Real Option Signaling Games of Debt Financing Using Equity Guarantee Swaps under Asymmetric Information

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Abstract

We analyze the real option signaling game models of debt financing of a risky project under information asymmetry, where the firm quality is only known to the firm management but not outsiders. The firm decides on the optimal investment timing of the risky project that requires upfront fixed funding cost and subsequent operating costs. The fixed funding cost is financed via either direct bank loan or entering into a three-party equity guarantee swap (EGS) that involves a bank granting the loan and third party guarantor. Under the EGS agreement, the guarantor is obligated to pay all the future coupon stream to the bank upon default of the firm. In return for the provision of the guarantee, the guarantor obtains certain proportional share of equity of the firm at the time when the swap agreement is signed. The share of equity demanded by the guarantor depends on the outside investors' belief on the firm quality. The low-type firm has the incentive to mimic the investment strategy of being high-type in terms of investment timing and share of equity. The high-type firm may adopt the appropriate separating strategy by speeding up investment or choosing an alternative financing choice. The resulting loss of the real option value of the investment opportunity represents the information cost under separating strategies. We examine the incentive compatibility constraints faced by the firm under different quality types and discuss characterization of the separating and pooling equilibriums. Unlike the usual assumption of perpetuity of investment opportunity, our real option model assumes the time window of the investment opportunity to be finite. We explore how the information cost and nature of separating and pooling equilibriums evolves over the finite time span of the investment opportunity. The information costs and investment thresholds exhibit interesting time dependent behaviors. We examine the firm's investment and financing choice between EGS and the direct bank loan against time and other parameters by comparing the corresponding information costs and investment thresholds.

Keywords: debt financing, signaling games, separating and pooling equilibriums, real options, information costs

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1 Introduction

Analysis of investment and financing choices under information asymmetry between the firm management and outsiders has been one of the focuses of research in corporate finance. The earliest static investment and financing choices models (Myers, 1984; Myers and Majluf, 1984) establish the pecking order theory under adverse selection, where debt is preferred to equity. The later works extend the model to the dynamic version to consider optimality of debt versus equity-like securities in financing choices. Under their dynamic signaling game model, Strebulaev *et al.* (2014) show that a project is financed by equity if the probability of success is low, otherwise debt financing is preferred. Yang and Zeng (2019) propose a theory of security design in financing entrepreneurial production. They show that debt is optimal when information is not valuable for production and an efficient combination of debt and equity is optimal when information is valuable.

The real option pricing theory for investment problems (Brennan and Schwartz, 1985; McDonald and Siegel, 1986) can be extended to solve dynamic signaling game problems of investment and debt-equity financing choices. Morellec and Schürhoff (2011) develop dynamic real option signaling game models of corporate investment and financing. In their models, a firm of either one of the two quality types is assumed to hold a real investment opportunity with a perpetual life and raises fund by issuing equity or debt. Assuming that the exact firm type is the private information held by the firm, they calculate the real option values and investment thresholds under separating and pooling equilibriums. They also show how the firm may signal its quality type via financing choices between equity and debt. Clausen and Flor (2015) extend the real option signaling games of Morellec and schürhoff (2011) by including the abandonment right and assets-in-place. They find that firms are more likely to choose debt against equity when their assets-in-place is higher. Fulghieri *et al.* (2015) show that when asymmetry information has a small impact on the right tail of the firm value distribution, debt is preferred for low capital needs while convertible debt is optimal for larger capital needs. In addition, Grenadier and Wang (2005), Grenadier and Malenko (2011), Xu and Li (2010), Bouvard (2014) consider the effect of asymmetric information on the investment and financing policies of different firm types under various real option signaling game models. Besides capital financing, real options signaling game models have been adopted to analyze various corporate finance issues, like liquidation timing of a distressed firm (Nishihara and Shibata, 2017), strategic investment games of incumbent and entrant firms (Watanabe, 2016), decisions on selling out IPO (Nishihara, 2016), mergers and acquisitions strategies of bidder and target firms (Leung and Kwok, 2018).

There exist several potential extensions of the above real option signaling game models in finance. First, most real option models of investment and financing assume investment opportunity to be perpetual, which may be queried since technologies have finite life spans. Gryglewicz *et al.* (2008) discard the perpetuality assumption and assume a finite project life in real option investment models. They observe the acceleration of investment when uncertainty increases as time comes closer to expiry. Wang and Kwok (2019) analyze the real option signaling games of equity financing with information asymmetry under a finite time horizon. Second, most of the previous works on corporate financing consider either debt or equity financing. Such capabilities are limited to listed companies with sufficient resources. However, small- or medium-sized firms cannot issue equity or debt and can only raise fund

via bank loans. In China, the innovative equity guarantee swaps (EGS) are introduced to overcome the difficulties that private small- and medium-sized firms may not be able to secure bank loans. To hedge the firm default risk faced by the bank, the EGS agreement introduces a third party profit seeking guarantor. The guarantor has the obligation to pay all the future coupon stream to the bank upon default of the firm. In return for the provision of the guarantee, the guarantor obtains a proportional share of equity of the firm at the time when the swap agreement is signed. Since EGS involves both the bank loan and share of equity, the nature of EGS is somehow similar to convertible bonds, which is a hybrid of debt financing and equity participation. Wang *et al.* (2015) initiate the theoretical studies of EGS in China. Their real option models show that these swaps provide substantial diversification benefits and tax advantages. Luo *et al.* (2016) construct more refined real option models to analyze the investment timing of EGS when the revenue flow dynamics follows the double exponential jump-diffusion models. Luo and Yang (2017) analyze the separating equilibrium in real option signaling game models for EGS under asymmetric information. Dybvig *et al.* (2016) perform empirical studies on the third party loan guarantees for small- and medium-sized enterprises in China. Their studies reveal that loans screened by a third party guarantor have low default rate, indicating the important informational role played by these equity guarantee swaps.

This paper analyze the real option signaling game models of debt financing using either direct bank loans or equity guarantee swaps under asymmetric information. We consider a small- or medium-sized firm facing an investment opportunity for a risky project when a new technology comes into existence. The investment opportunity expires within a finite time horizon. The firm has no ability to issue equity and can only raise capital to fund the project with a bank loan. The firm can choose to adopt a direct bank loan, or enter into an equity guarantee swap (EGS) with a guarantor. In our real signaling game model, the firm quality as characterized by the revenue flow generated from the project is assumed to be the private information held by the firm manager and is not accessible to outsiders. The outsiders update their belief on the firm type by observing the investment timing chosen by the firm. We characterize the time-evolving investment thresholds of different firm types for EGS and direct bank loan under separating and pooling equilibriums. We focus on the studies of the financing choice between EGS and direct bank loan under perfect and asymmetric information by examining the real option values and information costs.

This paper is organized as follows. Section 2 constructs the real option signaling game model of debt financing under finite time horizon and define the belief system of the loan counterparty. The values of the firm's equity, default right and liability are expressed under the benchmark case of complete information. The direct bank loan and equity guarantee swap are first considered separately and then compared between each other. Section 3 discusses the investment behaviors of the firm in separating equilibrium under EGS and the direct bank loan. Incentive compatibility constraints, binding thresholds and investment thresholds for separating strategies are determined for the high- and low-type firm. Section 4 examines pooling equilibriums under debt financing. Besides incentive compatibility constraints, optimal investment thresholds and belief systems, we focus on determining the share of equity under EGS and the unified coupon rate under the direct bank loan. Section 5 presents numerical results and discusses their implications. We examine the time dependence of investment thresholds and value functions under the finite time horizon. Also, we

consider the financing choice between EGS and direct bank loans by observing the corresponding information costs and real option values. Section 6 highlights the main results and concludes the paper.

2 Model formulation of debt financing

We assume that all agents in the financial markets are risk neutral and discount cash flows at the constant riskless interest rate r. The investment opportunity is assumed to have a finite life with a fixed time horizon T, while the revenue flow generated by the project have a perpetual life. Once the project is launched, it produces a continuous revenue shock variable that depends on the firm type k. In this paper, we consider two types of the firm quality: high-type ($k = h$) and low-type ($k = l$) firm. At time t, the net revenue shock variable is denoted by $\lambda X_t - f$, where $\lambda > 0$ is a multiplier which takes value of λ_h or λ_l ($\lambda_h > \lambda_l > 0$), $f > 0$ is the constant operating expenses of the investment project and X_t is the stochastic shock variable representing the revenue shock variable generated from the project. We assume that X_t is observable and evolves according to the following Geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \, \mathrm{d}t + \sigma X_t \, \mathrm{d}Z_t, \quad X_0 > 0,$$

where Z_t is the standard Brownian motion, $\mu < r$ is the constant drift rate and $\sigma > 0$ is the constant volatility.

To fund the investment on the project, the firm has to pay a constant upfront direct cost I. The investment is irreversible and the investor has the option to wait. The firm manager may choose to raise the capital amount I for funding the project by direct bank loan or entering into a three-party equity guarantee swap (EGS). The swap involves a bank that grants the loan and a third party guarantor.

Conditioned on no default, the present value of the perpetual revenue flow generated by the investment project at time t is given by

$$\mathbb{E}_t \left[\int_t^\infty \mathrm{e}^{-r(u-t)} \lambda X_u \, \mathrm{d}u \, \middle| \, X_t = X \right] = \frac{\lambda X}{r - \mu}, \tag{2.1}$$

where $\mathbb{E}_t$ denotes the expectation based on the information at time t and λ may assume the value λ_h or λ_l. We write

$$\Pi(X) = \frac{X}{r - \mu}$$

for notational convenience. Let F denote the present value of the future perpetual stream of operating expenses of the investment project, where

$$F = \int_t^\infty \mathrm{e}^{-r(u-t)} f \, \mathrm{d}u = \frac{f}{r}. \tag{2.2}$$

Upon default, the equity value of the firm becomes zero and the ownership of the firm is transferred to the bank under the direct bank loan or the equity guarantee swap guarantor under EGS. In either case, a fraction $\alpha \in (0, 1)$ of the firm equity value will be lost due to

bankruptcy cost. Thus, the revenue flow received by the bank or guarantor after taking over the firm becomes $(1 - \alpha)\lambda X_t - f$.

The asymmetric information in our real signaling game model is the firm quality, as characterized by the revenue shock variable multiplier λ. The firm quality, either as high-type or low-type, is the private information held only by the manager of the firm. The loan counterparty is either the bank (under direct bank loan) or guarantor (under EGS). Let Λ be the discrete Bernoulli random variable that characterizes the multiplier λ. The loan counterparty only forms a belief system of the firm type, denoted by the probabilistic representation: $\mathbb{P}[\Lambda = \lambda_h] = p$ and $\mathbb{P}[\Lambda = \lambda_l] = 1 - p$, where $\lambda_h > \lambda_l > 0$; p is deterministic and $p \in (0, 1)$. To signify the firm type, the signal sent by the firm is the threshold of X_t launching the investment. After receiving the signal, the loan counterparty's belief on Λ can be categorized into three types:

(i) $\Lambda = \lambda_l$, the true "low" type of the firm is revealed to the counterparty;

(ii) $\Lambda = \lambda_h$, the true "high" type of the firm is revealed to the counterparty;

(iii) $\Lambda = \lambda_p = p\lambda_h + (1 - p)\lambda_l$, a probabilistic belief on Λ since the signal fails to reveal the type of the firm to the counterparty.

To fix the notation in our later discussion, we adopt the following conventions to define various types of variables in our model:

- The subscript θ denotes the financing choice of the entrepreneur: $\theta = g$ for the equity-guarantee swap agreement and $\theta = b$ for the direct bank loan.

- The subscript k denotes the firm type: $k = h$ for the high-type firm and $k = l$ for the low-type firm. Under pooling, we only have probabilistic belief on the firm type.

- The superscript γ denotes the type of equilibrium: $\gamma = c$ for complete information, $\gamma = m$ for mimicking, $\gamma = s$ for separating and $\gamma = p$ for pooling.

- The superscript "$*$" denotes the optimal stopping threshold, the "overline" denotes the binding threshold, the "underline" denotes the default threshold and the "hat" denotes the threshold corresponding to zero net payoff.

For example, $I_{g,l}^m(X)$ represents the intrinsic value I at $X_t = X$ of the low-type firm (l) under mimicking strategy (m) when the financing choice is EGS (g). The firm has to pay the perpetual coupon stream c_θ to the bank under the financing choice θ. Similar to (2.2), the present value of the perpetual continuous coupon stream is found to be $\frac{c_\theta}{r}$, where $\theta = g$ or b.

