Evaluation of Forest Investment Using Real Options Theory

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February 15, 2017

Abstract

The modern financial literature recommends the real options approach to incorporate uncertainty and managerial flexibilities in forestry investment projects. This work aims to develop a valuation model for forestry projects in which the growth of tree inventory follows a logistic equation based on the estimated real growth of a forest. This paper aims to quantify the economic benefits of an optimal production policy driven by tree cutting in a eucalyptus forest. The model includes three independent state variables (inventory, time, and price, the last one modelled as a geometric Brownian motion) and two dependent variables: cutting rate and the value of the investment option. The results, obtained through the explicit finite difference method, are compared to other alternatives of inventory evolution and cutting rate decisions. The results show that adopting an optimal cutting policy based on real options theory has a great advantage. Moreover, the forest option value is higher when inventory growth is modelled by the deterministic logistic equation compared to the stochastic logistic equation.

Key words: Forestry Investment; Real Options Theory; Finite Difference Method; Logistic Growth Function; Natural Resource Investment.

1 Introduction

Real options theory (ROT) is broadly used to evaluate natural resource investment projects (Pindyck, 1984; Brennan and Schwartz, 1985; Morck, Schwartz and Stangeland, 1989; Levi, 1996; Moreira et al., 2000; Insley, 2002; Rocha et al., 2004; Baran, 2005; Kerr, 2008; Gastaldi and Minardi, 2012). In forestry projects, timber price is related to the core uncertainty as well as the amount of trees growing in a forest (Morck, Schwartz and Stangeland, 1989; Conrad, 1997; Forsyth, 2000; Rocha et al., 2004). Rational investors maximize the value of the project by choosing an optimal cutting policy according to the current price of wood, stock and time. In most cases, defining the volume of trees cut in each period is a problem.

We extend the model proposed by Morck, Schwartz and Stangeland (1989) to a forestry investment project that has the flexibility to employ an optimal price policy and stock-dependent cutting, presenting a more realistic temporal evolution of the stock. The price of wood follows a geometric Brownian motion (GBM) as in the mentioned paper. However, following the literature on the population of living organisms (Peroni and Hernández, 2011; Goes, 2012), we propose that the inventory of trees follows a logistic growth equation. Given this main contribution of the work, we present the application of this model to a hypothetical forest. An important consideration of the model is the maximum level of saturation, implying a more realistic approach since natural resources are finite. The purpose is to present a general solution to a stochastic optimal control problem for investments in renewable resources, such as forests, that is, the optimal cutting rate and the value of a forest investment project. The analysis carried out in this article may interest forest companies as well as governments for decision making on public policy, specifically for better evaluation of concessions.

Another contribution is the use of an explicit finite differences method (FDM) to determine the value of the project according to a similar approach proposed by Rocha et al. (2004). As the differential equation for the value of the investment option has no analytical solution, this work proposes the use of the finite difference method, which transforms a partial differential equation into difference equations that can be solved numerically. The necessary conditions for stability are applied in order to obtain convergence for the finite difference solution.

The present work is structured as follows. In chapter 2, we present a literature review about the main papers that use real options theory to evaluate natural resource projects. Chapter 3 presents the proposed model and the tools applied in the development of a practical case. Chapter 4 presents a discussion of the most relevant results. Chapter 5 presents the conclusions of this work, as well as recommendations for future research.

2 Literature Review

The evaluation of renewable resource projects in several sectors, including forestry investments, makes use of real options theory, considering the characteristics of uncertainty and flexibility of the projects. In these projects, uncertainty is commonly modelled on the price of the product being sold and on the resource stock.

Pindyck (1984) examines the uncertainty implications in the stock of a hypothetical resource, that is, how the volatility of this variable affects the value of an asset. He shows that an increase in the variance of stock fluctuations reduces the expected rate of inventory growth. One of the relevant results to be mentioned is that in this case the rate of extraction or cutting is reduced. The author presents two reasons: increased volatility increases product scarcity and reduces the expected growth rate.

Morck, Schwartz and Stangeland (1989) evaluate a forest concession in which wood price follows a GBM and the stock drift varies with the growth rate and the cutting rate. One of the main goals is to determine an optimal production policy that maximizes the value of the company. The cutting rate is determined as a function of the price, the current wood inventory and the time remaining until the end of the concession. Among the assumptions adopted, the cost function is given by a quadratic equation. Therefore, the function for the optimal production policy presents a maximum point in the quantity to be produced. The authors calculate the value of a forest in Canada and conduct a sensitivity analysis on some variables of the problem. A finding worth noting is that the less the stock, the lower the optimal cutting rate as the concession term approaches. They observe that (i) the higher the growth rate of the stock, the lower the cutting rate, so that cutting will be carried out for longer periods until the end of the stock; (ii) the higher the initial price, the greater the incentive to cut early; and (iii) the greater the price volatility, the greater the value of the forest option, so it is better to delay the cutting in order to make a larger profit.

Levi (1996) presents a real option application to eucalyptus forest exploration projects for cellulose production. The dynamics of price and inventory growth are the same as proposed by Morck, Schwartz and Stangeland (1989), but with the assumption of a linear rather than the original quadratic cost function. In this case, the optimal stochastic control problem has a *bang-bang* solution, “to produce the maximum capacity or not to produce,” which simplifies the numerical solution of the problem.

From another perspective, Conrad (1997) uses the real option approach to analyse the decision to preserve a 100-year-old forest in a North-American national park. Future values of park maintenance benefits (or amenities), such as visitation and habitat for the local fauna, are uncertain and follow a GBM. Considering the amenity flow to be proportional to park visitation, the author calculates the tendency and volatility parameters. Analytical solutions are derived for the value of the amenity that justifies preservation of the forest, besides harvesting it for commercialization.

Forsyth (2000) applies the same concept from the abovementioned paper to evaluate a wilderness area. In both articles, the option value approach is used to determine whether to harvest a forest or preserve it for recreational use. Unlike the work of Conrad (1997), Forsyth (2000) uses a logistic process to describe the amenities. The author observes, using the logistic growth function, that the critical amenity value decreases as volatility increases.

Moreira et al. (2000) evaluate a forest concession in Brazil, which depends on uncertainties in the estimation of commercial timber volume in the concession area, as well as on the future price of the log. Legal restrictions are imposed on the concession. The authors propose a methodology that values a forest concession based on real options theory and observe that the concession value is not proportional to current prices and inventories, being more sensitive to the uncertainty of prices than to inventories. Thus, the former must be measured more carefully. In addition, assuming a lack of knowledge of the initial stock level decreases the value of the concession, so that the greater the uncertainty regarding the initial quantity of wood in the region, the lower the value obtained for the concession.

Rocha et al. (2004) apply a model similar to that used by Morck, Schwartz and Stangeland (1989) to a concession in the Amazon rainforest. Two stochastic processes are proposed for the price: the GBM and the mean reversion model (MRM). The result of the Dickey-Fuller unit root statistical test does not reject the GBM hypothesis for the price. As in the work of Levi (1996), the cost is given as a linear function of the cut rate, so that the cut rate is always maximum or zero. Applying the explicit finite difference methodology for the calculation of the concession value, the authors analyse the sensitivity of the option value to some problem variables for the two stochastic price processes. The value of the forest concession calculated with the price following a GBM is at least 1.5 times greater than that obtained under the mean reversion assumption and 8 times higher than the value based on the traditional NPV method. In addition, the longer the expiration period, the lower the sensitivity of the option value over the concession period. As price volatility increases, the value of the concession increases. In contrast, the value of the concession decreases as stock volatility increases.

