Agency problems in PPP investment projects

Florina Silaghi[a,*], Sudipto Sarkar[b]

[a] *Universitat Autonoma de Barcelona, Campus de Bellatera, 08193, Barcelona, Spain*
[b] *McMaster University, 1280 Main Street West, ON L8S 4M4, Hamilton, Canada*

Abstract

This paper examines investment timing by a private firm in a public-private partnership with a government in the presence of agency conflicts arising from asymmetric information. The design of optimal contracts to provide incentives to the private firm to exert effort is analyzed. We show within a real options framework that although first-best investment timing can be implemented, this comes at the cost of a social welfare loss. The analysis is extended to incorporate an exit/bailout option for the private firm.

Keywords: Investment timing, Contracting, Agency, Real options, Public and private partnerships (PPPs)
JEL: G11, G13, G30, G31, G38, D82

[*] Corresponding author. Tel.: +34 93 581 4302.
Email addresses: florina.silaghi@uab.cat (Florina Silaghi), sarkars@mcmaster.ca (Sudipto Sarkar)

1. Introduction

There is an ongoing discussion regarding the performance of public-private-partnerships (PPPs) vis-à-vis government delivery of infrastructure services such as transportation, water and education. It used to be generally accepted that PPP is the best organizational form for infrastructure projects (Bradford, 2003; Vining and Boardman, 2008). However, recently there has been considerable dissatisfaction with PPP projects (Boothe et al., 2015, Engel et al., 2013; Spackman, 2002; Vining and Boardman, 2008). Governments seem to find it difficult to reduce their risk exposure and often end up making windfall payments to the private party (Alonso-Conde et al., 2007; Brown, 2005; Power et al., 2009), while private parties appear to suffer from insufficient or negative cash flows in some cases (Vining and Boardman, 2008). Alonso-Conde et al. (2007) and Brown (2005) claim that proper real-option valuation would reveal that government guarantees are often overly generous to the private party, mainly because of flawed contract design. The major problem seems to be the large exogenous demand risk, which is often not properly priced and allocated in the contract design (Boothe et al., 2015, Engel et al., 2013). Proper valuation of the embedded options, along with appropriate contract terms to ensure correct risk allocation and compensation, would significantly reduce these problems. In this paper, we analyze the optimal design of a PPP project within a real-options framework, taking into account incentive mechanisms to ensure optimal investment timing.

The existing PPP literature has two strands focusing on "real options" and "contract design". The former values real options embedded in the contract (revenue guarantee, revenue sharing, buyout clause, etc.), and determines the optimal investment policy in a real-option setting.[1] However, this literature does not address contract design and performance incentives.[2]

[1] Chiara et al. (2007) and Huang and Chou (2006) examine revenue guarantees, Brandao and Saraiva (2008) minimum traffic guarantees, Doan (2010) revenue and income guarantees, and Rose (1998) and Alonso-Conde et al. (2007) fee deferral and buyout options. Bowe and Lee (2004) value the deferral, abandonment and expansion options, citePoweretal2009 value revenue guarantee, revenue sharing and buyout option, Takashima et al. (2010) value investment subsidy, revenue guarantee and termination option, and Shan et al. (2010) and Adkins and Paxson (2017) show how to use option combinations (collars) to manage revenue risk.

[2] Sitruk (2009) models a high-speed rail PPP project identifying the optimal investment trigger. Although he does not consider a proper mechanism design with the corresponding

The "contract design" literature examines the optimality of the PPP organizational form versus the traditional contract (building, operating and maintaining are bundled in the former and unbundled in the latter).[3] However, this literature focuses on asymmetric information, principal-agent problems, and the theory of incomplete contracts (Dewatripont and Legros, 2005), and ignores the effect of the embedded real options which have a significant impact on project valuation and risk allocation (Bowe and Lee, 2004; Huang and Chou, 2006; Power et al., 2009).

This paper aims to combine the two strands of the literature mentioned above, to identify the optimal contract specifications in a model that explicitly takes into account the commonly-observed real options in a PPP project. This is of crucial importance since incorrect contract terms can be very expensive to the government and taxpayers, e.g., the governments of Mexico and Spain recently had to pay \$8.9 billion and \$2.5 billion respectively to their private partners because of inappropriate contract terms (Brandao and Saraiva, 2008).

We build on Grenadier and Wang (2005)'s model of investment timing by managers in a decentralized firm in the presence of asymmetric information. While they consider both a hidden information and hidden action problem, we focus on a hidden action problem for simplicity. In their model the agency conflict is between the shareholder and the manager of the firm. In our model, the agency problem is between the government and a private firm. In particular, the private firm can exert an unobservable effort that increases the probability of obtaining a high-quality investment project. In

optimization problem for the government, his study is the only real-option study to address the issue of giving incentives to the operator to achieve an exogenous quality level.

[3]Hart (2003) argues that the choice between PPP and traditional contract turns on whether it is easier to write contracts on service provision or on construction quality. For the former, PPP is optimal; for the latter, the traditional (unbundled) contract is better. Martimort and Pouyet (2008) show that bundling (unbundling) is optimal with positive (negative) externalities, and Iossa and Martimort (2012) show that bundling of tasks is generally the optimal organizational form. They also show that, if the productivity shock is common knowledge and verifiable, the optimal contract would (i) ensure that the private party is not exposed to negative cash flows, and (ii) require a revenue-sharing arrangement independent of the productivity shock. Iossa and Martimort (2009) show that bundling (or PPP) is optimal, while Iossa and Martimort (2008) show that PPP is suitable for traditional infrastructure projects such as transportation and water, but less so for others (e.g., schools).

order to induce the private firm to exert effort and invest at the first-best trigger, the government designs a contract to provide the firm with the appropriate incentives. A contract consists of a pair of investment triggers and a remuneration for the private firm. While in Grenadier and Wang (2005) the manager receives a one-time wage at the time of the investment, we model the remuneration of the private firm as a fraction of net revenues that the firm receives starting from the time of the investment. This is in line with the wide use of revenue-sharing contracts between the two parties in PPP projects (Iossa and Martimort, 2012).

We show that the government can induce the firm to exercise at the first-best investment trigger. Nevertheless, this comes at the cost of paying the firm some informational rents. This will in turn distort the government's effort choice, which leads to a social loss. Furthermore, we extend the basic model to incorporate an exit option into the investment project (as in Takashima et al., 2010). The firm then has the possibility of asking for a bailout from the government when the situation deteriorates. We provide a numerical analysis that illustrates the characteristics of the optimal contract, the social loss and the impact of the bailout option.

To our knowledge, this is the first model to incorporate a moral hazard problem into a real options PPP framework. In this way, our paper bridges the rich contract theory literature on PPPs with the real options literature on PPPs.

Most papers in the real options literature assume perfect, symmetric information. A notable exception is the paper by Soumare and Lai (2016). They compare two forms of government support, loan guarantee and direct investment through PPPs in a framework of asymmetric information: the government knows less about project quality than do private partners (the so-called plum problem). While their model is a model of hidden information, our model is one of hidden action.

Takashima et al. (2010) is one of the few key analytical studies in the real options literature. While they consider perfect, symmetric information in their model, we introduce asymmetric information. Nevertheless, they also derive a First-best and a second-best investment policy. In their First-best benchmark the investment time is chosen to maximize the value of the total project (the sum of firm and government values). This is similar to our approach assuming that the project is run solely by the government. However, their second-best investment policy is not a proper "second-best" scenario in the sense used in agency theory. Namely, by second-best case they mean that the investment time is chosen to maximize the value of the private firm only. Nevertheless, since there is no asymmetric information in their framework, the government could design the concession contract such that the private firm implements the First-best investment policy at no cost. On the contrary, this would not be possible in our framework due to the presence of asymmetric information.

The rest of the paper is organized as follows. Section 2 describes our benchmark model including its setup, the first-best solution and the principal-agent setting. In Section 3 we simplify the optimization program of the government and we derive the optimal contracts, i.e., the second-best solution. The exit or bailout option for the private firm is introduced in Section 4. Section 5 provides a numerical sensitivity analysis. Finally, Section 6 concludes.

2. Model

We now start by describing the model setup. We will then derive the first-best benchmark solutions assuming full symmetric information. Finally, we will provide the optimal contracts that result from the principal-agent maximization problem under asymmetric information.

2.1. Setup

The principal (government) owns an option to invest in a single project. We assume that the principal delegates the exercise decision to an agent (private firm). Once investment takes place, the project generates cash flows x observable and contractible to both the government and the private firm. The exercise price is given by $I - \theta$. Here I can be interpreted as the "gross" investment cost and θ as the amount by which the gross investment cost can be reduced depending on the private firm's unobservable effort. Let the cash flows follow a geometric Brownian motion:

$$dx(t) = \mu x(t)dt + \sigma x(t)dz(t), \tag{1}$$

where μ and σ represent the drift and standard deviation per unit time respectively, and dz is the increment of a standard Wiener process. Let x_0 equal the value of the cash flow at time zero. In addition, the operating cost stream is c per unit time. For our base case model we will assume $c = 0$ (Sections 2 and 3). In this case, it is optimal to operate the project perpetually. In Section 4 we will extend the base case model in order to introduce an exit option in the case of a positive operating cost $c > 0$. Thus, for consistency we will develop all our expressions taking into account the operating cost $c \geq 0$.