2.1   Complete information: direct bank loan versus equity guarantee swap

Under the assumption of complete information, the revenue shock variable multiplier λ is known to all parties. We follow the debt financing model of Morellec and Schürhoff (2011),

which assumes the debt holder to have the right to default. First, we derive the value functions of the default right, liability and equity held by various parties when the firm finances the investment via either the direct bank loan or equity guarantee swap under complete information. We provide financial interpretation of various terms in these value functions. We then derive the real option value function before investment under the assumption of investment opportunity of a finite term horizon.

2.1.1 Direct bank loan

Let $D_b(X; \lambda, c_b)$ denote the firm's real option value of the default right of the bank loan at $X_t = X$ and $\Lambda = \lambda$, and c_b is the coupon payment rate paid by the firm to the bank under the issuance of the perpetual bank loan. Let $\underline{X}_b(\lambda, c_b)$ be the optimal default threshold. The optimal default threshold under the perpetual direct bank loan is given by (Morellec and Schürhoff, 2011)

$$\underline{X}_b(\lambda, c_b) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} \left(F + \frac{c_b}{r}\right), \tag{2.3}$$

where

$$\xi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0.$$

The default right can be quantified as the option held by the firm to terminate the liabilities of paying the perpetual coupon stream and operating cost, while forfeit the benefit of receiving the revenue flow. Let $D_b(X; \lambda, c_b)$ denote the option value of the default right, which is the expected value of such net gain. The value function of the default right is found to be (Dixit and Pindyck, 1994)

$$D_b(X; \lambda, c_b) = \left[F + \frac{c_b}{r} - \lambda\Pi(\underline{X}_b(\lambda, c_b))\right] \left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^{\xi}, \tag{2.4}$$

where $\left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^{\xi}$ is interpreted as the probability of hitting the default threshold $\underline{X}_b(\lambda, c_b)$ when the revenue shock variable assumes the level X. Upon default, the bank takes over the ownership of the firm while the receipt of the coupon stream is discontinued. Let α be the fractional loss on the revenue shock variable due to the bankruptcy cost, where $\alpha \in (0, 1)$. The liabilities value borne by the bank arising from the firm's default right is given by

$$L_b(X; \lambda, c_b) = \left[F + \frac{c_b}{r} - (1 - \alpha)\lambda\Pi(\underline{X}_b(\lambda, c_b))\right] \left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^{\xi}. \tag{2.5}$$

Note that $L_b(X; \lambda, c_b) > D_b(X; \lambda, c_b)$ due to the fractional loss α arising from the bankruptcy cost. As expected, $L_b(X; \lambda, c_b)$ equals $D_b(X; \lambda, c_b)$ when $\alpha = 0$.

The equity value $E_b(X; \lambda, c_b)$ of the project after investment under the direct bank loan is the sum of the expected value of the revenue and the value of the default right minus the sum of values of the coupon stream and the operating cost. This gives

$$E_b(X; \lambda, c_b) = \lambda\Pi(X) - F - \frac{c_b}{r} + D_b(X; \lambda, c_b). \tag{2.6}$$

Since we assume that the firm has only one activity: operating on the project and holding the full ownership of the project, the firm's intrinsic value under direct bank loan is the same as the equity value. However, this is different from the scenario under the EGS arrangement, where the firm holds partial ownership of the project [see (2.17) later].

The firm raises the fund amount I to finance the investment. The bank charges the coupon rate c_b for the loan amount I according to the budget constraint:

$$\frac{c_b}{r} - L_b(X; \lambda, c_b) = I. \tag{2.7}$$

This gives an implicit equation for finding c_b, which has implicit dependence on X and λ. Putting all these relations together, we can simplify the firm's equity value as follows:

$$E_b(X; \lambda, c_b) = \lambda \Pi(X) - F - I - L_b(X; \lambda, c_b) + D_b(X; \lambda, c_b)$$

$$= \lambda \Pi(X) - F - I - \alpha \lambda \Pi(\underline{X}_b(\lambda, c_b)) \left[\frac{X}{\underline{X}_b(\lambda, c_b)} \right]^{\xi}. \tag{2.8}$$

Under our finite time horizon model, the investment opportunity faces with the mandated maturity date T. For direct bank loan financing with complete information, the real option value of the firm before investment $V_b^c(X, t; \lambda)$ is given by

$$V_b^c(X, t; \lambda) = \sup_{u \in [t,T]} \mathbb{E}_t \left[e^{-r(u-t)} E_b(X_u; \lambda, c_b) \big| X_t = X \right], \quad 0 < X < X_b^*(t; \lambda), \tag{2.9}$$

where $X_b^*(t; \lambda)$ is the first-best threshold for optimal entry into the bank loan at time t. The solution of $V_b^c(X, t; \lambda)$ involves a nonstandard optimal stopping problem since the exercise payoff is $E_b(X; \lambda, c_b)$, which is highly nonlinear in X.

2.1.2 Equity guarantee swap

The two active parties under the equity guarantee swap (EGS) agreement are the firm and the guarantor, while the bank plays a passive role since the bank bears no liability associated with the default right of the firm. Assuming no default risk of the guarantor, the loan is considered to be risk free for the bank. Therefore, the coupon rate charged by the bank on the bank loan under EGS is given by

$$c_g = rI, \tag{2.10}$$

which has the nice feature that it is independent of X and λ. The corresponding value functions of the firm's default right and guarantor's liability now take simpler analytic forms, namely,

$$D_g(X; \lambda) = \left[F + I - \lambda \Pi(\underline{X}_g(\lambda)) \right] \left[\frac{X}{\underline{X}_g(\lambda)} \right]^{\xi}, \tag{2.11}$$

and

$$L_g(X; \lambda) = \left[F + I - (1 - \alpha) \lambda \Pi(\underline{X}_g(\lambda)) \right] \left[\frac{X}{\underline{X}_g(\lambda)} \right]^{\xi}, \tag{2.12}$$

where the optimal default threshold under perpetuality of the project is given by

$$\underline{X}_g(\lambda) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} (F + I). \tag{2.13}$$

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The firm's equity value under EGS does not include liability $L_g(X; \lambda)$ since the liability arising from the default of the bank loan is borne by the guarantor. As a result, the corresponding firm's equity value under EGS is given by

$$E_g(X; \lambda) = \lambda\Pi(X) - F - I + D_g(X; \lambda). \tag{2.14}$$

In return, the guarantor is entitled to receive the proportional share $\phi(X; \lambda)$ of the firm's equity value $E_g(X; \lambda)$. To achieve a fair deal, the liability is compensated by holding share of equity. The fair value of the proportional share $\phi(X; \lambda)$ is then given by

$$\phi(X; \lambda) = \frac{L_g(X; \lambda)}{E_g(X; \lambda)}$$

$$= \frac{\left[F + I - (1-\alpha)\lambda\Pi(\underline{X}_g(\lambda))\right]\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}}{\lambda\Pi(X) - F - I + \left[F + I - \lambda\Pi(\underline{X}_g(\lambda))\right]\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}}$$

$$= \frac{\eta + \eta_\alpha}{[\lambda\Pi(X)]^{-\xi}(F + I)^{\xi-1}[\lambda\Pi(X) - F - I] + \eta}, \tag{2.15}$$

where

$$\eta = \frac{1}{1-\xi}\left(\frac{\xi}{\xi-1}\right)^{-\xi} > 0, \quad \text{and} \quad \eta_\alpha = \alpha\left(\frac{\xi}{\xi-1}\right)^{1-\xi} > 0. \tag{2.16}$$

Suppose that the firm enters into the bank loan and EGS simultaneously to fund the project, the firm's intrinsic value $I_g^c(X; \lambda)$ under EGS is given by

$$I_g^c(X; \lambda) = [1 - \phi(X; \lambda)]E_g(X; \lambda) = \lambda\Pi(X) - F - I + D_g(X; \lambda) - L_g(X; \lambda)$$

$$= \lambda\Pi(X) - F - I - \alpha\lambda\Pi(\underline{X}_g(\lambda))\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}. \tag{2.17}$$

It is seen that $I_g^c(X; \lambda)$ is an increasing function of X. Let $\hat{X}_g(\lambda)$ denote the revenue shock variable level such that $I_g^c(\hat{X}_g(\lambda); \lambda) = 0$. To observe non-negativity of $I_g^c(X; \lambda)$, we set $I_g^c(X; \lambda) = 0$ when $X \leq \hat{X}_g(\lambda)$. Indeed, non-negativity of $I_g^c(X; \lambda)$ is equivalent to $E_g(X; \lambda) \geq L_g(X; \lambda)$. This is consistent with the observation of the property $\phi(X; \lambda) \in [0, 1]$.

Similarly, we assume that the investment opportunity lasts until the maturity date T. For the EGS agreement under complete information, the real option value of the firm before investment $V_g^c(X, t; \lambda)$ is given by

$$V_g^c(X, t; \lambda) = \sup_{u \in [t,T]} \mathbb{E}_t\left[e^{-r(u-t)}I_g^c(X_u; \lambda)\big|X_t = X\right], \quad 0 < X < X_g^*(t; \lambda), \tag{2.18}$$

where $X_g^*(t; \lambda)$ is the first-best threshold for optimal investment entry under the EGS agreement at time t. The optimal stopping problem associated with $V_g^c(X, t; \lambda)$ can be solved at relative ease since $I_g^c(X; \lambda)$ only involves power functions of X.

2.1.3 Comparison of value functions and optimal thresholds

Once the value functions and optimal thresholds associated with the financing choices of either the direct bank loan or equity guarantee swap under complete information have been determined, we can compare the relative magnitudes of these quantities between the two financing choices of direct bank loan and EGS. First, it is obvious that $c_b > c_g$ since

$$\frac{c_b}{r} - \frac{c_g}{r} = L_b(X; \lambda, c_b) > 0. \tag{2.19}$$

Without the guarantee offered by the EGS agreement, the bank charges a higher coupon rate to compensate for the liability associated with the default right of the firm. Consequently, the optimal default threshold $\underline{X}_b(\lambda, c_b)$ under the direct bank loan should be higher than $\underline{X}_g(\lambda)$ under EGS since

$$\underline{X}_b(\lambda, c_b) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} \left(F + \frac{c_b}{r}\right) > \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} \left(F + \frac{c_g}{r}\right) = \underline{X}_g(\lambda). \tag{2.20}$$

Furthermore, since the coupon rate is higher under the direct bank loan, the values of the default right and liability would be higher when compared with those under EGS as verified mathematically below:

$$D_b(X; \lambda, c_b) = \left[F + \frac{c_b}{r} - \lambda \Pi(\underline{X}_b(\lambda, c_b))\right] \left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^{\xi}$$

$$= \eta \lambda^{\xi} \Pi(X)^{\xi} \left(F + \frac{c_b}{r}\right)^{1-\xi} > D_g(X; \lambda), \tag{2.21}$$

and

$$L_b(X; \lambda, c_b) = (\eta + \eta_\alpha) \lambda^{\xi} \Pi(X)^{\xi} \left(F + \frac{c_b}{r}\right)^{1-\xi} > L_g(X; \lambda). \tag{2.22}$$

In a similar manner, since $\underline{X}_b(\lambda, c_b) > \underline{X}_g(\lambda)$, we can establish that the intrinsic (equity) value under the direct bank loan is lower than the intrinsic value under EGS, where

$$E_b(X; \lambda, c_b) = \lambda \Pi(X) - F - I - \alpha \lambda \Pi(\underline{X}_b(\lambda, c_b)) \left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^{\xi}$$

$$< \lambda \Pi(X) - F - I - \alpha \lambda \Pi(\underline{X}_g(\lambda)) \left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi} = I_g^c(X; \lambda). \tag{2.23}$$

Since $I_g^c(X; \lambda) > E_b(X; \lambda, c_b)$, it would be more desirable for the firm to enter into the EGS agreement to achieve higher intrinsic value. Lastly, since $E_b(X; \lambda, c_b) < I_g^c(X; \lambda)$, the corresponding thresholds of optimal entry into investment under the above two loan arrangements observe

$$X_b^*(t; \lambda) > X_g^*(t; \lambda). \tag{2.24}$$

The calculations above show that the manager of the small- or medium-size firm strictly prefers to find a guarantor to enter into the EGS agreement over the choice of direct bank loan under complete information. The intrinsic value of the firm under either EGS or direct bank loan [shown in (2.8) and (2.17), respectively] can be expressed as:

$$\lambda \Pi(X) - F - I + D_\theta(X; \lambda) - L_\theta(X; \lambda), \quad \theta = g, b.$$

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In both loan arrangements, the firm's direct payoff from the project is the same. The payoff is given by $\lambda\Pi(X) - F - I$, which is the revenue flow of the project minus the sum of present value of future operating expenses and the direct cost. Also the firm holds a default right with value $D_\theta(X; \lambda) > 0$ after launching of the project. Upon default, the firm would also be penalized by the liability. We observe

$$D_\theta(X; \lambda) - L_\theta(X; \lambda) = -\alpha\lambda\Pi(\underline{X}_\theta)\left(\frac{X}{\underline{X}_\theta}\right)^\xi < 0, \qquad (2.25)$$

which arises due to the bankruptcy cost upon default with bankruptcy cost parameter α. With $\alpha > 0$, the firm's intrinsic value is less than the direct payoffs from the project.

