Switching from Oil to Gas Production - Decisions in a Field's Tail Production Phase☆

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Abstract

We derive an optimal decision rule with regards to making an irreversible switch from oil to gas production. The approach can be used by petroleum field operators to maximize the value creation from a petroleum field with diminishing oil production and remaining gas reserves. Assuming that both the oil and gas prices follow a geometric Brownian motion we derive an analytical solution for the exercise threshold and demonstrate with numerical examples the threshold for a generic petroleum field. The analytical solution and the general results may also be relevant for other real options cases with similar features.

Keywords: *OR in energy, Switching option, Petroleum, Investment under uncertainty*

1. Introduction

At the Prudhoe Bay field in Alaska, one of the largest oil fields in North America, operators have increased the recovery factor substantially due to gas injection, together with other techniques (Szabo and Meyers (1993)). The associated gas being produced together with the oil is re-injected into the reservoir. As oil production from the field falls, a gas pipeline to export the gas is being discussed; necessary infrastructure for large-scale gas export is not currently present. In the North Sea, on the Norwegian Continental Shelf (NCS), substantial investments have been made in the Statfjord Latelife project on the

☆The authors would like to thank Øystein Dahl Hem, Alexander Svendsen, and Vidar Gunnerud for discussions on reservoir dynamics and petroleum field cases.

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Statfjord field. The investments, including a new pipeline which connects the gas exports of the field to the UK market, have changed the primary function of the production facilities from predominantly oil production to gas production. On the Oseberg field, also located on the NCS, gas injection has also been used to enhance oil production. The field has been in a phase of declining oil production for many years, often refered to as the “tail production phase”. Discussions are ongoing as to what the optimal course for future action should be and producing the injected gas is one of the considered alternatives.

Injection of natural gas is one of a number of techniques employed by operators of petroleum fields to increase the recovery rate of oil. The gas used for injection may be associated gas produced together with the oil, gas transported to the field from other sources, or a combination of the two. From a business point of view this makes sense as long as the value of continuing oil production under the gas injection scheme is higher than the alternative value of stopping the gas injection and investing in producing and exporting the gas that has been injected (the term “export” here means the transportation of the gas to a market). As the oil field matures, and the amount of oil in the reservoir as well as the oil production rate decline, it may become optimal to export the gas rather than continuing the injection scheme. This could involve substantial investments in both the production facilities and in export solutions for the gas, as well as having a strong adverse effect on the oil production. Therefore, determining the optimal timing to start gas production and export is relevant for a number of stakeholders in a petroleum field. For the operators and owners of petroleum fields such models can contribute to maximizing the value of the asset both for themselves and the society in which they operate. Also, policymakers can make use of such models to avoid value-erosive regulations or approval decisions.

The type of optionality considered here falls naturally into a category of real options called switching options. One of the earliest examples of a valuation model for a switching option is that of Brennan and Schwartz (1985) who, using a copper mine as an example, value the combined options to temporarily shut down, re-opening after a temporary shutdown, and abandoning entirely. Hahn and Dyer (2008), also studying an oil-to-gas switching option, propose a binomial lattice approach for modeling real options when the underlying uncertainty factors follow correlated one-factor mean reverting processes. Using the Prudhoe Bay field previously mentioned as their case, including a research and development program with uncertain outcome, they apply their proposed approach to value the asset. The focus in their study is on asset value, rather than on a tractable decision rule for making a switch. Adkins and Paxson (2011a,b) and Dockendorf and Paxson (2013) study different types of switching options and all arrive at what they term a "quasi-analytical solution" to the decision rule problem. By assuming that asset prices follow geometric brownian motions and that a smooth pasting condition[1] holds, their approach results in an equation set that the authors solve numerically. Gahungu and Smeers (2011) study the more general problem of finding the optimal time to exercise an option to exchange a basket of assets for another, assuming the asset prices follow correlated geometric brownian motions. They show that an equation set such as the ones Adkins and Paxson (2011a,b) solve numerically (in their "single switch/renewal opportunity" cases) can be determined in closed form. Such a solution to another real-option case is presented in Heydari et al. (2012), who derive the optimal time in closed form for when to invest in two different emission-reduction technologies, each with two separate and correlated uncertainty factors as part of the income stream.

We model the switching option as a perpetual American style option and the decision to switch is considered irreversible. Although the negative effects on oil production from starting gas production depend on the characteristics of the oil field, we assume that the remaining oil is lost if the decision to switch is made[2]. This is a conservative assumption which will emphasize the trade-off effect between the two resources in the model. However, it is possible to relax this assumption within the same model framework. On the basis of a parameter

[1]This principle is sometimes called *high contact* or *smooth fit*. See Brekke and Øksendal (1991) for an introduction to the concept as well as a proof of sufficient and necessary conditions for the smooth pasting condition to produce the optimal solution to the stopping problem.

[2]The effect of gas injection on the oil production rate is dependent on the reservoir properties of each field, and placement of injecting and producing wells. Assuming that oil production drops to zero when the gas is produced might be a fair approximation if the oil layer in the reservoir is thin, where many wells can move below the oil-water contact if this shifts slightly upwards. In fields where gas is mostly used for moving the oil towards the wells this might be a poor approximation and more complex reservoir models may be necessary.

set that describes a representative large size oil field (initial reserves of 100–500 mill. barrels of oil) in the North Sea, we derive the region of oil and gas prices for which it is optimal to undergo a switch. We contribute to the existing literature by determining and applying an analytical solution to the decision to change from oil to gas production in the tail production phase of a petroleum field. In contrast to previous work on this particular type of switching problem, we focus on the optimal timing of this switch rather than the valuation of the asset.

