Estimation of guarantees in toll road projects: an application with real options analysis

Carlos Andrés Zapata*
Carlos Armando Mejía**

Abstract

This paper develops a proposal for the estimation of a Minimum Revenue Guarantee for a toll road project with private financing in Colombia, supported by Real Options Analysis (ROA). Specifically, we overcome the problem of guarantees valuation in a context where the Government assumes the risk of traffic in order to guarantee a minimum level of revenue and profitability to the investors. Based on this, we can estimate the value of the resources committed by modeling the traffic dynamic over time with an extended Ornstein-Uhlenbeck process. Finally, an alternative solution to finance projects that doesn't attract private investors is found.

Keywords: Real options analysis, stochastic process, Government guarantees.
Clasificación JEL: C14, C63, H54.

1. Introduction

Since the 1990s, Colombia has experienced a significant increase in infrastructure investments, where the private sector has played an important role, specifically in the construction of toll roads. This situation has been strengthened through the

*Research professor, Economy and Numerical Operations Observatory (ODEON), Externado University of Colombia. carlosa.zapata@uexternado.edu.co
***Research professor. Economy and Numerical Operations Observatory (ODEON), Externado University of Colombia carlos.mejia@uexternado.edu.co

implementation of private financing schemes, and more recently with the development of Public-Private Partnerships (PPP's).

PPP's represent alternatives for the financing and development of infrastructure projects. Their main objective is to increase Government efficiency by generating contractual incentives, as well as the allocation and transferences of risks (Gatti, 2008). Therefore, the Government can access to private capital, as well as the knowledge that allows it to build, operate, maintain and manage complex infrastructures (Grimsey and Lewis, 2002).

Under this financing scheme, both the construction and operation, as well as the maintenance of the complete infrastructure is delivered by the Government to a private investor through a concession agreement, for a period long enough to ensure the recovery of the investment and by considering the required return (Doan and Patel, 2010) and the management of the risks involved (Gatti et al., 2007). Based on this, is now the private investor the one that has the responsibility to provide public goods and services. Therefore, adequate management of the risks as well as their allocation are crucial elements and indispensable requirements to ensure its success.

Private financing, as an alternative for long-term financing, has played an important role in the provision of resources for the development of road infrastructure projects. This practice, although is relatively recent in developing countries, was born in the nineteenth century when financing structures with private participation were used in the United Kingdom for the renewal of the road network and the construction of railroads, based on a revenue generation scheme through toll collection (Yescombe, 2002). Since then, this practice has spread to several sectors such as mining, oil and gas, telecommunications, utilities and others.

Project financing has been consolidated as an intense practice and represents a useful tool to help reduce infrastructure gaps under the premise that the private sector has incentives to finance, build and operate this kind of projects, more efficiently than the public sector. According to Gatti (2008), this practice has certain distinctive characteristics compared to traditional corporate financing: project financing is characterized by high disbursements of resources (debt and equity), long repayment periods and multiple participants with different responsibilities and interests.

As mentioned above, the participation in this projects requires a careful analysis of all the risks throughout the project life, and their allocation along the different sponsors, since investors may be exposed to high levels of loss when the risks are not properly transferred and managed. In a road concession project, for example a regional toll road, the estimation of future traffic reflects a major concern, especially on those roads with a low potential for future traffic (Brandão and Cury, 2009), either because they are located in a remote region or because they are in focus on economically underdeveloped regions.

Also, if the Government wants to promote a project with these characteristics, the Government must incorporate contractual guarantees for the private investor, with the aim of motivating its participation. For instance, the Government may guarantee a minimum level of profitability whether the observed demand falls down to a minimum level. Considering all of this, the main problem that the Government faces is the estimation of the budget committed to the project, since its determination is a function of future uncertainty (Brandão and Saraiva, 2007).

In fact, recalling the example of the regional toll road, if the project does not reach an adequate traffic, Government can strongly compromise its budget[1] and affect the provision of other public goods and services. Furthermore, in the toll road concessions the Colombian Government granted high contractual guarantees to 11 concessions throughout its useful life. These committed resources represented a high contingent liability which was difficult to cover with the public budget[2]. Following

[1] Under the assumption that the Government participation depends only on the dynamics of the revenues of the project as the only source of uncertainty.

[2]Unlike other Latin American countries, Colombia did not have specific regulations for the implementation of PPP's projects before 2012. The applicable contracting process was based on a framework composed by: Law 80 of 1993, Law 1150 of 2007, Law 105 of 1993 and Law 185 of 1995. Thus, the framework of application was ambiguous, without clarity and lack of definition in many fundamental points of the contracts. This situation created obstacles in the financial closure, breaches and renegotiations of contracts, delays and cost overruns.

to Benavides (2009), these commitments have caused a deterioration of the Government's resources around the 0.5% of annual GDP.

When it incorporates guarantees, Government can be responsible for important project risks, so it has the need to quantify them adequately with the aim of determining an optimal level of committed resources and to avoid strong fiscal impacts as well as long-term contingent liabilities (Brandão and Saraiva, 2007; (Brandão and Cury, 2009). Additionally, we consider that the importance of valuing guarantees does not only lie in the determination of the Government budget that is compromised, but also in the definition of a level of guarantees that is high enough to make the project economically feasible.

An appropriate valuation requires an approach that incorporates future decisions in response to unexpected events in the project performance. This is where the Real Options Analysis (ROA) becomes a useful tool for making optimal investment decisions and better valuations (Trigeorgis, 1996; Amram y Kulatilaka, 1999). In fact, the different choices represent real options that may be valued when adapting the options pricing models developed by Black and Scholes (1973) and Merton (1973).

