade credit case.

Proposition 2: *The buyer's valuation of the project with trade credit is*

$$
\begin{aligned}
B^{TC}(c,\Delta;K) &= \left[\frac{P_0}{r-\mu} - \frac{c}{r}e^{-r\Delta}\right] - e^{-r\Delta}\left[\frac{P_0}{r-\mu}e^{\mu\Delta}N\left(-d_1(\mu,\Delta)\right) - \frac{c}{r}e^{-r\Delta}N\left(-d_2(\mu,\Delta)\right)\right] \\
&- \left[\frac{K}{r-\mu} - \frac{c}{r}e^{-r\Delta}\right]\left(\frac{P_0}{K}\right)^{-X} N\left(d_2\left(\mu - X\sigma^2, \Delta\right)\right), \qquad (4)
\end{aligned}
$$

The supplier's valuation of the project with trade credit is

$$S^{TC}(c,\Delta;K) = \left[e^{-r\Delta}\frac{c}{r} - \frac{\gamma}{r}\right]\left[1 - e^{-r\Delta} + e^{-r\Delta}N(d_2(\mu,\Delta)) - \left(\frac{P_0}{K}\right)^{-X} N(d_2(\mu - X\sigma^2,\Delta))\right] \tag{5}$$

with:

$$d_1(\mu,\Delta) = \frac{\ln(P_0/K) + (\mu + 1/2\sigma^2)\Delta}{\sigma\sqrt{\Delta}}, \qquad d_2(\mu,\Delta) = \frac{\ln(P_0/K) + (\mu - 1/2\sigma^2)\Delta}{\sigma\sqrt{\Delta}}. \tag{6}$$

The supply chain's valuation of the project with trade credit is

$$\begin{aligned} V^{TC}(c,\Delta;K) &= \left[\frac{P_0}{r-\mu} - \frac{\gamma}{r}\right] - e^{-r\Delta}\left[\frac{P_0}{r-\mu}e^{\mu\Delta}N(-d_1(\mu,\Delta)) - \frac{\gamma}{r}N(-d_2(\mu,\Delta))\right] \\ &\quad - \left[\frac{K}{r-\mu} - \frac{\gamma}{r}\right]\left(\frac{P_0}{K}\right)^{-X} N\left(d_2\left(\mu - X\sigma^2,\Delta\right)\right) \qquad (7) \\ &= V^{TC}(\Delta;K) \qquad (8) \end{aligned}$$

Proof: See Appendix. We show in Appendix that this valuation problem is very closely related to the valuation of deferred perpetual American puts in Gerber and Shiu (1993) and to that of forward start perpetuities in Shackleton and Wojakowski (2007). The above formulae critically depend on the threshold to decide to stop production. This decision threshold may be exogenously or endogenously chosen. If the buyer is "naive" (by being unable to internalize the trade credit terms in his decision to produce), then granting credit reduces the supplier's valuation of the project.[8]

Indeed, in case the buyer is choosing an exogenous stopping threshold, one has $\partial S^{TC}(c,\Delta;K)/\partial\Delta < 0$, thus:

$$S^{TC}(c,\Delta;K) \leq S^{NT}(c;K)$$

and there is no incentive for the seller to grant trade credit. This illustrates the cost of delay that granting trade credit implies for the supplier. Nevertheless, if the buyer internalizes the trade credit terms into his decision to stop production, granting trade credit does not only imply a cost of delay, but also a possible gain for the supplier. We will analyze this issue in the next section.

[8]This will likely be the case for small buyers. The argument also applies in a symmetric situation where the seller faces fluctuating instantaneous cash flows and decides to produce.

3. The advantages of offering trade credit for free

If the buyer internalizes the advantages of trade credit, he will optimally choose to stop production at an endogenous threshold that will depend on the maturity of the trade credit. The optimal stopping threshold is given by imposing:

$$\left.\frac{\partial B^{TC}(c,\Delta;K)}{\partial P_0}\right|_{P_0=K^{TC}} = 0. \tag{9}$$

Therefore, we have the following proposition:

Proposition 3: *The endogenous threshold to stop production is:*

$$K^{TC}(\Delta) = e^{-r\Delta}\frac{c(r-\mu)}{r}\frac{B(\Delta)}{A(\Delta)}, \tag{10}$$

where $A(\Delta)$ and $B(\Delta)$, with $A(\Delta) \geq B(\Delta)$, are given by:

$$A(\Delta) = 1 - e^{-(r-\mu)\Delta} + e^{-(r-\mu)\Delta} N\left(\frac{(\mu+\sigma^2/2)\Delta}{\sigma\sqrt{\Delta}}\right) + XN\left(\frac{(\mu - X\sigma^2 - \sigma^2/2)\Delta}{\sigma\sqrt{\Delta}}\right)$$

and

$$B(\Delta) = XN\left(\frac{(\mu - X\sigma^2 - \sigma^2/2)\Delta}{\sigma\sqrt{\Delta}}\right)$$

Proof: See Appendix.

Comparing the stopping threshold in the two scenarios, with and without trade credit, it can be shown that the buyer decides to postpone the stopping time in case of trade credit. We have the following proposition:

Proposition 4: *Comparing the stopping threshold with and without trade credit, one has:*

$$K^{TC}(\Delta) < K^{NT}$$

Proof: See Appendix.

Therefore, granting trade credit also implies a *direct* gain for the supplier, since she will now trade for a longer period of time with the buyer. That is, since the buyer stops production later with trade credit, the period of guaranteed business increases when granting trade credit. Whether granting trade credit is *finally* beneficial for the supplier depends on the trade-off between the cost of delay and the gain of extended business.