In summary, the strategy of entering into the EGS agreement dominates that of the direct bank loan since the firm pays lower coupon rate to the bank. This results in lower default threshold and thus lower expected bankruptcy cost when compared with the direct bank loan.

3   Real signaling games under separating equilibrium

This paper focuses on the analysis of the real signaling games on the financing decision of the firm with an investment opportunity of a risky project under asymmetric information. When the firm type is regarded as the private information and not accessible to the outsiders, the low-type firm may have an incentive to mimic the investment timing and financing choice of the high-type firm in order to reduce the share of equity under the EGS agreement or the coupon rate under the direct bank loan. On the other hand, the high-type firm has an incentive to speed up its investment or to adopt a different financing choice in order to separate from being visualized as low-type.

Assuming a finite time horizon of the investment opportunity, we examine the incentive compatibility constraints (ICCs) of both firm types and the properties of the corresponding investment thresholds under the EGS agreement. We then characterize the separating equilibrium under the EGS agreement. The high-type firm may choose to signal its true type through investment timing and separate from the low-type by investing earlier. Besides EGS, we consider the traditional direct bank loan as an alternative to EGS under inforamtion asymmetry and examine the separating equilibrium when the firm signals through financing choice. In this paper, we generalize analysis under perpetuality (Morellec and Schürhoff, 2011) to finite time horizon. Under both financing choices, we discuss the nature of separating equilibrium when the investment opportunity comes close to expiry.

3.1   Separating equilibrium through investment timing

We first consider the case where the firm finances the project through the EGS agreement. Since both firm types choose the financing choice of entering into EGS, the outsiders perceive the firm type through its investment threshold. Under complete information, the low-type firm would enter into the EGS agreement and invest optimally at threshold $X_g^*(t; \lambda_l)$ at time t. On the other hand, under asymmetric information, the low-type firm has the incentive to mimic the investment behavior of the high-type firm by speeding up its investment in

order to reduce the share of equity, which takes the value $\phi(X; \lambda_h)$ under the belief system $\mathbb{P}[\Lambda = \lambda_h] = 1$. The intrinsic value of the low-type firm under the mimicking strategy is given by

$$I_{g,l}^m(X) = [1 - \phi(X; \lambda_h)]E_g(X; \lambda_l) = \frac{E_g(X; \lambda_l)}{E_g(X; \lambda_h)}[E_g(X; \lambda_h) - L_g(X; \lambda_h)]. \quad (3.1)$$

The domain of definition of $I_{g,l}^m(X)$ is specified based on the following financial phenomena. Since $\phi(X; \lambda_h) \in [0, 1]$, non-negativity of $I_{g,l}^m(X)$ would be ensured. Therefore, the revenue shock variable level X should be above $\hat{X}_g(\lambda_h)$, which is the root obtained by solving (2.17) under $\lambda = \lambda_h$. On the other hand, the equity value of the low-type firm $E_g(X; \lambda_l)$ should be positive, which implies that X should be above the default threshold $\underline{X}_g(\lambda_l)$. Combining these results, we obtain the lower bound of the domain of definition: $\hat{\hat{X}}_{g,l} = \max(\hat{X}_g(\lambda_h), \underline{X}_g(\lambda_l))$. Besides, since the low-type firm speeds up its investment under the mimicking strategy, the revenue shock variable level X would stay below the first-best investment threshold $X_g^*(t; \lambda_l)$ at time t when $I_{g,l}^m(X)$ is well defined. The upper bound of the domain of definition is given by $X_g^*(t; \lambda_l)$. Therefore, $I_{g,l}^m(X)$ is defined within the interval $[\hat{\hat{X}}_{g,l}, X_g^*(t; \lambda_l)]$.

3.1.1 Incentive compatibility constraint for the low-type firm

At time t, the low-type firm chooses between mimicking the high-type firm and investing at its first-best threshold $X_g^*(t; \lambda_l)$ under complete information. The low-type firm prefers to mimic the high-type firm only when its corresponding intrinsic value $I_{g,l}^m(X)$ at the revenue shock variable X is larger than $V_g^c(X, t; \lambda_l)$, which is the real option value under complete information given by (2.18). The incentive compatibility constraint (ICC) for the low-type firm at revenue shock variable level X and time t is characterized by positivity of the difference of these two value functions, where

$$G_{g,l}(X, t) = I_{g,l}^m(X) - V_g^c(X, t; \lambda_l), \quad \hat{\hat{X}}_{g,l} \le X \le X_g^*(t; \lambda_l). \quad (3.2)$$

Let $\overline{X}_{g,l}(t)$ denote the binding threshold of (3.2), where $G_{g,l}(\overline{X}_{g,l}(t), t) = 0$. Only when $X > \overline{X}_{g,l}(t)$ at time t, positivity of $G_{g,l}(X, t)$ is observed. The incentive for the low-type firm to mimic the high-type becomes compatible since the ICC [see (3.2)] of the low-type firm is satisfied. In other words, the high-type firm can separate from the low-type only when the revenue shock variable level X is lower than $\overline{X}_{g,l}(t)$. Lemma 1 establishes the existence of $\overline{X}_{g,l}(t)$ so that the high-type firm can separate from the low-type firm and optimally chooses to speed up investment under separating equilibrium.

Lemma 1. *There exists a binding threshold $\overline{X}_{g,l}(t)$ of (3.2) at time t within the interval $[\hat{\hat{X}}_{g,l}, X_g^*(t; \lambda_l)]$ such that the ICC of the low-type firm is violated. Mathematically, we have*

$$I_{g,l}^m(X) \le V_g^c(X, t; \lambda_l)$$

when $X \le \overline{X}_{g,l}(t)$. As a result, the high-type firm can separate from the low-type when the revenue shock variable X is below $\overline{X}_{g,l}(t)$.

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The proof of Lemma 1 is shown in Appendix A. The relations between the binding and first-best thresholds, and satisfaction of the ICC for the low-type firm are depicted in Figure 1. Due to time dependence of the real option value functions and investment thresholds, there is no closed form solution of $V_g^c(X, t; \lambda_l)$ and the binding threshold $\overline{X}_{g,l}(t)$. We resort to numerical methods to compute these two time dependent value functions.
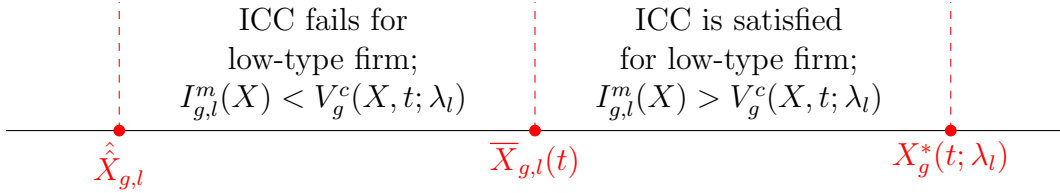

Figure 1: Relations between the binding and first-best thresholds, and satisfaction of the ICC of the low-type firm.

3.1.2 Incentive compatibility constraint for the high-type firm

The manager of the high-type firm may choose to separate from being perceived as low-type by speeding up investment. On the other hand, if the high-type firm fails to separate from the low-type, the belief system of the guarantor in the worst case should be $\mathbb{P}[\Lambda = \lambda_h] = 0$. Under failure of separation, the high-type firm has to accept $\phi(X; \lambda_l)$ so that the intrinsic value upon investment is given by

$$I_{g,h}^m(X) = [1 - \phi(X; \lambda_l)]E_g(X; \lambda_h) = \frac{E_g(X; \lambda_h)}{E_g(X; \lambda_l)}[E_g(X; \lambda_l) - L_g(X; \lambda_l)]. \qquad (3.3)$$

In other words, the high-type firm mimics the low-type firm in investment strategy under failure of separation. Given the intrinsic value $I_{g,h}^m(X)$, the corresponding real option value function before investment for the high-type firm under mimicking is given by

$$V_{g,h}^m(X, t) = \sup_{u \in [t,T]} \mathbb{E}_t \left[e^{-r(u-t)} I_{g,h}^m(X_u) \big| X_t = X \right], \quad 0 \le X \le X_{g,h}^{m*}(t), \qquad (3.4)$$

where $X_{g,h}^{m*}(t)$ is the optimal investment threshold of the high-type firm under mimicking strategy at time t. The optimal threshold $X_{g,h}^{m*}(t)$ is determined as part of the solution by the above optimal stopping problem. This is similar to finding the optimal exercise threshold in an American option model.

To analyze the incentive compatibility constraint (ICC) for the high-type firm, the necessary condition (ICC) under which the high-type firm prefers separating rather than mimicking is given by

$$G_{g,h}(X, t) = I_g^c(X; \lambda_h) - V_{g,h}^m(X, t) \ge 0, \quad \hat{X}_g(\lambda_h) \le X \le X_{g,h}^{m*}(t). \qquad (3.5)$$

Let $\overline{X}_{g,h}(t)$ denote the time-t binding threshold of (3.5) at which $G_{g,h}(\overline{X}_{g,h}(t), t) = 0$. Violation of the ICC implies that the high-type firm would not choose to separate from the low-type by speeding up investment when the revenue shock variable level X falls below $\overline{X}_{g,h}(t)$.

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A separating equilibrium under EGS exists only when $\overline{X}_{g,h}(t) \le X \le \overline{X}_{g,l}(t)$; that is, the ICC of low-type firm [(3.2)] is violated while the ICC of the high-type firm [(3.5)] is satisfied. The results of the general separating equilibrium and the least-cost separating equilibrium are summarized as follows:

Proposition 2. *There exists a unique least-cost separating equilibrium under EGS at time t when $\overline{X}_{g,h}(t) < \overline{X}_{g,l}(t)$, where the high-type firm enters into the EGS agreement and invests at the threshold $\min(\overline{X}_{g,l}(t), X_g^*(t; \lambda_h))$ and the low-type firm invests at its first-best threshold $X_g^*(t; \lambda_l)$. The separating equilibrium is sustained under the belief system:*

$$\Lambda(X_{inv}) = \begin{cases} \lambda_l, & \text{if } X_{inv} > \min(\overline{X}_{g,l}(t), X_g^*(t; \lambda_h)) \\ \lambda_h, & \text{otherwise} \end{cases}, \tag{3.6}$$

where X_{inv} is the investment threshold.

The proof of Proposition 2 can be established by following a similar approach as discussed in Wang and Kwok (2019). The separating equilibrium under the EGS agreement holds under the pessimistic belief system of the guarantor. At time t, the high-type firm invests after the revenue shock variable X reaches $\overline{X}_{g,h}(t)$ but before it exceeds $\overline{X}_{g,l}(t)$. The guarantor recognizes the firm as the high-type and requires the lower share of equity $\phi(X; \lambda_h)$, whose positive effect dominates the information cost due to early investment. By speeding up its investment, the high-type firm imposes a mimicking cost to the low-type firm, which is high enough for the low-type firm to choose its first-best threshold $X_g^*(t; \lambda_l)$ and offer higher share of equity to the guarantor. We illustrate the relative positions of the investment thresholds and investment strategies of both firm types under the separating equilibrium using Figure 2.
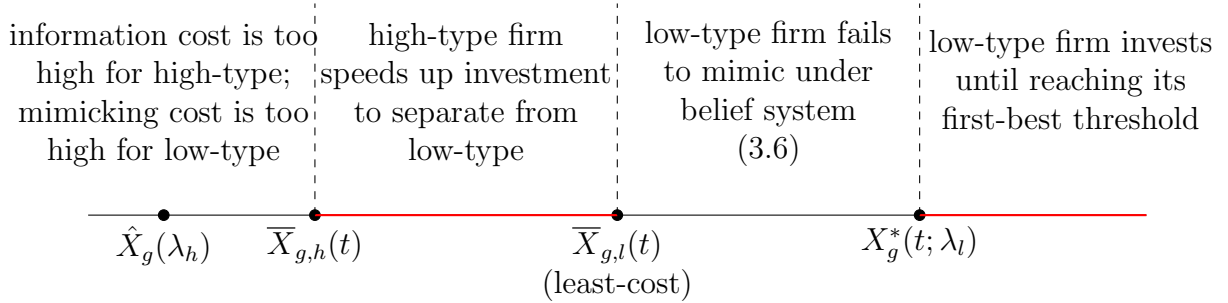

Figure 2: Relative positions of the investment thresholds and investment behaviors of both firm types under the separating equilibrium.

