Equity investment decision-making cost under the existence of convertible note holder in the second financing round

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Abstract

Although convertible note is favourably used for the early stage start-up financing, its usage creates a complicated situation among entrepreneur, convertible note holder and new equity investor in the second-round financing negotiation. The main objective of this paper is to build a model dealing with the interactions of these three key parties. This paper aims to figure out the cost for equity investment decision-making by incorporating the real option structure of the conversion of convertible note into equity, as well as the adverse selection problem in the financing negotiation. The results of case simulation suggest that the discount and valuation cap that are accompanied with the convertible note contract have great impacts on the cost for equity investment decision-makings, and entrepreneur should consider it when entering into the financing negotiation in the second financing round.

Keywords: convertible note, financing negotiation, adverse selection, real option

1. Introduction

For the early stage start-up financing, convertible note is favourably used. According to the survey by Marianne Hudson who is the ACA (Angel Capital Association) Executive Director in 2015, 78% of ACA members had used at least one convertible note in the recent 18 months until that year. Because of the high demand of the usage of convertible note, it is not difficult to find the websites or books for entrepreneurs, which clearly explain the details of how to use convertible note as a financing method and how to calculate the share after it converted into equity. This paper is also using the practical oriented books as a reference of the basic scheme of convertible note, such as the one by Feld and Mendeslon (2016) and by Poland (2017).

Although convertible note is a common financing method for start-ups and venture businesses, it is not easy to handle with. Convertible note is classified as 'mezzanine financing' which has both characteristics of debt and equity (e.g., Nijs (2014)). Convertible note has the feature of option, in other words, the debt holder has a right to convert it to equity if the situation becomes favourable to do so. Thus, the existence of convertible note holder could create a bit complicated situation, especially in the second financing round, if entrepreneur wants to raise funds as equity. The interests of these three parties are not always aligned. For new equity investor, as well as entrepreneur, whether convertible note holder exercises the conversion option or not is a grave concern because the equity share could be diluted after investment, at the expense of the increase of the share of convertible note holder. Therefore, considering these parties interactions is essential in the financing negotiation. However, so far, there are few academic researches and practical oriented guides that are dealing with this complicated relationship as the main topic.

In addition to the inherent complex feature by the usage of convertible note, the investment and financing activities by nature often bring about the information asymmetric problems. In the negotiation of investment and financing, the parties would normally be reluctant to reveal all the information they have in order to seal the deal as favourable for themselves as possible. Thus, the informational asymmetric situation is created because the parties are divided into the one who is informed and the one who is uninformed. Many academic researchers have addressed this issue related to the usage of convertible note. For example, Stein (1992) insists

that corporations may use convertible bonds when adverse selection problems make a conventional stock issue unattractive. Lewis (1998) suggests that some issuers design convertible debt to mitigate asset substitution problems, while others design it to reduce adverse selection problems. Related to it, Bascha (2001) argues that the ex-ante agreed optimal exit policy can be implemented with convertible securities. Krishnaswami and Yaman (2008) argue that moral hazard and adverse selection are important determinants of the likelihood of issuing convertible bonds over straight bonds. Wang et al. (2009) explain the reason why convertible notes would be chosen by introducing two academic approaches of 'asymmetric information approach' which focuses on its mitigation effect of convertible note and 'incomplete contract approach' which focuses on the renegotiation possibilities after investment.

As can be seen by the discussions above, it is obvious that the asymmetric information problem is critical for the financing negotiation especially when using convertible note. As will be explained, adverse selection is the one in this case. Nevertheless, those academic discussions are focusing only on the interactions between convertible note and entrepreneur in the early stage. These are not dealing with the relationship into which new equity investor enter in the second financing round, so-called the Series A round, for example. Therefore, one of the objectives of this paper is to build a model that is dealing with the interactions among these three key parties in the financing negotiation.

Furthermore, it can be logically predicted that new equity investor must bear a kind of additional cost under the existence of convertible note holder when entering the financing negotiation. In academic, in effect, the cost related to the investment and financing has been discussed. This topic seems traditionally to be recognised as post-investment monitoring cost. For example, Gompers (1995) examines the structure of staged venture capital investments when agency and monitoring costs exist. Neher (1999) explains that the venture capitalists cannot observe whether the project has become a failure without bearing a monitoring cost. Pagano and Röell (1998) insist that the optimal ownership structure generally involves some measure of dispersion, to avoid excessive monitoring by other shareholders.

Considering both post investment monitoring cost and procurement cost is critical. Lewis et al. (2003) point out in their empirical study that convertible debt can be designed to mitigate

different combinations of debt- and equity-related costs of external finance. However, it is not these kinds of costs but rather the cost for ‘investment decision-making’ that should be focused especially in the second-round financing negotiation. The main objective of this paper is to figure out this type of cost. In other words, this paper is trying to reveal the mathematical form of cost for equity investment decision-making. In particular, the unique feature of the model is trying to incorporate the real option structure, as well as the adverse selection problem. In addition, for better understanding the cost, the case simulation is also implemented on the basis of the effect of discount rate and valuation cap that are accompanied with the convertible note contract (these are also explained in the later section).

The structure of this paper is as follows. In section 2, the usage of convertible note as a financing method is explained, along the line of basic financing scheme. In section 3, the model of cost for equity investment decision making is developed. In section 4, the case simulation by using an actual start-up’s data is conducted. In the section 5, conclusion is remarked.

2. Usage of convertible note as a financing method

2.1. What is convertible note?

The convertible note is classified as one of mezzanine financing methods. As its name suggests, mezzanine financing is classified as a financing method with an intermediate characteristic between debt and equity. In the perspective of investor, the main feature of debt is financial obligation and contractual claim on the firms' assets, while the one of equity is the residual claim. Several famous mezzanine financing methods are known, such as convertible note, preferred share, option-linked bonds, step-up rate loans, second lien debt, PIK (Paid-in-kind) note, profit participating loans/rights, silent participation (Nijs, 2014). Although being categorised in the same 'mezzanine financing', the degree of characteristic between debt and equity of each financial instrument is different. Preferred share is similar to equity because it is usually not prior to senior debt, while option-linked bonds are close to debt because its basic characteristic is contractual claim, for example. Convertible note, which is sometimes called as convertible bond or convertible debt, has both features of debt and equity. Its basic feature is financial obligation, as its name represents. However, the convertible note holders have a right to convert it to equity under some conditions after obtaining it as debt. This means that they can choose whether they keep it as debt or change it to equity, depending on the situations. In practice, it would be rational that the convertible note holders exercise their right to convert when the economic situation has changed and they could benefit by doing so. The definition of convertible note is provided from this perspective, and the way of defining it is almost the same in academic and in practice. As an example of definition in academic, Tirole (2006) explains convertible note in his book as "one of the many claims that take the form of an option, which the holders can elect to exercise if circumstances are favourable. Convertible debt is basically debt, except that its holders can exchange it for the firm's shares at some predetermined conversion rate." On the other hand, from the practical perspective, for example, int his book for practitioners, Poland (2017) defines convertible note as follows: "In a convertible debt investment deal (also referred to as a convertible note), the investor makes a loan to the company (the debt), and that loan converts into equity at some point in the future, with an extra bonus to the investor for taking on higher risk of the early-stage startup." The "extra bonus" in the last sentence of his definition can be interpreted as the similar

meaning by Tirole of "at some predetermined conversion rate." The future convertible note holders can negotiate over the condition of conversion for their benefice as the compensation of bearing the debt with higher credit risk when convertible note is bought. (Here, credit risk is the one that borrowers fail to meet their financial obligation in the due date.) On the flip side, it can also be explained that the future convertible note holder can have a large expectation of the high valuation of the business in the second-round financing stage, which leads to the favourable condition for them to convert it to equity, thus they are willing to bear the credit risk. Therefore, convertible note is recognised as a tool for the holder to increase the future economic or financial benefit adjusting to its circumstance.