We can decompose the supplier's valuation of the project with trade credit in order to illustrate this trade-off:

$$
\begin{aligned}
S^{TC}\left(c,\Delta;K^{TC}\right) &= S^{NT}\left(c;K^{NT}\right) - \underbrace{\left[S^{NT}\left(c;K^{NT}\right) - S^{TC}\left(c,\Delta;K^{NT}\right)\right]}_{\text{cost of delay } \geq 0} \\
&+ \underbrace{\left[S^{TC}\left(c,\Delta;K^{TC}\right) - S^{TC}\left(c,\Delta;K^{NT}\right)\right]}_{\text{value of flexibility } \geq 0}
\end{aligned}
$$

As long as the maturity of the trade credit offered is not too large, the supplier's valuation of the project under trade credit will dominate the one without trade credit, as summarized in the following proposition. Note that to simplify the notation we will denote $S^{TC}\left(c,\Delta;K^{TC}\right)$ by $S^{TC}\left(c,\Delta\right)$ hereafter, and $S^{NT}(c;K^{NT})$ by $S^{NT}(c)$ (and similarly for the buyer, B, and total supply chain value, V).

Proposition 5: *For any cost c, there exists $\widehat{\Delta}$ such that the supplier is indifferent between granting trade credit or not*

$$
S^{NT}\left(c\right) = S^{TC}\left(c,\widehat{\Delta}\right)
$$

and such that, for any $\Delta < \widehat{\Delta}$, trade credit dominates

$$
S^{TC}\left(c,\Delta\right) > S^{TC}\left(c,\widehat{\Delta}\right) = S^{NT}\left(c\right)
$$

and for any $\Delta > \widehat{\Delta}$, no trade credit dominates

$$
S^{TC}\left(c,\Delta\right) < S^{TC}\left(c,\widehat{\Delta}\right) = S^{NT}\left(c\right).
$$

Figure 1 illustrates the previous proposition. We see that for $\Delta = 0$ the valuation of the supplier under trade credit is equal to her valuation in the no trade credit case, as expected. As the trade credit maturity increases and as long $\Delta \leq \widehat{\Delta}$, we have that trade credit dominates no trade credit from the supplier's perspective. In this region the gains of extended business more than compensate the cost of delay implied by granting trade credit. Therefore, it is optimal for the supplier to grant trade credit "for free", that is, without increasing the price c. Only if the trade credit maturity is set larger than $\widehat{\Delta}$ will the supplier suffer from offering trade credit. We thus provide an explanation for the puzzling recent empirical findings that challenge the common wisdom that trade credit is an expensive source of finance and that are in line with anecdotal evidence of cheap trade credit (Giannetti et al., 2011).

[Figure 1 about here.]

Finally, we have the following corollary:

Corollary 1: There exists an optimal Δ for the supplier S.

Since the supplier's profits are hump-shaped in the trade credit maturity, we conclude that there exists an optimal trade credit maturity for the supplier that maximizes her profit.

We end this subsection with the following remark. Note that the optimal stopping threshold that maximizes the total value of the supply chain (and not just of the buyer) is similarly given by: $K^{TC*} = \frac{\gamma(r-\mu)}{r}\frac{B(\Delta)}{A(\Delta)}$. Indeed, by substituting $ce^{-r\Delta}$ by γ in equation (4) we obtain equation (8).

4. Designing trade credit terms

The trade credit terms will be designed taking both parties' interests into account, their willingness to cooperate and their respective bargaining power. We will analyze three different possibilities: a cooperative solution corresponding to a vertically integrated supply chain, a non-cooperative Nash bargaining game, and a non-cooperative Stackelberg equilibrium in which the buyer acts as the leader and the supplier as a follower.

4.1. Cooperative solution

One possibility is the cooperative solution. The two parties can join their valuations and find the pair (c, Δ) that maximizes the sum $S^{TC}(c, \Delta) + B^{TC}(c, \Delta)$, that is, $V^{TC}(c, \Delta)$, the total supply chain value. This solution corresponds to a vertically integrated supply chain.

$$\operatorname*{Max}_{c,\Delta}\ V^{TC}(c, \Delta)$$

There exists an infinite number of pairs (c, Δ) that maximize the total supply chain value, with $\gamma \leq c \leq P_0$. Assume without loss of generality that the price c is set to be in line with other competing suppliers to $c = \underline{c}$. Then there exists a unique $\Delta^*(\underline{c})$ that maximizes the total supply chain value. Comparing to the no trade credit case, we note that the supply chain value is larger under trade credit: $V^{TC}(\underline{c}, \Delta^*(\underline{c})) \geq V^{NT}(\underline{c})$.[9] Indeed, the two

[9] To see that for any $\underline{c} > \gamma$ we have that $V^{TC}(\underline{c}, \Delta^*(\underline{c})) > V^{NT}(\underline{c})$ first note that when $\underline{c} = \gamma$, the optimal Δ is $\Delta^*(\gamma) = 0$. Then we have that $V^{TC}(\underline{c}, \Delta^*(\underline{c})) = V^{TC}(\gamma, \Delta^*(\gamma)) = V^{TC}(\gamma, 0) = V^{NT}(\gamma) > V^{NT}(\underline{c})$.

values are only equal for $\underline{c} = \gamma$. In this case, the wholesale price is set to be equal to the production cost, the retailer captures all the profits, and the supplier's valuation of the project is equal to zero. As previously explained, setting $c = \gamma$ in the no trade credit case corresponds to a vertically integrated supply chain. For any $c > \gamma$ the supply chain in the no trade credit case is not coordinated, thus the total supply chain value is lower than its maximum possible value. On the contrary, introducing trade credit adds extra flexibility in setting the contractual terms, which makes is possible to coordinate the supply chain for any wholesale price, c.

Recent empirical evidence shows that buyers prefer external suppliers to internal procurement (pure production arguments: weak economy of scale, financial arguments: conglomerate discount) and only after a change in regulation they internalize procurement to their own subsidiaries (Breza and Liberman, 2017). Therefore, we assume that internal suppliers are less efficient than external suppliers. Formally $\gamma_i > \gamma_e$, where the subindices i and e denote internal and external suppliers, respectively.