We define the real option value function of the high-type firm under the least-cost separating equilibrium before investment by $V_{g,h}^s(X, t)$. In the least-cost separating equilibrium, the high-type firm invests at the lower one between the two thresholds: the binding threshold $\overline{X}_{g,l}(t)$ of the ICC of the low-type firm and the first-best threshold $X_g^*(t; \lambda_h)$ of the high-type firm under complete information. When $X_g^*(t; \lambda_h) \le \overline{X}_{g,l}(t)$ at time t, the real-option value $V_{g,h}^s(X, t)$ of the high-type firm takes the same value as the case of complete information. When $X_g^*(t; \lambda_h) > \overline{X}_{g,l}(t)$, the high-type firm may speed up its investment in order to separate from the low-type. Therefore, $V_{g,h}^s(X, t)$ is not always dictated by the optimal stopping

rule. In fact, $V_{g,h}^s(X,t)$ can be defined as the real option value with intrinsic value $I_g^c(X;\lambda_h)$ but investment threshold $\min(\overline{X}_{g,l}(t), X_g^*(t;\lambda_h))$, which resembles an up-barrier call option defined on the domain $[0, \min(\overline{X}_{g,l}(t), X_g^*(t;\lambda_h))]$. The percentage drop of the real option value function of the high-type firm in the least-cost separating equilibrium is defined as the information cost due to investment distortion, namely,

$$C_g^s(X,t) = \frac{V_g^c(X,t;\lambda_h) - V_{g,h}^s(X,t)}{V_g^c(X,t;\lambda_h)}. \tag{3.7}$$

3.2 Separating equilibrium through financing choice

According to Section 2.1.3, EGS dominates the direct bank loan under complete information due to lower bankruptcy cost. However, it may not be the case under information asymmetry since EGS may cause higher information cost for the high-type firm and mimicking cost for the low-type firm. Therefore, it becomes plausible that the high-type firm may be able to separate from the low-type by adopting the direct bank loan. It is seen that under the EGS arrangement, the low-type firm may benefit from mimicking to achieve lower proportional share of equity. Under the direct bank loan, by mimicking the high-type firm, the low-type firm manages to lower the coupon rate. If investing at sufficiently low threshold, the bank would mistaken the low-type firm as being high-type with $\mathbb{P}[\Lambda = \lambda_h] = 1$. According to Morellec and Schürhoff (2011), the equity value of the low-type firm upon investment is given by

$$\begin{aligned} E_b(X;\lambda_l, c_{b,h}) = {} & \lambda_l\Pi(X) - F - \frac{c_{b,h}}{r} \\ & + \left[F + \frac{c_{b,h}}{r} - \lambda_l\Pi(\underline{X}_b(\lambda_l, c_{b,h}))\right]\left[\frac{X}{\underline{X}_b(\lambda_l, c_{b,h})}\right]^\xi, \end{aligned} \tag{3.8}$$

where the default threshold is

$$\underline{X}_b(\lambda_l, c_{b,h}) = \frac{\xi}{\xi - 1}\frac{r - \mu}{\lambda_l}\left(F + \frac{c_{b,h}}{r}\right)$$

and the coupon rate $c_{b,h}$ is determined by the budget constraint specified in (2.7) by setting $\lambda = \lambda_h$. The low-type firm prefers mimicking the high-type under the direct bank loan at the revenue shock variable level X to waiting until its first-best threshold under EGS when the following incentive compatibility constraint holds:

$$E_b(X;\lambda_l, c_{b,h}) \geq V_g^c(X,t;\lambda_l). \tag{3.9a}$$

On the other hand, the separating equilibrium exists only when the following incentive compatibility constraint of the high-type firm is satisfied:

$$E_b(X;\lambda_h, c_{b,h}) \geq V_{g,h}^m(X,t). \tag{3.9b}$$

Let $\overline{X}_{b,l}(t)$ and $\overline{X}_{b,h}(t)$ denote the binding thresholds under the incentive compatibility constraints (3.9a) and (3.9b), respectively. Given the current revenue shock variable level X,

the low-type firm would choose not to mimic the high-type through the direct bank loan when its corresponding equity value does not exceed its real option value in the benchmark case of EGS under complete information [shown by (3.9a)]. On the other hand, the high-type firm prefers to separate from the low-type by choosing the direct bank loan when its corresponding equity value is larger than its real option value under the mimicking strategy through EGS [shown by (3.9b)].

As a summary, the low-type firm would choose to wait until its first-best investment threshold $X_b^*(t; \lambda_l)$ only when the current revenue shock variable X is below $\overline{X}_{b,l}(t)$, where the negative effect of investment distortion dominates the positive effect of coupon reduction for the low-type firm. On the other hand, the high-type firm is willing to separate from low-type by imposing a mimicking cost for the low-type firm only when X stays above $\overline{X}_{b,h}(t)$. There is an information cost [see (3.7)] borne by the high-type firm in speeding up its investment in order to separate from the low-type firm.

We summarize the separating equilibrium strategies of both firm types under the direct bank loan in Proposition 3.

Proposition 3. *There exists a unique least-cost separating equilibrium under the direct bank loan at time t whenever $\overline{X}_{b,h}(t) \leq \overline{X}_{b,l}(t)$, where the high-type firm obtains the bank loan invests at the threshold $\min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h))$ and the low-type firm invests at its first-best threshold $X_g^*(t; \lambda_l)$ under the benchmark case of EGS.*

Define $V_{b,h}^s(X, t)$ as the real option value of the high-type firm with intrinsic value $E_b(X; \lambda_h, c_{b,h})$ and investment threshold $\min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h))$. The information cost of the high-type firm is then defined as

$$C_b^s(X, t) = \frac{V_g^c(X, t; \lambda_h) - V_{b,h}^s(X, t)}{V_g^c(X, t; \lambda_h)}. \tag{3.10}$$

Proposition 3 holds under the pessimistic belief system of the bank. In the separating equilibrium, the bank visualizes the firm as the high-type when it invests at or earlier than the threshold $\min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h))$. Under the direct bank loan, the high-type firm is willing to invest early. The resulting information cost of the high-type firm by investment distortion can be compensated by reduction of the coupon rate after signaling its quality type to the bank. On the other hand, by investing at the same threshold as the high-type, the low-type firm will be considered as high-type and make the same coupon payment only. However, it may find this choice too costly due to significant investment distortion which offsets the positive effect of lower coupon rate through mimicking.

3.3 Separating equilibrium near maturity through investment timing

We investigate the investment and financing behavior of the firm under the EGS when the investment opportunity is close to expiry. The corresponding first-best threshold $X_g^*(T^-; \lambda)$ can be found by solving the corresponding algebraic equations (see Appendix B for details).

Similar to American options, when $t \to T^-$, the real option value of the EGS agreement converges to the intrinsic value due to continuity. Under complete information, we have

$$\lim_{t \to T^-} V_g^c(X, t; \lambda) = I_g^c(X; \lambda). \tag{3.11}$$

At $t \to T^-$, the separating equilibrium exists only when the incentive compatibility constraint [(3.2)] of the low-type firm is violated, namely,

$$G_{g,l}(X, T^-) = I_{g,l}^m(X) - V_g^c(X, T^-; \lambda_l) = I_{g,l}^m(X) - I_g^c(X; \lambda_l) \le 0. \tag{3.12}$$

It is seen from Appendix A that $I_{g,l}^m(X) \ge I_g^c(X; \lambda_l)$, so (3.12) holds only when $I_{g,l}^m(X) = I_g^c(X; \lambda_l) = 0$. The binding threshold $\overline{X}_{g,l}(T^-)$ of (3.12) is the revenue shock level $\hat{X}_{g,l} = \max(\hat{X}_g(\lambda_h), \underline{X}_g(\lambda_l))$, where $I_{g,l}^m(X)$ first decreases to zero. Proposition 2 states that $\overline{X}_{g,l}(T^-)$ equals the investment threshold of the high-type firm in the least-cost separating equilibrium. When time is close to maturity of the investment opportunity, we deduce the following investment behavior of both firm types under the least-cost separating equilibrium:

1. When $\hat{X}_g(\lambda_h) \ge \underline{X}_g(\lambda_l)$, the binding threshold $\overline{X}_{g,l}(T^-)$ equals the zero intrinsic value threshold $\hat{X}_g(\lambda_h)$ of the high-type firm. The guarantor believes the firm as low-type whenever it invests after $\hat{X}_g(\lambda_h)$. It implies that the high-type firm will always be mimicked by the low-type as long as it invests with nonnegative intrinsic value. Then the high-type firm chooses not to invest and a separating equilibrium does not exist.

2. When $\hat{X}_g(\lambda_h) < \underline{X}_g(\lambda_l)$, the binding threshold $\overline{X}_{g,l}(T^-)$ equals the default threshold $\underline{X}_g(\lambda_l)$ of the low-type firm. The belief system of the guarantor becomes $\mathbb{P}[\lambda = \lambda_l] = 1$ if it invests after $\underline{X}_g(\lambda_l)$. The high-type firm is willing to invest at $\underline{X}_g(\lambda_l)$ and get positive equity value, while the low-type firm finds it indifferent between mimicking and waiting at $X = \underline{X}_g(\lambda_l)$ ($I_{g,l}^m(X) = V_g^c(X, T^-; \lambda_l) = 0$) and may choose to wait until its first-best investment threshold. Therefore, there exists a least-cost separating equilibrium where the high-type firm invests at $\min(X_g^*(T^-; \lambda_h), \underline{X}_g(\lambda_l))$ and the low-type firm invests at its first-best threshold $X_g^*(T^-; \lambda_l)$, where $X_g^*(T^-; \lambda_k)$ ($k = h, l$) can be calculated by the procedure shown in Appendix B.

4 Real signaling games under pooling equilibrium

When it is too costly for the high-type firm to separate from the low-type, it may rather choose to pool with the low-type firm. In the pooling equilibrium, the firm of either quality type adopts the same investment timing and financing choice. The outsiders are unable to distinguish the true firm quality. As a result, the corresponding belief on Λ is given by

$$\Lambda = \lambda_p = p\lambda_h + (1 - p)\lambda_l.$$

We start with the discussion of the ICCs and investment thresholds under the respective pooling equilibrium for the EGS agreement and direct bank loan. In particular, we analyze the pooling equilibrium when the investment opportunity comes closer to expiry.

4.1 Pooling equilibrium through EGS agreement

For the EGS agreement, the guarantor is unable to determine the exact firm type under the pooling equilibrium. For the guarantor, the expected liability $L_g^p(X)$ upon default is given by

$$L_g^p(X) = \mathbb{P}[\Lambda = \lambda_h]L_g(X;\lambda_h) + \mathbb{P}[\Lambda = \lambda_l]L_g(X;\lambda_l)$$

$$= p[F + I - (1-\alpha)\lambda_h \Pi(\underline{X}_g(\lambda_h))]\left[\frac{X}{\underline{X}_g(\lambda_h)}\right]^\xi$$

$$+ (1-p)[F + I - (1-\alpha)\lambda_l \Pi(\underline{X}_g(\lambda_l))]\left[\frac{X}{\underline{X}_g(\lambda_l)}\right]^\xi$$

$$= (\eta + \eta_\alpha)\lambda_{p,\xi}\Pi(X)^\xi (F + I)^{1-\xi},$$

where $\lambda_{p,\xi} = p\lambda_h^\xi + (1-p)\lambda_l^\xi$. As a fair deal, the guarantor demands the proportion $\phi_p(X)$ of the equity value $E_g^p(X)$ of the firm as dictated by

$$\phi_p(X)E_g^p(X) = L_g^p(X). \tag{4.1}$$

The expected equity value of the firm right after investment is given by

$$E_g^p(X) = \mathbb{P}[\Lambda = \lambda_h]E_g(X;\lambda_h) + \mathbb{P}[\Lambda = \lambda_l]E_g(X;\lambda_l)$$

$$= p\left\{\lambda_h\Pi(X) - F - I + [F + I - \lambda_h\Pi(\underline{X}_g(\lambda_h))]\left[\frac{X}{\underline{X}_g(\lambda_h)}\right]^\xi\right\}$$

$$+ (1-p)\left\{\lambda_l\Pi(X) - F - I + [F + I - \lambda_l\Pi(\underline{X}_g(\lambda_l))]\left[\frac{X}{\underline{X}_g(\lambda_l)}\right]^\xi\right\}$$

$$= \lambda_p\Pi(X) - F - I + \eta\lambda_{p,\xi}\Pi(X)^\xi (F + I)^{1-\xi}.$$

Under the pooling equilibrium, the EGS agreement specifies the same proportional share of equity for both firm types. To achieve a fair deal as dictated by (4.1), we obtain

$$\phi_p(X) = \frac{L_g^p(X)}{E_g^p(X)} = \frac{\eta + \eta_\alpha}{\lambda_{p,\xi}^{-1}\Pi(X)^{-\xi}(F + I)^{\xi-1}[\lambda_p\Pi(X) - F - I] + \eta}, \tag{4.2}$$

where the two coefficients η and η_α are defined in (2.16).

