Feeder Cattle Options: Where is the Beef?

Gláucia Fernandes[a,*], Luiz Brandão[a], Carlos Bastian Pinto[b]

[a]*Pontifical Catholic University of Rio de Janeiro*
[b]*Ibmec Business School*

Abstract

Feeder cattle is one of the most important sectors of the Brazilian livestock industry. Nonetheless, this sector is subject to many uncertainties, which requires firms to use risk management tools such as options. In this paper we discuss the real and financial options available to a feeder cattle producer, and analyse the option to switch livestock sales between the spot and the futures market. We also analyze put options contracts traded in the futures market of feeder cattle and compare prices derived from different option pricing models. We use actual put options prices on livestock futures from the BM&FBovespa, which is the main stock exchange in Brazil, and analyze if these are priced according to what classic models suggest. For the pricing of livestock options, we give particular attention to the results considering different types of volatility, different maturity months, degrees of moneyness and different maturity dates. Tests of mean differences are conducted to examine if there are statistically significant differences between the Longstaff and Schwartz, Barone-Adesi and Whaley, Bjerksund and Stensland and Cox, Ross and Rubinstein models. The results indicate that the switch is a valuable option, that prices derived from the theoretical models are close to the realized prices, and that the best pricing is obtained with the use of the implied volatility.

Keywords: Derivatives, feeder cattle, uncertainty, risk, option pricing

1. Introduction

Brazil is the second largest producer of beef in the world and was, according to the Confederation of Agriculture and Livestock of Brazil - CNA (2016), and the only segment

*Corresponding author at: Pontifical Catholic University of Rio de Janeiro, Brazil, Marquês de São Vicente, 225 - Gávea, Rio de Janeiro, 22451-900, Tel.: +55 21 21389358.

Email addresses: glaucia.fernandes@iag.puc-rio.br (Gláucia Fernandes), brandao@iag.puc-rio.br, CNPq-Brazil Scholarship 305422/2014-6 (Luiz Brandão), carbastian@gmail.com (Carlos Bastian Pinto)

of the Brazilian economy to show growth and a surplus in the trade balance of 2015. The growth of this sector can be observed by the volume of beef exports in February 2016, which had a 27% increase over the 78,000 tons of beef exported in January of the same year (CEPEA/UPS, 2016).

The growth of this commodity, however, is accompanied by numerous risks inherent to it, such as: (i) fluctuations in the price of beef and chicken (substitute goods); (ii) costs of inputs; (iii) interruption of exports of Brazilian beef due to the spread of disease; (iv) foreign exchange risk; (v) inability to cover operating costs; (vi) credit risk; (vii) difficulties in stock formation; (viii) the risk of production loss and others operating risks, and (ix) base risk, which is the difference between spot price and futures price. In the harvest months (December to May) there is a strengthening of the base, i.e., the difference of the base decreases; On the other hand, in the off season (June to November) there is a weakening of the base, i.e., the difference of the base increases.

In order to protect themselves against this risk, managers can resort to the use of derivative instruments, such as options which are contracts that provide the right to buy or sell an asset at a predetermined future date for a set price. According to Hull (2006), the derivatives market is widely used for hedging purposes, in particular for problems arising from financial risks and the seasonality of agricultural commodities (Lima et al., 2007). Derivative markets have expanded significantly in the last years due to the strong volatility of exchange rates and interest rate (Farhi, 2016). Derivative instruments include futures contracts, forward contracts, swaps and options market, each of which has specific characteristics. Options, for example, come in two categories: real options and the financial options.

Real options are opportunities to purchase real assets on favorable terms at a future time (Myers, 1977). According to Trigeorgis and Reuer (2017) these favorable terms hinge on adjustment costs, market power, or other imperfections in product or factor markets. Some authors such as Cunha et al. (2014) and Fernandes et al. (2015) analyze the real options associated with the market for livestock. Cunha et al. (2014) used the real option approach to analyze the economic feasibility of a feeder cattle experimental feedlot, and use the Black and Scholes (1973) and Merton (1973) (BSM) model to price this option. The study concluded that confinement is a financially attractive alternative. A similar analysis was made by Bastian-Pinto et al. (2015). Fernandes et al. (2015), on the other hand, evaluated the real option of storing bio-gas from swine biomass. According to the authors, storing bio-gas for future sale in the spot market can increase the revenue of rural investors who have in place power generation infrastructure and a connection to the grid.

Financial options contracts, on the other hand, are options on financial, as opposed to real assets, and are an important risk management mechanism, but the use of livestock options is scarce due to the unfamiliarity of the agents with option pricing models. The literature on financial options is vast. Vitiello Jr (2000) compared the option models of Black and Scholes (1973) and Merton (1973) (BSM), to the binomial model of Cox et al. (1979) (CRR) to determine which one would better approximate traded market values. The author reached the conclusion that option values at the Bolsa de Mercadorias & Futuros (BM&F Bovespa) from July 1994 to June 1997 were closer to the BSM model. Silva and Maia (2011) and Tonin and Coelho (2012) conducted a study on Arabica coffee pricing in the futures market of BM&FBovespa. Silva and Maia (2011) based their analysis exclusively on the BSM model, considering three different ways of estimating volatility. The tests showed that the implied volatility generated results closer to market prices. Tonin and Coelho (2012), on the other hand, tested the models of Black (1976), Barone-Adesi and Whaley (1987), Bjerksund and Stensland (1993) and the binomial and trinomial models for different calculations of volatility. The results showed that the best performance for option pricing occurred with the implied volatility. Regarding the pricing models, the best performance was achieved with the Barone-Adesi and Whaley (1987) model.

Clemente and Mattos (2011) compared the prices obtained in two models (Binomial and BSM) to those trading on the BM&FBovespa futures feeder cattle market. For comparison, the authors performed the tests with the use of two different types of volatility: historical and implied volatility. They concluded that the BSM model presented a better result compared to the binomial model for calls. For puts, the binomial model presented a better result. Close to this work, Pontes et al. (2013) analyzed the results of pricing feeder cattle options with the BSM model, for different types of volatility (historical, implicit and deterministic) and concluded that the pricing model with historical volatility performed better. Regarding the future market, Fraga and Neto (2016) test the existence of a long-term relationship between the physical and future markets of soybeans and also test the hypothesis of efficiency of the future price of this commodity. The results showed that the hypothesis of efficiency of the short-run market was rejected, but the existence of a long-term relationship between the spot and future series of soybeans was confirmed.

