The Use of Equity Financing in Debt Renegotiation

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Abstract

Debt renegotiation is often modeled as pure debt for equity or debt for debt swaps. In this paper we analyze the use of equity financing in addition to debt financing in debt repurchases. Firms with larger volatility, lower cash flow growth rates, or higher recovery rates are more likely to use equity financing in debt renegotiation. Flotation and renegotiation costs, the bargaining power of the creditors, and macroeconomic variables also influence this choice. When equity issuance is a possible source of financing in renegotiation, firms optimally choose larger debt reductions as compared to pure debt for debt swaps. The use of equity financing increases welfare. We provide closed-form solutions for the optimal use of funding and we derive novel testable empirical implications regarding the use of equity financing in debt repurchases.

Keywords: Debt renegotiation, Debt pricing, Strategic contingent claim analysis

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1. Introduction

Corporate debt renegotiation has been extensively studied in the literature. Different formulations of reorganization have been proposed starting from the well-known strategic debt service (Anderson and Sundaresan, 1996, Mella-Barral and Perraudin, 1997, Fan and Sundaresan, 2000), to the debt for equity swap (Fan and Sundaresan, 2000), and to the pure debt for debt swap (Mella-Barral, 1999, Lambrecht, 2001, Moraux and Silaghi, 2014).

Unlike bank debt that is relatively easy to renegotiate in a private workout, publicly traded debt is difficult or impossible to renegotiate outside of a formal bankruptcy procedure (Bolton and Scharfstein, 1996). In this context, Brandon (2013) argues that debt repurchases are a "market-based substitute for the renegotiation of corporate bonds". According to Kruse et al. (2014) the most common motives for debt tender offers are debt reduction and interest expense reduction. Other reasons are covenants relaxation and debt restructuring/distress. Moreover, firms which tender have more long term debt, less cash, and lower operating returns. Similarly, in Brandon (2013)'s analysis of debt repurchases, firms tend to repurchase their debt after periods of increasing leverage, negative shocks to cash flows, and bond rating downgrades. While in Kruse et al. (2014) firms improve their operating returns and interest coverage ratio after the tender offer, in Brandon (2013) after a debt repurchase firms improve their investments.

Debt repurchases use a variety of funding sources. According to Kruse et al. (2014), 39,9% of the debt tender offers in their sample use as a source of funds public debt. Other sources of financing used in debt tender offers are asset sales (14.9%), bank debt (13.9%), and common equity (13.9%). Furthermore, Brennan and Kraus (1987) argue that financing consisting of an equity issuance combined with debt retirement is quite common. This is consistent with the evidence of Masulis and Korwar (1986), in whose sample of 372 equity issuances, 179 use the proceeds for debt retirement. In the theoretical literature, Landier and Ueda (2012) find that a plan subsidizing common equity issues and buying back debt is close to optimal in bank restructuring, since asset sales are more costly to taxpayers.

Therefore, both the empirical and the theoretical literature suggest that a combination of debt and equity is used in debt renegotiation. However, the use of equity financing along with debt in debt renegotiation has received very little attention. A great part of the literature on debt renegotiation proposes strategic deb service which consists of temporary coupon reductions. Other studies model debt restructuring through debt for equity or debt for debt swaps, as previously mentioned. In a recent paper, Nishihara and Shibata (2016) study the decision to renegotiate debt or to proceed to direct liquidation, in a model in which the use of equity financing is allowed along with debt. Nevertheless, they focus on the choice between renegotiation and liquidation.

In this paper, we contribute to the literature by providing a first theoretical analysis, to our best knowledge, of the use of equity financing in debt renegotiation. We propose a structural model that incorporates taxes, bankruptcy and renegotiation costs. Renegotiation timing is optimally decided by the claimholders. Following Mella-Barral (1999), Lambrecht (2001), and Moraux and Silaghi (2014), renegotiation consists of a permanent coupon reduction. Unlike these studies however, we do not restrain the choice of the optimal reduced coupon, by allowing for transfers among claimholders. This implies that renegotiation is not just a pure debt for debt swap, but that it can also involve the use of new equity financing. We contribute to the literature by analyzing which firms are more likely to use equity financing in renegotiation, and how the use and amount of equity financing is influenced by the firm characteristics (volatility, cash flow growth rate, recovery rate), market variables (tax rate, interest rate), flotation and renegotiation costs. Moreover, we provide closed-form solutions for the optimal reduced coupon in debt renegotiation with equity financing, and study when renegotiation is preferred to liquidation. Furthermore, since forced asset sales are common in practice,[1], we extend our benchmark model to account for asset sales as a third source of financing for debt renegotiation.

We find that firms that have lower cash flow growth rates, larger volatility, and larger recovery rates, which are forced to sell assets, or which operate on markets with low corporate tax rates and interest rates, are more likely to use equity financing in renegotiation. On the other hand, firms with a relative large bargaining power for the equity holder, relatively larger flotation costs, and lower renegotiation costs are less likely to issue equity to repurchase debt. Regarding the coupon reduction, the model predicts even larger coupon reductions than previous literature (around 23% larger reductions for baseline

[1]Djankov et al. (2008) find that excessive forced asset sales of viable businesses make debt enforcement inefficient.

parameter values).[2] These reductions are in line with empirical findings regarding the size of tender offers. Kruse et al. (2014) find that for the average offer the issuer seeks to retire 89.9% of the outstanding debt issue, while based on book values in the year prior to the debt tender offer, the average offer represents and 80.2% of debt. Brandon (2013) shows that the average debt repurchase retires 53% of the face value of the targeted bond, and it reduces the repurchasing firm's leverage ratio by more than 16%.

Finally, we find that allowing for transfers between claimholders and thus for equity issuance in debt renegotiation leads to an increase in welfare. For reasonable parameter values, the firm value at renegotiation and the net renegotiation surplus increase by 1.76% and 2.69%, respectively.

The closest papers in the literature to the current one are Moraux and Silaghi (2014) and Nishihara and Shibata (2016). Moraux and Silaghi (2014) analyze the optimal number of debt renegotiations in a framework with multiple costly renegotiations, where transfers between claimholders are not allowed. Renegotiation in their model is thus a pure debt for debt swap. On the contrary, we relax this assumption, by allowing for transfers between the equity holder and the creditors, and we consider a single renegotiation in order to keep the analysis tractable.[3] When transfers are allowed the firm can issue equity in renegotiation and debt can be repurchased using both types of financing sources. This leads to larger, welfare increasing debt reductions. Nishihara and Shibata (2016) on the other hand, focus on the choice between renegotiating the debt using partial asset sales and direct full liquidation. They allow for equity financing to be used in renegotiation, however they do not investigate how the use of equity financing varies across firms, nor do they provide analytical solutions for the optimal reduced coupon. They note in their numerical analysis that equity financing always appears to be positive for reasonable parameter values. In this paper nevertheless, we provide a thorough analytical analysis of the use of equity financing in renegotiation, and show that, on the contrary, different costs can deter the use of equity financing in renegotiation. We also illustrate numerically the use of equity is-

[2] Moraux and Silaghi (2014) found that coupons were reduced at least until 67% of their initial value, and up to 27% of the original coupon value, depending on the bargaining power of the creditors.

[3] Although multiple rounds are common, Godlewski (2015b) finds that more than 65% of loans in his sample are renegotiated only once. Therefore, even analyzing this simple case of one single renegotiation round can have relevant implications in practice.

suance in renegotiation or its absence. Furthermore, we derive novel testable empirical implications regarding equity financing in renegotiation.

The rest of this paper is structured as follows: Section 2 describes our financial setup and valuation of financial claims. Section 3 presents our benchmark model regarding the use of equity financing in debt renegotiation. In section 4 we extend the benchmark model to allow for forced asset sales. Numerical simulations are discussed in section 5, while empirical implications are derived in section 6. Finally, section 7 concludes.

2. Financial setup and valuation

In this section we initially describe the continuous-time financial setup we use. We then introduce our model of debt renegotiation and present the valuation of financial claims. We consider a firm that is financed by equity and a consol debt only. The initial coupon value is denoted by c. The firms' EBIT (Earnings before interests and taxes), $X(t)$ follows a geometric Brownian motion:[4]

$$dX_t = \mu X_t dt + \sigma X_t dW_t, X_0 = x, \tag{1}$$

where $W = (W_t)_t$ is a standard Brownian motion, $x > 0$, and μ and σ represent the drift and volatility terms, respectively.

The firm pays income taxes at a rate τ. The interest rate is denoted by $r > \mu$ (see Dixit and Pindyck, 1994). In case of liquidation, the proceeds are $\frac{\alpha X_t}{r-\mu}$, where $\alpha \in (0, 1)$ represents the recovery rate.[5]

For the purpose of valuation, we will consider first the case in which there is no renegotiation, which will serve as a benchmark for the case with renegotiation.

2.1. No renegotiation

Let us assume that there exists no renegotiation and that the firm is directly liquidated. We denote the equity, debt, and firm values by $E(x, c)$, $D(x, c)$, and $V(x, c)$, respectively. Following the standard literature (see Leland, 1994, Goldstein et al., 2001), we obtain:

[4] The EBIT we consider is net of any running costs, which implies that the equity holder of a fully-equity financed firm would perpetually operate the firm without liquidation.

[5] In the main model we assume that partial asset sales are not possible. As an extension we will consider forced partial asset sales in Section 4.

$$E(x,c)=\frac{(1-\tau)x}{r-\mu}-\frac{(1-\tau)c}{r}-\left(\frac{(1-\tau)x_B(c)}{r-\mu}-\frac{(1-\tau)c}{r}\right)\left(\frac{x}{x_B(c)}\right)^{\gamma}, \tag{2}$$

$$D(x,c)=\frac{c}{r}\left(1-\left(\frac{x}{x_B(c)}\right)^{\gamma}\right)+\frac{\alpha x_B(c)}{r-\mu}\left(\frac{x}{x_B(c)}\right)^{\gamma}, \tag{3}$$

$$\begin{aligned} V(x,c) &= E(x,c)+D(x,c) \\ &= \frac{(1-\tau)x}{r-\mu}+\frac{\tau c}{r}-\left(\frac{(1-\tau)x_B(c)}{r-\mu}+\frac{\tau c}{r}-\frac{\alpha x_B(c)}{r-\mu}\right)\left(\frac{x}{x_B(c)}\right)^{\gamma}, \end{aligned} \tag{4}$$

with

$$x_B(c)=\frac{\gamma(r-\mu)c}{(\gamma-1)r}, \tag{5}$$

representing the default threshold. Here, the constant γ is given by $\gamma = 1/2-\mu/\sigma^2-\sqrt{(\mu/\sigma^2-1/2)^2+2r/\sigma^2} < 0$ and the EBIT value x is higher than the default threshold, which is endogenously chosen by the equity holder. Note that $x_B(c)$ is the optimal default threshold in the absence of renegotiation.

2.2. Debt renegotiation

Renegotiation consists of permanently reducing the initial coupon c to a lower payment of c_1. We thus have a lump-sum and permanent coupon reduction, following Moraux and Silaghi (2014). Indeed, continuous and infinitesimal coupon reductions as in Mella-Barral and Perraudin (1997) or Fan and Sundaresan (2000) are not likely to occur in practice due to renegotiation costs.[6] The renegotiation time is optimally chosen according to the

[6] In practice, renegotiation can consist of amending one or several contractual terms, such as the amount, the maturity, the covenants, etc. In our framework however, since we consider a perpetual debt, modeling renegotiation through a maturity extension or face value reduction is not feasible. Nevertheless, our permanent coupon reduction for a perpetual debt is similar to an amount amendment for a finite debt, which seems to be quite relevant in practice. Indeed, Godlewski (2015a) finds that the amount is the most often amended term in his sample of European loans.

bargaining power of the claimants (equity holder and creditors). We denote the renegotiation threshold by x_R. The claim values at renegotiation then become simple no-renegotiation values as expressed in the previous section, given by $E(x_R, c_1)$, $D(x_R, c_1)$, and $V(x_R, c_1)$. The final post-renegotiation liquidation time is endogenously chosen by the equity holder and denoted by $x_{B_1} \equiv x_B(c_1)$.

Debt renegotiation is costly and implies renegotiation costs proportional to the debt value just prior to renegotiation: $k_R D(x_R, c)$.[7] These renegotiation costs are suffered by the equity holder.[8]

The renegotiation surplus net of renegotiation costs is divided between the claimants according to their bargaining power. In particular, the creditors get $\beta D(x_R, c)$, where $\beta \geq 1$ represents the creditors' premium, and is an indicator of the bargaining power of the creditors.[9] If $\beta = 1$, then creditors are indifferent between renegotiation and liquidation, and the equity holder captures all the renegotiation surplus.