4.1.1 Incentive compatibility constraints

The pooling equilibrium exists only when the pooling strategies dominate the separating strategies for both firm types. The low-type firm prefers to pool with the high-type at time t when the following ICC holds:

$$I_{g,l}^p(X) \geq V_g^c(X,t;\lambda_l), \quad 0 \leq X \leq X_g^*(t;\lambda_l), \tag{4.3}$$

where the intrinsic value of the low-type firm under the pooling strategy is given by

$$I_{g,l}^p(X) = [1 - \phi_p(X)]E_g(X;\lambda_l) = \frac{E_g(X;\lambda_l)}{E_g^p(X)}[E_g^p(X) - L_g^p(X)].$$

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Denote the corresponding binding threshold of ineq. (4.3) by $\overline{X}_{g,l}^p(t)$. The ICC (4.3) indicates that the low-type firm finds the mimicking cost not too high so that it prefers to lower its investment threshold rather than waiting until its first-best threshold under the separating equilibrium. Non-negativity of $I_{g,l}^p(X)$ requires the share of equity $\phi(X) \in [0,1]$ and equity value $E_g(X; \lambda_l)$ to be nonnegative. The binding threshold $\hat{X}_{g,p}$ satisfies

$$E_g^p(\hat{X}_{g,p}) - L_g^p(\hat{X}_{g,p}) = 0,$$

where the expected equity value and liability are the same and the share of equity requested by the guarantor is zero. Similar to the discussion on the domain of definition of $I_{g,l}^m(X)$ [see (3.1)], the intrinsic value $I_{g,l}^p(X)$ stays positive when X is above the lower bound $\hat{\hat{X}}_{g,p} = \max(\hat{X}_{g,p}, \underline{X}_g(\lambda_l))$, and it is set to be zero when X is below $\hat{\hat{X}}_{g,p}$. The relations among the thresholds and satisfaction of the ICC of the low-type firm in the pooling equilibrium are depicted in Figure 3.
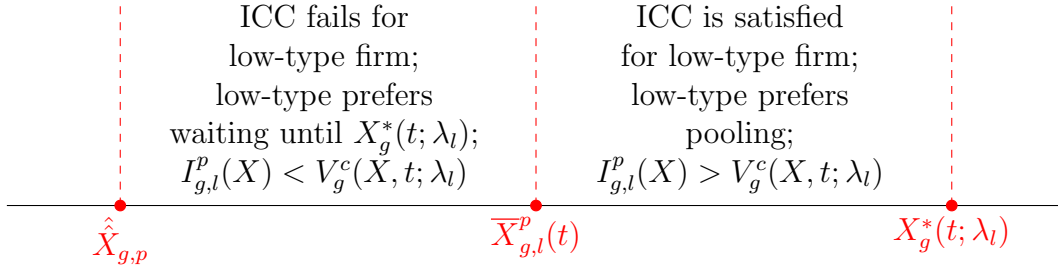

Figure 3: Relations among the thresholds and satisfaction of the ICC of the low-type firm in the pooling equilibrium.

4.1.2 Pooling equilibrium strategies

The high-type firm prefers the pooling equilibrium to separating equilibrium only when the pooling strategy dominates the least-cost separating equilibrium. As stated in Proposition 2, the high-type firm chooses to invest at the threshold $\min(\overline{X}_{g,l}(t), X_g^*(t; \lambda_h))$ under the least-cost separating equilibrium. When $X_g^*(t; \lambda_h) \leq \overline{X}_{g,l}(t)$ at time t, the high-type firm always prefers investing at $X_g^*(t; \lambda_h)$ under the pooling strategy due to optimality of the first-best threshold $X_g^*(t; \lambda_h)$. Therefore, the pooling equilibrium exists only when the binding threshold $\overline{X}_{g,l}(t)$ is below $X_g^*(t; \lambda_h)$; that is, $I_g^c(X_g^*(t; \lambda_h), t) > V_{g,h}^m(X_g^*(t; \lambda_h), t)$. The high-type firm prefers to adopt the pooling strategy when the following ICC holds at the threshold $\overline{X}_{g,l}(t)$:

$$I_g^c(\overline{X}_{g,l}(t); \lambda_h) \leq V_{g,h}^p(\overline{X}_{g,l}(t), t), \tag{4.4}$$

where the left-hand side is the intrinsic value of the high-type firm under the separating strategy [see (2.17)]. Here, the right-hand side is the real option value of the high-type firm under the pooling strategy with optimal threshold $X_g^{p*}(t)$ and intrinsic value as given by

$$I_{g,h}^p(X) = [1 - \phi_p(X)]E_g(X; \lambda_h) = \frac{E_g(X; \lambda_h)}{E_g^p(X)}[E_g^p(X) - L_g^p(X)]. \tag{4.5}$$

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Denote the binding threshold of ineq. (4.4) by $\overline{X}^p_{g,h}(t)$. The ICC depicted in (4.4) shows the choice of the high-type firm between the pooling and separating strategies. When the current revenue shock variable X reaches the separating threshold $\overline{X}_{g,l}(t)$, the high-type firm finds it too costly to separate from the low-type and prefers to wait until the optimal investment threshold $X^{p*}_g(t)$ of the pooling strategy. However, if the high-type firm waits for too long and the revenue shock variable X exceeds the optimal threshold $X^{p*}_g(t)$, the guarantor may misinterpret it as a low-type and request for higher share of equity. This implies that the high-type firm maximizes its value when it invests exactly at the threshold $X^{p*}_g(t)$, which constitutes a Pareto-dominant equilibrium.

We summarize the results of the pooling equilibrium in Proposition 4.

Proposition 4. *There exists a Pareto-dominant pooling equilibrium at time t when $\overline{X}_{g,l}(t) < X^*_g(t; \lambda_h)$ and the ICCs in (4.3) and (4.4) hold. In the pooling equilibrium, the firm of either quality type chooses to enter into the EGS agreement and invest when the revenue shock variable X reaches the optimal threshold $X^{p*}_g(t)$ of the high-type firm. The pooling equilibrium can be sustained under the belief system*

$$\Lambda(X_{inv}) = \begin{cases} \lambda_h, & \text{if } X_{inv} \leq \overline{X}_{g,l}(t) \\ \lambda_p, & \text{if } \overline{X}_{g,l}(t) < X_{inv} \leq X^{p*}_g(t) , \\ \lambda_l, & \text{otherwise} \end{cases} \tag{4.6}$$

where X_{inv} is the investment threshold.

Since the pooling equilibrium as stated in Proposition 4 should be Pareto-dominant, the high-type firm chooses the investment threshold in order to maximize its payoff. The optimal threshold $X^{p*}_g(t)$ is the highest value in the interval $(\overline{X}_{g,l}(t), X^{p*}_g(t)]$, where the guarantor cannot distinguish the firm type and the belief system is $\mathbb{P}[\Lambda = \lambda_h] = p$. Therefore, the high-type firm has no incentive to deviate from the equilibrium. On the other hand, the low-type firm also maximizes its payoff through investing at the same threshold $X^{p*}_g(t)$. It cannot deviate to its optimal pooling threshold $X^{p*}_{g,l}(t) > X^{p*}_g(t)$ since it falls into the region where the guarantor's belief system is $\mathbb{P}[\Lambda = \lambda_h] = 0$. Moreover, given that (4.3) and (4.4) are satisfied, the pooling equilibrium also dominates the separating equilibrium. Under Pareto-dominance, the guarantor requests for the lower share of equity from the low-type firm. This yields higher value of the low-type firm when compared to that under complete information, which dominates the mimicking cost of the low-type firm arising from investment distortion. Compared to the separating equilibrium, the high-type firm also achieves a higher payoff by pooling with the low-type by diminishing the negative effect of investment distortion, though it may be charged a higher share of equity by the guarantor. The relative positions of the investment thresholds and investment strategies of both firm types under the pooling equilibrium are illustrated in Figure 4.
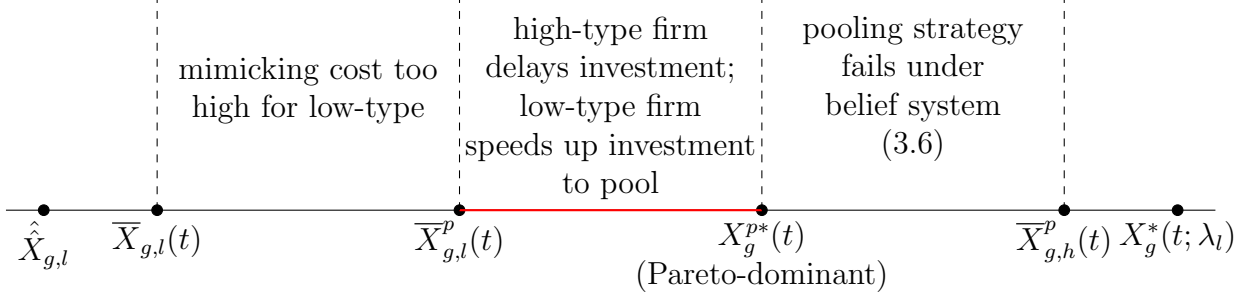
Figure 4: Relative positions of the investment thresholds and investment behaviors of both firm types under the separating equilibrium.

The optimal investment threshold $X_g^{p*}(t)$ under the pooling equilibrium strategy lies between the first-best thresholds $X_g^*(t; \lambda_h)$ and $X_g^*(t; \lambda_l)$. This implies that under the pooling equilibrium, the low-type firm invests more aggressively while the high-type firm delays its investment when compared with that under complete information. The high-type firm therefore incurs an information cost due to this form of investment distortion. The level of investment distortion can be quantified as the (relative) information cost of the high-type firm under the pooling equilibrium through the EGS agreement, which is defined by

$$C_g^p(X, t) = \frac{V_g^c(X, t; \lambda_h) - V_{g,h}^p(X, t)}{V_g^c(X, t; \lambda_h)}. \tag{4.7}$$

4.2 Pooling equilibrium under direct bank loan

When the high-type firm fails to separate from the low-type under the direct bank loan due to high information cost, it may resort to the pooling strategy. Since the bank cannot differentiate the type of the firm, the bank charges the same coupon rate $c_{b,p} \in (c_{b,h}, c_{b,l})$ as determined by the following budget constraint:

$$\frac{c_{b,p}}{r} - L_b^p(X; c_{b,p}) = I. \tag{4.8}$$

Here, the bank evaluates the default risk of the firm as quantified by the following expected liability value

$$
\begin{aligned}
L_b^p(X; c_{b,p}) &= \mathbb{P}[\Lambda = \lambda_h] L_b(X; \lambda_h, c_{b,p}) + \mathbb{P}[\Lambda = \lambda_l] L_b(X; \lambda_l, c_{b,p}) \\
&= p \left[F + \frac{c_{b,p}}{r} - (1 - \alpha)\lambda_h \Pi(\underline{X}_b(\lambda_h, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_h, c_{b,p})} \right]^\xi \\
&\quad + (1 - p) \left[F + \frac{c_{b,p}}{r} - (1 - \alpha)\lambda_l \Pi(\underline{X}_b(\lambda_l, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_l, c_{b,p})} \right]^\xi \\
&= (\eta + \eta_\alpha)\lambda_{p,\xi} \Pi(X)^\xi \left(F + \frac{c_{b,p}}{r} \right)^{1 - \xi}.
\end{aligned}
$$

The characterization of the pooling equilibrium for the direct bank loan is different from that of equity financing or EGS since the common coupon rate $c_{b,p}$ is dependent on the expected

liability value $L_b^p(X; c_{b,p})$. However, it is still feasible to calculate the expected liability value in eq. (4.8) since $c_{b,p}$ is independent of X.