According to Correa et al. (2014), the uncertainties regarding the prices that will be practiced in the commercialization of production expose the agents of the productive chain of feeder cattle to risks that can compromise the results of the activity. In this sense, knowing the behavior of the prices of feeder cattle in the market is an important differential for the planning of cattle production and the definition of trading strategies. Moreover, in view of the great representativeness of livestock in the economic context, the analysis of the behavior of cattle prices within the production chain is extremely important, since it is indispensable to the correct planning of the activity. Thus, it becomes evident the producer's need to find ways to protect himself against possible price fluctuations and use the most appropriate mechanisms for risk management (Guerra et al., 2013).

In this article we analyze the different types of options available to the feeder cattle producer. We consider that the producer has five real options and one financial option. Under the former, first he has the option to choose between producing milk or beef. Second, if he chooses beef he has the option of fattening the cattle in pasture or through confinement. Third, if he chooses confinement, he has the option to determine the best time to confine the cattle. He also has the option to choose the best time for sale and, finally, if the sale will be held in the spot market or in the futures market, which is a real option to switch between these two markets.

Regarding the financial option, the producer can buy put options in the future market. In this sense, we analyze different American option pricing models and examine whether these models differ significantly from traded market prices for feeder cattle put options on the BM&FBovespa for the period from January 2000 to February 2016. This period is important because according to Sachs and Pinatti (2015), the economic scenario changed since the Plano Real in 1994, whose consequences for the cattle ranchers were the revision of the productive chain. The producers started to face a tighter margin of profit in which the planning of the activity must be done in both the cost and the revenue. Also, since the 1990s, government policy began to provide new forms of support for agriculture. It was decided to create more modern instruments in partnership with a private initiative. Among them the put option contracts was implemented in the form of a price insurance for the rural producer (Pereira et al., 2015).

Note that feeder cattle options in futures market are American type options, i.e., it may be exercised by the holder at any time up to the maturity date. European options, on the other hand, can only be exercised at maturity. We consider the prices determined by the models of Longstaff and Schwartz (2001), Barone-Adesi and Whaley (1987), Bjerksund and Stensland (1993) and Cox et al. (1979), and show the results for different strata of the sample such as different maturities of the underlying assets, different degrees of moneyness and three volatility extraction techniques. Our analysis differs from the extant literature we consider American put options of futures contracts for feeder cattle under several different pricing models, while most commodities studies analyze only European type options.

This article is structured as follows: after this introduction, section 2 provides a brief review of the models used in this work and the research method. Section 3 presents the options associated with the feeder cattle market. Section 4 and 5 respectively discuss the results about the real option to switch markets and the financial put option. In section 6 we conclude.

2. Option Pricing Models

The holder of an option (long position) has the right to exercise it upon payment of the exercise price to the seller (short position). There are two basic types of options: call options and put options. The holder of a call option has the right to buy a certain underlying asset at a specific price at a predetermined date or over a period of time. On the other hand, the holder of a put option has the right to sell an underlying asset for a predetermined value at a predetermined date or over a period of time.

In the early 1970s, Black and Scholes (1973) and Merton (1973) (BSM) developed a model for pricing options that is widely used until this day to determine the price of options on stocks, indices and commodities. This model assumes that the returns on the asset follows a normal distribution and that asset prices follow a log-normal distribution, according to the stochastic diffusion process presented by equation 1:

$$\frac{dS_t}{dS} = \mu dt + \sigma dW_t \tag{1}$$

where S_t is the asset price diffusion process, W_t is a Wiener process, μ is the growth rate and σ is the volatility of the underlying asset. The solution to the differential equation that models the price of a European option is given by equations 2, 3 and 4:

$$p = Xe^{-rt}N(-d_2) - S_0N(-d_1) \tag{2}$$

where

$$d_1 = \frac{ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

where p is the put option price, $N(d_1)$ and $N(d_2)$ are, respectively, the cumulative normal distribution function for d_1 and d_2. S_0 is the price of the underlying asset at the moment 0, X is the exercise price, r is the risk-free rate, σ is the volatility of the underlying asset and T is the time to expiration of the option.

The BSM model for European options, however, cannot be used to price American options, as these are more complex to assess. Part of the pricing problem involves determining the optimal time of exercise. There are few analytical solutions for American-style options, such as the models of Barone-Adesi and Whaley (1987) and Bjerksund and Stensland (1993). The idea of the first model is to calculate the premium of early exercise and then add it to the value of European option calculated by the BSM model. According to Barone-Adesi and Whaley (1987) as the differential equations of European and American options differ only in the boundary conditions, then, the premium should also follow the same differential equation. Thus, the value of American option is given by equation 5:

$$F(S) = \begin{cases} f(S) + A\left(\frac{S}{S^*}\right)^\beta, & \text{if} \quad for S < S^*, \\ S - 1, & \text{if} \quad for S \geq S^*. \end{cases} \tag{5}$$

where $f(S)$ is the value of a European option calculated by the Black & Scholes model. However, the exercise price is replaced by investment. The calculation of the coefficient A depends on the trigger value (S^*). This value is obtained by a non-linear equation 6:

$$S^* - 1 = f(S^*) + \frac{S^*}{\beta}(1 - e^{-\delta T} N[d_1(S^*)]) \tag{6}$$

Note that the second term on the right is exactly the coefficient A.

On the other hand the model of Bjerksund and Stensland (1993) is considered computationally more efficient and accurate in pricing options with longer terms (Haug, 1998). This model requires imposing a price limit which if reached by the underlying asset, the option is exercised. Thus, with this model the American option can take the values shown in equations 7, 8 and 9:

$$F(S) = \begin{cases} \alpha S^\beta - \alpha\phi(S,T,\beta,S^*) + \phi(S,T,1,S^*) - \phi(S,T,1,I,S^*), & \\ \qquad -I\phi(S,T,0,S^*) + I\phi(S,T,0,I,S^*), & \text{if} \quad for S < S^*, \\ S - I, & \text{if} \quad for S \geq S^*. \end{cases} \tag{7}$$

where

$$\alpha = (S^* - I)(S^*)^{-\beta} \tag{8}$$

$$\beta = \left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right) + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{9}$$

The positive values of r and δ ensure that β must be greater than 1. The function $\phi(S, T, \gamma, H, S^*)$ is given by equations 10 to 14:

$$\phi = e^{\lambda T} S^* \left\{ N(\alpha) - \left(\frac{S^*}{S}\right)^{\kappa} N(b) \right\} \tag{10}$$

$$a = -\frac{ln\left(\frac{S}{H}\right) + \left[r - \delta + \left(\gamma - \frac{1}{2}\right)\sigma^2\right] T}{\sigma\sqrt{T}} \tag{11}$$

$$b = -\frac{ln\left(\frac{S^*}{SH}\right) + \left[r - \delta + \left(\gamma - \frac{1}{2}\right)\sigma^2\right] T}{\sigma\sqrt{T}} \tag{12}$$

where $H \leq S^*$ and the variables λ and κ are, respectively:

$$\lambda = -r + \gamma(r-\delta) + \frac{1}{2}\gamma(\gamma-1)\sigma^2 \tag{13}$$

and

$$\kappa = \frac{2(r-\delta)}{\sigma^2} + (2\gamma - 1) \tag{14}$$

In equation (6), the first two terms represent the discount component, i.e., the premium received by the prepayment of the option, while the last four terms represent the value of the European option.