4.2. Nash bargaining game

When the buyer trades with an external supplier, a cooperative solution will not be possible. If the two parties reject the cooperative solution, the outcome will be the solution of a non-cooperative bargaining game. This outcome depends on the parties' threat points and on their relative bargaining power. The threat points reflect the status quo between the negotiating parties. This status quo represents the utility gained by the parties if the bargaining breaks down (Binmore et al., 1986). The bargaining power of each party will depend on their market power, size and the competition they are facing (Fabbri and Klapper, 2016; Klapper et al., 2012). Let η denote the buyer's bargaining power and $1 - \eta$, the one of the supplier. Let Γ and Θ be the status quo of the supplier and the buyer respectively. In case the bargaining breaks down, the buyer might internalize procurement to its own subsidiaries, while the supplier might lose the business if he does not grant trade credit (her status quo could then be lower than her no trade credit valuation).

The buyer obtains $B^{TC} = \theta V^{TC}$ in negotiation, while the supplier gets $S^{TC} = (1-\theta)V^{TC}$, where θ is a parameter that reflects the sharing rule. The

Nash solution is characterized by:

$$\begin{aligned}\theta^* &= \underset{\theta}{argmax}\{\theta V\,(c,\Delta) - \Theta\}^\eta\{(1-\theta)V\,(c,\Delta) - \Gamma\}^{1-\eta} \\ \theta^* &= \eta - \frac{\eta\Gamma - (1-\eta)\Theta}{V\,(c,\Delta)}\end{aligned} \tag{11}$$

Thus, the buyer's stake is worth $\theta^* V\,(c,\Delta) = \theta^*[B\,(c,\Delta) + S\,(c,\Delta)]$. But the buyer' stake is also given by $B\,(c,\Delta)$. Equating the two expressions, $\theta^*[B\,(c,\Delta) + S\,(c,\Delta)] = B\,(c,\Delta)$, we get the Nash equilibrium relationship between the price c and the trade credit maturity Δ:

$$\eta S^{TC}\,(c,\Delta) - (1-\eta)B^{TC}\,(c,\Delta) = \eta\Gamma - (1-\eta)\Theta \tag{12}$$

There is an infinite number of pairs (c,Δ) satisfying this relationship. Among these pairs the parties will choose the one that maximizes the total value $V\,(c,\Delta)$:

$$\begin{array}{cl}\text{Max}_{c,\Delta} & V^{TC}\,(c,\Delta) \\ s.t. & \eta S^{TC}\,(c,\Delta) - (1-\eta)B^{TC}\,(c,\Delta) = \eta\Gamma - (1-\eta)\Theta\end{array} \tag{13}$$

From equation 12, we can express c as an implicit function of Δ, and substitute it into the objective function. The optimal Δ can then be obtained by taking the first order condition of $V\,(c(\Delta),\Delta)$ with respect to Δ. Since an analytical solution is not possible, this problem will be solved numerically.

In general, a cooperative solution is considered to be first best, while a non-cooperative solution is only the second best (whenever the parties can choose only one single variable, such as the wholesale price). That is, a non-cooperative solution in general leads to a lower total value than a cooperative one. Nevertheless, this is not the case in our model. Even with a non-cooperative bargaining game we can achieve first best. Indeed, note that the optimal pair (c^*,Δ^*) resulting from the Nash bargaining game is just one of the infinite pairs that maximize the total supply chain value in the cooperative solution. In particular, it is the pair (c,Δ) that satisfies equation (12), thus that splits the supply chain value according to the parties' bargaining power. Again, this result is due to the fact that trade credit adds flexibility to the supplier-buyer relationship (the parties can now choose two parameters, the wholesale price and the trade credit maturity). Under no trade credit the non-cooperative solution would be worse-off compared to the cooperative one.

A supply chain with external procurement under non-cooperative Nash bargaining can then achieve higher profits than with internal procurement under a cooperative solution. This is true as long as the external supplier is more efficient compared to the internal supplier, $\gamma_i > \gamma_e$, as we have previously assumed.

4.3. Stackelberg equilibrium

Finally, we analyze the non-cooperative Stackelberg equilibrium in which the buyer acts as the leader and the supplier as the follower. This is in contrast with the manufacturer Stackelberg supply chain strategy, where the manufacturer is the leader and the retailer is the follower (see Baron et al., 2016). Our choice is motivated by the fact that we are trying to explain the puzzling relationship where large retailers finance themselves off the back of small, weaker supplier evidenced in the recent trade credit empirical literature (Fabbri and Klapper, 2016; Klapper et al., 2012; Murfin and Njoroge, 2015). Although the buyer has the interest of paying the lowest possible price c in every period to the supplier, it has to consider the possibility that a too low price may become unattractive for the supplier, given his status quo, Γ.

The optimal pair (c, Δ) is obtained from the following maximization problem:

$$\begin{array}{cl} \text{Max}_{c,\Delta} & B^{TC}(c,\Delta) \\ s.t. & S^{TC}(c,\Delta) \geq \Gamma \end{array} \tag{14}$$

For a given reservation price Γ, the seller will not accept to contract if $S^{TC}_{\max} < \Gamma$. The supplier will always accept to contract if $\Gamma \leq S^{NT}(c)$ since $S^{TC}(c) > S^{NT}(c)$.. Finally, the supplier will accept to contract when $S^{NT}(c) < \Gamma \leq S^{TC}_{\max}$ only if the buyer accepts a trade credit with a delay in-between $\Delta_{\min}$ and $\Delta_{\max}$ the two solutions of $S^{TC}(\Delta) = \Gamma$.