2. Price Dynamics and Option Specification

In this section we introduce the model for the dynamics of the prices of oil and gas, as well as the valuation model that we will use in order to optimize the value and derive the exercise threshold in the switching case.

Let X_I, $I \in \{1, 2\}$, denote the spot price for oil ($I = 1$) and gas ($I = 2$), respectively. Assuming that both of these prices follow a geometric Brownian motion, their dynamics under the risk neutral measure are described by the following stochastic differential equation:

$$dX_{I,t} = \alpha_I X_{I,t} dt + \sigma_I X_{I,t} dZ_{I,t}. \tag{1}$$

Here α_I is the risk adjusted drift, σ_I is the volatility, and $dZ_{I,t}$ is the increment of a standard Brownian process. We allow the prices of oil and gas to be dependent, introducing the correlation parameter ρ, where $Cov[dZ_1, dZ_2] = \rho\sigma_1\sigma_2 dt$ represents the covariance between the two Brownian motions (Z_1 and Z_2), and with $|\rho| \leq 1$.

Although the production profile for an oil field depends on the field's physical characteristics and the chosen depletion strategy, there are in general three phases of production; build-up, plateau and decline (see e.g. Wallace et al. (1987) for a discussion of aggregate production profiles and examples). As can be seen in Figure 1, both the Oseberg and Prudhoe Bay fields previously mentioned are examples of fields whose production profiles[3] exhibit the typical

[3]Sources for the production numbers are the Norwegian Petroleum Directorate for the Oseberg field and the State of Alaska, Department of Revenue for Prudhoe Bay. The Prudhoe data is for the fiscal year July-June and is converted from daily average in thousand barrels by assuming it is averaged across 365 days per year.

characteristics of these three phases. When we consider the option to switch to gas production we assume that this is only relevant in the decline phase. Although it is possible to consider stopping oil production during the build-up or plateau phase, it is highly unlikely. The model we propose therefore needs to include a decline in the oil production rate in order to capture the characteristics of a representative field. We assume in the following that the production rate is exponentially declining, very much in line with the shape of the production curves in Figure 1. An exponentially declining production rate is a standard simplifying assumption used in literature addressing decision making related to petroleum extraction (see e.g. Paddock et al. (1988) for an early example). For each commodity I, we assume that when production is ongoing the production rate $R_{I,t}$ is exponentially declining over time, i.e. $R_{I,t} = R_{I,0}e^{-\theta_I t}$. Here $R_{I,0}$ and θ_I are constants and the former is the initial production rate while the latter is the exponential decline factor of the production. Furthermore, we assume that the production costs, E_I, are independent of the production rate, i.e. that the total costs of operation are fixed. Thus, the cash flow from production, when producing commodity I, is given by $(X_{I,t}R_{I,t} - E_I)dt$. Note that taxes and royalties are ignored, although this assumption can be relaxed.

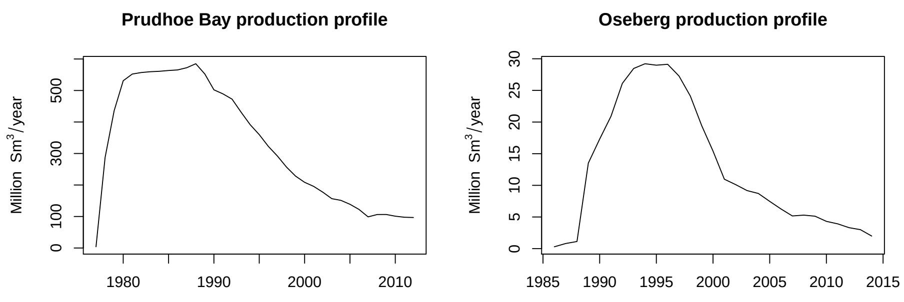

Figure 1: Historical oil production profiles for the Prudhoe and Oseberg fields

2.1. Switching Option

We let $F(\tau, x_1, x_2)$ denote the value of a petroleum field if it is decided to switch to gas production at time $t = \tau$:

$$\begin{aligned} F(\tau, x_1, x_2) =& \mathbb{E}\Bigg[\int_0^\tau (X_{1,t}R_{1,t} - E_1)\, e^{-rt} dt \\ &+ \int_\tau^\infty (X_{2,t}R_{2,t} - E_2)\, e^{-rt} dt - e^{-r\tau} S\Bigg] \\ =& \mathbb{E}\Bigg[\int_0^\tau X_{1,t}e^{-\theta_1 t} R_{1,0}e^{-rt} dt + \int_\tau^\infty X_{2,t}e^{-\theta_2 t}R_{2,0}e^{-rt}dt \\ &+ e^{-r\tau}\left(-S + \frac{E_1 + E_2}{r}\right)\Bigg] - \frac{E_1}{r}. \end{aligned}$$

Here x_i[4] is the current price of oil ($i = 1$) and gas ($i = 2$), S denotes the switching cost of converting from oil to gas production, and r is the risk free rate. For simplicity the switching is assumed to happen instantaneously and all of the switching costs are incurred immediately if the decision to switch is made. Note also that as long as oil is being extracted from the field, the oil production rate declines exponentially (at rate θ_1); however, when the switching occurs and gas production starts, the production rate for gas starts declining exponentially at rate θ_2. This means that the "potential" gas production rate is constant as long as no gas is being produced.