Both the government and the private firm are risk-neutral, with the risk-free rate denoted by $r > \mu$. We assume that the firm is protected by limited liability and other public guarantees. Unlike Adkins and Paxson (2017), we rule out the upfront sale of the project by the government to the private firm. This can be justified if the private firm is liquidity constrained.[4]

The variable θ can take on two possible values: θ_1 or θ_2, with $\theta_1 > \theta_2$, and $\Delta\theta = \theta_1 - \theta_2 > 0$. We can interpret a draw of θ_1 as a lower cost project, or higher quality project, and a draw of θ_2 as a higher cost project, or lower quality project. The effort that the private firm exerts influences the likelihood of obtaining a lower cost project. In particular, if the private firm exerts a one-time effort at time zero, the probability of drawing a higher quality project θ_1 is q_H. If no effort is exerted this probability is only $q_L < q_H$. Effort is costly, when exerting effort the private firm incurs a cost $\xi > 0$.

Since effort is unobservable, the government cannot contract on the effort of the private firm. However, it can contract on the observable cash flow of the project, $x(t)$. The firm's compensation will be contingent on the level of $x(t)$ at exercise. In Grenadier and Wang (2005) the agent receives a one-time payment at the time of exercise.[5] However, in PPP contracts, revenue-sharing agreements between the two parties are widely employed according to Iossa and Martimort (2012) (p. 447). We therefore assume that the net revenue from the PPP project is shared, with the private party getting a fraction b and the public party getting the rest (with $0 \leq b \leq 1$), as in Holmstrom and Milgrom (1991), Iossa and Martimort (2012), and Sitruk (2009).[6] The investment cost is shared according to the exogenous cost sharing ratio, α, with the private party investing αI and the government investing $(1 - \alpha)I$. Thus, the government provides an investment subsidy of $(1 - \alpha)I$.

[4]Nevertheless, the firm has enough liquidity to be able to co-finance the project, i.e., pay a fraction of the investment cost.

[5]This is also the case in Shibata (2009) and Shibata and Nishihara (2010).

[6]While in Iossa and Martimort (2012) the revenue-sharing coefficient is independent of the state variable, in our case, this coefficient is contingent on the cash flow value at exercise.

2.2. First-best benchmark

We first analyze the First-best benchmark, in which there is no agency problem. Suppose that there is no delegation of the exercise decision. Let $W(x, \theta)$ denote the government's option. As standard in the real options literature, W will solve the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 x^2 W_{xx} + \mu x W_x - rW = 0 \tag{2}$$

The boundary conditions are:

$$\begin{aligned} W(x^*(\theta), \theta) &= \frac{x^*}{r-\mu} - \frac{c}{r} - (I - \theta); \\ W_x(x^*(\theta), \theta) &= \frac{1}{r-\mu}; \\ W(0, \theta) &= 0, \end{aligned} \tag{3}$$

where $x^*(\theta)$ denotes the investment trigger. The first boundary condition is the value matching condition, while the second one is the smooth pasting condition. The third boundary condition reflects the fact that zero is an absorbing barrier for $x(t)$.

The government's option value at time zero and the investment trigger are given by:

$$W(x_0, \theta) = \left(\frac{x_0}{x^*(\theta)}\right)^{\beta_1} \left(\frac{x^*(\theta)}{r-\mu} - \frac{c}{r} - (I - \theta)\right), for\ x_0 < x^*(\theta) \tag{4}$$

and

$$x^*(\theta) = \frac{\beta_1}{\beta_1 - 1}\left(\frac{c}{r} + I - \theta\right)(r - \mu) \tag{5}$$

where

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} > 1 \tag{6}$$

The ex-ante value of the government's option in the First-best benchmark conditional on exerting effort, $V^*(x_0)$, is given by: $q_H W(x_0, \theta_1) + (1 -$

$q_H)W(x_0, \theta_2)$. Substituting the expressions for $W(x_0, \theta_1)$ and $W(x_0, \theta_2)$ we obtain:

$$V^*(x_0) = q_H \left(\frac{x_0}{x_1^*}\right)^{\beta_1} \left(\frac{x_1^*}{r-\mu} - \frac{c}{r} - (I - \theta_1)\right) + (1-q_H) \left(\frac{x_0}{x_2^*}\right)^{\beta_1} \left(\frac{x_2^*}{r-\mu} - \frac{c}{r} - (I - \theta_2)\right) \tag{7}$$

Note that it is optimal for the government to exert effort if :

$$q_H W(x_0, \theta_1) + (1 - q_H)W(x_0, \theta_2) - \xi \geq V^N(x_0) \equiv q_L W(x_0, \theta_1) + (1 - q_L)W(x_0, \theta_2) \tag{8}$$

That is, the government exerts effort if $\xi \leq V^*(x_0) - V^N(x_0)$. Substituting the expressions for $W(x_0, \theta_1)$ and $W(x_0, \theta_2)$ and simplifying, we get that effort exertion is optimal if:

$$\frac{\xi}{\Delta q} \leq \left[\left(\frac{x_0}{x_1^*}\right)^{\beta_1} \left(\frac{x_1^*}{r-\mu} - \frac{c}{r} - (I - \theta_1)\right) - \left(\frac{x_0}{x_2^*}\right)^{\beta_1} \left(\frac{x_2^*}{r-\mu} - \frac{c}{r} - (I - \theta_2)\right)\right] \tag{9}$$

Denoting by A the right hand side of the inequality, we have that effort exertion is optimal if $\frac{\xi}{\Delta q} \leq A$.[7] Thus, it is worth exerting effort only when the cost-benefit ratio $\frac{\xi}{\Delta q}$ is sufficiently low. To focus on the interesting case we assume that indeed $\frac{\xi}{\Delta q} \leq A$. We will see in the next section that when we have a moral hazard problem, this decision is distorted. We will find situations where even though it would be optimal for the government to exert effort in the First-best case, the government will optimally choose not to give incentives to the private firm to exert effort because it is too costly to do so.

2.3. A principal-agent setting

In order to give the private firm incentives to exert effort, the government offers a contract at time zero that commits the government to pay the firm a fraction b of the net cash flows of the project continuously starting with the time of exercise (investment).[8] As in Grenadier and Wang (2005), this type of contract is contingent on the value of the cash flows at the time of exercise,

[7]Note that $A = \frac{V^*(x_0) - V_N^*(x_0)}{\Delta q}$.

[8]Following Grenadier and Wang (2005) we assume that renegotiation is not allowed. Indeed, although commitment might be ex-post inefficient, it leads to a higher ex-ante value.

i.e., b is a function of the realized value of $x(t)$ at the time of exercise, $\hat{x}$. However, it differs from the contracts in Grenadier and Wang (2005), Shibata (2009) and Shibata and Nishihara (2010) since it is not a one-time payment at the time of exercise, but a linear revenue-sharing contract. This is in line with the contract theory literature on PPPs. Moreover, the firm pays a fraction α of the investment cost $I - \theta$.

Given a certain contract, the private firm chooses the exercise time in order to maximize the value of its option. The government will thus select the contract parameters such that it induces the private firm to exert effort and choose an exercise policy that maximizes the value of the government's option.

Since θ can only take two values, the private firm will choose at most two exercise triggers. Therefore, the contract offered by the government will be (b_1, x_1), the firm receives a fraction b_1 of the cash flows if it exercises at x_1, and (b_2, x_2), receiving a fraction b_2 when exercising at x_2.

The government's payoff is: $(1-b_1)(x-c)$, if $\theta = \theta_1$ and $(1-b_2)(x-c)$ if $\theta = \theta_2$. Then, conditional on the private firm exerting effort, the government's option value, $\Pi^G(x_0, b_1, b_2, x_1, x_2)$, is given by:

$$\begin{aligned} \Pi^G(x_0, b_1, b_2, x_1, x_2) &= q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} \left(G(x_1, b_1) - (1-\alpha)(I - \theta_1)\right) \\ &+ (1-q_H) \left(\frac{x_0}{x_2}\right)^{\beta_1} \left(G(x_2, b_2) - (1-\alpha)(I - \theta_2)\right), \end{aligned} \tag{10}$$

where

$$G(x_i, b_i) = (1 - b_i) \left(\frac{x_i}{r-\mu} - \frac{c}{r}\right) \tag{11}$$

Similarly, the private firm's payoff is: $b_1(x - c)$, if $\theta = \theta_1$ and $b_2(x - c)$ if $\theta = \theta_2$. Then, conditional on the private firm exerting effort, the private firm's option value, $\Pi^F(x_0, b_1, b_2, x_1, x_2)$, is given by:

$$\Pi^F(x_0, b_1, b_2, x_1, x_2) = q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - \alpha(I - \theta_1)) + (1 - q_H) \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - \alpha(I - \theta_2)), \tag{12}$$

where

$$F(x_i, b_i) = b_i \left(\frac{x_i}{r - \mu} - \frac{c}{r}\right) \tag{13}$$

The sum of the two option values simply equals the total value of the project.