An additional advantage identified under this application is that it allows the complex structures of investment projects to be reduced to simple analytical structures made up of different types of simple real options with easier applications (Trigeorgis, 1996). Therefore, this analysis may be accompanied by stochastic calculus along with the Monte Carlo simulation (MCS) and the incorporation of flexibility in decision making.

This paper proposes a model for estimating Government guarantees in a toll road project under the real options approach, and a simple application for a toll road concession is developed. An approach for evaluating the Government guarantees is proposed which represents a feasible alternative for the private investor.

2. Road infrastructure projects financing and the real options approach

2.1 Project financing and risk analysis

From a project financing perspective, the risk is understood as an undesirable event that can affect project performance associated with an occurrence probability - i.e., it is responsible for unexpected changes in the project capacity to repay costs, debt service, and dividends to shareholders (Boussabaine, 2014). Therefore, adequate risk management is a crucial factor, if the risk has not been anticipated and properly hedged, fact that can generate a cash shortfall and reach a default state (Gatti, 2008).

The design of a financing structure for an infrastructure project has one main principle: risks should be assigned to the counterparty that is best able to manage them (Gatti, 2008). The risk allocation defines the obligations of the counterparties in each of the project phases and, thereby, guarantees the quality and quantity of the services contracted, besides the return on investment. The process of managing risks and uncertainties must be comprehensive and efficient, so that, when carried out in real time, allows the management to advance a decision-making process.

Project financing involves designing a complex financial structure that links up multiple stakeholders: a group of sponsors (Government and private investors) and lenders who provide the resources needed to design, build and operate the project. Based on this, an independent Special Purpose Vehicle (SPV) is created, with debt and equity, where cash flows are considered as the primary source to cover the obligations incurred in the financing[3] such as, i) payment of debt service to lenders and, ii) dividends to sponsors.

The project's ability to generate enough cash flows during its operation phase determines the success of the financing, accompanied by the different contracts that

[3]Unlike the traditional (corporate) financing approach where lenders determine credit terms based on the company assets, its leverage and prior experience in other projects; in project financing, the lender evaluates the conditions from the expected cash flows for the payment of debt service, and the project assets only serve as collateral. Project financing, on the other hand, is based on the limited liability of the sponsors - i.e. in case the project fails, the banks are not entitled to recover the losses of any other party.

determine the allocation and transferences of risks and obligations. In addition, the financing agreement may require the incorporation of additional resources, such as guarantees, with the aim of hedging future project obligations or any over cost caused by uncertain events.

2.2 Traditional valuation model

Traditionally, the incorporation of the risks of a project into its valuation process have been included through a higher discount rate of the future expected cash flows (Dixit and Pindyck, 1994), nevertheless, this approach presents a notable limitation, given the own characteristics of an infrastructure project. Therefore, it is necessary to incorporate an adequate treatment of risks and uncertainties into the model which can also represent a useful approach to provide more accurate estimates and improve investments decision making.

In the case of a toll road concession, the future traffic can present strong changes, because of its own random nature, in such a way that the expected levels cannot be present, or even cash flows may be null or non-existent. Additionally, this high risk can produce a traffic level so low that can make the financing initiative non-viable (Brandão and Cury, 2009). Thereby, the concessionaire can execute real options inherent to the project (such as to contract or to expand, or even to receive government support), when the traffic turns out to be different than the expected. The traditional Discounted Cash Flow model (DCF) ignores the value of these flexibilities and uncertainties.

The valuation of an infrastructure project using the traditional model (DCF) should be improved. Furthermore, if the project incorporates government support as contractual guarantees to reduce demand risk, it is necessary to have adequate methodologies to model its dynamics and quantify their impacts on cash flows. Likewise, if the project counts with the participation of private investors in the infrastructure’s construction and operation, this would require a more complex valuation analysis.

Furthermore, the guarantees support has been traditionally quantified over the basis of sensitivity analysis and scenarios or through subjective criteria. However, the determination of the optimal level of guarantees requires a quantitative risk assessment, something that is not possible with the traditional model based on DCF. Brandão and Saraiva (2007) propose an estimation approach based on the ROA. Also, they suggest that guarantees support can be valued as options using the Black-Scholes pricing model or Monte Carlo simulation (MCS).

2.3 Real Options Analysis (ROA): incorporating the operational flexibility and uncertainty

As stated before, the ROA arises as a response to the limitations of the traditional valuation model (DCF) to value projects in projects in a dynamic environment. Since the ROA adapts the options pricing models developed by Black and Scholes (1973) and Merton (1973) is capable of incorporating the value of flexibility and uncertainty into investment decision making with the purpose of valuing real assets and investment projects.

Its origin goes back to the paper of Myers (1977), who finds that the company's value derives not only from its assets but also from its future investment opportunities or corporate growth options. Also, Myers (1977) states that in investment decisions, as well as in financial options, the Black-Scholes valuation model can be directly applied, which allows obtaining estimates that involve the optionality in corporate decisions and offers an alternative to DCF model.

Likewise, Pindyck (1991) suggest that traditional valuation models ignore the firm's flexibilities, and therefore new valuation methods are needed to incorporate them. Pindyck (1991) states that the value of growth options can represent a significant fraction of their market value, which increases with uncertainty in the company's future value. An increase of uncertainty can lead to a high asset value whether managers identify and use their investment options to respond flexibly to contingent states, while in the DCF model assumes the firm follows a fixed scenario.

