Rivalry with Market, Operating Efficiency and Technological Uncertainty in Technology Adoptions

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Abstract

We derive a multi-factor duopoly real option game model which optimizes the timing of new technology adoptions when there is uncertainty about market revenue, technological progress and operating efficiency after adoption. This last feature of our model has not yet been addressed in the real options literature. We find analytical solutions for the firms' value functions and analytical or numerical solutions for the investment thresholds of various alternative scenarios. We conclude that positive changes in the probability of technological progress sharply reduces the follower's sensitivity to changes in the leader's first mover advantage and, somewhat surprisingly, that for moderately low probability of a new technology arrival, market and operating efficiency uncertainty are no longer relevant factors determining the investment behaviour of the rival firm.

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1. Introduction

When should a leader adopt a new uncertain technology, if there is market and operating efficiency uncertainty? What is the real option value for a leader/follower with an investment opportunity to invest in alternative new technologies, when both technologies are available, or when only the slightly less efficient technology is currently available?

Firms face various types of uncertainties while evaluating new technology investments. The most common is market revenue uncertainty which is related to the unpredictability of the market demand and/or price changes, but often there is also operating efficiency (technical) and technological uncertainty which represent, respectively, the uncertainties related to the performance of the technology after the adoption and the arrival of new and more efficient technologies which makes the adopted technology (at least partially) obsolete. In this paper, we consider the simultaneous effect of market, technical and technological uncertainty on the timing of investments, for a leader-follower duopoly market. We provide analytical or quasi-analytical solutions for the optimal investment thresholds of the two firms, for various alternative investment scenarios, considering that technological uncertainty holds or is absent.

Suppose that two firms are considering the construction of a new production facility which can either be based on a technology that is in the market (tech 1) or a new technology that may arrive at a not yet know date (tech 2). Firms face symmetric market demand, and price and technological uncertainty, but asymmetric technical uncertainty due to the learning effect. More specifically, tech 2 is more advanced than tech 1 and so more efficient, with the efficiency after adoption (EAA) of the two technologies increasing at a constant rate over time due to learning effect. Therefore, for any given time t, with $t \in [0, \infty)$, the EAA of the tech 1 or tech 2, if adopted, is higher for the leader than is for the follower, resulting in a first-mover market share advantage. It is also assumed that firms play a "one-shot" investment game - i.e., they adopt either tech 1 or tech 2, depending on which of these strategies is the best.

An industry we have studied, natural gas fracking in the U.S., has some of the characteristics we seek to model, with substantial market volatility (both in prices and quantity produced), technical efficiency as frackers disclose periodic shifts in the quantity of gas developed per investment dollars, and also the periodic reduction in operating costs over time, as drillers become more experienced. Typically, each of the eight substantial independent frackers discloses regularly thousands of new development well investment opportunities (over one billion dollars), and the resulting quantity of reserves developed, and average operating costs for those completed investments. Each independent fracker offers guidance for develop opportunities and expectations over the next year, implying expected investment cost, expected future net cash flows, and operating efficiency levels. Finally, dramatic new technologies have transformed this industry into one of the U.S. energy success stories, with greater horizontal drilling, deeper wells, and greater volume developed per investment costs.[3]

McDonald and Siegel (1986), and Sick (1989) consider value and investment cost uncertainties for monopolies, and with Smets (1993) for duopolies, followed by Dixit and Pindyck (1994), Huisman (2001), Weeds (2002), Pawlina and Kort (2006) and Moretto (2008), among others. For monopoly markets, Murto (2007) studies the effect of revenues and technological uncertainty on the adoption of a new technology, where the arrival date of more advanced technologies is governed by a Poisson distribution. Grenadier and Weiss (1997) study the optimization of sequential investment opportunities due to technology innovations, where the value and the arrival of the innovations follow geometric Brownian motion (gBm) processes. Smith (2005) studies the simultaneous effect of revenue and investment cost uncertainty in the adoption of two complementary technologies, relating the cost uncertainty with technological progress.

[3] We will be pleased to supply our comparisons of eight independent frackers over the past few years.

For duopoly markets, Huisman (2001, ch. 9) studies the combined effect of revenue and technological uncertainty and shows that the timing optimization of investments is affected by the probability that a second technology becomes available. Paxson and Pinto (2005) study the effect of price and quantity uncertainty on firms' investment behaviour using similarity arguments. Armada et al. (2013) also consider price and quantity uncertainty without relying on similarity arguments. Azevedo and Paxson (2018) extend Smith (2005) model by considering duopoly rivalry. For a literature review on real option game models see Azevedo and Paxson (2014).

Technical uncertainty can also be related to the unpredictability of the investment cost, due to the complexity, size or other physical difficulties to concluding a project, following the Pindyck (1993) framework. It is usually modelled either as a unique source of uncertainty or in combination with market uncertainty.

We study the simultaneous effect on the investment timing of market, technical and technological uncertainty, considering duopoly competition. We treat the investment problem as a "one-shot" game, therefore, each firm invests only once, either in tech 1 or in tech 2. At the beginning of the investment game there is one technology available (tech 1) and the probability that a more advanced technology (tech 2) arrives in the next instant. Ex-ante, firms holds the option to adopt tech 1 and the option to adopt tech 2, but the latter option can only be exercised if tech 2 arrives.

We assume that market revenue and EAA follow geometric Brownian motion (gBm) processes, and technological uncertainty a Poisson process. Without further model constrains, it is not possible to know ex-ante which option exercise sequence firms will follow, but we identify the relevant "what-if" scenarios of this investment game and derive, for each firm, the respective value functions and investment thresholds, using the standard real options-backward induction framework. Our analytical derivations are organized in two main sections:

- First, we study the case where tech 2 is available (i.e., technological uncertainty is absent) and assume, in one case (*scenario 1*), that when tech 2 arrives both firms are idle. Another case (*scenario 2*) is considered for when tech 2 arrives and the leader is active with tech 1 and the follower is idle.
- Second, we study the case where tech 2 is not yet available (market, technical and technological uncertainties hold simultaneously) and characterize the scenario where at a given time *t* the leader is active with tech 1 and the follower is idle optimizing, in one case (*scenario 3*), the adoption of tech 2 and, in another (*scenario 4*), the adoption of tech 1.

Due to the high number of market variables and investment scenarios, to avoid unnecessary complexity, in section 4 we focus our analysis on the most relevant results only. However, other alternative and relevant analysis can also be provided. When we consider the joint effect of market, technical and technological uncertainty we find that, somewhat surprisingly, for the follower, a relatively low probability that a second technology arrives in the next instant means that market and technical uncertainty are no longer important factors determining its investment behaviour. Regarding the leader, we find that its investment behaviour is driven by a more balanced combination of the size of the first-mover advantage, and market, technical and technological uncertainty.

The paper is organized as follows. Section 2 introduces the duopoly investment game, describes the assumptions underlying the model for scenarios 1 and 2 and derives the firms' value functions and investment thresholds. Section 3 develops similar models for scenarios 3 and 4. In section 4 we provide some sensitivity analysis and comment on the most relevant results. In section 5 we conclude and give some suggestions for further research.

2. The Models: Scenarios 1 and 2

Suppose that there are two idle firms, i and j, considering the adoption of a new technology, tech 1 or tech 2, in a context where there is uncertainty about both market revenue, $x(t)$, and efficiency of the technologies after adoption, $E_k(t)$ with $k = \{1,2\}$, where "1" and "2" mean tech 1 and tech 2, respectively, and $E_k(t) \in [0, \infty)$, where the lower limit represents a *catastrophic scenario* (after adoption the technology operates with zero efficiency), and the upper limit represents a perfect scenario (after adoption the technology operates without inefficiencies)[4]. Tech 1 is available and tech 2 is not but it may arrive in the market at any moment in the near future. The arrival of tech 2 (at time τ) is governed by a Poisson process with intensity λ (there is a probability $\lambda \mathrm{dt} > 0$ that tech 2 arrives in the next interval dt). The firm that adopts first becomes the leader and gets a first-mover market share advantage (FMA). Finally, tech 2 is more expensive than tech 1 ($I_2 = \alpha I_1$, with $\alpha > 1$) and more efficient ($E_2(t) = \gamma E_1(t)$ with $\gamma > 1$) where I_1 and I_2 are the investment costs in tech 1 and tech 2, respectively.

Furthermore, market revenue and EAA follow geometric Brownian motion (gBm) processes, given by Equations (1) and (2): [5]

$$dX = \mu_X X dt + \sigma_X X dz \tag{1}$$

$$dE_k = \mu_{E_k} E_K dt + \sigma_{E_k} E_k dz_k \tag{2}$$

where, μ_X and μ_{E_k} are the instantaneous conditional expected percentage changes in X and E_k per unit of time, respectively; σ_X and $\sigma_{E_k}$are the instantaneous conditional standard deviation of X and E_k per unit of time, respectively; and dz and dz_k are the increment of a standard Wiener process for X and E_k, respectively, with k defined as previously. For convergence of the solution $r - \mu_X - \mu_{E_k} > 0$, where *r* is the riskless interest rate. For simplicity of notation we use $\delta_k = r - \mu_X - \mu_{E_k}$.

[4] Using our illustrative example of the introduction section, a catastrophic scenario represents the case of no economic natural gas development, which is rare but not unprecedented. The practical E_k limit is high (if not infinite) along the range of output quantity produced per I_k, and $X.E_k$ which we have studied. While not precisely geometric Brownian motion, an initial E_k well below 1, allows for substantial upside potential.