Unlike in Moraux and Silaghi (2014), debt renegotiation does not consist of a pure debt for debt swap. We relax their assumptions and allow for lump-sum transfers between the claimants. Although the creditors obtain in renegotiation $\beta D(x_R, c)$, the new debt value at renegotiation is $D(x_R, c_1)$, which could be different. Therefore, there is a lump-sum transfer of $\beta D(x_R, c) - D(x_R, c_1)$ from the equity holder to the creditors. Note that this transfer could be either positive or negative, depending on the bargaining power of the creditors β and the reduced coupon c_1. The total payment made by the equity holder at renegotiation is therefore:

$$EF(c_1) = (\beta + k_R) D(x_R, c) - D(x_R, c_1), \tag{6}$$

[7]We could also assume that the renegotiation costs are proportional to the firm value at restructuring like in Koziol (2010) or to the coupon reduction as in Hackbarth et al. (2007), or we could assume fixed renegotiation costs as in Moraux and Silaghi (2014). As long as the renegotiation costs are not proportional to the renegotiation surplus, we would obtain a finite number of renegotiations in a context of multiple renegotiations. The implications of the model are robust to this assumption.

[8]Nishihara and Shibata (2016) also assume that the equity holder suffers the renegotiation costs. Moraux and Silaghi (2014) allow for renegotiation costs to be suffered either by the party that has the bargaining power, or always by the equity holder. Since the former assumption brings no extra insights (the main implications of the model are robust) and comes at the cost of lower tractability, we assume the latter.

[9]Creditors would refuse renegotiation unless it is beneficial for them.

including the renegotiation costs and the transfer to the creditors. If this amount is positive, the equity holder will need to issue equity in order to raise these funds. However, the empirical evidence shows that using external equity financing is costly, in particular for small and young firms (Greenwald et al., 1984, Bernanke and Gertler, 1989, Bernanke et al., 1996). We thus assume that equity financing implies a proportional cost k_F. Nevertheless, if the total payment is negative, there is no need for the equity holder to issue equity, we have no equity financing in this case.[10]

We now derive the equity, debt, and firm values of a firm that proceeds to a debt renegotiation, denoted by $E_R(x)$, $D_R(x)$, and $V_R(x)$, respectively.

The equity value with renegotiation is given by:

$$\begin{aligned} E_R(x) = \frac{(1-\tau)x}{r-\mu} - \frac{(1-\tau)c}{r} + \{V(x_R, c_1) - (\beta + k_R)D(x_R, c) \\ -k_F \max\{EF(c_1), 0\} - \frac{(1-\tau)x_R}{r-\mu} + \frac{(1-\tau)c}{r}\right\} \left(\frac{x}{x_R}\right)^\gamma, \end{aligned} \tag{7}$$

The equity holder initially has a claim on the EBIT net of taxes and coupons (accounting for the tax shield) until renegotiation. At renegotiation, she exchanges this claim for a new equity claim with a reduced coupon, net of the total payment at renegotiation (transfer to creditors plus renegotiation costs and equity issuance costs).[11]

The debt and firm values in the case of renegotiation are:

$$D_R(x) = \frac{c}{r} - \left(\frac{c}{r} - \beta D(x_R, c)\right)\left(\frac{x}{x_R}\right)^\gamma, \tag{8}$$

$$\begin{aligned} V_R(x) &= E_R(x) + D_R(x) \\ &= \frac{(1-\tau)x}{r-\mu} + \frac{\tau c}{r} + \{V(x_R, c_1) - k_R D(x_R, c) \\ &\quad -k_F \max\{EF(c_1), 0\} - \frac{(1-\tau)x_R}{r-\mu} - \frac{\tau c}{r}\right\}\left(\frac{x}{x_R}\right)^\gamma, \end{aligned} \tag{9}$$

[10] A necessary condition for the amount to be negative would be for the new debt value $D(x_R, c_1)$ to be larger than the debt value without renegotiation $D(x_R, c)$. As Moraux and Silaghi (2014) show, this is likely to happen since despite having a lower coupon, we also have a lower probability of default, which could increase debt value.

[11] Note that since $V(x_R, c_1) = E(x_R, c_1) + D(x_R, c_1)$, we have that $E(x_R, c_1) - EF(c_1) = V(x_R, c_1) - (\beta + k_R)D(x_R, c)$.

The renegotiation threshold and the reduced coupon are optimally chosen by the claimholders. Consistent with the previous literature (Lambrecht, 2001, Moraux and Silaghi, 2014, and Nishihara and Shibata, 2016), it is optimal for the claimholders to renegotiate as late as possible. In the case when the equity holder has all bargaining power, she wants to delay renegotiation, as a later renegotiation implies a larger coupon reduction. As far as the creditors are concerned, they prefer to receive the full original coupon as long as possible. Therefore, it can be shown that the optimal renegotiation threshold x_R is equal to the original bankruptcy threshold $x_B(c)$ without debt renegotiation.

3. Equity financing in debt renegotiation

3.1. When is renegotiation possible?

Before determining the optimal reduced coupon, we need to answer a more important question: When is renegotiation possible? As we have seen above, renegotiation is designed such that the creditors accept it, since they receive at least as much as they had in case of liquidation. We need to check however, if the equity holder is willing to renegotiate or whether she prefers liquidation. Renegotiation is beneficial to her if the surplus she receives in renegotiation covers the total costs associated to it: renegotiation costs, transfers to the creditors, as well of equity financing costs, if new equity is issued. Formally, renegotiation is preferred to bankruptcy if in equation (7) the following condition is satisfied:

$$V(x_R, c_1) - (\beta + k_R)D(x_R, c) - k_F \max\{EF(c_1), 0\} \geq 0 \tag{10}$$

This means that the new firm value net of the creditors' part, and of renegotiation and equity financing costs has to be positive. Alternatively, given that $V(x_R, c_1) = E(x_R, c_1) + D(x_R, c_1)$ and that $EF(c_1) = (\beta + k_R)D(x_R, c) - D(x_R, c_1)$, we can rewrite the previous condition as:

$$E(x_R, c_1) \geq EF(c_1) + k_F \max\{EF(c_1), 0\} \tag{11}$$

A necessary condition for the firm to be able to raise funds at renegotiation is that the equity value has to be larger than the amount of funds that needs to be raised. Moreover, if equity issuance is costly, then the equity value has to be large enough to cover those costs as well.

A particular case appears when the new debt value at renegotiation $D(x_R, c_1)$ is large enough to cover the creditors' premium and the renegotiation costs. In this case, the firm does not need to raise funds at renegotiation, we have $EF(c_1) < 0$. Therefore, the condition above is always satisfied in that case since $E(x_R, c_1) \geq 0 > EF(c_1)$, and renegotiation is possible.

In case the firm needs to raise funds, i.e., $EF > 0$, renegotiation fails either when $E(x_R, c_1) < EF(c_1)$ because the renegotiation costs k_R and/or the creditors' premium β are too large, or when $EF(c_1) \leq E(x_R, c_1) < (1 + k_F)EF(c_1)$ because equity issuance costs k_F are too large. Of course, this is rather intuitive, the larger the costs (either k_R or k_F) and the larger the creditors' premium, the less likely it is that renegotiation will take place. This is also in line with Nishihara and Shibata (2016), who study the choice between renegotiation and liquidation by making a numerical comparative statics analysis with respect to these parameters. However, in their numerical analysis they highlight the fact that equity financing is always positive for the broad range of parameter values that they try, which is not the case for us. We show both analytically and numerically that equity financing can be both positive and negative. Therefore, equity financing is not always used in debt renegotiation. Moreover, in section 3.3 we investigate when equity financing is more likely to occur in renegotiation.

We contribute to this research question by presenting some quantitative evidence as well in section 5, where we numerically investigate the maximum size of renegotiation and equity issuance costs compatible with renegotiation.

The respective constraints for renegotiation to take place are expressed as a function of a general reduced coupon c_1. Of course, they should be evaluated at the optimal reduced coupon c_1^* that we will derive in the next subsection.

3.2. Optimal debt reduction

Regarding the optimal reduced coupon, since transfers between the two parties are allowed, there is no constraint regarding the choice of the reduced coupon,[12] and it is optimal to choose the coupon that maximizes the total

[12]If transfers were not allowed, as it is the case in Moraux and Silaghi (2014), then there would be a lower boundary below which the new coupon could not descend, since creditors would refuse renegotiation, i.e. c_{min} in Moraux and Silaghi (2014). When transfers are allowed, we can choose a reduced coupon that implies a lower debt value for the creditors, since we can compensate them by making them a lump-sum tranfer.

firm value. Looking at equation (9), we can see that this reduces to choosing the coupon that maximizes the new firm value at renegotiation net of equity issuance costs. Formally, the new coupon solves:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) - k_F \max\{EF(c_1), 0\} \tag{12}$$

Note that this is also equivalent to maximizing $E(x_R, c_1) - EF(c_1) - k_F \max\{EF(c_1), 0\}$.

The following proposition then applies.

Proposition 1. *The optimal reduced coupon is given by:*

(i) *Negative transfers*
If $(\beta + k_R)\alpha\gamma/(\gamma - 1) < A'$ *then* $EF(c_1^*) < 0$ *and:*

$$c_1^* \equiv c_1^A = c_0 * A \tag{13}$$

(ii) *Positive equity financing*
If $(\beta + k_R)\alpha\gamma/(\gamma - 1) > B'$ *then* $EF(c_1^*) > 0$ *and:*

$$c_1^* \equiv c_1^B = c_0 * B \tag{14}$$

(iii) *No equity issuance*
If $A' \leq (\beta + k_R)\alpha\gamma/(\gamma - 1) \leq B'$ *then* $c_1^* \equiv c_1^{EF}$ *is such that* $EF(c_1^*) = 0$ *and:*

$$c_1^* \in (c_1^A, c_1^B), \tag{15}$$

where

$$\begin{aligned}
A &= \left(\frac{\tau - (1-\alpha)\gamma}{\tau}\right)^{1/\gamma} \\
B &= \left(\frac{\tau + k_F - (1+k_F)(1-\alpha)\gamma}{\tau + k_F}\right)^{1/\gamma} \\
A' &= A + A^{1-\gamma}\left(\frac{\alpha\gamma}{\gamma - 1} - 1\right) \\
B' &= B + B^{1-\gamma}\left(\frac{\alpha\gamma}{\gamma - 1} - 1\right),
\end{aligned} \tag{16}$$

with $A \leq B$ and $A' \leq B'$, with equality $A = B$ and $A' = B'$ for $k_F = 0$.

Proof of Proposition 1. See appendix.

We have three possible reduced coupons depending on the parameter values, which will be illustrated later on in the numerical section. If the renegotiation costs and the bargaining power of the creditors are relatively low, the firm does not need to issue equity in order to raise funds to cover the debt renegotiation costs as well as the transfer to the creditors. On the contrary, the transfer to the creditors is negative. The optimal reduced coupon in this case is relatively low and does not depend on the renegotiation costs k_R, on the creditors' premium β, nor on the equity financing cost k_F, since there is no need for equity financing.

When the renegotiation costs and the bargaining power of the creditors are relatively high, the equity holder does not have enough funds to cover the renegotiation costs as well as the transfer due to the creditors. In this case we have positive equity financing, and the reduced coupon is relatively large. This means that we have a smaller coupon reduction, which implies a smaller transfer from the equity holder to the creditors. Indeed, since obtaining funds to finance the transfer is costly, it is optimal to try to minimize the transfer. Moreover, the reduced coupon depends on the costs of equity financing, k_F. Higher costs of equity financing lead to a higher reduced coupon, thus the firm retires less debt, we have a lower coupon reduction in order to minimize the amount of costly equity issuance. However, the reduced coupon does not depend on the renegotiation costs k_R, nor on the bargaining power of the creditors, β.

Finally, for intermediate values of the renegotiation costs and the bargaining power of the creditors, the reduced coupon will be chosen such that there is no equity issuance and the firm has the exact required funds to cover the renegotiation costs and the transfer to the creditors. In this case, the intermediate reduced coupon depends on the renegotiation costs k_R and the bargaining power of the creditors β, but not on the equity financing costs k_F.

Since it will be useful in the following subsections, we derive simple expressions for the total payment made by the equity holder to cover the renegotiation costs and the transfer to the creditors, $EF(c_1^*) = (\beta + k_R)D(x_R, c_0) - D(x_R, c_1^*)$, in the first two cases.[13] Using equation (3), we have that $D(x_R, c_0) =$

[13] The total payment can be either positive or negative, since the transfer to the creditors can be positive or negative depending on the optimal reduced coupon.

