Bouncing Back: Are Average Outage Factors Adequate for Valuing the Resilience of Infrastructure Projects?

Afzal S. Siddiqui* Ryuta Takashima†

22 June 2017

Abstract

Motivated by the need to bolster the resilience of infrastructure, such as bridges, nuclear power plants, and ports, in face of extreme weather events, we consider two types of projects: "riskier" and "safer." Each type of project, once constructed, earns identical instantaneous cash flows and is subject to the same risk of outage, which causes its cash flows to diminish. The only difference between the two projects is that the repair rate of the "safer" project is greater than that of the "riskier" one. Naturally, the "safer" project is more valuable to an investor due to its greater resilience. However, how much extra would an investor be willing to pay for this resilience? Under which circumstances would it be reasonable to replace the outage and repair rates with average outage factors? Using a real options approach, we show that even though the proportions of up- and down-times remain fixed, changes in the transition rates affect the willingness to pay for resilience. This implies that use of average outage factors will incorrectly inflate the resilience premium. In fact, only in the limit when transitions occur at infinitely high rates does the use of average outage factors accurately reflect

*Department of Statistical Science, University College London, London WC1E 6BT, U.K., Department of Computer and Systems Sciences, Stockholm University, Stockholm, Sweden, and Department of Decision Sciences, HEC Montréal, Montréal, QC H3T 2A7, Canada, e-mail address: afzal.siddiqui@ucl.ac.uk

†Department of Industrial Administration, Tokyo University of Science, 2641 Yamazaki, Noda-shi, Chiba-ken 278-8510, Japan, e-mail address: takashima@rs.tus.ac.jp

the investor's willingness to pay for resilience. Somewhat paradoxically, very frequent transitions reflect a situation in which average outage factors may be used.

Index terms— OR in environment and climate change, real options, investment analysis

1 Introduction

The occurrence of extreme weather events, such as floods and hurricanes, driven by climate change has become frequent. This poses risks for many infrastructure projects, such as bridges, nuclear power plants, and ports, which are necessarily located in areas prone to flooding or landslides. In fact, a U.S. Environmental Protection Agency white paper (EPA , 2008) stressed the need for adaptation measures that would bolster the resilience of ports specifically. Some of the impacts of climate change that it outlines are increased water levels and more severe storms. In response to these threats, adaptation measures posited by the white paper are protective seawalls and reinforcement of existing structures against stronger winds. Aware of its vulnerability to climate change, the County of Miami-Dade has invested in such measures as part of its Local Mitigation Strategy (Miami-Dade , 2016). Out of the $170 million earmarked for projects in 2015, over $10 million went towards the construction of a new seawall for the Miami Seaport along with the installation of concrete panels for greater resilience.

Naturally, we would conclude that a safer project, i.e., one that is able to resume operations more quickly after a contingency such as the newly retrofitted Miami Seaport, would be more valuable to an investor due to its greater resilience.[1] However, how much extra would such an investor be willing to pay for this resilience? In much of the analysis of the impacts of climate change and the benefits of adaptation or mitigation measures, this question is addressed by determining expected welfare based on probability estimates

[1]Resilience is defined by the Resilience Alliance (Walker et al., 2004) as "...the capacity of a system to absorb disturbance and reorganize while undergoing change so as to still retain essentially the same function, structure, identity, and feedbacks."

of catastrophes, i.e., regime switches in an ecosystem (Walker et al., 2010). More recent work takes an experimental economics approach to ascertain consumers' willingness to pay for more resilience (Richter and Weeks, 2016).

Yet, in these approaches, adaptation measures simply reduce the impact of damage from environmental factors. Via a now-or-never net present value (NPV) analysis that uses average outage factors, we first show that the willingness to pay for more resilient infrastructure depends only on the ratio of the outage and repair rates. Since adoption of such infrastructure is likely to be carried out by commercial decision makers (even if they use public funds) who will be concerned about the timing of the investment, under which circumstances would it be reasonable to replace the outage and repair rates with average outage factors in an economic analysis of resilience? Using a real options approach as suggested in Whitten et al. (2012), we demonstrate that even though the proportion of up- and down-times remains fixed, changes in the transition rates affect the willingness to pay for resilience. This implies that use of average outage factors will incorrectly overestimate the resilience premium. Intuitively, using average outage factors means complete elimination of the risk, whereas a resilience measure can only mitigate the risk. In fact, only in the limit when transitions occur at infinitely high rates will the use of average outage factors accurately reflect the investor's willingness to pay for resilience.

The rest of this paper is organised as follows. Section 2 discusses related work in order to provide context for our analysis. We develop the main mathematical model and derive analytical results in Section 3 for each type of project. Section 4 uses numerical examples to illustrate the insights, while Section 5 summarises the work and offers directions for future research. All proofs of the propositions may be found in the appendix.

2 Related Work

In contrast to the now-or-never NPV approach, the real options framework enables a decision maker to account for flexibility over investment timing and operations when undertaking the initial adoption (Dixit and Pindyck, 1994). The canonical setup assumes that the revenue from the completed project is exogenous, e.g., a geometric Brownian motion (GBM), and that cash flows from operations continue indefinitely after investment. Variations to the latter assumption posit that physical risk may cause the installed project to depreciate with subsequent re-investment options available to the decision maker. Ch. 6 of Dixit and Pindyck (1994) demonstrates that closed-form analytical solutions are possible in this context when physical outages occur according to a Poisson process. In the context of technology adoption under uncertainty, Farzin et al. (1998) and Doraszelski (2001) determine the optimal strategy of a firm that may upgrade to a new version given that technological performance advances randomly. Grenadier and Weiss (1997) examine alternative adoption strategies under both economic and technological uncertainty with finite upgrade opportunities, whereas Malchow-Møller and Thorsen (2005) extend the framework to consider infinite upgrade opportunities. Motivated by the problem of replacing wind turbines, Chronopoulos and Siddiqui (2015) extend the analysis of Grenadier and Weiss (1997) to derive further insights about the impact of the economic and technological uncertainties on the value of alternative adoption strategies. Likewise in the context of renewable energy investment, Boomsma et al. (2012) and Adkins and Paxson (2016) examine the impact of policy risk on technology choice.

A more general treatment of alternative investment strategies under uncertainty is taken up by Décamps et al. (2006). Considering two mutually exclusive projects with values depending on a common underlying price following a GBM, they show that, in contrast to an earlier analysis by Dixit (1993), the waiting region may be dichotomous. In particular, for an intermediate price range and with a relatively low level of volatility in the underlying price, it may be desirable to wait for more information before investing in the larger (smaller) project if the price increases (decreases) sufficiently. The analysis of alternative investment strategies (direct and stepwise) is conducted by Kort et al. (2010) for the same project. Somewhat surprisingly, although the stepwise strategy is more valuable, its relative advantage over the direct strategy diminishes as uncertainty increases. In effect, relatively high volatility in the underlying payoff tends to delay investment of any kind, thereby narrowing the benefit of using a stepwise strategy. Siddiqui and Takashima (2012) extend such an analysis to a duopolistic setting.

Our contribution to the literature is similar in spirit to that of Kort et al. (2010): we aim to determine the premium that an investor would be willing to pay in order to select one type of project over another. The main difference is that while two investment strategies were their focus, our aim is to determine an investor's the willingness to pay for resilience. Thus, in addition to modelling economic uncertainty via a GBM, we include the physical risk to an infrastructure project via a Poisson process. Consequently, we are able to cast the value of resilience in terms of an investment premium and illustrate why it is important to model physical risk explicitly in project valuation as opposed to via average outage factors.