Insley (2002) evaluates a forest investment and models the optimal harvest decision. The value of the cutting option is estimated according to the dynamic programming approach, with GBM used for the stochastic price process. According to Insley (2002), the relationship between option value and volatility can be understood by examining the problem solution, which describes how the option varies over time. For a longer time to expiration, the option value is greater for higher volatility values of the underlying asset.

Kerr (2008) applies the Insley (2002) methodology to a reforestation project, so that the optimal harvest decision is modelled as an American option. Variational inequalities from the American option problem are solved by the implicit finite difference method using the system of linear complementarity equations. The author observes that the option value and the optimal cutting rate are significantly influenced by wood price volatility, the risk-free rate, and harvest costs.

Gastaldi and Minardi (2012) evaluate the value of anticipating or postponing harvesting of a eucalyptus forest in the face of uncertainties about the price of timber. The valuation focuses on a comparison between use of the traditional discounted cash flow and valuation by real options. The authors compare the results using the GBM and the mean reversion process for the price. The results are quite intuitive, suggesting that the best option is to cut younger forests when prices are high.

Appendix 1 presents the articles in a consolidated form to facilitate comparison. The objective of the present research is to extend the Morck, Schwartz and Stangeland (1989) model considering a different process for the evolution tree inventory growth in a hypothetical forestry investment project. The present study proposes a logistic process to describe forest growth rather than considering a constant drift and or limiting growth with a barrier as in some abovementioned papers (Morck, Schwartz and Stangeland, 1989; Levi, 1996; Moreira et al., 2000; Rocha et al., 2004).

According to Goes (2012), the growth of general living organisms follows a sigmoidal curve, characterized by a relatively rapid initial phase, of the exponential type, a convex middle phase, and a concave curve at the end. Peroni and Hernández (2011) affirm that the populations do not grow exponentially, with rare exceptions. They affirm that the population size (or density) increases until it reaches a relatively stable maximum limit. This behaviour can be described by what is known as a logistic equation, also called the Pearl-Verhulst equation (see Dias, 2015, p.82, or Tuckwell, 1995, pp. 219-220). Figure 1 illustrates the logistic S-shaped growth curve.

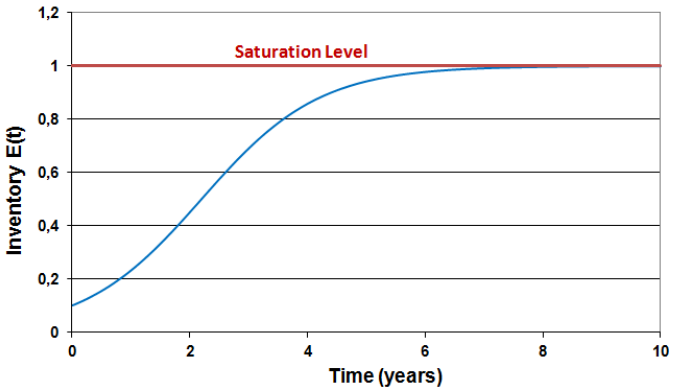

Figure 1 – The Logistic Growth Curve

As noted by Tsoularis (2001), unrestricted growth, similar to exponential growth (Malthusian model), is an unrealistic assumption for a population model. Therefore, Verhulst (1838) considered a stable population with a saturation level. This behaviour, typically called the carrying capacity, forms an upper bound on the growth size. The logistic curve incorporates this idea and can also be seen to model physical growth of a population. In particular, when the initial population size is much smaller than the carrying capacity, the resulting logistic growth rate curve is sigmoidal. The logistic model was rediscovered and popularized by Pearl and Reed (1920) and is widely used nowadays to model population growth.

Therefore, this work mainly contributes to the real options literature on forest resources by using the relatively more realistic logistic equation to model inventory growth. This means that without cutting, a population of trees will grow up to a limit, where it tends to stabilize.

Furthermore, we propose to use the explicit finite differences method to obtain a solution under the real options approach and calculate the optimum cutting rate for trees for each period. The explicit FDM is easier to implement, less numerically intensive and more intuitive than other finite difference methods (see, e.g., Dias, 2015). Since it may present convergence problems, a stability condition is added to attain convergence and to obtain reliable results. The proposed explicit FDM is presented in the Appendix 2.

3 Forest Concession Valuation

Since this is a methodological work, one of the goals is to calculate the market value of a commercial-purpose forest concession. This value will depend on selected parameters to reflect a range of perspectives on productivity or market evolution. The result will be the best estimate for the market value of the concession conditional on the set of parameters adopted. It is important to underline that the concession value is based on the assumption that the firm will act optimally in cutting trees. Therefore, the company that owns the concession will choose a cutting rate that maximizes its market value.

In the proposed model, the forest investment value – with an option of cutting trees in the concession period – and the cutting rate are the dependent variables; timber price, inventory of trees, and time are the independent variables.

The timber price follows a GBM, and its risk-neutral form can be written as shown in equation 1:

$$\frac{dP^Q}{P} = (r-\delta)dt + \sigma dz^Q \quad (1)$$

where P is the timber price at time *t*, $(r-\delta)$ is the risk-neutral drift rate, r is the risk-free rate, δ is the convenience yield of timber, σ is the price volatility, and *dz* is the increment of a standard Wiener process. *Q* indicates that the process is risk neutral.

In order to incorporate a more realistic element into the forest problem, we take into account the decreasing growth rate of the trees over time, which approaches zero as the forest inventory approaches the saturation level.

In this article, the tree inventory drift is given by the logistic growth equation presented below.

Considering that the forest initially has E_0 cubic metres of timber, for purposes of this work, the variation in stock is determined by the following equation 2[1]:

$$dE = [\eta[\bar{E} - E(t)]E(t)]dt - q(E, P, t)dt + \sigma_E E(t)dz \quad (2)$$

where *E(t)* represents the timber inventory at time *t*; $\eta[\bar{E} - E(t)]$ is the *drift* coefficient; η is the reversion speed (here a growth speed parameter); $\bar{E}$ is the saturation level; q is the cutting rate; and σ_E is the inventory volatility, which reflects the uncertainty about timber volume.

The cash flow of a company on timber production is determined by equation 3.

$$f(t) = (1 - \tau)\times(P\times q - C(t)) \quad (3)$$

where τ is the tax rate on the profit of production and $C(t)$ the cost.