The binomial method is considered more intuitive than the numerical methods, besides being very simple and flexible, and can be applied to both European and American options. Cox et al. (1979) showed that the distribution of continuous lognormal probability can be modeled by a discrete binomial tree, where at each time interval (Δt) there are two possible moves: up (u) and down (d), defined according to its individual probabilities. The equations and parameters developed in the binomial model are based on the assumption that there are no arbitrage possibilities, i.e., $d < 1+r < u$. Thus, changing from S to S_u corresponds to an upward movement, with probability of occurrence p, and changing from S to S_d corresponds to a downward movement, with probability $(1-p)$.

The model of Longstaff and Schwartz (2001) is an innovative alternative to traditional techniques of finite difference and binomial trees. The method uses dynamic programming techniques, where at every moment before the maturity of an American option the owner of the option compares the payoff from early exercise with its continuation value. The contribution of these authors was to identify the expectation conditional function to be estimated in the simulation using the least squares method. This technique is defined as the least squares method Monte Carlo (LSM) and can be used to price American options and options whose state variables follow any stochastic process or non-Markov process. The option value is the average of the cash flows from the optimal exercise in each simulation, discounted until the initial time, as shown in equation 15:

$$V_0 = \frac{1}{m}\sum_{j=1}^{m} V_{ij}e^{-t_j r} \tag{15}$$

where r is the risk free interest rate, m is the number of simulated price paths, t_j is a optimal exercise date of the option in the simulation i, V_{ji} is the cash flow generated by the exercise of the option at the moment t_j, in simulation i.

2.1. Volatility determination

The option premium can vary greatly depending on the volatility used in the pricing model. Thus, this work will consider three distinct volatilities: historical volatility, implied volatility and EWMA.

Historical volatility is the volatility that an asset presented in the past and can be observed in different periods of time. The calculation of the historical volatility (σ), shown in equation 16, is represented by the standard deviation (s) of the log-return of the underlying asset price series, i.e., on a series of future prices composed by the first opening exercising date, in percentage per annum,

$$s = \sqrt{\frac{\sum_{i=1}^{T}(r_i - \bar{r})^2}{n-1}} \tag{16}$$

with

$$\sigma = s\sqrt{252}$$

Where $n-1$ is the number of price changes, $r_i = ln(P_i/P_{i-1})$ is the return of the logarithmic series of prices with average $\bar{r}$, being P_i the price of the asset at the end of the $i-th$ period.

While the historical volatility measures past performance, current volatility of an asset is given by its implied volatility, i.e., it is a forward-looking approach. We can compare this approach to the historical volatility to verify if current market prices are consistent with what occurred in the past.

In order to determine the implied volatility we use the option premium observed during a given period, effectively reversing the pricing model, using the approach of Newton-Raphson shown in equation 17:

$$\sigma_{i+1} = \frac{\sigma_i - c(\sigma_i) - c_m}{\partial c/\partial \sigma_i} \tag{17}$$

where $c(\sigma_i)$ is the observed price and c_m the calculated price of the option. $\partial c/\partial \sigma_i$ is the sensitivity measure of option value with respect to volatility. Lanari et al. (1999) point out that the "volatility smile" is a bias of the BSM model, where different implied volatilities are obtained for options on the same underlying asset with different strike prices, forming a U shaped curved for the degrees of moneyness.

Moreover, Valls Pereira et al. (2003) consider the historical volatility not sufficient, because as it uses the same weight for the sample, the appearance of an outline return will raise the estimated volatility while the observation remains in the sample. In an attempt to minimize this problem, it was applied the moving average method by exponential smoothing (Exponencial Weighted Moving Average - EWMA), which is calculated based on the exponential damping methodology of the variance. The EWMA volatility is given by equation 18,

$$\sigma^2 = \sum_{i=1}^{n} \frac{\alpha(1-\alpha)^{i-1}}{1-(1-\alpha)^{n-1}} (r_i - \bar{r})^2 \tag{18}$$

where

$$\bar{r} = \textstyle\sum_{i=1}^{n} \frac{\alpha(1-\alpha)^{i-1}}{1-(1-\alpha)^{n-1}} r_i$$

This method assigns exponentially decreasing weights according to the age of each of the data. The weights are determined according to a smoothing parameter α, $0 < \alpha < 1$, with this, the weight of the return of one day is α times the weight of the return of the next day. In this paper we consider $\alpha = 0.84$ as used by BM&FBovespa.

3. Options associated with the feeder cattle market

The rural producer must choose which type of contract will be used to hedge against risks and also the optimal time to sell the cattle. In order to minimize risk, the producer can diversify the periods of sale of the product, rather than selling all his cattle at the same time. Also, the producer can resort to real and financial options to better manager the feeder cattle business.

As mentioned, the producer has five real options and one financial option at his disposal. The choice between working with the production of milk or beef (cattle for slaughter) can be easily analyzed with the traditional discounted cash flow method. Due to the focus of this article, we assume the producer has already on feeder cattle.

As a feeder cattle producer, the manager has the real option of fattening the cattle in pasture or through confinement. This option was studied by Bastian-Pinto et al. (2015). The uncertain variables of the model were the prices of the feeder cattle and the confinement costs, which were modeled according to an approximation of the recombinant binomial tree to the mean reversion process. The results indicated that there was a significant increase in financial returns through containment. The confinement increases cattle fattening speed and when compared against maintenance in pasture it maximizes return on investment for the producer.

According to Barbieri et al. (2016), the adoption of the cattle feedlot system has been a profitable and viable activity which allows greater cost control. Also, this strategy enables the producer to choose the best time to confine the cattle, which has a strong impact on the profitability of the business: if on the one hand the confinement increases the fattening speed, on the other hand it also increases the cost of production (Bastian-Pinto et al., 2015). The confinement decision, therefore, is also related to the time to sell the cattle for slaughter. In general, producers will trade their animals before or at the same time they are closing the cattle in the trough. According to Medeiros and Montevechi (2005), the fattening period lasts from six to eighteen months. At the end of this time, fattening options are extinguished and the sale of the animal is mandatory, characterizing the end of the fattening phase.