3 Mathematical Model and Analytical Results

3.1 Assumptions

Consider two types of infrastructure projects, $i \in \{R, S\}$, where R stands for "riskier" and S for "safer." Each type of project, once constructed, earns identical instantaneous cash flows of $\pi^i(P_t) = P_t - k$ in state 1 (Fig. 1) at time $t \geq 0$, where $dP_t = \alpha P_t dt + \sigma P_t dz_t$ is the GBM describing the exogenous revenues earned from operations, $k > 0$ is the operating cost, I^i is the investment cost for project type i, $\alpha \geq 0$ is the annual drift rate, $\sigma \geq 0$ is the annual volatility, and ξ^i is the optimal investment threshold for project i. All cash flows and rates are in real terms.

Each project is subject to the same physical outage at rate λ_1, which causes its cash

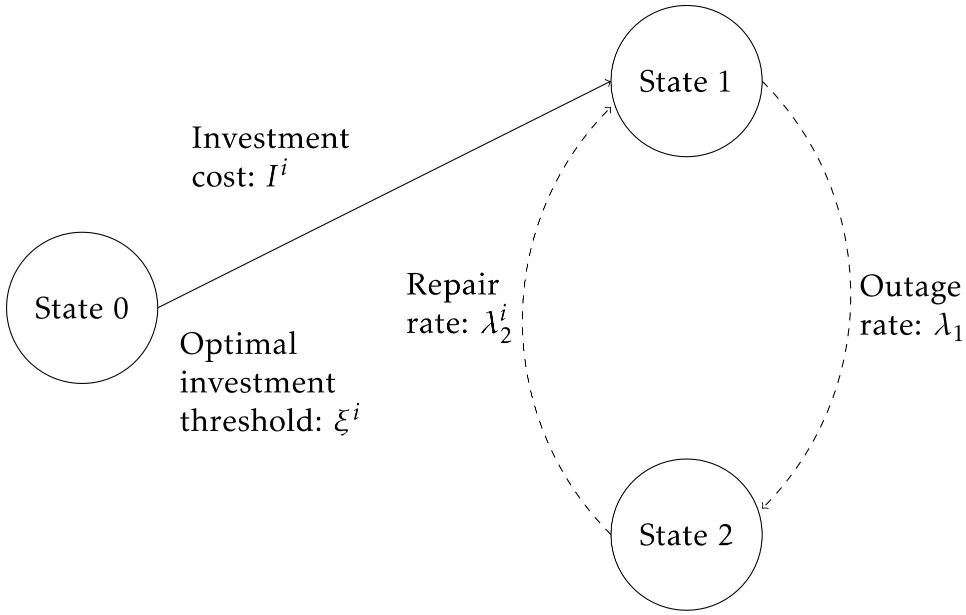

Figure 1: State-transition diagram for a project with physical risk

flows to diminish to $\pi^i(P_t) = -s$ in state 2, where $s > k$ represents the costs associated with repairs. The only difference between the two projects is that the repair rate of project S, $\lambda_2^S > 0$, is greater than that of project R, $\lambda_2^R > 0$. In the analysis, we assume no switching options after a project of a certain type has been constructed. This is because infrastructure projects such as bridges, nuclear power plants, and ports cannot be easily modified once constructed, especially for purely economic reasons.

3.2 Project Valuation with Average Outage Factors

Instead of modelling the risk of physical failures in a real options valuation of each project, suppose that a firm were to use average outage factors. Specifically, this means that for each project i, the proportions representing time spent in states 1 and 2, i.e., $\frac{\lambda_2^i}{\lambda_1+\lambda_2^i}$ and $\frac{\lambda_1}{\lambda_1+\lambda_2^i}$, would be used to determine average cash flows (Fig. 2). Thus, the average instantaneous cash flow for project i after investment is $\bar{\pi}^i(P_t) = \left(\frac{\lambda_2^i}{\lambda_1+\lambda_2^i}\right)(P_t - k) - \left(\frac{\lambda_1}{\lambda_1+\lambda_2^i}\right)s$. Consequently, the expected discounted value of project i at some arbitrary price, $P > 0$, using a subjective discount rate, $\rho > \alpha$, with average outage factors is:

$$\begin{aligned}\bar{V}_1^i(P) &= \mathbb{E}_P\left[\int_0^\infty \bar{\pi}^i(P_t)e^{-\rho t}dt\right] \\ \Rightarrow \bar{V}_1^i(P) &= \left(\frac{\lambda_2^i}{\lambda_1+\lambda_2^i}\right)\frac{P}{(\rho-\alpha)} - \frac{\left(\lambda_2^i k + \lambda_1 s\right)}{\left(\lambda_1+\lambda_2^i\right)\rho}\end{aligned} \tag{1}$$

Via dynamic programming (Dixit and Pindyck, 1994), the value of the option to invest in project i from state 0 may be found as:

$$\bar{V}_0^i(P) = \bar{a}_0^i P^{\beta_1}, \tag{2}$$

where $\beta_1 > 1$ is the root of the following characteristic quadratic:

$$\frac{1}{2}\sigma^2\beta(\beta-1)+\alpha\beta-\rho=0 \tag{3}$$

The optimal investment threshold for project i is:

$$\bar{\xi}^i = \left(\frac{\beta_1}{\beta_1-1}\right)\frac{(\rho-\alpha)}{\lambda_2^i}\left[I^i\left(\lambda_1+\lambda_2^i\right)+\frac{\left(\lambda_2^i k+\lambda_1 s\right)}{\rho}\right] \tag{4}$$

The endogenous constant in Eq. (2) is:

$$\bar{a}_0^i = \frac{1}{\beta_1}\left(\frac{\lambda_2^i}{\lambda_1+\lambda_2^i}\right)\frac{\left(\bar{\xi}^i\right)^{1-\beta_1}}{(\rho-\alpha)} \tag{5}$$

The latter two are obtained via standard value-matching and smooth-pasting conditions between $\bar{V}_0^i(P)$ and $\bar{V}_1^i(P) - I^i$ at $P = \bar{\xi}^i$.

Suppose now that we would like to determine how much extra an investor would be willing to pay for project type S rather than R. Keeping constant the investment cost of project type R and the expected discounted operating costs of the two projects, i.e.,