2.2. Why is convertible note used for start-up financing?

It would be rational to say that debt is not selected as an instrument for financing start-ups especially in the early stage, such as in the seed round, because the probability of failure of financial obligations is relatively high from the perspective of credit risk. In fact, however, convertible notes are favourably selected in this stage, as mentioned before. The potential convertible note holder can expect the business of the start-up will go well and the value of the firm will increase in the near future, in which they could be more economically compensated by holding as equity (receiving dividends and selling out the share) rather than as debt (receiving the interests and repayment of its principal). This is not the only reason why convertible note is used for start-up financing. Although there is no widely accepted explanation in academic so far, several technical advantages are pointed out in practice. According to the book of Poland (2017), for example, he pointed out the three advantages of speed (of obtaining money at hand), lower legal fees and delayed valuation. It is often said that the valuation of the firm for equity investment, in the case of start-ups in particular, is onerous and time-consuming because it requires severe and detailed negotiation between the entrepreneur(s) and investor(s) until they reach an agreement. On the other hand, the negotiation process for funding with convertible note is relatively simple. They must agree on only a few deal points and obtain available money quickly. Start-ups need funds for making their ends meet, and when they can have money at hand is a critical matter for them. Thus, the matter of speed is one advantage of using convertible note for funding start-ups. Closely related to this matter, it is also often said that the legal processes for equity investment,

including valuation and financing contract documentation, for example, is complicated, and it is necessary to ask the legal professionals such as attorney (or barrister). Hence, the fees become more expensive than in the case of the negotiation for funding with convertible note. Whilst these two matters are critical, the biggest advantage of using convertible note is for both entrepreneur(s) and investor(s) to delay the firm's or the project's valuation. Rather, it is to be able to avoid any complicated valuations at this moment. In the timing of funding with convertible note, the business has just begun, and only the prototype product and/or service have been prepared. In this stage, the exact valuation of the firm or the project is almost impossible because the information available is too little, even though the investors have a gut feeling of great growth potential of the entrepreneur's business ideas and plans. It is not too late for both entrepreneur(s) and investor(s) to put a valuation after observing the realisation of its planned product and/or service and evaluating the progress of the growth of the business.

2.3. Basic scheme of procurement with equity and convertible note

In this subsection, the basic scheme of investment with convertible note is reviewed. Fig.1 shows the valuation in the early stage, such as Seed round. When entrepreneurs need funds for the business in the seed round, the investors such as Angel would provide the necessary (or desired by entrepreneur) fund with a form of convertible note. I_C refers to the amount of investment as convertible note.

As mentioned above, while there would be no valuation agreed between entrepreneur and investor, it would be possible to think about the hypothetical 'pre-money value' and 'post-money value', though those concepts are generally used in the following financing round with equity such as Series A, B, C etc.

Introducing these concepts can explain why convertible note is favourable for entrepreneur in the early stage. For example, if entrepreneur can succeed in raising money of $100K by convertible note and the hypothetical 'pre-money value' of its business can be evaluated as $100K, then the share for entrepreneur is 50.00% and the one for convertible note holder is also 50.00%. If entrepreneur can succeed in raising money of $100K by convertible note and the hypothetical 'pre-money value' of its business can be evaluated as $900K because the

time passes a little and it turns out to be better, then the share for entrepreneur is 90.00% and the one for convertible note holder is also 10.00%. Although this story is theoretical, the takeaway from it is that too early valuation could bring significantly lower share to the entrepreneur. In addition to the difficulty of valuation itself, this is also the reason why the entrepreneur wants to avoid too early valuation.

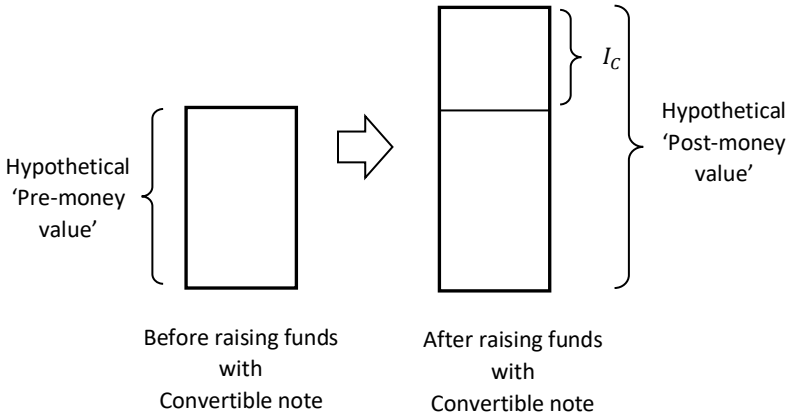


Fig.1 The valuation in the Seed round

The convertible note investors normally expect that entrepreneur will progress their project and its business will grow enough to be able to encourage new potential equity investors, or 'second-round equity investor' to invest and to establish a next 'Series A round'. The period between the Seed round and the Series A round seems to be 6 to12 months in many cases. Fig.2 shows what will be done at the Series A round.

Entrepreneur will explain the current situation and the prediction of its project and/or business to the potential investors, and will propose the amount which they need to and want to procure as equity (=I_E) in order to take a further step of its project and/or business. If the potential investors are interested and fascinated in the explanations, the negotiation about the price of the new equity (=s) will begin. At the same time, the amount of the pre-money value at this moment (=V_0) will be also negotiated and determined, on the basis of the hypothetical 'post-money value' in the Seed round. If they reach an agreement, the equity investment will be implemented.

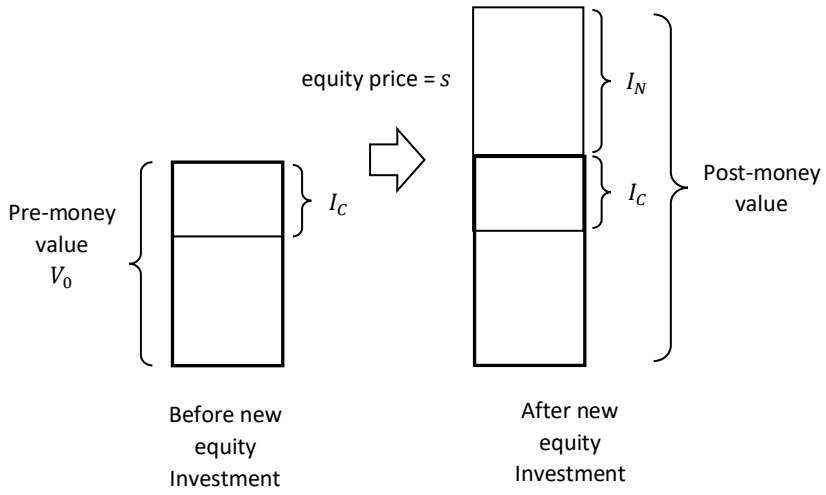


Fig.2 The valuation in the Series A round

2.4. Dilution problem for equity investor in the presence of convertible note

On the contrary to the procurement with debt instruments, entrepreneur does not need to repay the money when they procure with equity. However, it does not mean 'free-lunch.' The effect of share dilution for entrepreneur should be considered. The problem should also be critical for potential equity investors, when convertible note holders exit before they make their investment decision. The share of equity represents the degree of controlling power of running the firm. Thus, the dilution problem is quite sensitive for both entrepreneur and investors, and this problem is located as the central topic of this paper.

