Market Imperfections and Competitive Runs under Uncertainty

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Abstract

In a market where production has adverse externalities, policy makers may wish to increase welfare by imposing a cap on quantity. Previous literature has shown that while this cap lowers the long run adverse effects of investment in that market, it also makes these adverse effects appear earlier, as the cap speeds-up investment to that market. The current article finds that among these two contradicting effects – the latter is the dominant, rendering the cap harmful for welfare. In particular, the cap speeds-up investment by creating a "competitive run" where all the investment still allowed is done at one instant.

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1. Introduction

Intuitively, if investment has a social cost that is not fully internalized by the investor, then a cap on investment can increase social welfare as it prevents production in the range where output price is below the total social cost. Yet, this intuition hinges on static foundations as it ignores how the cap affects optimal investment timing. As this article shows, the cap may speed-up investments enough to have an overall negative effect on welfare. More specifically, the cap has contradicting effects on welfare: on the one hand – it indeed lowers the long-term magnitude of investment, on the other hand – in speeds-up investment and makes its adverse effects materialize earlier. The latter effect can be the dominant one – rendering the cap actually harmful for welfare.

Bartolini (1993) was the first to add to the literature on investment under uncertainty an analysis of how a cap on the quantity in the market affects investment timing. He has shown that due to the cap, optimal investment is no longer a gradual process, but a process that is gradual only until at a certain point in time when a "cap attack" occurs and the remaining allowed investment takes places at once. Bartolini has focused on how the cap creates this dynamic investment pattern, and ignored the issue of the motivation for imposing it. Specifically, his analysis takes the size of the cap as given, and he does not study its welfare implications.

Some of this void has been filled by several recent studies, with Moretto and Vergalli (2010) and Di Corato, Moretto and Vergalli (2013) being the most prominent ones.

Moretto and Vergalli (2010) is a theoretical study of how a cap on immigration in the host country affects the decisions of potential immigrants. Moretto and Vergalli view the immigration act as an irreversible investment and impose the standard investment under uncertainty analysis on it. The potential immigrant chooses to perform this irreversible costly act if its profitability, mostly based on labor market conditions at the host country, is sufficiently large. If the host country wishes to limit immigration via a cap – the Bartolini dynamic pattern emerges with a cap-attack at some point in time. In addition to applying these dynamics in the immigration context, Moretto and Vergalli show that the government can delay the cap-attack by creating uncertainty about the size of the cap.

Di Corato, Moretto and Vergalli (2013) apply the Bartolini analysis to the case of transforming forest land into agricultural land. Forest land generates welfare that the owners cannot fully charge for it, and mostly not for the utility derived from the beauty of the forested environment. Thus, the social loss of forest land is only partially internalized by the land owners when they convert their land to agriculture. This motivates a cap on the allowed amount of agricultural land. Yet, as Di Corato, Moretto and Vergalli show, the cap may create a cap-attack, which speeds-up the socially undesired land conversion. In addition to describing these dynamics, they also focus on conditions for land owners to voluntarily participate in government program meant to protect forestry, and also on the long-run average rate of investment in agricultural land.

Indeed, these two articles, as well as several related ones, do refer to policy-makers' welfare aims in setting the cap, and even point at how the resulting cap-attack harms welfare. Yet – they do not explicitly model welfare and therefore do not search for the welfare-maximizing size of the cap.

To fill this void I create in this article a model of investment under uncertainty which deviates from the typical models of that literature in just one aspect – the cost that the investor pays is only a part of the total cost that the investment inflicts on society. Uncommonly to this literature – I also add an explicit modeling of the welfare in the relevant market.

At the first stage of the analysis I assume no cap on investment. The analysis in this stage leads to the first main result of this study: despite the externality, the free market equilibrium may be the best one. The reason for that springs from one of the most well-known results of the literature on investment under uncertainty – that the stochastic nature of the profitability from the investment makes firms investment only when the output price is sufficiently above its marginal cost. This gap causes a welfare loss as output units that could add to welfare are not produced. On the other hand, the externality (in which the investor only pays part of the total social cost of the investment) promotes investment and output. There are two possible outcomes for the interaction between this externality and the under-investment that the stochastic nature of the profitability creates. In the first case the externality merely lowers the under-investment, but does not turn it into over-investment. In that case the externality in fact positively contributes to welfare and there is no need for policy measures against it. In the second case the externality dominates and leads to over-investment which indeed calls for policy measures. The analysis shows that the condition for the first case to prevail is that the externality is sufficiently small, in the sense that the part that the investor does not have to pay, out of the total investment cost, is sufficiently small.