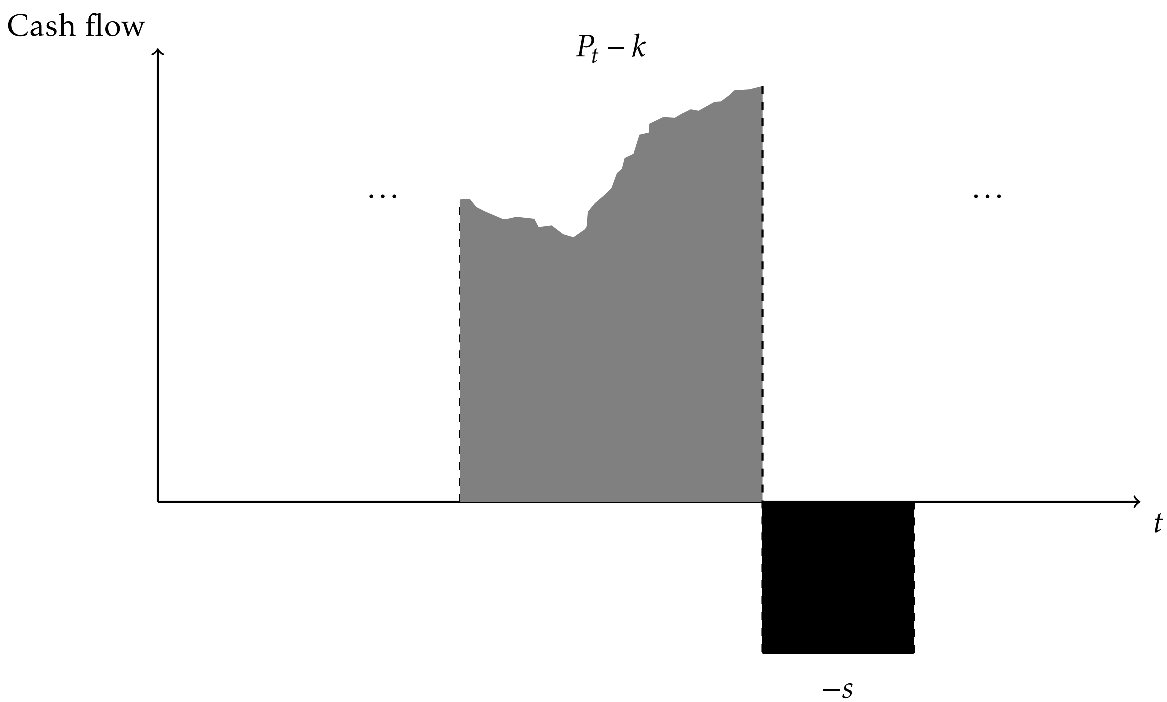

Figure 2: Cash flows during a typical cycle

$\frac{(\lambda_2^i k+\lambda_1 s)}{(\lambda_1+\lambda_2^i)\rho}$, we have the degree of freedom only to increase I^S until it hits a critical threshold, $\bar{I}$, such that the investor is indifferent between the two types of projects. In other words, we force $\bar{a}_0^R$ to equal $\bar{a}_0^S$ and determine the resilience premium, $\bar{\kappa}$:

Proposition 1. $\bar{\kappa} \equiv \frac{\left[\bar{I}+\frac{\left(\lambda_2^S k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^S\right)\rho}\right]}{\left[I^R+\frac{\left(\lambda_2^R k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^R\right)\rho}\right]} = \left\{\frac{\frac{\lambda_2^S}{\left(\lambda_1+\lambda_2^S\right)}}{\frac{\lambda_2^R}{\left(\lambda_1+\lambda_2^R\right)}}\right\}^{\frac{\beta_1}{\beta_1-1}}$

In a now-or-never NPV analysis, the corresponding resilience premium, $\bar{\kappa}^{NPV}$, has a similar form but does not depend on uncertainty:

Corollary 1. $\bar{\kappa}^{NPV} \equiv \frac{\left[\bar{I}+\frac{\left(\lambda_2^S k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^S\right)\rho}\right]}{\left[I^R+\frac{\left(\lambda_2^R k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^R\right)\rho}\right]} = \frac{\frac{\lambda_2^S}{\left(\lambda_1+\lambda_2^S\right)}}{\frac{\lambda_2^R}{\left(\lambda_1+\lambda_2^R\right)}}$

It is also straightforward to prove that the average resilience premium is, indeed, greater than one (Cor. 2) and that it is increasing as either the volatility of cash flows or project S's repair rate increases (Props. 2 and 3, respectively), *ceteris paribus*. Intuitively, a higher cash-flow volatility or repair rate of project S also implies a higher expected present value of revenues, which results in a greater resilience premium.

Corollary 2. $\bar{\kappa} > 1$

Proposition 2. $\frac{\partial \bar{\kappa}}{\partial \sigma} > 0$

Proposition 3. $\frac{\partial \bar{\kappa}}{\partial \lambda_2^S} > 0$

3.3 Project Valuation with Explicit Treatment of Physical Risk

If we treat the risks of outages and repairs explicitly, then we obtain the following expressions for each project i in state j, where $j = 1, 2$ (Makimoto, 2006):

$$V_j^i(P) = a_j^i P + b_j^i \tag{6}$$

where

$$\begin{aligned} a_1^i &= \frac{(\rho + \lambda_2^i - \alpha)}{(\rho + \lambda_1 - \alpha)(\rho + \lambda_2^i - \alpha) - \lambda_1 \lambda_2^i} > 0, \quad a_2^i = \frac{\lambda_2^i}{(\rho + \lambda_1 - \alpha)(\rho + \lambda_2^i - \alpha) - \lambda_1 \lambda_2^i} > 0 \\ b_1^i &= -\frac{(\lambda_1 s + (\rho + \lambda_2^i)k)}{(\rho + \lambda_1)(\rho + \lambda_2^i) - \lambda_1 \lambda_2^i} < 0, \quad b_2^i = -\frac{(\lambda_2^i k + (\rho + \lambda_1)s)}{(\rho + \lambda_1)(\rho + \lambda_2^i) - \lambda_1 \lambda_2^i} < 0 \end{aligned}$$

Again, through standard dynamic programming methods, the value of the option to invest from state 0 in plant i is:

$$V_0^i(P) \quad = \quad a_0^i P^{\beta_1}, \tag{7}$$

where

$$\xi^i = \left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{I^i - b_1^i}{a_1^i}\right) \tag{8}$$

and

$$a_0^i = \frac{a_1^i}{\beta_1}\left(\frac{\beta_1}{\beta_1 - 1}\right)^{1-\beta_1}\left(\frac{I^i - b_1^i}{a_1^i}\right)^{1-\beta_1} \tag{9}$$

are obtained via value-matching and smooth-pasting conditions between $V_0^i(P)$ and $V_1^i(P) - I^i$ at $P = \xi^i$ as defined in Eqs. (7) and (6), respectively.

As in Section 3.2, it is possible to define a resilience premium, κ, which here does not depend on average outage factors. Instead, it uses the actual repair rates and is obtained by equating the a_0^i terms from Eq. (9). Since $I^i - b_1^i$ may be thought of as the discounted investment and operating costs of project i and $\hat{I}$ is the maximum willingness to pay for project S, analogous to Prop. 1, we can express the resilience premium as a ratio.

Proposition 4. $\kappa \equiv \frac{\hat{I}-b_1^S}{I^R-b_1^R} = \left(\frac{a_1^S}{a_1^R}\right)^{\frac{\beta_1}{\beta_1-1}}$

Furthermore, we can prove that the resilience premium here is greater than one, increasing with volatility, and increasing with the repair rate of project S.