Tables 1 and 2 show this situation (in practice, it seems to be called as 'cap table'). The share for entrepreneur will decrease 100% to $\frac{V_0}{V_0+I_E} \times 100$ % in the case without convertible note holder, and to $\frac{V_0}{V_0+I_c+I_E} \times 100$ % in the case with convertible note holder. The share for second-round equity investor will also decrease from $\frac{I_E}{V_0+I_E} \times 100$ % to $\frac{I_E}{V_0+I_c+I_E} \times 100$ % in the presence of convertible note holder.

Table 1: Share at the second round (without convertible note holder)

	< Number of shares >	< Percentage of shares >
Entrepreneur	$\frac{V_0}{s}$	$\frac{V_0}{V_0 + I_E} \times 100$
Second-round equity investor	$\frac{I_E}{s}$	$\frac{I_E}{V_0 + I_E} \times 100$
< Total >	$\frac{V_0 + I_E}{s}$	100

Table 2: Share at the second round (with convertible note holder)

	< Number of shares >	< Percentage of shares >
Entrepreneur	$\frac{V_0}{s}$	$\frac{V_0}{V_0 + I_c + I_E} \times 100$
Convertible notes holder	$\frac{I_C}{s}$	$\frac{I_C}{V_0 + I_c + I_E} \times 100$
Second-round equity investor	$\frac{I_E}{s}$	$\frac{I_E}{V_0 + I_c + I_E} \times 100$
< Total >	$\frac{V_0 + I_c + I_E}{s}$	100

There are other factors that can be considered in this dilution problem when raising funds with convertible note: the business practice of 'discount' and 'valuation cap'. The former means the practice that the negotiated new equity price (=s) should be reduced to some extent when calculating the share for convertible note holder. The latter means the practice that the pre-money value is fixed at the pre-agreed value between entrepreneur and convertible note holder, no matter how much the pre-money value is agreed in the equity investment negotiation. These can be used as either a single practice or a combined one.

For better understanding, consider the following simple numerical example. An Angle investor invests \$25k in a start-up's seed round using a convertible note with a \$5M cap, 20% discount. The start-up succeeds in raising money as equity, with a pre-money valuation of \$10M and an equity price is \$5.00 at the Series A round. If the discount is applied, the equity price for calculating the convertible note holder's share should be \$5.00×(100 – 20%) =

$4.00. If the valuation cap is applied, the price should be $5.00×($5M cap ÷ $10M pre-money value) = $2.50, which is equivalent to 50% discount. When calculating the share, the latter is favourable for the convertible note holder because the share is determined by the equation of the investment amount of $25k ÷ the equity price calculated. Thus, this price would be adopted and the share would be 10,000. On the contrary, if a pre-money valuation is $6M, and the discount is applied, the price should be the same one of $4.00. If the valuation cap is applied, the price should be $5.00×($5M cap ÷ $6M pre-money value) = $4.17. In this case, the former would be adopted and the share would be 6,250.

As can be seen from the above, both 'discount' and 'valuation cap' can reward the convertible note investors who bear the high risk of the start-up's business failure by increasing the equity share after conversion. Hence, both the discount rate and the amount of cap are the important topics for convertible note investor in the negotiation with entrepreneur. The earlier they invest, the deeper discount and/or the lower amount of the valuation cap would be required. It is said in practice that this discount rate is set within the range between 15% to 25%.

Some might argue that the discount and the valuation cap are different. However, for modelling the situation from the viewpoint of the dilution problem, these can be expressed as the discount rate because both are represented with the equity price. In addition to this point, the discount rate is numerically easier to be handled than the valuation cap. Let the discount rate be α $(0.00 < \alpha < 1.00)$. The equity price for calculating the share for the convertible note holder should be changed from s into $s \times (1-\alpha)$. Table 2 is changed into Table 3.

Table 3: Share at the second round (with convertible note holder' discount)

	< Number of shares >	< Percentage of shares >
Entrepreneur	$\frac{V_0}{s}$	$\frac{V_0}{V_0 + I_c/(1-\alpha) + I_E} \times 100$
Convertible notes holder	$\frac{I_C}{s(1-\alpha)}$	$\frac{I_C/(1-\alpha)}{V_0 + I_c/(1-\alpha) + I_E} \times 100$
Second-round equity investor	$\frac{I_E}{s}$	$\frac{I_E}{V_0 + I_c/(1-\alpha) + I_E} \times 100$
< Total >	$\frac{V_0 + I_c/(1-\alpha) + I_E}{s}$	100

2.5. Conversion into equity: real option structure

According to the relationship of the equity price and the effect of discount and/or valuation cap above, it can be written as $s \times (1 - \alpha) = s \times (V_{cap}/V_0)$, or $1 - \alpha = V_{cap}/V_0$, where V_{cap} represents the amount of valuation cap that is determined through negotiation between entrepreneur and convertible note holder. It is obvious that the greater the pre-money value of V_0 becomes, the greater the discount α should be. This means that if the business goes well and the expectation of the future success increases, in other words, the pre-money value is evaluated to be high, the effect of valuation cap becomes larger. It is the very situation where convertible note holders are expecting because they can convert their debt into equity, which are more favourable in such a situation. New equity investors would also be attractive to the project and be willing to invest their funds. On the contrary, if the business does not go as desired, and the pre-money value is evaluated to be not so high as estimated, or lower, this situation is not suitable for convertible note holder to convert their debt right now. They would rather wait to execute the conversion, because whether they execute it or not is by nature a right but not an obligation.

This mechanism is the answer to the question of "when the conversion will be executed?" In practice, three conversion trigger cases are often pointed out. According to Poland (2017), these are on 'threshold financing', on IPO, and by 'elective conversion'. The first one means that conversion is executed when the entrepreneur succeeds in raising funds more than the pre-agreed amount in the next financing round. The second one means that conversion is executed only when IPO is realised. The last one means that conversion is executed at any time if 'threshold financing' is not implemented. Although there is no clear guidance of conversion in academia so far, the first one may be the most common in practice, especially in the United States. Along the 'elective conversion', in this case, the conversion will be executed when entrepreneur succeeds in obtaining the more amount of investment from new equity investors than the pre-determined 'threshold' amount, which is represented as I_X, in the Series A round.

Taking the situations above into consideration, the real options analysis can be applicable to this conversion mechanism. According to Copland and Antikarov (2003), the real options is the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or

abandoning) at a predetermined cost called the exercise price, for a predetermined period of time. In this case, the pre-determined amount of investment from new equity investors corresponds to the 'exercise price', and a 'predetermined period of time' is the one between the Seed round and the Series A round, which is around twelve months. The conversion mechanism can be said as the option to wait (or option to defer) for convertible note to convert their debt to equity. In effect, also from the academic perspective, Wang et al. (2009) explain that convertible notes give the firms a "back door" to equity and give investors an opportunity to wait and see if the project is worth investing in. This is represented in Fig.3 below.