The government wants to set the contract parameters b_1, b_2, x_1, x_2 in order to maximize its objective function, i.e., its option value, subject to several constraints. First, the government needs to give incentives to the firm to exert effort (ex-ante incentive compatibility constraint, IC). Second, the contract has to ensure that the total value to the firm of accepting the contract is non-negative (ex-ante participation constraint, PC). Finally, we need to ensure that the firm has incentives to invest ex-post (ex-post limited liability constraints, LL). Therefore, the government's maximization problem is the following:

$$\max_{b_1, b_2, x_1, x_2} q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} (G(x_1, b_1) - (1 - \alpha)(I - \theta_1)) + (1 - q_H) \left(\frac{x_0}{x_2}\right)^{\beta_1} (G(x_2, b_2) - (1 - \alpha)(I - \theta_2)) \tag{14}$$

$$s.t. \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - \alpha(I - \theta_1)) - \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - \alpha(I - \theta_2)) \geq \frac{\xi}{\Delta q} \tag{15}$$

$$q_H \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - \alpha(I - \theta_1)) + (1 - q_H) \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - \alpha(I - \theta_2)) - \xi \geq 0 \tag{16}$$

$$F(x_1, b_1) - \alpha(I - \theta_1) \geq 0, \quad F(x_2, b_2) - \alpha(I - \theta_2) \geq 0 \tag{17}$$

The first constraint is the ex-ante IC constraint that ensures the firm exerts effort. To see this, note that it is just a simplified version of the following constraint:

$$\begin{aligned}
&q_H\left(\frac{x_0}{x_1}\right)^{\beta_1}(F(x_1,b_1)-\alpha(I-\theta_1))+(1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1}(F(x_2,b_2)-\alpha(I-\theta_2))-\xi\geq\\
&\geq q_L\left(\frac{x_0}{x_1}\right)^{\beta_1}(F(x_1,b_1)-\alpha(I-\theta_1))+(1-q_L)\left(\frac{x_0}{x_2}\right)^{\beta_1}(F(x_2,b_2)-\alpha(I-\theta_2))
\end{aligned} \tag{18}$$

The left hand side of this inequality represents the value of the firm's option if it exerts costly effort minus the cost of effort, while the right hand side represents the value of the firm's option if it does not exert effort.

The second constraint represents the ex-ante PC constraint. Finally, the last two constraints are the LL constraints. This maximization problem can be further simplified, as we will show in the next section that some of the constraints do not bind at the optimum.

3. Model solution: optimal contracts

We now solve the government's maximization problem. We provide in the Appendix three propositions, Propositions 1-3 that will help us simplify the maximization problem.

Proposition 1 shows that the limited liability of the firm for a θ_1 project does not bind, i.e., $F(x_1,b_1)>\alpha(I-\theta_1)$. Proposition 2 shows that the ex-ante participation constraint does not bind either, while Proposition 3 shows that the limited liability constraint for a θ_2 type project binds, i.e. $F(x_2,b_2)=\alpha(I-\theta_2)$. Given these results, we now have a reduced maximization problem:

$$\begin{aligned}
\max_{b_1,b_2,x_1,x_2}\quad & q_H\left(\frac{x_0}{x_1}\right)^{\beta_1}(G(x_1,b_1)-(1-\alpha)(I-\theta_1))\\
+\quad & (1-q_H)\left(\frac{x_0}{x_2}\right)^{\beta_1}(G(x_2,b_2)-(1-\alpha)(I-\theta_2))\\
s.t.\quad & \left(\frac{x_0}{x_1}\right)^{\beta_1}(F(x_1,b_1)-\alpha(I-\theta_1))\geq\frac{\xi}{\Delta q}\\
& F(x_2,b_2)-\alpha(I-\theta_2)=0
\end{aligned} \tag{19}$$

In the reduced optimization problem we have only two constraints, the simplified IC constraint and the binding LL constraint for the θ_2 type project. We show in the Appendix that the IC constraint also binds at the optimum. The solution to this problem is then the following:

$$x_1 = x_1^*, \quad x_2 = x_2^* \tag{20}$$

and

$$b_1 = \left(\frac{\xi}{\Delta q} \left(\frac{x_1^*}{x_0} \right)^{\beta_1} + \alpha(I - \theta_1) \right) \left(\frac{x_1^*}{r - \mu} - \frac{c}{r} \right)^{-1}, \quad b_2 = \frac{\alpha(\beta_1 - 1)}{\beta_1 + \frac{c/r}{I - \theta_2}}.\text{[9]} \tag{21}$$

The investment triggers equal the first-best outcome, thus the private firm invests at the same thresholds as the government would. Nevertheless, in order to give incentives to the firm to invest at the optimal triggers, the government needs to pay the firm some rents. To see the value of these rents, note first that the government and firm's option values evaluated at the optimal contract parameters are given by:

$$\Pi^G(x_0) = V^*(x_0) - q_H \frac{\xi}{\Delta q} \tag{22}$$

and

$$\Pi^F(x_0) = q_H \frac{\xi}{\Delta q} > \xi \tag{23}$$

Now we can see that the firm is not only paid for the cost of effort, but also captures some rents to induce her to exert effort. The rents of the private firm are given by the difference between its option value and the effort cost: $\Pi^F(x_0) - \xi = q_L \frac{\xi}{\Delta q} > 0$.

3.1. When is it optimal to induce effort?

The existence of rents distorts the government's choice of effort. To see this, we first need to compute the government's value in case of no effort. Since for the private firm not exerting effort is costless, the government can offer a contract to the firm at no cost, as follows:

$$x_1 = x_1^*, \quad x_2 = x_2^* \tag{24}$$

[9] Note that one can also write $b_2 = \alpha(I - \theta_2) \left(\frac{x_2^*}{r-\mu} - \frac{c}{r} \right)^{-1}$.

and

$$b_1 = \frac{\alpha(\beta_1 - 1)}{\beta_1 + \frac{c/r}{I-\theta_1}}, \quad b_2 = \frac{\alpha(\beta_1 - 1)}{\beta_1 + \frac{c/r}{I-\theta_2}}, \tag{25}$$

where b_1 and b_2 are derived from the binding limited liability constraints.

The government's value under no effort is then simply given by $\Pi^{GN}(x_0) = V^N(x_0)$, where $V^N(x_0)$ was defined in equation (8).[10]

Therefore, under the second-best case with moral hazard, the government will induce the firm to exert effort if $\Pi^G \geq \Pi^{GN}$, that is, if:

$$V^*(x_0) - q_H \frac{\xi}{\Delta q} \geq V^N(x_0) \tag{26}$$

Thus, the government gives incentives to the firm to exert effort if:

$$\frac{\xi}{\Delta q} \leq \frac{V^*(x_0) - V^N(x_0)}{q_H} < \frac{V^*(x_0) - V^N(x_0)}{\Delta q} = A \tag{27}$$

This implies that for a cost-benefit ratio such that $\frac{\xi}{\Delta q} > \frac{(V^*(x_0) - V^N(x_0))}{q_H}$, the government's decision is distorted. While the government would exert effort under First-best, it does not induce the private firm to exert effort because it is too costly to give incentives.

The total social value in the second-best case, $V^{**}(x_0)$, is defined by the sum of the government's and private firm's values. Thus, the total social value at time zero is given by:

$$V^{**}(x_0) = \Pi^G(x_0) + \Pi^F(x_0) = \begin{cases} V^*(x_0), & \text{if } \frac{\xi}{\Delta q} < \frac{(V^*(x_0) - V^N(x_0))}{q_H} \\ V^N(x_0), & \text{if } \frac{\xi}{\Delta q} > \frac{(V^*(x_0) - V^N(x_0))}{q_H} \end{cases} \tag{28}$$

As a measure of the inefficiency caused by asymmetric information we define the social loss as:

$$L(x_0) = \begin{cases} (V^*(x_0) - \xi) - (V^{**}(x_0) - \xi), & \text{if } \frac{\xi}{\Delta q} < \frac{(V^*(x_0) - V^N(x_0))}{q_H} \\ (V^*(x_0) - \xi) - V^{**}(x_0), & \text{if } \frac{\xi}{\Delta q} > \frac{(V^*(x_0) - V^N(x_0))}{q_H} \end{cases} \tag{29}$$

[10]The reason for this is that the firm's value is zero since effort is costless and the participation constraint is binding: $\Pi^{FN}(x_0) = q_L \left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - (1-\alpha)(I - \theta_1)) + (1 - q_L) \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - (1-\alpha)(I - \theta_2)) = 0$.