From the participation constraint of the supplier, one can obtain c as a function of Δ. Replacing this expression into the objective function and taking the first order condition we then obtain the optimal trade credit maturity,

Δ, and the corresponding price, c. As before, the equation corresponding to the first order condition does not have an analytical solution, and will be solved numerically.[10]

The buyer will procure from the external supplier under trade credit as long as the profits obtained under the Stackelberg equilibrium exceed his reservation value Θ. Since empirical evidence suggests that the buyer's status quo is to internalize procurement, we assume that $\Theta = B^{TC}(c_i, \Delta_i)$, the buyer's profits under internal procurement.

5. The effects of regulation

We now turn our attention to the social planner's perspective and to regulation. From the social planner's perspective, the optimal pair (c, Δ) is chosen taking into account both the buyer and the supplier's valuations of the project. Moreover, the social planner can put different weights on the buyer and on the supplier:

$$\operatorname*{Max}_{c,\Delta} \ \alpha B^{TC}(c, \Delta) + (1-\alpha) S^{TC}(c, \Delta), \tag{15}$$

where α is the relative weight the planner puts on the buyer relative to the supplier. The lower α, the more the social planner wants to protect suppliers, thus the lower Δ. Indeed, as previously discussed, one common policy measure is for the policy maker to impose an upper limit on the trade credit maturity, Δ_n. By imposing an upper limit on the trade credit maturity, the social planner is trying to increase the supplier's bargaining power. However, we will see that this will not necessarily be attained, and that regulation might have undesired consequences. We now discuss the implications of such a measure. We consider first the effects of regulation in the case of internal procurement (cooperative solution), and then in the case of external procurement with weak suppliers (Stackelberg game).

Consider the case of internal procurement. In an integrated supply chain, the trade credit terms are set as the result of a cooperative solution. The objective of a vertically integrated supply chain is to maximize the total supply chain value, V. Moreover, we know that there exist an infinite number of pairs (c, Δ) that maximize V. Therefore, if one parameter is fixed since it

[10] Note that the Stackelberg solution coincides with the Nash bargaining solution for the polar case $\eta = 1$.

is imposed by regulation, Δ_n, the other one can still be adjusted such that the total supply chain value does not change. Therefore, regulation does not affect internal procurement since parties can undo regulation in a vertically integrated supply chain.

We move now to the case of external procurement with weak supplier. Let (c^*, Δ^*) be the solution to the Stackelberg game, where $\Delta^* > \Delta_n$. Once the regulation is implemented, the buyer will have to solve a constrained maximization problem:

$$\begin{aligned} Max \quad & B^{TC}(c, \Delta) \\ s.t. \quad & S^{TC}(c, \Delta) \geq \Gamma \\ & \Delta \leq \Delta_n \end{aligned} \tag{16}$$

Given the lower trade credit maturity imposed by regulation, the buyer now chooses $\Delta = \Delta_n$, decreasing the maturity until the limit allowed by regulation. However, a lower trade credit maturity does not imply an increase in the supplier's valuation since the trade credit maturity is not the only instrument that the two parties have available. Indeed, a price reduction could be used a complementary strategy to a decrease in the trade credit maturity (see Fabbri and Klapper, 2016 for evidence of such complementarity). The new price c will be set such that the supplier remains indifferent at his reservation value Γ. Thus, $c_n < c^*$. Nevertheless, the two instruments, price and maturity, are not perfect complements, in the sense that the two parties cannot "undo" regulation. That is, one cannot undo regulation by adjusting c so as to perfectly compensate for the changing Δ. One of the counterparty will see its profits decreased with respect to the pre-regulation situation. If c_n is chosen such that the supplier remains indifferent at his reservation value, the buyer will suffer: $B_n \equiv B^{TC}(c_n, \Delta_n) < B_{init} \equiv B^{TC}(c^*, \Delta^*)$. Moreover, if $B_n < \Theta$, then the buyer will prefer to internalize procurement given the new regulation. As we have seen above, unlike external procurement, regulation does not affect internal procurement since parties can undo regulation in a vertically integrated supply chain. Indeed, in the latter we have a cooperative solution, and not a non-cooperative bargaining game as in the case of external procurement. Similarly, if c_n is chosen such that the buyer is not affected by the regulation, $B_n = B_{init}$, then the supplier will suffer a loss, $S_n < S_{init} = \Gamma$.

The above discussion is summarized in the following proposition:

Proposition 6: *a) In the case of internal procurement (vertically integrated supply chain), regulation can be undone since we have a cooperative solution.*

- *The supply chain will choose c so that the supply chain value remains the same $V_n = V_{init}$. Since there exists an infinity of pairs (c, Δ) that maximize V, fixing one instrument (Δ_n imposed by regulation) will still allow achieving first best by simply adjusting the other instrument, c.*

b) In the case of external procurement, choosing Δ and c is not a zero-sum game since these parameters are the outcome of a non-cooperative bargaining game. One cannot undo the regulation by adjusting c so as to fully compensate for the changing Δ. One of the parties will be negatively affected by regulation. Consider S_{init}, B_{init}, c_{init}, Δ_{init}, and then a regulation shock Δ_n. Then one may change c_{init} to c_n:

- *First, such that $S_n = S_{init}$ (one expects a moderate decrease), then $B_n < B_{init}$, and the new value could be below the buyer's status quo, Θ;*
- *Second, such that $B_n = B_{init}$ (one expects a larger decrease), then $S_n < S_{init}$ and the new value could be below the supplier's status quo, Γ.*

Corollary 2: The model predicts that following a regulation that imposes an upper limit on trade credit maturity, trade credit will decrease and internal procurement will increase.

These predictions are in line with the empirical findings of Breza and Liberman (2017). When analyzing a change in regulation regarding Chilean supermarkets and their suppliers that limits the delay period from 90 to 30 days, they find that trade credit decreases by 11% and vertical integration is more likely, that is, the superstore procures from a wholly owned subsidiary. Nevertheless, they show that this is costly since the superstore is not able to replicate the pre-regulation market equilibrium. Moreover, they show that suppliers also adjust through prices: procurement prices decrease by 3.8% for treated firms relative to control firms in their sample. Furthermore, the size of this effect could be underestimated if firms become unprofitable below a threshold price causing them to exit the market, in line with our implications regarding the supplier's status quo, Γ that could be broken after the regulation.