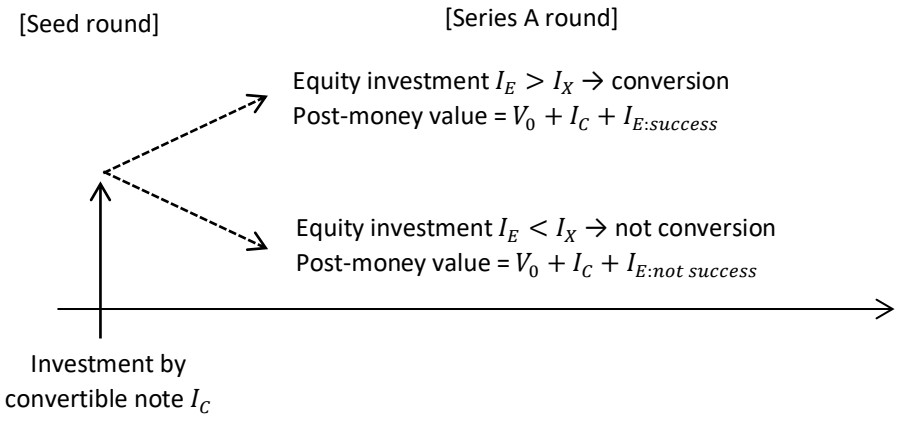


Fig.3 Real Options structure of conversion

3. New equity investors' concern: Adverse selection

3.1. New equity investors' concern

The methods of both discount and valuation cap in the previous subsection are to be a reward on the convertible note holder who has born credit risk and provided funds to the start-up business in the earlier stage. Due to them, nevertheless, the situation becomes unfavourable for new equity investors because they must reconcile to the lower share, comparing to the situation where convertible note holders do not exist. Thus, the negotiation between new equity investors and entrepreneur might not proceed smoothly in the second financing round. In fact, Feld and Mendelson (2016) point out this concern as follows: "Unlike equity, which is issued and can't be changed, the new equity investors could refuse to fund unless the debt investors remove or change the cap. Keep in mind that VCs will normally focus and peg their valuation of your company on that cap." The debt investors in this context means convertible note holders. They mention only about the valuation cap, however, the negative influence for new equity investors is not differentiated from the case of discount.

The most important point to be noticed in their statement is that "the new equity investors could refuse to fund." This statement could be backed by the following simple numerical example. Assume that $V_0 = 2.0$, $I_C = 0.5$, $I_E = 2.0$ (all are \$M). If $\alpha = 0.2$, the share of convertible note holder and the one of Second-round equity investor are 13.51% and 43.24% respectively, according to Table 3. If $\alpha = 0.6$ due to the effect of valuation cap etc., the shares are 23.81% and 38.10%. Thus, the share of Second-round equity investor is still greater. On the contrary, in the case where $V_0 = 2.0$, $I_C = 1.0$, $I_E = 2.0$ (all are \$M), if $\alpha = 0.2$, the share of convertible note holder and the one of Second-round equity investor are 23.81% and 38.10% respectively. If $\alpha = 0.6$, the shares changes into 38.46% and 30.77%, in other words, the share of Second-round equity investor can be lower than that of convertible note holder.

Even if this might be the extreme case, it can be said that the new equity investor shall always be cautious about the pre-determined term conditions of conversion when the convertible note holders exist. In fact, the new equity investor is not always willing to provide the full amount of fund requested by entrepreneur, and the amount that would be actually invested is normally

determined though the negotiation. However, bargaining parties generally would not like to reveal all the information they have during the negotiation. Material information should, of course, be opened for the better dealing. However, some of the private information would still remain unveiled as a consequence of trying to obtain as favourable term conditions as possible. It is also true for the financing contract negotiation between entrepreneur and new equity investor in the Series A round. As Hsu (2010) points out, it may be because the main goal of an entrepreneur is to maximize a probability of raising funds in the next financing round, while the aim of equity investor is to maximise the value of firm or project. Thus, when the new equity investor knows that the situation is as such, but if they still have an interest in the venture project and an expectation of the success, it is rational to assume that they would propose the reduced amount. Introducing a reducing investment coefficient β ($0.00 < \beta \leq 1.00$), this can represent the degree of concern for new equity investor, as also shown in Fig.4 and Table 4.

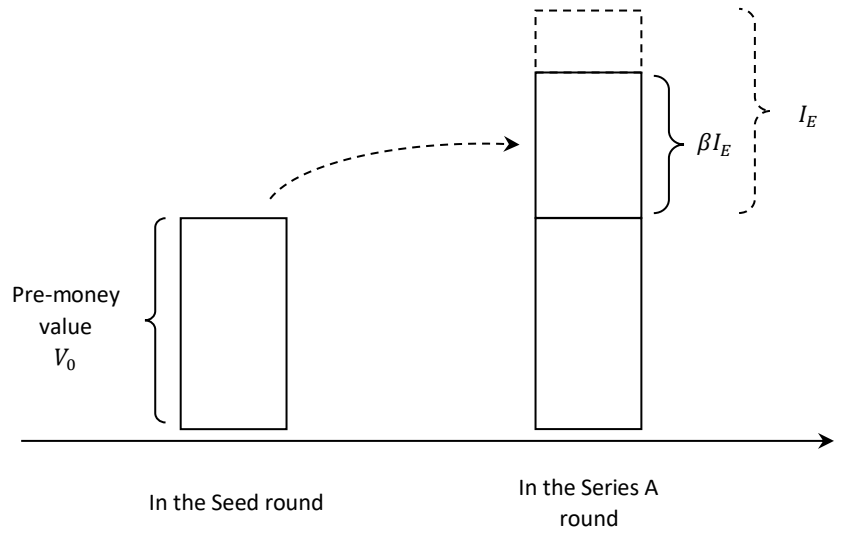


Fig.4 The valuation in the Series A round
Under the existence of convertible note holder

Table 4: Share at the second round (with new equity investor's concern)

	< Number of shares >	< Percentage of shares >
Entrepreneur	$\frac{V_0}{s}$	$\frac{V_0}{V_0 + I_c/(1-\alpha) + \beta I_E} \times 100$
Convertible notes holder	$\frac{I_C}{s(1-\alpha)}$	$\frac{I_C/(1-\alpha)}{V_0 + I_c/(1-\alpha) + \beta I_E} \times 100$
Second-round equity investor	$\frac{\beta I_E}{s}$	$\frac{\beta I_E}{V_0 + I_c/(1-\alpha) + \beta I_E} \times 100$
< Total >	$\frac{V_0 + I_c/(1-\alpha) + \beta I_E}{s}$	100

If they propose the reduced amount to the entrepreneur, the share of the new equity investor becomes lower. Using the numerical example above again for better understanding, in the case where $V_0 = 2.0$, $I_C = 1.0$, $I_E = 2.0$ (all are \$M), if $\alpha = 0.2$ and $\beta = 0.5$, the share of convertible note holder and the one of second-round equity investor are 29.41% and 23.53% respectively, while if $\alpha = 0.6$, the shares become 45.45% and 18.18%. In both situations, the share of new equity investor is lower than that of convertible note holder. However, if the new equity investor evaluates that the project is economically attractive as one component of their portfolio, they could still provide some amount of funds and obtain some equity share, though they must give up the controlling power as a majority (This can be represented as over 33.33%, for example).