If, on the other hand, the part that the investor does not have to pay is sufficiently large, then each additional investment harms welfare. This result suggests the possibility that a cap on investment may be beneficial for welfare and leads to the second part of the analysis where the existence of such a cap is added to the model. The analysis shows that the cap leads to the Bartolini cap-attack dynamic pattern of investment. Searching for the size of the cap that brings welfare to its maximum, reveals that it is best to push the cap as high as possible – which actually means having no cap at all.

This study contributes to the literature on investment under uncertainty beyond the results it provides about the desirability of a cap. In particular, the explicit modeling of welfare and the technique for calculating it are not common to this literature. In addition, the derivation of the emergence of the attack on the cap was fully carried out in detail, in order to take care of some flaws in the original derivation of this result by Bartolini (1993).

The article is organized as follows. Section 2 presents a static toy model where costs are not fully internalized and therefore a cap maximizes welfare. The analysis shows the intuition underlying this result, but also highlights its static origins. Section 3 presents and analyzes the dynamic model which is in the heart of this article, and derives its main results. Section 4 offers some concluding remarks.

2. A static model

In this short section a static toy model is constructed in order to present the motivation for imposing a cap, as well as to show how this motivation is deeply rooted in the static nature of this model.

Within a static single-period setting, consider a market for a good that its demand is:

$$(1) \qquad P = \frac{B}{Q}$$

There is a cost M for producing each unit of that good. Yet, the producers have to pay only the part $\lambda \cdot M$, where $0 < \lambda < 1$, while the rest of the cost is external to them.

Under this setup, the total welfare in this market is:

$$(2) \qquad W(Q) = B \cdot \ln(Q) - M \cdot Q .$$

By straightforward differentiation, welfare is maximized if Q satisfies:

$$(3) \qquad Q = \frac{B}{M} \equiv Q_1 .$$

Yet, in the free market equilibrium, the quantity is:

$$(4) \qquad Q = \frac{B}{\lambda \cdot M} > Q_1 ,$$

where the inequality follows from the market imperfection captured by $\lambda < 1$. Thus, maximizing welfare can be done via a cap on Q at the value of Q_1.

Note that concavity of welfare is sufficient to generate an optimal finite cap. Alternatively, if welfare was linear like the production cost, than it would be optimal to push the cap either to zero or to infinity. Setting the cap at zero implies banning production of X altogether, and pushing the cap to infinity means having no cap at all.

3. **The model**

Within a continuous time setting, consider a market for a perfectly durable good, named X, that at each point in time its demand is given by:

(5) $$P_t = \frac{B_t}{Q_t},$$

where Q_t and P_t are, respectively, the quantity and the price of X at time t. Demand changes stochastically over time according the swings in the process B_t. All producers face the same cost structure where supplying the quantity q_t at time t entails the instantaneous total cost to society as a whole:

(6) $$STC(q) = M \cdot q,$$

where M is constant. Part of this cost is an externality that the producers of X do not incur, and the instantaneous total cost of a producer that supplies the quantity q is:

(7) $$TC(q) = \lambda \cdot M \cdot q,$$

where $0 < \lambda < 1$.

Due to the perfect durability, the quantity Q_t is a stock and producing an additional unit of X is an investment that is based on the expected discounted flow of profits from that unit. This flow is a stochastic process due to stochastic nature of B_t. More specifically, B_t is the following Geometric Brownian Motion:

(8) $$dB_t = \mu \cdot B_t \cdot dt + \sigma \cdot B_t \cdot dZ_t,$$

where μ and σ are constants which measure, respectively, the drift and the variance of B_t, and dZ_t is the increment of the standard Wiener process satisfying at each instant:

(9) $$E(dZ_t) = 0, \quad E(dZ_t)^2 = 1.$$

By properties of the Geometric Brownian Motion, at time intervals when Q_t is unchanged, P_t is also a Geometric Brownian Motion with the same parameters as those of B_t. The interest rate, denoted r, is constant over time. Convergence of the value of owning a unit of X requires that the expected rate of growth of B_t does not exceed the discount rate, i.e., that $r > \mu$.