6. Numerical simulations

We now illustrate our model using numerical simulations. The parameter values are set as follows: $r = 0.04$, $\mu = 0$, $\sigma = 0.2$, $P_0 = 10$, $\gamma = 8$. These values are standard in real options literaure in corporate finance.

6.1. The no trade credit case

Figure 2 plots in panel a) S^{NT}, B^{NT}, and V^{NT} as a function of the coupon $c \geq \gamma$. We can see that the total supply chain value, V^{NT}, decreases with c and is maximized for $c = \gamma$. This is precisely one mechanism to coordinate the supply chain, setting the wholesale price equal to the production cost. For other values of the wholesale price the supply chain is not coordinated, and the buyer chooses to stop earlier than optimal, $K^{NT} > KNT$, as we can see in panel b) of the same figure.

[Figure 2 about here.]

6.2. The trade credit case

We now consider the trade credit case. Figure 3 panel a) plots the supplier value under both trade credit and no trade credit as a function of the trade credit period, Δ, for a wholesale price $c = 9$. For $\Delta = 0$, the two values coincide as expected. We note that the supplier value under trade credit is a hump-shape function of Δ. Although offering trade credit implies a cost of delay for the supplier, it also implies an advantage due to the added value of flexibility. The buyer now stops later than in the case without trade credit, as we can see in panel b). Since the threshold is lower, this shows that trade credit is a coordination technique as we are getting closer to the supply chain optimal threshold K^{NT*}. For $\Delta \leq 0.16$, we have that $S^{TC} \geq S^{NT}$, thus the value of flexibility dominates the cost of delay implied by trade credit. In this case it is optimal for the supplier to offer trade credit for free.

[Figure 3 about here.]

6.3. Designing trade credit terms

When designing the trade credit terms in an integrated supply chain there is an infinite number of pairs (c, Δ) that maximize the total supply chain value. When the wholesale price is equal to the production cost, $c = \gamma = 8$, the optimal trade credit maturity is zero, $\Delta = 0$. In this case the values of the supply chain with or without trade credit coincide, $V^{TC} = V^{NT}$ and are equal to 90 for our base case parameter values. For higher values of the wholesale price, the supply chain under no trade credit is not coordinated, thus its value will be lower. Under trade credit however, an increase in the wholesale price can be compensated by an increase in the trade credit maturity, hence achieving the maximum supply chain value. When trade credit occurs under external procurement with non-cooperative Nash bargaining, the optimal pair (c, Δ) is determined by the bargaining power of the two parties, η (supplier) and $1 - \eta$ (buyer). We present optimal trade credit terms in Table 1 for Δ ranging between 0 and 180 days, the most common trade credit maturity in practice. The corresponding bargaining power of the parties and values for the buyer, supplier and total supply chain are also presented, starting from the Stacklerberg equilibrium corresponding to $\eta = 1$.[11] A decrease of 36 days in the trade credit maturity from 72 to 36 days is equivalent to a price discount of 3.09%.

[Table 1 about here.]

6.4. The effects of regulation

Let us now illustrate the effects of regulation numerically, assuming that the trade credit maturity is decreased from 90 to 30 days. This corresponds to the change in regulation affecting Chilean supermarkets analyzed by Breza and Liberman (2017). Thus we assume that $\Delta_{init} = 0.25$ years, that is, a maturity of 90 days. Then the initial optimal trade credit terms are $(c_{init}, \Delta_{init}) = (8.86, 0.25)$. The corresponding initial values of the supplier and buyer are respectively $S^{TC}_{init} = 11.5725$ and $B^{TC}_{init} = 78.4275$. Assume that now by regulation we have $\Delta_n = 0.0833$ years (30 days). For this reduced trade credit maturity, and assuming no change in the initial coupon, the new values of the buyer and supplier are: $B^{TC}(c_{init}, \Delta_n) = 77.8577 < B^{TC}_{init}$ and

[11] We assume that the status quo of the two parties is zero, $\Gamma = \Phi = 0$.

$S^{TC}(c_{init}, \Delta_n) = 12.0419 > S^{TC}_{init}$.[12] Thus in principle, the regulator seems to achieve its purpose, increase the value of the supplier at the expense of the buyer.

However, if the buyer's status quo is larger than his new reduced value, the buyer could stop the relationship with the supplier and switch to internal procurement. In that case, in order not to lose the business with the buyer, the supplier will agree to decrease the wholesale price asked to the buyer to compensate him for the decrease in trade credit maturity, so as to at least guarantee a value for the buyer equal to his status quo. If the supplier wants to compensate the buyer for the loss, the new wholesale price would be set to maintain the value of the buyer constant, $B_n = B_{init} = 78.4275$. In this case the new wholesale price is $c_n = 8.8192 < c_{init}$, corresponding to a decrease of 0.46%. Nevertheless, the decrease in the wholesale price will imply that the value of the supplier decreases to $S^{TC}_n = 11.4905 < S^{TC}_{init}$. Intuitively, the initial trade credit terms were optimally set to maximize the supply chain value, which was splitted among the two parties according to their bargaining power. When one of the terms of the contract is exogenously imposed by regulation, the trade credit terms are no longer optimal, leading to a decrease in total supply chain value. If the wholesale price is chosen to keep the value of the buyer constant, the supplier value will decrease. The supplier will accept the new situation as long as his status quo is below this value.