[5] For convenience of notation, henceforth, we drop the "t".

Firm i's revenue flow if it operates with tech k is given by:

$$X.E_k.D_{k_i k_j} \tag{3}$$

where $X.E_k$ is the efficiency weighted revenue (EWR) of firm i if it is active with tech k, and $D_{k_i k_j}$ is a deterministic competition factor which represents the percentage of the market revenue of firm i for a given investment scenario, with $i, j = \{L, F\}$, where L means "leader" and F "follower".[6]

The following conditions on the above parameter $D_{k_i k_j}$ hold:

$$D_{2_L 0_F} = D_{1_L 0_F} > D_{1_L 1_F} = D_{2_L 2_F} > D_{1_L 2_F} \tag{4}$$

where $D_{2_L 0_F} = D_{1_L 0_F} = 1.0$ and $D_{1_L 2_F} < 0.5$, which ensures that: i) the leader gets 100 percent of the market share if it is active alone; ii) if the two firms are active with the same technology the leader gets a market share advantage, because of the learning effect; iii) the follower's market share is higher than that of the leader if the follower operates with tech 2 and the leader operates with tech 1, because tech 2 is more efficiency; iii) $D_{k_L k_F} + D_{k_F k_L} = 1.0$, because the sum of the market shares of the two firms when both are active is equal to 100 percent of the market revenue.

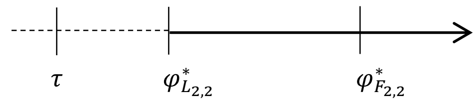
2.1 *Tech* 2 is Available

2.1.1 Both Firms are Inactive

We start our analysis by the scenario where tech 1 and tech 2 are both available in the market. As tech 2 is more efficient than tech 1, it is never optimal to adopt tech 1. Therefore, both firms adopt tech 2 at their optimal times. Figures 1 illustrates firm's investment thresholds for this scenario:

[6] As an illustration about how our notation works: $D_{2_L 0_F}$ represents the leader's percentage of the market revenue if it operates with tech 2 and the follower is inactive, and $D_{1_L 2_F}$represents the leader's percentage of the market revenue if it operates with tech 1 and the follower operates with tech 2. Similarly, $D_{2_F 2_L}$represents the follower's percentage of the market revenue if both firms operate with tech 2.

Figure 1: Thresholds line to adopt in tech 2.



where τ is the arrival time of tech 2, and $\varphi^*_{L_{2,2}}$ and $\varphi^*_{F_{2,2}}$ are the leader and the follower thresholds to adopt *tech* 2, respectively.

2.1.1.1 Follower

Let $F_{2,2}(X, E_2)$ be the follower's option value to adopt tech 2 if there is no technological uncertainty $(\lambda = 0)$ and the leader is active with tech 2. Setting the returns on this option equal to the expected capital gain on the option and using Ito's lemma, we obtain the partial differential equation (PDE) (5), which represents the follower's value function for the region where it is inactive.

$$\frac{1}{2}\frac{\partial^2 F_{2,2}}{\partial X^2}\sigma_X^2 X^2 + \frac{1}{2}\frac{\partial^2 F_{2,2}}{\partial E_2^2}\sigma_{E_2}^2 {E_2}^2 + \frac{\partial^2 F_{2,2}}{\partial X \partial E_2} X E_2 \sigma_X \sigma_{E_2} \rho_{XE_2} + \frac{\partial F_{2,2}}{\partial X}\mu_X X + \frac{\partial F_{2,2}}{\partial E_2}\mu_{E_2} E_2 - rF_{2,2} = 0 \tag{5}$$

We can reduce the dimensionality of the PDE from two to one using the similarity method, through the following variable change: $\varphi_2 = X.E_2$.[7] Doing the respective substitutions in (5) we get the ordinary differential equation (ODE) (6), which represents the option value as a function of φ_2.

$$\frac{1}{2}{\varphi_2}^2 {\sigma_{m_2}}^2 \frac{\partial^2 F_{2,2}(\varphi_2)}{\partial {\varphi_2}^2} + \varphi_2\left(\sigma_X \sigma_{E_2}\rho_{XE_2} + \mu_X + \mu_{E_2}\right)\frac{\partial F_{2,2}(\varphi_2)}{\partial \varphi_2} - rF_{2,2}(\varphi_2) = 0 \tag{6}$$

where ${\sigma_{m_2}}^2 = {\sigma_X}^2 + {\sigma_{E_2}}^2 + 2\rho_{XE_2}\sigma_X\sigma_{E_2}$.

ODE (6) has an analytical solution whose general form is:

$$F_{2,2}(\varphi) = A_1\varphi_2^{\beta_1} + B_1\varphi_2^{\beta_2} \tag{7}$$

[7] For a detailed discussion on similarity methods see Bluman and Cole (1974).

where A_1 and B_1 are constants to be determined using the value-matching (VM) and smooth-pasting (SP) conditions. β_1 and β_2 are the roots of the characteristic quadratic function of a Euler's type ODE given by:

$$\frac{1}{2}\sigma_{m_2}^2\beta(\beta-1)+\left(\rho_{XE_2}\sigma_X\sigma_{E_2}+\mu_X+\mu_{E_2}\right)\beta-r=0 \tag{8}$$

Solving (8) for β we get two roots, one positive (β_1) and one negative (β_2):

$$\beta_{1(2)}=\frac{0.5\sigma_{m_2}^2-\left(\rho_{XE_2}\sigma_X\sigma_{E_2}+\mu_X+\mu_{E_2}\right)+(-)\sqrt{\left(-0.5\sigma_{m_2}^2+\rho_{XE_2}\sigma_X\sigma_{E_2}+\mu_X+\mu_{E_2}\right)^2+2r\sigma_{m_2}^2}}{\sigma_{m_2}^2} \tag{9}$$

Notice that as if φ_2 approaches zero the option is worthless, therefore, in (7) $B_1=0$. Equations (11) and (12) are the VM and SP conditions.

$$F_{2,2}(\varphi)=0 \tag{10}$$

$$F_{2,2}\left(\varphi_{F_{2,2}}^*\right)=\frac{\varphi_{F_{2,2}}^* D_{2_{F2_L}}}{\delta_2}-I_2 \tag{11}$$

$$\frac{\partial F_{2,2}(\varphi_{F_{2,2}}^*)}{\partial\varphi_2}=\frac{D_{2_{F2_L}}}{\delta_2} \tag{12}$$

Solving together Equations (11) and (12), after some algebraic manipulation, we get the constant A_1 and the threshold to adopt tech 2.

$$A_1=\frac{\varphi_{F_{2,2}}^{*\,(1-\beta_1)}}{\beta_1}\frac{D_{2_{F2_L}}}{\delta_2} \tag{13}$$

$$\varphi_{F_{2,2}}^*=\frac{\beta_1}{\beta_1-1}\frac{\delta_2 I_2}{D_{2_{F2_L}}} \tag{14}$$

The follower's value function is:

$$F_{2,2}(\varphi_2) = \begin{cases} A_1 {\varphi_2}^{\beta_1} & \varphi_2 < \varphi^*_{F_{2,2}} \\ \frac{\varphi_2 D_{2_F 2_L}}{\delta_2} - I_2 & \varphi_2 \geq \varphi^*_{F_{2,2}} \end{cases} \tag{15}$$

In the first row of (15) is the follower's option value to invest in tech 2. In the second row is the follower's payoff from operating in the market with leader, both with tech 2, from the moment $\varphi^*_{F_{2,2}}$ is reached onwards, less the investment cost.

2.1.1.2 Leader

If the follower adopts tech 2 when $\varphi^*_{F_{2,2}}$ is reached the first time, the leader's payoff is given by:

$$E\left[\int_{t=0}^{T} \varphi_2 D_{2_L 0_F} e^{-rt} dt - I_2 + \int_{t=T}^{\infty} \varphi^*_{F_{2,2}} D_{2_L 2_F} e^{-rt} dt\right] \tag{16}$$

The first integral represents the leader's payoff for the period where it is active alone, where $t = 0$ is the moment when the leader adopts tech 2 and $t = T$ the moment when the follower adopts tech 2. The second integral is the leader's payoff for the period where both firms are active with tech 2. The leader's value function is:

$$L_{2,2}(\varphi_2) = \begin{cases} \frac{\varphi_2 D_{2_L 0_F}}{\delta_2} - I_2 + \frac{\varphi_2\left(D_{2_L 2_F} - D_{2_L 0_F}\right)}{\delta_2}\left(\frac{\varphi_2}{\varphi^*_{F_{2,2}}}\right)^{\beta_1} & \varphi_2 < \varphi^*_{F_{2,2}} \\ \frac{\varphi_2 D_{2_L 2_F}}{\delta_2} - I_2 & \varphi_2 \geq \varphi^*_{F_{2,2}} \end{cases} \tag{17}$$

where $\frac{\varphi_2 D_{2_L 0_F}}{\delta_2}$ is the leader's payoff at the moment it adopts tech 2 if it operates alone forever; $\frac{\varphi_2 (D_{2_L 2_F} - D_{2_L 0_F})}{\delta_2}\left(\frac{\varphi_2}{\varphi^*_{F_{2,2}}}\right)^{\beta_1}$ is derived using the continuity condition of $L_{2,2}(\varphi_2)$ at $\varphi^*_{F_{2,2}}$. It is negative given that $\left(D_{2_L 2_F} - D_{2_L 0_F}\right) < 0$ (see inequality 4)[8] and corresponds to the correction factor which

[8] This term equals the leader's loss discounted back from the (random) time at which the follower adopt tech 2. The term $\left({}^{\varphi}/_{\varphi^*_{F_{2,1}}}\right)^{\beta_1}$ is interpreted as a stochastic discount factor equal to the present value of \$1 received when the variable φ hits $\varphi^*_{F_{2,1}}$ (see Pawlina and Kort, 2006, p. 10).

incorporates the fact that in the future if $\varphi^*_{F_{2,2}}$ is reached the follower adopts tech 2 and the leader's profits are reduced. $\frac{\varphi_2 D_{2_L 2_F}}{\delta_2}$ is the leader's payoff if active in the market with the follower from the moment $\varphi^*_{F_{2,2}}$ is reached onwards.