$c_0/r\,[\alpha\gamma/(\gamma-1)]$. Using the same equation (3), the fact that $c_1^A = c * A$, $c_1^B = c * B$ and the expressions of A' and B' from equation (16), we have that $D(x_R, c_1^A) = c_0/rA'$, and $D(x_R, c_1^B) = c_0/rB'$. If we denote $Q \equiv (\beta + k_R)\alpha\gamma/(\gamma-1)$, then we obtain that:

$$\begin{aligned} EF(c_1^A) &= (Q - A')\,c_0/r < 0 \\ EF(c_1^B) &= (Q - B')\,c_0/r > 0 \end{aligned} \tag{17}$$

3.2.1. Limiting cases: no costs

We now analyze what happens in the limiting cases when renegotiation and equity issuance are costless. In the absence of equity financing costs, $k_F = 0$, the three cases above collapse to a single case, there is a unique reduced coupon $c_1^* = c_1^A = c_1^B$, with equity financing either negative for relatively low renegotiation costs and bargaining power of the creditors, or positive for relatively high renegotiation costs and bargaining power of the creditors. The optimal reduced coupon is independent of the bargaining power of the creditors and of the renegotiation costs.

Even in the absence of renegotiation costs $k_R = 0$ and full bargaining power for the equity holder $\beta = 1$, we cannot exclude the possibility of equity financing. Although no funds are needed to cover the renegotiation costs, the firm could still need funds to finance the transfer to the creditors. Even though the creditors have no bargaining power, renegotiation cannot be detrimental to them. Whenever the new optimal reduced coupon does not guarantee that the new debt value at renegotiation with the reduced coupon is at least as high as the debt value with the original coupon, there is a positive transfer from the equity holder to the creditors which needs to be financed through equity issuance. Therefore, we still have the three possibilities for the coupon reduction. However, in this case, renegotiation is always possible irrespective of the equity financing costs.[14]

[14] To show this, take the coupon that makes creditors indifferent between renegotiation and liquidation, denoted by c_{min} in Moraux and Silaghi (2014) and for which $D(x_R, c_{min}) = D(x_R, c_0)$. For $k_R = 0$ and $\beta = 1$ we have that $EF(c_{min}) = D(x_R, c_0) - D(x_R, c_{min}) = 0$, therefore $E(c_{min}) - EF(c_{min}) - k_F \max\{EF(c_{min}, 0\} = E(c_{min}) > 0$. But the optimal reduced coupon, c_1^*, maximizes $Eq(c_1) - EF(c_1) - k_F \max\{EF(c_1, 0\}$. Thus we have that $E(c_1^*) - EF(c_1^*) - k_F \max\{EF(c_1^*, 0\} \geq E(c_{min}) - EF(c_{min}) - k_F \max\{EF(c_{min}, 0\} > 0$, therefore, according to equation (11), renegotiation is always possible.

3.2.2. Comparative statics

As we have previously seen, depending on the relative size of renegotiation and equity financing costs, as well as on the bargaining power of the creditors, we obtain different optimal reduced coupons. At a closer look, we notice that the optimal reduced coupon which implies a negative transfer from the equity holder to the creditors, c_1^A (first case in Proposition 1), does not depend on the costs, nor on the bargaining power of the creditors. In the second case where the firm needs to issue equity to raise funds, we have that, on the contrary, the optimal reduced coupon c_1^B does depend on the equity financing costs. In particular, we have that $\partial c_1^B/\partial k_F > 0$, i.e., the firm will proceed to a lower debt reduction when equity issuance costs are high in order to minimize them. This adjustment is also numerically observed by Nishihara and Shibata (2016). Finally, when the optimal reduced coupon is set such that the equity financing need is exactly equal to zero, the reduced coupon c_1^{EF} depends both on the renegotiation costs k_R and on the creditors' premium β. As before, we observe an adjustment: the larger the renegotiation costs or the creditors' premium, the lower will be the coupon reduction. The firm optimally decides to reduce the coupon in a smaller proportion, in order to avoid the need of issuing equity.

3.2.3. Constrained versus unconstrained coupon choice: the impact of transfers

We would like to end this section by discussing the effect of allowing for transfers on the renegotiation process. First, we have seen that allowing for transfers between the claimholders has resulted in situations in which the firm proceeds to a debt renegotiation financed both by a new debt and by new equity issuance. Thus we no longer have a simple debt for debt swap.

Secondly, regarding the optimal reduced coupon, we compare with Moraux and Silaghi (2014), where transfers were not allowed. They provide an interval for the optimal reduced coupon, $[c_{min}, c_{max}]$. The two limits of this interval correspond to the two polar cases in which the equity holder has all the bargaining power, and the creditors have all the bargaining power respectively. More specifically, c_{min} was defined such that the creditors are indifferent between renegotiation and liquidation (our case of $\beta = 1$) and c_{max} such that it maximizes debt value at renegotiation. A coupon below c_{min} was not possible since creditors would refuse renegotiation. When transfers are allowed, the coupon choice is not constrained anymore. The equity holder selects the coupon that maximizes the net surplus of renegotiation, although this might imply $c_1^* < c_{min}$ and $D(x_R, c_1) < D(x_R, c_0)$. The difference is that now this lower debt value can be compensated by a transfer from the equity holder to the creditors.

In general, with a costly renegotiation and positive premium for the creditors, c_1^B could be either below or above c_{min}, but for sure below c_{max}. We can also show that $c_1^A \in [c_{min}, c_{max}]$. This is logical since for this coupon value there is no equity issuance. The coupon such that equity financing is exactly zero, c_1^{EF} also belongs to that interval. The optimal reduced coupon would never be above c_{max} since, as Moraux and Silaghi (2014) argue, those values would be Pareto dominated, in the sense that we could improve both the equity and debt value by further reducing the coupon.

In the particular case of $k_R = 0$ (no renegotiation costs) and $\beta = 1$, we can show that $c_1^B < c_{min}$, which implies that it is optimal for the firm to further decrease the coupon and issue equity to finance the transfer to the creditors, since the extra surplus obtained with the additional reduction more than compensates for the extra costs of equity financing. Moreover, we can also show that $c_1^{EF} = c_{min}$.

To sum up, allowing for transfers increases the range of possible coupon reductions, with even lower reductions being optimal under certain conditions. These reductions below the coupon that makes the creditors indifferent (c_{min}) are possible precisely because the firm issues equity to raise funds needed to finance the transfers to the creditors in order to compensate them for the extra coupon reduction. We have a less constrained choice that increases the total firm value. Thus, eliminating restrictions on transfers leads to increased welfare. We will numerically quantify this effect in section 5.

3.3. When is equity financing more likely in renegotiation?

We have seen that allowing for transfers between the claimholders can lead to equity issuance in debt renegotiation. We now analyze how likely it is for the firm to issue equity in debt renegotiation. Whether the firm issues equity or not depends on the size of the renegotiation costs and creditors' premium relative to the equity issuance costs. We know from Proposition 1 that when $(\beta + k_R)\alpha\gamma/(\gamma - 1) > B'$ the firm will have positive equity financing, $EF(c_1^B) > 0$. On the contrary, when renegotiation costs are relatively low, $(\beta + k_R)\alpha\gamma/(\gamma - 1) < A'$, the firm will not issue equity, the transfers from the firm to the creditors being negative, $EF(c_1^A) < 0$. For intermediate costs, $A' \leq (\beta + k_R)\alpha\gamma/(\gamma - 1) \leq B'$, the firm will set the reduced coupon such that the equity issuance is exactly equal to zero.

Therefore, we present comparative statics of these three quantities ($Q \equiv (\beta + k_R)\alpha\gamma/(\gamma - 1)$, A', and B') with respect to the parameters of the model (α, τ, k_R, β, k_F and γ). These comparative statics will allow us to derive empirical implications regarding the use of equity financing in debt renegotiation, and to contrast them with the empirical evidence.

For intuitive purposes, note that Q is proportional to the debt value at the renegotiation threshold with the original coupon, $D(x_R, c_0)$, while A' and B' are proportional to the new debt value at renegotiation with the reduced coupon, $D(x_R, c_1^*)$.[15]

Recovery rate, α . We can show regarding the proportion recovered in liquidation α that an increase in this parameter leads to an increase in A', B' and Q. Intuitively, since these quantities are proportional to the debt values at x_R with or without renegotiation, it is logical that an increase in the recovery rate leads to an increase in the debt values. In order to know whether issuing equity is more likely for a firm with a higher recovery value (lower bankruptcy costs) we would need to know which of these quantities increases more. Although it is not possible to answer this question analytically, we will show numerically in section 5 that Q increases more than A' and B' for low values of the recovery rate, i.e., $D(x_R, c_0)$ increases more with the recovery rate α than $D(x_R, c_1)$. This is due to the fact that the impact of the recovery rate is larger the closer the firm is to bankruptcy. Indeed, in the absence of renegotiation, we know that the firm would default at $x_R = x_B(c_0)$, this is why the impact is larger on $D(x_R, c_0)$. Therefore, it is more likely that $Q > B'$, i.e., the renegotiation costs together with the creditors' premium are larger than the debt value with the reduced coupon, and the firm needs to make a positive payment. Hence, a firm with a larger recovery value will more likely issue equity in renegotiation to finance the required funds.

Tax rate, τ . We can see that the renegotiation costs as well as the creditors' premium do not depend on the tax rate since renegotiation takes place at the optimal no-renegotiation bankruptcy threshold, $x_B(c_0)$. At this threshold, the initial debt value $D(x_R, c_0)$ is simply equal to the liquidation value of the firm, and does not depend on the tax rate. Thus, Q will not depend on τ. On the other hand, we can show that both A' and B' increase with the tax

[15] We remind the reader that $Q = (\beta + k_R)D(x_R, c_0)/(c_0/r)$, $A' = D(x_R, c_1^A)/(c_0/r)$ and $B' = D(x_R, c_1^B)/(c_0/r)$.

rate. Intuitively, a larger tax rate makes the firm choose a larger reduced coupon value to benefit from the tax advantage of debt, which will increase the value of the debt value just after renegotiation, $D(x_R, c_1)$.[16] Thus, it is more likely that the firm will not issue equity when the tax rate increases, since it will not need additional funds to make transfers to the creditors.

Renegotiation costs and creditors' premium, k_R and β. An increase in the renegotiation costs and the creditors' premium leads to an increase in the total payments that the equity holder will have to make at renegotiation, by increasing Q and not affecting A' or B'. It is more likely then for a firm with larger renegotiation costs or larger bargaining power for the creditors to need to issue equity to finance these costs and transfers.

Equity issuance costs, k_F. A firm which faces larger equity issuance costs will adjust the reduced coupon such that it limits the amount of funds it needs to raise. Therefore, we have a larger reduced coupon c_1 (a smaller coupon reduction) which implies a larger debt value $D(x_R, c_1)$, which will reduce the transfers to the creditors. Formally, Q and A' are not affected by the issuance costs k_F, while B' increases with k_F. Thus, it is more likely that when facing higher issuance costs, the firm will be less likely to issue equity at renegotiation (it is more likely for Q to be lower than B').

Drift, volatility, and interest rate, γ. We remind the reader that $\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2}$, thus it actually incorporates three parameters: μ, r and σ, the first two decreasing with γ and the last one increasing with γ. It is straightforward to show that Q decreases with γ, that is, it increases with μ and r and it decreases with σ. Intuitively, a firm with a larger volatility will decide to renegotiate (or to default in case of no renegotiation) at a lower threshold. The liquidation value of the firm will therefore be lower,[17] and so will be the renegotiation costs and the creditors' premium. However, we cannot show analytically how an increase in γ affects A' or B'. Nevertheless, we can show numerically (for reasonable parameter

[16] In general we know that debt is a hump-shape value of the coupon. However, since we know that the reduced coupon is below the coupon that maximizes debt value, c_{max}, we can conclude that debt is increasing in the reduced coupon.

[17] Note that the debt value without renegotiation at the renegotiation threshold x_R, $D(x_R, c_0)$ is simply equal to the liquidation value of the firm, since x_R is the threshold at which the firm would bankrupt in the absence of renegotiation.

values) that they also decrease with γ. Intuitively, when volatility increases, the new debt value with the reduced coupon decreases. We will show later on in the numerical section that the larger the volatility (or the lower the drift or the interest rate), the more likely it is for the firm to issue equity at renegotiation. A firm with larger volatility or lower growth rate of EBIT is then more likely to use equity financing to repurchase debt.

4. Extension:Forced asset sales

We now extend the previous framework in order to account for forced asset sales. Following other papers in the literature, Mella-Barral (1999), and Nishihara and Shibata (2016), we assume economies of scale, which implies that partial liquidation is inefficient, i.e. assets sold piecemeal are less valuable than the same assets sold as a going concern. Therefore, it is optimal for the firm not to partially sell assets. Nevertheless, *forced* asset sales of viable businesses do occur in practice, and, as documented by Djankov et al. (2008), they make debt enforcement inefficient. Moreover, according to the evidence of Kruse et al. (2014), 14.9% of debt tender offers use asset sales as a financing source.

Formally, we assume that by selling a fraction $\phi \in (0,1)$ of the assets at time t, the equity holder receives the proceeds $P(X(t),\phi)$ after taxes. We let $P(x,\phi) = F(\phi)x$, where F is a non-decreasing convex function with $F(0) = 0$. The convexity implies that full liquidation will always be preferred to partial liquidation. Therefore, if the firm could optimally choose the fraction of assets to liquidate at renegotiation, it would choose not to sell assets, i.e., $\phi = 0$, which is the case of our baseline framework.