Under the same coupon rate $c_{b,p}$, the equity values of the high-type and low-type firm right after investment are given by

$$
\begin{aligned}
E_b(X; \lambda_h, c_{b,p}) = {} & \lambda_h \Pi(X) - F - \frac{c_{b,p}}{r} \\
& + \left[F + \frac{c_{b,p}}{r} - \lambda_h \Pi(\underline{X}_b(\lambda_h, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_h, c_{b,p})} \right]^\xi
\end{aligned}
$$

and

$$
\begin{aligned}
E_b(X; \lambda_l, c_{b,p}) = {} & \lambda_l \Pi(X) - F - \frac{c_{b,p}}{r} \\
& + \left[F + \frac{c_{b,p}}{r} - \lambda_l \Pi(\underline{X}_b(\lambda_l, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_l, c_{b,p})} \right]^\xi,
\end{aligned}
$$

respectively. The low-type firm prefers pooling with the high-type at time t if the following ICC is satisfied:

$$
E_b(X; \lambda_l, c_{b,p}) \geq V_g^c(X, t; \lambda_l), \quad \underline{X}_b(\lambda_l, c_{b,p}) \leq X \leq X_g^*(t; \lambda_l). \tag{4.9}
$$

By adopting the financing choice of the direct bank loan and the same investment timing as the high-type, the low-type firm is charged by the common coupon rate $c_{b,p}$, which is lower than the coupon rate $c_{b,l}$ charged under complete information. The low-type firm prefers the pooling strategy since its corresponding equity value exceeds the real option value of waiting until its first best threshold $X_g^*(t; \lambda_l)$ under the benchmark case of EGS.

Similar to Section 4.1, the high-type firm always prefers separating when $X_b^*(t; \lambda_h) \leq \overline{X}_{b,l}(t)$ and it can invest at its first-best threshold. Therefore, a debt pooling equilibrium exists only when $\overline{X}_{b,l}(t) < X_b^*(t; \lambda_h)$. At the separating threshold $\overline{X}_{b,l}(t)$ of the high-type firm, ICC of the high-type firm is given by

$$
E_b(\overline{X}_{b,l}(t); \lambda_h, c_{b,h}) \leq V_{b,h}^p(\overline{X}_{b,l}(t), t), \tag{4.10}
$$

where the real option value $V_{b,h}^p(X, t)$ of the high-type firm under the pooling strategy is governed by

$$
V_{b,h}^p(X, t) = \sup_{u \in [t,T]} \mathbb{E}_t[e^{-r(u-t)} E_b(X_u; \lambda_h, c_{b,p}) \mid X_t = X], \quad 0 \leq X \leq X_b^{p*}(t) \tag{4.11}
$$

and $X_b^{p*}(t)$ is defined as the optimal threshold of the high-type firm under the pooling strategy. The high-type firm abandons the separating strategy at the threshold $\overline{X}_{b,l}(t)$ when (4.10) holds since the real option value of waiting until the optimal pooling threshold $X_b^{p*}(t)$ is higher than the equity value of investing immediately though it can separate from the low-type by doing this.

The debt pooling equilibrium under the direct bank loan is summarized in Proposition 5:

Proposition 5. *There exists a Pareto-dominant pooling equilibrium under the direct bank loan at time t when $\overline{X}_{b,l}(t) < X_b^*(t; \lambda_h)$ and (4.9) and (4.10) hold. In the pooling equilibrium, both firm types choose to invest when the revenue shock variable X reaches the optimal pooling threshold $X_b^{p*}(t)$. The bank charges the common coupon rate $c_{b,p}$ for both firm types as determined by (4.8).*

The (relative) information cost of the high-type firm under the pooling equilibrium through the direct bank loan is given by

$$C_b^p(X,t) = \frac{V_g^c(X,t;\lambda_h) - V_{b,h}^p(X,t)}{V_g^c(X,t;\lambda_h)}. \tag{4.12}$$

As a summary, when the pooling equilibrium is preferable, the low-type firm bears a lower coupon rate from the bank, which results in higher equity value since this dominates the mimicking cost due to investment distortion. On the other hand, the high-type firm also adopt the pooling equilibrium since it chooses to delay its investment further in order to avoid facing the high cost of investment distortion when separating strategies are adopted, though it is charged at a higher coupon rate compared to that under complete information.

4.3 Pooling equilibrium near maturity under EGS agreement

We consider the pooling strategy of the firm under the EGS agreement when the investment opportunity is going to expire very soon. Similar to Section 3.3, by virtue of continuity of the real option value function $V_{g,h}^p(X,t)$ with respect to t, the asymptotic value of $V_{g,h}^p(X,t)$ when $t \to T^-$ is given by

$$V_{g,h}^p(X,T^-) = I_{g,h}^p(X). \tag{4.13}$$

To examine the pooling strategy of the firm near maturity of the investment opportunity, we notice that the ICC of the low-type firm [(4.3)] always holds for $t \to T^-$ since $I_{g,l}^p(X) \geq I_g^c(X;\lambda_l) = V_g^c(X,T^-;\lambda_l)$ is always true. We deduce that the low-type firm always prefers to pool with the high-type to achieve higher equity value when the investment opportunity expires soon. Under $\overline{X}_{g,l}(T^-) < X_g^*(T^-;\lambda_h)$, it remains to check the satisfaction of the ICC of the high-type firm at $t \to T^-$, namely,

$$I_g^c(\overline{X}_{g,l}(T^-); \lambda_h) \leq I_{g,h}^p(\overline{X}_{g,l}(T^-)). \tag{4.14}$$

The high-type firm chooses whether to invest or not when the current revenue shock variable X reaches the separating threshold near maturity. Recall from Section 3.3 that $\overline{X}_{g,l}(T^-) = \hat{\hat{X}}_{g,l} = \max(\hat{X}_g(\lambda_h), \underline{X}_g(\lambda_l))$. We then determine the pooling equilibrium near maturity by considering the following two cases:

1. When $\hat{X}_g(\lambda_h) \geq \underline{X}_g(\lambda_l)$, the separating equilibrium does not exist since the binding threshold $\overline{X}_{g,l}(T^-)$ for the separating equilibrium equals the zero-NPV threshold $\hat{X}_g(\lambda_h)$ and the high-type firm chooses not to invest. After the revenue flow shock variable X exceeds $\hat{X}_g(\lambda_h)$, the guarantor's belief system changes to $\mathbb{P}[\lambda = \lambda_p]$. Then the high-type firm will rather wait until the revenue flow shock variable reaches the optimal pooling threshold $X_g^{p*}(T^-)$. Therefore, there exists a Pareto-dominant pooling equilibrium where the firm chooses to invest when X reaches the threshold $X_g^{p*}(T^-)$ regardless of its type.

2. When $\hat{X}_g(\lambda_h) < \underline{X}_g(\lambda_l)$, the separating threshold of the high-type firm is $\overline{X}_{g,l}(T^-) = \underline{X}_g(\lambda_l)$, where $I_g^c(\overline{X}_{g,l}(T^-); \lambda_h)$ is positive and larger than $I_{g,h}^p(\overline{X}_{g,l}(T^-))$. Then the high-type firm chooses to invest whenever the revenue flow shock variable X reaches $\underline{X}_g(\lambda_l)$, where it is still perceived as high-type. In this case, a pooling equilibrium fails to exist.

In summary, a Pareto-dominant pooling equilibrium exists if and only if $\hat{X}_g(\lambda_h) \geq \underline{X}_g(\lambda_l)$. Under such condition, separating equilibrium does not exist and the high-type firm prefers to adopt the pooling strategy and wait until the optimal pooling threshold $X_g^{p*}(T^-)$ to invest.

5 Numerical results

In this section, we present the numerical studies on the investment thresholds and information costs of the firm of either firm type under separating and pooling equilibriums. We focus on the time dependence of the firm's separating and pooling strategies, emphasizing the firm's financing choice between the EGS agreement and direct bank loan by comparing the real option values and information costs. To compute the value functions and optimal thresholds in the associated stopping models, the numerical calculations are based on the fully implicit finite difference scheme and the Projected Successive-Over-Relaxation method (Kwok, 2008; Wang and Kwok, 2019). The base parameters for the numerical plots are set to be $r = 5\%$, $\mu = 1\%$, $\sigma = 25\%$, $\lambda_h = 1.25$, $\lambda_l = 0.8$, $F = 200$, $I = 100$, $T = 5$ and $p = 0.5$.

5.1 Separating equilibrium

Figures 5 and 6 plot the investment thresholds of the high-type firm in the least-cost separating equilibrium with respect to time $t \in [0, T]$ at three levels of λ_l. The high-type firm chooses to invest at the lower one between its first-best and binding thresholds in the separating equilibrium. The figure shows that the first-best and binding thresholds of the firm are decreasing as time gets closer to maturity. It implies that the low-type firm's incentive of mimicking increases with time, which forces the high-type firm to invest earlier. The reason is that the firm's value of the option to invest decreases and the firm tends to invest earlier when remaining life span of the investment opportunity becomes narrower. We observe a concave function of the first-best threshold and a convex function of the binding threshold with respect to time. Driven by this fact, Figure 5(a) shows two intersection points of the first-best and the binding thresholds, which implies that the high-type firm chooses its first-best threshold $X_g^*(t; \lambda_h)$ at beginning of the time, changes to the binding threshold $\overline{X}_{g,l}(t)$ at the intermediate region of time and reverts to its first-best threshold when time is close to maturity. Intuitively, the first-best threshold is affected by the value of the firm's option to invest, which decreases at a slower rate in time when t is sufficiently far from maturity T but the rate of decrease rises sharply when t is about to reach the maturity date. On the other hand, the binding threshold of the high-type firm in the separating equilibrium is affected by the low-type firm's incentive of mimicking, which reaches a level sufficiently high and does not change much when time is close to expiry of the investment opportunity. By examining Figures 5(a) − (c) and Figures 6(a) − (c) for $\lambda_l = 0.4$, 0.5 and 0.8, we find that the

binding threshold increases with decreasing λ_l. The numerical plots are consistent with the financial intuition that the low-type firm finds it more costly to mimic the high-type for a smaller multiplier λ_l. Then the high-type firm is allowed to invest optimally at its first-best threshold in order to separate from the low-type.

Figure 7 plots the information costs of the high-type firm in the least-cost separating equilibrium against time $t \in [0, T]$ at three levels of λ_l. It shows strong time dependence of the firm's information costs in the separating equilibrium and thus the financing choice of the firm between EGS and the direct bank loan. The percentage costs under both financing choices are calculated at the zero-NPV threshold of the high-type firm $\hat{X}_g(\lambda_h) = 10.16$ under the EGS agreement. Figures $7(a, b)$ show that for low values of the multiplier λ_l, the information cost of the high-type firm under EGS first increases until reaching its maximum at intermediate time, and then decreases for time close to maturity. It is driven by the gap between the binding and first-best thresholds of the high-type firm under EGS, which becomes larger at longer time to expiry and narrower when time is near maturity. It implies that the high-type firm is the most aggressive to separate from the low-type at middle of the time but is less aggressive at the beginning and near maturity. In other cases, the information cost increases with time before maturity. At maturity T, since the real option value of the high-type firm converges to its intrinsic value, the information cost will surely jump to zero. The firm chooses its optimal financing strategy corresponding to lower cost. We can deduce from the figure that the high-type firm prefers to separate from the low-type through EGS when the multiplier λ_l is low, while separate through the direct bank loan when λ_l is high. For intermediate level of λ_l, the high-type firm prefers to separate through the direct bank loan when time is far from maturity and changes to EGS when time is near maturity. It is because when λ_l is low enough, it becomes more difficult for the low-type firm to distort its investment strategy. Also, the effect of investment distortion on the high-type firm lessens at a certain time before maturity, so that the high-type firm becomes easier to separate through investment timing.
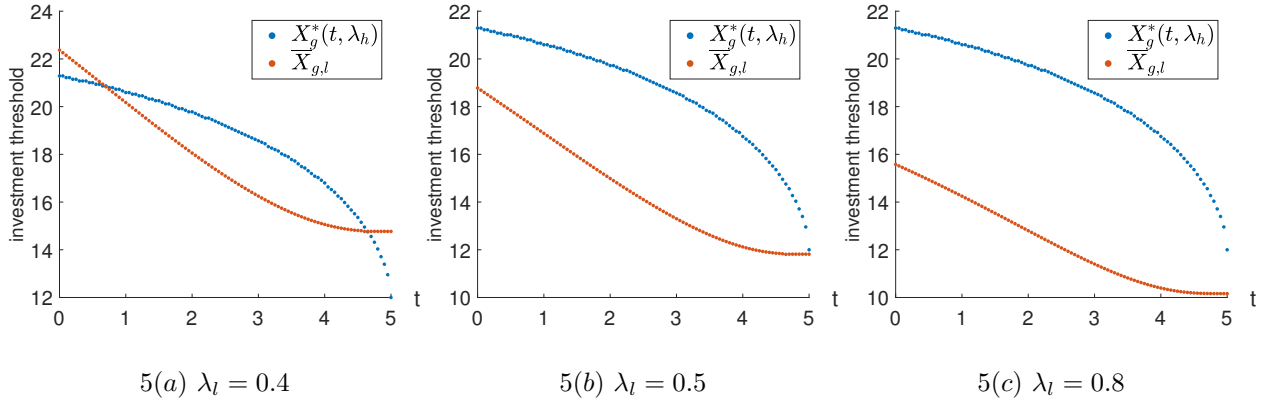

