Risk Sharing with Collar Options in Infrastructure Investments

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Abstract

A real option model is formulated for infrastructure investments with collars, which are devised to guarantee a floor cash flow for an active real asset while capping any abnormally high cash flows. Composed of pairs of put and call American perpetuity options, feasible collars perform a similar role as investment subsidies by yielding a lower investment threshold, thereby inducing an earlier exercise than the without-collar variant. While the investment threshold for the with-collar model is governed only by the floor, the investment option value is influenced positively by the floor but negatively by the cap, so by appropriately adjusting the floor and cap, the with-collar investment option value can be engineered to equal that for the without-collar variant, making it effectively "costless". A volatility increase makes the with-collar variant less valuable due to the greater chance of hitting the cap. The "profits" of the concessionaire are compared to those of the concession granting government under collar, and floor or ceiling only, viewing the arrangement as a real option game between principal and agent. The collar analysis is extended to two more complex collar designs, and also compared with floor only and ceiling only arrangements.

JEL Classifications: D81, G31, H25

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1 Introduction

We present a collar option as a suitable policy device for a government (GOV) granting a concession to induce Public-Private Partnership (PPP) infrastructure investment by a concessionaire (CON) by guaranteeing a floor in the face of adverse circumstances, and simultaneously capturing abnormally high returns when the circumstances are sufficiently favourable. Implementing a collar results in an earlier exercise than for an investment opportunity without a collar due to the guarantee, while its cost may be partially recouped from significantly high profits. The analysis of collars adopts a real option formulation because the guarantee on the downside and bonus compensation for the government on the upside are expressible as real options, the sunk cost is partly irretrievable, deferral flexibility is present, and uncertainty prevails. Using an American perpetuity model, we show that while the minimum revenue guarantee enhances the attractiveness of the with-collar for the CON compared to the without-collar opportunity and reduces its threshold resulting in an earlier exercise, the compensation ceded to the GOV on the upside only reduces the real option value (ROV). This finding produces a straightforward method for engineering a collar because the guarantee level can first be ascertained from knowing the desired threshold prompting exercise, and the compensation level can then be determined from deriving the appropriate ROV (which may, or may not, be paid by the CON for the concession to the GOV).

With a significant decrease in the investment threshold and increase in the investment opportunity value, private capital may be motivated not only to undertake these projects but to implement them early. However, these policies are alleged to distort the risk-return profile in favour of the private party and may be seen to be too generous. According to Shaoul et al. (2012), PPPs are expensive and have failed to deliver value for public money.

Most of the authors considering PPP arrangements as a set of real options embedded in an active project adopt numerical techniques like Monte-Carlo simulations, sometimes in conjunction with a binomial lattice for obtaining their findings, but a few base their conclusions on an analytical real option framework. By evaluating numerically an actual toll road concession involving both a guarantee and compensation, Rose (1998) shows that the minimum revenue guarantee contributes significant value to the Melbourne CityLink (toll road) project. The project reverts to the government if the internal rate of return is very high, which is a type of cap option. Brown (2005) also provides some details on the CityLink arrangements, along with several other Australian toll road PPP, some of which have only ceilings (or an increasing share of the revenues past a benchmark which are paid to the government). An alternative analysis of CityLink is provided by Alonso-Conde et al. (2007), who show that the guarantee not only acts as an incentive but also potentially transfers significant value to the investor.

The implied value of several interacting flexibilities for a rail concession are investigated by Bowe and Lee (2004), while Huang and Chou (2006) appraise minimum revenue guarantees and abandonment rights for a similar concession using a European-style framework. Brandão and Saraiva (2008) evaluate the real option value of a minimum traffic guarantee in Brazil combined with a limit on government exposure, using a Monte Carlo simulation. They propose and evaluate a floor and ceiling guarantee model ("it is only fair"). Blank et al. (2009) investigate the role of a graduated series of guarantees and penalties incurred when operating another Brazilian toll road concession as a risk transfer device for avoiding bankruptcy that benefits both the investor and lender. Shan et al. (2010) value sharing of revenue risks in transportation, which involve European collars of a revenue guarantee and upside compensation to the government. Carbonara et al. (2014) evaluate the real option value of revenue guarantee for an Italian toll road project, also using a Monte Carlo simulation.

Others consider a type of written call option for a successful PPP project which consists of a transfer back to the government for a nil, minimum or residual price at the end of the concession period. Atlantica (2015) has invested in a Polish toll road which has a profit sharing scheme with the State share rising in line with increases in the shareholder returns, and on the termination of the concession the infrastructure must have at least 50% of its remaining useful life. Other possible benefits for a government are reductions in the feed-in-tariff for electricity if construction costs are below expected levels, as in the proposed Hinkley Point C arrangements in the U.K., National Audit Office (2016). Not all authors investigate the incentives for the concessionaire, for instance to control construction costs, or to operate just short of the level that triggers the upside call option, or to reduce the project volatility by hedging or issuing risk sharing debt instruments.

Besides these numerical investigations, there are two key analytical studies. Takashima et al. (2010) design a PPP deal involving government debt participation that incorporates a floor on the future maximum loss level where the investor has the right to sell back the project whenever adverse conditions emerge. Using an analytical model, they show the effect of such deals on the investment timing decision. Also, Armada et al. (2012) make an analytical comparison of various subsidy policies and a demand guarantee scheme. In summary, literature is full of examples of floor only, ceiling only, and collar arrangements for PPP projects, which, however, are often more complicated than the analytical models presented below.

Several authors focus on the conflict between a principal GOV and agent concessionaire CON implicit in contracts. Chevalier-Roignant et al. (2011) and Azevedo and Paxson (2014) survey many real option game problems between principal and agent. Páez-Pérez and Sándhez-Silva (2016) focus on the conflictive roles in a PPP infrastructure arrangement. Scandizzo and Ventura (2010) is the closest paper to ours with a focus on "calculating a baseline to organize a concession contract…to measure the balance of power between the public and the private party", especially in Autostrade S.P.A.

Our contribution consists of analytical models for a post-investment (ACTIVE) collar and a pre-investment (INVEST) collar, so the costs and benefits to the CON and GOV can be clearly identified, initially and as the parameter values evolve over time. Also, it is easy to see what initial parameter values the CON and GOV are likely to over (under) estimate or emphasize, and what basic incentives are evident for the two parties to a PPP arrangement. The basic game theory applicable to a principal and an agent is that the incentives for the agent should be allied to the objectives of the principal, and that the principal monitors periodically the performance of the agent to see whether those objectives are being met.

This paper is organized in the following way. The fundamental investment opportunity model (without a collar) is reproduced to act as a benchmark for comparing the qualities of the with-collar model. We then proceed to formulate the with-collar model analytically and examine its key properties. This requires developing the collar representation for an active project and incorporating its features within an investment opportunity model. In section 4, further insights are gained from performing a numerical sensitivity analysis. Section 5 presents some of the more interesting aspects of "who wins, who loses, why" between the CON and GOV as parameter values change. The versatility of the analytical representation is demonstrated in section 6 through extensions to two additional complex extensions. The last section is a conclusion.

2 Fundamental Model

For a firm in a monopolistic situation confronting a single source of uncertainty due to output price[1] variability, and ignoring operating costs and taxes, the opportunity to invest in an irretrievable project at cost K depends solely on the price evolution, which is specified by the geometric Brownian motion process:

$$\mathrm{d}P = \alpha P \mathrm{d}t + \sigma P \mathrm{d}W \tag{1}$$

where α denotes the expected price risk-neutral drift, σ the price volatility, and $\mathrm{d}W$ an increment of the standard Wiener process. Using contingent claims analysis, the option to invest in the project $F(P)$ follows the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 F}{\partial P^2} + (r-\delta) P \frac{\partial F}{\partial P} - rF = 0 \tag{2}$$

where $r > \alpha$ denotes the risk-free interest rate and $\delta = r - \alpha$ the rate of return shortfall. The generic solution to (2) is:

$$F(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} \tag{3}$$

where A_1, A_2 are to be determined generic constants and β_1, β_2 are, respectively, the positive and negative roots of the fundamental equation, which are given by:

$$\beta_1, \beta_2 = \left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right) \pm \sqrt{\left(\tfrac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{4}$$

In (3), if $A_2 = 0$ then F is a continuously increasing function of P and represents an American perpetual call option, Samuelson (1965), while if $A_1 = 0$ then F is a decreasing function and represents a put option, Merton (1973), Merton (1990) and Alvarez (1999).

In the absence of other forms of optionality and a fixed output volume Y, a firm optimally invests when the value matching relationship linking the call option value and the net proceeds $PY/\delta - K$ holds:

$$A_0 P^{\beta_1} = PY/\delta - K\,. \tag{5}$$

[1] This model can easily be altered to involve quantity (Y) uncertainty, for toll roads with stochastic traffic and tolls, where R=X=P*Y, as in CON vs. GOV and Case A and B.

Following standard methods, the without-collar optimal price threshold level triggering investment $\hat{P}_0$ is:

$$\hat{P}_0 = \frac{\beta_1}{\beta_1 - 1}\frac{\delta}{Y}K \ , \tag{6}$$

and the value function is:

$$F_0(P) = \begin{cases} = \dfrac{K}{\beta_1 - 1}\left(\dfrac{P}{\hat{P}_0}\right)^{\beta_1} & \text{for } P < \hat{P}_0 \\ = \dfrac{PY}{\delta} - K & \text{for } P \geq \hat{P}_0, \end{cases} \tag{7}$$

with:

$$A_0 = \frac{\hat{P}_0^{1-\beta_1}Y}{\delta\beta_1} = \frac{K\hat{P}_0^{-\beta_1}}{\beta_1 - 1}. \tag{8}$$

3 Investment and Collar Option

3.1 Real Collar Option for an ACTIVE Project

A collar option is designed to confine the output price for an active project to a tailored range, by restricting its value to lie between a floor P_L and a cap P_H. Whenever the price trajectory falls below the floor, the received output price is assigned the value P_L, and whenever it exceeds the cap, it is assigned the value P_H. By restricting the price to this range, the firm benefits from receiving a price that never falls below P_L and obtains protection against adverse price movements, whilst at the same time, it is being forced never to receive a price exceeding P_H to sacrifice the upside potential. Protection against downside losses are mitigated in part by sacrificing upside gains. If a government offers a firm a price collar in its provision of some output Y, the government compensates the firm by a positive amount equalling $(P_L - P)Y$ whenever $P < P_L$, but if the cap is breached and $P > P_H$, then the firm reimburses the government by the positive amount $(P - P_H)Y$. It follows that for an active project, the revenue accruing to the firm is given by $\pi_C(P) = \min\{\max\{P_L, P\} P_H\} \times Y$ and its value V_C is described by the risk-neutral valuation relationship:

$$\tfrac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_C}{\partial P^2} + (r-\delta) P \frac{\partial V_C}{\partial P} - rV_C + \pi_C(P) = 0 . \qquad (9)$$

The relationship (9) and (2) are identical in form except for the revenue function.