Substituting the total social value under the second-best case, V^{**}, we obtain:

$$L(x_0) = \begin{cases} 0, & \text{if } \frac{\xi}{\Delta q} < \frac{(V^*(x_0) - V^N(x_0))}{q_H} \\ \Delta q(W(x_0,\theta_1) - W(x_0,\theta_2)) - \xi, & \text{if } \frac{\xi}{\Delta q} > \frac{(V^*(x_0) - V^N(x_0))}{q_H} \end{cases} \tag{30}$$

4. Optimal contracts with a bailout option

So far we have considered a zero operating cost and a perpetually operated project. We now consider the more realistic case of a positive operating cost, $c > 0$, and incorporate an exit option into the model. Indeed, the firm could ask the government for a bailout if the project turns out to be unprofitable. Following Takashima et al. (2010) and Adkins and Paxson (2017) we assume that whenever the prospect of cash flows diminishes below a certain threshold, x_g, the firm has the right to transfer its ownership to the government in exchange for an amount K.

We now need to adjust the expressions of $G(x, b)$ and $F(x, b)$ in order to account for the exit option. Following standard arguments in real option theory, we have the following 2 ODEs:

$$\frac{1}{2}\sigma^2 x^2 G_{xx} + \mu x G_x - rG + (1-b)(x-c) = 0 \tag{31}$$

$$\frac{1}{2}\sigma^2 x^2 F_{xx} + \mu x F_x - rF + b(x-c) = 0 \tag{32}$$

The general solutions of equations (31) and (32) are given by:

$$G(x, b) = a_1 x^{\beta_2} + (1-b)\left(\frac{x}{r-\mu} - \frac{c}{r}\right), \tag{33}$$

and

$$F(x, b) = a_2 x^{\beta_2} + b\left(\frac{x}{r-\mu} - \frac{c}{r}\right), \tag{34}$$

where

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0 \tag{35}$$

and a_1, a_2 and x_g are constants to be determined using the following boundary conditions:

$$F(x_g, b) = K \tag{36}$$

$$F'(x_g, b) = 0 \tag{37}$$

$$G(x_g, b) = \frac{x_g}{r-\mu} - \frac{c}{r} - K \tag{38}$$

Equation (36) represents the value matching condition that requires the value of the private firm at the transfer trigger to equal K. Equation (37) is the smooth pasting condition that ensures optimality of the transfer trigger, x_g. Finally, equation (38) is the value matching condition that ensures the value of the government at the transfer threshold is equal to the operating value of the project after the transfer minus the transfer cost. The value of the government and of the private firm are then given by:

$$G(x, b) = (1-b)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) - \frac{1}{1-\beta_2}\left(K + \frac{bc}{r}\right)\left(\frac{x}{x_g}\right)^{\beta_2} \tag{39}$$

and

$$F(x, b) = b\left(\frac{x}{r-\mu} - \frac{c}{r}\right) + \frac{1}{1-\beta_2}\left(K + \frac{bc}{r}\right)\left(\frac{x}{x_g}\right)^{\beta_2} \tag{40}$$

and the threshold is

$$x_g \equiv x_g(b) = \frac{\beta_2}{\beta_2 - 1}\frac{\left(K + \frac{bc}{r}\right)(r-\mu)}{b} \tag{41}$$

The maximization problem of the government is expressed in equations (14-17) and can be simplified to the reduced problem given by equation (19), with $G(x, b)$ and $F(x, b)$ given by equations (39) and (40). The solution in this case is the following:

$$x_1 = x_1^*, \quad x_2 = x_2^*, \tag{42}$$

and b_1, b_2 satisfy the following equations:

$$b_1\left(\frac{x_1^*}{r-\mu} - \frac{c}{r}\right) + \frac{1}{1-\beta_2}\left(K + \frac{b_1 c}{r}\right)\left(\frac{x_1^*}{x_g(b_1)}\right)^{\beta_2} = \frac{\xi}{\Delta q}\left(\frac{x_1^*}{x_0}\right)^{\beta_1} + \alpha(I - \theta_1) \tag{43}$$

$$b_2\left(\frac{x_2^*}{r-\mu} - \frac{c}{r}\right) + \frac{1}{1-\beta_2}\left(K + \frac{b_2 c}{r}\right)\left(\frac{x_2^*}{x_g(b_2)}\right)^{\beta_2} = \alpha(I - \theta_2) \tag{44}$$

Equations (43) and (44) must be solved numerically for the firm's fractions of cash flows b_1 and b_2.

As in the base case model without an exit option, the investment triggers equal the First-best outcome, thus the private firm invests at the same thresholds as the government would. However, giving incentives to the firm is costly, thus the government's decision might be distorted by the rents paid to the firm.

We end this section with a brief note on limited liability. In Grenadier and Wang (2005) limited liability meant that the wages received by the manager at the time of exercise, w_1, w_2 were positive. In their case there was a one-time payment at the time of exercise. However, we have a continuous payment (a fraction b of the net cash-flows) starting with the time of exercise. Limited liability could then imply that the cash flows of the private firm have to be positive at all times (similar to the minimum revenue guarantee, also called floor in Adkins and Paxson, 2017, or risk share option in Takashima et al., 2010). A simple way to guarantee positive cash flows at all times, without having to introduce revenue subsidies from the government would be to set $K \geq K_{min}$, where $x_g(b_2, K_{min}) - c = 0$, $K_{min} = \frac{b_2 c(r-\beta_2\mu)}{-\beta_2 r(r-\mu)}$.

5. Numerical analysis

In this section we illustrate the dependence of our results on the main parameters of the model. In particular, we focus on the optimal contract (investment thresholds and private firm's fraction of the net revenue), the first and second-best values, as well as the social loss due to moral hazard. We will also look at how the bailout option affects the optimal contract. Finally, since volatility is a key parameter in real options we will analyze its impact separately. The base case parameters used in this analysis are as follows: $\mu = 0.03$, $\sigma = 0.3$, $r = 0.06$, $c = 0.15$, $I = 10$, $\theta_1 = 9$, $\theta_2 = 3$, $\alpha = 0.7$, $q_H = 0.7$, $q_L = 0.5$, $\xi = 0.05$, and $K = 0.1$.

5.1. Optimal contract

We plot in Figure 1 the comparative statics of the optimal contract (b_1, b_2, x_1, x_2) with respect to the operating cost c. The fraction of net revenues obtained by the private firm decreases with the operating cost. This is due to the fact that the discounted value of future net revenues at the time of the investment increases with the operating cost.[11] Therefore, a lower fraction of the net revenues is needed to compensate the firm for its shared investment and effort. Moreover, both investment triggers increase with the operating cost. A higher cost thus implies a delayed investment. For relatively high operating costs it is not optimal for the government to induce effort, and the fraction of net revenue received by the firm of a high-type project decreases substantially.

[Figure 1 about here.]

Figure 2 depicts the comparative statics of the optimal contract with respect to the investment cost I. The fraction of net revenues obtained by the private firm increases with the investment cost since the private firm shares this investment cost with the government in a proportion equal to α. Therefore, a larger investment cost implies that the firm requires a higher fraction of the net revenues to participate in the project. As before, both investment triggers increase with the investment cost. Thus an increased investment cost delays investment. Again, for a relatively high investment cost the government will not give incentives to the firm to exert effort, hence the drop in b_1.

[Figure 2 about here.]

Figures 3 and 4 present the comparative statics with respect to the θ_1 and θ_2. In general the optimal contract of a θ_i-project depends only on θ_i, where $i \in \{1, 2\}$. Both the fraction of net revenues obtained by the private firm and the investment trigger decrease with θ. This is because θ represents the amount by which the gross investment cost can be reduced. However, we can see that b_1 depends indirectly on θ_1 since when the projects are very similar (θ_2 gets closer to θ_1, and the difference $\Delta\theta$ diminishes) the government will decide not to induce effort.

[Figure 3 about here.]

[Figure 4 about here.]

[11]Note that the investment trigger increases with c.

As expected, in Figure 5 we note that the fraction of net revenues obtained by the private firm increases with the cost sharing ratio α. However, the government's effort choice does not depend on this parameter, nor does the investment trigger.

[Figure 5 about here.]

Figure 6 shows that the fraction of net revenues received by the private firm of a θ_1 project increases with the effort cost ξ in order to provide the firm with incentives to exert effort. Moreover, as expected, for a large effort cost the government no longer wants to induce the private firm to exert effort.

[Figure 6 about here.]

Finally, in Figure 7 we can see that when Δq (the benefit of exerting effort) is relatively small, the government will not induce effort. For a relatively large benefit of effort, when the government induces effort, the fraction of net revenues obtained by the private firm decreases with Δq. When the benefit of exerting effort increases, it is less costly for the government to provide incentives since the firm will naturally have more incentives to exert effort.

[Figure 7 about here.]