Another possibility is to set the new wholesale price to maintain the value of the supplier equal, $S_n = S_{init} = 11.5725$. Since the supplier benefits from the imposed reduction in maturity, a reduction in the wholesale price charged to the buyer is needed to maintain the supplier's value constant, although not such a large reduction as before. The new wholesale price will be $c_n = 8.8253 < c_{init}$. A reduction of 60 days in the wholesale price will thus lead to a decrease of 0.39% in the wholesale price. The buyer's new value will nevertheless decrease $B_n = 78.3429 < B_{init} = 78.4275$. When one

[12] An increase in trade credit maturity, as explained before, has two opposite effects on the supplier value: a negative effect, the cost of delay, and a positive one, the value of flexibility since it guarantees the business for a longer period. Therefore, a decrease in maturity imposed by the regulator could have both a positive or a negative effect on the supplier value, depending on which of the effects dominates. However, the optimal trade credit maturity that maximizes the supply chain value is much larger than the optimal one from the supplier's perspective. Hence, a decrease in trade credit maturity leads to an increase in supplier's value, assuming c is unchanged.

of the trade credit parameters is fixed by regulation, first best is no longer achievable, leading to a loss in value.

Comparing to the reduction in wholesale prices documented by Breza and Liberman (2017) for the case of Chilean supermarkets, 3.8%, the reduction in our numerical example is smaller. The limited impact can be due to the fact that we model the supply chain using perpetual cash flows. In practice, trade credit relationships have an average maturity of 8 years (Murfin and Njoroge, 2015). As we argue in the following section, our model is a first attempt to explain recent empirical findings in trade credit, and we have modeled it in infinite time for tractability. Future research could try to address this limitation, considering trade credit in finite time.

7. Conclusions

In this paper we provide a real options analysis of trade credit design. Our model offers a first theoretical explanation for the recent empirical findings regarding trade credit from small suppliers to large buyers and the effects of regulation. We show that suppliers might find it optimal to offer trade credit at zero cost. Moreover, trade credit acts as a coordination device for the supply chain. Regarding regulation, we show that the two parties, buyer and supplier, cannot undo regulation, despite the complementarity of price discount with trade credit. Our implications are in line with empirical evidence that shows that following regulation that imposes a limit on trade credit maturity, trade credit decreases and internal procurement increases.

There are several implicit assumptions behind our framework. First, in deriving our results we have assumed a perpetual demand. In practice however, the average length of trade credit contracts is of 8 years (Murfin and Njoroge, 2015). An interesting avenue for future research would be to model trade credit within a finite time framework. This could potentially lead to obtaining larger impacts of regulation on the values of the buyer and the supplier. Second, we have assumed that the supplier's cash flows and production costs are deterministic. Future research might consider introducing two sources of risk, one for the supplier and one for the buyer, although at the cost of losing tractability. Third, we have abstracted from default risk. We know nevertheless that financial risk is an important element of trade credit contracts. Klapper et al. (2012) suggested that the financial risk matters and that this issue may impact the trade credit design. For trade credit relationships with large buyers and small suppliers, the buyer might take into account the supplier's default risk when setting the contractual terms. This is left for future research.

From a policy perspective, our model highlights that offering trade credit has not only costs, but also benefits. In particular, trade credit can be favorable for the firm extending it since it enables and guarantees trade. Thus policy makers considering a change in regulation affecting trade credit should consider both its benefits and costs in their analysis.

Appendix

Proof of Proposition 1: Following standard option theory (Dixit and Pindyck, 1994), we have:

$$
\begin{aligned}
B^{NT} &= E_0\left(\int_0^{\tau_B}(P_s-c)e^{-rs}ds\right) && (.1)\\
&= E_0\left(\int_0^{\infty}(P_s-c)e^{-rs}ds - \int_{\tau_B}^{\infty}(P_s-c)e^{-rs}ds\right) && (.2)\\
&= \left[\frac{P_0}{r-\mu}-\frac{c}{r}\right]-\left[\frac{K^{NT}}{r-\mu}-\frac{c}{r}\right]\left(\frac{P_0}{K^{NT}}\right)^{-X} && (.3)
\end{aligned}
$$

Then the smooth-pasting condition applied to $B^{NT}\left(K^{NT}\right)$. $\Diamond$

Proof of Proposition 2:

The buyer's valuation is given by:

$$
\begin{aligned}
B^{TC} &= E_0\left(\int_0^{\Delta}P_se^{-rs}ds\right)+E_0\left(1_{\{P_\Delta<K\}}\int_{\Delta}^{2\Delta}-ce^{-rs}ds\right) && (.4)\\
&+ E_0\left[1_{\{P_\Delta>K\}}\left(\int_{\Delta}^{\tau}P_se^{-rs}ds+\int_{\Delta}^{\tau+\Delta}-ce^{-rs}ds\right)\right] && (.5)
\end{aligned}
$$

We now compute each of these terms separately.

$$E_0\left(\int_0^{\Delta}P_se^{-rs}ds\right)=\frac{P_0}{r-\mu}\left(1-e^{-(r-\mu)\Delta}\right)$$

Following Shackleton and Wojakowski (2007), the second term is a cash-or-nothing put option on $c/r(1-e^{-r\Delta})$ with exercise price K and maturity Δ:

$$
\begin{aligned}
E_0\left(1_{\{P_\Delta<K\}}\int_\Delta^{2\Delta}-ce^{-rs}ds\right) &= E_0\left(e^{-r\Delta}1_{\{P_\Delta<K\}}\int_\Delta^{2\Delta}-ce^{-r(s-\Delta)}ds\right)\\
&= -E_0\left(e^{-r\Delta}1_{\{P_\Delta<K\}}\frac{c(1-e^{-r\Delta})}{r}\right)\\
&= -\frac{c}{r}e^{-r\Delta}(1-e^{-r\Delta})N(-d_2(\mu,\Delta))
\end{aligned}
$$