The optimal value of the field is now given by

$$V(x_1, x_2) = sup_\tau F(\tau, x_1, x_2), \tag{2}$$

where V has to satisfy, in the continuation region, the partial differential equation

$$-rV(x_1, x_2) + LV(x_1, x_2) + x_1R_{1,0} - E_1 = 0, \tag{3}$$

with the infinitesimal generator L given by:

$$\begin{aligned} LV(x_1, x_2) =& \frac{1}{2}\sigma_1^2 x_1^2 \frac{\partial^2 V(x_1, x_2)}{\partial x_1^2} + \frac{1}{2}\sigma_2^2 x_2^2 \frac{\partial^2 V(x_1, x_2)}{\partial x_2^2} \\ &+ \rho\sigma_1\sigma_2 x_1 x_2 \frac{\partial^2 V(x_1, x_2)}{\partial x_1 \partial x_2} + (\alpha_1 - \theta_1)x_1 \frac{\partial V(x_1, x_2)}{\partial x_1} + \alpha_2 x_2 \frac{\partial V(x_1, x_2)}{\partial x_2}. \end{aligned} \tag{4}$$

[4] In the rest of the paper we denote random variables by an uppercase letter, while their realizations will be denoted by a lowercase letter.

We remark again that the production rate for gas only declines when one starts extracting gas, and therefore in the continuation region (i.e., before the switching) there is no declining behavior. Consequently, in the continuation region, the drift for the gas price is just α_2, whereas for the oil price it is $\alpha_1 - \theta_1$.

We propose a solution to (3) with the following functional form:

$$v(x_1, x_2) = A x_1^\beta x_2^\eta + \frac{x_1 R_{1,0}}{r + \theta_1 - \alpha_1} - \frac{E_1}{r}, \tag{5}$$

where A, β and η are parameters still to be determined. The first term of the solution is the switching option value while the second and third terms represent the present value of perpetual oil production. Computing the derivatives of this function, and plugging into (3), we conclude that β and η are the roots of the following characteristic root equation:

$$\frac{1}{2}\sigma_1^2\beta(\beta-1) + \frac{1}{2}\sigma_2^2\eta(\eta-1) + \rho\sigma_1\sigma_2\beta\eta + (\alpha_1 - \theta_1)\beta + \alpha_2\eta - r = 0. \tag{6}$$

Note that as $\lim_{x_1\to\infty} v(x_1, x_2) = 0$, it means that $\beta < 0$, whereas $\lim_{x_2\to\infty} v(x_1, x_2) = \infty$ means that $\eta > 0$.

We let x_1^* and x_2^* denote the threshold switching values for the processes X_1 and X_2 respectively. Then the value-matching condition that must hold at the switching threshold results in:

$$A {x_1^*}^\beta {x_2^*}^\eta + \frac{x_1^* R_{1,0}}{r + \theta_1 - \alpha_1} - \frac{E_1}{r} = \frac{x_2^* R_{2,0}}{r + \theta_2 - \alpha_2} - \frac{E_2}{r} - S, \tag{7}$$

and the smooth-pasting conditions are

$$A\beta {x_1^*}^{\beta-1} {x_2^*}^\eta + \frac{R_{1,0}}{r + \theta_1 - \alpha_1} = 0, \tag{8}$$

$$A\eta {x_1^*}^\beta {x_2^*}^{\eta-1} = \frac{R_{2,0}}{r + \theta_2 - \alpha_2}. \tag{9}$$

This implies that

$$-\frac{R_{1,0} x_1^*}{\beta(r + \theta_1 - \alpha_1)} = \frac{R_{2,0} x_2^*}{\eta(r + \theta_2 - \alpha_2)}, \tag{10}$$

and therefore

$$x_1^* = -\frac{\beta(r + \theta_1 - \alpha_1)}{\eta(r + \theta_2 - \alpha_2)} \frac{R_{2,0}}{R_{1,0}} x_2^*, \tag{11}$$

$$A = -\frac{R_{1,0}}{\beta(r + \theta_1 - \alpha_1){x_1^*}^{\beta-1}{x_2^*}^\eta}. \tag{12}$$

Using these results in the value-matching condition, we derive the following useful relation:

$$x_1^* \frac{R_{1,0}}{r+\theta_1-\alpha_1}\left(\frac{\eta+\beta-1}{\beta}\right) + S - \frac{E_1-E_2}{r} = 0. \tag{13}$$

This leads to an equation set with the following three equations which must be solved to find the switching threshold:

$$\frac{1}{2}\sigma_1^2\beta(\beta-1) + \frac{1}{2}\sigma_2^2\eta(\eta-1) + \rho\sigma_1\sigma_2\beta\eta + (\alpha_1-\theta_1)\beta + \alpha_2\eta - r = 0, \tag{14}$$

$$x_1^* = -\frac{\beta(r+\theta_1-\alpha_1)}{\eta(r+\theta_2-\alpha_2)}\frac{R_{2,0}}{R_{1,0}}x_2^*, \tag{15}$$

$$x_1^* \frac{R_{1,0}}{r+\theta_1-\alpha_1}\left(\frac{\eta+\beta-1}{\beta}\right) + S - \frac{E_1-E_2}{r} = 0. \tag{16}$$