However, this approach requires the assumption that the real asset or investment project has its own market. Mason and Merton (1985) argue that this assumption is based on the hypothesis that the real asset contributes to the company's market value and therefore may be treated as a financial asset. Thus, the asset value may be used as the underlying asset and modeled from a Geometric Brownian Motion (GBM). This analysis is expanded by Copeland and Antikarov (2001) who, under the *Marketed Asset Disclaimer* (MAD) assumption, suggest that the project value, as a result from the applying of the FCD model, represents an objective estimate of its value, as if the project were traded on the market.

Brennan and Schwartz (1985) and McDonald and Siegel (1986) extend this approach by incorporating stochastic control theory to find optimal exercise policies. In addition, the incorporation of flexibilities in the analysis of investment decisions is studied in more detail by Dixit and Pindyck (1994), Trigeorgis (1996) and Amram and Kulatilaka (1999). They suggest that real assets can be considered as options when the flexibility and uncertainty are incorporated and recognize that management has the ability to influence the trajectory of free cash flows[4] in response to uncertainty and the market interactions.

By the adaptation of the options pricing model, the ROA has found numerous applications in the field of project financing, when it is necessary an appropriate treatment in projects with high uncertainties including the road infrastructure sector. Rose (1998), Bowe and Lee (2004), Brandão and Saraiva (2007), Brandão and Cury (2009), Blank *et al.* (2009), Doan and Patel (2010), Wibowo, Permana and Kochendörfer (2012), Rakić and Rađenović (2014) present different applications about it.

[4]For instance, when you have the possibility to execute an investment or the right to sell or change the implied asset.

3. Valuation model

In infrastructure projects through PPP's scheme, the Government may consider future commitments with the aim to ensure financial feasibility from the private investor. This makes an important difference with the traditional model of toll road concessions since the Government participation can be limited to risk management to facilitate project implementation. The estimation of the optimal level of guarantees represents a real challenge for the Government and generates an important interest in the field of the stochastic modeling.

The concern of the Government is to estimate an adequate level of future resources that will be compromised in its budget if the risks involved are not quantified and analyzed correctly. For example, the road construction in a remote region with little expectation of increased traffic could hardly represent a profitable investment for private investors. The best option to develop the project for the Government is to commit to make payments to the concession when traffic does not reach an adequate level. This is known as Minimum Revenue Guarantee (MRG)[5].

The incorporation of the ROA provides an adequate estimation from contingent guarantees value. This valuation approach requires explicit modeling of project's cash flows, which can be directly simulated by assuming a stochastic process. However, it is possible to isolate the joint effect of the underlying uncertainties (or risk factors) that drives the value of the project's cash flows (Rose, 1998).

Similarly, according to Brandão and Saraiva (2007) some components of the financial model may be modeled from stochastic processes. For instance, in a road concession project, it is possible to simulate traffic and then determine their impact on their cash flows. If the traffic can be described a diffusion process, it can be simulated using MCS. The integration of these methodologies allows the application of ROA.

[5]Although this has been the most used scheme, there are other types of guarantee such as shadow tolls or extension of the concession period.

3.1. Stochastic modeling of traffic and revenues

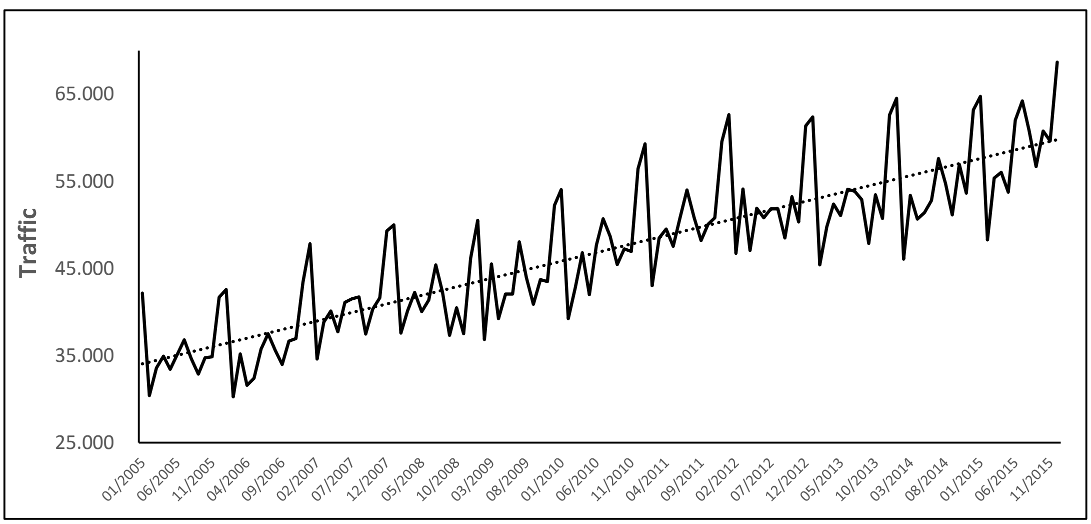
Estimation of future traffic is critical to determine the concession revenue, besides the profitability of the sponsors. Thus, Brandão and Saraiva (2007) consider that, under a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, traffic (T) can be represent as a geometric Brownian motion -GBM - before:

$$\boldsymbol{dT_t = \mu T_t dt + \sigma_t T_t \; dW_t} \quad (\mathbf{3.1})$$

Where, $\boldsymbol{\mu}$ reflects the drift component of the instantaneous changes of $\boldsymbol{T}$, **σ** its diffusion component and $\boldsymbol{dW_t}$ is an increment, during the interval $(t, t + dt)$ of a Wiener process under the Real Probability Measure $\mathbb{P}$, which follows a normal distribution with mean zero and variance t.