5.2. Social loss

Figure 8 depicts the comparative statics of the social loss with respect to the parameters of the model. As previously discussed, for relatively high operating or investment costs the government's choice of effort is distorted under asymmetric information. Therefore, a social loss arises. This social loss is decreasing with both costs. In the limit, for very high costs the government would not exert effort under the first-best scenario either, thus the loss would be null.

[Figure 8 about here.]

A social loss also arises either for a relatively low θ_1 or for a relatively high θ_2. A lower θ_1 quality keeping θ_2 constant implies a lower difference in the quality of the two projects. The government would then be less likely to incentivize effort since the benefit of effort (a higher probability of obtaining a high quality project) is not so valuable when the difference in quality is low. A similar argument holds for a larger θ_2 keeping θ_1 constant. Then the higher the difference in quality of the two projects the higher the social loss.

Asymmetric information also generates a social loss either when the cost of effort is relatively high, or when the benefit of effort is relatively low. The higher the cost-benefit ratio of effort the more expensive it will be for the government to give incentives to the form to exert effort, thus the more likely it is that his effort choice is distorted.

Note that in all cases the highest losses are obtained not for extreme project parameters, but for intermediate ones. Indeed, in the extreme (e.g., for very high operating costs) both under first-best and under second-best the government will choose not to induce effort exertion. Therefore, losses are particularly important for average projects, not for extreme/outlier projects.

5.3. The bailout option

[Figure 9 about here.]

We now analyze how the optimal contract changes when we add a bailout option. As expected, the bailout triggers increase with the bailout price. Thus a higher bailout price will lead to an earlier bailout (panel c) of Figure 9). Note that a firm with a low-type project will ask for a bailout later than a firm with a high-type project ($x_g(b_2) < x_g(b_1)$). The fraction of net revenues received by the private firm decreases with the bailout price. Naturally, since now the firm has an exit option, the government does not need to offer such a high fraction of revenues to induce effort.

5.4. The impact of volatility

Since volatility is one of key parameters in real options, we analyze it separately. Regarding the investment triggers, as asymmetric information does not distort the investment timing, we have the same comparative statics as in standard real options. A higher volatility leads to a delay in investment as we can note in panel c) of Figure 10. Moreover, the values of the options to invest at time zero also increase with volatility (panel d)). Note that the value of the option to invest in the lower quality project, θ_2 increases at a faster pace.

[Figure 10 about here.]

The fraction of net revenue obtained by the private firm decreases with volatility. This is due to the fact that this fraction is inversely related to the investment trigger. The higher the investment trigger the lower the fraction of revenues needed to compensate the private firm.

[Figure 11 about here.]

Since the values of the options to invest increase with volatility, the ex-ante value of the government's option (V^* under First-best and V^{**} under second-best) also increases with volatility (see Figure 11). However, for large volatilities the government will not provide incentives to the firm to exert effort. This is because for large volatility values the difference in the values of the option to invest in the two projects decreases. Thus the gain from effort is smaller. Therefore we have that the social loss decreases with volatility.

[Figure 12 about here.]

Finally, we can see that the bailout triggers decrease with volatility (Figure 12). The larger the volatility the later will the private firm ask for a bailout. For large levels of volatility it is not optimal for the government to induce effort. This leads to a jump in the bailout trigger, the firm asks for a bailout earlier.

6. Conclusions

In this paper, we present a real-option model for a PPP investment project under moral hazard, thus combining the real-options literature on PPPs with the contract theory literature on PPPs. We derive the optimal contract that provides incentives to the private firm to exert effort. The first-best investment timing is induced. However, the asymmetric information distorts the government's decision to induce effort with respect to the first-best policy. This leads to a social welfare loss. Additionally, we have incorporated a bailout option into the basic model, and analyzed its impact on the optimal contract.

In the future, we will extend the model to introduce a revenue guarantee. Such an option might alter the optimal investment timing chosen by the private firm.

Appendix: Solution to the optimal contracting problem

We provide a derivation of the optimal contracts presented in Section 3. We start by proving three propositions that help us simplify the maximization problem of the government. Proposition 1 shows that the limited liability for the private firm with a θ_1-type project does not bind.

Proposition 1. *The limited-liability condition for a manager of a θ_1-type project does not bind. That is, $F(x_1, b_1) - \alpha(I - \theta_1) > 0$.*

Proof.

$$\left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - \alpha(I - \theta_1)) \geq \frac{\xi}{\Delta q} + \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - \alpha(I - \theta_2)) \geq \frac{\xi}{\Delta q} > 0 \quad (.1)$$

The first and second inequalities follow from the ex-ante IC and ex-post LL constraints (equations 15 and 17), respectively.

Proposition 2 shows that the ex ante participation constraint in equation (16) does not bind.

Proposition 2. *The ex-ante participation constraint does not bind.*

Proof.

$$\left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - \alpha(I - \theta_1)) + \frac{1 - q_H}{q_H} \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - \alpha(I - \theta_2)) \geq \frac{\xi}{\Delta q} \geq \frac{\xi}{q_H} > 0 \quad (.2)$$

where the first inequality follows from the ex ante incentive constraint and the limited liability condition for the type-θ_2 project (equations 15 and 17).

Given these two propositions, we can express the government's maximization problem as maximizing his objective function subject only to two constraints: the ex-ante IC constraint (equation 15) and the LL constraint for the θ_2 type project (the second equation in 17). We can form the Lagrangian, using the Kuhn-Tucker method, with λ_1 and λ_2 denoting the Lagrange multipliers for the two constraints respectively:

$$\begin{aligned} L &= \left(\frac{x_0}{x_1}\right)^{\beta_1} (G(x_1, b_1) - (1 - \alpha)(I - \theta_1)) + \frac{1 - q_H}{q_H} \left(\frac{x_0}{x_2}\right)^{\beta_1} (G(x_2, b_2) - (1 - \alpha)(I - \theta_2)) \\ &+ \lambda_1 \left[\left(\frac{x_0}{x_1}\right)^{\beta_1} (F(x_1, b_1) - \alpha(I - \theta_1)) - \left(\frac{x_0}{x_2}\right)^{\beta_1} (F(x_2, b_2) - \alpha(I - \theta_2)) - \frac{\xi}{\Delta q} \right] \\ &+ \lambda_2 (F(x_2, b_2) - \alpha(I - \theta_2)), \end{aligned} \quad (.3)$$

with corresponding complementary slackness conditions for the two constraints.

The first-order condition with respect to b_1 gives: $\lambda_1 = 1 > 0$. This implies that the ex-ante IC constraint is binding.

The first-order condition with respect to b_2 gives:

$$\left(-\lambda_1 - \frac{1-q_H}{q_H}\right)\left(\frac{x_0}{x_2}\right)^{\beta_1} + \lambda_2 = 0 \tag{.4}$$

Since we have from above $\lambda_1 = 1$, we obtain $\lambda_2 = (x_0/x_2)^{\beta_1}/q_H > 0$. This implies that the LL constraint for the θ_2-type is binding. Thus we have:

$$F(x_2, b_2) = \alpha(I - \theta_2) \tag{.5}$$

We summarize this in Proposition 3.

Proposition 3. *The ex-post LL constraint for a θ_2-type project binds.*

The first-order condition with respect to x_1 implies:

$$x_1 = \frac{\beta_1}{\beta_1 - 1}\left(\frac{c}{r} + I - \theta_1\right)(r - \mu) \tag{.6}$$

Finally, the first-order condition with respect to x_2 implies:

$$x_2 = \frac{\beta_1\left[(1 - q_H - b_2)c/r + (1 - q_H - \alpha)(I - \theta_2)\right]}{\beta_1(1 - q_H - b_2) - (1 - q_H)}(r - \mu) \tag{.7}$$

We solve for b_2 and x_2 using equations (.5) and (.7), and we obtain:

$$b_2 = \frac{\alpha(\beta_1 - 1)}{\beta_1 + \frac{c/r}{I - \theta_2}} \tag{.8}$$

and

$$x_2 = \frac{\beta_1}{\beta_1 - 1}\left(\frac{c}{r} + I - \theta_2\right)(r - \mu) \tag{.9}$$

Since we have shown that the ex-ante IC constraint given by equation (15) binds ($\lambda_1 = 1 > 0$), we can obtain b_1 given by equation (21).

The proof is similar for the case with a bailout option.

References

Adkins, R., Paxson, D., 2017. Risk sharing with collar options in infrastructure investments. Working Paper.

Alonso-Conde, A.B., Brown, C., Rojo-Suarez, R., 2007. Public-private partnerships: Incentives, risk transfer and real options. Review of Financial Economics 16, 335–349.

Boothe, P., Boudreault, F., Hudson, D., Moloney, D., Octaviani, S., 2015. The procurement of public infrastructure: Comparing P3 and traditional approaches. Lawrence National Centre for Policy and Management, Ivey School of Business, University of Western Ontario.

Bowe, M., Lee, D., 2004. Project evaluation in the presence of multiple embedded real options: Evidence from the Taiwan high-speed rail project. Journal of Asian Economics 15, 71–98.