The valuation of a with-collar active project is conceived over three mutually exclusive exhaustive regimes, I, II and III, specified on the P line, each with its own distinct valuation function. Regimes I, II and III are defined by $P \le P_L$, $P_L < P \le P_H$ and $P_H \le P$, respectively. Over Regime I, the firm is granted a more attractive fixed price P_L compared with the variable price P, but also possesses a call-style option to switch to the more favourable Regime II as soon as P exceeds P_L. This switch option increases in value with P and has the generic form AP^{β_1}, where A denotes a to be determined generic coefficient. Over Regime III, the firm is not only obliged to accept the less attractive fixed price P_H instead of P but also has to sell a put-style option to switch to the less favourable Regime II as soon as P falls below P_H. This switch option decreases in value with P and has the generic form AP^{β_2}. Over Regime II, the firm receives the variable price P, possesses a put-style option to switch to the more favourable Regime I as soon as P falls to P_L, but sells a call-style option to switch to the less favourable Regime III as soon as P attains P_H. The various switch options are displayed in Table 1, where A denotes a generic coefficient.

Table 1: The Various Switch Options

From – To	**Option Type**	**Value**	**Sign of A**
I – II	Call	AP^{β_1}	+
II – I	Put	AP^{β_2}	+
II – III	Call	AP^{β_1}	-
III – II	Put	AP^{β_2}	-

If the subscript C denotes the with-collar arrangement, then after paying the investment cost, the valuation function for the firm managing the ACTIVE project is formulated as:

$$V_C(P) = \begin{cases} \dfrac{P_L Y}{r} + A_{C11} P^{\beta_1} & \text{for } P < P_L \\ \dfrac{PY}{\delta} + A_{C21} P^{\beta_1} + A_{C22} P^{\beta_2} & \text{for } P_L \le P < P_H \\ \dfrac{P_H Y}{r} + A_{C32} P^{\beta_2} & \text{for } P_H \le P. \end{cases} \tag{10}$$

In (10), a coefficient's first numerical subscript denotes the regime $\{1 = I, 2 = II, 3 = III\}$, while the second denotes a call if 1 or a put if 2. The coefficients A_{C11}, A_{C22} are expected to be positive because the firm owns the options and a switch is beneficial. In contrast, the A_{C21}, A_{C32} are expected to be negative because the firm is writing the options and is being penalized by the switch. The real collar is composed of a pair of both call and put options. The first pair facilitates switching back and forth between Regime I and II, which results in an advantage for the concessionaire, while the second pair facilitates switching back and forth between Regime II and III, which results in a disadvantage for the firm. The real collar design differs from the typical European collar, which only involves buying and selling two distinct options.

A switch in either direction between Regime I and II occurs when $P = P_L$. It is optimal provided the value-matching relationship:

$$\frac{P_L Y}{r} + A_{C11} P^{\beta_1} = \frac{PY}{\delta} + A_{C21} P^{\beta_1} + A_{C22} P^{\beta_2} \tag{11}$$

and its smooth-pasting condition expressed as:

$$\beta_1 A_{C11} P^{\beta_1} = \frac{PY}{\delta} + \beta_1 A_{C21} P^{\beta_1} + \beta_2 A_{C22} P^{\beta_2} \tag{12}$$

both hold when evaluated at $P = P_L$. Similarly, a switch in either direction between Regime II and III occurs when $P = P_H$. It is optimal provided the value-matching relationship:

$$\frac{PY}{\delta} + A_{C21} P^{\beta_1} + A_{C22} P^{\beta_2} = \frac{P_H Y}{r} + A_{C32} P^{\beta_2} \tag{13}$$

and its smooth-pasting condition expressed as:

$$\frac{PY}{\delta} + \beta_1 A_{C21} P^{\beta_1} + \beta_2 A_{C22} P^{\beta_2} = \beta_2 A_{C32} P^{\beta_2} \tag{14}$$

both hold when evaluated at $P = P_H$. A novel expression for the option coefficients is:

$$A_{C11} = \left[\frac{P_H Y}{P_H^{\beta_1}} - \frac{P_L Y}{P_L^{\beta_1}}\right] \times \frac{\left(r\beta_2 - r - \delta\beta_2\right)}{\left(\beta_1 - \beta_2\right) r\delta} > 0,\ A_{C21} = \frac{P_H Y\left(r\beta_2 - r - \delta\beta_2\right)}{P_H^{\beta_1}\left(\beta_1 - \beta_2\right) r\delta} < 0,$$
$$A_{C22} = \frac{-P_L Y\left(r\beta_1 - r - \delta\beta_1\right)}{P_L^{\beta_2}\left(\beta_1 - \beta_2\right) r\delta} > 0,\ A_{C32} = \left[\frac{P_H Y}{P_H^{\beta_2}} - \frac{P_L Y}{P_L^{\beta_2}}\right] \times \frac{\left(r\beta_1 - r - \delta\beta_1\right)}{\left(\beta_1 - \beta_2\right) r\delta} < 0. \tag{15}$$

The signs of the four option coefficients comply with expectations. Other findings can also be derived. The coefficient A_{C22} for the option to switch from Regime II to I, which depends on only P_L and not on P_H, increases in size with P_L. This switch option becomes more valuable to the firm as the floor level increases. Similarly, the coefficient A_{C21} for the option to switch from Regime II to III, which depends on only P_H and not on P_L, decreases in magnitude with P_H. This switch option becomes less valuable to the government as the cap level increases. The coefficients A_{C11} and A_{C32} for the switch option from Regime I to II and from Regime III to II, respectively, depend on both P_L and P_H.

3.2 Investment Option

We conjecture that the with-collar optimal price threshold $\hat{P}_C$ triggering an investment lies between the floor and cap limits, $P_L \le \hat{P}_C \le P_H$. The floor limit holds because no optimal solution exists in its absence, that is for $\hat{P}_C < P_L$. We subsequently demonstrate that $\hat{P}_C$ attains a minimum of $P_L = rK/Y$ and a maximum of $\hat{P}_0$ for $P_L = 0$, so the introduction of a price floor always produces at least an hastening of the investment exercise and never its postponement. The cap limit holds because of the absence of any effective economic benefit from exercising at a price exceeding the cap. Initially the price can be presumed to be near zero and the investment option treated as out-of-the-money. With the passage of time, the price trajectory can be expected to reach the cap P_H before reaching some level exceeding P_H, and since the value outcome $P_H Y/r$ is the same for both $P = P_H$ and $P > P_H$, there is no gain in waiting. The following analysis treats the threshold $\hat{P}_C$ as lying between the lower and upper limits.

When $P_L \le \hat{P}_C \le P_H$, the optimal solution is obtained from equating the investment option value with the active project net value at the threshold $P = \hat{P}_C$. The optimal solution is determined from both the value-matching relationship as in Clark and Easaw (2007):

$$A_{C0}P^{\beta_1} = \frac{PY}{\delta} + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} - K \tag{16}$$

and its smooth-pasting condition expressed as:

$$\beta_1 A_{C0}P^{\beta_1} = \frac{PY}{\delta} + \beta_1 A_{C21}P^{\beta_1} + \beta_2 A_{C22}P^{\beta_2} \tag{17}$$

when evaluated for $P = \hat{P}_C$. This reveals that:

$$\frac{\hat{P}_C Y}{\delta} = \frac{\beta_1}{\beta_1 - 1}K - \frac{\beta_1 - \beta_2}{\beta_1 - 1}A_{C22}\hat{P}_C^{\beta_2} \tag{18}$$

$$\begin{aligned} A_{C0} &= \frac{K\hat{P}_C^{-\beta_1}}{\beta_1 - 1} - \left(\frac{1-\beta_2}{\beta_1 - 1}\right)A_{C22}\hat{P}_C^{\beta_2 - \beta_1} + A_{C21} \\ &= \frac{1}{\beta_1 - \beta_2}\left[\left(1-\beta_2\right)\frac{\hat{P}_C Y}{\delta} + \beta_2 K\right]\hat{P}_C^{-\beta_1} + A_{C21}. \end{aligned} \tag{19}$$

The absence of a closed-form solution requires $\hat{P}_C$ to be solved numerically from (18), and A_{C0} from (19). The investment option value INVEST $F_{C0}\left(P\right)$ for the project is:

$$F_{C0}\left(P\right) = \begin{cases} A_{C0}P^{\beta_1} & \text{for } P < \hat{P}_C \\ \dfrac{PY}{\delta} - K + A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2} & \text{for } \hat{P}_C \le P < P_H, \end{cases} \tag{20}$$

where $P_L \le \hat{P}_C \le P_H$.

From (18), the threshold $\hat{P}_C$ depends only on the floor P_L through A_{C22}, but not on the cap P_H. Adjusting the cap of the collar has no material impact on the threshold, so the timing decision is affected by the losses foregone by having a floor but not by the gains sacrificed by having a cap. Since A_{C22} is non-negative, the with-collar threshold $\hat{P}_C$ is always no greater than the without-collar threshold $\hat{P}_0$, and an increase in the floor produces an earlier exercise due to the reduced threshold level. However, the floor cannot increase without bound and consequently the with-collar threshold has a lower limit. In (18), if $P_L = 0$, then $A_{C22} = 0$ and $\hat{P}_C = \hat{P}_0$, the optimal investment threshold without a floor. Further, if $\hat{P}_C = P_L$, then $P_L Y = rK$ and consequently the investment threshold equals the zero NPV (Net Present Value) solution, since the project remains being viable whatever price trajectory emerges subsequent to exercise due to the presence of the floor level. It follows that the corresponding bounds for the optimal investment trigger level $\hat{P}_C$ and the price floor level P_L are $\left(P_L, \hat{P}_0\right)$ and $\left(0, rK/Y\right)$, respectively, and that $\hat{P}_C$ is a decreasing function of P_L.