There is free entry to this market with an infinite amount of potential investors. Yet, the investment, i.e. producing a new unit of X, commits the producer to permanently offer it and therefore to an infinite flow of the cost $\lambda \cdot M$. The discounted present value of this flow is $\frac{\lambda \cdot M}{r}$ and it can be viewed as an irreversible investment cost.

3.1 Optimal investment in the absence of a cap

Under the setup described above, the potential investor in this model is facing the same situation as the investors in Leahy (1993). In this sub-section I use Leahy's analysis to present the potential investors' optimal investment policy.

At each instant, each potential investor has to decide whether to produce and supply a new increment of X, or not. The decision depends on the expected profitability of this investment, and therefore takes place only when B_t is sufficiently large, where $B^*(Q)$ denotes the investment threshold. a larger level of Q implies, ceteris paribus, lower profitability, so the threshold $B^*(Q)$ is an increasing function of Q.

Let $V(Q, M)$ be the value of owning a unit of X. The following no-arbitrage condition states that the instantaneous profit, $\frac{B}{Q} - \lambda \cdot M$, along with the expected instantaneous capital gain from a change in B, must equal the instantaneous normal return:

(10) $$r \cdot V(Q, B) \cdot dt = \frac{B}{Q} - \lambda \cdot M + E[dV(Q, B)].$$

By Ito's lemma:

(11) $$E[dV(Q, B)] = \frac{1}{2} \cdot \sigma^2 \cdot B^2 \cdot V_{BB}(Q, B) + \mu \cdot B \cdot V_B(Q, B)$$

Applying (11) in (10) yields:

$$\tag{12} \tfrac{1}{2}\cdot\sigma^2\cdot B^2\cdot V_{BB}(Q,B)+\mu\cdot B\cdot V_B(Q,B)-r\cdot V(Q,B)+\frac{B}{Q}-\lambda\cdot M=0$$

Trying a solution of the type B^x for the homogenous part of this differential equation and a linear form as a particular solution to the entire equation, yields:

$$\tag{13} V(Q,B)=Z(Q)\cdot B^{\alpha}+Y(Q)\cdot B^{\beta}+\frac{B}{Q\cdot(r-\mu)}-\frac{\lambda\cdot M}{r},$$

where α and β are the roots of the quadratic:

$$\tag{14} \tfrac{1}{2}\cdot\sigma^2\cdot x^2+\left(\mu-\tfrac{1}{2}\cdot\sigma^2\right)\cdot x-r=0.$$

The assumption that $r>\mu$ asserts that $\beta>1$ and $\alpha<0$.

$\frac{B}{Q\cdot(r-\mu)}-\frac{\lambda\cdot M}{r}$ describes the expected extra value this unit generates if Q remains forever in its current level. The two other elements of the RHS of (7) represent therefore how the changes in Q over time are expected to affect the value of the unit.

By properties of the Geometric Brownian Motion, when B goes to 0 the probability of it ever rising to $B^*(Q_t)$, and Q consequently changing, approaches 0. Thus implies:

(15) $$\lim_{B\to\infty}\left[Z(Q)\cdot B^{\alpha}+Y(Q)\cdot B^{\beta}\right]=0,$$

which leads to $Z(Q)=0$, since $\alpha < 0$, and therefore to:

(16) $$V(Q,B)=Y(Q)\cdot B^{\beta}+\frac{B}{Q\cdot(r-\mu)}-\frac{\lambda\cdot M}{r}$$

Additional boundary conditions are required for finding $Y(Q)$ and the threshold function $B^{*}(Q_t)$. The first one is the following *Value Matching Condition*:

(17) $$V\left[Q,B^{*}(Q)\right]=0.$$

The second one is the following *Smooth Pasting Condition*:

(18) $$V_B\left[Q,B^{*}(Q)\right]=0.$$

Applying (16) in (17) and (18) yields:

(19) $$B^{*}(Q)=\overline{\beta}\cdot(r-\mu)\cdot\lambda\cdot\frac{M}{r}\cdot Q,$$

where $\overline{\beta}\equiv\frac{\beta}{\beta-1}$. Note that $\overline{\beta}>1$ since $\beta > 1$.