A quadratic function is adopted (Morck, Schwartz and Stangeland, 1989), instead of a linear function (Levi, 1996; Rocha et al., 2004), to model the cost. According to Levi (1996), if the cost function is linear in relation to the cutting rate, the problem of stochastic optimal control will have a solution of the type "to cut at the maximal rate or not to cut," simplifying the numerical solution of the problem. When the cost function is quadratic, the optimal cutting policy assumes values between an interval, and not necessarily zero or maximum values. The function considers a linear variable cost and a quadratic variable cost that reflects the marginal cost increase and can be represented as follows:

$$C(t) = \begin{cases} c_1 \times q + \frac{c_2 \times q^2}{2}, \ se \ f(t) > 0 \\ 0, \ se \ f(t) \leq 0 \end{cases} \quad (4)$$

Given equations 1 and 2 for price *P(t)* and inventory *E(t)*, respectively, according to the Itô-Doeblin equation for two stochastic variables, the partial differential equation (PDE) of the forest value is given by

$$dF = F_t dt + F_P dP + \frac{1}{2}F_{PP}(dP)^2 + F_E dE + \frac{1}{2}F_{EE}(dE)^2 \quad (5)$$

[1] This stochastic version of logistic equation is a mean reversion process (geometric Ornstein-

Applying the Itô-Doeblin formula, it can be shown that the market value of the forest concession, *F(P,E,t),* is a contingent asset of the price and underlying inventory and grows according to PDE given by equation 6.

$$rF = F_t + (r - \delta_P)F_P P + \frac{1}{2}F_{PP}\sigma^2 P^2 + F_E\{[\eta[\bar{E} - E(t)]]E(t) - q(t)\} + \frac{1}{2}F_{EE}{\sigma_E}^2 E^2 + f(t) \quad (6)$$

For this problem, the following boundary conditions are applied:

$$F(P,E,T) = 0, \quad (7)$$
$$F(0,E,t) = 0, \quad (8)$$
$$F(P,0,t) = 0, \quad (9)$$
$$\lim_{P \to P_{máx}} F_P = E - E_{min}, \quad (10)$$

Equation 7 ensures that the value of the concession is zero at the end of the concession. Equations 8 and 9 ensure that the concession value is zero when wood or stock prices fall to zero. According to equation 10, changes in market value due to changes in price are linearly proportional to the inventory available for production because when the price tends to infinity, we attempt to exploit any available reserve immediately. In this case, the project value is a function of inventory variation only.
For each period of time, price and inventory, the optimal cutting rate *q** is calculated to obtain the option value. In order to find this optimum production policy, we substitute equations 3 and 4 in equation 6 and take the partial derivative in relation to the quantity *q*. Imposing the constraints that (i) production should be non-negative and (ii) production can be suspended or restarted without additional costs, we can solve the optimal cutting rate using equation 11.

$$q^*(P,E,t) = \begin{cases} max\left[0, \frac{-F_E}{(1-\tau)\times c_2} + \frac{P - c_1}{c_2}\right], \ se \ f(t) > 0 \\ 0, \ se \ f(t) \leq 0 \end{cases} \quad (11)$$

The PDE given by equation 6, with the boundary conditions in equations 7 to 10 and the constraints of *q* (P,E,t)* in 11, must be solved numerically for the option value solution.
Besides the original model described, additional assumptions are made to compare different models, as special cases. For example, if the volatility of the inventory is zero, the inventory follows a deterministic equation, ignoring the stochastic term regarding timber volume. This particular example of the main model is analysed more specifically, and the results acquired in both cases are compared.

We compare the market value given by the real options methodology with the traditional net present value (NPV) approach. The intrinsic flexibility of being able to temporarily suspend production or change the cutting rate cannot be quantified according to the traditional approach, and is given by the expected cash flow discounted by the project risk-adjusted discount rate (μ) as shown below:

$$\sum_{t=0}^{T} \frac{FC(P_0,q,t)}{(1+\mu)^t} = \sum_{t=0}^{T} \frac{(1-\tau)[(P_0 q)-(cost)]}{(1+\mu)^t} \qquad (12)$$

$$cost = c_1 q + \frac{c_2 q^2}{2} \qquad (13)$$

In this work, two applications are realized for this model. In the first application the cutting rate *q* is fixed and the cutting process is performed at each period. Then, a second application based on cash flow maximization was made to obtain an optimum cutting rate, which varies with price and costs. In addition, inventory variation is considered for past cuts made.

4 Hypothetical Project and Results

The proposed model is applied to a hypothetical forest investment project and analysed. The parameters used in this analysis are shown in Table 1.

Table 1 – Parameters

Risk-free rate of return	r	5% per year
Current price of timber	P_0	R$ 50/m^3
Maximum price	P_{max}	R$ 500/ m^3
Volatility of price	σ	18% per year
Convenience yield	δ	4.6%per year
Current inventory*	E_0	100 m^3/ha
Minimum inventory*	E_{min}	30 m^3/ha
Saturation level of inventory*	$\bar{E}$	600m^3/ha
Reversion speed	η	0.0004
Expiration	T	10 years
Tax rate**	τ	15% per year
Linear variable cost	c_1	R$ 10/m^3
Quadratic variable cost	c_2	R$ 5/m^3

* Variables in thousand m^3/ha.

** Income tax rate obtained according to Law No. 9,393 (Brazil, 1996) for a forest with a total area of 500 to 1000 hectares and a land utilization rate of at least 80%. The tax rate was considered as a tax on profit. Any other tax that affects profit can be added without compromising the results.

Historical eucalyptus prices were deflated and converted to constant real prices, on April 2015 basis, by the General Price Index (IGP-DI), an inflation index in Brazil. The Dickey-Fuller (DF) unit root test was performed, and the t-statistic (= -2.17) for the price series does not allow rejection of the null hypothesis (at a significance level of 1%). Therefore the GBM is a suitable process to represent the price series. The series of prices and the statistical results are shown in Appendix 3 and 4.

Although MRM is usually used for modelling commodity prices, the GBM is used as the price diffusion process in the forest sector literature in most cases (Morck, Schwartz and Stangeland, 1989; Levi, 1996; Moreira et al., 2000; Baran, 2005), and the null hypothesis was not rejected by the unit root test.

The parameters of volatility and convenience rate are estimated based on the historical series of wood prices. The estimated drift α is 4.8% per year, and the volatility σ is 18% per year. The risk-free rate of return and the convenience yield are the same as those used by Moreira et al. (2000). The maximum price used in the finite difference grid is R$ 500 / m^3, since the series does not exceed R$ 95.00 in the analysed period.

In the main model proposed, the inventory follows a stochastic logistic process. Subsequently, we analyse a particular model where the inventory follows an equivalent process without the stochastic term, which is here called the logistic model. Furthermore, we treat another case using a fixed cutting rate instead of an optimum value.

Data of the eucalyptus population on the level of saturation and minimum inventory levels of the eucalyptus population were obtained from Rodriguez, Bueno and Rodrigues (1997). The average reversion velocity was chosen to better represent forest growth behaviour as a logistic equation.

In addition, the linear and quadratic variable costs considered are, respectively, R$ 10/m^3 and R$ 5/m^3, and sensitivity analysis is conducted for both. The linear variable cost, which incorporates implementation and maintenance costs, is the most commonly used parameter in the real options literature. However, the quadratic variable cost, used by Morck, Schwartz and Stangeland (1989), has an economic importance because it incorporates possible additional costs, such as extraordinary charges and costs of storage and displacement, among others.

For application of the explicit finite differences method (Explicit FDM), a Java-based software[2] was developed from the discretized equations corresponding to the proposed model as presented in Appendix 2.

To explore the optimal cutting policy, some assumptions about the future price and inventory levels are necessary. On the assumption that these variables grow by their historical rates, an optimum cutting rate policy in

[2] Available on request in case of interest.

relation to time can be obtained. The optimal cutting rates are based on two assumptions: (i) the initial timber price (R$ 50 in the base case) increases by the risk-neutral rate, (ii) the inventory is calculated based on the logistic equation, which is the value of inventory incorporating logistic growth. In addition, the harvest made in the previous period is deducted from the value of the stock over time, *E(t)*.