An investment opportunity with a collar having only a floor is always more valuable than one without, and this value increases as the floor becomes increasingly more generous. We show this by establishing that the investment option coefficient A_{C0} with $A_{C21} = 0$, (19), is always at least greater than A_0, (8), and that A_{C0} is an increasing function of $\hat{P}_C$. Since $\hat{P}_0 \geq \hat{P}_C$ then from (19):

$$A_{C0} \geq \frac{1}{\beta_1 - \beta_2}\left[\left(1-\beta_2\right)\frac{\hat{P}_0 Y}{\delta} + \beta_2 K\right]\hat{P}_0^{-\beta_1} = \frac{K\hat{P}_0^{-\beta_1}}{\beta_1 - 1}$$

In the absence of a cap, having a floor is always at least as valuable as not having a floor. Further, by differentiating (19) with respect to $\hat{P}_C$, A_{C0} is an increasing function of $\hat{P}_C$. However, if a collar contains both a floor and a cap, then the sign and magnitude of the switch option coefficient A_{C21} have to be taken into account. This coefficient is negative and its magnitude decreases towards zero as P_H becomes increasingly large, so the negative effect of a cap on A_{C0} is strongest and most significant for relatively low P_H levels. This means that for sufficiently low P_H levels, $A_{C0} < A_0$ implying that an investment opportunity without a collar is more valuable than one with a collar despite the latter having a lower investment threshold and an earlier exercise time.

3.3 Floor and Cap Options

The analogous results, investment threshold and investment option value, for the floor only and the cap only are reproduced in the Appendix.

4 Numerical Illustrations

Although the analytical results reveal some interesting properties, further insights into model behaviour are obtainable from numerical evaluations. The base case parameter values are K=100, Y=1, σ=25%, r=4%, and δ=4%. The evaluated power parameters for these values are $\beta_1 = 1.7369$ and $\beta_2 = -0.7369$ from (4), with $\hat{P}_0 = 9.4279$ and $A_0 = 2.7547$, from (6) and (8), respectively. In this section, we consider first the behaviour of the switch options for the collar model before proceeding to the properties of the investment threshold and option value of the investment opportunity for the collar model, and ending with an investigation of changing volatility on the investment decision.

4.1 Collar ACTIVE Switch Options

Using the base case parameter values, we illustrate in Table 2 the evaluated switch option coefficients, $A_{C11}, A_{C21}, A_{C22}, A_{C32}$ in Panels A-D, respectively, for various floor and cap levels. The floor levels are chosen to vary between a minimum $P_L = 0$ and a maximum $P_L = rK/Y = 4$, and the cap levels between a minimum $P_H = 10$, slightly in excess of $\hat{P}_0$, and a maximum $P_H = \infty$. As expected, all the four coefficients adopt the correct sign, A_{C21} is independent of P_L and A_{C22} of P_H, while A_{C11}, A_{C32} depend on both. Further, A_{C11}, the coefficient for the option to switch from Regime I to II, decreases with P_L but increases with P_H, since for any feasible Regime I price level, the switch option is more valuable for lower P_L levels because of the time value of money and that the price level is closer to P_L, and for higher P_H levels because less is being sacrificed. Similarly, A_{C32}, the negative coefficient for the option to switch from Regime III to II increases in magnitude with P_H because of the time value of money and decreases with P_L because less is being sacrificed. Finally, A_{C21}, the negative coefficient for the option to switch from Regime II to III decreases in magnitude with P_H because less is being sacrificed at higher P_H levels, while A_{C22}, the coefficient for the option to switch from Regime II to I increases with P_L because more is being gained for higher P_L levels. Note that the coefficients for the price floor are also available from Table 2 in the rows where $P_H = \infty$, while those for the price cap model are available from the columns where $P_L = 0$.

****Table 2 about here****

The switch option value $F_C(P)$ is derived from the active asset value $V_C(P)$, (10) (15):

$$F_C(P)=\begin{cases} A_{C11}P^{\beta_1} & \text{for } P<P_L \\ A_{C21}P^{\beta_1}+A_{C22}P^{\beta_2} & \text{for } P_L\le P<P_H \\ A_{C32}P^{\beta_2} & \text{for } P_H\le P. \end{cases} \tag{21}$$

The difference between V_C, (10), and F_C, (21), is the present value in the absence of any optionality. Since V_C-F_C differs for each of the 3 regimes, F_C would normally experience a discontinuity jump at $P=P_L$ and $P=P_H$. However, in our case, since r and δ are selected to be equal, the discontinuity jumps are absent. Figure 1 illustrates the effect of P_L and P_H variations on $F_C(P)$ for constant P_H and P_L, respectively. These profiles tend to follow a similar pattern, being positive for P values around P_L where the owned option to switch between Regime I and II dominates, and negative around P_H where the sold option to switch between Regime II and III dominates. In Figure 1 where P_H is held constant, a P_L increase shifts the profile upwards for $P>P_L$ that reflects the enhanced switch option value due to the gain in downside protection.

**** Figure 1 about here****

4.2 Investment Option

Using base case values, the investment threshold and investment value option coefficient solutions for variations in P_L and P_H, where P_L varies between 0 and rK/Y and P_H between 10 and infinity are illustrated in Table 3. Panel A exhibits the threshold $\hat{P}_C$, (18), and Panel B the option coefficient A_{C0} from (19). As expected, the threshold declines as P_L increases within its allowable range, showing that an earlier exercise is achievable only for improvements in the floor. The locus relating the threshold $\hat{P}_C$ with the floor P_L defined by (18) is illustrated in Figure 2, which reveals not only the feasible limits of $\hat{P}_C$ and P_L, but also their negative relationship. In contrast, the choice of cap P_H has no effect on the threshold and the timing decision. In Panel B of Table 3, the option coefficient is observed to move in line with positive changes in P_L or P_H. A P_L increase raises the extent of the downside protection thereby making the investment option more attractive, while a P_H increase reduces the extent of the upside sacrifice thereby making it more valuable. In Table 3, the results for the floor model are obtainable from the row where $P_H = \infty$, and for the cap model from the column where $P_L = 0$

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****Table 3 and Figure 2 about here****

The relationship between the before and after exercise investment value, with and without a collar, and price is illustrated in Figure 3. We select the collar levels as $P_L = 4.0$ and $P_H = 20.0$, which yield a threshold of $\hat{P}_C = 4.000$ and option coefficient $A_{C0} = 2.5270$. Despite having a higher threshold level, which suggests an earlier exercise for the collar variant if exercised, the collarless variant is always more favourable for the concessionaire by having a greater option coefficient.

****Figure 3 about here****

The cost of the subsidy can be neutralized and the collar made "costless" by suitably engineering its floor and cap levels. For the ACTIVE concessionaire, or for an investor owning an ACTIVE project, a "costless collar" might be obtained from a third party equating the written call and protective put $A_{C21}P^{\beta_1} + A_{C22}P^{\beta_2}$ for $P_L \leq P < P_H$. For instance, for base case parameter values when P=6, P_L=4, P_H=15.6, $-A_{C21}P^{\beta_1} = A_{C22}P^{\beta_2}$.

For the INVEST opportunity, a "costless collar " might be designed in the following way: (i) the without-collar option coefficient A_0 is evaluated from (8), (ii) for some pre-specified value of the collar threshold $\hat{P}_C$, perhaps equalling the prevailing price, the implied floor P_L can be determined from (18) (19) because of its invariance with P_H, and finally (iii), by setting $A_{C0} = A_0$ the implied cap is determined from (19). Some illustrative "costless" P_L and P_H pairs are presented in Table 4. The pairs are inversely related, as expected, since for the collar to remain "costless", any increase in the floor and reduction in downside risk has to be compensated by an additional sacrifice in upside potential.

****Table 4 about here****

In the presence of a stochastic output price, a collar option can be designed that protects the investor from downside risk by limiting adverse prices to a floor while simultaneously compelling the investor to forego favourable prices above a cap. This trade-off between upside potential and downside risk can be engineered to make the collar-variant to be more valuable as well as supporting an earlier exercise. The floor and cap affect the solution in distinct ways. Variations in P_H have no effect at all on the investment threshold, but the sacrifice of additional upside potential is reflected in decreases in the investment option coefficient. In contrast, an improvement in P_L and reduction in downside risk produces both a fall in investment threshold prompting an earlier exercise and a rise in the investment option coefficient making it more valuable. When designing a collar, initial attention focuses on the floor in determining the threshold for ensuring the investment has a timely exercise, and then on the cap in assessing the extent of the value created by the floor is to be sacrificed. While a viable floor increase for a collar motivates early exercise as well as enhancing its attractiveness, a cap decrease incurs a sacrifice leading to a reduction in its attractiveness.

4.2.1 Changes in Volatility

In the absence of a collar, a volatility increase is known to accompany a rise in both the investment threshold and investment option value, Dixit and Pindyck (1994). By using base case values except that the volatility σ varies incrementally up to a maximum of 50%, we compare the impact of volatility changes on the with- and without-collar solutions for $P_L = 3$ and $P_H = 500$. The threshold for the without-collar variant is shown in Figure 4 to increase at a faster rate as volatility increases as expected because $P_L > 0$ and $\hat{P}_C < \hat{P}_0$, so the with-collar variant possesses a lower threshold and an earlier timing for all positive σ.

The comparative timing decisions for the with- and without-collar variants remain essentially unaltered in the presence of a volatility change, because if $P_L > 0$ then $\hat{P}_C < \hat{P}_0$ while if $P_L = 0$ then $\hat{P}_C = \hat{P}_0$ for all positive σ. However, a volatility increase can produce a distinctive change in the with-collar option value, which can result in a change of the more preferred variant. If for low σ, the chance of a price trajectory penetrating the cap is insignificant, then the magnitude of the switch option coefficient A_{C21} is similarly insignificant and consequently the option coefficient is virtually unaffected. However, as σ increases, the chance of penetrating the cap becomes increasingly significant and likewise the coefficient A_{C21}, with the consequence that increases in the with-collar option coefficient begin to retard and falter enabling the without-collar option coefficient to assume dominance. In the design of a collar, if a concessionaire perceives a likely volatility increase to be imminent, then the cap has to be adjusted upwards to ensure its acceptance by the investor community.

**** Figure 4 about here ****

5 Who Wins, Who Loses, Why?

In the principal-agent problem (GOV-CON) and risk-sharing aspects of collar and floor or ceiling only arrangements, who wins, and who loses, as parameter values change are likely to be the indicators for CON versus GOV incentives after the initial transaction. For an ACTIVE project post-investment with various PPP arrangements, it is assumed that the CON pays the "fair value" of the concession to the GOV initially. In the base case, we also assume that the GOV has offered the CON a "costless" real collar arrangement, where the value of a CALL written by the CON to the GOV on upside revenues higher than the ceiling R_H and a PUT written by the GOV to the CON on downside revenues lower than a floor R_L are equal and $R_L<R<R_H$. Both the CON and the GOV agree on the initial parameter values. The effect of changes in the parameter values can be divided generally into *zero-sum games* (where the CON gain/loss is equal to the GOV loss/gain, so that the CON plus GOV profit is zero), *constant-sum games*, and *variable-sum games* (where the CON gain/loss less the GOV loss/gain varies). Changes in revenue volatility, interest rates, floor level, and ceiling level are *zero-sum games*, while changes in revenue and yield are *variable-sum games*, that is both CON and GOV benefit or both lose as the parameter values change, but not always by the same magnitude.