The third term can be split as follows:

$$
\begin{aligned}
E_0\left[1_{\{P_\Delta>K\}}\left(\int_\Delta^{\tau}P_se^{-rs}ds+\int_\Delta^{\tau+\Delta}-ce^{-rs}ds\right)\right] &= E_0\left[1_{\{P_\Delta>K\}}\left(\int_\Delta^{\tau}P_se^{-rs}ds-\int_\Delta^{\tau}ce^{-rs}ds\right)\right.\\
&- \left.\int_\tau^{\tau+\Delta}1_{\{P_\Delta>K\}}ce^{-rs}ds\right]\\
&= E_0\left[1_{\{P_\Delta>K\}}\left(\int_\Delta^{\tau}(P_s-c)e^{-rs}ds\right)\right.\\
&- \left.\int_\tau^{\tau+\Delta}1_{\{P_\Delta>K\}}ce^{-rs}ds\right]
\end{aligned}
$$

The first part of the third term represents the expected value at time zero of receiving the cash flow P_s-c starting at time Δ and for as long as the price is above the stopping threshold K, conditional on the price at time Δ being above K. Using standard option theory, this value at time Δ is equal to: $\frac{P_\Delta}{r-\mu}-\frac{c}{r}-\left(\frac{K}{r-\mu}-\frac{c}{r}\right)\left(\frac{P_\Delta}{K}\right)^{-X}$. Similarly, the value of the second part of the third term at time Δ is equal to: $-\frac{c}{r}(1-e^{-r\Delta})\left(\frac{P_\Delta}{K}\right)^{-X}$. Then we can rewrite the third term as follows:

$$
e^{-r\Delta}E_0\left[1_{\{P_\Delta>K\}}\left(\frac{P_\Delta}{r-\mu}-\frac{c}{r}-\left(\frac{K}{r-\mu}-\frac{c}{r}\right)\left(\frac{P_\Delta}{K}\right)^{-X}-\frac{c}{r}(1-e^{-r\Delta})\left(\frac{P_\Delta}{K}\right)^{-X}\right)\right]
$$

Following Shackleton and Wojakowski (2007), the first two terms of the previous expression represent the difference between an asset-or-nothing call option on $\frac{P_\Delta}{r-\mu}$, and a cash-or-nothing call option on $\frac{c}{r}$, with exercise prices

K, and maturity Δ. Moreover, from the Appendix A of Shackleton and Wojakowski (2007):

$$e^{-r\Delta}E_0\left[1_{\{P_\Delta>K\}}P_\Delta^{-X}\right]=P_0^{-X}N(d_2(\mu-X\sigma^2,\Delta))$$

Therefore, the third term of equation (.5) is equal to:

$$\begin{aligned}
&= \frac{P_0e^{-(r-\mu)\Delta}}{r-\mu}N(d_1(\mu,\Delta))-\frac{ce^{-r\Delta}}{r}N(d_2(\mu,\Delta))-\left(\frac{K}{r-\mu}-\frac{c}{r}\right)\left(\frac{P_0}{K}\right)^{-X}\\
&* \quad N(d2(\mu-X\sigma^2,\Delta))-\frac{c}{r}(1-e^{-r\Delta})\left(\frac{P_0}{K}\right)^{-X}N(d2(\mu-X\sigma^2,\Delta))\\
&= \frac{P_0e^{-(r-\mu)\Delta}}{r-\mu}N(d_1(\mu,\Delta))-\frac{ce^{-r\Delta}}{r}N(d_2(\mu,\Delta))-\left(\frac{K}{r-\mu}-\frac{ce^{-r\Delta}}{r}\right)\left(\frac{P_0}{K}\right)^{-X}\\
&* \quad N(d2(\mu-X\sigma^2,\Delta))
\end{aligned}$$

Suming these three terms, and using the property that $1-N(x)=N(-x)$, we obtain equation (4). In this proof we have used Shackleton and Wojakowski (2007)'s valuation of forward start perpetuities, as well as floors and caps. This valuation problem is also very closely related to the valuation of deferred perpetual American puts in Gerber and Shiu (1993). A deferred perpetual American put can only be exercised after a delay of n years. In our case, this delay is Δ.

Proof of Proposition 3: From equation (9) we obtain:

$$\frac{A}{r-\mu}=\frac{ce^{-r\Delta}B}{rK}$$

where A and B are given by the following expressions:

$$\begin{aligned}
A(\Delta) &= 1-e^{-(r-\mu)\Delta}+e^{-(r-\mu)\Delta}N\left(\frac{(\mu+\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right)+\frac{e^{-(r-\mu)\Delta}}{\sigma\sqrt{\Delta}}\phi\left(\frac{(\mu+\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right)\\
&- \left[-XN\left(\frac{(\mu-X\sigma^2-\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right)+\frac{1}{\sigma\sqrt{\Delta}}\phi\left(\frac{(\mu-X\sigma^2-\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right)\right]
\end{aligned}$$

that simplifies to the expresion in the proposition and

$$\begin{aligned}
B(\Delta) &= \frac{e^{-r\Delta}}{\sigma\sqrt{\Delta}}\phi\left(\frac{(\mu-\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right)-\frac{1}{\sigma\sqrt{\Delta}}\phi\left(\frac{(\mu-X\sigma^2-\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right) \qquad (.6)\\
&+ XN\left(\frac{(\mu-X\sigma^2-\sigma^2/2)\,\Delta}{\sigma\sqrt{\Delta}}\right)
\end{aligned}$$

and the associated expression follows.