We therefore adjust our initial equity, debt, and firm value under no renegotiation, to account for asset sales. Consider a firm that is operating with the asset size ϕ. Its equity, debt and firm value are given by the following equations:

$$E(x,\phi,c) = \frac{(1-\tau)\phi x}{r-\mu} - \frac{(1-\tau)c}{r} - \left(\frac{(1-\tau)\phi x_B(\phi,c)}{r-\mu} - \frac{(1-\tau)c}{r}\right)\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma} \tag{18}$$

$$D(x,\phi,c) = \frac{c}{r}\left(1-\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma}\right) + P(x_B(\phi,c),\phi)\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma}, \tag{19}$$

$$
\begin{aligned}
V(x,\phi,c) &= E(x,\phi,c) + D(x,\phi,c) \\
&= \frac{(1-\tau)\phi x}{r-\mu} + \frac{\tau c}{r} - \left(\frac{(1-\tau)\phi x_B(\phi,c)}{r-\mu} + \frac{\tau c}{r} - P(x_B(\phi,c),\phi)\right)\left(\frac{x}{x_B(\phi,c)}\right)^{\gamma},
\end{aligned}
\tag{20}
$$

with

$$
x_B(\phi,c) = \frac{\gamma(r-\mu)c}{(\gamma-1)r\phi}, \tag{21}
$$

representing the default threshold.

If the firm sells a fraction ϕ of its assets at renegotiation, then the amount of equity financing needed in renegotiation will depend on the proceeds from the asset sales:

$$
EF(c_1,\phi) = (\beta + k_R)D(x_R,1,c) - D(x_R,1-\phi,c_1) - P(x_R,\phi), \tag{22}
$$

where $x_R = x_B(1,c)$.

If the new debt value of a firm operating with an asset size $1-\phi$ plus the proceeds from selling a fraction ϕ of the assets are not enough to cover the creditors' premium and the renegotiation costs, then the firm will have to issue equity financing.

Renegotiation is possible only if:

$$
V(x_R,1-\phi,c_1) + P(x_R,\phi) - (\beta+k_R)D(x_R,1,c) - k_F \max\{EF(c_1,\phi),0\} \geq 0 \tag{23}
$$

The optimal coupon is chosen in order to maximize the new firm value at renegotiation after the asset sale, net of equity issuance costs:

$$
c_1^*(\phi) = \arg\max_{c_1} V(x_R,1-\phi,c_1) - k_F \max\{EF(c_1,\phi),0\} \tag{24}
$$

In order to obtain closed-form solutions for the optimal reduced coupon, we take an explicit function of the proceeds obtained through asset sales:[18]

$$
P(x_R,\phi) = \frac{\alpha\phi^{1.01}x_R}{r-\mu}, \tag{25}
$$

[18] Nishihara and Shibata (2016) use the same liquidation function.

which is a convex function. Note that for $\phi = 1$, the full liquidation value is given by $\frac{\alpha x_R}{r-\mu}$, which is the same as in the benchmark model.

The following proposition then applies.

Proposition 2. *The optimal reduced coupon in the general case in which we allow for asset sales in renegotiation is given by:*

(i) *Negative transfers*
If $(\beta + k_R - \phi^{1.01})\alpha\gamma/(\gamma - 1) < A'(\phi)$ *then* $EF(c_1^*(\phi), \phi) < 0$ *and:*

$$c_1^*(\phi) \equiv c_1^A(\phi) = c_0 * A(\phi) \tag{26}$$

(ii) *Positive equity financing*
If $(\beta + k_R - \phi^{1.01})\alpha\gamma/(\gamma - 1) > B'(\phi)$ *then* $EF(c_1^*(\phi), \phi) > 0$ *and:*

$$c_1^*(\phi) \equiv c_1^B(\phi) = c_0 * B(\phi) \tag{27}$$

(iii) *No equity issuance*
If $A'(\phi) \leq (\beta + k_R - \phi^{1.01})\alpha\gamma/(\gamma - 1) \leq B'(\phi)$ *then* $c_1^*(\phi) \equiv c_1^{EF}(\phi)$ *is such that* $EF(c_1^*(\phi), \phi) = 0$ *and:*

$$c_1^*(\phi) \in (c_1^A(\phi), c_1^B(\phi)), \tag{28}$$

where

$$\begin{aligned}
A(\phi) &= (1-\phi)\left(\frac{(\tau-\gamma)(1-\phi) + \alpha\gamma(1-\phi)^{1.01}}{\tau(1-\phi)}\right)^{1/\gamma} \\
B(\phi) &= (1-\phi)\left(\frac{\tau + k_F - (1+k_F)(\gamma - \alpha\gamma(1-\phi)^{0.01})}{\tau + k_F}\right)^{1/\gamma} \\
A'(\phi) &= A(\phi) + A(\phi)\left(\frac{\alpha\gamma(1-\phi)^{0.01}}{\gamma-1} - 1\right)\left(\frac{1-\phi}{A(\phi)}\right)^{\gamma} \\
B'(\phi) &= B(\phi) + B(\phi)\left(\frac{\alpha\gamma(1-\phi)^{0.01}}{\gamma-1} - 1\right)\left(\frac{1-\phi}{B(\phi)}\right)^{\gamma},
\end{aligned} \tag{29}$$

with $A(\phi) \leq B(\phi)$ and $A'(\phi) \leq B'(\phi)$, with equality $A(\phi) = B(\phi)$ and $A'(\phi) = B'(\phi)$ for $k_F = 0$. Note that for $\phi = 0$, i.e., no asset sales in renegotiation, we obtain the same results as in proposition 1 in our benchmark model.

Proof of Proposition 2. See appendix.

5. Numerical analysis

We study the numerical implications of our benchmark model on the value of the optimal reduced coupon, the conditions under which renegotiation is possible, and the use of equity financing in renegotiation, in the first three subsections. We will also analyze the impact of forced asset sales on the renegotiation process in the fourth subsection.

As far as the parameter values are concerned, we choose orders of magnitude similar to those assumed by previous models of debt renegotiation, in order to facilitate comparison between models. For our baseline case, we set the riskless interest rate to 6% (as Leland, 1994, Mella-Barral and Perraudin, 1997, and Nishihara and Shibata, 2016 did),[19] the drift to 1% (as in Bruche and Naqvi, 2010), the tax rate to 35% (as in Leland, 1994), the volatility to 20% (Leland, 1994 and Fan and Sundaresan, 2000 set it to 25%, while Nishihara and Shibata (2016) sets it at 20%), the recovery rate to 60% (Mella-Barral and Perraudin, 1997 chose bankruptcy costs of 20%, while Leland, 1994 chose bankruptcy costs of 50%), the renegotiation costs to 5%, the equity issuance costs to 10%, and the creditors' premium β to 1.05 (the last three in line with Nishihara and Shibata, 2016). Finally, without loss of generality, we consider an initial coupon value of 2 and we set the initial cash flow value equal to 2.

[Table 1 about here.]

Our baseline case parameter values are presented in Table 1. These parameter values are used in all the tables and figures presented in this paper, unless specified otherwise. Nevertheless, we also present comparative statics for every parameter, thus we will let each variable vary along a quite large interval around the baseline values. This is done with a twofold aim: in order to be able to illustrate the different solutions we obtain for our model (renegotiation versus bankruptcy, equity financing in renegotiation or pure debt for debt swap) and for robustness purposes.

5.1. Optimal reduced coupon

In this subsection we illustrate the three cases for the optimal reduced coupon, we present its comparative statics, and we analyze the impact of

[19] Since a risk-free rate of 6% seems very high compared to current rates, we also analyze the numerical implications of the model for any value of r between 0 and 6%.

allowing for transfers between the claimholders, thus having an unconstrained coupon choice.

We start by presenting three different set of parameters under which we obtain three different cases for the optimal reduced coupon: c_1^A, where we have no equity financing, c_1^{EF}, where the coupon is exactly set such that there is no need to issue equity financing, and c_1^B, where there is positive equity financing in renegotiation. Table 2 illustrates the variables of interest for three different values of the tax rate: 15%, 25% and 35%. We focus on the following variables: Q, A' and B', which will tell us whether equity financing occurs in renegotiation or not and what the optimal reduced coupon is, the reduced coupon c_1^*, the equity financing value $EF(c_1^*)$, the equity and debt value at renegotiation $E(x_R, c_1^*)$ and $D(x_R, c_1^*)$, the total equity, debt and firm value at time 0 with and without renegotiation, $E_R(x_0)$, $D_R(x_0)$, $V_R(x_0)$, and $E(x_0, c_0)$, $D(x_0, c_0)$, $V(x_0, c_0)$, respectively.

[Table 2 about here.]

In order to simplify the analysis, we set the renegotiation costs $k_R = 0$ and the creditors' premium, $\beta = 1$. The rest of the parameters are the baseline parameters. In the first case in which the tax rate is equal to 35% (second column of Table 2), we see that $Q < A'$, since the creditors' share and renegotiation costs are relatively low, thus the firm does not need to issue equity financing, and the optimal coupon is c_1^A. Indeed, we observe that for this optimal reduced coupon, the new debt value obtained with the new coupon at the renegotiation threshold, $D(x_R, c_1^*)$ is larger than the debt value without renegotiation at the same threshold, $D(x_R, c_0)$. Since $\beta = 1$ and creditors obtain no surplus at renegotiation, they will have to transfer to the equity holder an amount of $D(x_R, c_1^*) - D(x_R, c_0) = 1.092$. Moreover, as renegotiation costs are equal to zero, the total payment that the equity holder has to make at renegotiation (renegotiation costs plus transfer to the creditors) is negative. Comparing the debt value at time 0 with and without renegotiation, we can see that the creditors are indifferent between renegotiating or not, which is normal given that we set $\beta = 1$. The equity holder has all the bargaining power and takes all the renegotiation surplus, $E_R(x_0) - E(x_0, c_0) = 1.625$.

For a low tax rate of 15%, we have that $B' < Q$, which implies that the firm will issue equity at renegotiation in order to finance the payment that the equity holder needs to make. Since the renegotiation costs are null, this payment only consists of the transfers to the creditors. Since the tax rate is low, it is optimal for the firm to choose a lower reduced coupon, as the tax advantage of debt is reduced. This leads to a lower debt value at renegotiation, $D(x_R, c_1^*) = 11.610$. Given that the debt value without renegotiation at x_R is $D(x_R, c_0^*) = 12.000$, the creditors would only accept renegotiation if they receive a positive transfer from the equity holder of at least 0.390. This is precisely the amount of funds that the firm has to raise through equity issuance, as renegotiation costs are null. As before, the equity holder captures all the surplus from renegotiation, and the creditor is indifferent between renegotiation and liquidation.

For an intermediate tax rate of 25%, we have that $A' < Q < B'$, which means that the firm will optimally choose the reduced coupon such that it avoids equity issuance, $EF(c_1^*) = 0$. Given that there are no renegotiation costs, the new coupon is chosen such that the new debt value at renegotiation $D(x_R, c_1^*)$ is exactly equal to the debt value without renegotiation $D(x_R, c_0)$, and there are no transfers from the equity holder to the creditors. The optimal coupon in this case coincides with the coupon that Moraux and Silaghi (2014) find, c_{min} (the case of one costless renegotiation, full bargaining power to the equity holder). Indeed, they assumed that lump-sum transfers between the two claimholders were not possible.

[Figure 1 about here.]

We now represent graphically the optimal reduced coupon and its comparative statics with respect to the parameters of the model. Figure 1 plots the optimal reduced coupon as a function of the equity issuance costs for different values of the renegotiation costs. In panel a), for low renegotiation costs, we have that for $k_F < 0.34$ the optimal reduced coupon is c_1^B. In this case there exists positive equity financing in renegotiation, which implies that the optimal coupon is increasing in the issuance costs. For higher issuance costs (above 0.34), it is optimal for the firm to set the coupon such that the equity issuance amount is exactly equal to zero, thus the optimal coupon is c_1^{EF}, and does not depend on the issuance costs. In panel b), for higher renegotiation costs, the optimal reduced coupon is always c_1^B.[20]

[Figure 2 about here.]

[20] Nevertheless, renegotiation is only possible for $k_F < 0.45$ (see Figure 5). For $k_F > 0.45$ the firm is liquidated.

We also illustrate how the reduced coupon varies with the bargaining power of the creditors in Figure 2. In panel a), for a tax rate of 15%, it is optimal for the firm to reduce significantly the coupon as the tax advantage of debt is low. The optimal coupon is c_1^B with equity financing, and does not vary with the creditors' premium, β. For a higher tax rate of 35%, the optimal coupon obtained depends on the value of the creditors' premium. When the latter one is relatively large, the firm issues equity in renegotiation, the optimal coupon is c_1^B, and does not vary with β. For relatively low values of the creditors' premium, there is no equity financing in renegotiation, the optimal coupon is c_1^A, and again, it does not vary with β. For intermediate values of the creditors' premium, the firm chooses the coupon such that there is no need to issue equity financing in renegotiation. The optimal coupon is c_1^{EF} and it increases in the creditors' premium. The larger the bargaining power of the creditors, the larger the optimal new coupon in order to reduce the need for a transfer to the creditors, and thus the need for equity financing.