The selling strategy is also an important issue for commodities, since uncertainty is greater in the off-season. In this case the producer has the option to choose the best timing for trade the production. According to Caetano (2014), the sale time is associated with the risk aversion of the producer. The farmer is faced with a situation of perfect competition, where the maximum profit is achieved by negotiating the best price. As the price is not constant throughout the year, in order to sell at higher prices the producer should accept a greater risk, considering his risk expectation. The author classifies the sales into seven groups according to the farmer's risk aversion.

Finally, the producer needs to define if the sale will be in the spot market or in the futures market, which is characterized by a switch option. In the futures market the participants undertake to buy or sell a fixed amount of an asset for a predetermined price at a future date. Then, if the producer decides to operates in this market he can work with financial put option contracts on the future market of feeder cattle. This two options (switch real option and financial put option) will be discussed in more detail in the following sections.

4. The real option to switch markets

Feeder cattle prices have increased in the past decade, as shown in Figure 1. The upward movement of prices, accompanied by the country's potential to produce large quantities of beef at low cost and uniform quality has attracted many investors to this industry. However, despite this growth trend, especially when considering the country's trade balance, both domestic prices and international commodity prices tend to fall in the long term and the margin of gains (profits) may become tight or non-existent.

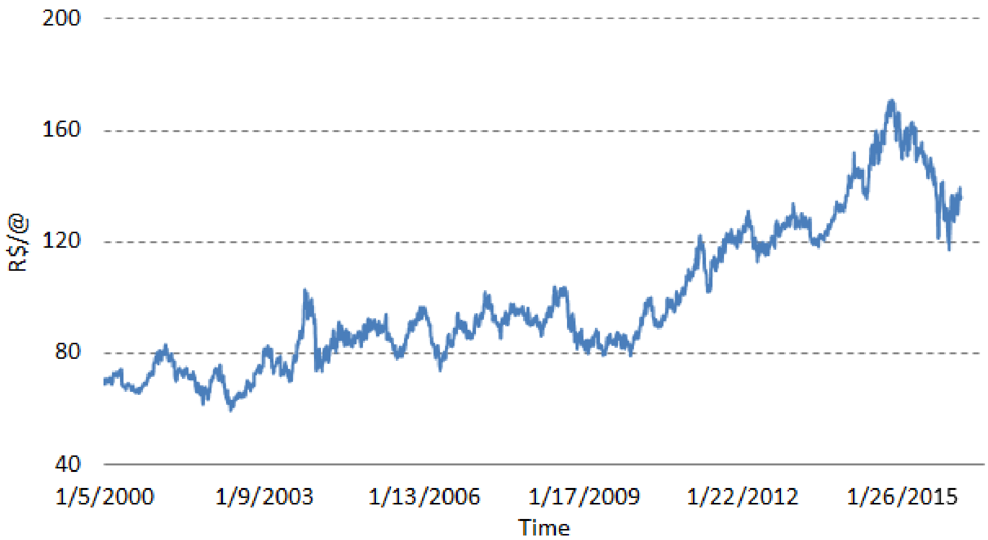

Figure 1: Spot price of cattle feeder.

The risk of price fluctuation can be minimized through transactions in futures market. The producer can mitigate the price risk, for example, by means of a hedge in which he fixes on the BM&FBovespa the price of the final product (sale hedge). According to Santos and Aguiar (2015), the feeder cattle commodity has a significant volume of contracts traded in the Brazilian future market. The future market is characterized by strong standardization, high liquidity and transparent trading. On the other hand, the operational disadvantages are demanding high financial movement due to daily adjustments and need deposit of guarantees.

In view of the high variability of prices in the spot market, the rural producer has two alternatives: continue to sell in the spot market or switch to the futures market. Aside from the uncertainty about the prices, the livestock industry is also subject to three other types of risks: climate, credit and operating risk. Climate risk arises from the variations in rain, temperature and other naturally occurring phenomena. This type of risk can be covered by production insurance. Credit risk is the risk that prior commitments to customers and banks will not be be honored due to poor results of the business. Operational risk stems from problems that may arise from the day to day operation of the business, such as equipment or management failures. For the purposes of this article we will consider only price risk when determining the option values.

Figure 2 illustrates the switch option trough a simple binomial tree, according to the initial position of producer at the beginning of decision; if he is in the spot market (SM) (on the left of figure) or in the future market (FM) (on the right of figure). Note the switch is advantageous if the producer is acting in the future market and the market goes up; or if he is in the spot market and the market goes down. This option can be exercised whenever there is a lot of cattle for sale with no correlation with the past time, which works as a bundle of European options.

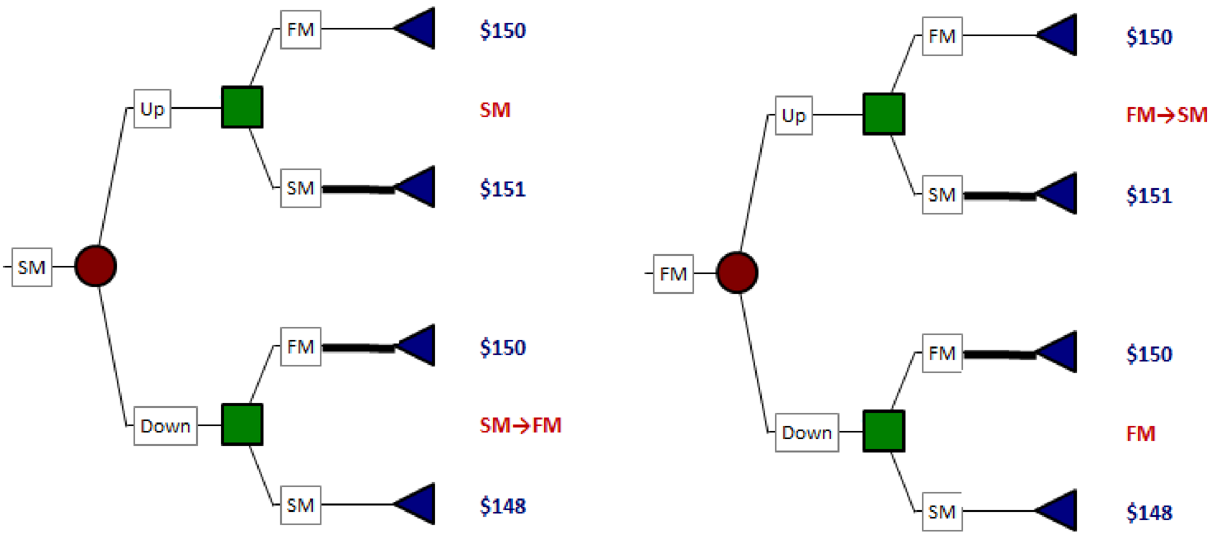

Figure 2: Illustration of put option.

To show the value of this option to a producer let's consider, for example, a rural producer in January 2014 that is new in the feeder cattle sector and is concerned about the market. The two first years can be decisive and may lead either to bankruptcy or to large financial returns. Thus, we will determine the value of the switch option in this period.