Although the GBM has been widely used to model underlying variables, the application of this process has a drawback that is identified by observing certain characteristics of the traffic (see figure 3.1). By observing the historical series of traffic, for the period 2005-2015, we can see a mean-reverting behavior along a tendency, which may imply a limitation when using the GBM. Therefore, a suitable application requires incorporating a stochastic process that meets these characteristics.

Figure 3.1. Historical series of traffic, period 2005-2015



Source: National Infrastructure Agency - NIA, Colombia.

The choice of a mean reversion stochastic process may represent a better adaptation of the behavior that has historically represented the traffic. In this way, the extended Ornstein-Uhlenbeck (OU) is adopted to model future traffic. Under a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, this second model is an extension of Ornstein-Uhlenbeck (OU) process:

$$dT_t = \{\alpha\,(\bar{T}_t - T_t)\}\,dt + \sigma\,dW_t \quad (3.2)$$

Where, $\bar{T}_t$ is the long-term value which is time dependent; α is the speed at which traffic reverts to $\bar{T}_t$, σ is the diffusion term of the process and dW_t is an increment, during the interval $(t, t + dt)$ of a Wiener process under the Real Probability Measure $\mathbb{P}$, which follows a normal distribution with mean zero and variance t. A characteristic of this model is that it assumes σ constant and known in time[6]. The adoption of an extended OU process seeks to ensure the mean reversion property that characterizes the dynamics of this type of variables.

On the other hand, the estimated revenue for a period is obtained as the product of the traffic in t and the established toll rate. Now, if the toll rate is adjusted annually at a constant rate[7] during the life of the concession, then the revenues also follow an extended OU process and have the same parameters of the traffic process. Therefore, traffic and revenues change stochastically over time, being indifferent to use one or another process, so that the traffic guarantee is equivalent to a revenue guarantee.

3.2 Calibration of parameters

The extended OU process where the mean is time-dependent can also be view as a two – part model (Carmona and Ludkovski, 2004):

$$T_t = x_t + y_t \quad (3.3)$$

[6] Although under this model $\mathbf{T_t}$ can take negative values, as we will see later this is in part avoid with the trend component obtained.
[7]Regularly the rate of adjustment is established by the Government in the bidding process of the concession and reflects the expectation of the adjustment of market prices.

Where $\boldsymbol{x_t}$ follows a mean-zero OU process and $\boldsymbol{y_t}$ a deterministic time-dependent process. Based on this, we can calibrate the first part as simple OU process where the long-term mean is zero (Carmona and Ludkovski, 2004), and then construct the second component in order to identify the trend and calibrate with this the long-term mean.

For the $\boldsymbol{x_t}$ component, we first obtained the exact solution of the mean-zero OU process by defining the function $\boldsymbol{\alpha_t e^{\theta t}}$, applying Ito's lemma and integrating between $\boldsymbol{s}$ and $\boldsymbol{t}$, where $\boldsymbol{0 \leq s < t}$:

$$\boldsymbol{x_t = x_s e^{-\alpha(t-s)} + \sigma \int_s^t e^{-\alpha(t-u)}\, dW_u} \quad (\mathbf{3.4})$$

From here we can conclude that α_t follows a normal distribution with expected value and variance conditional to $\boldsymbol{\mathfrak{I}_s}$:

$$\boldsymbol{E(x_t|\mathfrak{I}_s) = x_s e^{-\alpha(t-s)}} \quad (\mathbf{3.5})$$

$$\boldsymbol{Var(x_t|\mathfrak{I}_s) = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha\Delta t})} \quad (\mathbf{3.6})$$

With this, we can discretize this equation over a partition $0 < t_0 < t_1 < \cdots$ with a constant interval $\Delta t = t_i - t_{i-1}$:

$$\boldsymbol{x_t = x_{t-1} e^{-\alpha\Delta t} + \sigma \sqrt{\frac{1}{2\alpha}(1 - e^{-2\alpha\Delta t})} Z_t} \quad (\mathbf{3.7})$$

And we can construct the Log-Likelihood function

$$\boldsymbol{Ln[\gamma(\alpha, \sigma|x_t)] = -\frac{n}{2}Ln(2\pi) - nLn(\hat{\sigma}) - \frac{1}{2\hat{\sigma}^2}\sum_{i=1}^{n}[(x_i - x_{i-1}e^{-\alpha t})^2]} \quad (\mathbf{3.8})$$

Where

$$\boldsymbol{\hat{\sigma}^2 = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha\Delta t})} \quad (\mathbf{3.9})$$

And we proceed to take the partial derivatives and equal to zero. We begin with the partial derivative respect to $\boldsymbol{\alpha}$, we equal to zero and we obtain the following equation:

$$\boldsymbol{\alpha = -\frac{1}{\Delta t} Ln\left[\frac{\sum_{i=1}^{n}(x_i x_{i-1})}{\sum_{i=1}^{n}({x_{i-1}}^2)}\right] \quad (3.10)}$$