Following the principle of rent equalization of Fudenberg and Tirole (1985), the leader adopts tech 2 at the time when the value functions of the two firms cross first time. Therefore, equalizing Equations (15) and (16), for $\varphi_2 < \varphi^*_{F_{2,2}}$, we get:

$$\frac{\varphi_2 D_{2_L 0_F}}{\delta_2} - I_2 + \frac{\varphi_2 \left(D_{2_L 2_F} - D_{2_L 0_F}\right)}{\delta_2} \left(\frac{\varphi_2}{\varphi^*_{F_{2,2}}}\right)^{\beta_1} - A_1 {\varphi_2}^{\beta_1} = 0 \qquad (18)$$

Replacing in (18) φ_2 by $\varphi^*_{L_{2,2}}$, and using standard numerical methods to solve for $\varphi^*_{L_{2,2}}$, we obtain the leader's threshold to adopt tech 2.

2.1.2 Leader Active and Follower Inactive

2.1.2.1 Follower

Now we derive the value function and threshold to adopt *tech* 2 for the follower if when tech 2 arrived the leader was already active with tech 1. The follower's value function is:

$$F_{2,1}(\varphi_2) = \begin{cases} A_2 {\varphi_2}^{\beta_1} & \varphi_2 < \varphi^*_{F_{2,1}} \\ \frac{\varphi_2 D_{2_F 1_L}}{\delta_2} - I_2 & \varphi_2 \geq \varphi^*_{F_{2,1}} \end{cases} \qquad (19)$$

with β_1 is given by Equations (9) and the constant A_2 is given by:

$$A_2 = \frac{{\varphi^*_{F_{2,1}}}^{(1-\beta_1)}}{\beta_1} \frac{D_{2_F 1_L}}{\delta_2} \qquad (20)$$

The threshold to adopt tech 2 is:

$$\varphi^*_{F_{2,1}} = \frac{\beta_1}{\beta_1 - 1} \frac{\delta_2 I_2}{D_{2_F 1_L}} \qquad (21)$$

2.1.2.2 Leader

The leader's value function is:

$$L_{1,2}(\varphi_1) = \begin{cases} \frac{\varphi_1 D_{1_L 0_F}}{\delta_1} - I_1 + \frac{\varphi_2\left(D_{1_L 2_F} - D_{1_L 0_F}\right)}{\delta_2}\left(\frac{\varphi_2}{\varphi^*_{F_{2,1}}}\right)^{\beta_1} & \varphi_2 < \varphi^*_{F_{2,1}} \\ \frac{\varphi_2 D_{1_L 2_F}}{\delta_1} - I_1 & \varphi_2 \geq \varphi^*_{F_{2,1}} \end{cases} \tag{22}$$

Equalizing (19) and (22), for $\varphi_2 < \varphi^*_{F_{2,1}}$, we obtain:

$$\frac{\varphi_1 D_{1_L 0_F}}{\delta_1} - I_1 + \frac{\varphi_2\left(D_{2_L 2_F} - D_{2_L 0_F}\right)}{\delta_2}\left(\frac{\varphi_2}{\varphi^*_{F_{2,1}}}\right)^{\beta_1} - A_1 {\varphi_2}^{\beta_1} = 0 \tag{23}$$

Replacing in (23) φ_2 by $\varphi^*_{L_{1,2}}$, and following standard numerical methods to solve for $\varphi^*_{L_{1,2}}$, we get the leader's threshold to adopt tech 1 conditioned on the follower adopting tech 2. The economic interpretations of (22) and (23) is similar to those we described for (17) and (18).

3. The Models: Scenarios 3 and 4

3.1 *Tech* 2 is Not Available: leader active and follower inactive

3.1.1 Follower's threshold to adopt tech 2

Notice that, under technological uncertainty the arrival date of tech 2 in the market is not known in advance. Thus, the follower can only adopt tech 2 is it arrives in the market and the threshold to adopt the technology is reached. In this section we assume that the leader is active with tech 1 (so it cannot adopt tech 2). Thus, the follower's value is given by:

$$\frac{1}{2}{\varphi_2}^2{\sigma_{m_2}}^2\frac{\partial^2 F_{2,1}(\varphi_2)}{\partial {\varphi_2}^2} + \varphi_2\left(\sigma_X\sigma_{E_2}\rho_{XE_2} + \mu_X + \mu_{E_2}\right)\frac{\partial F_{2,1}(\varphi_2)}{\partial \varphi_2} - rF_{2,1}(\varphi_2) + \lambda\left[F_{2,1}(\varphi_2) - F_{1,1}(\varphi_1)\right] = 0 \tag{24}$$

Using the two possible expressions for $F_{2,1}(\varphi_2)$ (see Equation 19), we get the following solution:

$$F_{2,1}(\varphi_2)|\varphi_1 > \varphi_{L_1}^* = \begin{cases} A_1{\varphi_2}^{\beta_1} + Y{\varphi_1}^{\beta_3} & \varphi_2 < \widehat{\varphi_{F_{2,1}}^*} \\ W{\varphi_2}^{\beta_4} + \frac{\varphi_2 D_{2_{F}1_L}}{\delta_2}\frac{\lambda}{\delta_2-\lambda} - \frac{\lambda I_2}{r-\lambda} & \varphi_2 \geq \widehat{\varphi_{F_{2,1}}^*} \end{cases} \tag{25}$$

where β_1 and A_1 are given by Equations (9) and (13), respectively. The constants A_2 and W are given by Equations (25) and (26), respectively - derived by solving an equation system with the continuity and differentiability conditions for $F_{2,1}(\varphi_2)|\varphi_1 > \varphi_{L_1}^*$ at $\varphi_2 = \widehat{\varphi_{L_{1,2}}^*}$.

$$Y = \frac{\left(\varphi_{F_{2,1}}^*\right)^{-\beta_3}[r\delta_2\beta_4 + (r-(\mu_X+\mu_{E_2})\beta_1)\lambda\beta_4 - \delta_2(r-\lambda)\beta_1]I_2}{(r-\lambda)(\delta_2+\lambda)(\beta_1-1)(\beta_3-\beta_4)} < 0 \tag{26}$$

$$W = \frac{\left(\varphi_{F_{2,1}}^*\right)^{-\beta_4}[r\delta_2\beta_3 + (r-(\mu_X+\mu_{E_2})\beta_1)\lambda\beta_3 - \delta_2(r-\lambda)\beta_1]I_2}{(r-\lambda)(\delta_2+\lambda)(\beta_1-1)(\beta_3-\beta_4)} > 0 \tag{27}$$

See proof in Appendix.

In the first row of (25), $A_1{\varphi_2}^{\beta_1}$ is the option value to adopt tech 2 and $Y{\varphi_1}^{\beta_3}$ is a (negative) correction factor that reflects the fact that $\varphi_{F_{2,1}}^*$ can be reached with tech 2 not yet available. In the second row, $\frac{\varphi_2 D_{2_{F}1_L}}{\delta_2}\frac{\lambda}{\delta_2-\lambda}$ is the present value of the follower's revenues from operating with tech 2 from the moment $\widehat{\varphi_{F_{2,1}}^*}$ is reached onwards; $\frac{\lambda I_2}{r-\lambda}$ is the present value of the investment cost; and $W{\varphi_2}^{\beta_4}$ is the option value to adopt tech 2, where β_3 and β_4 are, respectively, the positive and the negative roots of the following quadratic equation: $0.5\sigma_{m_2}^2\beta(\beta-1) + \left(\rho_{XE_2}\sigma_X\sigma_{E_2} + \mu_X + \mu_{E_2}\right)\beta - (r-\lambda) = 0$, given by:

$$\beta_{3(4)} = \frac{0.5\sigma_{m_2}^2 - (\rho_{XE_2}\sigma_X\sigma_{E_2} + \mu_X + \mu_{E_2}) + (-)\sqrt{\left(-0.5\sigma_{m_2}^2 + \rho_{XE_2}\sigma_X\sigma_{E_2} + \mu_X + \mu_{E_2}\right)^2 + 2(r+\lambda)\sigma_{m_2}^2}}{\sigma_{m_2}^2} \tag{28}$$

For the follower's investment threshold, we use the following VM and SP conditions, respectively:

$$F_{2,1}\left(\widehat{\varphi_{F_{2,1}}^*}\right) = \frac{\widehat{\varphi_{F_{2,1}}^*}\, D_{2_{F}1_L}}{\delta_2} - I_2 \tag{29}$$

$$\frac{\partial F_{2,1}\left(\widehat{\varphi_{F_{2,1}}^*}\right)}{\partial \widehat{\varphi_{F_{2,1}}^*}} = \frac{D_{2_{F}1_L}}{\delta_2} \tag{30}$$