The main assumptions concerning this hypothetical producer and the stochastic process we use to model the price uncertainty are presented in Table 1. For the sake of simplicity, we consider that prices follows a Geometric Brownian Motion (GBM). A GBM is a continuous-time stochastic process in which the logarithm of the randomly variable follows a Brownian motion (Wiener process) with drift. A stochastic process S_t is said to follow a GBM if it satisfies the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where W_t is a Wiener process or Brownian motion, and μ (the percentage drift) and σ (the percentage volatility) are constants.

Also, we consider the producer has technology to fatten the animal within one year. Thus, each cattle confinement cycle lasts one year. In addition, we assume in the future market, on 01/16/2014, the feeder cattle for April 2014 is quoted at R$ 150.00. For April 2015 and 2016 we assume the values R$ 160.00 and R$ 172.00. Each contract is traded with a minimum of 20 head of cattle, therefore, we assume that given the farmer's risk aversion he will trade 40% of his cattle in the future market. In addition, each contract has a minimum required of 330 arrobas (@ = 15kg).

Table 1: Model assumptions

Producer			GBM		
Livestock	20,000	heads	σ	11%	per year
Contract	400	unit	r_f	5%	per year
@/contract	330	$/@	S_0	150	$/@

Figure 3 shows our results. For this example, the producer should start the production by selling in the future market and change to the spot market if the spot market goes up. In this case he would change again to the future market only if the market goes down, and then to the spot market if this market goes up. Otherwise, he would only change for the spot market if this market goes successively down, up and up. This option has a value of $3.96/@, which corresponds to an increase of approximately 31.93% of the animal's price.

It is important to note that according to Mattos et al. (2009), the transaction costs may represent about 9.7% of the average price of feeder cattle. Nevertheless, we are not considering transaction costs in this paper once these costs are small when compared to the amount that is traded in the stock exchange.

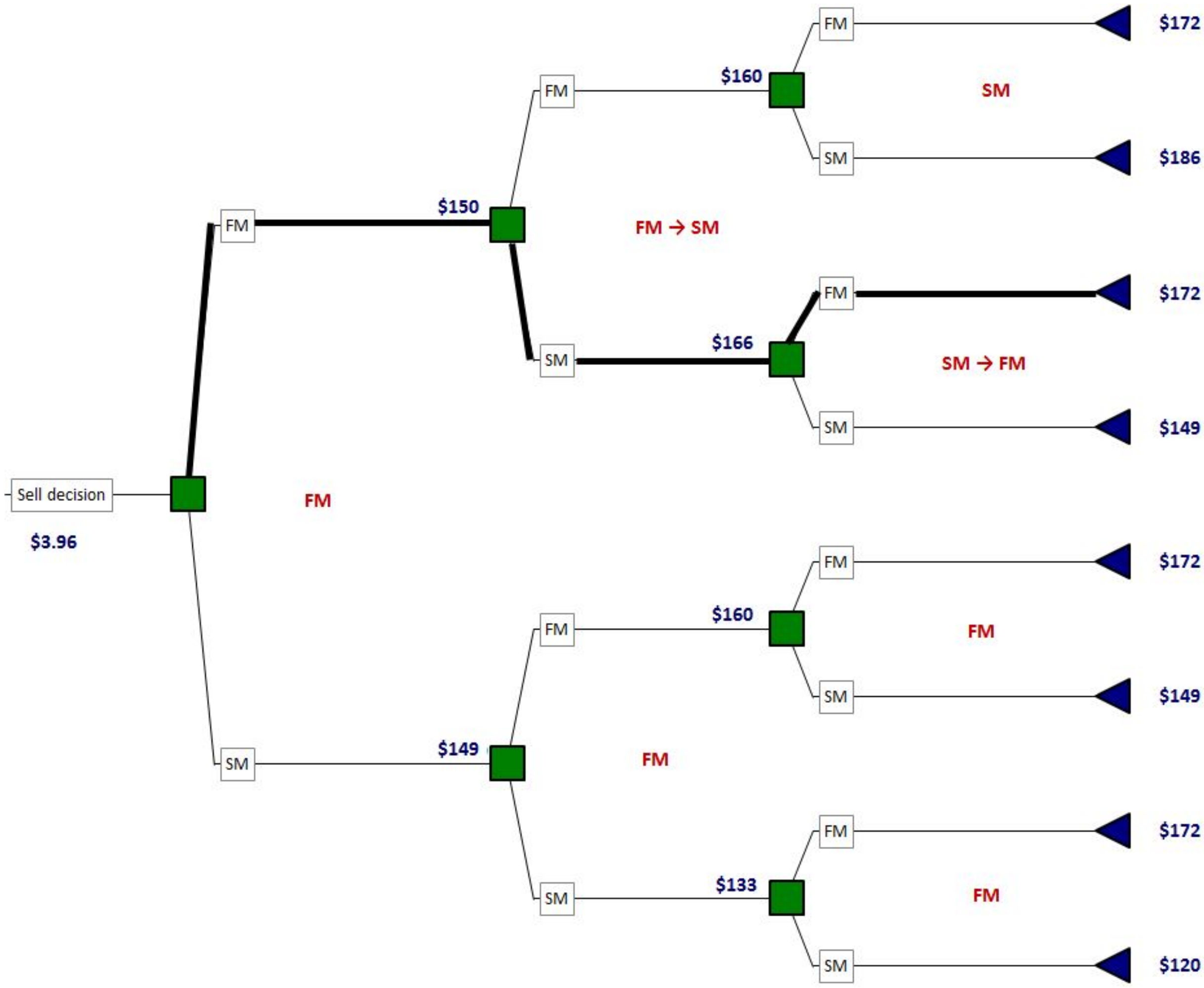

Figure 3: Switch option.

5. The financial put option

The contract of options over the future market of feeder cattle was introduced by BM&FBovespa in 1994 with the trades occurring in US dollars (BM&FBovespa, 2017). In 2000, this contract started to have the Brazilian real as the base currency. This contract is available for trading in the form of American options with maturity in the last business day of each month of the year. The option premium is given in $R\$/@$ and the standard size of contracts is 330@, where each @ (arroba) corresponds to 15Kg.

The options market is an alternative to fix the price of sale for a future date. Suppose that a cattle feeder rancher intends to protect the sale price of the livestock that are in fattening process. He verifies that there is a possibility to buy a put option with the premium of $3/@, which will give him the right to sell the cattle in October to $150/@, which will guarantee his profit margin. In October, the price of the feeder cattle is being traded at \$140/@. The rancher will exercise his right and receive the difference of \$10/@ (\$150/@ - \$140/@) on the Stock Exchange. Even delivering the beef at \$140 to the refrigerator, it will return to the previously fixed sale price of \$147/@, for which he paid the premium (\$3/@).