3.2. Adverse selection problem in equity financing contract

One question arises: what kind of problem should be incorporated? Many academic text books of economics deal with asymmetric information problems. The common subtitles of asymmetric information are moral hazard, adverse selection and signalling. According to the text book of microeconomics by Mankiw (2007), for example, these are defined as follows: Moral hazard is "the tendency of a person who is imperfectly monitored to engage in dishonest or otherwise undesirable behaviour." Adverse selection is "the tendency for the mix of unobserved attributes to become undesirable from the standpoint of uninformed party." Signalling is "an action taken by an informed party to reveal private information to an

uninformed party." Macro-Stadler and Pérez-Castrillo (2001) set forth these in a more precise way: "a moral hazard problem exists when the agent's action is not verifiable, or when the agent receives private information after the relationship has been initiated." "an adverse selection problem appears when the agent holds private information before the relationship is begun." "(signalling) this situation is similar to adverse selection. However, after learning his type, and before signing the contract, the agent can send a signal that is observed by the principal." Salanié (2005) explains these terms for the purpose of modelling: Moral hazard refers to "the uninformed party moves first and is imperfectly informed of the actions of the informed party." Adverse selection refers to "the uninformed party is imperfectly informed of the characteristics of the informed party; the uniformed party moves first." Signalling refers to "the informational situation is the same (with adverse selection) but the informed party moves first."

Macro-Stadler and Pérez-Castrillo are explaining these terms from the principle-agent model perspective. They are at the same time explaining the following three basic features for modelling principle-agent relationship: (1) The principal designs the contract, or set of contracts, that she will offer to the agent. (2) The agent accepts the contract if she so desires, that is if the contract guarantee him greater expected utility than other opportunities available to him. (3) The agent carries out an action or effort on behalf of the principal. Looking at the contract negotiation in the Series A round from the perspective of principle-agent model, it is new equity investor who offers equity (as I_E or βI_E), and it is entrepreneur who accepts the offer and carries out the effort to grow the venture business by using this equity. Therefore, new equity investor corresponds to principal and entrepreneur corresponds to agent. It is new equity investor who has concern about the possibility of dilution problem due to the existence of convertible note holder, and it is entrepreneur who has all the information about the term conditions with convertible note holder and the more private (insider) information about their business. In other words, it is new equity investor who is 'uninformed party' and it is entrepreneur who is the 'informed party'. In addition, int the Series A round, new equity investor as 'uninformed party' moves first, though entrepreneur provides the draft offer before going into negotiation. Taking all these into consideration, this situation can be said to have adverse selection problem.

4. Modelling the cost for equity investment decision making

4.1. Modelling of adverse selection effect: Two-type model

The modelling of adverse selection is often discussed on the basis of the principal-agent model which assumes two types of agents. The model constructed in this subsection is inspired by the one of Macho-Stadler and Pérez-Castrillo (2001) and Salanié (2005).

In the adverse selection model, entrepreneur (agent) is imperfectly observed by new equity investor (principal). In this paper, the two types of entrepreneurs (agents) are defined as 'good' type and 'bad' type. This concept is in line with the modelling by Koufopoulos (2009) of securitizing under the existence of information asymmetry. The former 'good' type can be interpreted as the entrepreneur who is relatively willing to revel the information about the contract with convertible note holders to new equity investor in the financing negotiation in the Series A round. This could be because such an entrepreneur would often be confident to the success of the venture business, thus, convertible note holder would not have asked for the deep discount and/or small amount of valuation cap, and the entrepreneur has fewer reasons to be reluctant to share the information he/she has in the negotiation. Therefore, such 'good' type of entrepreneur could obtain the higher equity. On the contrary, the latter can be interpreted as an entrepreneur who has the opposite characteristics. Such 'bad' type of entrepreneur would be relatively reluctant to reveal the information, not only about the contract with convertible note holder but also about the likelihood of the venture business success. The convertible note holder may have asked for the deep discount and/or small amount of valuation cap. The new equity investor may also have some kind of anxiety and the amount of investment would be lower.

For starting to construct the adverse selection model, it is normal to set the utilities for both new equity investor (principal) and entrepreneur (agent). In the context of second-round financing, entrepreneur obtains an economic benefit with the form of fund at the expense of giving up some proportion of control benefit which is represented as equity share. Borrowing the idea proposed by Salanié (2005), the utility for entrepreneur can be written as follows:

$$U_{entrepreneur} = \theta q - t$$

Where

q: equity investment amount that entrepreneur obtains (or, investor offers: $q_g > q_b$)

θ: index of entrepreneur type

t: control benefit associated with the share of equity

The first term of θq represents the economic benefit. θ represents the index of entrepreneur type (or the agent's private characteristics), and θ_g indicates the 'good' type, θ_b is the 'bad' type ($\theta_g > \theta_b$). In the context of financing, the more shares new equity investor grasps, the more deeply they can become engaged in the venture business. Many academic researches show that the engagement of venture capitalist can have positive influences on the venture business and become helpful for entrepreneur to progress its business. (e.g., Bertoni et al. (2011), Croce et al. (2013)) As defined above, the 'good' type of entrepreneur is relatively willing to reveal the information, and in exchange for it, they will be able to succeed in the financing negotiation contract. As a consequence, they can obtain more amount of investment and easily gain the more advice for the business success in proportion to the equity share of new investor, such as an experienced venture capitalist. On the other hand, the 'bad' type of entrepreneur does not have a willingness to reveal the information actively, and it might be difficult to gain the investment and support. Therefore, the degree of θ can be interpreted as the degree of help obtained from venture capitalist as represented the proportion of its equity share, in exchange for the tender of the private information that entrepreneur has.

For financing, new equity investor provides the fund, and in exchange for it, he/she can obtain the control benefit as the equity share. Borrowing the idea proposed by Salanié (2005) as well, the utility of new equity investor can be written as follows:

$$U_{new\ equity\ investor} = t - C(q)$$

Where

$C(q)$: cost for decision-making of new equity investor

This cost means the necessity for new equity investor to implement due diligence when they scrutinise whether they invest or not. As mentioned before, new equity investor does not know well about the characteristics of entrepreneur nor about to what degree entrepreneur is

willing to reveal the private information. This due diligence is not an easy task. Therefore, it is rational to think of this kind of cost. Needless to say, the cost required for new equity investor is not limited to the due diligence of entrepreneur's characteristics. The due diligence of the product and/or service is also necessary, for example. However, for simplicity, this paper focuses only on the cost due to the information asymmetry caused by the characteristics of entrepreneur or willingness of unveiling the private information. In this sense, this cost could also be called as "additional" cost.