Using Equations (29)-(30), after some algebraic manipulation, we obtain:

$$\widehat{\varphi_{F_{2,1}}^{*}} = \frac{\beta_3}{\beta_3 - 1}\frac{\delta_2}{D_{2_{F1_L}}} I_2 \tag{31}$$

3.1.2 Follower's threshold to adopt tech 1

In this section, we determine the follower's threshold to adopt tech 1 considering that tech 2 is not available but it is likely to arrive in the near future. Thus, the follower's value function is given by:

$$F_{1,1}(\varphi_k) = \begin{cases} J\varphi_1{}^{\psi_1} + W\varphi_2{}^{\beta_3} + H\varphi_2{}^{\beta_3} & \varphi_2 \in [0, \widehat{\varphi_{F_{2,1}}^{*}}) \\ J\varphi_1{}^{\psi_1} + P\varphi_2{}^{\beta_3} + \frac{\varphi_2 D_{2_{F1_L}}}{\delta_2}\frac{\lambda}{\delta_2 - \lambda} - \frac{\lambda I_2}{r-\lambda} & \varphi_2 \in [\widehat{\varphi_{F_{2,1}}^{*}}, \widehat{\varphi_{F_{1,1}}^{*}}) \\ \frac{\varphi_1 D_{1_{F1_L}}}{\delta_1} - I_1 & \varphi_1 \in [\widehat{\varphi_{F_{1,1}}^{*}}, \infty) \end{cases} \tag{32}$$

In the first row, $J\varphi_1{}^{\psi_1}$ is the option value to adopt tech 1, with ψ_1 and the constant J given by Equations (34) and (35), respectively; $W\varphi_2{}^{\beta_3}$ is the option value to adopt tech 2, with the constant W given by Equation (27); $H\varphi_2{}^{\beta_3}$ is a negative correction parameter which takes into account the fact that $\widehat{\varphi_{F_{2,1}}^{*}}$ can be reached with tech 2 not available, with constant H given by Equation (36). In the second row, $P\varphi_2{}^{\beta_3}$ reflects the fact that $\widehat{\varphi_{F_{2,1}}^{*}}$ was reached, so $P > 0$. The meaning of the other factors is the same as those described in previous sections.

Solving (33) for ψ we get two roots, one positive (ψ_1) and one negative (ψ_2):

$$\frac{1}{2}\sigma_{m_1}^2\psi(\psi - 1) + \left(\rho_{XE_1}\sigma_X\sigma_{E_1} + \mu_X + \mu_{E_1}\right)\psi - r = 0 \tag{33}$$

Solving (8) for ψ we get two roots, one positive (ψ_1) and one negative (ψ_2):

$$\psi_{1(2)} = \frac{0.5\sigma_{m_1}^2 - \left(\rho_{XE_1}\sigma_X\sigma_{E_1} + \mu_X + \mu_{E_1}\right) + (-)\sqrt{\left(-0.5\sigma_{m_1}^2 + \rho_{XE_1}\sigma_X\sigma_{E_1} + \mu_X + \mu_{E_1}\right)^2 + 2r\sigma_{m_1}^2}}{\sigma_{m_1}^2} \tag{34}$$

with $\sigma_{m_1}{}^2 = \sigma_X{}^2 + \sigma_{E_1}{}^2 + 2\rho_{XE_1}\sigma_X\sigma_{E_1}$.

Solving simultaneously the continuity and differentiability conditions for $F_{1,1}(\varphi)$ at $\varphi = \widehat{\varphi^*_{F_{2,1}}}$ (expression 29) and the "value matching" and the "smooth pasting" conditions for $F_{1,1}(\varphi_k)$ at $\varphi_k = \widehat{\varphi^*_{F_{1,1}}}$ (expression 42, second row), we determine the constants H, J and P, and the follower's investment threshold $\widehat{\varphi^*_{F_{1,1}}}$:

$$J = \frac{1-\beta_4}{\beta_3-1} W \left(\widehat{\varphi^*_{F_{1,1}}} \right)^{(\beta_4-\beta_3)} + \frac{r}{(\beta_3-1)(r-\lambda)} I_1 \left(\widehat{\varphi^*_{F_{1,1}}} \right)^{-\beta_3} \tag{35}$$

$$H = J + \frac{\beta_4}{\beta_3} W \left(\widehat{\varphi^*_{F_{2,1}}} \right)^{(\beta_4-\beta_3)} - \frac{D_{2_F 1_L}}{\beta_3(\delta+\lambda)} \left(\widehat{\varphi^*_{F_{2,1}}} \right)^{(1-\beta_3)} \tag{36}$$

$$P = W \tag{37}$$

There is no closed-form solution for the follower's investment threshold, but using the VM condition at $\widehat{\varphi^*_{F_{1,1}}}$ and the information from Expression (33) and Equations (35)-(37) we obtain the Equation (38) from which we determine $\widehat{\varphi^*_{F_{1,1}}}$.

$$(\beta_3 - \beta_4) P (\widehat{\varphi^*_{F_{1,1}}})^{\beta_4} + \frac{(\beta_3-1)\lambda \widehat{\varphi^*_{F_{1,1}}} D_{2_F 1_L}}{(\delta+\lambda)\delta} - \frac{(\beta_3-1)\widehat{\varphi^*_{F_{1,1}}} D_{1_F 1_L}}{\delta} + \frac{r\beta_3 I_2}{r+\lambda} = 0 \tag{38}$$

3.1.3 Leader: threshold to adopt tech 1

The value function is:

$$L_{1,2}(\varphi_1) = \begin{cases} E\varphi_1{}^{\beta_3} + \frac{\varphi_1 D_{1_L 0_F}}{\delta_1} - I_1 + \frac{\varphi_2 \left(D_{1_L 2_F} - D_{1_L 0_F} \right)}{\delta_2} \left(\frac{\varphi_2}{\widehat{\varphi^*_{F_{2,1}}}} \right)^{\beta_3} & \varphi_1 \in [0, \widehat{\varphi^*_{F_{2,1}}}) \\ G\varphi_1{}^{\beta_4} + \frac{\varphi_1 D_{1_L 0_F}}{\delta_1+\lambda} \frac{\varphi_1 D_{1_L 2_F}}{\delta_1} \frac{\lambda}{\delta_1+\lambda} & \varphi_1 \in [\widehat{\varphi^*_{F_{2,1}}}, \infty) \end{cases} \tag{39}$$

where $E\varphi_1{}^{\beta_3}$ and $G\varphi_1{}^{\beta_4}$ are both positive (see proof in Appendix) and correct for the fact that the follower's threshold to adopt tech 2 can be reached before tech 2 is available, which favours the payoff of an active leader. The constants E and G are given by Equations (40) and (41), respectively, derived using the continuity and differentiability conditions for $L_{1,2}(\varphi_1)$ at $\varphi_2 = \widehat{\varphi^*_{F_{2,1}}}$; $\frac{\varphi_1 D_{1_L 0_F}}{\delta_1} - I_1$

is the leader's payoff at the time of the adoption of tech 1 if it operates alone forever; $\frac{\varphi_2\left(D_{1_L2_F}-D_{1_L0_F}\right)}{\delta_2}\left(\frac{\varphi_2}{\widehat{\varphi^*_{F_{2,1}}}}\right)^{\beta_3}$ is derived using the continuity condition of $L_{1,2}(\varphi_1)$ at $\widehat{\varphi^*_{F_{2,1}}}$, it is negative given that $\left(D_{1_L2_F}-D_{1_L0_F}\right)<0$ (see inequality 4), and corresponds to the correction factor that incorporates the fact that in the future if $\widehat{\varphi^*_{F_{2,1}}}$ is reached the follower adopts tech 2 and so the leader's payoff will be reduced. The rest of the terms are defined as in the previous sections - with the necessary natation adjustments for the firm and technology that is being adopted. The constants E and G are given by:

$$E=\frac{\left(\widehat{\varphi^*_{F_{2,1}}}\right)^{1-\beta_3}[\delta(\beta_1-\beta_4)+\lambda(\beta_1-1)]\left(D_{1_L0_F}-D_{1_L2_F}\right)}{(\delta+\lambda)\delta(\beta_3-\beta_4)} \tag{40}$$

$$G=\frac{\left(\widehat{\varphi^*_{F_{2,1}}}\right)^{1-\beta_4}[\delta(\beta_1-\beta_3)+\lambda(\beta_1-1)]\left(D_{1_L0_F}-D_{1_L2_F}\right)}{(\delta+\lambda)\delta(\beta_3-\beta_4)} \tag{41}$$

The threshold to adopt tech 1, $\widehat{\varphi^*_{L_{1,2}}}$, is determined by equalizing Equations (25) to (39) for $\varphi_2<\widehat{\varphi^*_{F_{2,1}}}$, from where we obtain Equation (42):

$$A_1{\varphi_1}^{\beta_3}+Y{\varphi_1}^{\beta_3}-E{\varphi_1}^{\beta_3}-\frac{\varphi_1 D_{1_L0_F}}{\delta_1}+I_1-\frac{\varphi_2\left(D_{1_L2_F}-D_{1_L0_F}\right)}{\delta_2}\left(\frac{\varphi_2}{\widehat{\varphi^*_{F_{2,1}}}}\right)^{\beta_3}=0 \tag{42}$$

Replacing in (42) φ_1 by $\widehat{\varphi^*_{L_{1,2}}}$ and solving in order to $\widehat{\varphi^*_{L_{1,2}}}$ we get the leader's investment threshold.