This equation set is very similar to those stated in Adkins and Paxson (2011a, eq. 2.4, 3.4 and 3.5) and Adkins and Paxson (2011b, eq. 4, 15 and 20) and these are solved numerically by the authors. However, as was shown for the general case by Gahungu and Smeers (2011) and for a particular two-dimensional case by Heydari et al. (2012), the set should have an analytical solution. Note that there are four unknowns (x_1^*, x_2^*, β, and η) and three equations in this equation set. Although this seemingly makes the solution indetermined that is not the case. The solution we are looking for is not a particular point, but rather pairs of critical oil and gas prices. When determining whether it is optimal to switch it only makes sense to consider the two prices jointly and therefore we can first assume a critical oil/gas price and find the corresponding critical gas/oil price. Analytical solutions for η and β could be expressed in terms of either x_1^* or x_2^*. However, one can chose so that the model can be interpreted unambiguously for all prices. Consider the following expression:

$$C(x_1^*) \equiv 1 + \left[\frac{r+\theta_1-\alpha_1}{x_1^* R_{1,0}}\right]\left[S - \frac{(E_1-E_2)}{r}\right]. \tag{17}$$

When $\left[S - \frac{(E_1-E_2)}{r}\right] > 0$ it means that $C(x_1^*) > 1$ and that the switching threshold intercepts the gas price axis. Therefore the model is in this case defined for all oil prices, but not for all gas prices. We assume this condition is satisfied in the following, but will also show a solution for the alternative case ($C(x_1^*) < 1$). It can then be shown that the solution to $\beta(x_1^*)$ from the above equation set must be

$$\beta(x_1^*) = \frac{f(x_1^*)}{2g(x_1^*)} - \sqrt{\left(\frac{f(x_1^*)}{2g(x_1^*)}\right)^2 + 2\frac{(r-\alpha_2)}{g(x_1^*)}}, \tag{18}$$

where $f(x_1^*) \equiv \sigma_1^2 - 2(\alpha_1 - \theta_1) - 2\rho\sigma_1\sigma_2 + C(x_1^*)(2\alpha_2 + \sigma_2^2)$ and $g(x_1^*) \equiv \sigma_1^2 + \sigma_2^2 C(x_1^*)^2 - 2\rho\sigma_1\sigma_2 C(x_1^*)$. Assuming that $r > \alpha_2$ (otherwise it is never optimal to exercise the option) it must always be true that

$$\frac{f(x_1^*)}{2g(x_1^*)} < \sqrt{\left(\frac{f(x_1^*)}{2g(x_1^*)}\right)^2 + 2\frac{(r-\alpha_2)}{g(x_1^*)}}, \tag{19}$$

when $g(x_1^*) > 0$. Recognizing that $g(x_1^*)$ is equivalent to a weighted variance expression, $\mathrm{Var}\,(x_2 C(x_1^*) - x_1 | \, x_1^*)$, and that variances for non-constant variables are stricly positive (i.e. $g(x_1^*) > 0$), then it must also be true that $\beta(x_1^*) < 0$ for all values of x_1^*. Rearranging (16) shows that $\eta(x_1^*) = 1 - \beta(x_1^*)C(x_1^*)$ and that the parameter η no longer needs to explicitly be part of the analytical solution. It follows that $\eta(x_1^*) > 1$ (since $\beta(x_1^*) < 0$ and $C(x_1^*) > 1$) and that $\eta(x_1^*) + \beta(x_1^*) > 1$:

$$\eta(x_1^*) + \beta(x_1^*) = 1 - \beta(x_1^*)(C(x_1^*) - 1) > 1 \tag{20}$$

This result is the same as in switching option cases studied by Adkins and Paxson (2011a,b). The analytical solutions for $x_2^*(x_1^*)$ and $A(x_1^*)$, expressed as functions of x_1^*, are found by substituting η with $1 - \beta(x_1^*)C(x_1^*)$ in (12) and (15) and rearranging the latter expression:

$$x_2^*(x_1^*) = -\frac{(1 - \beta(x_1^*)C(x_1^*))(r + \theta_2 - \alpha_2)R_{1,0}}{\beta(x_1^*)(r + \theta_1 - \alpha_1)R_{2,0}} x_1^* \tag{21}$$

$$A(x_1^*) = -\frac{R_{1,0}}{\beta(x_1^*)(r + \theta_1 - \alpha_1){x_1^*}^{\beta(x_1^*)-1} x_2^*(x_1^*)^{1-\beta(x_1^*)C(x_1^*)}}. \tag{22}$$

Note that if $\left[S - \frac{(E_1 - E_2)}{r}\right] < 0$ (i.e. $C(x_1^*) < 1$ and the threshold has an intercept on the oil price axis) the solution is defined for all gas prices, but not all oil prices. If this is the case, and to make sure that the solution is defined for all prices, similar expressions can be found for a given x_2^* (see Appendix A for this version of the solution). In the special case that $\left[S - \frac{(E_1 - E_2)}{r}\right] = 0$ the problem collapses to a version of the solution derived by McDonald and Siegel (1986), where all the parameters are constant. Another special version of the suggested solution is found when the present values of producing oil or gas (rather than the prices themselves) are assumed to follow geometric Brownian motions, and the production decline rates are set to zero (this ensure that the dynamics of the present value of gas is the same in the stopping and continuation region). In this scenario, the payout from the option is equal to the difference between the value of two assets following a geometric Brownian motion, minus a fixed switching cost. In the finance literature this is often refered to as a spread option. Using the same approach as outlined above for the switching option, an analytical solution can be expressed for the exercise threshold of a perpetual spread option (see Appendix B for this version of the solution).