Bradford, N., 2003. Public-private partnership? Shifting paradigms for economic governance in Ontario. Canadian Journal of Political Science 36, 1005–1033.

Brandao, L., Saraiva, E., 2008. The option value of government guarantees in infrastructure projects. Construction Management and Economics 26, 1171–1180.

Brown, C., 2005. Financing transportation infrastructure: For whom the road tolls. The Australian Economic Review 38, 431–438.

Chiara, N., . Gavin, M., Vecer, J., 2007. Valuing simple multiple-exercise real options in infrastructure projects. Journal of Infrastructure Systems 13, 97–104.

Dewatripont, M., Legros, P., 2005. Public-private partnerships: Contract design and risk transfer. European Investment Bank Papers 10, 120–145.

Doan, P. and. Patel, K., 2010. Toll road investment under uncertainty. Working Paper.

Engel, E., Fischer, R., Galetovic, A., 2013. The basic public finance of public-private partnerships. Journal of the European Economic Association 11, 83–111.

Grenadier, S., Wang, N., 2005. Investment timing, agency, and information. Journal of Financial Economics 75, 493–533.

Hart, O., 2003. Incomplete contracts and public ownership: Remarks, and an application to public-private partnerships. Economic Journal 113, C69–C76.

Holmstrom, B., Milgrom, P., 1991. Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. Journal of Law, Economics, & Organization 7, 24–52.

Huang, Y.L., Chou, S.P., 2006. Valuation of the minimum revenue guarantee and the option to abandon in BOT infrastructure projects. Construction Management and Economics 24, 379–389.

Iossa, E., Martimort, D., 2008. The simple micro-economics of public-private partnerships. CEIS Tor Vergata Research Paper.

Iossa, E., Martimort, D., 2009. The theory of incentives applied to the transport sector. Working Paper.

Iossa, E., Martimort, D., 2012. Risk allocation and the costs and benefits of public-private partnerships. Rand Journal of Economics 43, 442–474.

Martimort, D., Pouyet, J., 2008. To build or not to build: Normative and positive theories of public-private partnerships. International Journal of Industrial Organization 26, 393–411.

Power, G., Burris, M., Vadali, S., Vedenov, D., 2009. Valuation of strategic options for public-private partnerships. Working Paper.

Rose, S., 1998. Valuation of interacting real options in a toll-road infrastructure project. Quarterly Review of Economics and Finance 38, 711–723.

Shan, L., Garvin, M., Kumar, R., 2010. Collar options to manage revenue risks in real toll public-private partnership transportation projects. Construction Management and Economics 28, 1057–1069.

Shibata, T., 2009. Investment timing, asymmetric information, and audit structure:a real options framework. Journal of Economic Dynamics and Control 33, 903–921.

Shibata, T., Nishihara, M., 2010. Dynamic investment and capital structure under manager-shareholder conflict. Journal of Economic Dynamics and Control 34, 158–178.

Sitruk, J., 2009. Real options and incentives in a PPP railway project. Working paper.

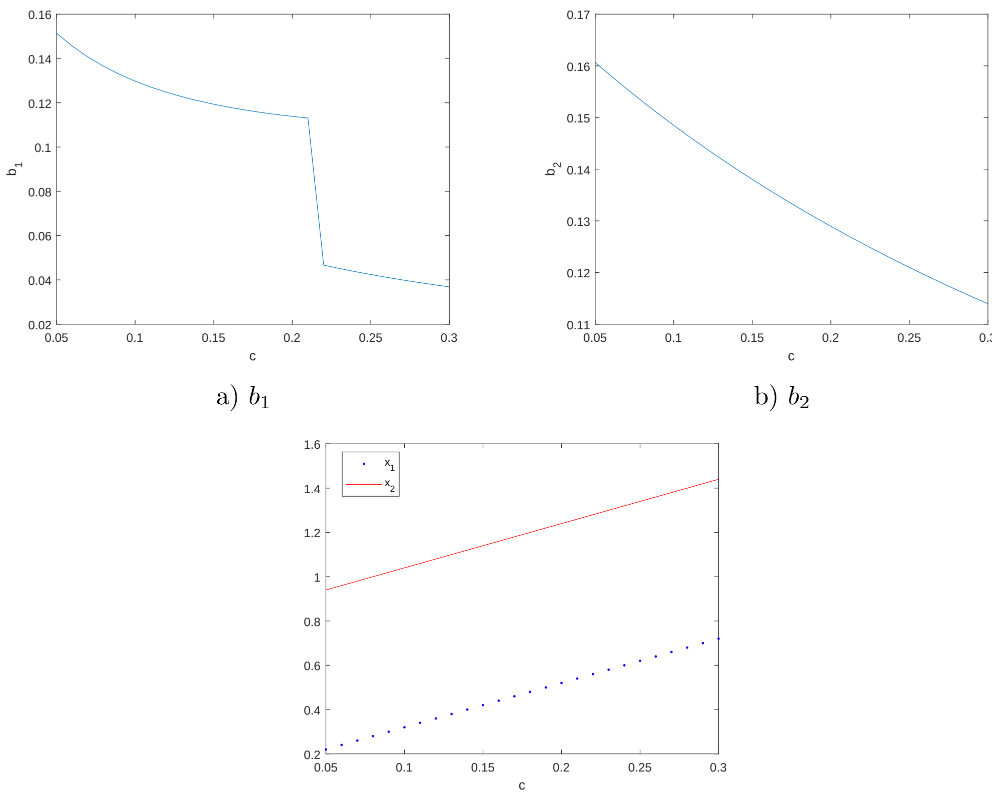
Soumare, I., Lai, V.S., 2016. An analysis of government loan guarantees and direct investment through public-private partnerships. Economic Modelling 59, 508 – 519.

Spackman, M., 2002. Public-private partnerships: Lessons from the British approach. Economic Systems 26, 283–301.

Takashima, R., Yagi, K., Takamori, H., 2010. Government guarantees and risk-sharing in public-private partnerships. Review of Financial Economics 19, 78–83.

Vining, A., Boardman, A., 2008. Public-private partnerships in Canada: Theory and evidence. Canadian Public Administration 51, 9–44.

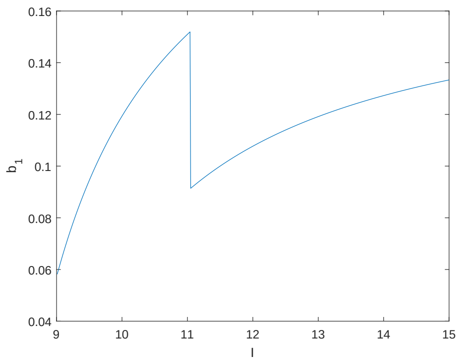
Figures



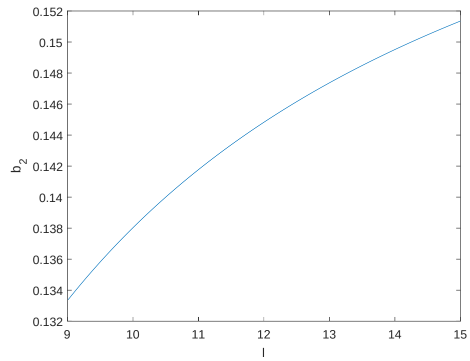
a) b_1 b) b_2

c) x_1 and x_2

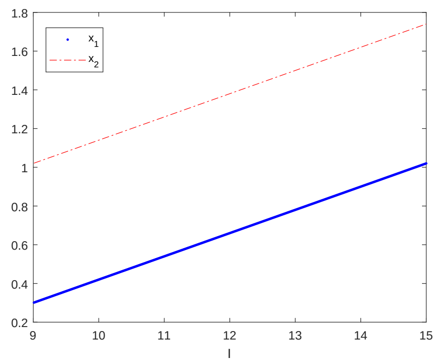
Figure 1: Optimal contract as a function of the operating cost c.



a) b_1

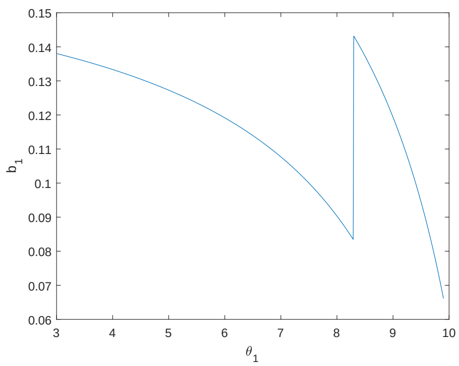


b) b_2

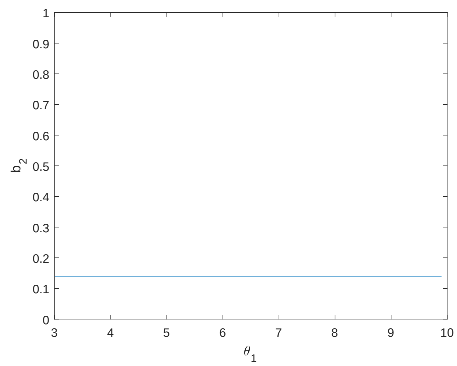


c) x_1 and x_2

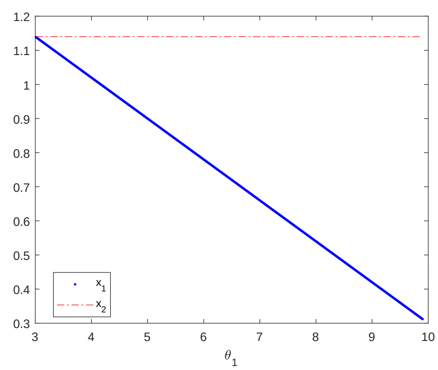
Figure 2: Optimal contract as a function of the investment cost I.