3.2 Both Firms Inactive

3.2.1 Leader's threshold to adopt tech 1

Assuming that the leader adopts tech 1 before tech 2 arrives and the follower is optimizing the adoption of tech 1 (adopts tech 1 when the respective threshold is reached), for when both firms are active with tech 1 ($\varphi_1 \geq< \widehat{\varphi^*_{F_{1,1}}}$), the leader's payoff is given by:

$$F_{L_{1,1}}(\varphi_1) = \frac{\varphi_1 D_{1_L 1_F}}{\delta} \tag{43}$$

At the moment the leader adopts tech 1 (T_{1_L}) its expected payoff is given by:

$$F_{L_{1,1}}(\varphi_1) = E\left[\int_{t=T_{1_L}}^{T_{1_F}} \varphi_1 D_{1_L 0_F} e^{-rt} dt - I_1 + F_{L_{1,1}}(\varphi_1) e^{-rt} \middle| t \le T_{1_F} + \int_{T_{1_F}}^{\infty} \varphi_1 D_{1_L 1_F} e^{-rt} dt\right] \tag{44}$$

The first integral represents the leader's payoff while alone in the market; the second integral is the leader's payoff for the period where both firms are active with tech 1; $F_{L_{1,2}}(\varphi_1)e^{-rt}\big|t \le T_{1_F}$ is the present value of the leader's payoff given by expression (43), which takes into account the fact that, before the follower adopts tech 1, there is the possibility that tech 2 arrives and the respective threshold reached, which would reduce the value of the leader. Thus, the leader's value function is:

$$L_{1,1}(\varphi) = \begin{cases} L\varphi^{\beta_3} + \frac{\varphi\left(D_{1_L 2_F} - D_{1_L 0_F}\right)}{\delta}\left(\frac{\varphi}{\widehat{\varphi^*_{F_{2,1}}}}\right)^{\beta_1} + \frac{\varphi D_{1_L 0_F}}{\delta} - I_1 & \varphi \in \left[<\widehat{\varphi^*_{L_{1,2}}}, <\widehat{\varphi^*_{F_{2,1}}}\right) \\ M\varphi^{\beta_3} + G\varphi^{\beta_4} + \frac{\varphi D_{1_L 0_F}}{\delta+\lambda} + \frac{\varphi D_{1_L 2_F}}{\delta}\frac{\lambda}{\delta+\lambda} & \varphi \in \left[<\widehat{\varphi^*_{F_{2,1}}}, <\widehat{\varphi^*_{F_{1,1}}}\right) \\ \frac{\varphi D_{1_L 1_F}}{\delta} - I_1 & \varphi \in \left[<\widehat{\varphi^*_{F_{1,1}}}, \infty\right) \end{cases} \tag{45}$$

where, the first row represents the leader's value at the instant it adopts tech 1; $L\varphi^{\beta_3}$ corrects the fact that tech 2 has to arrive for the follower to adopt it, which favours the leader; $\frac{\varphi\left(D_{1_L 2_F} - D_{1_L 0_F}\right)}{\delta}\left(\frac{\varphi}{\widehat{\varphi^*_{F_{2,1}}}}\right)^{\beta_1}$ is negative and represents the fact that if tech 2 arrives and $\widehat{\varphi^*_{F_{2,1}}}$ is reached the follower adopts tech 2, reducing the leader's value; $\frac{\varphi D_{1_L 0_F}}{\delta} - I_1$ is the present value of the leader's payoff when it operates alone with tech 1 forever; in the second row, $M\varphi^{\beta_3}$ values the possibility that φ rises above $\widehat{\varphi^*_{F_{1,1}}}$ before tech 2 arrives. This has both a positive and a negative effect on the leader's value. There is a negative effect, because if the follower adopts tech 1 the leader loses its monopoly, but a positive effect, because if the follower adopts tech 1 it loses the option to adopt tech 2. Hence, the signal for the constant M depends on the market conditions; $G\varphi^{\beta_4}$ and $\frac{\varphi D_{1_L 2_F}}{\delta}\frac{\lambda}{\delta+\lambda}$ have the same meanings as those described for expression (39); $\frac{\varphi D_{1_L1_F}}{\delta}$ is the leader's payoff when both firms operate with tech 1 from $\widehat{\varphi^*_{F_{1,1}}}$ onwards.[9]

The leader's threshold to adopt tech 1, considering that the follower will adopt tech 1, is derived by equalizing the leader's and the follower's value functions, expressions (45) for $\varphi \in \left[\widehat{\varphi^*_{L_{1,2}}}, \widehat{\varphi^*_{F_{2,1}}}\right)$ and (32) for $\varphi_1 \in [0, \widehat{\varphi^*_{F_{2,1}}}$ respectively, from which we obtain:

$$L\varphi^{\beta_3} + \frac{\varphi\left(D_{1_L2_F} - D_{1_L0_F}\right)}{\delta}\left(\frac{\varphi}{\widehat{\varphi^*_{F_{2,1}}}}\right)^{\beta_1} + \frac{\varphi D_{1_L0_F}}{\delta} - I_1 - J\varphi_2^{\beta_3} - W\varphi_2^{\beta_3} - H\varphi_2^{\beta_3} \tag{45}$$

Replacing in (45) φ by $\widehat{\varphi^*_{L_{1,1}}}$, we determine a numerical solution for the leader's investment threshold.

4. Results and Sensitivity Analyses

In this section, we provide a sensitivity analysis which examines the effect on firms' investment thresholds of changes in our model parameters. Table 1 clarifies our scenarios and notation.

Technological Uncertainty?	**(Equation)**	**Notation**	**Scenario**	**Description**
		τ		Arrival time of tech 2
NO Tech 2 is available $t \geq \tau$	**(14)**	$\varphi^*_{F_{2,2}}$	**(1)**	Follower's threshold to adopt tech 2 if the leader is active with tech 2
	(18)	$\varphi^*_{L_{2,2}}$	**(1)**	Leader's threshold to adopt tech 2 if the follower adopts tech 2 in the future
	(21)	$\varphi^*_{F_{2,1}}$	**(2)**	Follower's threshold to adopt tech 2 if the leader is currently active with tech 1
	(23)	$\varphi^*_{L_{1,2}}$	**(2)**	This is the leader's threshold to adopt tech 1 considering that it cannot adopt tech 2 and the follower can
YES Tech 2 is not available $t < \tau$	**(31)**	$\widehat{\varphi^*_{F_{2,1}}}$	**(3)**	Follower's threshold to adopt tech 2 when the leader is active with tech 1 and tech 2 is not yet available
	(39)	$\widehat{\varphi^*_{L_{1,2}}}$	**(3)**	Leader's threshold to adopt tech 1 considering that the follower adopts tech 2 when if it arrives
	(35)	$\widehat{\varphi^*_{F_{1,1}}}$	**(4)**	Follower's threshold to adopt tech 1 when the leader is active with tech 1 and tech 2 is not yet available
	(43)	$\widehat{\varphi^*_{L_{1,1}}}$	**(4)**	Leader's threshold to adopt tech 1 considering that the follower adopts tech 1

Table 1 – this table defines our investment scenarios and clarifies our notation relating it to the investment scenarios. In the previous sections we derived the expression for the investment thresholds above, which guide firms in their investment timing optimization.

[9] See full derivation of expression (45) and the expressions for the constants L and M in the Appendix, section 2.

In our illustrative results we use the following base parameter values:

X	E_1	E_2	φ_1	φ_2	I_1	I_2	σ_X	σ_{E_1}	σ_{E_2}	μ_X	μ_{E_1}	μ_{E_2}	r	λ	ρ_{XE_k}
10	0.7	0.85	70	85	100	100	0.30	0.30	0.30	0.05	0.0	0.0	0.1	0.2	0.0

Table 2 – Model Inputs

$D_{1_L0_F}$	$D_{0_F1_L}$	$D_{1_L1_F}$	$D_{1_F1_L}$	$D_{2_L2_F}$	$D_{2_F2_L}$	$D_{1_L2_F}$	$D_{2_F1_L}$
1.0	0.0	0.60	0.40	0.60	0.40	0.55	0.45

Table 3 - Competition Factors. Notice that we assume that, due to the efficiency asymmetry between the two technologies, the leader's FMA is lower in the scenario where it is active with tech 1 and the follower is active with tech 2 than in the scenario where both firms are active with the same technology, because the follower benefits from operating with a more efficient technology which reduces the leader's FMA.

4.1 Tech 2 is Available

4.1.1 Results: Scenarios 1 and 2

Figures 1 and 2 show the sensitivity of the investment thresholds of the leader and the follower, respectively, to changes in the correlation coefficient between the market revenues and the EAA of technology k, for the investment scenarios 1 and 2.