Corollary 3. $\kappa > 1$

Proposition 5. $\frac{\partial\kappa}{\partial\sigma} > 0$

Proposition 6. $\frac{\partial\kappa}{\partial\lambda_2^S} > 0$

Now, suppose that we change all transition rates by some factor $\eta > 0$, which clearly does not affect the proportion of time spent in state 1 or 2. Such a transformation would not affect any of the results in the previous section concerning $\bar{\xi}^i$ or Props. 1–3. Yet, if physical risks were explicitly treated, then they would affect not only optimal investment thresholds but also the resilience premium, which is $\kappa(\eta)$ and dependent on the maximum amount that the investor is willing to pay, $\hat{I}$, for project S. In the limit, as cycles become more frequent, i.e., $\eta \to \infty$, we can show that the resilience premium converges to $\bar{\kappa}$:

Proposition 7. $\lim_{\eta\to\infty} \kappa(\eta) = \bar{\kappa}$

Intuitively, it is only in a situation with very frequent state transitions that it is acceptable to determine the resilience premium on the basis of average outage factors. By contrast, when state transitions hardly occur, i.e., $\eta \to 0$, there is no willingness to pay for resilience:

Proposition 8. $\lim_{\eta \to 0} \kappa(\eta) = 1$

In general, the resilience premium is an increasing function of the transition rate:

Proposition 9. $\frac{\partial \kappa(\eta)}{\partial \eta} > 0$

The next two propositions and Lemma 1 summarise the economic implications of simplifying the real options analysis. First, Prop. 10 indicates that the use of average outage factors postpones investment *vis-à-vis* the explicit treatment of risk because the latter recognises that risk has not been eliminated and invests only in an active project that commences operations in state 1. Thus, real options analysis with explicit treatment of risk will not require as high of a threshold as one with average outage factors. Second, Prop. 11 states that the value of the investment opportunity when risk is treated explicitly is higher than when average outage factors are used. The intuition for this result is similar to that for Prop. 10: explicitly accounting for physical risk means that no project would ever start in state 2, i.e., the payoff of the option is inherently more valuable. Therefore, an investor who uses average outage factors in a real options analysis will (incorrectly) be willing to pay more for resilience as shown in Lemma 1.

Proposition 10. $\xi^i(\eta) < \bar{\xi}^i$

Proposition 11. $a_0^i > \bar{a}_0^i$

Lemma 1. $\bar{\kappa} > \kappa(\eta)$

Finally, Prop. 12 reveals that the discrepancy between the resilience premia increases as uncertainty increases. Indeed, the impact of using average outage factors is magnified in a more economically volatile situation.

Proposition 12. $\frac{\partial \bar{\kappa}/\kappa(\eta)}{\partial \sigma} > 0$

4 Numerical Examples

We use the following parameter values to illustrate the properties of the solutions and value functions: $\alpha = 0, \rho = 0.05, \lambda_1 = 0.5, \lambda_2^R = 0.2, I^R = 10, \lambda_2^S = 0.3, I^S = 15, k = 1, s = 2, \sigma = 0.2, P_0 = 1$. First, we plot the value functions for the two plant types with average outage factors (Fig. 3). As expected, for an arbitrarily low initial price, the option to invest in the "safer" project is more valuable and triggers earlier investment. Next, we do the same when physical risks are correctly modelled (Fig. 4). Here, there are two sets of post-investment value functions for each type of project depending on the state of operations. Thus, the physical risk is reflected in the value of the investment opportunity even as the "safer" project is worth more than the "riskier" one.

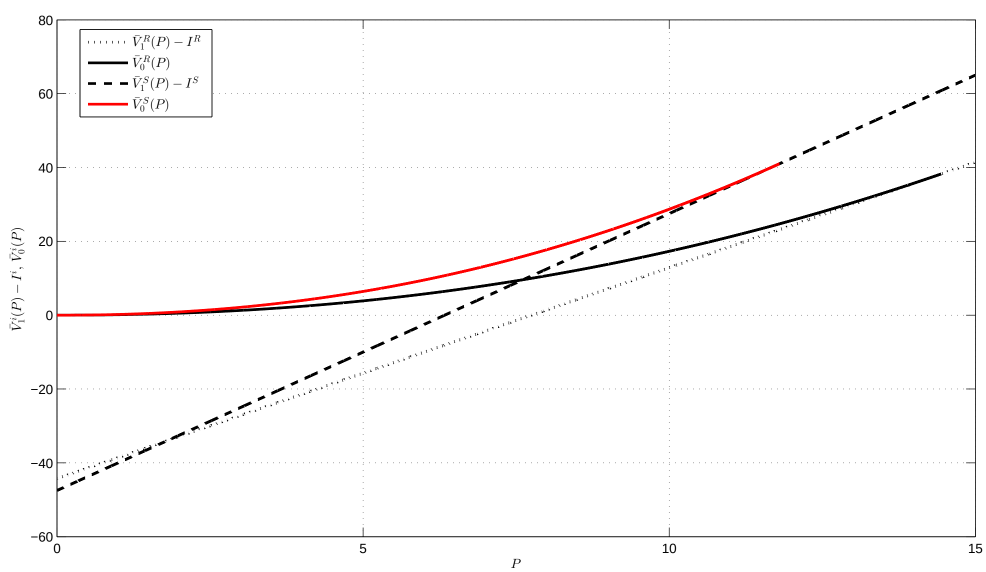

Figure 3: Value functions with average availability factors for $\sigma = 0.20$

The optimal investment thresholds and corresponding option value coefficients for various volatilities, i.e., $\sigma \in [0.20, 0.40]$, are plotted in Figs. 5 and 6 to verify Props. 10 and 11, respectively. In particular, the value of the option to invest is higher when modelling physical risk for each type of project with a lower corresponding investment threshold.

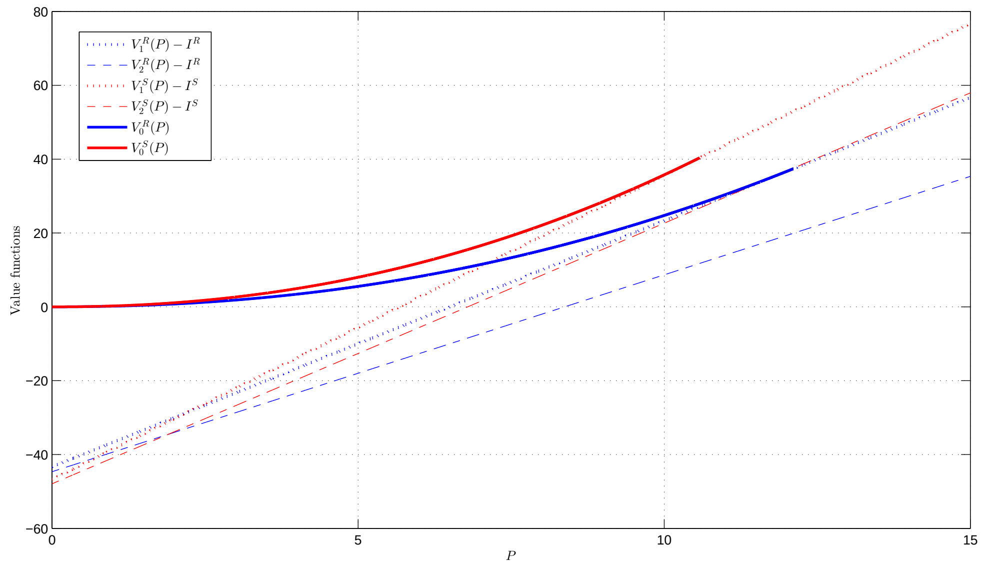

Figure 4: Value functions with physical risk for $\sigma = 0.20$

As argued in Section 3.3, this discrepancy follows from the higher intrinsic value of the post-investment project in state 1 as opposed to some value based on an average outage factor.

The next plot in Fig. 7 illustrates Props. 1, 4, 7, 8, and 9, Cors. 2 and 3, and Lemma 1, where the horizontal line indicates the resilience premium under the assumption of average outage factors, $\bar{\kappa} = 1.66$. Here, I^R is fixed at 10, but I^S is varied until the option values of the two projects are equal. Thus, while $\bar{\kappa}$ is unaffected by η, $\kappa(\eta)$, which is the resilience premium in a model with explicit treatment of physical risk, increases monotonically until it asymptotically approaches $\bar{\kappa}$. Finally, Fig. 8 indicates that both resilience risk premia increase with volatility along with their ratio as proven in Props. 2, 5, and 12. Hence, when using average factors, the investor typically overvalues the resilience premium except when cycles become exceptionally short.