a) b_1

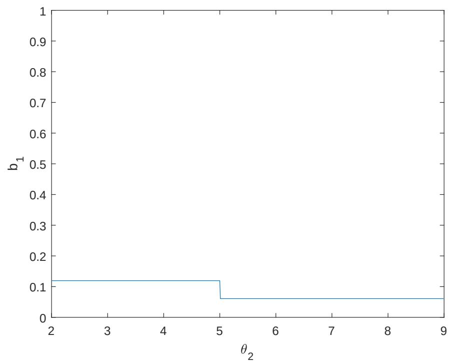


b) b_2

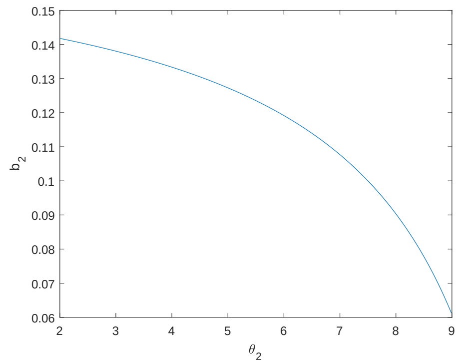


c) x_1 and x_2

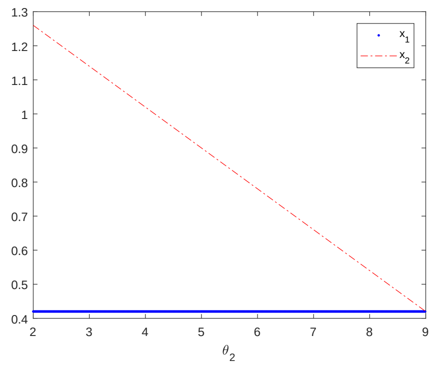
Figure 3: Optimal contract as a function of the θ_1.



a) b_1

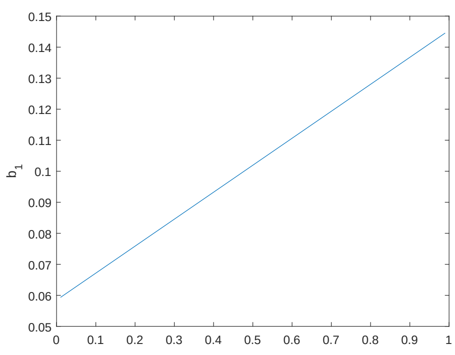


b) b_2

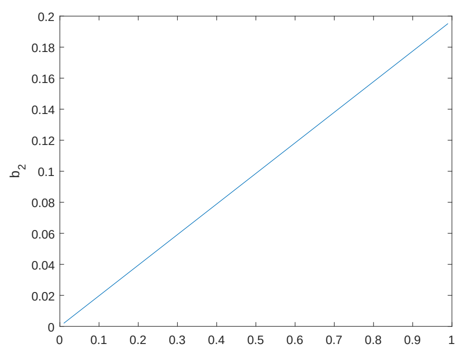


c) x_1 and x_2

Figure 4: Optimal contract as a function of the θ_2.

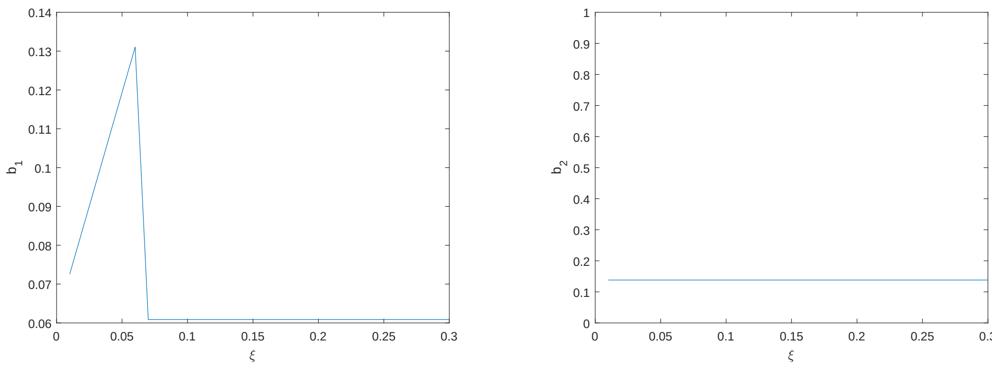


a) b_1



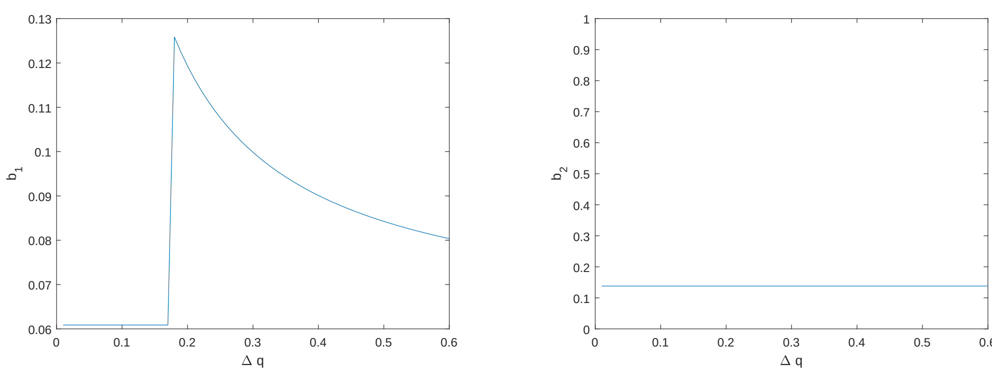
b) b_2

Figure 5: Optimal contract as a function of the cost sharing rate α.