And then we take the partial derivative respect to $\boldsymbol{\hat{\sigma}}$ and we obtain the following equation:

$$\boldsymbol{\hat{\sigma} = \frac{1}{n}\sum_{i=1}^{n}({x_i}^2) - 2e^{-\alpha\Delta t}\sum_{i=1}^{n}(x_i x_{i-1}) + e^{-2\alpha\Delta t}\sum_{i=1}^{n}({x_{i-1}}^2) \quad (3.11)}$$

And

$$\boldsymbol{\sigma^2 = \hat{\sigma}^2\left(\frac{2\alpha}{1-2^{-2\alpha\Delta t}}\right) \ (3.12)}$$

To calibrate the second term, we need first to define a time depend function. As we have an OU process around zero, the form of this function is motivated by the fact that we need to identify the trend component of the model. Based on this, we can define y_t in the following way:

$$\boldsymbol{y_t = \bar{T}t \ \ (3.13)}$$

Where $\boldsymbol{\bar{T}}$ is basically the slope of the line around which the values oscillated following the OU process. Based on this, we need to obtain the value for $\boldsymbol{\bar{T}}$ so we can run a regression between each value and a time index (beginning in 1).

3.3 Incorporation of the real option

The incorporation of a guarantee on traffic can constitute an effective mechanism to manage the demand risk for the concessionaire by eliminating the unfavorable states of the distribution of the returns of the project (or downside risk). Thus, Government participation is contingent and it's limited to the high uncertainty: if traffic is below a minimum level, the Government provide resources, otherwise, the Government must not.

The valuation of minimum revenue guarantees is made taking as a reference the Brandão and Saraiva (2007) model. In this way, if the revenue in year t ($\mathbf{R_t}$) is given by:

$$\boldsymbol{R_t = T_t \ x\, Toll\, rate} \quad (\mathbf{3.14})$$

Where, we assumed $Toll\,rate$ constant and $\boldsymbol{T_t}$ is the annual stochastic traffic. It should be noted that the performance of traffic determines the conditioned revenues for the concessionaire, as follows:

i) If during the year t, the traffic is less than minimum traffic ($T_t < \tau$), the Government will guarantee a minimum level of revenue.

ii) If during the year t, the traffic is less or equal than expected ($T_t \geq \tau$), the Government won't contribute with payments to the concessionaire.

Where $\tau = E(T_t) * \alpha$. In this way, the effective revenue (ER_t) of the concessionaire in each year is given by:

$$\boldsymbol{ER_t = max \begin{Bmatrix} R_t, & if\, T_t \geq \tau \\ P_t, & if\, T_t < \tau \end{Bmatrix}} \quad (\mathbf{3.15})$$

Where P_t is the revenue guaranteed by the Government. This allows the establishment of the *pay-off*, which finally determines the revenue for the concessionaire in each year t:

$$\boldsymbol{ER_t = max\,(R_t;\, P_t)} \quad (\mathbf{3.16})$$

Likewise, the guarantee value (G_t) -Government support- in year t is given by:

$$\boldsymbol{G_t = max\,(0;\, P_t - R_t)} \quad (\mathbf{3.17})$$

The Government guarantees can be valued as a series of European put options with maturities of 1 year. Thus, options should be valued, depending on the lifetime of the concession (*n* years). In addition, the options valuation can be done by applying the MCS technique by assuming that the option will be exercised as long as the annual traffic is below the established minimum level.

As the traffic is modeled monthly according to the model described in equation (3.7), the calculation of total traffic for each year *(t)* is done by accumulating the monthly traffic (Jan-Dec), this value is compared with the expected level, according to the agreement between the concessionaire and the Government, which determines the exercise or not (of the option).

By applying the above process for *n* years of the concession and adding up the present value of all options, the total value of the revenue guarantee (equation 3.18) is found.

$$\boldsymbol{Value\ of\ guarantee = \sum_{t=3}^{17} \frac{G_t}{(1+r)^t}} \quad \textbf{(3.18)}$$

Where, $\boldsymbol{r}$ is the social discount rate which is assumed constant and is defined by Colombian Government under the resolution 446/2010.

4. Valuation model applied

4.1. About Concession: Toll road concession project

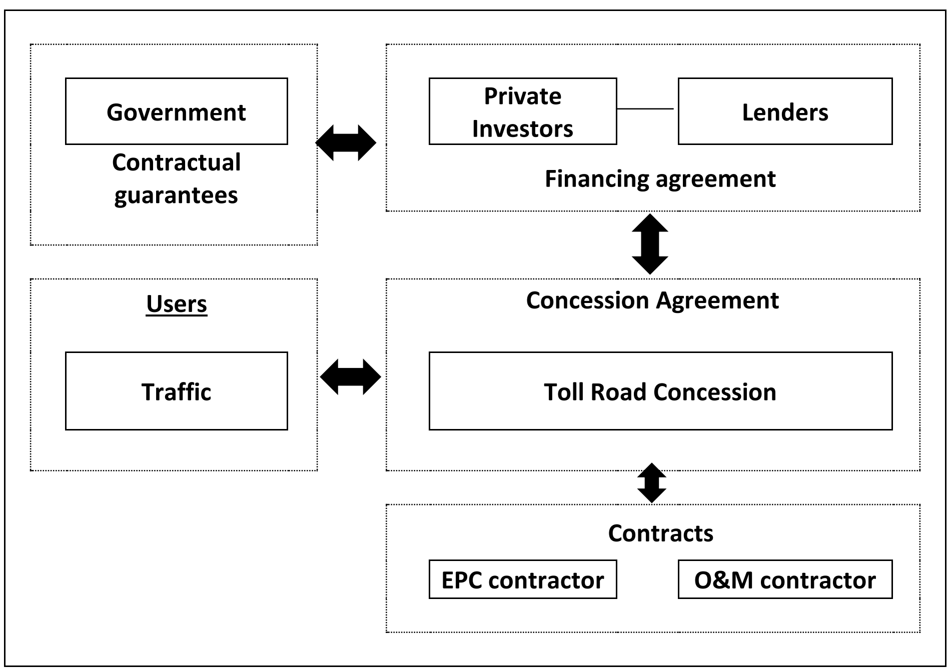
The toll road concession project involves the construction, operation and maintenance of the road of 79 km. The project is considered as one of the most important in the region as it seeks to promote, not only its connection with others regions but it also competitiveness regional. Likewise, it seeks to optimize public resources and risk management, as well as to consolidate best practices in the structuring and contracting processes. The Government's main concern is to make feasible the concession to attract private capital without compromising strongly its budget.