ACTIVE

We show here six examples of CON versus GOV results, as parameter values change (see a complete description of these and other results in the Supplementary Appendix A (ACTIVE) and B (INVEST). In Figure 5, from an initial "costless" collar if R_L=4 and R_H =15.6 and R=6, if R increases, both CON and GOV benefit in a *variable-sum game*, but on the upside the CON benefit less than the GOV due to the negative CALL increasing more than the PUT. But, the CON loses less than the GOV on the downside due to the minimum revenue guarantee.

**** Figure 5 about here ****

In Figure 6, from an initial volatility of 25%, if volatility increases, in a *zero-sum game* (where the benefits and costs change, but remain equal to each other) the CON suffers from the negative CALL increasing more than the PUT, and GOV benefits. If volatility decreases from 25%, CON benefits from the negative CALL decreasing less than the PUT, but when the volatility is close to zero, neither is of any value, thereby reverting to a costless collar.
**** Figure 6 about here ****

Since the GOV profit is increased when volatility increases past 25%, the GOV should welcome R volatility increases, and the CON strive for decreased volatility, perhaps through dynamic pricing or issuing debt instruments tied to revenue or traffic levels, or through hedging if possible.

In Figure 7, from an initial interest rate of 4%, if the interest rate decreases, CON benefits from the negative CALL decreasing and the PUT increasing significantly. If the interest rate increases from 4%, CON suffers from the negative CALL increasing and the PUT decreasing , and GOV experiences the opposite effect. So the CON might seek to protect herself from interest rate increases by entering into fixed rate loans to fund infrastructure investments, but with prepayment conditions which allow refinancing if interest rates decrease.
**** Figure 7 about here ** **

In Figure 8, changes in the asset yield result in a highly *variable-sum game* for the CON and GOV. From an initial yield of 4%, if the yield δ decreases, CON benefits from the PV (R/δ) increasing less the negative CALL increasing (which benefits the GOV). If the yield increases from 4%, CON benefits from the negative CALL decreasing and the PUT increasing , offset by the PV decreasing. GOV suffers from the negative CALL decreasing. The so-called asset yield, dividend, or convenience yield, or return "shortfall" is a difficult concept to interpret in most applications, illustrated in this case. The GOV might seek to benefit protect herself by restricting the payouts of the CON, or by hedging using a term structure of revenue futures, but since the revenue is probably not a traded security, it is hard to imagine how GOV could realize this benefit practically.
**** Figure 8 about here ** **

It is interesting to compare collar arrangements with different floors and ceilings, and with floor only or ceiling only arrangements. Figure 9 shows the risk sharing collar arrangements between CON and GOV as a function of different levels of the floor. It is natural that the CON benefits (and the GOV suffers) from higher floors, in a *zero-sum game*.

**** Figures 9 and 10 about here ** **

Equally dramatic in a *zero-sum game* is the effect of changes in revenue volatility on risk sharing when there is only a floor, or alternatively only a ceiling. Figure 10 shows the risk allocation with a floor only as a function of R volatility, but with the CON benefit increasing with volatility up to a certain point (about 45%), when thereafter volatility increases result in a decline of the CON benefit.

6 Two Additional Cases

We now consider two illustrative cases, case-A and case-B, to investigate whether the findings for the plain vanilla collar formulation concerning the nature of the investment threshold and option coefficient extend to more complicated collar designs. In case-A, we increase the number of regimes by 1 and formulate the shared revenue for the outer regimes of the collar to depend on a proportion of the revenue and not on a constant as for the vanilla version. Our findings for this revision demonstrate that an analytical solution is obtainable despite the increase in complexity. Some of the sensitivities to changes in parameter values are similar to the previous collar model, but some are surprising. Similarly, the number of regimes for case-B, is also increased by 1, but there is also the possibility of giving the investor a "sell-out" or exit option. This revision does produce a notable change in the resulting solution compared with the plain vanilla findings, which is due to the altered switch option structure. The notation we use in section 6.1 and 6.2 are specific to each of those 2 sections, except that β_1 and β_2 are specified by (4).

6.1 Case-A Partial Put and Partial Call

Shaoul et al. (2012) report that for a U.K. rail franchise agreement, investors are reimbursed for 50% of any revenue shortfall below 98% of forecast and 80% below 96%, but suffer a claw-back of 50% of revenue exceeding 102%, equivalent to partial puts and calls. In case-A, we amend this arrangement as follows. The actual revenue generated from operating the franchise through making an irrecoverable investment with cost K is denoted by X. For the purpose of determining the revenue to be received by the investor, the agreement with the government divides the revenue schedule into 4 distinct exhaustive regimes. The 3 junctions for neighbouring regimes occur at $X = X_{LL}$, where X_{LL} represents the lowest limit, at $X = X_L$

where X_L is the lower limit, and at $X = X_U$ where X_U is the upper limit, with $X_{LL} \le X_L < X_U$. Under Regime I with $X < X_{LL}$, the "revenue received" by the concessionaire is the actual revenue X plus a proportion $1-w_{LL}$ of the shortfall below forecast, under Regime II with $X_{LL} \le X < X_L$, revenue received is X plus a proportion $1-w_L$ of the shortfall below forecast, where $0 \le w_{LL} \le w_L \le 1$, under Regime III with $X_L \le X < X_U$, revenue received is X, and under Regime IV with $X_U \le X$, the revenue received is X less a proportion $1-w_U$ of the excess over forecast where $0 \le w_U \le 1$. In the absence of any fixed costs and taxation, the regime value is determined not only from the revenue schedule but also from the presence of any switch options.

For each regime, if there exist opportunities for switching to an upper or lower neighbouring regime, then these are represented by options, a call-style option for upward switching and a put-style option for downward switching, so both Regime II and III are characterized by both call and put options, while Regime I by a call and Regime IV by a put. Also, a switch producing a revenue advantage is represented by a positive option value coefficient, while that for a revenue disadvantage by a negative coefficient. The specification and associated revenue values for each of the four regimes are listed in Table 5.

Table 5

Regime Specification and Revenue Schedule for Case-A

Regime	**Specification**	**Value**
I	$X < X_{LL}$	$V_I(X) = A_{11}X^{\beta_1} + \frac{w_{LL}X}{\delta} + \frac{(1-w_L)X_L}{r} + \frac{(w_L - w_{LL})X_{LL}}{r}$
II	$X_{LL} \le X < X_L$	$V_{II}(X) = A_{21}X^{\beta_1} + A_{22}X^{\beta_2} + \frac{w_L X}{\delta} + \frac{(1-w_L)X_L}{r}$
III	$X_L \le X < X_U$	$V_{III}(X) = A_{31}X^{\beta_1} + A_{32}X^{\beta_2} + \frac{X}{\delta}$

IV	$X_U \le X$	$V_{IV}(X) = A_{42}X^{\beta_2} + \dfrac{w_U X}{\delta} + \dfrac{(1-w_U)X_L}{r}$

The six unknown switch option coefficients, $A_{11}, A_{21}, A_{22}, A_{31}, A_{32}, A_{42}$, are determined from the value matching relationships and associated smooth pasting conditions. The value matching relationships, defined at each of the 3 junctions of neighbouring regimes are, respectively:

$$\left[V_{II}(X) - V_I(X)\right]\Big|_{X=X_{LL}} = 0 \tag{22}$$

$$\left[V_{III}(X) - V_{II}(X)\right]\Big|_{X=X_L} = 0 \tag{23}$$

$$\left[V_{IV}(X) - V_{III}(X)\right]\Big|_{X=X_U} = 0 \tag{24}$$

Equations (22)-(24) together with the 3 associated smooth pasting conditions are sufficient to solve for the unknowns. The resulting solutions together with their signs are presented in Table 6 in their order of calculation. The coefficients having a positive value indicate that the corresponding switch options are owned by the investor and contribute to their investment value, whilst those having a negative sign are sold and detract from the investment value.

Table 6

Case-A Solutions and Conditions for the Switch Option Coefficients Partial Collar Model

Coefficient Solution	**Condition**
$A_{22} = \dfrac{(w_L - w_{LL})X_{LL}\left(r(1-\beta_1)+\delta\beta_1\right)}{r(\beta_1-\beta_2)\delta X_{LL}^{\beta_2}}$	$A_{22} \ge 0$
$A_{31} = -\dfrac{(1-w_U)X_U\left(r(1-\beta_2)+\delta\beta_2\right)}{r(\beta_1-\beta_2)\delta X_U^{\beta_1}}$	$A_{31} \le 0$
$A_{21} = A_{31} + \dfrac{(1-w_L)X_L\left(r(1-\beta_2)+\delta\beta_2\right)}{r(\beta_1-\beta_2)\delta X_L^{\beta_1}}$	$A_{21} \ge 0$
$A_{32} = A_{22} + \dfrac{(1-w_L)X_L\left(r(1-\beta_1)+\delta\beta_1\right)}{r(\beta_1-\beta_2)\delta X_L^{\beta_2}}$	$A_{32} \ge 0$
$A_{11} = A_{21} + A_{22}\dfrac{X_{LL}^{\beta_2}}{X_{LL}^{\beta_1}} + \dfrac{(w_L - w_{LL})X_{LL}(r-\delta)}{r\delta X_{LL}^{\beta_1}}$	$A_{11} \ge 0$

$A_{42} = A_{31}\dfrac{X_U^{\beta_1}}{X_U^{\beta_2}} + A_{32} + \dfrac{(1-w_L)X_U(r-\delta)}{r\delta X_U^{\beta_2}}$	$A_{42} \le 0$

The optimal exercise of the investment opportunity is characterized by the unknown revenue threshold denoted by $\hat{X}_0$, which is derived from the value matching relationship and optimality condition. At $X = \hat{X}_0$, the opportunity value, $A_0\hat{X}_0^{\beta_1}$ with unknown coefficient $A_0 > 0$, is sufficient to compensate the value of net revenue generated by the project, less the investment cost K, plus the values of any available switch options. For the purpose of analysis, we presume that exercise occurs for $X_L \le X < X_U$, where the revenue enjoys its greatest incremental rate. The value matching relationship is:

$$A_0\hat{X}_0^{\beta_1} = \frac{\hat{X}_0}{\delta} - K + A_{31}\hat{X}_0^{\beta_1} + A_{32}\hat{X}_0^{\beta_2} \tag{25}$$