5(a) $\lambda_l = 0.4$ 5(b) $\lambda_l = 0.5$ 5(c) $\lambda_l = 0.8$

Figure 5: The binding threshold $\overline{X}_{g,l}(t)$ and the first-best investment threshold $X_g^*(t; \lambda_h)$ are plotted against time t at three values of λ_l.
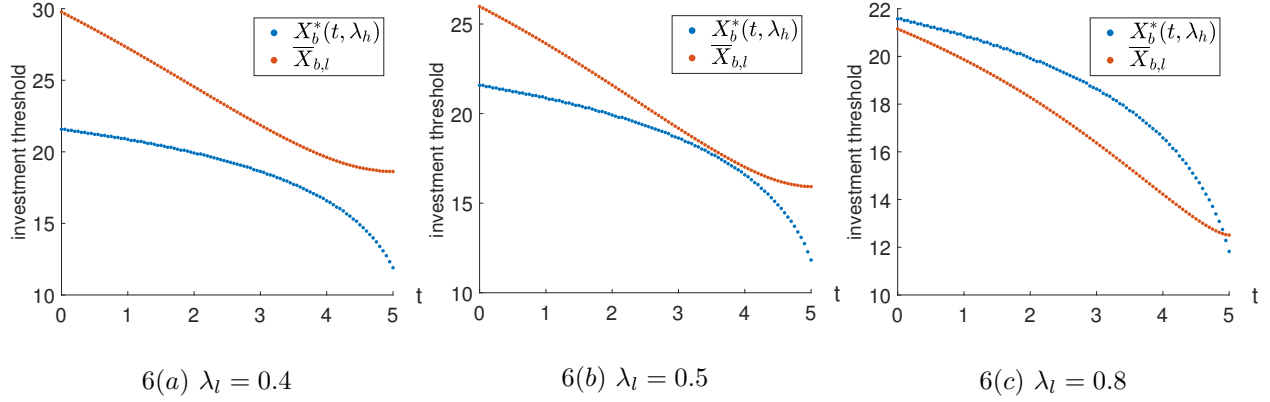
6(a) $\lambda_l = 0.4$ 6(b) $\lambda_l = 0.5$ 6(c) $\lambda_l = 0.8$

Figure 6: The binding threshold $\overline{X}_{b,l}(t)$ and the first-best investment threshold $X_b^*(t; \lambda_h)$ are plotted against time t at three values of λ_l.
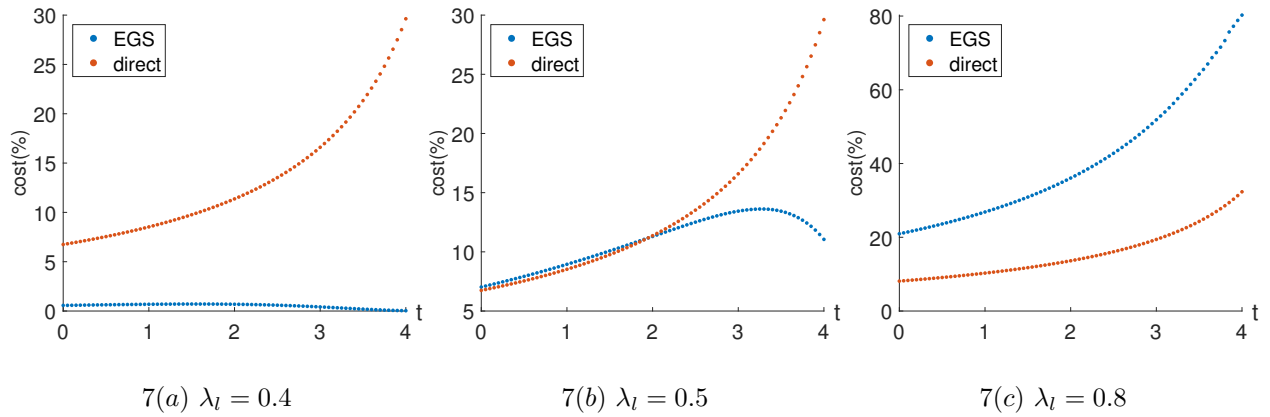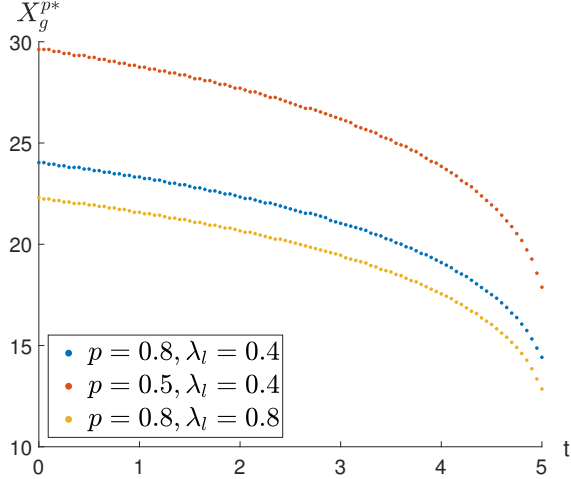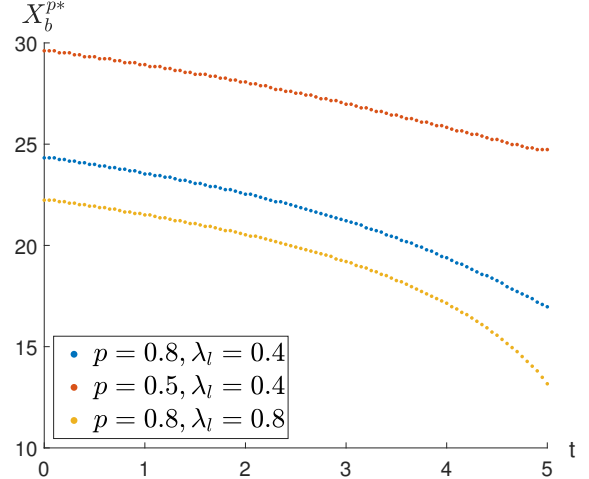

7(a) $\lambda_l = 0.4$ 7(b) $\lambda_l = 0.5$ 7(c) $\lambda_l = 0.8$

Figure 7: The information costs in the least-cost separating equilibrium are plotted against time t at three values of λ_l for loan arrangement under the EGS and direct bank loan.

5.2 Pooling equilibrium

Figures 8 and 9 plot the investment thresholds and corresponding information costs (in percents) of the high-type firm in the pooling equilibrium with respect to time under different sets of parameter values: (i) $p = 0.8$, $\lambda_l = 0.4$, (ii) $p = 0.5$, $\lambda_l = 0.4$ and (iii) $p = 0.8$, $\lambda_l = 0.8$. The two financing choices of EGS and the direct bank loan are plotted separately. Figure 8 and Figure 9 show that for larger probability p, the optimal threshold of the high-type firm under the pooling strategy gets lower and the information cost is lower; for lower multiplier λ_l, the optimal threshold gets higher and the information cost is higher. Similar to Figure 5, the information cost of the high-type firm under pooling strategy increases with time and finally jump to zero at maturity T.
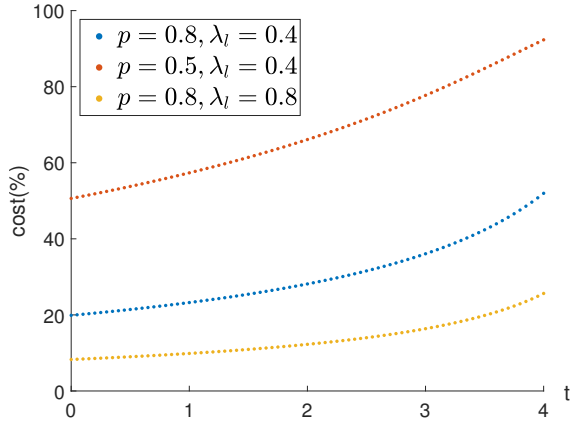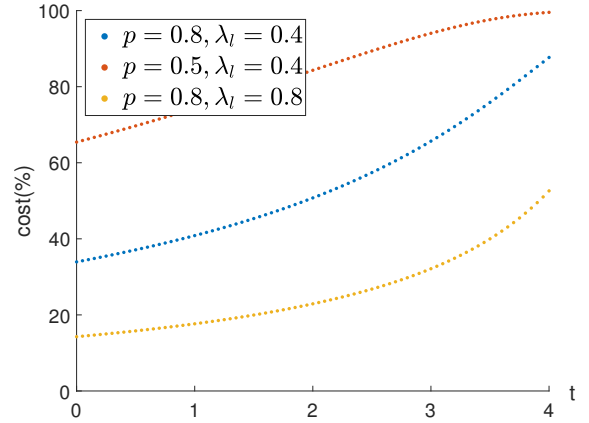
8(a) EGS

8(b) direct bank loan

Figure 8: The optimal thresholds $X_g^{p*}(\lambda_h)$ and $X_b^{p*}(\lambda_h)$ are plotted against time t at three sets of parameter values of λ_l and p.

9(a) EGS

9(b) direct bank loan

Figure 9: The information costs of the high-type firm in the Pareto-dominant pooling equilibrium are plotted for EGS and the direct bank loan against time t at three sets of parameter values of λ_l and p.

Table 1 lists the information costs (in percents) of the high-type firm under EGS separating, EGS pooling, direct bank loan separating and direct bank loan pooling with respect to the multiplier λ_l and two maturity dates. The firm chooses the investment and financing strategy maximizing its real option value (or equivalently, minimizing its information cost). The figure shows that the high-type firm chooses to separate through EGS when λ_l is low since the gap between the revenue flows of the two firm types is large and the high-type firm finds it less costly to separate by speeding up its investment. In particular, the high-type firm invests at its first-best threshold when the corresponding information cost is zero. When the multiplier λ_l is high and close to λ_h, the high-type firm resorts to the pooling strategy through EGS since the information cost of separating is large. For intermediate level of λ_l,

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the high-type firm chooses to separate through the direct bank loan. This result is consistent with Morellec and Schürhoff (2011), though EGS is not quite the same as issuing equity but rather a mixture of equity and debt financing. The table also shows the effect of the maturity date of the investment opportunity. For intermediate value of the multiplier λ_l ($\lambda_l = 0.5$ for instance), the high-type firm would choose to separate through the direct bank loan for shorter maturity ($T = 5$) but to separate through EGS for longer maturity ($T = 10$). The result is consistent with that showed in Figure 7(b).

Table 1: The information costs (%) of the high-type firm under EGS separating, EGS pooling, direct bank loan separating and direct bank loan pooling are listed against the multiplier λ_l with two maturity dates.

λ_l	$T = 5$				$T = 10$			
	EGS_s	EGS_p	loan_s	loan_p	EGS_s	EGS_p	loan_s	loan_p
0.125	0	78.67	6.68	99.08	0	58.31	3.07	88.20
0.250	0	64.55	6.68	87.83	0	43.77	3.07	61.16
0.375	0.12	52.75	6.66	69.09	0.03	33.77	3.06	41.31
0.500	7.14	42.42	6.67	51.93	2.38	26.04	3.07	28.69
0.625	18.80	33.22	6.94	38.62	6.68	19.75	3.15	20.43
0.750	21.36	25.00	7.76	28.62	8.21	14.49	3.42	14.73
0.875	19.28	17.61	8.34	20.98	8.05	10.01	3.63	10.57
1.000	14.47	11.06	7.85	15.15	6.55	6.18	3.51	7.46
1.125	7.99	5.19	6.68	10.46	3.90	2.87	3.08	5.01

In Table 2, we show the effect of probability p and fractional bankruptcy loss α on information costs of the high-type firm under different strategies. It suggests that the high-type firm would choose to separate from the low-type for low probability p since it is more likely to be perceived as low-type under the pooling strategy. It would instead choose to adopt the pooling strategy for high probability p. The EGS agreement is shown to be more sensitive to the level of probability compared with the direct bank loan. Considering the pooling equilibrium, the firm prefers to pool through the direct bank loan for low probability p while through EGS for high probability p. Table 2 also suggests that the high-type firm prefers separating for low bankruptcy cost while pooling for high bankruptcy cost, since pooling with the low-type will mitigate the loss upon default when the bankruptcy cost is too high. The direct bank loan is shown to be more sensitive to change to the fractional bankruptcy loss α. Under pooling equilibrium, the firm prefers the direct bank loan for low bankruptcy cost while EGS for high bankruptcy cost.