For the toll road project, a concession contract type BOMT (build, operate, maintenance and transfer) is agreed in a total time of 17 years. With the BOMT scheme, private investors commit to financing, design, build, operate and manage infrastructure, and then transfer it (to the end of the project), free of charge, to the host Government after a specified concession period. The first 2 years represent to the design and construction phase, and the next 15 years of operation and maintenance (O&M) phase. From the third year, the concessionaire will oversee the operation of the toll collection booths.

The net revenue collected from the toll (without costs operation, and additional expenses) will be handled through a special trust account, which must be used to finance the contributions of the syndicated loan and the payment of dividends. The design of the project financing structure is shown in figure 4.1.

The structure incorporates a contractual guarantee where the Government is bound to make certain payments to the concessionaire if the traffic falls below a pre-established level (each year).

Figure 4.1. Concession financing model



4.2 Financial analysis and cash flow models

The main assumptions of the financial model of the project are summarized in table 4.1. The financial model presents a series of own characteristics to this type of financing, so that its construction requires the incorporation of previous constraints.

These cash flows are evaluated from the point of view of the investors and lenders of the project.

Table 4.1. Assumptions of the financial model

Project duration:	17 years
Currency:	COP ($)
Annual inflation expected:	4.5%
CAPEX (millions) [i]:	$95.000
O&M costs (millions):	$3.100
Administration fees (% of Toll revenues)	10%
Average toll rate [ii]:	$15.500
Month traffic volume:	59.250
Annual traffic grows:	4.5%
Admin expensive	10%
Taxes:	34%
Financing	
Equity [iii]	60%
Debt [iv]	40%

i. Total investment represents the resources needed by the infrastructure and includes pre-operational costs, studies and designs, financial costs and others. These funds will be disbursed as follows: year 1: 60%; year 2: 40%.
ii. The average toll rate was estimated taking into account the category, and the fee established for each category, in addition to the adjustment involving road improvements.
iii. Equity represents the resources provided by private investors (60%). It assumes a cost of equity (Ke) of 12%.
iv. The financing of the project is made with senior debt guaranteed for a period of 10 years and with an interest rate of 8.8%. The financing also involves a number of clauses and conditions for its payment, as well as the distribution of dividends, for example, only dividends can be paid when the debt balance is less than 50% of total debt acquired and the accumulated balances guarantee two years of O&M and payment of debt service.

The estimations of expected toll revenues in each year *t* are obtained by multiplying the traffic estimate by the corresponding toll rate (equation 3.14). The forecasting of the different components of the cash flow without financial leverage (or operating cash flows) are found in Table 4.2.

Table 4.2. Cash flows from operating activities (Without debt)

Year	Traffic month [i]	Operating revenue	O&M	Admin. expenses	Taxes	Cash flow (without debt)
1	-	-	-	-	-	(59,565)
2	-	-	-	-	-	(62,245)
3	743.0	26,284	4,756	2,628	6,237	12,461
4	776.4	28,703	4,970	2,870	6,885	13,966
5	811.4	31,345	5,193	3,134	7,596	15,408
6	847.9	34,229	5,427	3,423	8,375	16,990
7	886.0	37,379	5,671	3,738	9,230	18,724
8	925.9	40,819	5,926	4,082	10,167	20,627
9	967.6	44,575	6,193	4,458	11,195	22,712
10	1011.1	48,677	6,472	4,868	12,321	24,997
11	1056.6	53,157	6,763	5,316	13,556	27,502
12	1104.2	58,048	7,067	5,805	14,908	30,246
13	1153.9	63,390	7,385	6,339	16,390	33,253
14	1205.8	69,224	7,718	6,922	18,013	36,546
15	1260.0	75,594	8,065	7,559	19,790	40,153
16	1316.7	82,551	8,428	8,255	21,736	44,102
17	1376.0	90,147	8,807	9,015	23,867	48,427

Also, operating cash flows determine the ability to pay the company's financial obligations as debt service (syndicated loan). When the debt service is incorporated into the financial model, cash flows with financial leverage are obtained. It should also be borne in mind that the financing of the project incorporates financial costs, before the O&M phase, this costs corresponds to the interest, commitment commission and payment of the financing arrangement (table 4.3).