3.2 Welfare in the absence of a cap

Following the same procedure as that conducted for the value of a unit of X, yields that given the current levels of *B* and *Q* the value of social welfare satisfies:

(20) $$W(Q, B) = C(Q) \cdot B^{\beta} + \frac{B \cdot \ln(Q)}{r - \mu} - \frac{M \cdot Q}{r},$$

where $C(Q)$ is to be determined by boundary conditions. The first such condition is the following *Value Matching Condition* at times of hitting the investment threshold:

(21) $$W_Q\left[Q, B^{*}(Q)\right] = 0$$

Applying (19) in (20), partially differentiating with respect to *Q*, applying (19), and rearranging terms, yields:

(22) $$C'(Q) = \frac{M}{r} \cdot \frac{1 - \overline{\beta} \cdot \lambda}{B^{*}(Q)^{\beta}}.$$

Applying (17), integrating and simplifying, yields:

(23) $$C(Q) = \frac{M}{r} \cdot \frac{\overline{\beta} \cdot \lambda - 1}{B^{*}(Q)^{\beta}} \cdot \frac{Q}{\beta - 1} + G,$$

To find the value of the integration constant, *G*, note from (19) that when $Q \to \infty$ the threshold $B^{*}(Q)$ goes to infinity too, and the probability of *B* hitting it goes to 0. In that case no further changes in Q are expected, and therefore the value of the possibility of such changes is zero. Formally put:

(24) $$\lim_{Q\to\infty} C(Q) = 0 .$$

Since $\beta > 1$, condition (23) implies that $G = 0$, and therefore:

(25) $$C(Q) = \frac{M}{r} \cdot \frac{\overline{\beta}\cdot\lambda - 1}{B^{*}(Q)^{\beta}} \cdot \frac{Q}{\beta - 1} .$$

Note from (23) that if, and only if, λ is sufficiently small, specifically – below $\frac{1}{\overline{\beta}}$, then $C(Q) < 0$. This implies that if, and only if, the market imperfection is sufficiently strong then the value of the possibility of further investments in Q is negative.

To have a better insight into the role that $\overline{\beta}\cdot\lambda - 1$ plays, it is convenient to look at the case of $\mu = 0$, i.e., the case where the dynamics in B are purely stochastic, as there is no deterministic drift. In that case the optimal investment rule of investing when $B \geq B^{*}(Q)$ which can be presented, by applying (5) and (19) that $P \geq \overline{\beta}\cdot\lambda\cdot M$. Thus, if $\overline{\beta}\cdot\lambda - 1 > 0$, then investment takes place when $P \geq M$ which means that the marginal utility gained from Q exceeds the total social cost M and therefore increase welfare. It can be concluded then that the market imperfection lowers the price that triggers producing additional unit below the total social cost, but investors' reaction to uncertainty raises it back above the total social cost.[1]

3.3 Optimal investment policy with a cap on *Q*

The possibility that $C(Q)$ may be negative could lead policy makers to limit future investments with a cap on the level of *Q*. We denote the size of the cap by $\overline{Q}$. The analysis in this case follows Bartolini (1993). Similar to the analysis conducted in sub-section 3.1 above for the case with no cap on *Q*, the analysis in this case too starts with the definition of $V(Q,M)$ as the value of owning a unit of *Q* and continues through equation (10) to (16). Then, to find $Y(Q)$ and the threshold function $B^*(Q)$, Bartolini too uses the *Value Matching Condition* (17). From here on the analysis for the case of a cap departs from that conducted in sub-section 3.1 as the other boundary condition that Bartolini uses is:

(26) $$V_Q\left[Q, B^*(Q)\right] = 0$$

Bartolini (1996) proves the existence of condition (26) in *Proposition 1* of his article. As he shows there, the condition springs from:

(27) $$V\left[Q, B^*(Q+\Delta Q)\right] = V\left[Q+\Delta Q, B^*(Q+\Delta Q)\right].$$

[1]For a similar explanation for the case where $\mu \neq 0$ it is helpful to use the manner by which Kongstead (1996) separates form one another the effects that the drift component and the uncertainty component have on the optimal investment thresholds.