As put options allow agents to reduce the risk from low future market prices, the number of put option contracts had a significant growth in the past two decades, as shown in Figure 4. Put option contracts represent approximately 7% of all contracts traded in the last sixteen years, and increased to 18% in recent years.

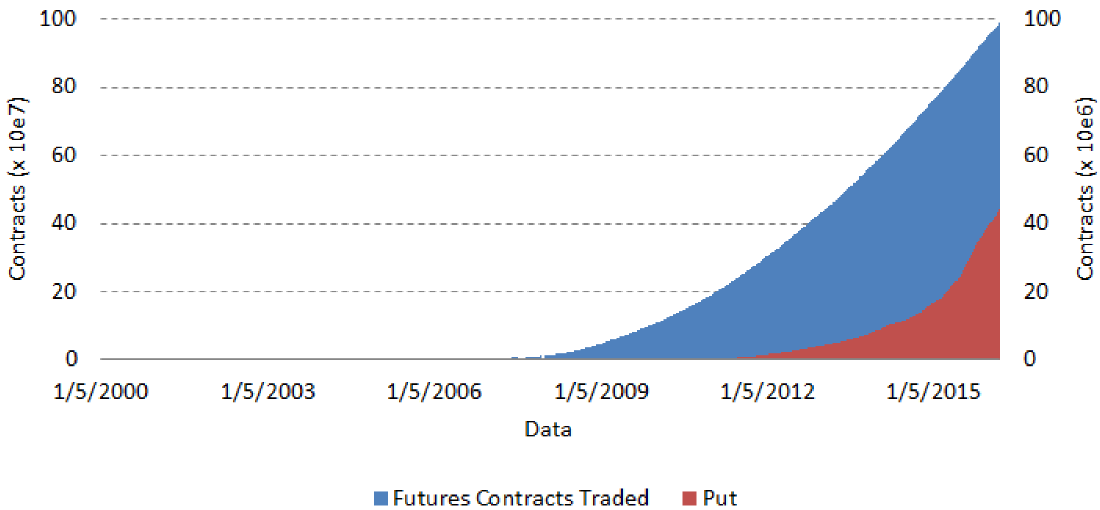

Figure 4: Contracts negotiated.

The increase in put option contracts in recent years suggests a concern of livestock managers with fluctuations in the price of feeder cattle and the low price cycles. This fact confirms the importance of providing adequate risk management tools for these agents. Pricing these options, however, is not trivial and there are a number of models in the literature that can be used for this purpose. Thus, in order to clarify to the manager of feeder cattle the mainly models that can be used to price American put options and which one of these models stands out, we compare the results of the theoretical models with those of BM&FBovespa. We use the theoretical models of Longstaff and Schwartz (LSM), Barone-Adesi and Whaley (BAW), Bjerksund and Stensland (BSA) and Cox, Ross and Rubinstein (CRR).

The variables for the pricing is retrieved from the BM&FBovespa website: historical prices of feeder cattle futures contracts, the option prices of these contracts, the exercise price and the number of days to maturity for the period running from January 2000 to February

2016. This period includes changes in the underlying asset after the internationalization of agricultural markets in 1999. As a proxy for risk free interest rate we adopt the interest rate of certificates of interbank deposits (known as Certificados de Depósitos Interbancários - CDI), which is obtained from Bloomberg for the period. As in Hull (2006) and Albanese and Campolieti (2006) in this article we do not consider payment of dividends.

For the selection of the put options sample we remove from the analysis options that are very close to expiration (maturing in 5 days or less), and also exclude from the sample options that are issued in months where there are no future contract maturing. This same process is done by Malz (2000).

The pricing of these options is analyzed considering the perspective of maturity, the deadline of the options and the moneyness degree. With respect to the maturity, the sample is divided into three distinct periods: short term - $ST(n < 30)$, midterm - $MT(30 \leq n \leq 90)$ and long term - $LT(N > 90)$. The deadline varies from January to December. According to Ederington and Guan (2000) the moneyness degree - MN - is given by equation 19:

$$MN_{put} = \frac{X_{j,t}}{S_t} - 1 \tag{19}$$

with

$$MN_{put} > 0 = ITM$$
$$MN_{put} < 0 = OTM$$

where $X_{j,t}$ is the option exercise price j in time t and S_t is the future price of the underlying asset on day t. ITM or in-the-money is an option in the money, OTM or out-of-the-money indicates an option out of the money. Similar approach was used by Vitiello Jr (2000), Mikoszewki (2003), Luccas (2007) and Tonin and Coelho (2012). This categorization makes it possible to analyze the options according to how close the price of the underlying asset is to the exercise price.

Figure 5 shows the characteristics of the put contracts of the sample. The moneyness index shows that most of the put options are out-of-the-money (OTM), and the deadline indicate a predominance of maturity in the medium term (MP). Moreover, note that despite the fact that negotiations of the feeder cattle options on futures market have maturities in all months of the year, most occur in the transition from the off-season to harvest, i.e., between the months of October to December.

After collection and data processing we compare the theoretical model prices with the actual market prices. The sample of 10,220 observations is priced with 3 volatilities and

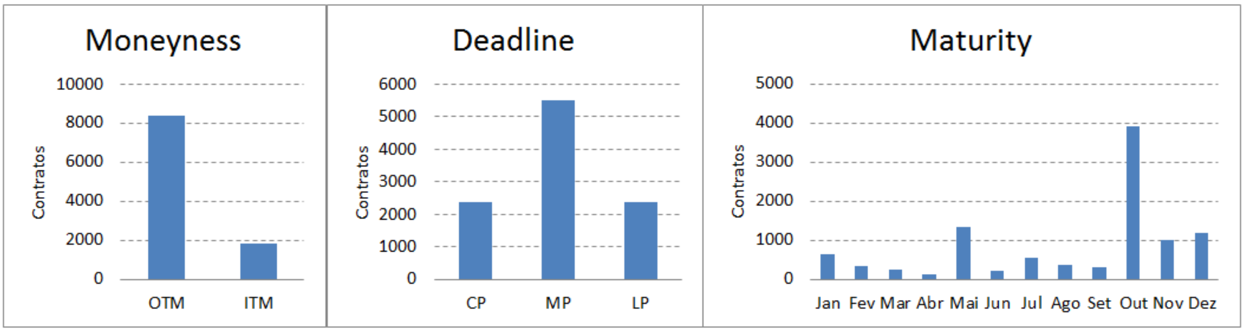

Figure 5: Characteristics of put options.

the 4 different models, as previously mentioned. To evaluate the predictive ability of the models, we use the mean squared error (MSE), which represents the sum of the differences between the estimated value and the actual value of the data, weighted by the number of terms:

$$MSE = \frac{1}{n} \sum_{i=1}^{N} (p_i - \hat{p}_i)^2$$

where p_i and $\hat{p}_i$ are the market price and the estimated price of the put option, respectively, and n is the number of contracts traded. According to Valença (2005) the use of MSE is a metric that penalizes large errors, and their partial derivatives with respect to weights can be easily determined.