According to the microeconomic theory, if a seller of some kind of goods (e.g. smart phone) as principal can observe the type θ_i (e.g. i = 'big fun of the seller' as 'good' type or 'normal type' as 'bad' type) of the buyer as agent, the principal can charge higher price to 'good' type of buyer than to 'normal' type because the former type want to obtain the goods literally at any cost, even the price is higher. This is called as first-best or perfect discrimination, and the principal's surplus is maximised. For the equity financing round, the same structure can be thought. In the case that new equity investor can observe whether entrepreneur is 'good' type or 'bad' type, the new equity investor as principal will solve the following problem:

$$\max_{q_i, t_i} \big(t_i - C(q_i)\big),$$

Subject to

$$\theta_i q_i - t_i \geq 0$$

For the principal, zero surplus left for the agent can lead the optimum of the utility, thus, it can be thought as:

$$t_i^* = \theta_i q_i^* \quad (i = \text{'good' or 'bad'})$$

In the adverse selection model, new equity investor (principal) is assumed to not directly be able to observe the type of entrepreneur (agent). Thus, the perfect discrimination is infeasible, and it is necessary to consider the second-best. In this situation, the principal will design the menu of contract: It is (q_g, t_g) (q_b, t_b). According to the revelation principle, the 'good' type will choose the former, and the 'bad' type will do the latter. Assuming that the principal only knows that the probability of encountering the 'bad' type entrepreneur, which is represented

as π. The focus is on the best pair of contracts (the second-best optimum), and this is obtained by solving the following:

$$\max_{q_b,t_b,q_g,t_g} \left[\pi \times \left(t_b - C(q_b)\right) + (1-\pi) \times \left(t_g - C\left(q_g\right)\right)\right]$$

Subject to

$$\theta_b q_b - t_b \geq \theta_b q_g - t_g \text{ (IC}_1\text{)}$$

$$\theta_g q_g - t_g \geq \theta_g q_b - t_b \text{ (IC}_2\text{)}$$

$$\theta_b q_b - t_b \geq 0 \text{ (IR}_1\text{)}$$

$$\theta_g q_g - t_g \geq 0 \text{ (IR}_2\text{)}$$

The first two constraints are called as the incentive compatibility constraints (IC_1, IC_2). They state that each agent prefers the contract that was designed for him. The last two constraints are called as the individual rationality or participation constraints (IR_1, IR_2). They guarantee that each type of agent accepts his designed contract.

If IR_1 is inactive, so would be IR_2, and if it can be assumed to increase t_b and t_g by the same amount. This would increase the principal's utility without any effect on incentive compatibility. Thus, IR_1 should be active and $\theta_b q_b = t_b$. In a similar way, if IC_2 is inactive, then $\theta_g q_g - t_g > \theta_g q_b - t_b \geq \theta_b q_b - t_b = 0$. Thus, it is possible to increase t_g without breaking the incentive compatibility constraints or the individual rationality, and can lead to increase the principal's utility. It is not optimal. Therefore, IC_2 should be active, and $\theta_g q_g - t_g = \theta_g q_b - t_b \Leftrightarrow t_g = t_b + \theta_g\left(q_g - q_b\right)$. Considering the case of first best contract for 'good' type, $q_g = q_g^*$, $t_g = t_b + \theta_g\left(q_g - q_b\right)$ can be $t_g = t_b + \theta_g\left(q_g^* - q_b\right)$

Then, $\max_{q_b,t_b,q_g,t_g} \left[\pi \times \left(t_b - C(q_b)\right) + (1-\pi) \times \left(t_g - C\left(q_g\right)\right)\right]$ can be rewritten as follows:

$$\Leftrightarrow \max_{q_b,t_b,q_g} \left[\pi \times \left(\theta_b q_b - C(q_b)\right) + (1-\pi) \times \left(t_b + \theta_g\left(q_g^* - q_b\right) - C\left(q_g^*\right)\right)\right]$$

$$\Leftrightarrow \max_{q_b,q_g} \left[\pi \times \left(\theta_b q_b - C(q_b)\right) - (1-\pi) \times \left(\left(\theta_g - \theta_b\right)q_b + \theta_g q_g^* - C\left(q_g^*\right)\right)\right]$$

In the optimal situation, $\theta_g q_g^* - C\left(q_g^*\right) = 0$, thus,

$$\Leftrightarrow \max_{q_b} \left[\left(\theta_b q_b - C(q_b)\right) - \frac{1-\pi}{\pi} \times \left(\left(\theta_g - \theta_b\right)q_b\right)\right]$$

$$\Leftrightarrow C'(q_b) = \theta_b - \frac{1-\pi}{\pi} \times \left(\theta_g - \theta_b\right) = \frac{1}{\pi}\theta_b - \frac{1-\pi}{\pi}\theta_g$$

In the perspective of mathematics, a constant value should be necessary when reverting the first derivative to its original function. In the course of discussion above, it can be interpreted as the minimum cost for equity investment decision-making. However, this value can be zero because the due diligence is not necessary when the investment amount q_b is zero, in other words, the equity financing is not provided. Therefore, for new equity investor, the form of the cost of decision-making under the informational asymmetry situation can be expressed as follows:

$$C(q_b) = \left(\frac{1}{\pi}\theta_b - \frac{1-\pi}{\pi}\theta_g\right) \times q_b$$

In this context, $q_b = \beta I_E$. Furthermore, this cost should be positive, and the following non-negative condition is added:

$$\frac{1}{\pi}\theta_b - \frac{1-\pi}{\pi}\theta_g > 0 \Leftrightarrow 1 - \pi < \frac{\theta_b}{\theta_g} < 1$$

4.2. Incorporation of real option structure into modelling the index of entrepreneur type

Assuming that the 'good' type entrepreneur will settle the second-round financing negotiation successfully and obtain the full investment amount that they have desired beforehand, the 'bad' type will not be able to do as they have expected. The consequence influences on whether convertible note is converted into equity. As also mentioned in the section of 2.5, this structure can be the real option. If entrepreneur fails to obtain the more equity amount than the pre-determined 'threshold' I_X, convertible note holder will exercise the option to defer to convert. At the same time, θ represents the characteristics of entrepreneur as defined above. Considering that θ is a component of the economic benefit term of θq, it is the indicator or index of the beneficial effect of advice for the business success, which can be obtained in proportion to the equity share of new equity investor. The 'bad' type entrepreneur is difficult to gain enough supports from venture capitalist, comparing to the 'good' type. Being based on

the real option structure, θ can be corresponded to the percentage of share of new equity investor as follows:

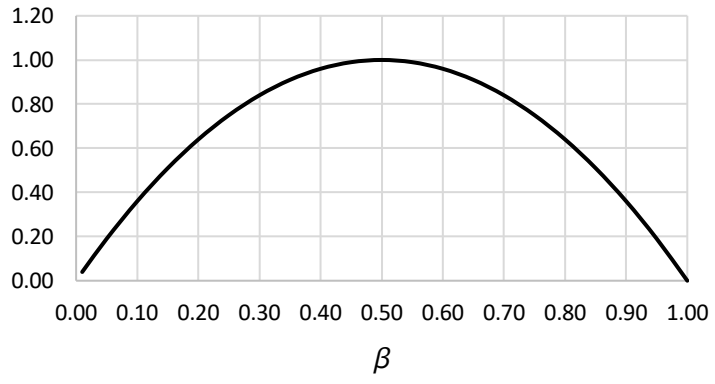
$\theta_g \rightarrow \frac{I_E}{V_0+I_c/(1-\alpha)+I_E}$,

$\theta_b \rightarrow \frac{\beta I_E}{V_0+I_c/(1-\alpha)+\beta I_E}$ (*conversion*), or $\theta_b \rightarrow \frac{\beta I_E}{V_0+\beta I_E}$ (*non-conversion*)

Whether the conversion is implemented or not is a quite critical concern for new equity investor because their share will decrease from $\frac{\beta I_E}{V_0+\beta I_E}$ to $\frac{\beta I_E}{V_0+I_c/(1-\alpha)+\beta I_E}$, especially when considering the case of the 'bad' type entrepreneur. This influence of conversion can be expressed by using the rate of change of $\left(\frac{\beta I_E}{V_0+\beta I_E} - \frac{\beta I_E}{V_0+I_c/(1-\alpha)+\beta I_E}\right) / \frac{\beta I_E}{V_0+\beta I_E}$.