Table 2: The information costs (%) of the high-type firm with varying values of probability p and fractional loss α due to bankruptcy using the loan arrangement of either EGS or direct bank loans under separating and pooling equilibriums.

p	EGS_s	EGS_p	loan_s	loan_p	α	EGS_s	EGS_p	loan_s	loan_p
0.1	20.95	42.34	8.11	39.18	0.1	19.29	21.42	4.47	19.35
0.2	20.95	36.98	8.11	35.76	0.2	20.41	21.76	6.87	23.31
0.3	20.95	31.89	8.11	32.43	0.3	21.46	22.07	9.38	27.26
0.4	20.95	26.85	8.11	28.93	0.4	22.46	22.44	12.02	31.30
0.5	20.95	21.92	8.11	25.29	0.5	23.41	22.70	14.70	35.14
0.6	20.95	17.19	8.11	21.65	0.6	24.30	22.94	17.45	38.88
0.7	20.95	12.66	8.11	18.00	0.7	25.14	23.17	20.23	42.50
0.8	20.95	8.28	8.11	14.27	0.8	25.94	23.38	23.02	45.96
0.9	20.95	4.05	8.11	10.50	0.9	26.68	23.66	25.87	49.44

6 Conclusion

We examine equity guarantee swaps (EGS), a new financing choice for a small- or medium-sized firm without sufficient ability to issue equity or debt and can only finance an investment project through bank loans. We develop the dynamic real option signaling game models of debt financing using EGS together with direct bank loans as an alternative. This paper assumes asymmetric information, where the firm quality is private information for the firm management and not accessible to outsiders including the guarantor and the bank. The low-type firm may have an incentive to mimic the investment timing and financing choice of the high-type in order to be charged by a lower proportional share of equity or coupon rate. Correspondingly, the high-type firm may have an incentive to separate from the low-type by invest earlier or adopting a different financing choice. Under the separating equilibrium, the high-type firm is faced with information costs due to investment distortion. On the other hand, if the high-type firm fails to separate due to high information costs, it may choose a pooling equilibrium, where both firm types invest at the same time and adopt the same financing choice. We examine characteristics of separating and pooling equilibriums under the EGS agreement and direct bank loan and specify the binding and optimal investment thresholds in different scenarios.

This paper extends previous real option signaling game models from perpetuity assumption to a finite time horizon, where the life span of the investment opportunity is limited. We find significant time dependence of the firm's investment and financing behavior under separating and pooling equilibriums. By comparing corresponding information costs and investment thresholds, we show that the firm's investment choice between its first-best and binding thresholds and financing choice between EGS and direct bank loans may alter as time evolves. We also examine the firm's choice among EGS separating, EGS pooling, direct bank loan separating and direct bank loan pooling against parameters other than time, obtaining similar results to previous literature about equity-debt decisions. It turns out that EGS is a hybrid of debt and equity financing and shares the properties of both. In summary, this paper extends the real option signaling game model of debt financing into the generalized fi-

nite time horizon and applies this model to a new financial product, equity guarantee swaps, which fits better to the real world and provides a possible solution to corporate financing problems of small- and medium-sized firms.

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Appendix A - Proof of Lemma 1

We would like to prove the existence of a root for $G_{g,l}(X) = 0$ [see (3.2)], where

$$G_{g,l}(X,t) = I_{g,l}^m(X) - V_g^c(X,t;\lambda_l), \quad \hat{\hat{X}}_{g,l} \le X \le X_g^*(t;\lambda_l).$$

Since $I_l^m(\hat{\hat{X}}_{g,l}) = 0$, and the real option value function $V_g^c(X,t;\lambda_l)$ is non-negative, we have $G_{g,l}(\hat{\hat{X}}_{g,l},t) < 0$. At $X = X_g^*(t;\lambda_l)$, the low-type firm invests at its first-best threshold under complete information. According to the value-matching condition [see (2.17) and (2.18)], we have

$$V_g^c(X_g^*(t;\lambda_l),t;\lambda_l) = I_g^c(X_g^*(t;\lambda_l);\lambda_l) = \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I - \alpha\lambda_l\Pi(\underline{X}_g(\lambda_l))\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi.$$

The value of liabilities under EGS can be deduced to be [replacing $\frac{c_b}{r}$ in $L_b(X;\lambda)$ by I]

$$L_g(X;\lambda) = (\eta + \eta_\alpha)\lambda^\xi\Pi(X)^\xi(F+I)^{1-\xi}.$$

Obviously, $L_g(X;\lambda_h) < L_g(X;\lambda_l)$ and $E_g(X_g^*(t;\lambda_l);\lambda_l) < E_g(X_g^*(t;\lambda_l);\lambda_h)$ since $\lambda_h > \lambda_l$ and $\xi < 0$. We consider

$$I_{g,l}^m(X_g^*(t;\lambda_l))$$
$$= \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + D_g(X_g^*(t;\lambda_l);\lambda_l) - \frac{E_g(X_g^*(t;\lambda_l);\lambda_l)}{E_g(X_g^*(t;\lambda_l);\lambda_h)}L_g(X_g^*(t;\lambda_l);\lambda_h)$$
$$> \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + D_g(X_g^*(t;\lambda_l);\lambda_l) - \frac{E_g(X_g^*(t;\lambda_l);\lambda_l)}{E_g(X_g^*(t;\lambda_l);\lambda_h)}L_g(X_g^*(t;\lambda_l);\lambda_l)$$
$$> \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + D_g(X_g^*(t;\lambda_l);\lambda_l) - L_g(X_g^*(t;\lambda_l);\lambda_l)$$
$$= \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + \left[F + I - \lambda_l\Pi(\underline{X}_g(\lambda_l))\right]\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi$$
$$- \left[F + I - (1-\alpha)\lambda_l\Pi(\underline{X}_g(\lambda_l))\right]\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi$$
$$= \lambda_k\Pi(X_g^*(t;\lambda_l)) - F - I - \alpha\lambda_l\Pi(\underline{X}_g(\lambda_l))\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi = V_g^c(X_g^*(t;\lambda_l),t;\lambda_l).$$

Therefore, we have $G_{g,l}(X_g^*(t;\lambda_l),t) = I_{g,l}^m(X_g^*(t;\lambda_l)) - V_g^c(X_g^*(t;\lambda_l),t;\lambda_l) > 0$. According to the mean value theorem, there exists a solution of the algebraic equation: $G_{g,l}(X,t) = 0$ within the interval $[\hat{\hat{X}}_{g,l}, X_g^*(t;\lambda_l)]$ since the function $G_{g,l}(X,t)$ continuous with respect to X. In other words, there exists a threshold $\overline{X}_{g,l}(t)$ binding (3.2) within the interval $[\hat{X}_g(\lambda_h), X_g^*(t;\lambda_l)]$, such that the high-type firm can separate from the low-type at time t when $X \le \overline{X}_{g,l}(t)$.

Appendix B - Optimal thresholds near maturity

At $t \to T^-$, the investment opportunity remains alive and thus the real option value V satisfies the following equation:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}X^2\frac{\partial^2 V}{\partial X^2} + \mu X\frac{\partial V}{\partial X} = rV.$$

The continuity of the real option value functions implies that they converge to their corresponding exercise payoffs when the investment opportunity is about to evaporate. Similar to the analysis of the exercise threshold at time close to expiry for an American option (Kwok, 2008), the optimal stopping threshold X^* when $t \to T^-$ is obtained by setting

$$\left.\frac{\partial V(X,t)}{\partial t}\right|_{t=T^-} = -\frac{\sigma^2}{2}X^2\frac{\partial^2 V(X,T^-)}{\partial X^2} - \mu X\frac{\partial V(X,T^-)}{\partial X} + rV(X,T^-) = 0, \tag{B1}$$

where $V(X,T^-)$ is set to be the exercise payoff.

First-best threshold under equity guarantee swap

If the firm chooses EGS, the real option value under complete information at $t = T^-$ becomes $I_g^c(X;\lambda)$, where

$$V_g^c(X,T^-;\lambda) = I_g^c(X;\lambda) = \lambda\Pi(X) - F - I - \eta_\alpha\lambda^\xi\Pi(X)^\xi(F+I)^{1-\xi}.$$

The first- and second-order derivatives of $V_g^c(X,T^-;\lambda)$ with respect to X are found to be

$$\frac{\partial I_g^c(X)}{\partial X} = \frac{\lambda}{r-\mu} - \xi\eta_\alpha\lambda^\xi X^{\xi-1}(r-\mu)^{-\xi}(F+I)^{1-\xi},$$

$$\frac{\partial^2 I_g^c(X)}{\partial X^2} = -\xi(\xi-1)\eta_\alpha\lambda^\xi X^{\xi-2}(r-\mu)^{-\xi}(F+I)^{1-\xi}.$$

Substituting these relations into (B1), the first-best threshold $X_g^*(T^-;\lambda)$ is given by the solution of the following equation:

$$\lambda X + \left[\frac{\sigma^2}{2}\xi(\xi-1) + \mu\xi - r\right]\eta_\alpha\lambda^\xi\Pi(X)^\xi(F+I)^{1-\xi} - r(F+I) = 0.$$

First-best threshold under direct bank loan

If the firm chooses the direct bank loan, the real option value under complete information becomes $E_b(X;\lambda,c_b)$ at $t \to T^-$:

$$V_b^c(X,T^-;\lambda) = E_b(X;\lambda,c_b) = \lambda\Pi(X) - F - I - \eta_\alpha\lambda^\xi\Pi(X)^\xi\left(F + \frac{c_b}{r}\right)^{1-\xi}, \tag{B2}$$

where the coupon rate c_b is determined by the following budget constraint:

$$\frac{c_b}{r} - (\eta+\eta_\alpha)\lambda^\xi\Pi(X)^\xi\left(F + \frac{c_b}{r}\right)^{1-\xi} = I. \tag{B3}$$

Since c_b has implicit dependence on X, we apply the Implicit Function theorem to (B3) to obtain

$$\frac{\partial c_b}{\partial X} = \left[\frac{X}{\xi(c_b - rI)} - \frac{(1/\xi - 1)X}{c_b + rF}\right]^{-1}.$$

Combining (B3) and (B2), the value function can be expressed as

$$V_b^c(X, T^-; \lambda) = \lambda\Pi(X) - F - I - \frac{\eta_\alpha}{\eta + \eta_\alpha}\left(\frac{c_b}{r} - I\right).$$

The first- and second-order derivatives of $V_b^c(X, T^-; \lambda)$ are found to be

$$\frac{\partial V_b^c}{\partial X} = \frac{\lambda}{r - \mu} - \frac{\frac{\eta_\alpha}{\eta+\eta_\alpha}\xi X^{-1}\left(\frac{c_b}{r} - I\right)}{1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}},$$

$$\frac{\partial^2 V_b^c}{\partial X^2} = \frac{\frac{\eta_\alpha}{\eta+\eta_\alpha}(1 - \xi)\xi X^{-2}\left(\frac{c_b}{r} - I\right)\left[\frac{r(F+I)}{c_b+rF}\right]^2}{\left[1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}\right]^3}.$$

Substituting V_b^c, $\frac{\partial V_b^c}{\partial X}$ and $\frac{\partial^2 V_b^c}{\partial X^2}$ into (B1), the first-best threshold $X_b^*(T^-; \lambda)$ is given by the solution of the following equation:

$$\frac{\eta + \eta_\alpha}{\eta_\alpha}\frac{\lambda X - r(F + I)}{c_b/r - I} = \frac{\frac{\sigma^2}{2}(1 - \xi)\xi\left[\frac{r(F+I)}{c_b+rF}\right]^2}{\left[1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}\right]^3} - \frac{\mu\xi}{1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}} + r.$$

Since the coupon rate c_b is a function of X, we resort to numerical method to solve the above algebraic equation.