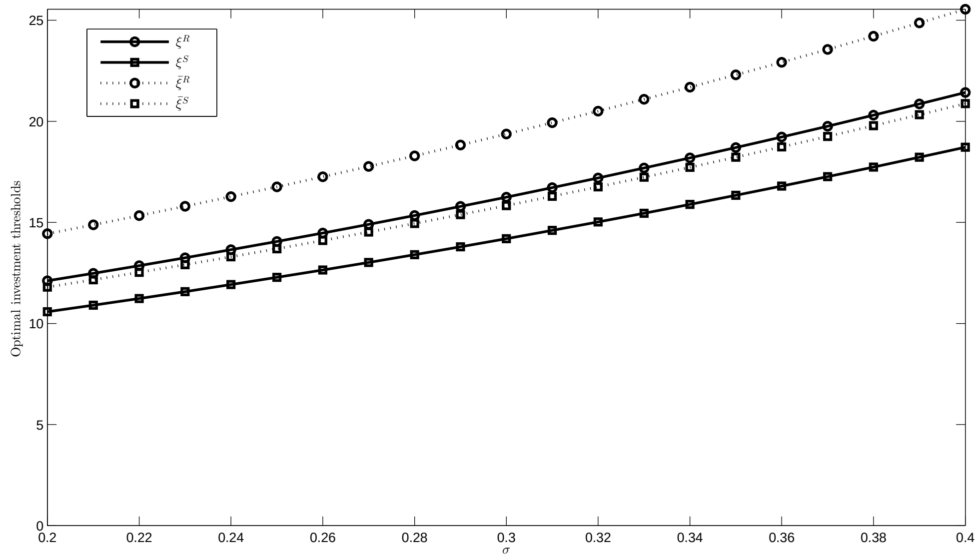

Figure 5: Effect of volatility on the optimal investment thresholds

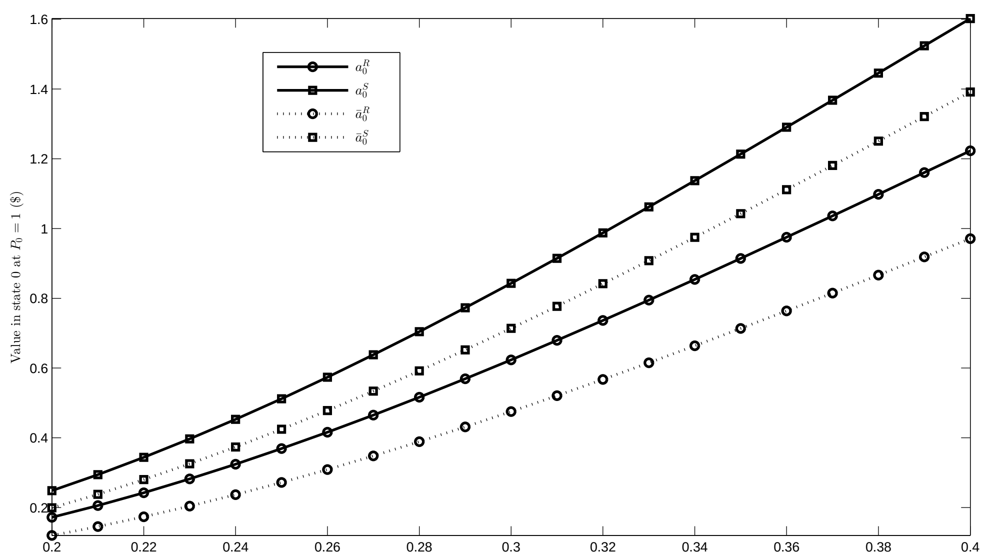

Figure 6: Effect of volatility on the option values

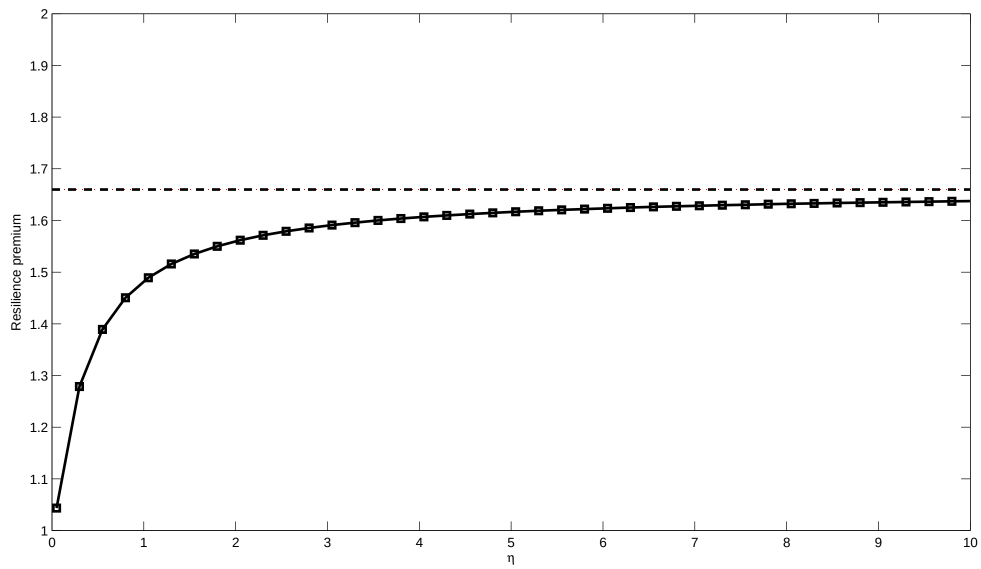

Figure 7: Effect of cycle length on the resilience premium, $\kappa(\eta)$

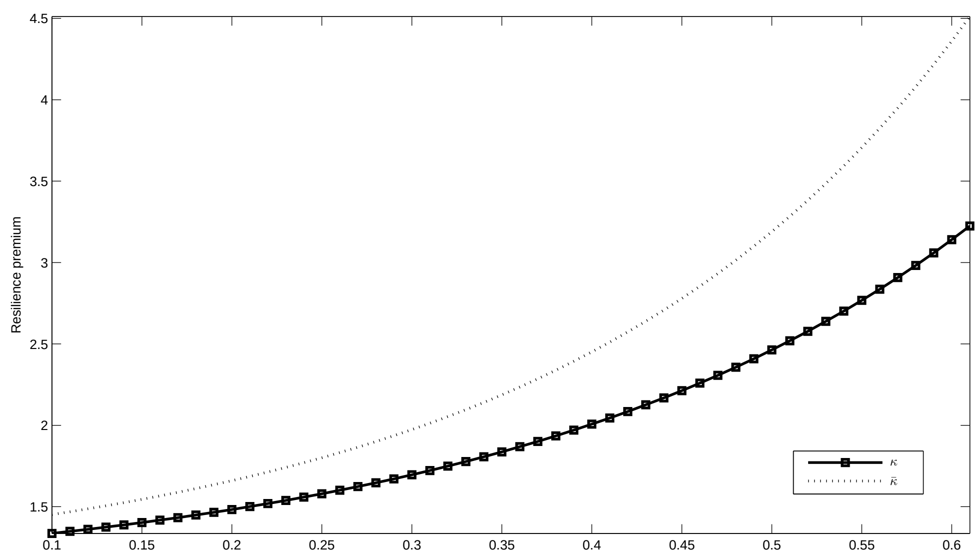

Figure 8: Effect of volatility on the resilience premium, κ

5 Conclusions

Given the increased occurrence of extreme weather events linked to climate change, owners of infrastructure are considering investment in more resilient plants. Although policymakers typically offer incentives to improve investment in and retrofit of infrastructure that is better adapted to climate risk, ultimately, the adoption decision is undertaken by private investors who aim to maximise the value of their projects. Thus, it is desirable to consider the willingness of decision makers to pay for enhanced resilience in a market environment with both economic uncertainty and physical risk.