Some analyses related to the cutting policy are presented in Figures 2 to 6. Figure 2 shows the cutting rate over time for different initial stock values. Most of the inventory is cut during the first half of the investment period. In addition, the higher the initial stock, the longer is the cutting period. This result is similar to the finding by Morck, Schwartz and Stangeland (1989), because the cutting rate decreases over time and the cutting time increases with the inventory.

Figure 3 shows that the higher the initial price, the greater the incentive to cut the forest faster in the early periods. In contrast to Morck, Schwartz and Stangeland's (1989) results, cutting is done faster in the present work, and the forest is cut in a shorter period of time.

Figure 4 shows that higher volatility implies a higher option value, so postponing can be better than cutting right away.

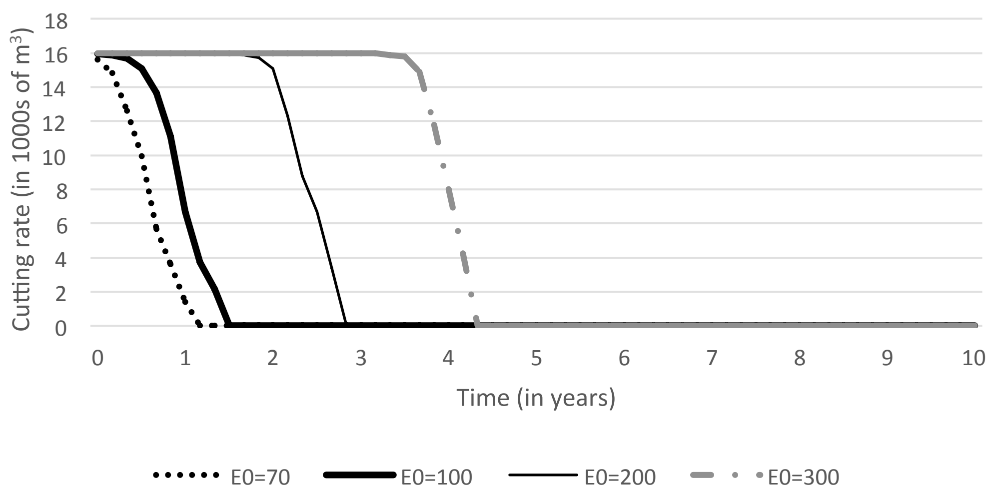

Figure 2 – Cutting rate vs. time for different initial inventory levels

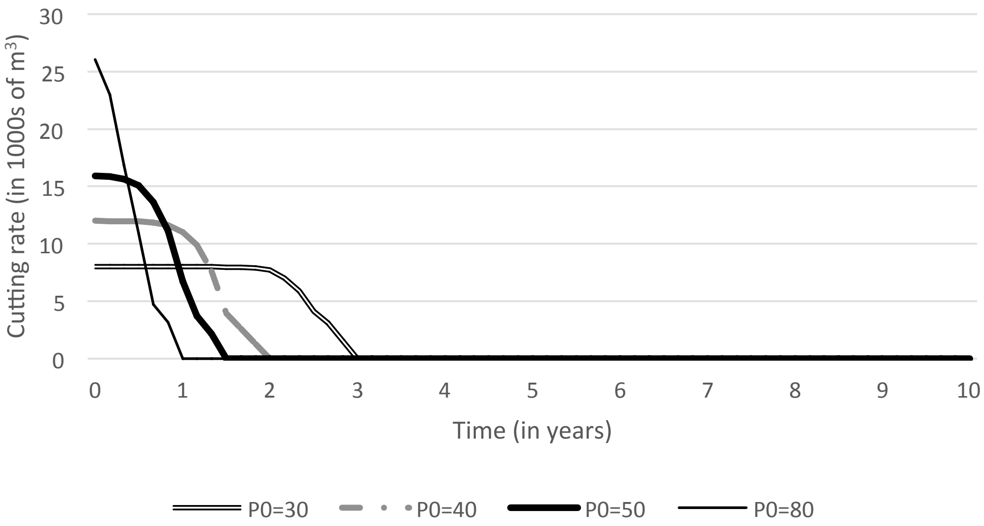

Figure 3 – Cutting rate vs. time for different initial price levels

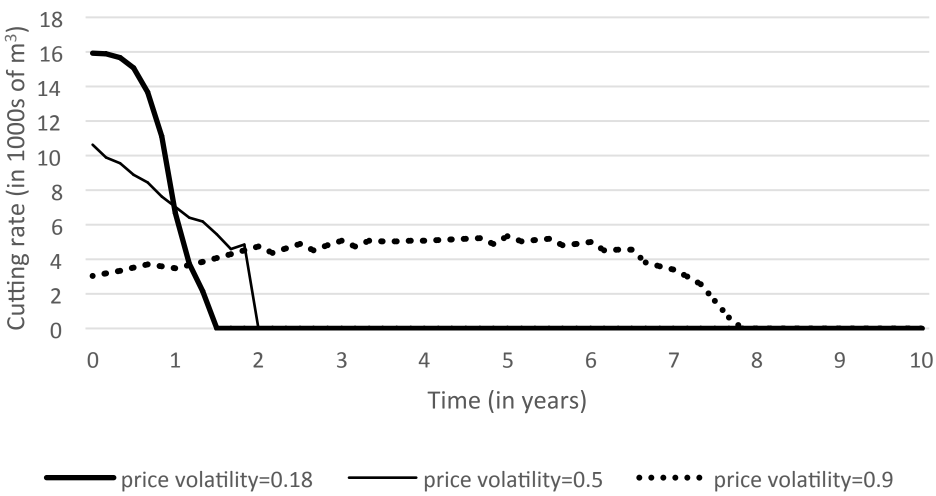

Figure 4 – Cutting rate vs. time for different price volatility levels

Figures 5 and 6 show how the cutting rate varies over time for different variable cost values, both linear and quadratic. The cutting rate decreases with an increase in both linear and quadratic costs, and cutting itself is performed for a longer period. The cutting rate is more sensitive to the quadratic than the linear variable cost, which means that additional costs throughout the project can have a negative effect on the final investment value. This indicates that a detailed analysis of this variable must be conducted to ensure that the project is correctly evaluated.