3. Numerical Examples

The numerical examples are constructed around a base case for the switching option. Parameter values for the base case are chosen to reflect a "representative" case for a large size (initial reserves of 100–500 mill. barrels of oil) oilfield in the North Sea. This means that oil and gas prices from this region are used to estimate price process parameters. The example case is considered to be an offshore field in the decline phase. Therefore, the decline rate of production should be realistic for a representative offshore field in the North Sea. The International Energy Agency (IEA (2008)) estimates the average decline rate post-plateau to 15.5% for OECD Europe (only North Sea fields included). Based on this study we assume a 15.5% decline rate for both oil and gas in the base case.

3.1. Price process parameters

The data used for estimating the price process parameters are daily observations of futures prices from the Intercontinental Exchange (ICE) for the time period August 12th 2010 to June 16th 2015. For the oil prices the Brent crude futures are used and for the gas prices we use UK Natural gas futures. As a proxy for the spot price for oil and gas the front month contract price is used. The gas prices, which are quoted in GBP, are converted to USD using USD/GBP forward rates quoted by Thomson Reuters. When annualizing the volatility estimates, 251 tradings days per year is assumed. Moreover, since the estimates for volatility are conducted using log returns on the data, we adjust for rollover effects. Table 1 summarizes the estimation results.

Table 1: Estimated price process parameters

	Estimated values(S.E.)
α_1	0.0042(0.0013)
σ_1	0.3380(0.0056)
α_2	0.0051(0.0007)
σ_2	0.2674(0.0054)
ρ	0.1838(0.0278)

To estimate the risk adjusted drifts, a pair of futures were chosen for each commodity such that the difference in time to maturity between the two contracts is constant. We use the 12th position future relative to the observation day (approximately one year to maturity) and the 36th position (approximately 3 years to maturity) with a constant 2 year timespan between them in terms of time to maturity. Using no-arbitrage arguments, it is assumed that futures prices are equal to the risk adjusted expected spot prices. Since we assume geometric Brownian motion, the following must therefore hold true: $\alpha_i = \frac{\ln\left(\frac{F_{s,T}}{F_{s,t}}\right)}{T-t}$. Here α_i is the risk adjusted drift for commodity i, T and t are times of maturity with $T > t$, so that $F_{s,T}$ is a contract with a longer time to maturity than $F_{s,t}$, and finally $s < t$ is the time of observation. Using this relationship to calculate observed α_i for both oil and gas the risk adjusted drifts α_1 and α_2 are estimated as the mean of each observed set respectively.

3.2. Switching Option

For the numerical results a set of parameters, summarized in Table 2, are assumed in the base case for the switching option. In the following, the effects of changing key parameters of the model is demonstrated through a sensitivity analysis. Unless otherwise noted, only one parameter at the time is allowed to change and the other parameter values are assumed equal to those set in the base case.

The current switching threshold for the base case, as well as the thresholds one and five years ahead (assuming a deterministic and exponential decline in the oil production) are shown in Figure 2. The thresholds should be interpreted such that for a given oil price, it is optimal to switch to gas production if the market price of gas is above the corresponding critical gas price. Alternatively, for a given gas price it is optimal to switch from oil to gas production if the price of oil drops below the critical price.

Table 2: Base case parameters switching option

	Values	Units	Description
$R_{1,0}$	2.0	mill. Sm^3	Yearly oil production
$R_{2,0}$	15	bill. Sm^3	Yearly gas production
θ_1	0.155		Oil production decline rate
θ_2	0.155		Gas production decline rate
E_1	500	mill. USD	Yearly oil production costs
E_2	500	mill. USD	Yearly gas production costs
r	0.03		Risk free rate
S	1000	mill. USD	Cost of switching

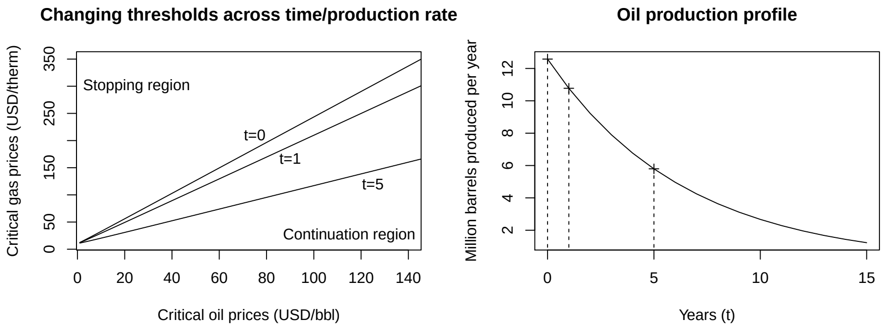

Figure 2: Switching threshold across time/production rate for the base case

The changing threshold across time due to the deterministic decrease in production is similar to the effect of changing the initial production $R_{1,0}$. Changing the oil production (either the initial production or as an effect of the deterministic decline rate) produces a monotonic change in the entire threshold, decreasing the size of the continuation region as production decreases. Changing other parameters of the model can produce more complicated results. Consider the drift rates α_1 and α_2 of the oil and gas prices. For the range of values illustrated in Figure 3 the continuation region always decreases when either of the drift rates decrease. However, while this monotonic behavior is always true for α_1 this is not the case for α_2. As the drift rate of gas decreases the continuation region also always decreases given $\alpha_2 > 0$, but for some negative value of α_2 the behavior is reversed. This occurs due to two competing effects when α_2 is decreasing; the present value of gas production decreases (switch "later"), and expected future gas price decrease (switch "earlier"; standard option pricing result). The absolute changes in the threshold values are also much more sensitive to changes in α_2 than in α_1.