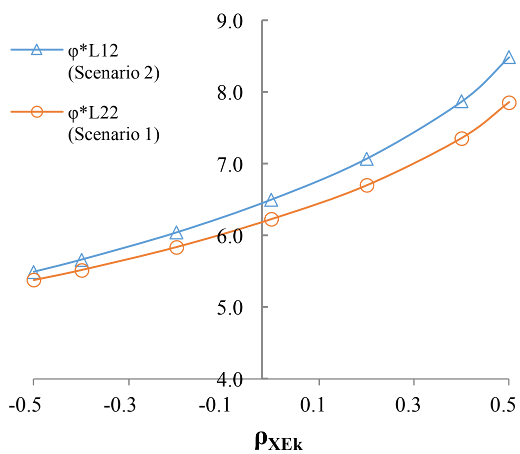


Figure 1 - shows the sensitivity of the leader's threshold to adopt tech k to changes in ρ_{XE_k} if there is no technological uncertainty.

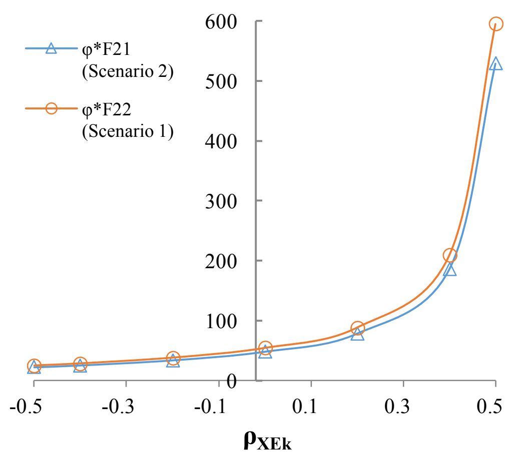


Figure 2 - shows the sensitivity of the follower's threshold to adopt tech 2 to changes in ρ_{XE_k} if there is no technological uncertainty.

For both figures 1 and 2, the line marked with a circle represents the thresholds for scenario 1 whereas the line marked with a triangle represents the thresholds for scenario 2. Thus, we conclude the investment thresholds of the leader and the follower increase with the correlation between the market revenues and the EAA of the technology k (ρ_{XE_k}) – a higher correlation delays the technology k adoption. Also, the investment thresholds of both firms become more sensitive as ρ_{XE_k} increases for both scenarios (1 and 2), particularly for the follower.

Figures 3 and 4 show the effect of changes in the volatility of the EAA of technology k on the investment thresholds of the leader and the follower, respectively. Again, the line marked with a circle represents scenario 1, whereas the line marked with a triangle represents scenario 2.

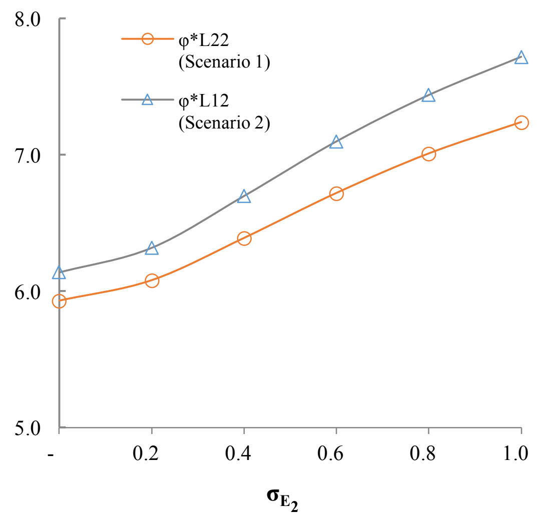


Figure 3 – shows the sensitivity of the leader's threshold to adopt tech 2 to changes in σ_{E_2}, if there is no technological uncertainty.

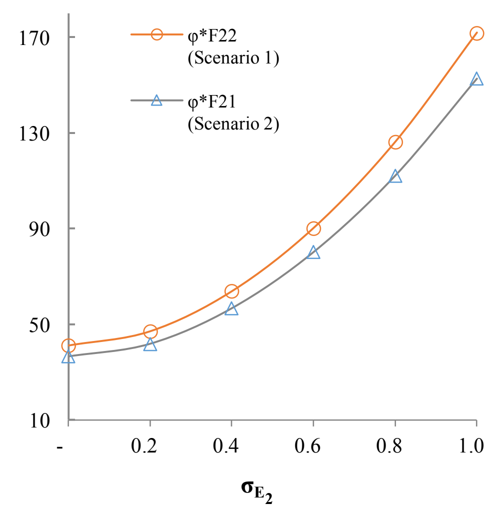


Figure 4 – shows the sensitivity of the follower's threshold to adopt tech k to changes in σ_{E_2}, if there is no technological uncertainty.

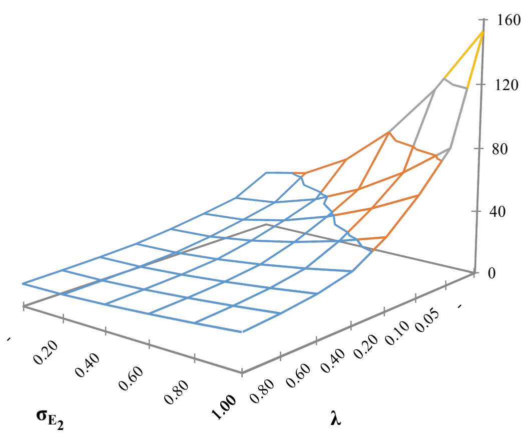
As expected, our results above show that, for the two firms and both scenarios, the investment thresholds increase with the tech 2's EAA volatility - higher EAA uncertainty delays the adoption.

4.2 Tech 2 is not yet available

Now we show our results for when tech 2 is not yet available. There are several investment scenarios available. Below we show our results for those we have analysed in the previous section.

4.2.1 Scenario 3

Figures 5 and 6 show, respectively, the sensitivity of the follower's threshold to adopt tech 2 ($\widehat{\varphi^*_{F_{2,1}}}$) and the leader's threshold to adopt tech 1 ($\widehat{\varphi^*_{L_{1,2}}}$) to changes in λ and σ_{E_2}, in both cases for scenario 3 - where the leader is active with tech 1, and tech 2 is not yet available but there some probability (λ) that it may arrive in the next instant.



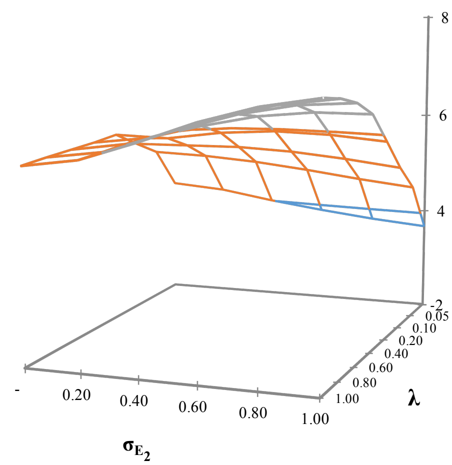


Figure 5: shows the sensitivity of the <u>follower's threshold</u> to adopt tech 2 for scenario 3 ($\widehat{\varphi^*_{F_{2,1}}}$) (where tech 2 is not available and the leader is active with tech 1) to changes in both λ and the volatility of the EAA of tech 2 (σ_{E_2}).

Figure 6 shows the sensitivity of the <u>leader's threshold</u> to adopt tech 1 for the scenario 3 ($\widehat{\varphi^*_{L_{1,2}}}$) (where tech 2 is not available and the follower adopts tech 2 if it arrives) to changes in λ and the volatility of the EAA of tech 2 (σ_{E_2}).

Figure 5 shows that the follower's investment behaviour regarding the adoption of tech 2 is mainly driven by λ - its threshold to adopt tech 2 decreases significantly with λ. Hence, a higher technological uncertainty accelerates the investment. It also shows that a higher EAA uncertainty delays the adoption. Moreover, somewhat surprisingly, we also find that, from low to moderately low λ values, a rise in λ increases significantly the follower commitment to adopting tech 2 and turns much less relevant the effect of the EAA uncertainty on the follower's investment behaviour. Figure 6 shows that a rise in λ also delays the leader's adoption of tech 1, which is in line with what we would expect because the more likely is the arrival of tech 2 the less attractive is the decision to adopt tech 1 because tech 2 is a more efficient technology. The effect of the volatility of the EAA of tech 2 (σ_{E_2}) on the leader's threshold to adopt tech 1 is however complex. For low values of λ (from zero up to around 0.15), a rise in σ_{E_2} hasten slightly the adoption of tech 1 but, as λ increases, a rise in σ_{E_2} delays slightly the adoption. There is therefore a trigger value for λ which once reached, if it decreases, a rise in σ_{E_2} accelerates the adoption of tech 1, whereas, if it increases, a rise in σ_{E_2} delays the adoption of tech 1. It appears that, for some range of technological uncertainty (λ), the more unreliable is the expected EAA of the technology that is not yet available, the less likely is the adoption by the leader of the technology that is currently available.

Figures 7 and 8 show, respectively, the sensitivity of the follower's threshold to adopt tech 2 ($\widehat{\varphi^*_{F_{2,1}}}$) and the leader's threshold to adopt tech 1 ($\widehat{\varphi^*_{L_{1,2}}}$) to changes in λ and the FMA, in both cases for scenario 3 - where the leader is active with tech 1, and tech 2 is not yet available but there some probability (λ) that it may arrive in the next instant.