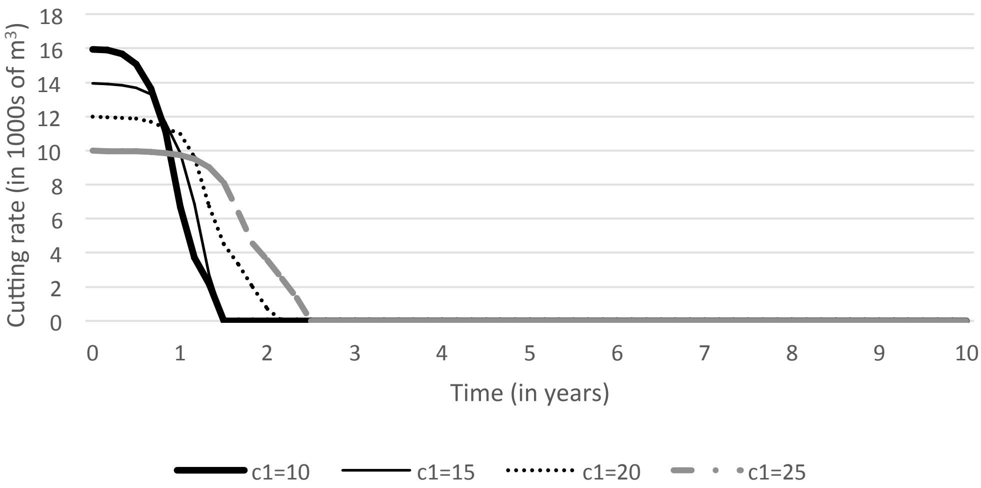

Figure 5 – Cutting rate vs. time for different linear variable costs levels

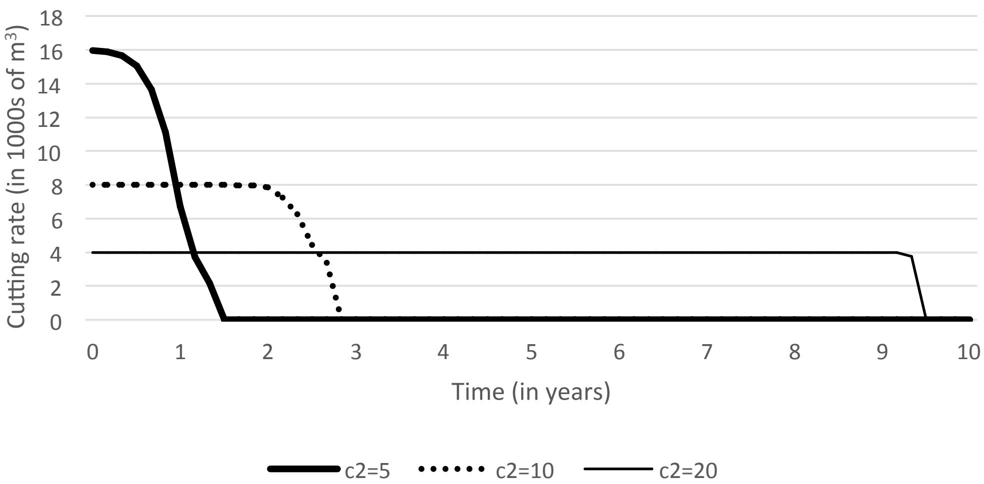

Figure 6 – Cutting rate vs. time for different quadratic variable costs levels

Figures 7 to 9 show the option values related to the initial price for different parameter values of the stochastic price evolution processes and inventory levels. Figure 7 shows that the higher the initial inventory, the greater the value of the option, as in Morck, Schwartz and Stangeland (1989).

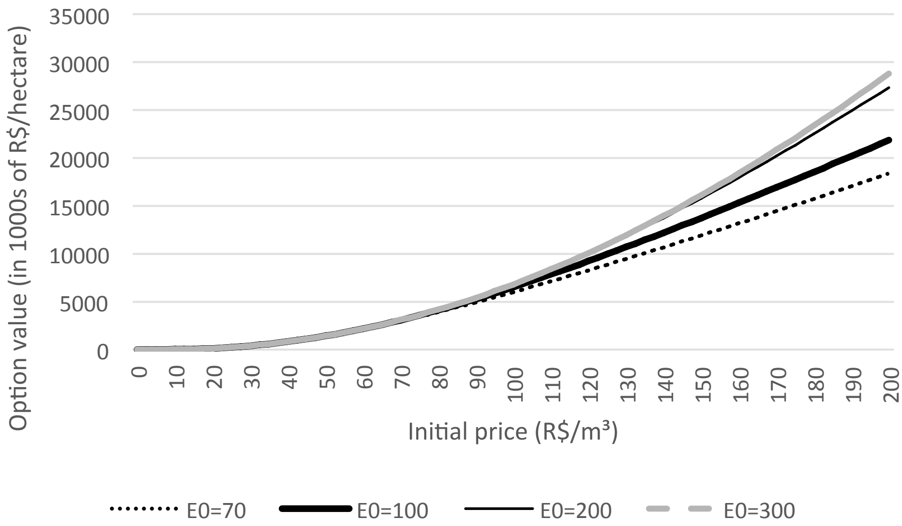

Figure 7 – Option value vs. price for different initial inventory levels

Figure 8 shows that the convenience yield of the price has an inverse relationship with the option value, because the higher the price, the larger is the convenience yield, and the lower the option value. This result can be examined mathematically because this is a factor that diminishes the risk-neutral drift of the price process. In addition, the convenience yield is also called the *rate of return of shortfall* (term coined by McDonald and Siegel, 1984), so that high product inventory reduces the probability of supply disruption, decreasing the convenience yield. Therefore, the lower the convenience rate, the greater the option value.

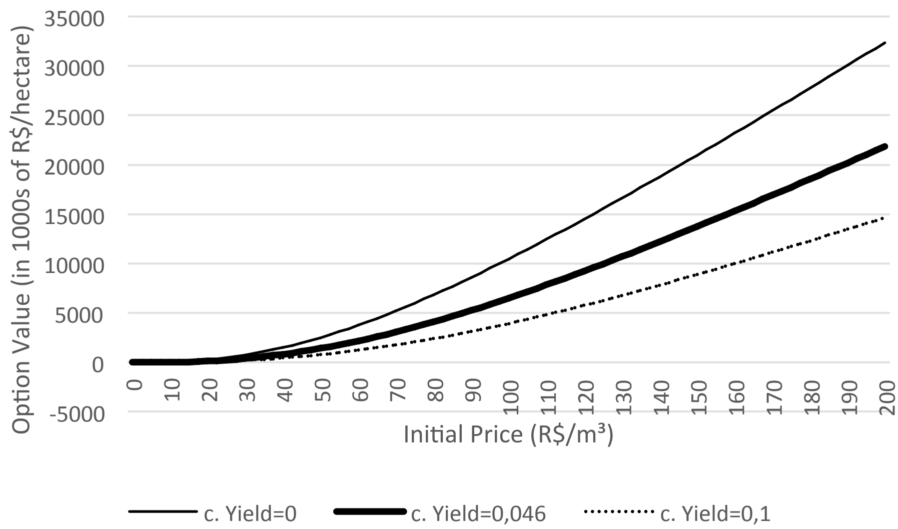

Figure 8 – Option value vs. price for different convenience yield levels

Figure 9 shows the option value variation in relation to the initial inventory for different timber volume volatilities. Increasing the uncertainty in relation to the inventory decreases the option value. Moreira et al. (2000) point to the negative sign of the EDP in equation 6 as vindication of this fact. Therefore, the result is similar to that presented by these authors.