Due to the similarity between (16) and (25), it is straightforward to deduce that $\hat{X}_0$ and $A_0 > 0$ are given by, respectively:

$$\frac{\hat{X}_0}{\delta} = \frac{\beta_1}{\beta_1 - 1}K - \frac{\beta_1 - \beta_2}{\beta_1 - 1}A_{32}\hat{X}_0^{\beta_2} \tag{26}$$

$$\begin{aligned} A_0 &= \frac{K\hat{X}_0^{-\beta_1}}{\beta_1 - \beta_2} - \left(\frac{1-\beta_2}{\beta_1 - 1}\right)A_{32}\hat{X}_0^{\beta_2-\beta_1} + A_{31} \\ &= \frac{1}{\beta_1 - \beta_2}\left[(1-\beta_2)\frac{\hat{X}_0}{\delta} + \beta_2 K\right]\hat{X}_0^{-\beta_1} + A_{31}. \end{aligned} \tag{27}$$

Equations (26) and (27) reveal that while the investment threshold $\hat{X}_0$ depends only on A_{32}, the option coefficient A_0 depends on both A_{31} and A_{32}. This result echoes the findings for the plain vanilla collar formulation. The investment threshold depends on A_{32}, which depends on the floor-like attributes X_L and w_L, and on A_{22}, which also depends on the floor-like attributes X_{LL} and w_{LL}. The threshold is determined by only floor-like attributes. Similarly, the option value depends not only on A_{32} but also A_{31}, which depends on the cap-like attributes X_U and w_U. The investment option value is determined by both floor- and cap-like attributes. A systematic approach for a government in deciding suitable values for the floor- and cap-like attributes is identify the threshold level, which may be aligned to the prevailing level to ensure immediate exercise, in order to determine the floor-like attributes, and then to invoke policy for identifying a subsidy level, which defined by the difference between the without- and with-collar option values is used to determine the cap-like attributes. It is interesting to note that although these findings are based on assuming that $X_L \le \hat{X}_0 < X_U$, they also result when assuming that $X_{LL} \le \hat{X}_0 < X_L$ (for numerical illustrations see the Supplementary Appendix).

6.2 Case-B

We now turn to a more sophisticated version of the collar option also having 4 regime layers involving a differential tax structure and exit option. Its development combines some aspects of the models proposed by Rose (1998) and Takashima et al. (2010). A firm in a monopolistic situation possesses the opportunity to invest in an irretrievable project having a capital expenditure of K. The revenue generated by the active project denoted by X is described by a geometric Brownian motion process having identical parameter values as before. Out of revenue is paid a constant fixed cost f yielding after-tax net revenue of $(X-f)(1-\tau_0)$, where τ_0 is the relevant corporate tax rate. The firm negotiates a contractual agreement with the government, which offers the firm protection against adverse revenue movements but at the risk that favourable movements incur higher tax rates. If an adverse movement produces a net revenue loss $(X-f)<0$, initially the government then reimburses the firm for the difference so the net revenue is specified by $\max\{X-f,0\}$. For subsequent adverse movements, the firm has the right to dispose of the project asset to the government for the amount K_D, where $K_D < K$. Optimal disposal occurs as soon as revenue falls to the exit threshold, $\hat{X}_D$ where $\hat{X}_D < f$. In contrast, if the movement is favourable, then the project attracts a higher tax rate $\tau_1 > \tau_0$ for revenues exceeding some pre-specified upper limit X_U. For subsequent favourable movements where $X \ge X_{UU}$, with $X_{UU} > X_U$, the revenue is capped at the pre-specified top upper limit X_{UU}.

There are four identifiable distinct and exhaustive regimes for this collar arrangement, defined over $(\hat{X}_D, \infty]$. Regime I is specified by $\hat{X}_D \le X < f$; Regime II by the $f \le X < X_U$; Regime III by $X_U \le X < X_{UU}$; and Regime IV by $X_{UU} \le X$. For Regimes II and III, embedded options exist for switching to the neighbouring lower and upper regimes, while for Regime I, there exists options for switching to Regime II and for disposal, and for Regime IV, an option for switching to Regime III. The regime specifications and values $V_J, J = I, II, III, IV$ are reproduced in Table 7.

Table 7

Regime, Specification and Value for Case-B

Regime	**Specification**	**Value**
I	$\hat{X}_D \le X < f$	$V_I(X) = A_{11}X^{\beta_1} + A_{12}X^{\beta_2}$
II	$f \le X < X_U$	$V_{II}(X) = A_{21}X^{\beta_1} + A_{22}X^{\beta_2} + \frac{X(1-\tau_0)}{\delta} - \frac{f(1-\tau_0)}{r}$
III	$X_U \le X < X_{UU}$	$V_{III}(X) = A_{31}X^{\beta_1} + A_{32}X^{\beta_2} + \frac{X(1-\tau_1)}{\delta} - \frac{f(1-\tau_1)}{r} + \frac{(X_U - f)(\tau_1 - \tau_0)}{r}$
IV	$X_{UU} \le X < \infty$	$V_{IV}(X) = A_{42}X^{\beta_2} + \frac{(X_{UU} - f)(1-\tau_1)}{r} + \frac{(X_U - f)(\tau_1 - \tau_0)}{r}$

At the disposal junction and at each of the three junctions having neighbouring regimes, there is a value matching relationship:

$$\left[V_I(X) - K_D\right]\Big|_{X=\hat{X}_D} = 0 \tag{28}$$

$$\left[V_{II}(X) - V_I(X)\right]\Big|_{X=f} = 0 \tag{29}$$

$$\left[V_{III}(X) - V_{II}(X)\right]\Big|_{X=X_U} = 0 \tag{30}$$

$$\left[V_{IV}(X) - V_{III}(X)\right]\Big|_{X=X_{UU}} = 0 \tag{31}$$

The 4 equations (28)-(31) together with the 4 associated smooth pasting conditions are sufficient for solving the 8 unknowns $A_{11}, A_{12}, A_{21}, A_{22}, A_{31}, A_{32}, A_{42}, \hat{X}_D$ [2]. The solutions, which are evaluated in the order of presentation, are presented in Table 8 together with any conditions.

[2] Note that K_D has to be specified so that $0 \le \hat{X}_D < f$.

Table 8

Case-B Solutions and Conditions for the Switch Option Coefficients

$A_{31} = -\dfrac{X_{UU}\left(r\left(1-\beta_2\right)+\delta\beta_2\right)\left(1-\tau_1\right)}{r\left(\beta_1-\beta_2\right)\delta X_{UU}^{\beta_1}}$	≤ 0
$A_{21} = A_{31} - \dfrac{X_U\left(r\left(1-\beta_2\right)+\delta\beta_2\right)\left(\tau_1-\tau_0\right)}{r\left(\beta_1-\beta_2\right)\delta X_U^{\beta_1}}$	≤ 0
$A_{11} = A_{21} + \dfrac{f\left(r\left(1-\beta_2\right)+\delta\beta_2\right)\left(1-\tau_0\right)}{r\left(\beta_1-\beta_2\right)\delta f^{\beta_1}}$	≥ 0
$\hat{X}_D = \left(\dfrac{-\beta_2 K_D}{A_{11}\left(\beta_1-\beta_2\right)}\right)^{1/\beta_1}$	$0 \le \hat{X}_D < f$
$A_{12} = \dfrac{\beta_1 K_D}{\left(\beta_1-\beta_2\right)\hat{X}_D^{\beta_2}}$	≥ 0
$A_{22} = -\dfrac{f\left(1-\tau_0\right)}{\beta_2\delta f^{\beta_2}} + \dfrac{\beta_1\left(A_{11}-A_{21}\right)f^{\beta_1}}{\beta_2 f^{\beta_2}} + A_{12}$	≥ 0
$A_{32} = \dfrac{X_U\left(\tau_1-\tau_0\right)}{\beta_2\delta X_U^{\beta_2}} + \dfrac{\beta_1\left(A_{21}-A_{31}\right)X_U^{\beta_1}}{\beta_2 X_U^{\beta_2}} + A_{22}$	≤ 0
$A_{42} = \dfrac{X_{UU}\left(1-\tau_1\right)}{\beta_2\delta X_{UU}^{\beta_2}} + \dfrac{\beta_1 A_{31} X_{UU}^{\beta_1}}{\beta_2 X_{UU}^{\beta_2}} + A_{32}$	≤ 0

The investment threshold $\hat{X}_0$ is determined as before. At $X = \hat{X}_0$, the opportunity value $A_0\hat{X}_0^{\beta_1}$ where $A_0 \ge 0$ equals the generated net value plus any switching options. The net value depends on the relevant regime at exercise, which we presume occurs during Regime II because of its higher net revenue and lower tax rate, so the net value is given by $V_{II}\left(X\right)-K$. The value matching relationship is given by:

$$A_0\hat{X}_0^{\beta_1} = \frac{\hat{X}_0\left(1-\tau_0\right)}{\delta} - \frac{f\left(1-\tau_0\right)}{r} + A_{21}\hat{X}_0^{\beta_1} + A_{22}\hat{X}_0^{\beta_2} - K \qquad (32)$$

From (32) and its associated smooth pasting condition, the investment threshold is obtained numerically from:

$$\frac{\hat{X}_0\left(1-\tau_0\right)}{\delta}=\frac{\beta_1}{\beta_1-1}\left(K+\frac{f\left(1-\tau_0\right)}{r}\right)-\frac{\beta_1-\beta_2}{\beta_1-1}A_{22}\hat{X}_0^{\beta_2} \tag{33}$$

and the option coefficient from:

$$\begin{aligned}A_0&=\frac{\left(Kr+f\left(1-\tau_0\right)\right)\hat{X}_0^{-\beta_1}}{r\left(\beta_1-\beta_2\right)}-\left(\frac{1-\beta_2}{\beta_1-1}\right)A_{22}\hat{X}_0^{\beta_2-\beta_1}+A_{21}\\&=\frac{1}{\beta_1-\beta_2}\left[\left(1-\beta_2\right)\frac{\hat{X}_0\left(1-\tau_0\right)}{\delta}+\beta_2\frac{Kr+f\left(1-\tau_0\right)}{r}\right]\hat{X}_0^{-\beta_1}+A_{21}.\end{aligned} \tag{34}$$

In the absence of any collar arrangement, but retaining the lower tax rate, the investment threshold $\hat{X}_{00}$ and option coefficient A_{00} are given by standard theory as, respectively:

$$\frac{\hat{X}_{00}\left(1-\tau_0\right)}{\delta}=\frac{\beta_1}{r\left(\beta_1-1\right)}\left(rK+f\left(1-\tau_0\right)\right) \tag{35}$$

$$A_{00}=\frac{\hat{X}_0\left(1-\tau_0\right)}{\delta\beta_1\hat{X}_0^{\beta_1}}=\frac{Kr+f\left(1-\tau_0\right)}{r\left(\beta_1-1\right)\hat{X}_0^{\beta_1}}. \tag{36}$$

From (33) and (34), respectively, the investment threshold $\hat{X}_0$ depends on A_{22} while A_0 depends on both A_{22} and A_{21}. This result is similar to the plain vanilla collar formulation, but with an important exception. Whilst for the plain vanilla collar, A_{21} depend on the floor and cap attributes and A_{22} on floor property alone, respectively, for case-B, they both depend on both attributes. The value of A_{22} is composed of 3 components: the first depends on the fixed cost, a basis for the floor specification, the second on the difference between A_{11} and A_{21}, and third on A_{12}. The difference $A_{11}-A_{21}$ similarly depends on only the fixed cost, but A_{12} depends on $\hat{X}_D, A_{11}, A_{21}, A_{31}$, where A_{21} and A_{31} depend on X_U and X_{UU}, bases for the cap specification. The values of both the investment threshold and option coefficient are influenced by both the floor and cap attributes.