Table 4.3. Cash flows (with debt)

Year	Debt service	Pre-operating financial costs	Cash flow (with debt)
1	-	2,703	(64,704)
2	-	4,412	(66,658)
3	9,160	-	3,301
4	8,731	-	5,234
5	8,303	-	7,105
6	7,874	-	9,116
7	7,445	-	11,279
8	7,016	-	13,610
9	6,588	-	16,124
10	6,159	-	18,839
11	5,730	-	21,772
12	5,301	-	24,945

13	-	-	33,253
14	-	-	36,546
15	-	-	40,153
16	-	-	44,102
17	-	-	48,427

The NPV of cash flows is estimates using a risk-adjusted discount rate (WACC), which reflects its financial structure. The project's valuation results are presented:

WACC: 9,7%	**Results**
NPV	($5.654,6)
IRR	9,2%

With a discount rate of 9.7% is found the project is not feasible. Given the very low traffic level and its corresponding expected growth the NPV is ($ 5,911.6), while the IRR is only 9.2%. Therefore, the private investor won't be willing to participate in its financing. Project's conditions turn out to be unfavorable and show a complex scenario to the Government since the project are won't be implemented.

Now, if we consider that this result depends on, mainly, future traffic, then financing of the project requires the incorporation of contractual guarantees.

4.3 Valuation of guarantee as put options

In the O&M phase, it is likely that the concessionaire’s revenues to be lower than expected given low traffic (90%). This scenario will improve significantly by incorporating a minimum revenue guarantee (MRG) based on equations (3.15 - 3.18). Also, the expected value of the payments made each year by the Government to meet the minimum revenue guarantee is estimated by modeling the stochastic process according to equation (3.7).

The first step of the valuation procedure is the simulation of traffic; this procedure is performed monthly (twelve steps each year) during the 17 years of operation of the project.

By applying equations (3.10), (3.11) and (3.12), we obtain the following results, mean-zero OU process (the $\boldsymbol{x_t}$ component):

$$\alpha = -\frac{1}{\Delta t} Ln\left[\frac{\sum_{i=1}^{n}(x_i x_{i-1})}{\sum_{i=1}^{n}(x_{i-1}{}^2)}\right]$$

$$\alpha \ = 0{,}0029$$

$$\hat{\sigma} = \frac{1}{n}\sum_{i=1}^{n}(x_i{}^2) - 2e^{-\alpha\Delta t}\sum_{i=1}^{n}(x_i x_{i-1}) + e^{-2\alpha\Delta t}\sum_{i=1}^{n}(x_{i-1}{}^2)$$

$$\hat{\sigma} \ = 35.485{,}320$$

$$\sigma^2 = \hat{\sigma}^2\left(\frac{2\alpha}{1 - 2^{-2\alpha\Delta t}}\right)$$

$$\sigma^2 \ = 35.588.241$$

$$\sigma = \sqrt{\sigma^2}$$

$$\sigma \ = 5.965{,}59$$

Also, we obtain the following equation for the trend component of the model (equation 3.13)

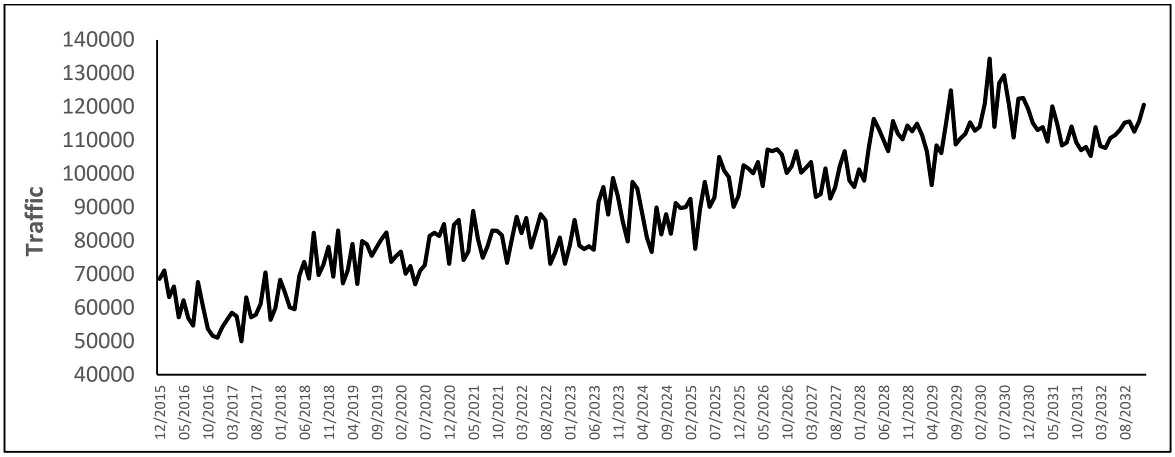
$$y_t = 196{,}64t + 33.829$$

With $R^2 = 0{,}7395$. From here we can conclude that $\overline{\boldsymbol{T}} = 196{,}64$. In conclusion, we obtain the following parameters:

$\boldsymbol{\alpha}$**=**	0,0029
$\boldsymbol{\sigma}$**=**	5.965,59
$\overline{\boldsymbol{T}} =$	196,64

Then, we proceed to model the trajectories that characterize the traffic stochastic, which is modeled, as indicated in equation (3.7), with MCS. The figure 4.2 presents a simulation (of the many possibilities) of traffic to period 2016-2032.