In addition, mean difference tests is conducted to examine if there is statistically significant differences between the values generated by each option pricing models. In order to do this, the Kolmogorov-Smirnov test is used to check if the data had a normal distribution and Levene's test to see if the variance is homogeneous. After verification of these assumptions, the appropriate parametric or non-parametric test is used.

Table 2 show the results of the cattle put options pricing for all models, considering the full sample period and no stratification. In this case, the mean square error is lower in all models considering the implied volatility. Furthermore, for the historic volatility and the implied volatility the model prices are in most cases smaller than the market value, while for the EWMA are in most cases higher than the market value. A considerable portion of the results is within the error range of 10% of the market value, which indicates a fair market pricing.

The results of the different models is also analyzed by considering different types of sample stratification. This analysis is done to check the adjustment of pricing in smaller samples and under different conditions. In addition, the data in the tables 3 and 4 are subjected to analysis of variance (ANOVA) aimed mainly at checking whether there is a significant difference between the means, followed by the Tukey's test, which involves comparing means in pairs with equal sample size (Hoffmann, 2006). The null hypothesis for Tukey's test states that all means being compared are from the same population. This test is done after the normal distribution and homogeneity of variance is confirmed by the Kolmogorov-Smirnov and Levene tests.

Table 3 shows the result of the pricing for different degrees of moneyness. The results are divided into out-of-the-money (OTM) and in-the-money (ITM). In this case, the models of Cox, Ross and Rubinstein and Barone-Adesi and Whaley for ITM data shows comparatively the best results among the analyzed models. For OTM data the best model is Bjerksund and Stensland (BSA). In all cases the MSE is greater for EWMA volatility, suggesting this is the worst model. Also, the null hypothesis of this model is rejected in the Tukey's test.

Table 4 shows the result of pricing for different deadlines. The results are separated in short term ($n < 30$), medium term ($30 \leq n \leq 90$) and long term ($n > 90$), where n is the number of days to maturity. In this case the model shows the best result is Bjerksund and Stensland (BSA) for all deadlines. The models calculated with the implied volatility provided a better fit of the estimated prices, and the long-term model is the one that had the lowest MSE.

Table 5 shows the results of the sample for different maturities of futures contract. The feeder cattle options have maturity in every month of the year, thus this table shows the mean square error for the different models between January and December. As before, the models calculated with the implied volatility outperformed those with the historical volatility and EWMA volatility. In this case, the best pricing results occurs between the months of December to April to the BSA model, since all results of ANOVA test do not reject the null hypothesis that the average of the models differ and the mean squared errors are low. The LSM model outperformed better for the model calculated with the historical volatility for the months between September to January, which are much of the off-season of cattle feeder, i.e., this model is more accurate in periods of greater uncertainty in the series.

In general, the pricing models generated results very close to the put options contracts traded on the feeder cattle futures market of BM&FBovespa. Also, among the four models analyzed, the one that showed the best performance was the Bjerksund and Stensland (BSA) model for the entire sample, the model of Cox, Ross and Rubinstein (CRR) and Barone-Adesi and Whaley (BAW) to ITM data, the model of Bjerksund and Stensland (BSA) for data OTM, and the model of Bjerksund and Stensland (BSA) for all maturities. Note that the model of Bjerksund and Stensland (BSA) was the most repeated between those models with better performance, therefore, in case of impossibility of comparing models, the manager should choose this model.

The results also indicate that the prices traded in feeder cattle futures contracts in the Brazilian market (BM&FBovespa) present distortions with respect to the fair price determined by traditional options pricing models in some periods. Finally, all options pricing models were better performed considering the implied volatility, which has the lowest mean square error.

Table 2: Result of the pricing of put options

	Statistics	Historical Volatility	Implied Volatility	EWMA Volatility
LSM	MSE	0.314	0.331	1280
	$Theoretical > Market$	37.68%	63.12%	97.09%
	$Theoretical < Market$	51.28%	5.60%	2.91%
	Interval (1%)	2.07%	6.36%	0.06%
	Interval (5%)	11.04%	31.28%	0.33%
	Interval (10%)	21.11%	59.29%	0.63%
CRR	MSE	0.319	0.147	1198
	$Theoretical > Market$	34.12%	42.98%	97%
	$Theoretical < Market$	54.56%	5.63%	3%
	Interval (1%)	2.43%	12.26%	0.04%
	Interval (5%)	11.32%	51.39%	0.34%
	Interval (10%)	21.48%	81.30%	0.70%
BAW	MSE	0.317	0.144	11202
	$Theoretical > Market$	34.43%	44.08%	97%
	$Theoretical < Market$	54.54%	4.74%	3%
	Interval (1%)	2.27%	12.58%	0.03%
	Interval (5%)	11.03%	51.18%	0.35%
	Interval (10%)	21.16%	80.22%	0.71%
BSA	MSE	0.327	0.091	1126
	$Theoretical > Market$	32.40%	30.55%	96.93%
	$Theoretical < Market$	56.41%	9.18%	3.07%
	Interval (1%)	2.43%	14.47%	0.05%
	Interval (5%)	11.19%	60.27%	0.31%
	Interval (10%)	20.91%	88.19%	0.69%

Note: The models are represented in the table with the following abbreviations: Longstaff and Schwartz (LSM), Cox, Ross and Rubinstein (CRR), Barone-Adesi and Whaley (BAW) and Bjerksund and Stensland (BSA).

Table 3: Pricing of the results for different degrees of moneyness

Moneyness		OTM			ITM		
		Hist Vol.	Impl. Vol.	EWMA	Hist Vol.	Impl. Vol.	EWMA
LSM	MSE	0.341	0.023	1247	0.306	0.076	1033
	p-value	0.000*	0.000*	0.000*	0.35	0.004*	0.000*
CRR	MSE	0.309	0.097	1236	0.367	0.038	1022
	p-value	0.000*	0.000*	0.000*	0.023	0.152	0.000*
BAW	MSE	0.306	0.014	1240	0.371	0.031	1025
	p-value	0.000*	0.000*	0.000*	0.01	0.245	0.000*
BSA	MSE	0.316	0.058	1160	0.381	0.024	966
	p-value	0.000*	0.107	0.000*	0.002*	0.5	0.000*

Note: * Means differ at 1% probability.