In the perceptive of convertible note holder, the likelihood of conversion is not constant, rather it depends on the equity investment amount which is represented with βI_E. In the situation of lower β where new equity investor is reluctant to invest, convertible note holder would wait to convert its debt almost certainly. In the situation of higher β where new equity investor is willing to invest, convertible note holder would certainly convert. Whilst these situations are not so problematic for new equity investor, and the degree concern is low, the situation of neither low nor high β makes their degree of concern be increased. Thus, the influence of conversion should be modified by this factor. Let call this factor be an impact modification factor, and it can be assumed as $-4(\beta - 0.5)^2 + 1$, shown in Fig.5.

Fig.5 Impact modification factor



Recalling the situation how entrepreneur obtains the advice and support from equity investor, such as venture capitalist, the 'good' type will be able to enjoy fully the benefits of advice and support from venture capitalist, while the 'bad' type will not be able to do so and its index can be set as the reduced value. thus, the index of this 'good' type can be set as a benchmark of $\theta_g = 1$.

In addition to those above, the non-negative condition of $1 - \pi < \frac{\theta_b}{\theta_g} < 1$ should be taken into account. For mapping a variable x in the range of $0 < x < 1$ into the range of $1 - \pi < x' < 1$, the mathematical manipulation of $x' = \pi \times x + (1 - \pi)$ is added.

Combining all the discussions above, θ_b can be obtained as follows:

$$\theta_b = \pi \times \left[\left(\frac{\beta I_E}{V_0+\beta I_E} - \frac{\beta I_E}{V_0+I_c/(1-\alpha)+\beta I_E}\right) / \frac{\beta I_E}{V_0+\beta I_E} \times (-4(\beta - 0.5)^2 + 1)\right] + (1 - \pi)$$

Plugging θ_b and θ_g into the cost function of decision-making, how the cost will be affected by the discount rate α and the reducing investment coefficient β can be evaluated. In this process, the effect of π can be eliminated.

In the next section, the case simulation is implemented for the purpose of better understanding of this cost which is theoretically derived from the discussions above.

5. Case simulations

In this section, a case simulation is implemented by using the data example based on actual medical start-up. This company was able to succeed in raising funds as convertible note of $3.85 million, and later, called for the equity of $11.80 million as shown in Table 5.

Table 5: The realised investment amounts

Date	Type	Offered (million$)	Sold (million$)
31/Jul/20X7	Convertible Securities	3.85	3.85
13/Nov/20X7	Series A	11.80	4.00

The values for this simulation are as follows:

$V_0 = 4.00$ (million$)

$I_C = 3.85$ (million$)

$I_E = 11.80$ (million$)

Although the actual pre-money value is not possible to be known, the estimated value of $4.00 million is used.

5.1. Simulation result of the effect of discount rate for convertible note holder

As mentioned in the section 3.1, new equity investor is quite concerned about the existence of convertible note holder because whether the debt is converted into equity is uncertain. In addition, how much of the discount rate or valuation cap have been determined is also quite difficult to know for new equity investor because this is one of the materials for financing negotiation. Thus, the effect of discount rate (or valuation cap) is a critical element to understand. The simulation results are shown in Fig.6 and 7.

Fig.6 shows the simulation result in the case of the discount rate $\alpha = 0.20$. This rate may be the normal setting in the negotiation between entrepreneur and convertible note investor. If this discount rate is applied, the share for new equity investor will be diluted by the range from 23%

to 54% when convertible note holder exercises their conversion option. This effect is large in the range of lower coefficient β, however, the conversion will normally be deferred. Thus, the effect of dilution shall be thought around 25% to 30%. As shown Fig.7, it is not uncommon that convertible note investor requires the deep discount $\alpha = 0.70$, for example, as the small amount of valuation cap against pre-money valuation. In this case, the share for new equity investor will be diluted by around 45% to 55% in similar manner. We can verify even in the practical financing situation that the dilution effect by the discount rate and valuation cap is significant, and it is no doubt that new equity investor will have concerns about the contract details with convertible note holder when negotiating the amount of equity investment with entrepreneur in the second financing round, such as the Series A.

Fig.6 Equity Share (α=*0.20*)

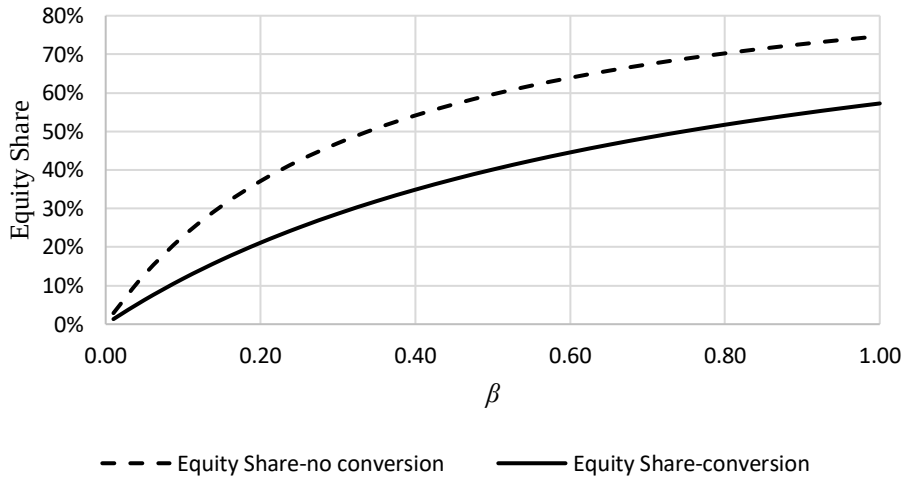
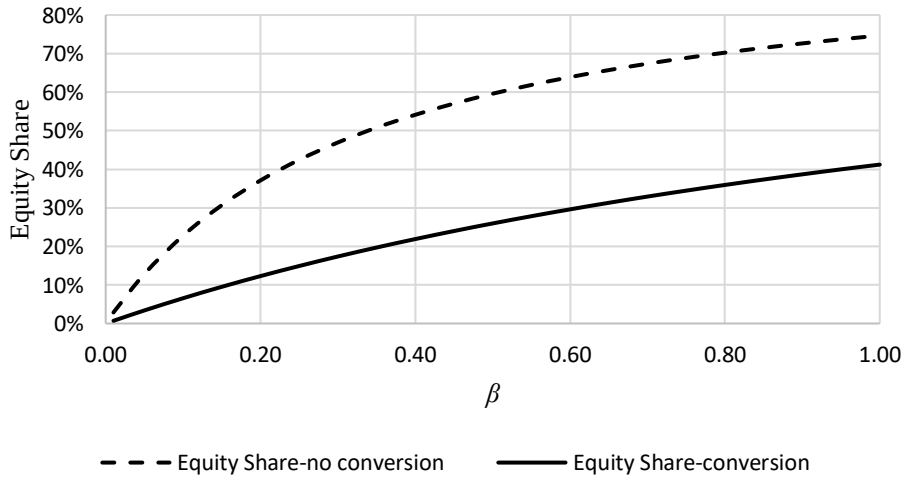


Fig.7 Equity Share *(α=0.70)*



5.2. Simulation result of the cost of equity investment decision-making

As being verified above, the impact on the dilution of equity share by the discount rate and valuation cap is significant. If entrepreneur reveals this kind of contract details with convertible note holder, new equity investor's concern will be largely resolved. Nevertheless, if entrepreneur's main goal was just to maximise a probability of raising funds in the next financing round, the information would be revealed only within the necessity of negotiation, and it may be still kept unveiled. Thus, the cost of equity investment decision-making becomes unignorable for new equity investor under such a situation of asymmetric information. In particular, the relationship between the effect of discount rate and valuation cap and the cost for equity investment decision-making becomes essential.