Proof of Proposition 4:

Remember that

$$K^{TC} = e^{-r\Delta}\frac{c(r-\mu)}{r}\frac{B}{A}, \tag{.7}$$

and

$$K^{NT} = \frac{c\,(r-\mu)}{r}\frac{X}{X+1}$$

Then we have that

$$\begin{aligned} K^{TC} &\leq K^{NT} \Leftrightarrow \frac{B}{A} \leq \frac{X}{X+1} \qquad (.8) \\ \Leftrightarrow &\quad \frac{XN\left(\frac{\left(\mu - X\sigma^2 - \sigma^2/2\right)\Delta}{\sigma\sqrt{\Delta}}\right)}{1 - e^{-(r-\mu)\Delta}(1 - N\left(\frac{(\mu+\sigma^2/2)\Delta}{\sigma\sqrt{\Delta}}\right)) + XN\left(\frac{(\mu - X\sigma^2 - \sigma^2/2)\Delta}{\sigma\sqrt{\Delta}}\right)} \leq \frac{X}{X+1} \end{aligned}$$

Denote $n_2 \equiv N\left(\frac{\left(\mu - X\sigma^2 - \sigma^2/2\right)\Delta}{\sigma\sqrt{\Delta}}\right)$ and $n_1 \equiv N\left(\frac{\left(\mu + \sigma^2/2\right)\Delta}{\sigma\sqrt{\Delta}}\right)$. Since we know that $\mu - X\sigma^2 - \sigma^2/2 < \mu + \sigma^2/2$, and the normal cdf is an increasing function, we have that $n_2 < n_1$.

Using this notation in the previous inequality we obtain:

$$\begin{aligned} K^{TC} &\leq K^{NT} \Leftrightarrow \frac{Xn_2}{1 - e^{-(r-\mu)\Delta}(1-n_1) + Xn_2} \leq \frac{X}{X+1} \qquad (.9) \\ \Leftrightarrow &\quad X^2 n_2 + Xn_2 \leq X - Xe^{-(r-\mu)\Delta}(1-n_1) + X^2 n_2 \\ \Leftrightarrow &\quad e^{-(r-\mu)\Delta}(1-n_1) \leq 1 - n_2 \end{aligned}$$

Since we have $n_2 < n_1$, this implies $1 - n_1 < 1 - n_2$. Therefore, the previous inequality is true. So we have that $\frac{B}{A} \leq \frac{X}{X+1}$ and thus $K^{TC} \leq K^{NT}$.

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Figures

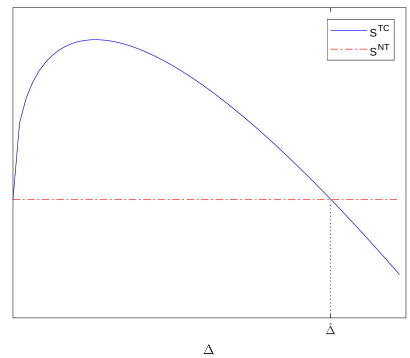
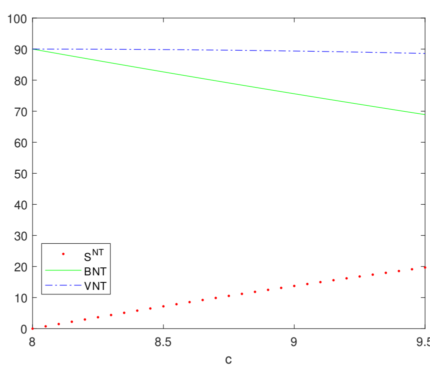
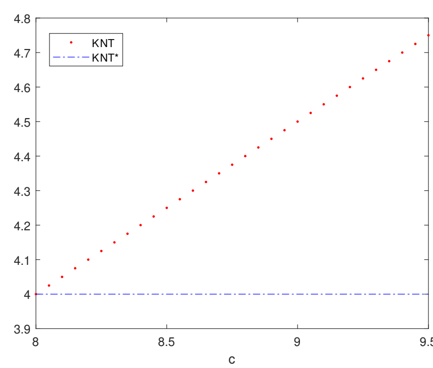


Figure 1: Supplier's valuation with and without trade credit as a function of the trade credit maturity Δ.

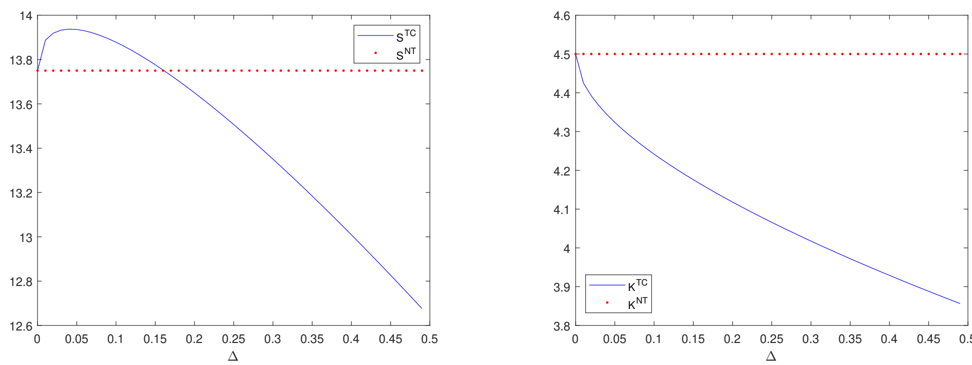


a) Values



b) Stopping thresholds

Figure 2: The no trade credit case



a) Supplier value

b) Stopping threshold

Figure 3: The trade credit case ($c = 9$)

Tables

Table 1: Design of trade credit terms (c, Δ) according to bargaining power η

c	Δ	η	S^{TC}	B^{TC}	V^{TC}	V^{NT}
8.00	0 years (0 days)	1	0	90	90	90
8.48	0.1 years (36 days)	0.9256	6.6959	83.3041	90	89.8560
8.75	0.2 years (72 days)	0.8867	10.1977	79.8022	90	89.7598
8.96	0.3 years (108 days)	0.8578	12.7976	77.2024	90	89.6280
9.16	0.4 years (144 days)	0.8309	15.2218	74.7782	90	89.1590
9.35	0.5 years (180 days)	0.8058	17.4775	72.5225	90	88.8609