Drawing upon the real options approach for assessing the premium associated with a more flexible investment strategy (Kort et al., 2010), we determine the willingness to pay for resilience in the context of infrastructure projects. Specifically, we include physical risk to the operations of infrastructure projects as a Poisson process and determine the maximum additional capital expenditure that an investor would be willing to outlay in exchange for a more resilient project, i.e., one that resumes operations from an inactive state more quickly. We show analytically that the resilience premium obtained via explicit treatment of physical risk is always less than that implied by a simpler real options analysis that uses average outage factors. Intuitively, the latter analysis with average outage factors assumes that greater resilience essentially means eliminating all of the risk. However, the analysis with explicit treatment of risk acknowledges that risk can only be mitigated. Consequently, its resilience premium is less than that yielded by the analysis with average outage factors. Yet, sensitivity analysis with respect to cycle lengths indicates that this discrepancy between the two premia dissipates as frequency of transitions increases. Indeed, in the limit as cycle lengths become very short, the use of average outage factors becomes acceptable as an approximation. Hence, our work demonstrates that policy initiatives based on average outage factors to bolster investment in more resilient infrastructure may overestimate industry's willingness to pay.

Our analysis is necessarily stylised in order to obtain analytical results. It is, therefore, limited by its choice of stochastic processes capturing both economic uncertainty and physical risk. Numerical approaches using more realistic stochastic processes applied to case studies would, thus, be welcome extensions. Likewise, analyses with mutually exclusive investment between "safer" and "riskier" options (Décamps et al., 2006) or under rivalry (Siddiqui and Takashima, 2012) would provide richer insights.

Acknowledgements

Siddiqui is grateful to the Japan Society for Promotion of Science (JSPS) Furusato Award and a Tokyo University of Science Invitational Fellowship for support.

References

Adkins, R. and D. Paxson (2016), "Subsidies for Renewable Energy Facilities under Uncertainty," *The Manchester School* 84(2): 222–250.

Boomsma, T.K., N. Meade, S.-E. Fleten (2012), "Renewable Energy Investments under Different Support Schemes: a Real Options Approach," *European Journal of Operational Research* 220(1): 225-237.

Chronopoulos, M. and A.S. Siddiqui (2015), "When is it Better to Wait for a New Version? Optimal Replacement of an Emerging Technology under Uncertainty," *Annals of Operations Research* 235(1): 177–201.

Décamps, J.-P., T. Mariotti, and S. Villeneuve (2006), "Irreversible Investment in Alternative Projects," *Economic Theory* 28: 425–448.

Dixit, A.K. (1993), "Choosing Among Alternative Discrete Investment Projects under Uncertainty," *Economics Letters* 41: 265–268.

Dixit, A.K. and R.S. Pindyck (1994), *Investment under Uncertainty*, Princeton University Press, Princeton, NJ, USA.

Doraszelski, U. (2001), “The Net Present Value Method Versus the Option Value of Waiting: A Note on Farzin, Huisman and Kort (1998),” *Journal of Economic Dynamics and Control* 25: 1109–1115.

Farzin, Y.H., K. Huisman, and P. Kort (1998), “Optimal Timing of Technology Adoption,” *Journal of Economic Dynamics and Control* 22(5): 779–799.

Grenadier, S.R. and A.M. Weiss (1997), “Investment in Technological Innovations: An Option Pricing Approach,” *Journal of Financial Economics* 44: 397–416.

Kort, P., P. Murto, and G. Pawlina (2010), “Uncertainty and Stepwise Investment,” *European Journal of Operational Research* 202: 196–203.

The Miami-Dade Local Mitigation Strategy (2016), *Whole Community Hazard Mitigation Part 6: Completed Projects*, Miami, FL, USA (available at http://www.miamidade.gov/fire/library/OEM/local-mitigation-strategy-part-6-completed-projects.pdf).

Makimoto, N. (2006), “Optimal Investment Policy for Real Option Problems with Regime Switching,” in *The Sixth International Symposium on Operations Research and Its Applications*, Xinjiang, P. R. China.

Malchow-Møller, N. and B.J. Thorsen (2005), “Repeated Real Options: Optimal Investment Behaviour and a Good Rule of Thumb,” *Journal of Economic Dynamics and Control* 29(6): 1025–1041.

Richter, L.-L. and M. Weeks (2016), “Flexible Mixed Logit with Posterior Analysis: Eliciting Willingness to Pay for Grid Resilience,” EPRG Working Paper 1615 and Cambridge Working Paper in Economics 1631, University of Cambridge, U.K. (available at http://www.eprg.group.cam.ac.uk/wp-content/uploads/2016/05/1615-Text.pdf).

Siddiqui, A.S. and R. Takashima (2012), "Capacity Switching Options under Rivalry and Uncertainty," *European Journal of Operational Research* 222(3): 583–595.

U.S. Environmental Protection Agency (2008), *Planning for Climate Change Impacts at U.S. Ports*, White Paper, Office of Policy, Economics, and Innovation Sector Strategies Division, Washington, DC, USA (available at https://archive.epa.gov/sectors/web/pdf/ports-planing-for-cci-white-paper.pdf).

Walker, B., C.S. Holling, S.R. Carpenter, and A. Kinzig (2004), "Resilience, Adaptability, and Transformability in SocialEcological Systems," *Ecology and Society* 9(2): 5 (available at http://www.ecologyandsociety.org/vol9/iss2/art5/).

Walker, B., L. Pearson, M. Harris, K.-G. Maler, C.-Z. Li, R. Biggs, and T. Baynes (2010), "Incorporating Resilience in the Assessment of Inclusive Wealth: An Example from South East Australia," *Environmental and Resource Economics* 45(2): 183–202.

Whitten, S.M., G. Hertzler, and S. Strunz (2012), "How Real Options and Ecological Resilience Thinking Can Assist in Environmental Risk Management," *Journal of Risk Research* 15(3): 331–346.