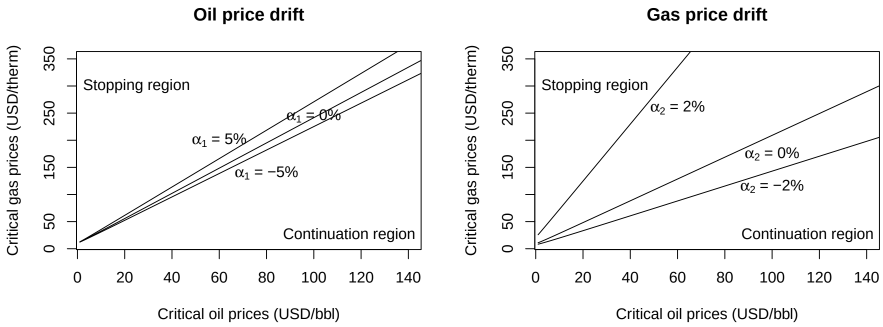

Figure 3: Effects on the switching threshold from changing the drift rate parameters

Increasing the volatilities of either the gas or oil price *generally* increases the volatility of the payout from the switching option. This is always the case when the correlation $\rho \leq 0$. However, when $\rho > 0$ increasing one of the volatilities can have a negative effect for very low volatility values. This effect is due to the fact that the payout of the switching option is a function of the difference between two stochastic elements (the present values of gas and oil) and the variance expression for such a payout has a negative term for the covariance/correlation. In general, when the volatility of the payout of the function increases the value of the option increases and consequently the continuation region for the option should increase. These effects are in line with the observations made by Adkins and Paxson (2011b) and McDonald and Siegel (1986). The effects on the switching threshold of changing the volatility levels are illustrated in Figure 4, both for the base case and for $\rho = 1$.

The effects of changing some of the other key parameters to the model; θ_2, r, S, and ρ, are summarized and depicted in Figure 5. Increasing either the switching cost S or the gas production decline rate θ_2 both increase the size of the continuation region. The intuition is straightforward; both of these effects decrease the value received when switching, making a switch to gas less valuable in general. Increasing the correlation ρ or the risk free discount rate r decreases the size of the continuation region. The effect from correlation can be interpreted

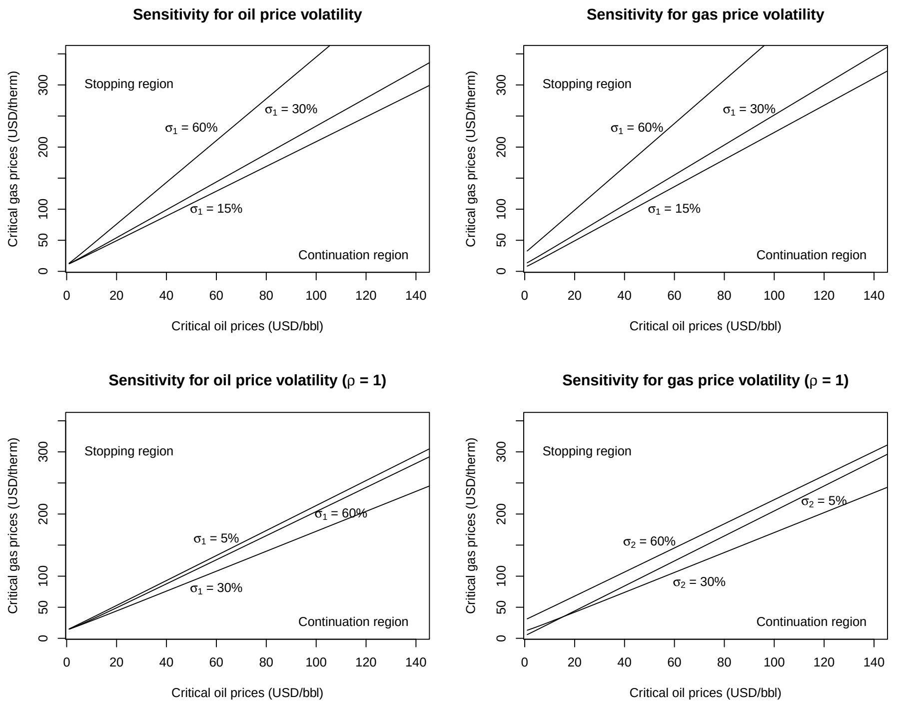

Figure 4: Effects on the switching threshold from changing the volatility parameters

as a volatility effect; increasing the correlation decreases the volatility of the payout of the option and therefore the continuation region shrinks. Although the effect of increasing r is also a monotonically shrinking continuation region, the interpretation is not straightforward. This effect changes both the present value of gas production and oil production, as well as the discount rate for the option payout.