We have thus provided three numerical examples of the three cases we had previously characterized analytically. For reasonable parameter values, we can either have positive equity issuance in renegotiation, or no equity issuance in renegotiation (negative, or exactly equal to zero). This evidence is in contrast with the findings of Nishihara and Shibata (2016), who argue that for the vast range of parameter values that they have used, equity financing always appears to be positive, although the only results reported are for a tax rate of 15%. For higher tax rates, we have seen that the firm does not issue equity in renegotiation. Unlike Nishihara and Shibata (2016), we characterize analytically the conditions under which the firm issues equity in renegotiation. We thus offer a complete characterization of equity financing in renegotiation.

Finally, we quantify numerically the impact of allowing for transfers between claimholders, and thus issuing equity in renegotiation, by comparing to Moraux and Silaghi (2014), where transfers are not allowed. In Table 3 we compare the optimal reduced coupon when not allowing for transfers, with the optimal reduced coupon when equity issuance is possible. We can see that when equity financing is used in renegotiation the debt reduction is even larger. In our numerical example the coupon is 23.55% lower with equity financing than in a pure debt for debt swap. Moreover, as argued previously, we observe that allowing for transfers increases welfare. The firm value at renegotiation increases by 1.76%, while the renegotiation surplus net of costs increases by 2.69%.[21]

[Table 3 about here.]

5.2. Conditions for renegotiation

We now make a qualitative and quantitative analysis regarding the conditions under which renegotiation is possible.

We illustrate graphically the first condition that tells us whether renegotiation is possible or not, inequality (10). According to the latter, renegotiation is possible whenever the firm value at renegotiation is larger than the total costs of renegotiation (equity financing costs plus renegotiation costs plus creditors' premium). In Figure 3 we plot the firm value at renegotiation and the total costs, as a function of equity issuance costs parameter, k_F, for three different values of the renegotiation costs, k_R. The premium of the creditors is fixed ($\beta = 1.05$) and the tax rate is set to 15%. In panel a), for low renegotiation costs $k_R = 0.05$ we have that renegotiation is always possible, irrespective of the value of the equity issuance costs. For $k_F < 0.34$, we notice that the total costs are hump shaped in k_F. This is due to the fact that the issuance costs are given by $k_F \max(EF, 0)$, and the amount of equity finance is decreasing in k_F. For $k_F > 0.34$, we notice that neither the total costs, nor the firm value vary with k_F. This is because the firm does not issue equity in renegotiation for high equity issuance costs.[22]

[Figure 3 about here.]

In panel b) for renegotiation costs of $k_R = 0.3$, the firm will renegotiate as long as the equity financing costs are below $k_F = 0.45$. For higher renegotiation costs renegotiation is not possible. We will see later on in Figure 5 that although the equity value at renegotiation is larger than the amount of funds the firm needs to raise ($E(x_R, c_1) > EF(c_1)$), equity issuance costs are too large and prevent renegotiation ($E(x_R, c_1) < (1+k_F)EF(c_1)$). Finally, in panel c) for renegotiation costs of $k_R = 0.5$, the firm will not renegotiate, irrespectively of the equity issuance costs. In this case, renegotiation costs are too large and prevent renegotiation since the equity value at renegotiation is smaller than the funds needed to be raised ($0 < E(x_R, c_1) < EF(x_R, c_1)$).

[21] Since in Moraux and Silaghi (2014) renegotiation costs are not covered with equity financing, we assume $k_R = 0$, to facilitate the comparison.

[22] See panel a) of Figure 5, where for $k_F > 0.34$ we have that $EF = 0$.

In Figure 4 we look at the decision to renegotiate or to liquidate from the perspective of the creditors' premium, β, the volatility, σ, the recovery rate, α, the drift, μ, the tax rate, τ, and the interest rate, r.

What is the maximum creditors' premium for which the firm would still renegotiate its debt and not be liquidated? As we can see in panel a), the maximum creditors' premium that allows for a renegotiation to take place is $\beta = 1.31$. An increase in β, the creditors' premium, increases the transfers that the equity holder has to make to creditors, thus increasing the total costs of renegotiation, and making liquidation more likely. In panel b), we can see that a larger volatility makes it more likely for the firm to enter direct liquidation without renegotiating its debt previously. This is due to the fact that the convexity of the equity holder's option to default is stronger in the case of liquidation than in the case of renegotiation.

[Figure 4 about here.]

The first two comparative statics we present in Figure 4 are consistent with the findings of Nishihara and Shibata (2016) who study the decision of renegotiation versus liquidation. We extend their analysis by presenting new evidence on this decision in the next four panels. In panel c), we observe that firms with higher recovery rates are more likely to liquidate rather to renegotiate. This is in line with the fact that the scope of renegotiation is larger when bankruptcy costs are larger, since renegotiation permits to save and share those costs. Regarding the growth rate of cash flows (panel d)), firms that have a higher growth rate are more likely to renegotiate their debt rather than to be liquidated since a higher drift increases the continuation value of the firm more than the liquidation value.[23] As far as the macroeconomic variables τ and r are concerned, on the one hand, a larger tax rate makes it more likely for firms to be liquidated directly. This is due to the fact that a larger tax rate increases the claim of the government at the expense of the equity holder (despite the increases in the tax benefit just as in Goldstein et al., 2001), and thus decreases the continuation value of the firm, while it does not affect the liquidation value. Therefore, the firm value at

[23]We remind the reader that a part of the total costs of renegotiation comprised of the creditors' premium and the renegotiation costs is proportional to the original debt value at renegotiation, $D(x_R, c_0)$, which is in fact the liquidation value of the firm, if the firm is liquidated at this threshold.

renegotiation decreases more than the total costs of renegotiation (see panel e)). On the other hand, a larger interest rate makes it more likely for firms to renegotiate their debt, as it decreases the continuation value of the firm less than its liquidation value (see panel f)).

5.3. Equity financing in renegotiation

Whether renegotiation is possible or not can also be studied comparing the equity value at renegotiation with the amount of equity financing and its cost, as showed in inequality (11). Comparing the equity value at renegotiation with the amount of equity financing and its cost does not only allow us to see under what conditions renegotiation is preferred to liquidation, but also to analyze the evolution of equity financing with respect to the parameters of the model. We thus plot similar figures to those presented in the previous section, this time using equity values and the amount of equity financing. Our goal is to analyze numerically how the use and amount of equity financing in renegotiation depend on the parameters of the model.

In Figure 5 we plot the equity value at renegotiation, $E(x_R, c_1)$, the equity financing amount, $EF(c_1)$, and the equity financing amount plus the cost it involves, $EF(c_1) + \max(EF(c_1), 0)$. All these three quantities are plotted as a function of the equity financing costs k_F for different levels of the renegotiation costs, k_R.

[Figure 5 about here.]

In panel a), for low renegotiation costs, $k_R = 0.05$, we can see that renegotiation is possible irrespectively of the value of the equity issuance costs, just as in Figure 3. Additionally, we can see that the amount of equity financing decreases with the costs of issuing new equity, as discussed in section 3.3. When equity issuance costs are large the firm will optimally adjust the level of the coupon in order to reduce the costs of equity issuance. The firm therefore increases the level of the reduced coupon (we have a lower coupon reduction) to be able to issue a lower amount of equity (see panel a) of Figure 1). Furthermore, we observe that as issuance costs increase, the firm is less likely to use equity financing in renegotiation. For issuance costs larger than $k_F = 0.34$ the firm renegotiates its debt through a pure debt to debt swap, without issuing equity.

In panel b) of Figure 5 we notice that renegotiation is only possible for $k_F < 0.45$, since renegotiation costs are larger, $k_R = 0.3$. As mentioned before, although the equity value at renegotiation exceeds the amount of equity financing needed, the presence of large equity issuance costs ($k_F > 0.45$) prevents renegotiation: $EF(c_1) < E(x_R, c_1) < EF(c_1) + k_F \max(EF(c_1), 0)$. Finally, in panel c) it is the presence of large renegotiation costs, $k_R = 0.5$, that prevents renegotiation. The firm is not able to renegotiate its debt for any value of the equity issuance costs, k_F (we have $E(x_R, c_1) < EF(c_1)$). Fixing $k_F = 0.4$ and comparing the first two panels (across renegotiation costs), we notice that equity financing is more likely to occur in renegotiation as renegotiation costs increase. The firm needs more funding to finance the larger costs and the transfers to the creditors as renegotiation costs increase.

[Figure 6 about here.]

Regarding the use of equity financing as a function of the creditors' premium we plot the same variables as before as a function of β for different values of the tax rate. For a low tax rate of 15% we can see in panel a) of Figure 6 that renegotiation is possible whenever $\beta < 1.4$ and that for any value of the creditors' premium, there exists positive equity financing in renegotiation. A low tax rate leads to a low optimal coupon, and therefore the need to issue equity to compensate the creditors. The amount of equity financing increases in the creditors' premium as expected. In panel b) of Figure 6, similarly to panel b) of Figure 2, we obtain the three cases of: no equity issuance (negative $EF(c_1)$ for $\beta < 1.04$), equity issuance exactly equal to zero (for $\beta \in [1.04, 1.08]$), and positive equity issuance (for $\beta > 1.08$). Whenever there exists equity financing in renegotiation, it will be increasing in the creditors' premium as shown analytically. Comparing the two panels (across tax rates), we observe that as the tax rate decreases it is more likely that the firm issues equity in renegotiation.

Whether the firm will use equity financing to renegotiate its debt also depends on the recovery rate, α. First, in order to see how the use of equity financing in renegotiation changes with the recovery rate, we plot in panel a) of Figure 7 the three quantities A', B', and Q that determine the three cases of negative ($Q < A'$), zero ($A' \leq Q \leq B'$) or positive ($B' < Q$) equity financing. We notice that, as mentioned before in section 3.3, all the three quantities are increasing in the recovery rate since they are proportional to the debt value which increases with the recovery rate. We observe that for relatively low values of the recovery rate, Q increases faster than A' and B'. Although for relatively high values of the recovery rate, B' increases faster than Q, it will always stay below Q, as we increase the recovery rate from $\alpha = 0.47$ onwards. In the limit, for $\alpha = 1$ (no bankruptcy costs), we obtain $A' = B' = \frac{\gamma}{\gamma-1}$ and $Q = (\beta + k_R)\frac{\gamma}{\gamma-1}$. This means that $Q \geq B'$ since $\beta \geq 1$, so the firm uses equity financing in renegotiation, if renegotiation takes place.[24] This implies that it is more likely that $Q > B'$ as the recovery rate increases. Therefore, it is also more likely that the firm will use equity financing in renegotiation for larger recovery rates.

[Figure 7 about here.]

Panel b) of Figure 7 plots the same variables in the case where there are no costs in renegotiation: $k_R = k_F = 0$ and $\beta = 1$. Since there are no issuance costs, $k_F = 0$ we obtain that $A' = B'$, and for the limit case $\alpha = 1$, $Q = A' = B' = \frac{\gamma}{\gamma-1}$. For a recovery rate below 0.41 we have that $Q < A' = B'$ and the firm renegotiates through a pure debt to debt swap. When the recovery rate is above 0.41, $Q > A' = B'$ and thus the firm uses equity financing in renegotiation.

[Figure 8 about here.]

Second, we plot in Figure 8 the evolution of the amount of equity financing with changes in the recovery rate. In order to know when renegotiation is preferred to liquidation, we also plot the equity value at renegotiation, and the equity financing plus the issuance costs for different values of the tax rate. In panels b) and d) there are no costs related to renegotiation ($\beta = 1$, $k_R = 0$, and $k_F = 0$), therefore, renegotiation is always preferred to liquidation. In panels a) and c), for baseline values of the parameters regarding the costs, renegotiation is only possible for a recovery rate below 0.83 at a tax rate of 15%, and below 0.79 for a tax rate of 35%. For the values of the recovery rate for which renegotiation is preferred to liquidation, and there exists positive equity financing in renegotiation, the amount of equity financing is either increasing in the recovery rate (panel c)) or hump-shaped (panels a), b), and d)). Firms with larger recovery rates are thus more likely to issue equity in renegotiation, however, they do not necessarily issue a higher amount of equity.

[24] We will see in panel c) of Figure 8 that for the baseline parameters and $\alpha = 1$ the equity holder optimally prefers liquidation to renegotiation. This is intuitive since the scope of renegotiation is to avoid bankruptcy costs and in the case of $\alpha = 1$ there are no bankruptcy costs, and thus no scope for renegotiation.

[Figure 9 about here.]