Appendix: Proofs of Propositions

Proof of Proposition 1: Use the definitions of $\bar{a}_0^i$ and equate them:

$$\begin{aligned}\left(\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}\right)^{\beta_1}\left[I^R+\frac{\left(\lambda_2^R k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^R\right)\rho}\right]^{1-\beta_1} &= \left(\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}\right)^{\beta_1}\left[I^S+\frac{\left(\lambda_2^S k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^S\right)\rho}\right]^{1-\beta_1} \\ \Rightarrow \bar{\kappa} &= \left\{\frac{\frac{\lambda_2^S}{\left(\lambda_1+\lambda_2^S\right)}}{\frac{\lambda_2^R}{\left(\lambda_1+\lambda_2^R\right)}}\right\}^{\frac{\beta_1}{\beta_1-1}} \qquad (10)\end{aligned}$$

□

Proof of Corollary 1: Use the definitions of $\bar{V}_1^i(P)$, equate them at $P = \xi_{NPV}^R \equiv \frac{(\rho-\alpha)}{\lambda_2^i}\left[I^i\left(\lambda_1+\lambda_2^i\right)+\frac{(\lambda_2^i k+\lambda_1 s)}{\rho}\right]$ and solve for $\bar{\kappa}^{NPV} \equiv \frac{\left[\bar{I}+\frac{\left(\lambda_2^S k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^S\right)\rho}\right]}{\left[I^R+\frac{\left(\lambda_2^R k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^R\right)\rho}\right]}$:

$$\begin{aligned} \frac{\lambda_2^S}{\lambda_1+\lambda_2^S}\left[I^R+\frac{\left(\lambda_2^R k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^R\right)\rho}\right] &= \frac{\lambda_2^R}{\lambda_1+\lambda_2^R}\left[I^S+\frac{\left(\lambda_2^S k+\lambda_1 s\right)}{\left(\lambda_1+\lambda_2^S\right)\rho}\right] \\ \Rightarrow \bar{\kappa}^{NPV} &= \frac{\frac{\lambda_2^S}{\left(\lambda_1+\lambda_2^S\right)}}{\frac{\lambda_2^R}{\left(\lambda_1+\lambda_2^R\right)}} \end{aligned} \tag{11}$$

□

Proof of Corollary 2: Since $\bar{\kappa} > 1 \Leftrightarrow \frac{\frac{\lambda_2^S}{\left(\lambda_1+\lambda_2^S\right)}}{\frac{\lambda_2^R}{\left(\lambda_1+\lambda_2^R\right)}} > 1$, we need the following:

$$\begin{aligned} \frac{\lambda_2^S}{\left(\lambda_1+\lambda_2^S\right)} &> \frac{\lambda_2^R}{\left(\lambda_1+\lambda_2^R\right)} \\ \Rightarrow \lambda_2^S\lambda_1+\lambda_2^S\lambda_2^R &> \lambda_2^S\lambda_2^R+\lambda_2^R\lambda_1 \\ \Rightarrow \lambda_2^S &> \lambda_2^R, \end{aligned}$$

which is true by assumption. □

Proof of Proposition 2: From Eq. (10), we have:

$$\frac{\partial\bar{\kappa}}{\partial\sigma} = -\log\left(\frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}}\right)\left(\frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}}\right)^{\frac{\beta_1}{\beta_1-1}}\frac{1}{(\beta_1-1)^2}\frac{\partial\beta_1}{\partial\sigma}.$$

Since $\frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}} = \frac{\lambda_1\lambda_2^S+\lambda_2^S\lambda_2^R}{\lambda_1\lambda_2^R+\lambda_2^S\lambda_2^R} > 1$, $\log\left(\frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}}\right) > 0$. In addition, from $\left(\frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}}\right)^{\frac{\beta_1}{\beta_1-1}} > 0$, $-\frac{1}{(\beta_1-1)^2} < 0$, and $\frac{\partial\beta_1}{\partial\sigma} < 0$ (Dixit and Pindyck, 1994), we can obtain $\frac{\partial\bar{\kappa}}{\partial\sigma} > 0$. □

Proof of Proposition 3: We partially differentiate Eq. (10) with respect to λ_2^S:

$$\frac{\partial \bar{\kappa}}{\partial \lambda_2^S} = \left(\frac{\beta_1}{\beta_1 - 1}\right)\left\{\frac{\frac{\lambda_2^S}{(\lambda_1+\lambda_2^S)}}{\frac{\lambda_2^R}{(\lambda_1+\lambda_2^R)}}\right\}^{\frac{1}{\beta_1-1}}\left(\frac{\lambda_1+\lambda_2^R}{\lambda_2^R}\right)\frac{\lambda_1}{\left(\lambda_1+\lambda_2^S\right)^2} \tag{12}$$

Since each term on the right-hand side of Eq. (12) is positive, we have the result. □

Proof of Proposition 4: Use the definitions of a_0^i and equate them:

$$\begin{aligned} \frac{a_1^R}{\beta_1}\left(\frac{\beta_1}{\beta_1-1}\right)^{1-\beta_1}\left(\frac{I^R-b_1^R}{a_1^R}\right)^{1-\beta_1} &= \frac{a_1^S}{\beta_1}\left(\frac{\beta_1}{\beta_1-1}\right)^{1-\beta_1}\left(\frac{\hat{I}-b_1^S}{a_1^S}\right)^{1-\beta_1} \\ \Rightarrow \kappa &\equiv \frac{\hat{I}-b_1^S}{I^R-b_1^R} = \left(\frac{a_1^S}{a_1^R}\right)^{\frac{\beta_1}{\beta_1-1}} \end{aligned} \tag{13}$$

□

Proof of Corollary 3: Since $\kappa > 1 \Leftrightarrow a_1^S > a_1^R$, we need to demonstrate the following:

$$\begin{aligned} \frac{\partial}{\partial x}\left(\frac{\rho+x-\alpha}{(\rho+\lambda_1-\alpha)(\rho+x-\alpha)-\lambda_1 x}\right) &> 0 \\ \Rightarrow \frac{(\rho+\lambda_1-\alpha)(\rho+x-\alpha)-\lambda_1 x-(\rho+x-\alpha)(\rho-\alpha)}{((\rho+\lambda_1-\alpha)(\rho+x-\alpha)-\lambda_1 x)^2} &> 0 \\ \Rightarrow \frac{\lambda_1(\rho-\alpha)}{((\rho+\lambda_1-\alpha)(\rho+x-\alpha)-\lambda_1 x)^2} &> 0, \end{aligned}$$

which holds because both the numerator and the denominator are positive. □

Proof of Proposition 5: From Eq. (13), we have:

$$\frac{\partial \kappa}{\partial \sigma} = -\log\left(\frac{a_1^S}{a_1^R}\right)\left(\frac{a_1^S(\eta)}{a_1^R(\eta)}\right)^{\frac{\beta_1}{\beta_1-1}}\frac{1}{(\beta_1-1)^2}\frac{\partial \beta_1}{\partial \sigma}. \tag{14}$$

In Eq. (14), since we have $a_1^S > a_1^R$ from Eq. (14), we obtain $\log\left(\frac{a_1^S}{a_1^R}\right) > 0$. Thus, $\frac{\partial \kappa}{\partial \sigma} > 0$. □

Proof of Proposition 6: Partially differentiating κ in Eq. (13) with respect to λ_2^S, we have:

$$\frac{\partial \kappa}{\partial \lambda_2^S} = \left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{a_1^S}{a_1^R}\right)^{\frac{1}{\beta_1 - 1}} \frac{1}{a_1^R}\frac{\partial a_1^S}{\partial \lambda_2^S},$$

which is greater than zero because each term is greater than zero and $\frac{\partial a_1^S}{\partial \lambda_2^S} > 0$ follows from the proof of Corollary 3. $\square$

Proof of Proposition 7: First, use l'Hôpital's rule to show that $\lim_{\eta\to\infty} a_1^i(\eta) = \frac{\lambda_2^i}{(\rho-\alpha)(\lambda_2^i+\lambda_1)}$. Next, take the limit of the expression in Prop. 4:

$$\begin{aligned} \lim_{\eta\to\infty} \kappa(\eta) &= \lim_{\eta\to\infty}\left(\frac{a_1^S(\eta)}{a_1^R(\eta)}\right)^{\frac{\beta_1}{\beta_1-1}} \\ \Rightarrow \lim_{\eta\to\infty} \kappa(\eta) &= \left\{\frac{\frac{\lambda_2^S}{(\lambda_1+\lambda_2^S)}}{\frac{\lambda_2^R}{(\lambda_1+\lambda_2^R)}}\right\}^{\frac{\beta_1}{\beta_1-1}} \end{aligned} \tag{15}$$