Table 4: Pricing of the results for different maturity periods

Deadline		ST			MT			LT		
		Hist Vol.	Impl. Vol.	EWMA	Hist Vol.	Impl. Vol.	EWMA	Hist Vol.	Impl. Vol.	EWMA
LSM	MSE	0.190	0.090	901	0.297	0.029	1056	0.475	0.064	1873
	p-value	0.35	0.0044*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
CRR	MSE	0.183	0.031	890	0.302	0.012	1046	0.495	0.030	1860
	p-value	0.0226	0.1517	0.000*	0.000*	0.0719	0.000*	0.000*	0.0029*	0.000*
BAW	MSE	0.182	0.027	893	0.301	0.017	1050	0.489	0.032	1868
	p-value	0.0102	0.245	0.000*	0.000*	0.0789	0.000*	0.000*	0.0011*	0.000*
BSA	MSE	0.181	0.002	849	0.030	0.008	985	0.416	0.001	1734
	p-value	0.0019*	0.5001	0.000*	0.000*	0.6022	0.000*	0.000*	0.1347	0.000*

Note: * Means differ at 1% probability.

Table 5: Pricing of the results for different maturities of futures contract

Maturity		Historical Volatility				Implied Volatility				EWMA			
		LSM	BIN	BAW	BSA	LSM	BIN	BAW	BSA	LSM	BIN	BAW	BSA
Jan	MSE	0.484	0.497	0.496	0.507	0.020	0.005	0.005	0.003	832	825	827	788
	p-value	0.000*	0.000*	0.000*	0.000*	0.576	0.973	0.978	0.998	0.000*	0.000*	0.000*	0.000*
Fev	MSE	0.505	0.509	0.509	0.523	0.002	0.004	0.004	0.003	1113	1096	1101	1042
	P-value	0.001*	0.000*	0.000*	0.000*	0.929	0.997	0.998	0.999	0.000*	0.000*	0.000*	0.000*
Mar	MSE	0.616	0.646	0.645	0.659	0.002	0.005	0.005	0.003	1685	1672	1678	1582
	P-value	0.073	0.023	0.022	0.011	0.962	0.999	0.999	1	0.000*	0.000*	0.000*	0.000*
Abr	MSE	0.349	0.342	0.344	0.345	0.001	0.006	0.005	0.003	2061	2032	2041	1899
	P-value	0.999	1	1	1	0.997	0.999	0.999	1	0.000*	0.000*	0.000*	0.000*
May	MSE	0.312	0.296	0.295	0.290	0.002	0.001	0.001	0.008	2496	2482	2494	2301
	p-value	0.143	0.766	0.726	0.985	0.064	0.532	0.484	0.887	0.000*	0.000*	0.000*	0.000*
Jun	MSE	0.130	0.120	0.120	0.121	0.002	0.007	0.006	0.004	2115	2082	2090	1937
	p-value	0.735	0.926	0.936	0.975	0.967	0.998	0.998	0.999	0.000*	0.000*	0.000*	0.000*
Jul	MSE	0.107	0.009	0.009	0.009	0.001	0.008	0.007	0.005	1408	1398	1402	1322
	p-value	0.101	0.425	0.453	0.692	0.638	0.93	0.94	0.991	0.000*	0.000*	0.000*	0.000*
Ago	MSE	0.107	0.104	0.105	0.108	0.002	0.007	0.006	0.004	1184	1169	1172	1099
	p-value	0.99	1	1	1	0.935	0.996	0.997	0.999	0.000*	0.000*	0.000*	0.000*
Set	MSE	0.166	0.175	0.175	0.188	0.002	0.001	0.009	0.006	597	577	578	555
	p-value	0.501	0.275	0.272	0.169	0.926	0.99	0.991	0.999	0.000*	0.000*	0.000*	0.000*
Out	MSE	0.294	0.305	0.303	0.320	0.005	0.002	0.002	0.001	1096	1087	1091	1021
	p-value	0.000*	0.000*	0.000*	0.000*	0.000*	0.022	0.017	0.322	0.000*	0.000*	0.000*	0.000*
Nov	MSE	0.295	0.309	0.308	0.318	0.002	0.008	0.008	0.005	540	530	531	510
	p-value	0.000*	0.000*	0.000*	0.000*	0.2954	0.91	0.913	0.992	0.000*	0.000*	0.000*	0.000*
Dec	MSE	0.412	0.420	0.418	0.426	0.001	0.006	0.006	0.003	702	693	694	664
	p-value	0.000*	0.000*	0.000*	0.000*	0.212	0.853	0.856	0.989	0.000*	0.000*	0.000*	0.000*

6. Conclusion

Option models can be useful to managers who face many uncertainties and high risk, and have embedded managerial flexibilities in their line of business, such as feeder cattle producers. Among others, the beef producer has the option to switch the market where his product is sold (spot or futures markets) and also the flexibility to buy financial put options in the futures market. Both these options can be used to minimize risk.

In this paper we analyse these two options and compare, in the second case, with the premium values obtained in the Brazilian stock exchange. We analyze the adherence of feeder cattle put options premiums traded in the Brazilian BMF&Bovespa derivatives market with the theoretical prices determined by classical models in the literature: Longstaff and Schwartz, Barone-Adesi and Whaley, Bjerksund and Stensland and Cox, Ross and Rubinstein.

The results confirm that the real switch option is an important tool for value maximization and protection against uncertainties faced by the cattle rancher. Moreover, the results indicate that depending on the sample strata, the theoretical models have equal distributions, which shows that their prices do not differ significantly. The main changes are due to the use of different volatility extraction methods, whereas the use of different pricing models slightly alters the results. Furthermore, it was observed that for implied volatility, the mean square error of the models was close to zero, indicating that the market is trading their prices to a level close to what the classics models indicate as fair price. It is also clear that the best performance in pricing options were obtained using implied volatility. Regarding the pricing models, the best performance was of that of the approach of the Bjerksund and Stensland model.

Note that the production activity of feeder cattle is characterized by a standardized product market and with low entry barriers. High profitability in any giver cycle will attract new producers to the business, which will eventually increase supply and depress prices. Thus, feeder cattle producers can benefit from the use of the risk management instruments discussed in this article in order to reduce risk and increase profits, or even choosing the optimal time to exit the market. Our results may also be of interest to public and private agencies involved in monitoring, planning, determining public policies, decision-making and industry analysis in order to foster the sustainable evolution of feeder cattle production in Brazil

While in this article we adopted the widely used Geometric Brownian Motion stochastic diffusion model to mimic price uncertainty, suggestions for future research in this area could involve different stochastic price models, such as Mean Reverting Models, which may be appropriate for some types of commodities, also include the impact of transaction cost on the value of the value of the switch option.

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