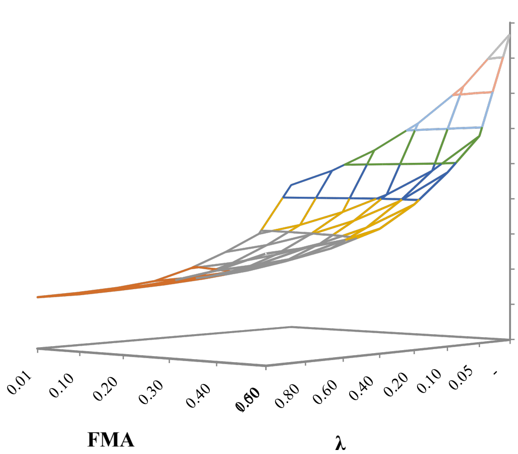


Figure 7 - shows the sensitivity of the follower's threshold to adopt tech 2 to changes in both λ and the first-mover advantage (FMA), for scenario 3 ($\widehat{\varphi^*_{F_{2,1}}}$), where tech 2 is not available and the leader is active with tech 1.

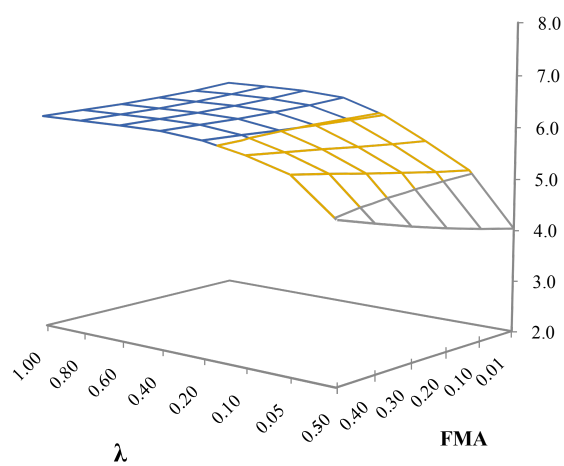


Figure 8 - shows the sensitivity of the leader's threshold to adopt tech 1 to changes in λ and the first-mover advantage (FMA), for the scenario 3 ($\widehat{\varphi^*_{L_{1,2}}}$), where tech 2 is not available and the follower adopts tech 2 if it arrives.

Figure 7 shows that, for the follower, a rise in the leader's FMA turns the adoption of tech 2 by the follower less attractive and that the follower's threshold to adopt tech 2 decreases significantly with λ. Figure 8 shows a rise in the FMA accelerates adoption of tech by the leader and that the leader's threshold to adopt tech 1 is only sensitive to changes in the technological uncertainty for relatively low value of λ. This is a very interesting result because it means that for high or moderately high technological uncertainty, technological uncertainty changes do not affect the leader's commitment to the adoption of the available technology.

4.2.2 Scenario 4

In this scenario, we consider the case where tech 2 is not yet available, the leader is active with tech 1 and the follower is committed to the adoption of tech 1. We provide below the investment thresholds of the leader and the follower for scenario 4 and compare them with those we obtained for scenario 3.

	Efficiency Weighted Revenues: φ_k	$X.E_1 = \varphi_1$	7.0
		$X.E_2 = \varphi_2$	*8.5*
Tech 2 is not available t τ <	*Scenario 3*	$\widehat{\varphi^*_{L_{1,2}}}$	5.98
		$\widehat{\varphi^*_{F_{2,1}}}$	21.58
	Scenario 4	$\widehat{\varphi^*_{L_{1,1}}}$	5.50
		$\widehat{\varphi^*_{F_{1,1}}}$	31.29

Table 4 - this table compares our results for both scenario 3 and scenario 4

Based on the on above results we conclude that, for the leader, the threshold to adopt tech 1 was reached in both scenarios. Thus, it should adopt tech 1 immediately. As soon as the leader adopts tech 1, the follower should monitor both the threshold to adopt tech 2 ($\widehat{\varphi^*_{F_{2,1}}}$) and the threshold to adopt tech 1 ($\widehat{\varphi^*_{F_{1,1}}}$), and adopt tech 1 or tech 2 depending on which of these thresholds is reached first - notice that tech 2 can only be adopted if the threshold is reached and it is available. According to the information above, none of the follower's thresholds were yet reached, so it should wait. Notice that, if tech 2 arrives and both firms are inactive, they should monitor the thresholds derived for scenarios 1 and 2 and adopt tech 2 when their thresholds are reached.

From Table 4 we can also see that the leader's thresholds for scenarios 3 and 4 are very similar, which means that the leader is almost indifferent about what the follower does (adopt tech 1 or tech 2). This is because, the FMA plays a very important role in the leader's investment behavior and has no effect of the follower's. However, the follower's threshold for scenarios 3 and 4 differs significantly, which means that, it is very relevant for the follower which technology the leader adopts. It is more likely that it adopts the more efficient technology (if it arrives), when the leader operates with the less efficient technology.

5. Conclusions

This is the first two-firm multi-option real options game model studying the simultaneous effect of rivalry (through a duopoly game) and market, technical and technological uncertainty. Our results show that the "probability that a second -and more efficient- technology arrives in the next instant" (λ) has a significant effect on the investment behaviour of the leader and the follower.

When we consider the joint effect of market, technical and technological uncertainty we find that, somewhat surprisingly, a relatively low "probability that a second technology arrives in the next instant" (technological uncertainty) reduces significantly the importance of the market and technical uncertainty on the investment behaviour of rival firms. Any positive probability of technological innovation sharply reduces the follower's sensitivity to changes in the leader's FMA and the reliability of the second technology. The follower's investment behaviour is driven mainly by the size of λ and the leader's investment behaviour by a more balanced combination of other model factors.

When there is no technological uncertainty, negative or relatively low positive correlations affects slightly the investment threshold of both firms and high positive correlations affect slightly the investment threshold of the leader and significantly the investment threshold of the follower. The follower is highly sensitive to changes in σ_{E_k}, and the leader is not.

Our real option game setting is based on the assumption that there is a duopoly market with a FMA. It would be interesting to relax this assumption and extend our model to cases where there is a second-mover advantage (attrition game). We use a competition framework where the FMA is based on ex-ante determined competition factors, defined as proportions of the market revenues. Although mathematically challenging, it would be interesting to refine this assumption allowing dynamic market share.

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Appendix

1. Proofs

1.1 Proof #1: $Y<0$

Rewriting equation (39) as (A5),

$$Y=\frac{\left(\varphi_{F_{21}}^{*}\right)^{-\beta_3}\left[r\delta\beta_4+\left(r-(\mu_X+\mu_E)\beta_1\right)\lambda\beta_4-\delta\left(r+\lambda\right)\beta_1\right]I_2}{\left(r+\lambda\right)\left(\delta+\lambda\right)\left(\beta_1-1\right)\left(\beta_3-\beta_4\right)} \tag{A1}$$

We know that $\varphi_{F_{21}}^{*}$, δ, r, λ, β_1, β_3 and I_2 are all positive, and $\beta_4<0$.

Simplifying the numerator: let $v_1=\left(\varphi_{F_{21}}^{*}\right)^{-\beta_3}$, $a=r\delta$, $b=\left(r-(\mu_X+\mu_E)\beta_1\right)\lambda$, $c=\delta\left(r+\lambda\right)\beta_1$. Simplifying the denominator: let $d=\left(r+\lambda\right)\left(\delta+\lambda\right)\left(\beta_1-1\right)$, and $e=\left(\beta_3-\beta_4\right)$. Substituting these terms in (A1) and rewriting yields:

$$Y=\frac{v_1(a\beta_4+b\beta_4-c)I_2}{d(e)} \tag{A2}$$

From the information above we conclude that a, b, c and d (given that $\beta_1>1$) are all positive. From Equation (32) we can see that $\beta_3>0$ and $\beta_4<0$, so $e>0$ and the denominator is positive. The nominator is negative since v_1, a, b, c and I_2 are positive and $\beta_4<0$. Hence, $Y<0$.

1.2 Proof #2: $W>0$

Rewriting equation (40) as (A7),

$$W=\frac{\left(\varphi_{F_{21}}^{*}\right)^{-\beta_4}\left[r\delta\beta_3+\left(r-(\mu_X+\mu_E)\beta_1\right)\lambda\beta_3-\delta\left(r+\lambda\right)\beta_1\right]I_2}{\left(r+\lambda\right)\left(\delta+\lambda\right)\left(\beta_1-1\right)\left(\beta_3-\beta_4\right)} \tag{A3}$$

We know that $\varphi_{F_{21}}^{*}$, $\delta>0$, r, λ, β_1, β_3 and I_2 are all positive and $\beta_4<0$. Simplifying the numerator: let $v_2=\left(\varphi_{F_{21}}^{*}\right)^{-\beta_4}$, $a=r\delta$, $b=\left(r-(\mu_X+\mu_E)\beta_1\right)\lambda$, $c=\delta\left(r+\lambda\right)\beta_1$. Simplifying the denominator: let $d=\left(r+\lambda\right)\left(\delta+\lambda\right)\left(\beta_1-1\right)$ and $e=\left(\beta_3-\beta_4\right)$. Substituting in (A3) and rewriting yields:

$$W=\frac{v_2(a\beta_3+b\beta_3-c)I_2}{d(e)} \tag{A4}$$

From the information above we conclude that a, b, c and d (given that $\beta_1>1$) are all positive. From Equation (32) we can see that $\beta_3>0$ and $\beta_4<0$, so $e>0$. Therefore, both the numerator and the denominator are positive. Hence $W>0$.