4. Conclusion

We propose a model to determine the optimal time to switch from oil to gas production. Assuming that the oil and gas prices follow geometric Brownian motions with correlated increments, we derive an analytical solution for the switching strategy. The analytical solution, or some version of it, can be

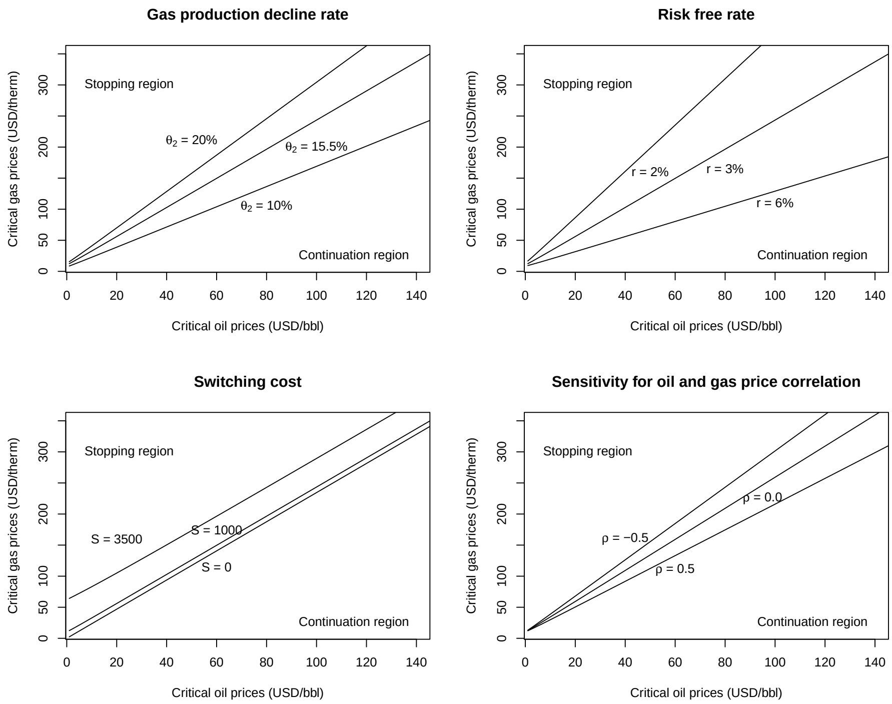

Figure 5: Effects on the switching threshold from changing key parameters

applicable to other options with similar features as the switching option considered here. We implement the model using numerical examples, focusing on the effects on the threshold prices from changing key parameters to the models. This approach can be used to maximize the value-creation from aging oil fields with remaining gas reserves.

Appendix A. Analytical solution expressed in gas prices

Defining

$$C_2(x_2^*) \equiv 1 + \left[\frac{r+\theta_2-\alpha_2}{x_2^* R_{2,0}}\right]\left[\frac{(E_1-E_2)}{r} - S\right] \tag{A.1}$$

then $\beta(x_2^*)$ is given analytically by

$$\beta(x_2^*) = \frac{m(x_2^*)}{2n(x_2^*)} - \sqrt{\left(\frac{m(x_2^*)}{2n(x_2^*)}\right)^2 + \frac{(2(C_2(x_2^*)^2 r - \alpha_2 C_2(x_2^*)) + C_2(x_2^*)\sigma_2^2 - \sigma_2^2)}{n(x_2^*)}}, \tag{A.2}$$

where $m(x_2^*) \equiv C_2(x_2^*)^2\sigma_1^2 - 2(\alpha_1-\theta_1) - 2C_2(x_2^*)\rho\sigma_1\sigma_2 + 2\alpha_2 + \sigma_2^2$ and $n(x_2^*) \equiv C_2(x_2^*)^2\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$. Assuming that $C_2(x_2^*) > 1$ (i.e. $C(x_1^*) < 1$) and $r > \alpha_2$ then it follows that $\beta(x_2^*) < 0$ as long as $n(x_2^*) > 0$. Recognizing that $n(x_2^*) \geq \mathrm{Var}\,(x_2 - x_1 |\, x_2^*)$, and that variances for non-constant variables are stricly positive (i.e. $n(x_2^*) > 0$), then it must also be true that $\beta(x_2^*) < 0$ for all values of x_2^*. Using the same assumptions it can be shown that $\eta(x_2^*) = \frac{1-\beta(x_2^*)}{C_2(x_2^*)}$ and it then follows that $\eta(x_2^*) > 0$ and $\eta(x_2^*) + \beta(x_2^*) < 1$. The analytical solutions for $x_1^*(x_2^*)$ and $A(x_2^*)$, expressed as functions of x_2^*, are:

$$x_1^*(x_2^*) = -\frac{\beta(x_2^*)(r+\theta_1-\alpha_1)R_{2,0}}{((1-\beta(x_2^*))/C(x_2^*))(r+\theta_2-\alpha_2)R_{1,0}} x_2^* \tag{A.3}$$

$$A(x_2^*) = -\frac{R_{1,0}}{\beta(x_2^*)(r+\theta_1-\alpha_1)x_1^*(x_2^*)^{\beta(x_2^*)-1} {x_2^*}^{(1-\beta(x_2^*))/C_2(x_2^*)}}. \tag{A.4}$$