The simulation result of the cost for equity investment decision-making is shown in Fig.8. When new equity investor provides the full amount that entrepreneur desires or calls for beforehand ($\beta = 1.00$), the cost will be zero because it is assumed that entrepreneur is willing to reveal all the information, thus, no concern is generated for equity investment decision-making. In a similar manner, the cost will also be zero when investor do not provide any funds ($\beta = 0.00$). On the other hand, when new equity investor is wondering what amount of equity should be provided, the cost for decision-making becomes increased. These are the extreme situation that is almost free from the information asymmetry problem.

Fig.8 Cost of equity investment decision-making

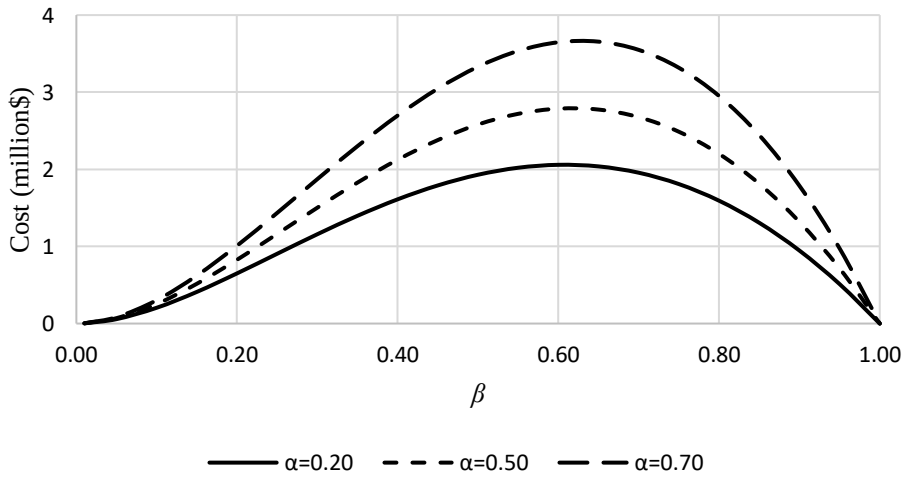


Fig.8 shows that the cost becomes expensive when new equity investor is thinking of reducing the investment amount, in which the information asymmetry problems stand out. According to the simulation result, the cost becomes maximised at $\beta = 0.61$ when $\alpha = 0.20$, at $\beta = 0.62$ when $\alpha = 0.50$, and at $\beta = 0.63$ when $\alpha = 0.70$, respectively. The result shows that under the existence of convertible note holder and informational asymmetry, the new equity investor's cost for investment decision-making becomes maximised if they think of reducing the amount by around 40% comparing to the one that has been asked for by entrepreneur. This is compatible to the practical situation. As mentioned before, the convertible note holder's conversion option is not mandatory and new equity investor is normally unable to the details. On top of that, convertible note holder will exercise the option only when entrepreneur succeed in fund raising of more than the threshold amount I_X. Therefore, it is rational that when the likelihood of conversion becomes largest, the new equity investor's concern becomes maximised and the most careful due diligence is necessary. This leads to the situation where the cost for decision-making becomes maximised. The simulation result of this start-up's case is the point of around 40% reduction. Taking one step further, it might be able to guess that I_X was βI_E = 0.62×\$11.80 million = \$7.32 million, for example.

This result gives entrepreneur an insight for financing negotiation. The result that the cost for new equity investor becomes maximised at around $\beta = 0.6$ means that setting the negotiation goal at the 60% investment amount is quite difficult to realise. Even if this goal is being targeted, new equity investor would not like to agree at this amount, rather, they would try to settle either at lower or higher amount in order to avoid the higher cost for investment decision-making. New equity investor would normally agree at lower amount. In effect, in this start-up case, the deal was settled at $\beta = 0.34$. Although there are other reasons why the deal was settled at this figure, such as consideration of the prospective of the business, the higher cost for investment decision-making could become a rational explanation.

The great impact of α on the cost is also to be noticed. The maximum value is \$2.06 million when $\alpha = 0.20$, \$2.79 million when $\alpha = 0.50$, and \$3.67 million when $\alpha = 0.70$, respectively. The deeper the discount is, the larger the cost becomes. Comparing the maximum cost when $\alpha = 0.70$ and the one when $\alpha = 0.20$, the former is 1.78 times. This is also compatible to the practical situation. As the simulation result of the equity share shows, the impact of discount and valuation cap on the equity share is significant. In addition to the concern about whether

convertible note holder exercises the conversion option, the concern about how much the discount and valuation cap has been agreed between convertible note holder and entrepreneur would be grown. Therefore, new equity investor must bear the cost for investment decision-making much more, especially when the deeper discount and smaller valuation cap is suspected. In this sense, the simulation result showing that cost will increase by 1.78 times would be serious for this start-up's case.

As for the financing negotiation, entrepreneur can obtain another insight from this result. If entrepreneur had accepted the contract that allows the deep discount and/or small valuation cap with convertible note holder in the early stage, it might make the equity financing negotiation quite difficult to be settled as desired in the later financing round. Convertible note holder expects to gain a lot in exchange for bearing risks with the investment as convertible note in the early stage, and thus, they require the deep discount and/or small valuation cap. Entrepreneur who has neither bargaining power nor confidence in the business may agree on such a contract because it does not affect the debt amount itself. However, it will impose new equity investor the higher cost for investment decision-making. Therefore, entrepreneur had better recognise that it might be a pitfall in the second financing round.

6. Conclusion

Convertible note is often used for the early stage start-up financing. However, its usage creates the complicated situation among entrepreneur, convertible note holder and new equity investor in the second-round financing negotiation because the conversion of convertible note into equity causes the dilution problem of the equity share of new equity investor and entrepreneur. One of the objectives of this paper is to build a model that is dealing with the interactions of these three key parties. Another important objective of this paper is to figure out the cost for equity investment decision-making by incorporating the real option structure of the conversion of convertible note into equity, as well as the adverse selection problem in the financing negotiation.

According to the results of case simulation with actual start-up data, it can be verified that the discount and valuation cap have great impacts on the cost for equity investment decision-makings. The results show that, under the existence of convertible note holder, if entrepreneur aims to seal the financing negotiation at around 40% reduction of investment, the cost for decision-making becomes highest, and thus, new equity investor may reduce more (or increase in some cases). The results also show that the deeper the discount and the smaller the valuation cap, the larger the cost becomes, and thus, the likelihood of success of the financing negotiation becomes difficult. Therefore, entrepreneur should take this into consideration when entering into the financing negotiation in the second financing round.

This model has even more rooms to be developed. For example, the model assumes that the impact modification factor is symmetric. If there are some researches or data that show the practical distribution pattern, the model would be more precise. This model assumes three parties. In practice, even within the convertible note holder and equity investor, there are sometimes different types. Thus, this matter could also be incorporated in the future research.

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