The explanation underpinning the dependence of the case-B investment threshold on both the floor and cap attributes hinges on its distinctive collar design. Unlike the plain vanilla variant which permits switching between all neighbouring regimes, there is no recourse in the case-B design to revert back to operating the active asset following its disposal. The plain vanilla and case-B designs are subtly different, a distinction causing the threshold for the latter to depend on both the floor and cap attributes (see the Supplementary Appendix).

The ACTIVE Case A and B collars have somewhat different sensitivities to changes in P and P volatility. ACTIVE Case B partial collars combined increase more or less linearly with increases in P, while Case A partial collars with the parameter values in the Supplementary Appendix, the same proportional sharing over the regimes, show a decreasing sensitivity to increases in P. The sensitivity of Case A and B to changes in P volatility ("vegas") are substantially different, with the VC partial collar A hardly changing as volatility increases, but the VC partial collar B decreases sharply with volatility increases (see the Supplementary Appendix).

The INVEST Case A and B collars have substantially different sensitivities to changes in P and P volatility. INVEST Case B partial collars combined increase with increases in P, while Case A partial collars first increase and then decrease with increases in P. The sensitivity of Case A and B to changes in P volatility are opposite, with the VC partial collar A increasing as P volatility increases, but the VC partial collar B decreases sharply with volatility increases. The implications are that a prospective concessionaire expecting volatility increases in the future would not expect to be compensated post-investment in Case A, but would for Case B, but for pre-investment combined options the concessionaire would appreciate increased volatility in the underlying P in Case A arrangements, but not for Case B schemes (see the Supplementary Appendix).

Note that the incentives for volatility management of the concessionaire who is interested in maximizing ROV pre-investment are completely different for the case A and B arrangements. The concessionaire should prefer to reduce volatility both pre-investment for Case B (increases ROV) and post-investment (ACTIVE), but not necessarily for Case A arrangements. Governments seeking early investment should prefer reduced volatility in both cases.

7 Conclusion

In a mainly analytical way, the properties of a plain vanilla collar, made up of a floor and cap, are investigated for an active asset using a real option formulation. The collar is composed of pairs of American perpetuity put and call options that confine a focal variable, such as revenue, price or volume, to a designated field specified by the floor and cap. We demonstrate that provided that the floor is positive and selected from its feasible domain, then the investment threshold for a with-collar is always less than that for a without-collar asset opportunity. Under these conditions, the with-collar investment opportunity is exercised earlier and by inducing investment, the collar acts in a similar way as an investment subsidy. Whether the with-collar opportunity should be exercised in preference to the without-collar alternative depends on the relative magnitudes of their investment option values. Although only the floor governs the investment timing, both the floor and the cap are crucial in determining the real option value of the investment opportunity, but in opposing ways. The floor provides partial if not complete protection against the downside risk of the net cash flows rendered by the with-collar asset being insufficiently viable, and any feasible increase in the floor is associated with improvements in the investment value. In contrast, the cap affects only the investment value by controlling the magnitude of the upside potential that the investor foregoes, and any cap reduction enhances this sacrifice with a consequential loss in option value.

Only the floor of the collar governs the investment threshold while both the floor and cap impact on the option value, but in opposing directions. Normally, the threshold exceeds the zero NPV level in order to moderate the extent that future net cash flows are non-viable, and greater volatility values are reflected in higher thresholds. Since a floor mitigates this extent, its presence necessarily lowers the threshold while simultaneously enhancing the opportunity value. In contrast, the cap representing a sacrifice to the investor depresses the opportunity value and reduced cap levels are reflected in lower opportunity values. Further, since a specified cap level gains in significance as the volatility increases, a with-collar variant may be preferred at lower volatilities while the without-collar variant may be preferred by the concessionaire at higher volatilities.

As a form of subsidy, the collar can be designed to clawback high profits as well as inducing early investment or even immediate investment. The role of the cap is to mitigate the cost to the government of guaranteeing a floor, and thereby inhibits the spread of any allegations of being over-generous. Governments can even create a "costless" collar by selecting a floor to induce investment and a cap that neutralizes the additional value it creates. A collar shares the benefits of a more conventional subsidy-taxation model for inducing investment with the additional merit of not having to incur an immediate subsidy payment.

We provide an analytical framework for viewing the real option value of various PPP arrangements, ranging from no collar, floor only, ceiling only, and a collar (both floor and ceiling). One use of this framework is to identify clearly the gains and losses for a principal (GOV) and agent (CON) participants in a PPP infrastructure project as parameter values change. Different real option games are envisioned, where changes in the parameter values after an initial deal result in *zero-sum games*, or *constant-sum games*, or *variable-sum games*. This basic framework may be useful in viewing the intended consequences of different appropriate PPP arrangements, and in identifying incentives for the agent in holding an investment opportunity or in operating an infrastructure facility.

The plain vanilla collar is extended in two directions. The first considers a design where the floor and cap attributes are not constant but depend on the focal variable. This demonstrates that the previous findings continue to hold, that the threshold is determined by only the floor attributes while the option value is determined by both the floor and cap attributes. This facilitates the engineering of the collar design, since adjustments to the floor attributes are first made to yield the desired threshold and then the cap attributes are adjusted to meet the desired government contribution. The second extension involves an exit option, which does not allow any return to operating the active asset following its disposal. For this design, the plain vanilla findings do not hold as both the floor and cap attributes influence the threshold and option value.

There are several implicit assumptions behind our analytical framework. (i) The arrangements are perpetual American call or put options, and a perpetual series of cash flows, viewed in continuous time. Real arrangements may not perpetual, so both the options and the cash flows would have to be reformatted as perpetuals less forward start options, or finite annuities, especially for short-term arrangements and low discount rates. This may not be a significant problem for 100 year arrangements when discount rates are high. (ii) Parameter values such as interest rates, yield, revenue volatility, revenue floors and ceilings are considered constant or deterministic. Relaxing some of these assumptions is an interesting extension. (iii) Sometimes PPP arrangements specify that the concession termination is based on a specified achieved internal rate of return, or cumulative net present value, or accumulated net cash flows. We do not focus on negotiated exit prices or for CON or GOV determined exit timing, except for Case B. (iv) PPPs are assumed to be monopolies, without competition or unexpected failures or physical disasters. (v) The framework models are viewed in continuous time whereas revenue (especially traffic), minimum and maximum revenue compensations and payments are likely to occur in discrete time. (vi) We do not allow for operating costs that are not already included in net revenues, or for periodic maintenance requirements. (vii) The revenue stream ignores other possible real options such as project cancellation, downsizing, renegotiation, expansion and resale, dynamic pricing for times of usage, and extensions into other activities such as retail activities for motorway operators. (viii) PPP arrangements are envisioned as enforceable, without credit or default risk for either party, and investments are irrevocable, immediate, and terms cannot be re-negotiated over time. (ix) While many of the PPP infrastructure arrangements cited herein concern transportation, other PPP arrangements such as building and operating hospitals and educational establishments may not have clear objectives such as sharing revenue risks and benefits. Suitably designed optional elements may incorporate some of same, or conceivably completely different objectives. Most of these issues present interesting aspects for future research.

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Appendix

Price Floor Model

We use the additional subscript f to indicate a model with only a floor. From (10) the active project valuation function becomes:

$$V_{Cf}(P) = \begin{cases} \dfrac{P_L Y}{r} + A_{Cf11} P^{\beta_1} & \text{for } P \le P_L \\ \dfrac{PY}{\delta} + A_{Cf22} P^{\beta_2} & \text{for } P_L \le P, \end{cases} \tag{A1}$$

with:

$$A_{Cf11} = \frac{-P_L Y \left(r\beta_2 - r - \delta\beta_2 \right)}{P_L^{\beta_1} \left(\beta_1 - \beta_2 \right) r\delta} \ge 0,\ A_{Cf22} = \frac{-P_L Y \left(r\beta_1 - r - \delta\beta_1 \right)}{P_L^{\beta_2} \left(\beta_1 - \beta_2 \right) r\delta} \ge 0. \tag{A2}$$

The investment option value is specified by:

$$F_{C0f}(P) = \begin{cases} A_{Cf0} P^{\beta_1} & \text{for } P \le \hat{P}_{Cf} \\ \dfrac{PY}{\delta} + A_{Cf22} P^{\beta_2} - K & \text{for } \hat{P}_{Cf} \le P, \end{cases} \tag{A3}$$

with $\hat{P}_{Cf} = \hat{P}_C$ determined from (18) with A_{C22} replaced by A_{Cf22}, and the investment option coefficient by:

$$A_{Cf0} = \frac{1}{\beta_1 - \beta_2} \left[\left(1 - \beta_2\right) \frac{\hat{P}_C Y}{\delta} + \beta_2 K \right] \hat{P}_C^{-\beta_1} \ge A_0 . \tag{A4}$$

A feasible floor for an active asset yields both a more valuable investment opportunity and one that is exercisable at an earlier time. Consequently, a floor represents a government granted subsidy, Armada et al. (2012).