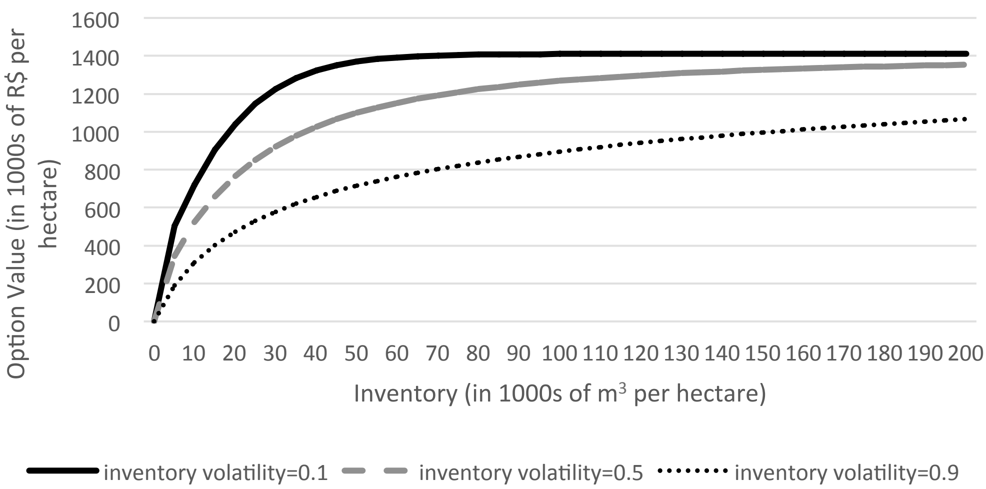

Figure 9 – Option value vs. inventory for different inventory volatilities

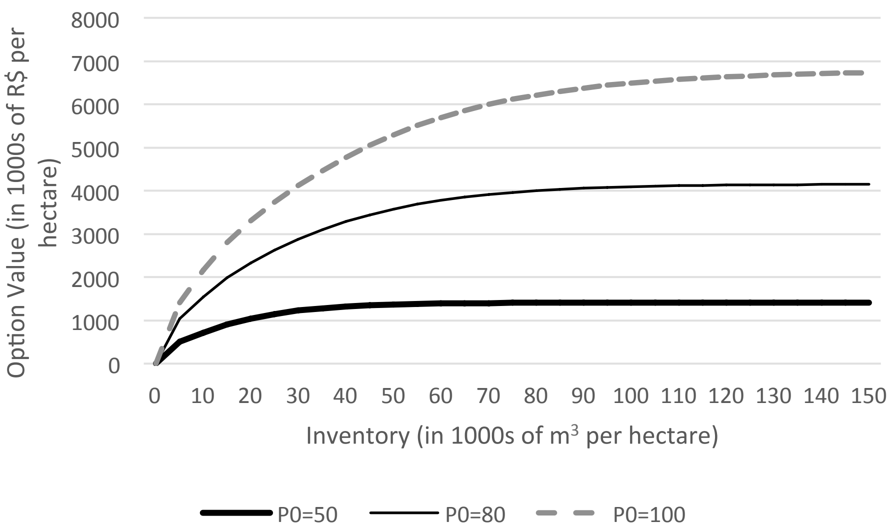

Figure 10 – Option value vs. inventory for different initial price levels

Figure 10 shows that the value of the option tends to a constant for very high inventory levels. Similarly, this result confirms the findings of Morck, Schwartz and Stangeland (1989) and Levi (1996), who use another growth model.
A comparative analysis based on the project value is carried out for the three models described here as (i) the logistic model with a stochastic component, (ii) the logistic model and (iii) the logistic model with a fixed cutting rate. Case (i) is the main model proposed in this paper. In the logistic model with a stochastic component, the inventory follows the

geometric Ornstein-Unlenbeck model given by equation 2, and the cutting rate is optimized. In models (ii) and (iii), the inventory presents a growth path according to the deterministic logistic equation, removing the stochastic part. In model (ii), however, the cutting rate is optimized as in model (i), while case (iii) focuses on the decision of whether or not to cut at a given maximum cut rate (q_{max}).

Table 2 summarizes the value of a logging company, that is, the value of the investment option, based on the finite difference method, with the parameters given in Table 1. The table shows the value of the project for different initial price levels. Each starting price has five different results. In addition to the cases involving real options theory, two cases based on the traditional net present value method are compared, one in which the cutting rate is q_{max} = 15 m³/year and another where cutting rate is optimized (q^*), taking the partial derivative of the equation 3 in relation to the quantity q. As expected, the net present value criterion underestimates the forest value in all cases compared to the real options methodology.

Table 2 – Market value in thousands of reals for different initial prices

	(i) Logistic Stochastic	(ii) Logistic	(iii) Logistic fixed cutting**	NPV $q=15$	NPV q^*
P0=40	818	818	366	-709	567
P0=50	1,410	1,411	890	236	1,009
P0=100	6,495	6,561	5,495	4,965	5,107
P0=200	21,866	22,225	15,808	14,423	20,052
P0=300	39,330	40,020	27,034	23,880	30,307

**This model considered a maximum cutting rate of q_{max} = 15 m^3/year.

The model with the stochastic component provides a smaller option value than the logistic model, as evident from the sign of the second derivative F_{II}, which is negative (see the partial derivative of equation 6). In this case, the greater the inventory volatility, the lower is the option value. This specific analysis is not reported in the literature, but other authors (Rocha et al., 2004; Moreira et al., 2000) have presented the case of varying inventory volatility, using an exponential classic stochastic model.

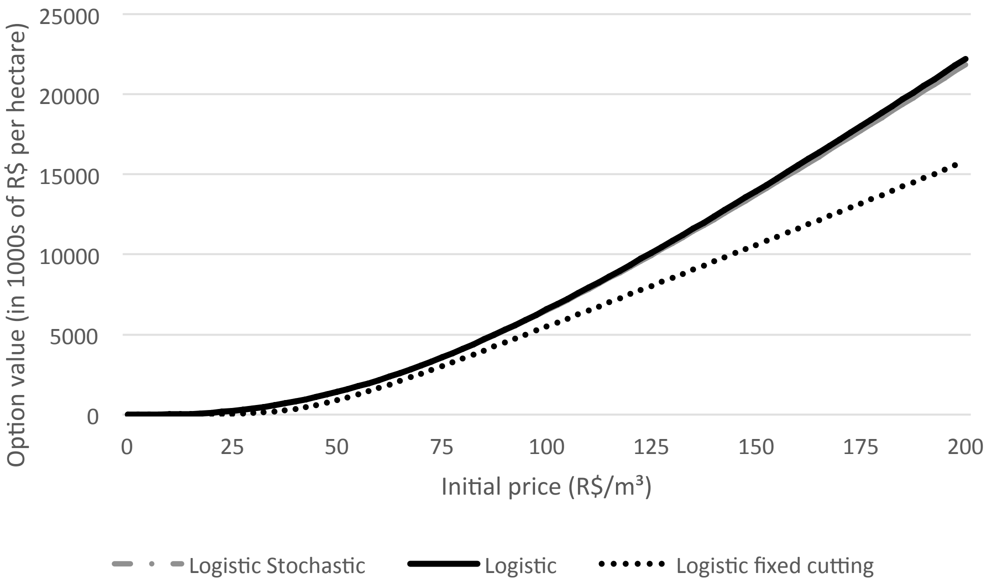

Figure 11 – Option value vs. price inventory for different models

Figure 11 shows that the value of the option is higher for the logistic model. However, the difference between the results obtained with the stochastic logistic model and the logistic model is small. As already seen in the analysis of Figure 7, for the stochastic model, the sensitivity of the option value to the price is very low with initial stocks lower than 150 m³. The same is the case with the logistic model. As for the base case of an initial stock of 100 m³, this difference is not very sensitive, as shown in the graphic.