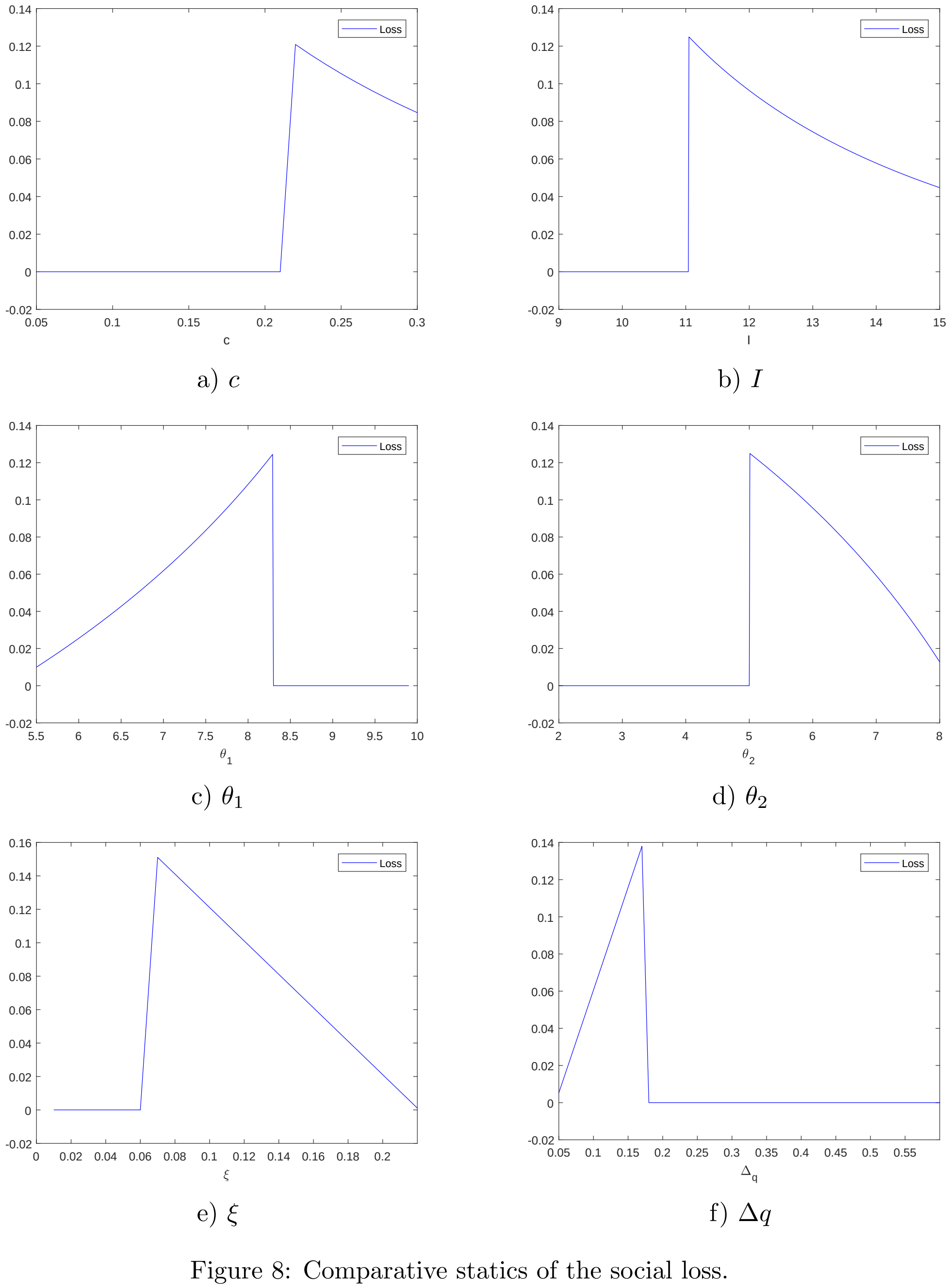
a) b_1 b) b_2

Figure 6: Optimal contract as a function of the effort cost ξ.



a) b_1 b) b_2

Figure 7: Optimal contract as a function of the effort benefit Δq.

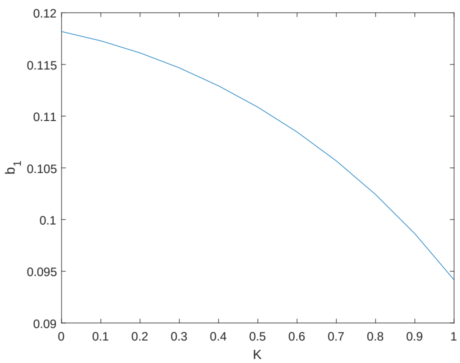


a) c b) I

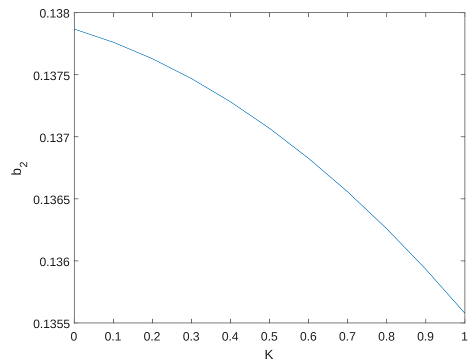
c) θ_1 d) θ_2

e) ξ f) Δq

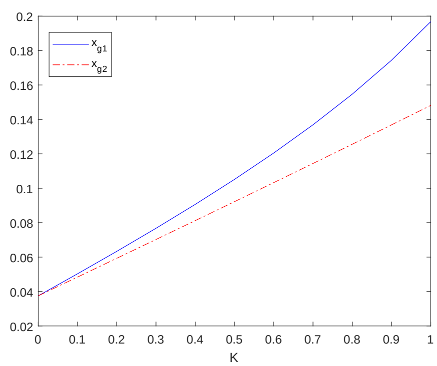
Figure 8: Comparative statics of the social loss.



a) b_1

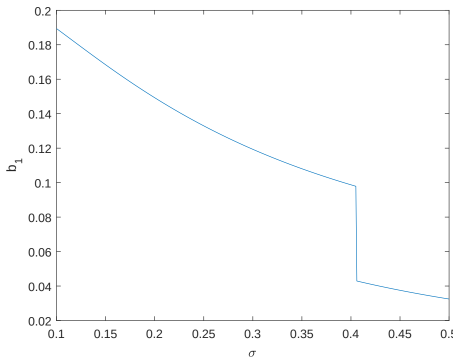


b) b_2

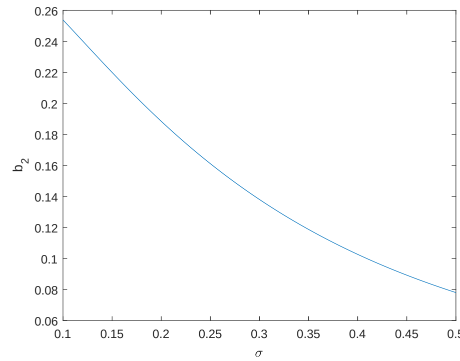


c) $x_g(b_1)$ and $x_g(b_2)$

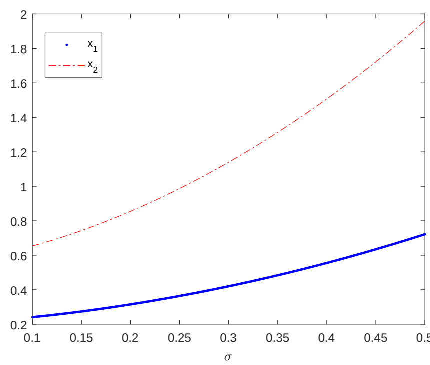
Figure 9: Optimal contract and buyout thresholds as a function of the buyout price K.



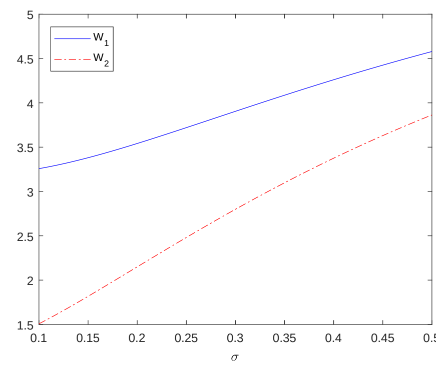
a) b_1



b) b_2

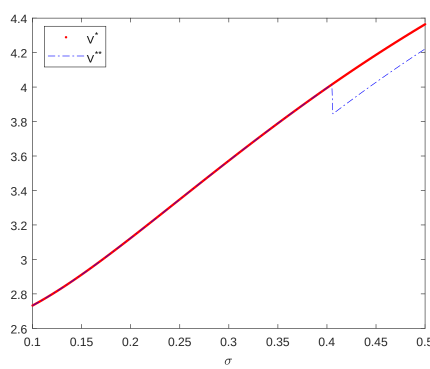


c) x_1 and x_2

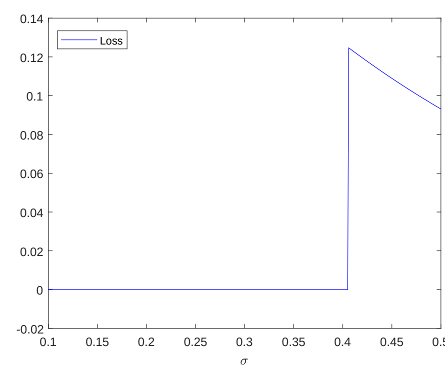


d) W_1 and W_2

Figure 10: Optimal contract and option values as a function of the volatility σ.

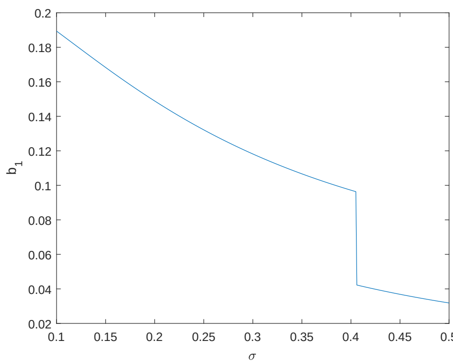


a) Values

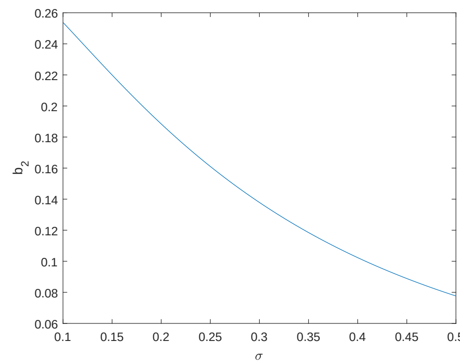


b)Social loss

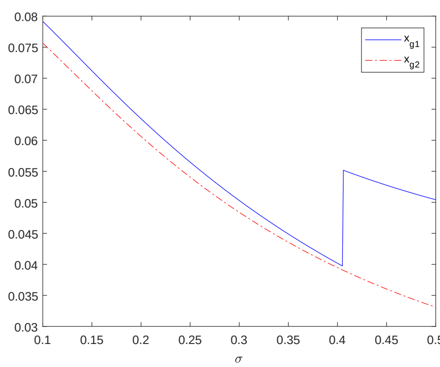
Figure 11: First and second-best values and social loss as a function of volatility σ.



a) b_1



b) b_2



c) $x_g(b_1)$ and $x_g(b_2)$

Figure 12: Optimal contract and buyout thresholds as a function of volatility σ.