Price Cap Model

We use the additional subscript c to indicate a model with only a cap. From (10) the active project valuation function becomes:

$$V_{Cc}(P) = \begin{cases} \dfrac{PY}{\delta} + A_{Cc21} P^{\beta_1} & \text{for } P < P_H \\ \dfrac{P_H Y}{r} + A_{C32} P^{\beta_2} & \text{for } P_H \le P, \end{cases} \tag{A5}$$

with:

$$A_{Cc21} = \frac{P_H Y}{P_H^{\beta_1}} \frac{r\beta_2 - r - \delta\beta_2}{r(\beta_1 - \beta_2)\delta} \le 0, \quad A_{Cc32} = \frac{P_H Y}{P_H^{\beta_2}} \frac{r\beta_1 - r - \delta\beta_1}{r(\beta_1 - \beta_2)\delta} \le 0\,. \tag{A6}$$

The investment option value is specified by:

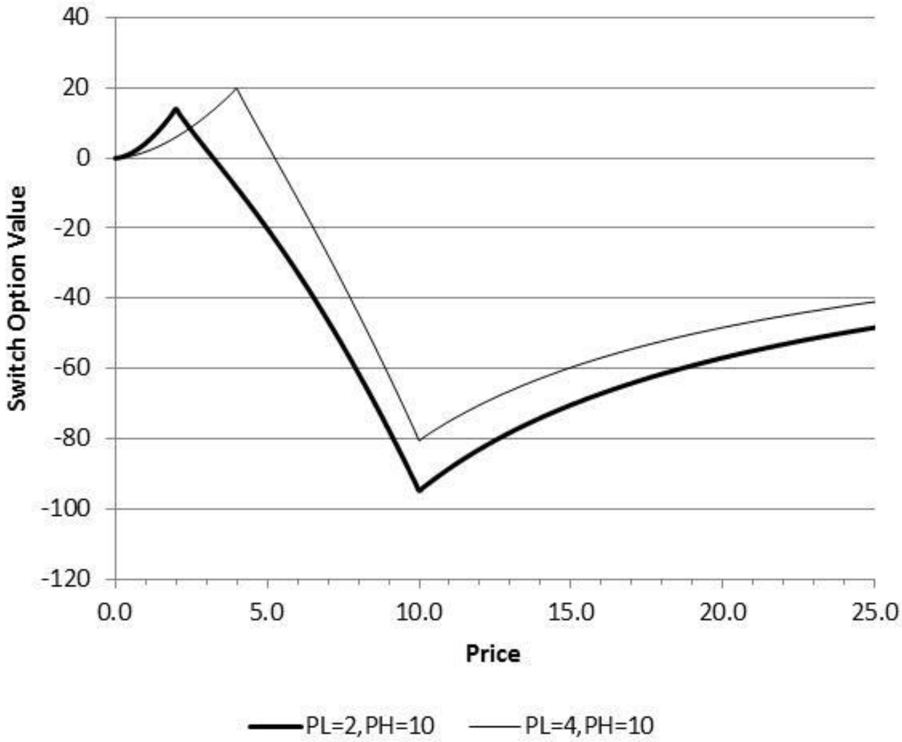
$$F_{C0c}(P) = \begin{cases} A_{Cc0} P^{\beta_1} & \text{for } P \le \hat{P}_{Cc} \\ \dfrac{PY}{\delta} + A_{Cc21} P^{\beta_1} - K & \text{for } \hat{P}_{Cc} \le P \le P_H, \end{cases} \tag{A7}$$

with $\hat{P}_{Cc} = \hat{P}_0$ determined from (6), and investment option coefficient:

$$A_{Cc0} = \frac{K\hat{P}_C^{\beta_1}}{\beta_1 - 1} + A_{Cc21} \le A_0\,. \tag{A8}$$

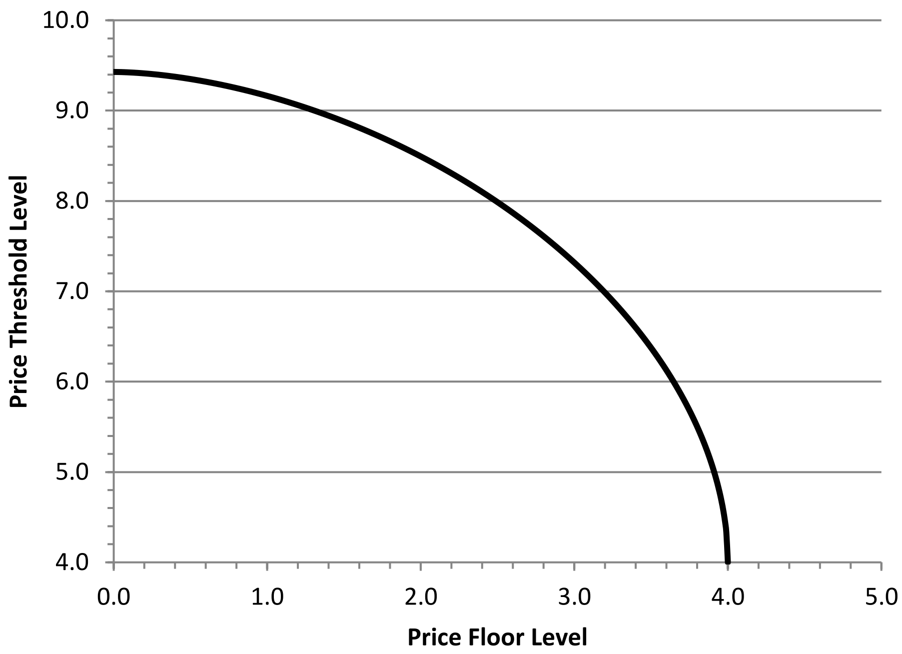
The imposition of a cap has no effect on the investment threshold and the timing, but it does produce a less valuable investment option. It is significantly less desirable for the concessionaire than an opportunity without a cap, and consequently it is imposed by, for example, a government intent on offering a subsidy while reducing its cost, or by limits to growth due to firm or market characteristics.

Figure 1
Effect of Price on Switch Option Value
at Two Different Floor Levels



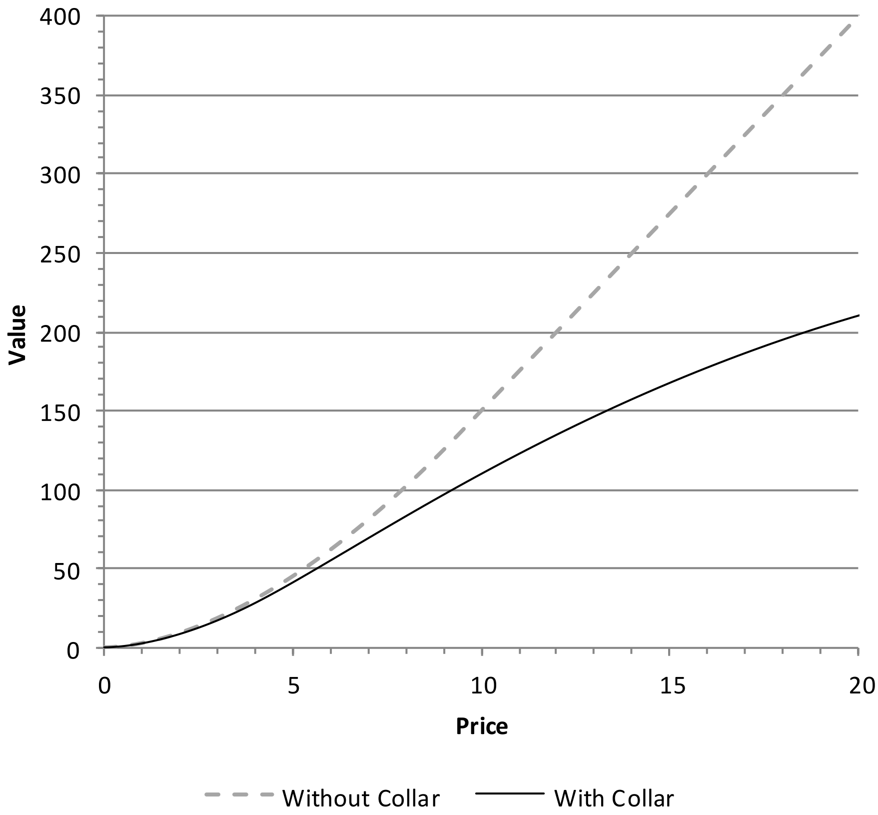
Using the baseline data, the switch option value is evaluated from (21) for the indicated P_L and P_H values.

Figure 2
Relationship between Floor and Threshold for the Collar Model



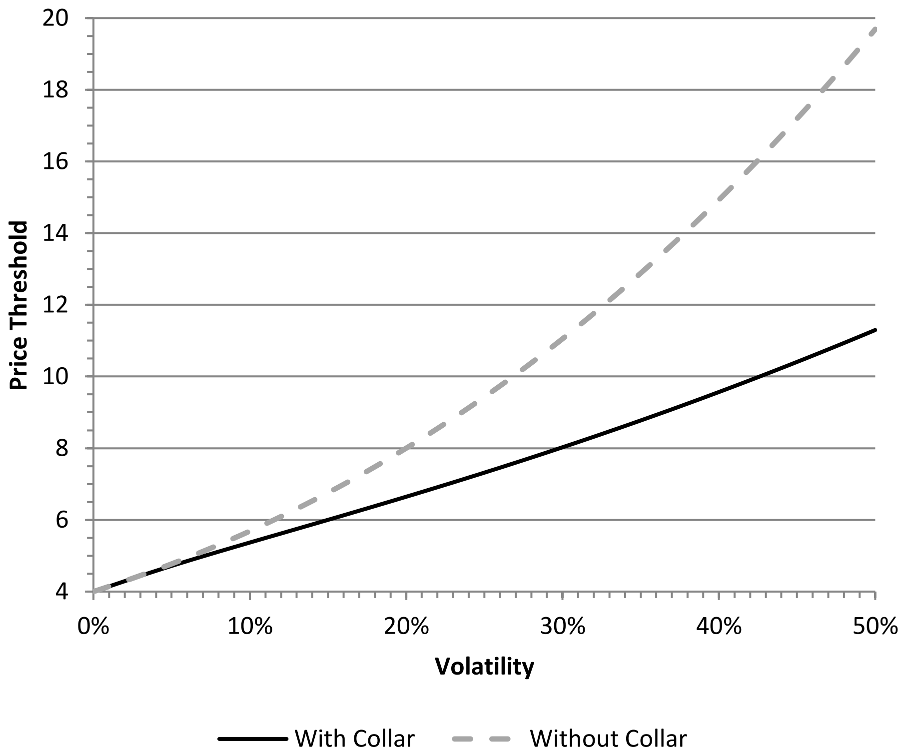
Using the baseline data, the relationship between P_L and $\hat{P}_C$ is evaluated from (15) and (18).