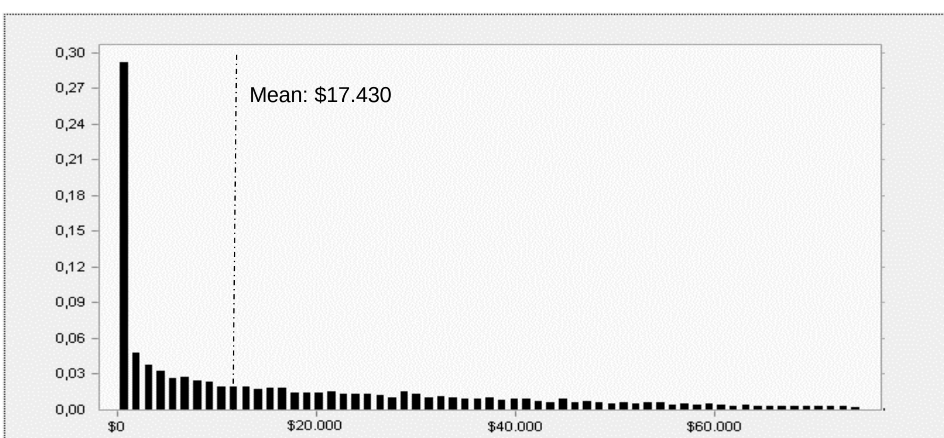
Figure 4.2. Simulation of traffic (O&M phase)



Source: Own elaboration

As can be observed, the traffic follows an OU process during the concession's life. Following a process through the MCS technique with 10,000 iterations of guarantee value is calculated for each year. The figure 4.3 shows the discounted value of concession payments. As a result, the present value of the resources committed by the Government during the 15 years of operation of the concession corresponds to $ 17,430 million.

Figure 4.3. Valuation of minimum revenue guarantee



In addition, the MRG implementation significantly improves the concessionaire's financial performance and functions as an effective risk traffic management. While in the initial scenario, the concessionaire presents a loss probability of 42% and NPV that can decrease up to ($50,000) million, in the other scenario (with traffic risk management), the MRG incorporation improves all the contingent scenarios of the concessionaire with an expected NPV of $ 11,776 million. Figures 4.4a and 4.4b show the above results.

Figure 4.4a. Concessionaire's NPV without MRG

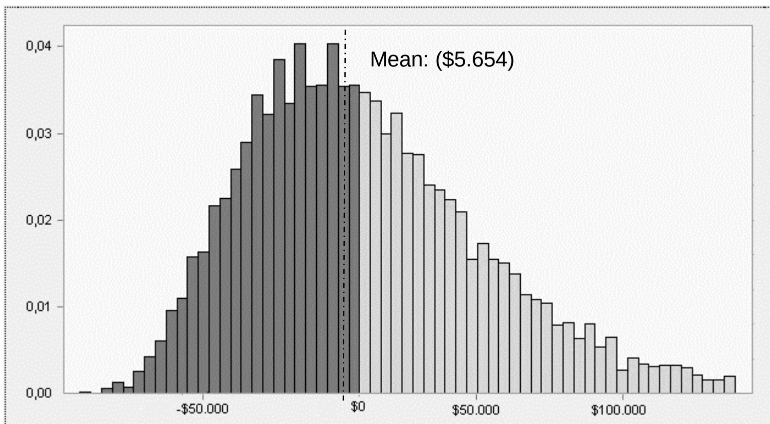
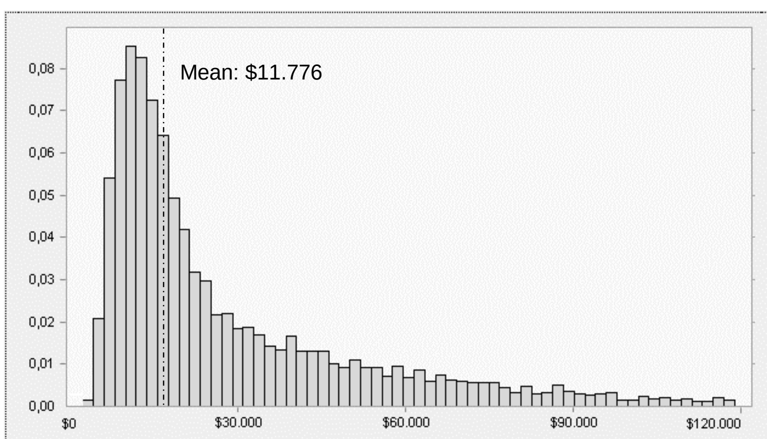


Figure 4.4b. Concessionaire's NPV with MRG



These results show that the strategy implemented hedge the traffic risk is consistent and significantly improves financial indicators for the private investor, without compromising the Government budget. Finally, undertaking the concession of the toll road may be feasible for the Government and the incorporation of the MRG becomes a very useful strategy.

4. Conclusions

Private investment in infrastructure projects represents a major challenge for the Government since the public budget is not enough and the Government must look at the private sector to finance, build and operate it. In this scenario, the risks of the project must be shared and the Government assumes its costs through governmental support, mainly when it comes to non-viable investments. However, the determination of the optimal level of resources provided as guarantees represents a complex task.

Some kinds of governmental support, such as contractual guarantees, can be identified and valuated as real options, since the guarantees are triggered when some conditions are met. The value of these real options has to be properly valuated for, so as to strike a better balance between risk and benefit. To achieve this, an estimation model of minimum revenue guarantees was proposed adopting an extended OU stochastic process supported in the MCS technique.

As a result, we found that the incorporation of MRG generates a feasible result for the private investor, while the Government can assume the costs of traffic risk without compromising strongly its budget. The methodology that we proposed is accurate, consistent and uses a stochastic model that adjusts to the dynamics of traffic.

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