1.3 Proof #3: $E>0$

Rewriting equation (39) as (A5),

$$E=\frac{\left(\varphi_{F_{21}}^{*}\right)^{1-\beta_3}\left[\delta\left(\beta_1-\beta_4\right)+\lambda\left(\beta_1-1\right)\right]\left(d_{1_L0_F}-d_{1_L2_F}\right)}{\left(\delta+\lambda\right)\delta\left(\beta_3-\beta_4\right)} \tag{A5}$$

We know that $\varphi_{F_{21}}^{*}$, $\delta>0$, r, λ, β_1 and β_3 are all positive and $\beta_4<0$. Simplifying the numerator: let $v_3=\left(\varphi_{2_F}^{*}\right)^{(1-\beta_3)}$, $a=\delta$, $b=\beta_1-\beta_4$, $c=\lambda\left(\beta_1-1\right)$, $d=\left(d_{1_L0_F}-d_{1_L2_F}\right)$. Simplifying the denominator: let $u=\left(\delta+\lambda\right)$ and $e=\left(\beta_3-\beta_4\right)$. Substituting in (A5) and rewriting yields:

$$E = \frac{v_3\left[a(b)c)\right]d}{u(a)e} \tag{A6}$$

We conclude that v_3, a, b, c and d are positive (for *c* note that $\beta_1 > 1$). From equation (32) we can see that $\beta_3 > 0$ and $\beta_4 < 0$, so $e > 0$). Hence $E > 0$.

1.4 Proof #4: $G > 0$

Rewriting equation (40) as (A7),

$$G = \frac{\left(\varphi^*_{F_{21}}\right)^{1-\beta_4}\left[\delta\left(\beta_1-\beta_3\right)+\lambda\left(\beta_1-1\right)\right]\left(d_{1_L 0_F}-d_{1_L 2_F}\right)}{\left(\delta+\lambda\right)\delta\left(\beta_3-\beta_4\right)} \tag{A7}$$

We know that $\varphi^*_{F_{21}}$, $\delta > 0$, r, λ, β_1 and β_3 are all positive and $\beta_4 < 0$. Simplifying the numerator: let $v_4 = \left(\varphi^*_{F_{21}}\right)^{1-\beta_4}$, $a = \delta$, $b = \beta_1 - \beta_3$, $c = \lambda\left(\beta_1 - 1\right)$, $d = \left(de_{1_L 0_F} - de_{1_L 2_F}\right)$. Simplifying the denominator: let $u = (\delta + \lambda)$ and $e = \left(\beta_3 - \beta_4\right)$. Substituting in (A7) and rewriting yields:

$$G = \frac{v_4\left[a(b)c)\right]d}{u(a)e} \tag{A8}$$

Notice that for $\lambda = 0$, $b = \beta_1 - \beta_3 = 0$ (i.e., Eq. 32 is equal to Eq. 12). Defining the numerator of (A8) with β_3 as a function of λ and taking its second derivative we can see that it is positive. In addition, we know that v_4, a, c and d are all positive (for *c* note that $\beta_1 > 1$). From equation (32) we can see that $\beta_3 > \beta_4$, so $e > 0$. Hence $G > 0$.

2. Derivation - Expression (49)

Let the first integral of Equation (48) be:

$$Z(\varphi) = E\left[\int_{t=T_{1_L}}^{T_{1_F}} \varphi(t)\left(d_{1_L 0_F}\right)e^{-rt}dt\right] \tag{A9}$$

$$rZ(\varphi) = \varphi\left(d_{1_L 0_F}\right) + \lim_{dt\to 0}\frac{1}{dt}E\left[dZ(\varphi)\right] \qquad \text{A10)}$$

Ito's lemma gives:

$$E\left[dZ(\varphi)\right] = (1-\lambda dt)\left(\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2 Z(\varphi)}{\partial\varphi^2}dt + (\sigma_X\sigma_E\rho + \mu_X + \mu_E)\varphi\frac{\partial Z(\varphi)}{\partial\varphi}dt\right) + \lambda dt(0 - Z(\varphi)) \tag{A11}$$

Leading to:

$$\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2 Z(\varphi)}{\partial\varphi^2} + \left(\sigma_X\sigma_E\rho_{XE} + \mu_X + \mu_E\right)\varphi\frac{\partial Z(\varphi)}{\partial\varphi} - \left(r+\lambda\right)Z(\varphi) + \varphi\left(de_{1_L 0_F}\right) = 0 \tag{A12}$$

With solution:

$$Z(\varphi) = C_1\varphi^{\beta_3} + C_2\varphi^{\beta_4} + \frac{\varphi d_{1_L 0_F}}{\delta + \lambda} \tag{A13}$$

$C_2 = 0$ since $\beta_4 < 0$ and as φ increases the value of the leader should increase. Using the absorbing barrier condition $Z(0) = 0$ and the condition that ensures that at the follower's investment threshold the leader's option value is null, i.e., $Z(\varphi^*_{F_{11}}) = 0$ we conclude that and C_1 is given by,

$$C_1 = (\varphi^*_{F_{11}})^{1-\beta_3}\frac{-d_{1_L 0_F}}{\delta+\lambda} \tag{A14}$$

Let the second integral of equation (48) be:

$$W(\varphi) = \left[e^{-rT} \Big|_{T \le T_{1_F}} \right] F_{L_{12}}(\varphi) + \int_{t=\min(t, T_{1_F})}^{\infty} \left[\varphi\left(d_{1_L 1_F}\right) e^{-rt} \Big|_{T > T_{1_F}} \right] dt \quad \text{(A15)}$$

The function $W(\varphi)$ must satisfy the Bellman equation for $\varphi < \varphi^*_{F_{11}}$:

$$rW(\varphi) = (1 - \lambda dt)\left(\frac{1}{2}\sigma_m^2 \varphi^2 \frac{\partial^2 W(\varphi)}{\partial \varphi^2} dt + \left(\sigma_X \sigma_E \rho_{XE} + \mu_X + \mu_E\right)\varphi \frac{\partial W(\varphi)}{\partial \varphi} dt \right) + \lim_{dt \to 0} \frac{1}{dt}\left(\lambda dt (F_{F_{21}}(\varphi) - W(\varphi) \right) \quad \text{(A16)}$$

Leading to:

$$\frac{1}{2}\sigma_m^2 \varphi^2 \frac{\partial^2 Z(\varphi)}{\partial \varphi^2} + \left(\sigma_X \sigma_E \rho + \mu_X + \mu_E\right)\varphi \frac{\partial Z(\varphi)}{\partial \varphi} - (r - \lambda) W(\varphi) + \lambda F_{F_{21}}(\varphi) = 0 \quad \text{(A17)}$$

With solution:

$$W(\varphi) = \begin{cases} B_1 \varphi^{\beta_3} + B_2 \varphi^{\beta_4} + A_{12} \varphi^{\beta_1} + \dfrac{\varphi d_{1_L 0_F}}{\delta} \dfrac{\lambda}{(\delta + \lambda)} & \varphi < \varphi^*_{F_{21}} \\ B_3 \varphi^{\beta_3} + B_4 \varphi^{\beta_4} + \dfrac{\varphi d_{1_L 2_F}}{\delta} \dfrac{\lambda}{(\delta + \lambda)} & \varphi \ge \varphi^*_{F_{21}} \end{cases} \quad \text{(A18)}$$

Using the boundary conditions: $W(0) = 0$ we get the constant $B_2 = 0$. The rest of the constants are determined by solving the continuity and differentiability condition at $\varphi = \varphi^*_{F_{21}}$ and using the boundary condition $W(\varphi^*_{F_{11}}) = \frac{\varphi^*_{F_{11}} d_{1_L 1_F}}{\delta}$, leading to:

$$B_1 = B_3 + E \quad \text{(A19)}$$

$$B_4 = G \quad \text{(A20)}$$

where E and G are given by equations (39) and (40), respectively, and B_3 is given by:

$$B_3 = (\varphi^*_{F_{11}})^{-\beta_3} \left(\frac{\varphi^*_{F_{11}} d_{1_L 1_F}}{\delta} - \frac{\lambda \varphi^*_{F_{11}} d_{1_L 2_F}}{(\delta + \lambda)\delta} \right) - G(\varphi^*_{F_{11}})^{(\beta_4 - \beta_3)} \quad \text{(A21)}$$

Combining equations (47), (A14) and (A18) we get equation (49), rewritten here as (A22)

$$F_{L_{11}}(\varphi) = \begin{cases} L\varphi^{\beta_3} + \dfrac{\varphi(d_{1_L 2_F} - d_{1_L 0_F})}{\delta} \left(\dfrac{\varphi}{\varphi^*_{F_{21}}} \right)^{\beta_1} + \dfrac{\varphi d_{1_L 0_F}}{\delta} - I_1 & \varphi \in \left[\varphi^*_{L_{12}}, \varphi^*_{F_{21}} \right) \\ M\varphi^{\beta_3} + G\varphi^{\beta_4} + \dfrac{\varphi d_{1_L 0_F}}{(\delta + \lambda)} + \dfrac{\varphi d_{1_L 2_F}}{\delta} \dfrac{\lambda}{(\delta + \lambda)} & \varphi \in \left[\varphi^*_{F_{21}}, \varphi^*_{F_{11}} \right) \\ \dfrac{\varphi d_{1_L 1_F}}{\delta} & \varphi \in \left[\varphi^*_{F_{11}}, \infty \right) \end{cases} \quad \text{(A22)}$$

where,

$$L = C_1 + B_3 + E \quad \text{(A23)}$$

$$M = C_1 + B_3 \quad \text{(A24)}$$

With C_1, B_3 and E given by (A14), (A21) and (39), respectively.