Finally, we illustrate graphically how the use and amount of equity financing in renegotiation relate to the three parameters contained in γ: the volatility σ, the drift μ, and the interest rate r.

The effect of volatility on the use of equity financing in renegotiation is plotted in Figure 9. We observe that all three quantities A', B', and Q decrease with volatility, since they are proportional to the debt value which decreases with volatility. Equity financing is used in renegotiation for volatility values above 12% in the case of a low tax rate (panel a)), and above 24% for a high tax rate (panel b)). Firms with larger volatilities are more likely to use equity financing in renegotiation. This is due to the fact that a larger volatility decreases the new debt value at renegotiation, $D(x_R, c_1)$ not only by lowering the renegotiation threshold, but also by decreasing the coupon. Therefore, the firm needs to issue equity to be able to make the transfers to the creditors to compensate for the reduction in the debt value and to cover the renegotiation costs.[25]

[Figure 10 about here.]

Similarly to the comparative statics with respect to the recovery rate, the amount of equity financing is either increasing or hump-shaped in the volatility (see panel b) and a) of Figure 10, respectively).

[Figure 11 about here.]

Unlike the volatility, the drift is negatively related to the parameter γ. We observe in Figure 11 that as the drift increases it is less likely for the firm to use equity financing in renegotiation. In panel a), the firm issues positive equity financing as long as the drift is below 0.025. For larger drifts the firm has a pure debt for debt swap, without issuing equity in renegotiation.

[Figure 12 about here.]

[25]The value of the initial debt at renegotiation, $D(x_R, c_0)$, is only affected by a higher volatility through the impact on the renegotiation threshold, but not on the coupon. Consequently, this debt value and the creditors' premium and renegotiation costs proportional to it decrease less when volatility increases, as compared to $D(x_R, c_1)$.

In Figure 12 we can see that the amount of equity financing used in renegotiation can be either decreasing, hump-shaped on increasing in the drift (see panels a), b), and c) respectively), depending on the parameter values.

[Figure 13 about here.]

Regarding the last parameter, the interest rate, this is also negatively related with the parameter γ. We observe in Figure 13 that the higher the interest rate the less likely it is that the firm will issue equity in renegotiation. In panel b), whenever the interest rate is above 0.037 the firm will not issue equity to renegotiate the debt.

[Figure 14 about here.]

The amount of equity financing decreases in the interest rate (see Figure 14). This is due to the fact that besides the indirect effect r has on the equity financing amount through γ, r also directly and negatively affects the amount of equity financing as shown in equation (17).[26]

5.4. Forced asset sales

The previous analysis has been made for our benchmark model in which the firm optimally chooses not to sell assets in renegotiation. We now analyze the impact of allowing for forced asset sales on the debt renegotiation process. In panel a) of Figure 15, we plot the value of the firm at the renegotiation threshold plus the proceeds from the asset sales, and the total costs of renegotiation, as a function of the fraction of asset sales, ϕ. We observe that the larger the fraction of asset sales, the less likely it is that the firm value plus the proceeds from asset sales will cover the renegotiation costs. Therefore, firms with larger fractions of asset sales are more likely to proceed to direct liquidation rather than to renegotiate their debt due to the inefficiency of partial asset sales in renegotiation.

[26] The comparative statics analysis regarding the decision to renegotiate or to liquidate and the use of equity financing in renegotiation was conducted keeping the initial coupon fixed to our baseline parameter. However, we also made the same analysis for the optimal initial coupon that maximizes firm value at time zero, and we obtain the same comparative statics. This is why they are omitted here. Furthermore, in terms of the optimal capital structure we obtain that the optimal leverage ratio increases with renegotiation since renegotiation reduces the bankruptcy costs, confirming the evidence from the previous studies, such as Christensen et al. (2014) and Nishihara and Shibata (2016).

[Figure 15 about here.]

In panel b) we notice that the optimal reduced coupon decreases with the fraction of asset sales. Naturally, a firm with a lower operating size chooses a lower optimal coupon. In panel c) we observe that the higher the fraction of asset sales, the more likely it is that the firm issues equity in renegotiation (it is more likely that $Q > B'$). These first two implications regarding the decision to renegotiate and the coupon reduction are in line with the findings of Nishihara and Shibata (2016).

[Figure 16 about here.]

Regarding the amount of equity financing, we find that depending on the parameter values, this can be either increasing, decreasing or hump-shaped with respect to the fraction of asset sales. Indeed, an increase in the fraction of asset sales has two opposite effects on the amount of equity financing (see equation 22). On the one hand when the fraction of asset sales increases, the proceeds from asset sales will be larger, which means that the firm needs to issue less equity. On the other hand, a higher fraction of asset sales also implies that the firm's scale is reduced and the value of the new debt with the reduced coupon is smaller, $D(x_R, 1-\phi, c_1)$. This implies that the firm needs to issue more equity in order to compensate the creditors for the decrease in their new debt claim. Depending on which effect dominates, the amount of equity financing could be decreasing or increasing in the fraction of asset sales. Two examples are illustrated in Figure 16. Our finding in contrast with the results of Nishihara and Shibata (2016), who find that selling assets increases the amount of equity financing in renegotiation.

6. Empirical implications

Our model has a number of testable implications for debt renegotiation, several of which are novel with respect to the existent literature. Given that the main contribution of the paper is to account for the use of equity financing, we will focus in this section on the implications regarding equity financing. We will also briefly discuss the implications regarding the coupon reduction and the new implications regarding the decision to renegotiate or to liquidate.

6.1. *Equity financing*

Our analysis implies first of all that firms which have more intangible assets (firms from the technological sector for example that have low recovery rates) are less likely to use equity financing in renegotiation. Second, smaller/younger firms (which tend to have larger cash flows volatility) are more likely to issue equity financing to renegotiate their debt (if the volatility is not so high as to push them into direct liquidation). Third, firms with higher cash flow growth rates are less likely to issue equity financing in order to repurchase debt. These implications are consistent with the empirical evidence of Kruse et al. (2014) who study the cumulative absolute returns (CARs) of debt tender offers. They find that when debt is used to repurchase debt, but equity is not, the mean CAR is positive and significant. That is, the market values more firms which do not use equity financing to repurchase debt. Our implications are also consistent with the evidence of Brennan and Kraus (1987) who shows that using equity proceeds to retire debt has a more negative share price reaction than other uses.

The model predicts that small firms, which have a concentrated group of creditors who closely monitor them (in which creditors' premium is higher), are more likely to use equity financing to repurchase debt compared to large firms with a high number of institutional investors, or in which a high percentage of the equity is held by institutional investors (in which the creditors' premium is lower). At the same time, firms which have more public rather than private debt and a complex capital structure with international creditors (which implies higher renegotiation costs) are more likely to issue equity in renegotiation.

Another prediction is that firms are less likely to use equity financing in renegotiation on markets where equity financing is more expensive (higher flotation costs) or the corporate tax rate and the risk free interest rate are higher.

Finally, our model predicts that firms which are forced to sell assets in renegotiation are more likely to use equity financing than those firms which do not sell assets in renegotiation.

6.2. *Coupon reduction*

A second set of implications of our model concerns the coupon reduction. We show that for firms which use equity financing in renegotiation, the size of the debt reduction does not depend on the renegotiation costs, nor on the creditors' premium. This is in contrast to the implication of Moraux and Silaghi (2014), who showed that bargaining power plays a critical role in determining the size of coupon reductions in renegotiation in a framework where the possibility of equity financing is excluded. Indeed, by allowing for transfers between claimholders, and thus for equity to be issued in renegotiation, we enlarge the set of choices available for the debt reduction.

This unconstrained choice also makes larger debt reductions possible. This implication is consistent with the empirical evidence of Kruse et al. (2014) and Brandon (2013). Kruse et al. (2014) find that in debt tender offers the average and median offers represent 80.2% and 40.5% respectively, of the debt book value in the year prior to the tender offer. Brandon (2013) finds that the repurchase retires 53% of the face value of the bond on average, and reduces the firm's leverage ratio by more than 16%. These very large fractions of debt retired are consistent with our large coupon reductions.

The size of the debt reduction in renegotiations that involve equity financing also depends on the cost of equity financing and the fraction of asset sales. The debt reduction made by firms using equity financing in renegotiation will be larger on markets where equity financing is cheaper. The debt reduction is also larger for firms with larger amounts of forced asset sales.

6.3. Renegotiation versus liquidation

Finally, our model has new implications for the decision to renegotiate the debt or to liquidate the firm directly. In particular, our model implies that firms with more tangible rather than intangible assets are more likely to be liquidated directly. This is consistent with the results of Goto and Suzuki (2015) who show that the equity holder might want to liquidate the firm rather than renegotiate the debt if the tangible asset value is sufficiently high. Another implication is that high growth firms are more likely to renegotiate their debt rather than to be liquidated. As far as the macroeconomic variables are concerned, our model implies that firms from markets with low corporate tax rates and large riskless interest rates are more likely to renegotiate rather than to be liquidated.

7. Conclusions

In this paper we have analyzed the use of equity financing in debt renegotiation. To the best of our knowledge, this is the first paper to provide a thorough analysis of equity issuance in renegotiation, both analytically and numerically. We have studied the conditions under which a firm repurchases debt using a combination of debt financing and equity financing (along with forced asset sales) rather than a pure debt for debt swap. Depending on the firm characteristics, the market variables or different costs involved in the renegotiation process, some firms will choose to issue equity in order to finance part of the proceeds of the debt repurchase, while others will prefer not to use equity financing in renegotiation. If new equity is issued, the optimal reduced coupon will be adjusted in order to minimize flotation costs.

When transfers between the equity holder and the creditors are allowed, and equity financing can be used in renegotiation, the firm optimally chooses to reduce the coupon even further than previously documented in the literature. This extra debt reduction in line with empirical evidence is welfare increasing. An unconstrained choice of the optimal reduced coupon increases the net surplus from renegotiation.

Several novel empirical implications regarding the use of equity financing in debt renegotiation are also derived. Moreover, new implications on the choice between debt renegotiation and direct liquidation of a firm have been provided. They could motivate further empirical evidence on the use of equity financing in debt repurchases. Indeed, despite a few recent empirical studies on debt repurchases, the literature remains quite scarce.

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Appendix A.

Proof of Proposition 1. Our maximization problem is the following:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) - k_F \max\{EF(c_1), 0\} \tag{A.1}$$

This can be rewritten as two maximization problems:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) \tag{A.2}$$

s.t.

$$EF(c_1) \le 0$$

and

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) - k_F EF(c_1) \tag{A.3}$$

s.t.

$$EF(c_1) \ge 0$$

We know that $EF(c_1) = (\beta + k_R)D(x_R, c) - D(x_R, c_1)$. Since the first term does not depend on c_1, the second maximization problem reduces to:

$$c_1^* = \arg\max_{c_1} V(x_R, c_1) + k_F D(x_R, c_1) \tag{A.4}$$

s.t.

$$EF(c_1) \ge 0$$

We can compute the two derivatives that interest us $\frac{\partial V(x_R,c_1)}{\partial c_1}$ and $\frac{\partial D(x_R,c_1)}{\partial c_1}$.