The condition shows that when the quantity is Q and B passes its corresponding threshold level, then, by definition of B^* as a threshold level, Q is increased by another increment with probability 1. This probability, together with the no-arbitrage condition, equates the value function between the two states. Dividing both sides by ΔQ and taking the limit $\Delta Q \to 0$ leads to (26). Note that (17) and (26) are not optimality conditions and should hold for any $B^*(Q)$, not necessarily the optimal one, as they merely reflect the no-arbitrage condition on the value of the firm, given a certain threshold. This means that (17) holds for all levels of Q, which implies that the derivatives with respect to Q of both its sides should equal one another, i.e.:

(28) $$\frac{dV\left[Q, B^*(Q)\right]}{dQ} = 0 .$$ [2]

Expanding (28) and applying (26) in it yield the condition:

(29) $$V_B\left[Q, B^*(Q)\right] \cdot \frac{dB^*(Q)}{dQ} = 0$$

For (29) to hold it requires either $V_B\left[Q, B^*(Q)\right] = 0$ or $\frac{dB^*(Q)}{dQ} = 0$. In the former case – the *Smooth Pasting Condition* (18) holds, and the threshold $B^*(Q)$ is given by (19), as in the case of no cap.

[2]The derivation of (26) and (29) was fully carried out here, instead of merely referring to the similar equations, (9) and (12), in Bartolini (1993). The reason for that is that Bartolini's proof for (26) lacks mentioning that it follows also from a no-arbitrage assumption, and his proof for (29) lacks explaining that it follows from differentiation of <u>both sides</u> of the *Value Matching Condition*.

In the latter case, the *Smooth Pasting Condition* (18) and the resulting (19) do not hold. To further understand this case, recall that $Y(Q)$ represents how the changes in Q over time are expected to affect the value of an increment of X. At the upper limit of Q no such changes can happen and the value of this possibility is 0. Thus:

(30) $$Y(\overline{Q}) = 0 .$$

Applying (30) and (16) and in (17) yields:

(31) $$B^*(\overline{Q}) = (r - \mu) \cdot \lambda \cdot \frac{M}{r} \cdot \overleftrightarrow{Q}$$

Thus, Smooth pasting does not hold in $\overline{Q}$ and, by continuity, also not within a sufficiently close vicinity of $\overline{Q}$. This vicinity is $\left[\tilde{Q}, \overline{Q}\right]$ where $\tilde{Q}$ satisfies:

(32) $$B^*(\tilde{Q}) = B^*(\overline{Q}),$$

due to $\frac{dB^*(Q)}{dQ} = 0$. Applying (19) and (31) in (32) yields:

(33) $$\tilde{Q} = \frac{1}{\overline{\beta}} \cdot \overline{Q} .$$

To summarize the resulting investment dynamics:

- As long as $Q < \tilde{Q}$, when B hits the threshold $B^*(Q)$ investment occurs. The rising Q makes $B^*(Q)$ rise too, so that B is once again below its threshold and investment stops, until the next time B hits the threshold. In *Figure 1* below this is described by the move from point E to point F.

- If $Q = \tilde{Q}$, then when B hits the threshold $B^*(Q)$ investment occurs, but in this case the threshold is not increased by the rising Q as $\frac{dB^*(Q)}{dQ} = 0$. Thus, B is still at the threshold and investment continues and Q immediately hits its cap. In *Figure 1* below this is described by the move from point G to point H.