In general, the option pricing methodology is useful to help investors estimate the fair value of an asset under uncertainties, quantify the economic benefit of the investment and determine the feasibility of forest management.

5 Conclusion

This paper uses real options theory to estimate the value of a forest concession for timber production, proposing an alternative process to describe the growth of tree inventory based on the logistic equation. The explicit finite difference numerical method was applied, and a sensitivity analysis based on the results was carried out to compare different cases and analyse the variables. Compared to the traditional NPV criteria, real options theory allows us to evaluate the gains from decision flexibility for scenarios of uncertainty. In the base case considered, the project value obtained through real options theory is more than twice as large as the one obtained through NPV approach with the fixed cutting rate and approximately 40% higher with the NPV with optimized cutting rate.

Another relevant result of the analysis is that the option value is greater for the logistic model. The results show that the value of a forest investment project is more sensitive to price uncertainty than inventory uncertainty, as in Moreira et al. (2000), so the estimation of the former must be even more important.

Considering the lack of data availability and the precariousness of specific data on real projects, the results obtained are only indicative of the project value. However, these results are sufficiently suggestive and revealing to drive an upgrade of the proposed methodology with a set of parameters considered more realistic.

Therefore, the cutting rate for wood in forest concessions can be determined in the context of an optimal control policy. This optimal control policy would determine the cutting pattern of the trees, depending on the price of the wood, the amount of wood in the forest reserve and the current time. However, the fact that the optimal time for cutting trees could be technically determined was not considered. Failure to consider that the cutting rate and the ideal age for cutting trees are technically determined can lead to results that do not represent the reality of the forest market. A technical and economic analysis of planting operations and industrial processes would be an important tool for management because the two activities are linked. This application, as well as an analysis of a forest concession with greater flexibility regarding the appropriate time for cutting, is suggested for future research.

This research can be extended to different approaches and may interest both forest sector companies and governments, as an aid to public policy decision making, for better pricing of concessions. As a suggestion for future work, other numerical methods can be applied to evaluate options. Moreover, other stochastic processes can be used to describe wood price – mean reversion process, for example. It would also be interesting to extend the problem to consider the threshold price $P^*(E, t)$, the price above which it is optimal to have some cutting of trees, that is, the price so that $q^* > 0$. We leave this analysis for future work.

In fact, focusing on the optimal investment policy allows us to study social and regulatory problems as well, which is also a motivating theme for the use of the approach presented here. Parameters involving the inventory variable could also be estimated. In addition, with more in-depth research on the cost of forest market articles, one could consider the fixed cost of maintaining forest production and adopting a more assertive approach to variable costs. As this is a methodology applied to a natural resource, other types of resources could be considered.

One expects that the application of this methodology to general problems involving inventory management has significant importance for academic works that would influence the development of a sustainable natural resource policy by companies and the regulators. Viewed mainly from the aspect of environmental preservation, the conscious use of natural resources is a promising field to be explored by future works.

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Appendix 1 – Main references of real options in the area of renewable resources

Year	Authors	Title	Modeled variables	Corresponding processes	Option type	Decision	Relevant results
1984	Pindyck	Uncertainty in the Theory of Renewable Resource Markets	Inventory	Classic Stochastic Model with General Growth Function	Simple Wait	to cut or not to cut at a certain q rate	In the application of a logistics function of inventory growth the cutting rate is reduced.
1985	Brennan and Schwartz	Evaluating Natural Resource	Price and Production	GBM and Deterministic Model	Temporary Stop and abandon	optimum exercise timing	The higher the inventory in the mine the greater the incentive to extract the copper and the lower the price under which the mine is opened, closed or abandoned.
1989	Morck, Schwartz and Stangeland	The Valuation of Forestry Resources under Stochastic Prices and Inventories	Price and Inventory	GBM and Classic Stochastic Model	Expansion	optimum cutting rate	The optimal cutting rate decreases as the inventory decreases. The higher the inventory growth rate, the lower the cutting rate.
1996	Levi	Avaliação Econômica de Projetos de Exploração Florestal	Price and Inventory	GBM and Classic Stochastic Model	Simple Wait	to cut or not to cut at a certain q rate	The cost of transportation is the main differential to accept or rejecting a project.
1997	Conrad	On the option value of old-growth forest	Amenity	GBM	Simple Wait	to preserve a forest or not	As the inventory volatility increases the higher the preservation value.
2000	Moreira et al.	A valoração das concessões nas florestas nacionais da Amazônia: uma abordagem com opções reais	Price and Inventory	GBM and Classic Stochastic Model	Simple Wait	to cut or not to cut at a certain q rate	The greater the uncertainty regarding the inventory, the lower the value of the concession.
2000	Forsyth	On estimating the option value of preserving a wilderness area	Amenity	Stochastic Logistic Growth Process	Simple Wait	to preserve a forest or not	The critical amenity value decreases as volatility increases.
2002	Insley	A Real Options Approach to the Valuation of a Forestry Investment	Price and Inventory	GBM/MRM and Model of Rollins & Forsyth (1995)	Simple Wait	corta a floresta ou não corta	The option value is higher for higher volatility values of the amenity.
2005	Baran	Avaliação de uma Floresta de eucaliptos na presença de um mercado de certificados para reduções de emissões de carbono: uma abordagem por opções reais	Price and Inventory	GBM and Model of Schumacher (1939)	Simple Wait	to preserve a forest or not	The forestry activity becomes more profitable when taking into account the reward for the dioxide carbon absorption.
2004	Rocha et al.	The market value of forest concessions in the Brazilian Amazon: a Real Option approach	Price and Inventory	GBM/MRM and Classic Stochastic Model	Simple Wait	to cut or not to cut at a certain q rate	The greater the price volatility, the value of the concession increases; however, the opposite occurs with the inventory, when the volatility is increased, the value of the concession decreases.
2008	Kerr	Decisão ótima de corte de uma floresta de eucalipto, utilizando diferenças finitas totalmente implícitas com algoritmo PSOR	Price and Inventory	GBM/MRM and Model of Schumacher (1939)	Simple Wait	to cut a forest or not	The harvest cost significantly influences the option value and the optimal cutting rate.
2012	Gastaldi and Minardi	Opções Reais em Investimentos Florestais	Price and Inventory	GBM/MRM	Simple Wait	to cut a forest or not	A project considering a single rotation of 7 years values the asset.

Appendix 2 – Explicit Difference Method

We propose to use the explicit finite difference method. The first step is to establish a uniform grid where time, price, and inventory are discretized. The option *F(P,E,t)* is a function of three variables; therefore, a grid of three dimensions is represented. The option value is obtained through a backwards process, from the expiration date to the initial date.

To develop the grid, we consider the maximum value for each of the following variables: P_{max}, the maximum price value; *T*, the expiration date; and $\bar{E}$, the saturation level. Then, the variables are divided into steps: *o* = *T/dt*, *m* = P_{max}/dP and $n = \frac{\bar{E}}{dE}$. Thus, the option value is calculated for each grid node (*m,n,o*) = (*i.ΔP, j.ΔE, o.Δt*), and the optimal cutting rate is determined.