$$\begin{aligned}
\frac{\partial V(x_R, c_1)}{\partial c_1} &= \frac{\tau}{r} - \left(\frac{(1-\tau)\partial x_B(c_1)/\partial c_1}{r-\mu} + \frac{\tau}{r} - \frac{\alpha \partial x_B(c_1)/\partial c_1}{r-\mu}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&+ \left(\frac{(1-\tau)x_B(c_1)}{r-\mu} + \frac{\tau c_1}{r} - \frac{\alpha x_B(c_1)}{r-\mu}\right)\frac{\gamma}{x_B(c_1)}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \frac{\partial x_B(c_1)}{\partial c_1} \\
&= \frac{\tau}{r} - \frac{1-\tau}{r-\mu}\frac{\gamma(r-\mu)}{(\gamma-1)r}(1-\gamma)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma - \frac{\tau}{r}\left(1 - \frac{c_1\gamma}{x_B(c_1)}\frac{\gamma(r-\mu)}{(\gamma-1)r}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&+ \frac{\alpha}{r-\mu}\frac{\gamma(r-\mu)}{(\gamma-1)r}(1-\gamma)\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&= \frac{\tau}{r} + \frac{(1-\tau)\gamma}{r}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma - \frac{\tau(1-\gamma)}{r}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma - \frac{\alpha\gamma}{r}\left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \\
&= \frac{\tau}{r} + \left(\frac{x_B(c)}{x_B(c_1)}\right)^\gamma \frac{\gamma(1-\alpha)-\tau}{r} \\
&= \frac{1}{r}\left(\tau + \left(\frac{c}{c_1}\right)^\gamma (\gamma(1-\alpha)-\tau)\right)
\end{aligned} \tag{A.5}$$

$$
\begin{aligned}
\frac{\partial D(x_R, c_1)}{\partial c_1} &= \frac{1}{r} - \left(\frac{1}{r} - \frac{\alpha}{r-\mu}\frac{\partial x_B(c_1)}{\partial c_1}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} \\
&+ \left(\frac{c_1}{r} - \frac{\alpha x_B(c_1)}{r-\mu}\right)\frac{\gamma}{x_B(c_1)}\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\frac{\partial x_B(c_1)}{\partial c_1} \\
&= \frac{1}{r} - \left(\frac{1}{r} - \frac{\alpha}{r-\mu}\frac{\gamma(r-\mu)}{(\gamma-1)r}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} \\
&+ \left(\frac{c_1}{r} - \frac{\alpha}{r-\mu}\frac{\gamma(r-\mu)c_1}{(\gamma-1)r}\right)\frac{\gamma(\gamma-1)r}{\gamma(r-\mu)c_1}\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\frac{\gamma(r-\mu)}{(\gamma-1)r} \\
&= \frac{1}{r} - \left(\frac{1}{r} - \frac{\alpha\gamma}{(\gamma-1)r}\right)\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} + \left(\frac{c_1}{r} - \frac{\alpha\gamma c_1}{(\gamma-1)r}\right)\frac{\gamma}{c_1}\left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma} \\
&= \frac{1}{r} - \left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\left(\frac{1}{r} - \frac{\alpha\gamma}{(\gamma-1)r} - \frac{\gamma}{r} + \frac{\alpha\gamma^2}{(\gamma-1)r}\right) \\
&= \frac{1}{r} - \left(\frac{x_B(c)}{x_B(c_1)}\right)^{\gamma}\left(\frac{1}{r} - \frac{\gamma}{r} + \frac{\alpha\gamma}{r}\right) \\
&= \frac{1}{r}\left(1 - \left(\frac{c}{c_1}\right)^{\gamma}(1-(1-\alpha)\gamma)\right)
\end{aligned}
\tag{A.6}
$$

We start by solving the first maximization problem. Assume the constraint is not binding, i.e. $EF(c_1) < 0$. Then the optimal coupon is such that $\frac{\partial V(x_R, c_1)}{\partial c_1} = 0$. Letting the derivative from equation (A.5) be equal to zero and solving for c_1 we obtain the optimal coupon c_1^A. Since we assumed that the constraint is not binding, we are in the case in which $EF(c_1^A) < 0$, which is equivalent to $Q < A'$.

We solve the second maximization problem assuming again that the constraint is not binding, i.e., $EF(c_1) > 0$. Then the optimal coupon is such that $\frac{\partial(V(x_R, c_1) + k_F D(x_R, c_1))}{\partial c_1} = 0$. Solving for c_1 we obtain the optimal coupon c_1^B. As before, the constraint is not binding, and we are in the case in which $EF(c_1^B) > 0$, which is equivalent to $Q > B'$.

Finally, when the constraint is binding, $EF(c_1) = 0$, the optimal coupon is such that this condition is satisfied, and we have that $c_1^{EF} \in (c_1^A, c_1^B)$. □

Proof of Proposition 2. The proof is similar to the proof of Proposition 1, there exists just an extra parameter, ϕ, for the asset sales.

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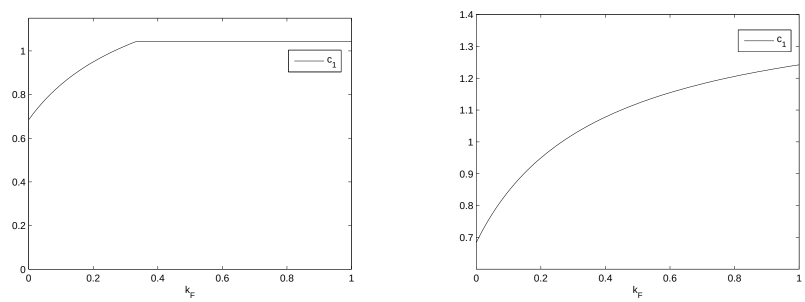
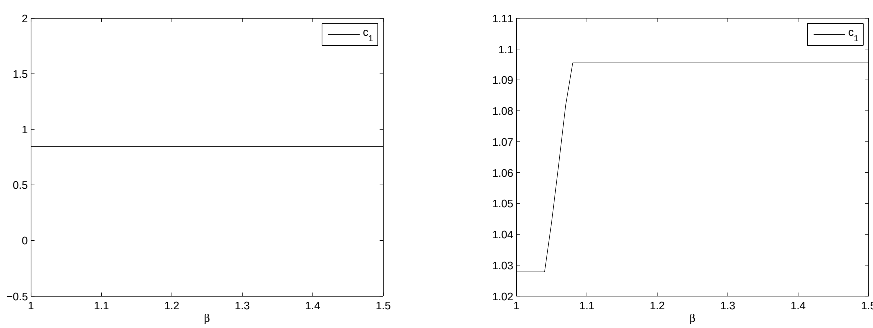
Figures



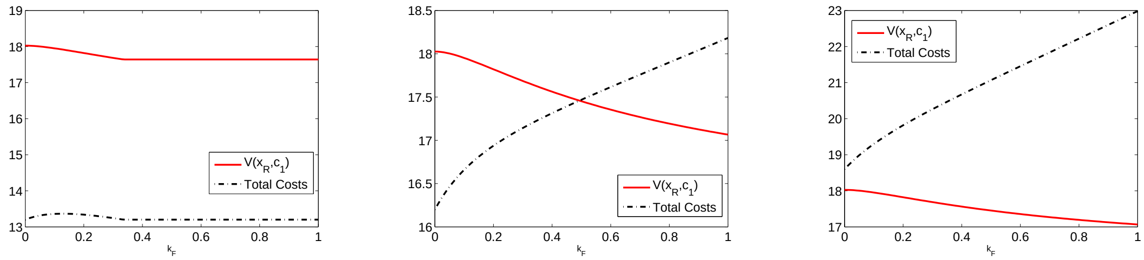
a) $k_R = 0.05$ b) $k_R = 0.3$

Figure 1: Optimal reduced coupon and issuance costs. The figure plots the optimal reduced coupon c_1 as a function of the equity issuance costs parameter k_F for different values of the renegotiation costs k_R. The tax rate is equal to $\tau = 0.15$.



a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 2: Three cases for the optimal reduced coupon. The figure plots the optimal reduced coupon as a function of β for two levels of the interest rate. The issuance costs are set to their baseline value $k_F = 0.1$.



a) $k_R = 0.05$ b) $k_R = 0.3$ c) $k_R = 0.5$

Figure 3: Renegotiation versus liquidation and equity issuance costs. The figure plots the firm value at renegotiation and the total costs as a function of the equity issuance costs parameter k_F. The different panels correspond to different values of the renegotiation costs k_R. The tax rate is $\tau = 0.15$.

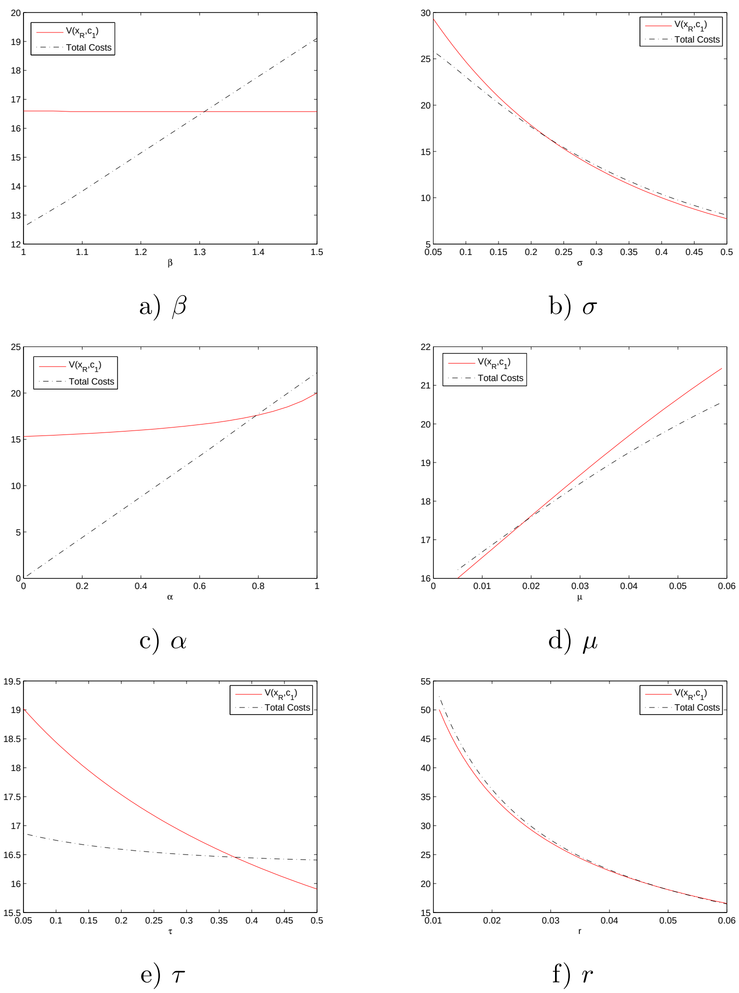
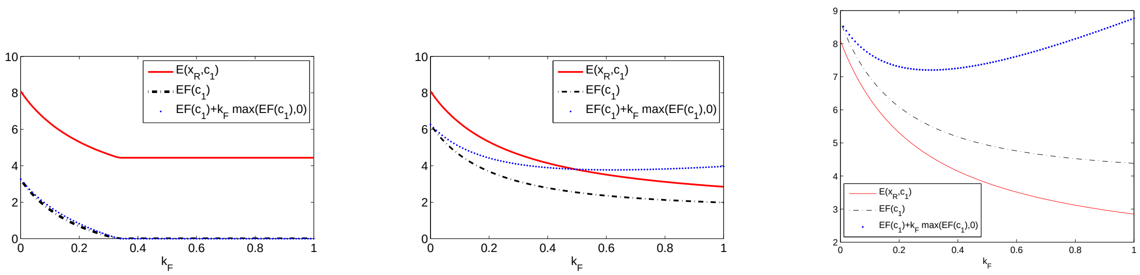
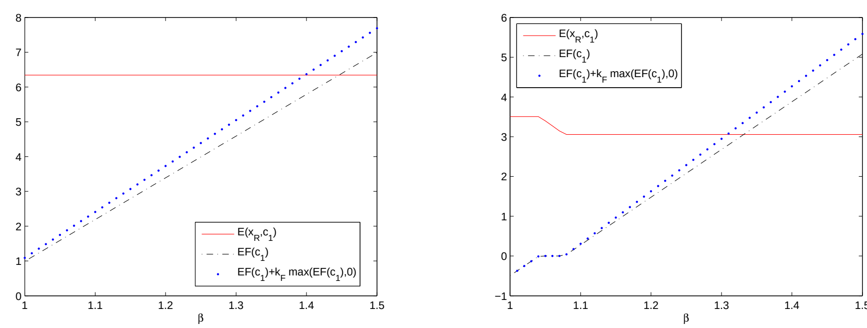


Figure 4: Renegotiation versus liquidation. The figure plots the firm value at renegotiation, and the total costs of renegotiating as a function of the parameter β in panel a), σ in panel b) (for $k_F = 0.2$, $k_R = 0.1$, $\beta = 1.3$, and $\tau = 0.15$), α in panel c), μ in panel d) (for $k_F = 0.2$, and $\beta = 1.3$), τ in panel e) (for $\beta = 1.3$), and r in panel f) (for $\beta = 1.3$).



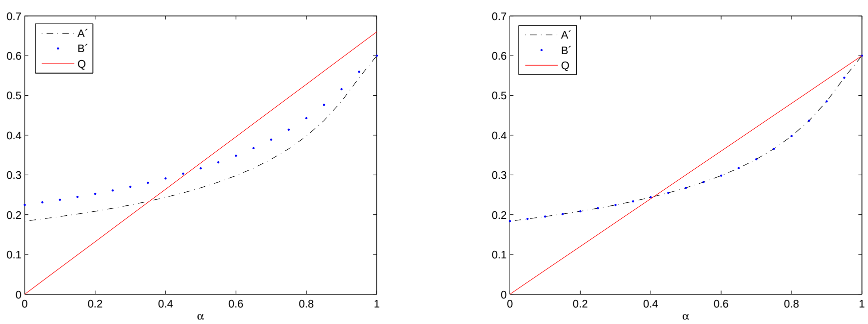
a) $k_R = 0.05$ b) $k_R = 0.3$ c) $k_R = 0.5$

Figure 5: Equity financing and issuance costs. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the equity issuance costs parameter k_F for different values of the renegotiation costs k_R. The tax rate is equal to $\tau = 0.15$.



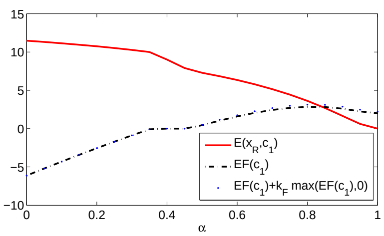
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 6: Equity financing and creditors' premium. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the creditors' premium β. The issuance costs are set at their baseline value $k_F = 0.1$.