Appendix B. Analytical solution for the spread option

Assume that two asset prices X_1 and X_2 follow correlated geometric Brownian motions under the risk neutral measure:

$$dX_{I,t} = \alpha X_{I,t}dt + \sigma_I X_{I,t}dZ_{I,t}, \tag{B.1}$$

using similar notation as in section 2. Consider an option with no maturity date that would give a payout of $(X_2 - X_1 - S)$ if exercised, where S is a constant strike price. Assume further that the value of such an option can be expressed in the functional form

$$v_s(x_1, x_2) = Bx_1^{\gamma}x_2^{\nu}, \tag{B.2}$$

where the subscript s is used to signify that this is the value of a spread option. Following the same type of argument as in section 2, the exercise threshold for the option can be determined by the characteristic root equation

$$\frac{1}{2}\sigma_1^2\gamma(\gamma-1) + \frac{1}{2}\sigma_2^2\nu(\nu-1) + \rho\sigma_1\sigma_2\gamma\nu + \alpha_1\gamma + \alpha_2\nu - r = 0. \tag{B.3}$$

and the value-matching and smooth-pasting conditions:

$$Bx_1^{*\gamma}x_2^{*\nu} = x_2^* - x_1^* - S \tag{B.4}$$

$$B\gamma x_1^{*\gamma-1}x_2^{*\nu} = -1 \tag{B.5}$$

$$B\nu x_1^{*\gamma}x_2^{*\nu-1} = 1, \tag{B.6}$$

where x_1^* and x_2^* denote the threshold switching values for the processes X_1 and X_2 respectively. Solving this equation set, the analytical solution to γ, given a value for x_1^*, is:

$$\gamma(x_1^*) = \frac{p(x_1^*)}{2q(x_1^*)} - \sqrt{\left(\frac{p(x_1^*)}{2q(x_1^*)}\right)^2 + 2\frac{(r-\alpha_2)}{q(x_1^*)}}, \tag{B.7}$$

where $p(x_1^*) \equiv \sigma_1^2 - 2\alpha_1 - 2\rho\sigma_1\sigma_2 + (1 + S/x_1^*)(2\alpha_2 + \sigma_2^2)$ and $q(x_1^*) \equiv \sigma_1^2 + \sigma_2^2(1+S/x_1^*)^2 - 2\rho\sigma_1\sigma_2(1+S/x_1^*)$. Given the assumption that $r > \alpha_2$ (otherwise the option is never exercised) it must always be true that $\gamma(x_1^*) < 0$ for all values of x_1^*. It can also be shown that $\nu(x_1^*) = 1 - \gamma(x_1^*)(1 + S/x_1^*) > 1$ and $\gamma(x_1^*) + \nu(x_1^*) > 1$. The analytical solutions for $x_2^*(x_1^*)$ and $B(x_1^*)$, expressed as functions of x_1^*, are:

$$x_2^*(x_1^*) = -\frac{1-\gamma(x_1^*)(1+S/x_1^*)}{\gamma(x_1^*)} x_1^* \tag{B.8}$$

$$B(x_1^*) = -\frac{1}{\gamma(x_1^*) {x_1^*}^{\gamma(x_1^*)-1} {x_2^*(x_1^*)}^{1-\gamma(x_1^*)(1+S/x_1^*)}}. \tag{B.9}$$

References

Adkins, R., Paxson, D., 2011a. Reciprocal energy-switching options. Journal of Energy Markets 4 (1), 91–120.

Adkins, R., Paxson, D., 2011b. Renewing assets with uncertain revenues and operating costs. Journal of Financial and Quantitative Analysis 46 (03), 785–813.

Brekke, K. A., Øksendal, B., 1991. The high contact principle as a sufficiency condition for optimal stopping. In: Lund, D., Øksendal, B. K. (Eds.), Stochastic Models and Option Values: Applications to Resources, Environment, and Investment Problems. North-Holland, pp. 187–208.

Brennan, M., Schwartz, E., 1985. Evaluating natural resource investments. Journal of Business 58 (2), 135–157.

Dockendorf, J., Paxson, D., 2013. Continuous rainbow options on commodity outputs: what is the real value of switching facilities? The European Journal of Finance 19 (7-8), 645–673.

Gahungu, J., Smeers, Y., 2011. Optimal time to invest when the price processes are geometric brownian motions. A tentative based on smooth fit. CORE Discussion Paper (2011/34), Universit catholique de Louvain, Center for Operations Research and Econometrics (CORE).

Hahn, W. J., Dyer, J. S., 2008. Discrete time modeling of mean-reverting stochastic processes for real option valuation. European Journal of Operational Research 184 (2), 534–548.

Heydari, S., Ovenden, N., Siddiqui, A., 2012. Real options analysis of investment in carbon capture and sequestration technology. Computational Management Science 9 (1), 109–138.

IEA, 2008. World energy outlook. International Energy Agency.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. The Quarterly Journal of Economics, 707–728.

Paddock, J. L., Siegel, D. R., Smith, J. L., 1988. Option valuation of claims on real assets: The case of offshore petroleum leases. The Quarterly Journal of Economics, 479–508.

Szabo, J., Meyers, K., 1993. Prudhoe Bay: development history and future potential. In: SPE Western Regional Meeting. Society of Petroleum Engineers.

Wallace, S. W., Helgesen, C., Nystad, A. N., 1987. Generating production profiles for an oil field. Mathematical Modelling 8, 681–686.