To simplify the notation, assume that *F(P,E,t)* at *(P,E,t)* is represented by $F_{i,j,k}$, where *P = i.dP* and *i* ∈ *(0, m)*, *E = j.dE* and *j* ∈ *(0, n)*, *t = k.dt* and *k* ∈ *(0, o)*. Applying the approximations of the finite difference method in equation 6, we can write $F_{i,j,k}$ as a function of $F_{i^*,j^*,k+1}$, given in the equation 12 (where *i* = i, i-1, i+1 e j* = j, j-1, j+1*):

$$F_{i,j,k} = \frac{1}{r+\frac{1}{\Delta t}}\left[p_i^+ F_{i+1,j,k+1} + p_i^- F_{i-1,j,k+1} + p^0 F_{i,j,k+1} + p_j^+ F_{i,j+1,k+1} + p_j^- F_{i,j-1,k+1} + f(t)\right] \quad (12)$$

where the values p_i, p_j and p^0 depend on the current states *P* and *E*, and are given by

$$p_i^+ = \frac{\sigma_P^2 i^2 + (r-\delta)i}{2} \quad (13)$$

$$p_i^- = \frac{\sigma_P^2 i^2 - (r-\delta)i}{2} \quad (14)$$

$$p^0 = \frac{1}{\Delta t} - \sigma_P^2 i^2 - \sigma_E^2 j^2 \quad (15)$$

$$p_j^+ = \frac{\sigma_E{}^2}{2} + \frac{[\eta[\bar{E} - j\Delta E]](j\Delta E) - q}{2\Delta E} \quad (16)$$

$$p_j^- = \frac{\sigma_E{}^2}{2} - \frac{[\eta[\bar{E} - j\Delta E]](j\Delta E) - q}{2\Delta E} \quad (17)$$

Then the unknown value $F_{i,j,k}$ is an explicit function of the values of F, known at time k + 1. For the values of the upper and lower borders of the variables P and T (P_{max}, *P = 0* and *t = T)*, the option value is given by boundary conditions in the equations 7 to 10.

Then, the discretization of dP, dE and dt is chosen to provide an adequate balance between precision and computational time in order to avoid numerical problems, and the process converges to the correct value.

This work follows the approach used by Dias (2015) to calculate dP and dE. The input values are not the steps *(m, n and o),* but the expected error ε. The steps in *P* and *E* were defined as percentages of accuracy of their respective current values. Thus, ε is the expected error, and the discretization is calculated as follows:

$\Delta P = P_0 \times \varepsilon$ (18)

$\Delta E = E_0 \times \varepsilon$ (19)

The non-negativity of the value p^0 in equation 15 is a necessary and sufficient condition to guarantee the stability of explicit DFM. Therefore, the value of Δt is given by

$$\Delta t = \frac{1}{\frac{\sigma_P^2 {P_{max}}^2}{\Delta P^2} + \frac{\sigma_E^2 \bar{E}^2}{\Delta E^2}} \quad (20)$$

Appendix 3 – Historical Series of Prices

The historical series of eucalyptus spot prices was obtained from the forestry sector of the Advanced Studies Centre in Applied Economics (CEPEA), a Brazilian institution. Monthly data for the Sorocaba region in São Paulo were collected in units of (R$/st), or real per stereo, a volume measure of wood in stacked logs. The unit of measurement was then converted to real per cubic metre (R$/m³). Figure A.1 shows a historical monthly series based on 204 observations spanning approximately 18 years, from June 1998 to April 2015. The deflation of the historical series of wood prices is graphically presented in Figure A.2.

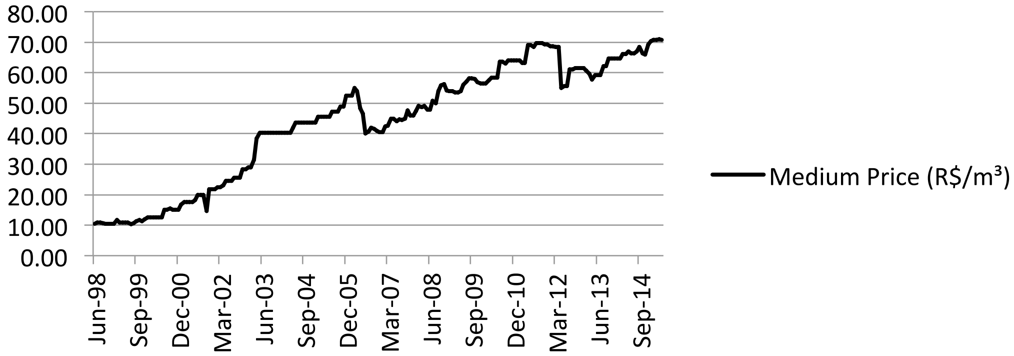

Figure A.1 – Historical average prices of eucalyptus in Brazil

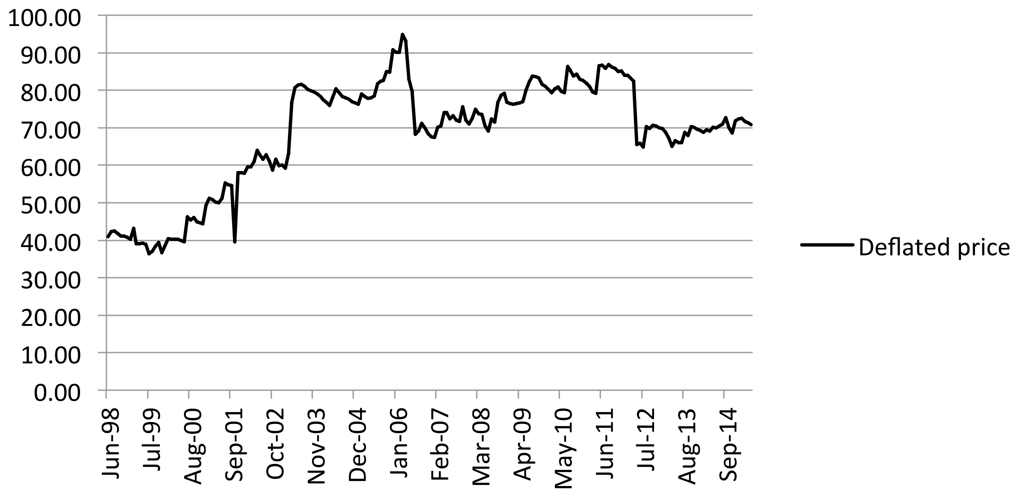

Figure A.2 – Historical deflated prices of eucalyptus in Brazil

Appendix 4 – Results presented by Dickey-Fuller test for price series using the EViews software

Null Hypothesis: LNPRECO has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=0)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2,17076	0,2177
Test critical values:	1% level	-3,462574	
	5% level	-2,875608	
	10% level	-2,574346	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LNPRECO)
Method: Least Squares
Date: 07/18/16 Time: 15:16
Sample (adjusted): 2 204
Included observations: 203 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNPRECO(-1)	-0,03197	0,014727	-2,17076	0,0311
C	0,13697	0,061958	2,210741	0,0282

R-squared	0,02291	Mean dependent var	0,00271
Adjusted R-squared	0,01805	S.D. dependent var	0,05211
S.E. of regression	0,05164	Akaike info criterion	-3,07931
Sum squared resid	0,53597	Schwarz criterion	-3,04667
Log likelihood	314,55	Hannan-Quinn criter.	-3,06611
F-statistic	4,7122	Durbin-Watson stat	2,35873
Prob(F-statistic)	0,03112		