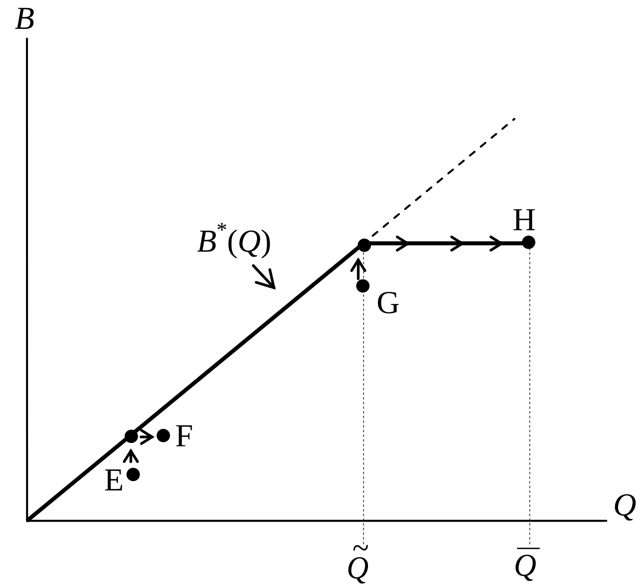

Figure 1: Investment dynamics.

3.4 Welfare with a cap on *Q*

Much of the analysis of welfare in the case of no cap is relevant with a cap too. In particular, welfare is still given by (20), the boundary condition (21) still holds, and so does expression (23) for $C(Q)$. Yet, the introduction of the cap changes the welfare after the run starts. When this happens, everything is immediately transformed and Q hits $\overline{Q}$, so that:

(34) $$W\left[\tilde{Q}, B^*\left(\tilde{Q}\right)\right] = \frac{B^*\left(\overline{Q}\right)\cdot \ln\left(\overline{Q}\right)}{r-\mu} - \frac{M\cdot \overline{Q}}{r} .$$

Evaluating (20) at $\left[\tilde{Q}, B^*\left(\tilde{Q}\right)\right]$, equating it to (34), applying (32) and (33) and rearranging terms, yields:

(35) $$C\left(\tilde{Q}\right) = \frac{M}{r}\cdot \frac{\ln\left(\overline{\beta}\right)\cdot \lambda - \frac{1}{\beta}}{B^*\left(\overline{Q}\right)^{\beta}}\cdot \overline{Q} .$$

Evaluating (23) at $\tilde{Q}$ and equating it to (35), yields that in the case of a cap:

(36) $$G = \left[\beta\cdot \ln\left(\overline{\beta}\right) - \overline{\beta}\right]\cdot \frac{M}{r}\cdot \frac{\lambda}{B^*\left(\overline{Q}\right)^{\beta}}\cdot \frac{\overline{Q}}{\beta} .$$

Applying (36) in (23) yields that $C(Q)$ is given by:

(37) $$C(Q) = \frac{M}{r}\cdot \frac{\overline{\beta}\cdot \lambda - 1}{B^*(Q)^{\beta}}\cdot \frac{Q}{\beta - 1} - \left[\overline{\beta} - \beta\cdot \ln\left(\overline{\beta}\right)\right]\cdot \frac{M}{r}\cdot \frac{\lambda}{B^*\left(\overline{Q}\right)^{\beta}}\cdot \frac{\overline{Q}}{\beta}$$

By a standard algebraic analysis, the function $\overline{\beta} - \beta \cdot \ln\left(\overline{\beta}\right)$ is strictly positive within its definition range, $\beta > 1$. This, together with (19) and (20) leads to the conclusion that welfare is strictly increasing in $\overline{Q}$. Pushing the cap as high as possible is optimal therefore, but actually implies – giving up on the cap.

4. Conclusion

This study has looked at the case of a market with adverse externalities to production and a cap on the quantity in that market. The analysis has shown that while the cap indeed lowers the long term adverse effects by lowering the long term quantity in the market – it also speeds up investment in that market and therefore hastens the appearance of these adverse effects. In particular it was shown that due to the cap, gradual incremental investment is replaced by a process which is gradual only up to a point where a run on the remaining amount still allowed for investment takes place. It was found that the speeding up effect dominates the long-term lower quantity effect, and that therefore the cap harms welfare.

Typically, models of investment under uncertainty do not include a modeling of welfare, and the modeling of welfare in the current article, with the technique used to calculate it, are another novelty of this article.

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