$\square$

Proof of Proposition 8: First, use l'Hôpital's rule to show that $\lim_{\eta\to 0} a_1^i(\eta) = \frac{1}{(\rho-\alpha)}$. Next, take the limit of the expression in Prop. 4:

$$\begin{aligned} \lim_{\eta\to 0} \kappa(\eta) &= \lim_{\eta\to 0}\left(\frac{a_1^S(\eta)}{a_1^R(\eta)}\right)^{\frac{\beta_1}{\beta_1-1}} \\ \Rightarrow \lim_{\eta\to 0} \kappa(\eta) &= \left\{\frac{1}{1}\right\}^{\frac{\beta_1}{\beta_1-1}} \end{aligned} \tag{16}$$

$\square$

Proof of Proposition 9: Using the definition of a_1^i from Eq. (6), we have:

$$\frac{\partial a_1^i(\eta)}{\partial \eta} = -\frac{\lambda_1(\rho-\alpha)^2}{\left[(\rho+\eta\lambda_1-\alpha)(\rho+\eta\lambda_2^i-\alpha)-\eta^2\lambda_1\lambda_2^i\right]^2} < 0 \tag{17}$$

$$
\begin{aligned}
\frac{\partial a_1^R(\eta)}{\partial \eta} &> \frac{\partial a_1^S(\eta)}{\partial \eta} \\
\Rightarrow \frac{\frac{\partial a_1^S(\eta)}{\partial \eta}}{\frac{\partial a_1^R(\eta)}{\partial \eta}} &< 1
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\frac{\partial \kappa(\eta)}{\partial \eta} &= \left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{a_1^S(\eta)}{a_1^R(\eta)}\right)^{\frac{1}{\beta_1 - 1}} \left[\frac{\frac{\partial a_1^S(\eta)}{\partial \eta} a_1^R(\eta) - a_1^S(\eta)\frac{\partial a_1^R(\eta)}{\partial \eta}}{(a_1^R(\eta))^2}\right] \\
&= -\left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{a_1^S(\eta)}{a_1^R(\eta)}\right)^{\frac{1}{\beta_1 - 1}} \left[\frac{1}{(a_1^R(\eta))^2}\right]\left(\frac{\partial a_1^R(\eta)}{\partial \eta}\right)\left(a_1^S(\eta) - a_1^R(\eta)\frac{\frac{\partial a_1^S(\eta)}{\partial \eta}}{\frac{\partial a_1^R(\eta)}{\partial \eta}}\right)
\end{aligned} \tag{19}
$$

From Eqs. (17) and (18), $-\left(\frac{\partial a_1^R(\eta)}{\partial \eta}\right)\left(a_1^S(\eta) - a_1^R(\eta)\frac{\frac{\partial a_1^S(\eta)}{\partial \eta}}{\frac{\partial a_1^R(\eta)}{\partial \eta}}\right)$ in Eq. (19) is positive. Thus, $\frac{\partial \kappa(\eta)}{\partial \eta} > 0$. □

Proof of Lemma 1: From $\bar{\kappa} > 1$, $\lim_{\eta \to 0} \kappa(\eta) = 1$ in Prop. 8, $\lim_{\eta \to \infty} \kappa(\eta) = \bar{\kappa}$ in Prop. 7, and $\frac{\partial \kappa(\eta)}{\partial \eta} > 0$ in Prop. 9, we can obtain $\bar{\kappa} > \kappa(\eta)$ for $\forall \eta$. □

Proof of Proposition 10: Using the definition of b_1^i from Eq. (6), we have:

$$
\frac{\partial b_1^i(\eta)}{\partial \eta} = \frac{\rho^2 \lambda_1 (k - s)}{\left[(\rho + \eta\lambda_1)(\rho + \eta\lambda_2^i) - \eta^2 \lambda_1 \lambda_2^i\right]^2} < 0 \tag{20}
$$

$$
\lim_{\eta \to \infty} \xi^i(\eta) = \bar{\xi}^i \tag{21}
$$

$$
\lim_{\eta \to 0} \xi^i(\eta) = \frac{\beta_1}{\beta_1 - 1}(\rho - \alpha)\left(I^i + \frac{k}{\rho}\right) < \bar{\xi}^i \tag{22}
$$

From Eqs. (17) and (3),

$$
\frac{\partial \xi^i(\eta)}{\partial \eta} = -\left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{\left(\frac{\partial b_1^i(\eta)}{\partial \eta} a_1^i (I^i - b_1^i(\eta))\frac{\partial a_1^i(\eta)}{\partial \eta}\right)}{(a_1^i(\eta))^2} > 0 \tag{23}
$$

Thus, from Eqs. (21)–(23), $\xi^i(\eta) < \bar{\xi}^i$ for $\forall \eta$. □

Proof of Proposition 11: From Prop. 10, we know that:

$$\xi^i(\eta)^{1-\beta_1} > \bar{\xi}^{i\,1-\beta_1} \tag{24}$$

$$a_1^i(\eta) - \frac{\eta\lambda_2^i}{(\eta\lambda_1 + \eta\lambda_2^i)(\rho-\alpha)} = \frac{(\rho-\alpha)^2\eta\lambda_1}{(\rho-\alpha)(\eta\lambda_1+\eta\lambda_2^i)\left\{(\rho-\alpha)(\eta\lambda_1+\eta\lambda_2^i)+(\rho-\alpha)^2\right\}} > 0 \tag{25}$$

From Eqs. (24) and (25),

$$a_0^i(\eta) - \bar{a}_0^i = \frac{1}{\beta_1}\left(a_1^i(\eta)\xi^i(\eta)^{1-\beta_1} - \frac{\eta\lambda_2^i}{(\eta\lambda_1+\eta\lambda_2^i)(\rho-\alpha)}\bar{\xi}^{i\,1-\beta_1}\right) > 0 \tag{26}$$

□

Proof of Proposition 12: Using Eqs. (10) and (13), we obtain:

$$\frac{\bar{\kappa}}{\kappa(\eta)} = \left(\frac{\frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}}}{\frac{a_1^S(\eta)}{a_1^R(\eta)}}\right)^{\frac{\beta_1}{\beta_1-1}}. \tag{27}$$

When $\Lambda = \frac{\frac{\lambda_2^S}{\lambda_1+\lambda_2^S}}{\frac{\lambda_2^R}{\lambda_1+\lambda_2^R}} / \frac{a_1^S(\eta)}{a_1^R(\eta)}$, $\frac{\partial\bar{\kappa}/\kappa(\eta)}{\partial\sigma} = \frac{\partial}{\partial\sigma}\Lambda^{\frac{\beta_1}{\beta_1-1}} = -\log\Lambda\Lambda^{\frac{\beta_1}{\beta_1-1}}\left\{\frac{1}{(\beta_1-1)^2}\right\}\frac{\partial\beta_1}{\partial\sigma}$. From Lemma 1, we can obtain $\log\Lambda > 0$. Thus, $\frac{\partial\bar{\kappa}/\kappa(\eta)}{\partial\sigma} > 0$. □