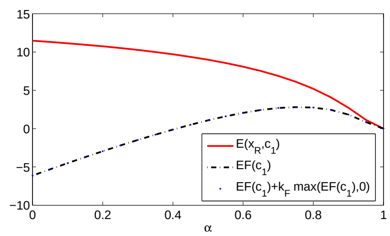


a) Costs   b) No costs

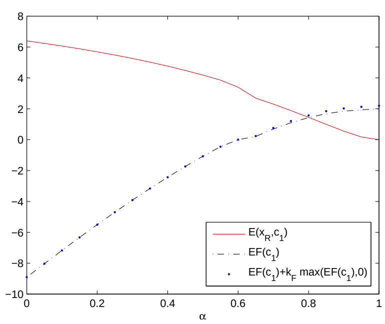
Figure 7: The use of equity financing and the recovery rate. The figure plots the three quantities A', B' and Q as a function of the recovery rate α. The tax rate is set to $\tau = 0.15$. In panel a), the parameters related to the total costs of renegotiation take the baseline values: $\beta = 1.05$, $k_R = 0.05$, and $k_F = 0.1$. In panel b) the total costs of renegotiation are equal to zero: $\beta = 1$, $k_R = 0$, and $k_F = 0$.



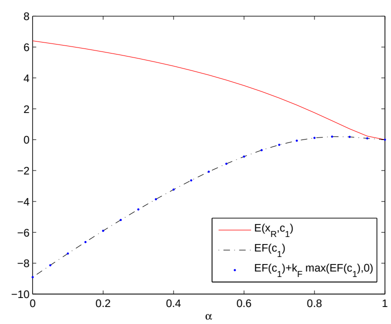
a) $\tau = 0.15$ Costs



b) $\tau = 0.15$ No costs

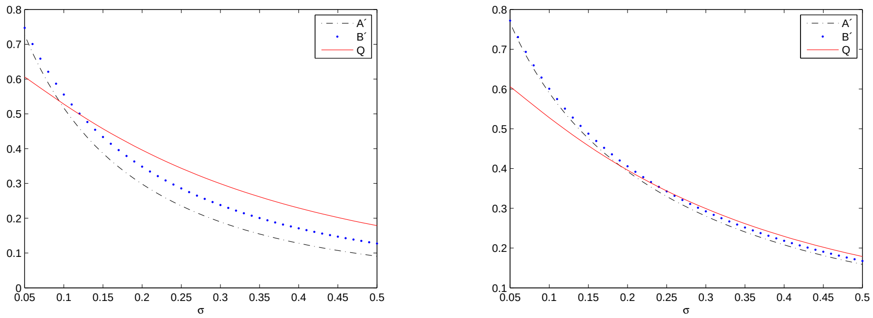


c) $\tau = 0.35$ Costs



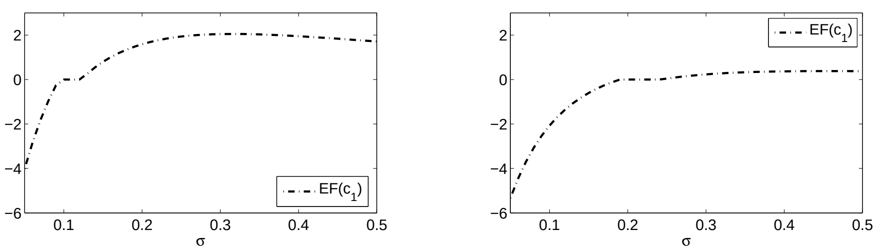
d) $\tau = 0.35$ No costs

Figure 8: The amount of equity financing and the recovery rate. The figure plots the equity value at renegotiation, the equity financing amount and the equity financing amount plus the issuance costs as a function of the recovery rate α. In panels a) and b) the tax rate is set to $\tau = 0.15$, while in panels c) and d) it is set to $\tau = 0.35$. In panels a) and c) the parameters related to the total costs of renegotiation take the baseline values: $\beta = 1.05$, $k_R = 0.05$, and $k_F = 0.1$. In panels b) and d) the total costs of renegotiation are equal to zero: $\beta = 1$, $k_R = 0$, and $k_F = 0$.



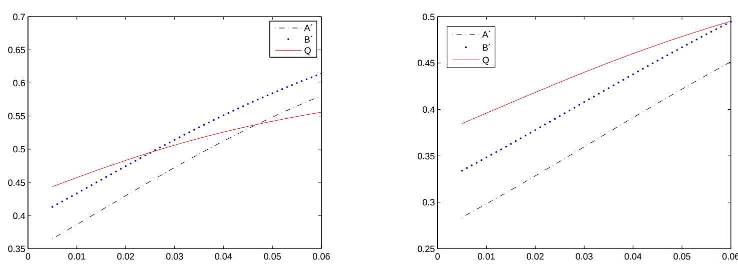
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 9: The use of equity financing and the volatility. The figure plots the three quantities A', B' and Q as a function of the volatility σ. The tax rate is set to $\tau = 0.15$ in panel a), and to $\tau = 0.35$ in panel b).



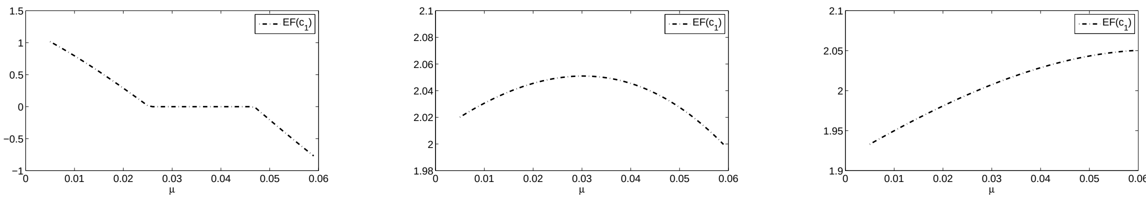
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 10: The amount of equity financing and the volatility. The figure plots the amount of equity financing $EF(C_1)$ as a function of the volatility σ. The tax rate is set to $\tau = 0.15$ in panel a), and to $\tau = 0.35$ in panel b).



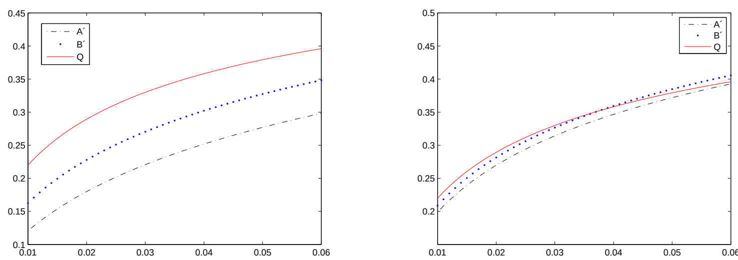
a) $\sigma = 0.15$ b) $\sigma = 0.20$

Figure 11: The use of equity financing and the drift. The figure plots the three quantities A', B' and Q as a function of the drift μ. The volatility is set to $\sigma = 0.15$ in panel a), and $\sigma = 0.20$ in panel b). The tax rate is equal to $\tau = 0.15$ in both panels.



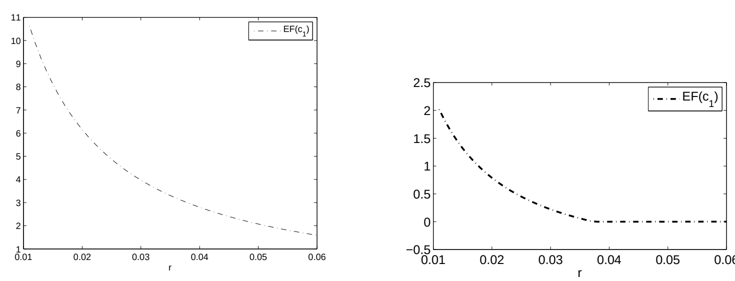
a) $\sigma = 0.15$ b) $\sigma = 0.35$ c) $\sigma = 0.40$

Figure 12: The amount of equity financing and the drift. The figure plots the amount of equity financing $EF(C_1)$ as a function of the drift μ. The volatility is set to $\sigma = 0.15$ in panel a), $\sigma = 0.35$ in panel b), and $\sigma = 0.40$ in panel c). The tax rate is equal to $\tau = 0.15$ in all panels.



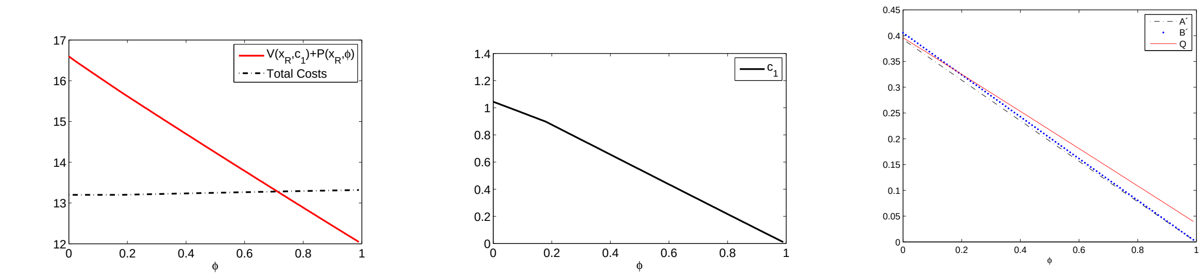
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 13: The use of equity financing and the interest rate. The figure plots the three quantities A', B' and Q as a function of the interest rate r. The tax rate is set to $\tau = 0.15$ in panel a), and $\tau = 0.35$ in panel b).



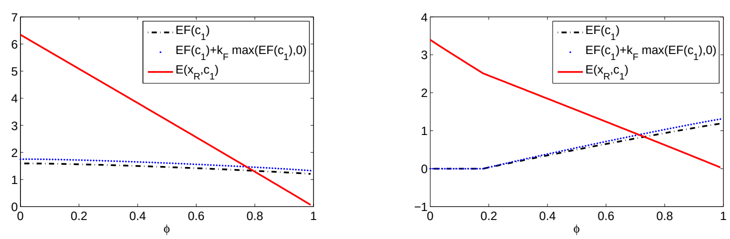
a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 14: The amount of equity financing and the interest rate. The figure plots the amount of equity financing $EF(C_1)$ as a function of the interest rate r. The tax rate is set to $\tau = 0.15$ in panel a), and $\tau = 0.35$ in panel b).



a) Renegotiation versus liquidation b) Reduced coupon c)Use of equity financing

Figure 15: Forced asset sales and renegotiation. The figure plots the value of the firm at renegotiation and the total costs of renegotiation as a function of the fraction of asset sales ϕ in panel a). In panel b) the optimal reduced coupon is plot. In panel c) the three variables $A'(\phi)$, $B'(\phi)$, and $Q(\phi)$ are plot, indicating whether equity financing is used in renegotiation.



a) $\tau = 0.15$ b) $\tau = 0.35$

Figure 16: The amount of equity financing and the fraction of asset sales. The figure plots the amount of equity financing $EF(C_1)$ as a function of the fraction of asset sales ϕ. The tax rate is set to $\tau = 0.15$ in panel a), and $\tau = 0.35$ in panel b).

Tables

Table 1: Parameter values for the baseline case

Parameter	Value
x_0	2
r	6%
μ	1%
τ	35%
σ	20%
α	60%
c_0	2
β	1.05
k_R	5%
k_F	10%

Table 2: Numerical results of the baseline case ($k_R = 0$ and $\beta = 1$)

Results	$\tau = 0.35\%$	$\tau = 0.25\%$	$\tau = 0.15\%$
Q	0.360	0.360	0.360
A'	0.393	0.359	0.298
B'	0.406	0.384	0.348
c_1^*	c_1^A =1.028	c_1^{EF} =0.888	c_1^B =0.845
$EF(c_1^*)$	-1.092	0.000	0.390
$E(x_R, c_1^*)$	3.506	5.211	6.342
$D(x_R, c_1^*)$	13.092	12.000	11.610
$D(x_R, c_0^*)$	12.000	12.000	12.000
$E_R(x_0)$	9.023	10.378	11.764
$D_R(x_0)$	25.791	25.791	25.791
$V_R(x_0)$	34.814	36.169	37.555
$E(x_0, c_0)$	7.398	8.536	9.674
$D(x_0, c_0)$	25.791	25.791	25.791
$V(x_0, c_0)$	33.189	34.327	35.465

Table 3: Impact of equity financing ($k_R = 0$, $\beta = 1.1$, $\tau = 0.15$)

Variable	Pure debt/debt swap	Equity financing	Relative difference
c_1^*	0.9607	0.8450	-23.55%
$V(x_R, c_1^*)$	17.6414	17.9520	1.76%
Net surplus at renegotiation	5.6414	5.7930	2.69%