Figure 3
The Effect of Price on the Investment Value
for the With- and Without-Collar Variants



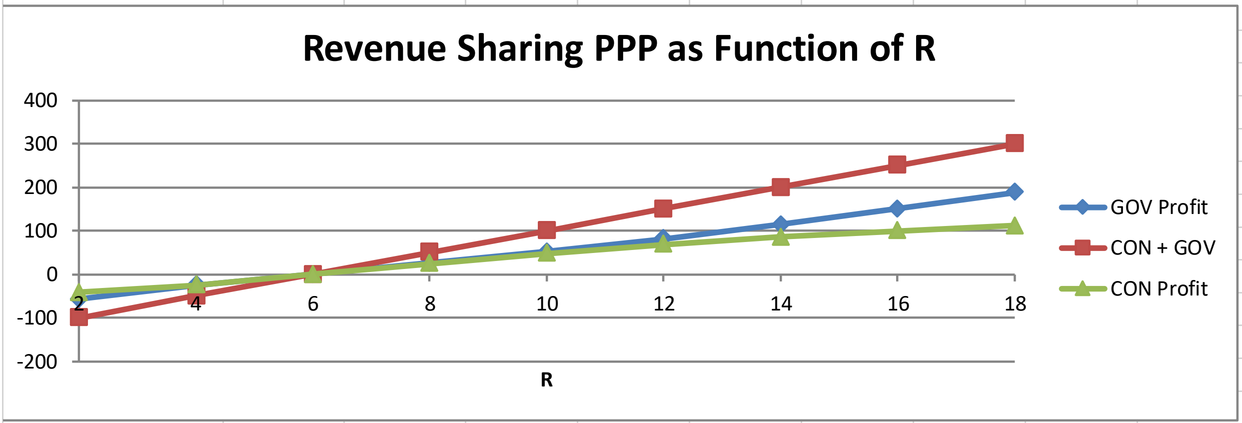
The selected floor and cap prices for the collar variant are $P_L = 4.0$ and $P_H = 20.0$, respectively. The evaluations for the two variants are based on base case values. The solution values for the collarless variant are $A_0 = 2.7547$ and $\hat{P}_0 = 9.4273$, while those for the collar variant are drawn from Tables 2 and 3.

Figure 4
Effect of Volatility Variations on the Price Thresholds for the
With- and Without-Collar Variants



The evaluations for the two variants are based base case values, and the floor and cap prices for the collar variant are $P_L = 3.0$ and $P_H = 500.0$, respectively.

Figure 5



Assumes

At R=2, the CON would have lost -150+50=-100 without the Min R guarantee and ROV, but instead loses -42.

At R=18, the CON would have gained 450-150=300 without the ceiling ceded to the GOV, but instead gains 111.

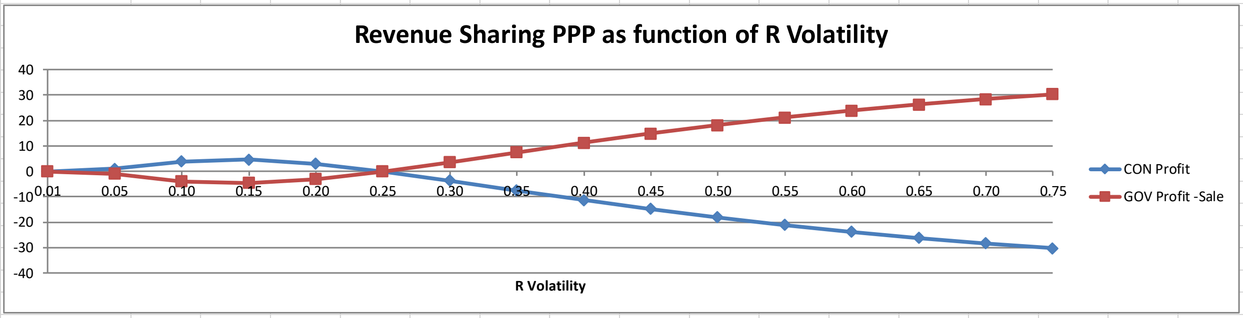
Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when R=6.

ACTIVE infrastructure is sold to Concessionaire (CON) at fair value R/δ when R=6 and
Government (GOV) guarantees a minimum R of 4 and receives all R over 15.60.

At R=6 and the other parameter values, -CALL=PUT for a "costless collar",
so the combined "profit" over the fair value of the CON and GOV is 0.

Figure 6

CON Profit	**0.00**	**0.98**	**3.95**	**4.62**	**2.99**	**0.00**	**-3.64**	**-7.49**	**-11.27**	**-14.84**	**-18.13**	**-21.13**	**-23.82**	**-26.23**	**-28.38**	**-30.29**
GOV Profit -Sale	**0.00**	**-0.98**	**-3.95**	**-4.62**	**-2.99**	**0.00**	**3.64**	**7.49**	**11.27**	**14.84**	**18.13**	**21.13**	**23.82**	**26.23**	**28.38**	**30.29**



Interpretation

At σ=.01, the CALL and PUT for both the CON and GOV would have been of little value, when R=6.

At σ=.75, the CALL would be worth 106.14 for the GOV (and -106.4 for the CON), while the PUT would be worth -75.85 for the GOV.

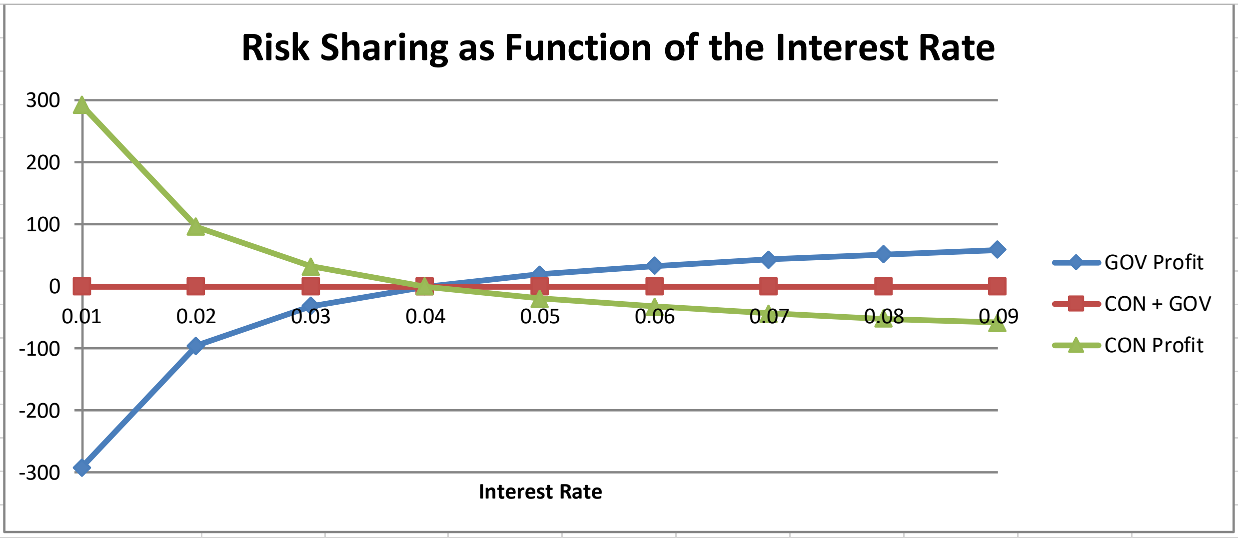
So with these values, the CON would welcome R volatility below 25%, and the GOV benefit from higher volatility.

Assumes the monopoly over which the GOV cedes control is of no value to the GOV.

ACTIVE infrastructure is sold to Concessionaire (CON) at fair value R/δ when
Government (GOV) guarantees a minimum R of 4 and receives all R over 15.60.

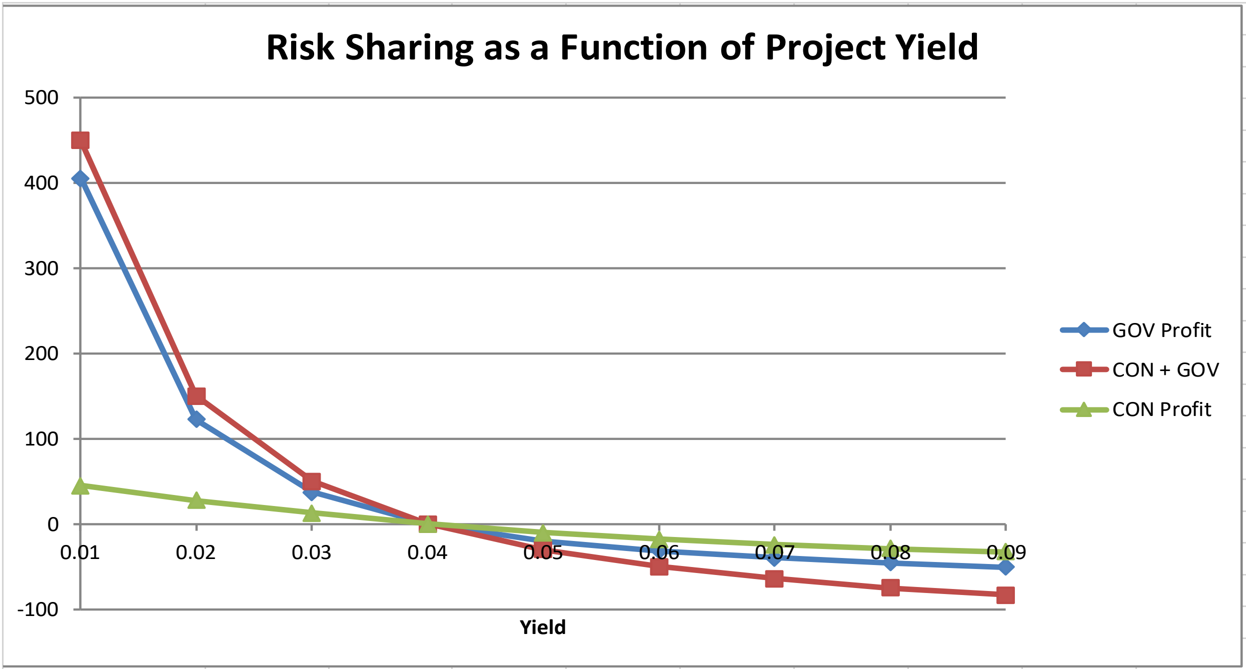
At R=6 and the other parameter values, -CALL=PUT for a "costless collar",
so the combined "profit" over the fair value of the CON and GOV is 0.

Figure 7



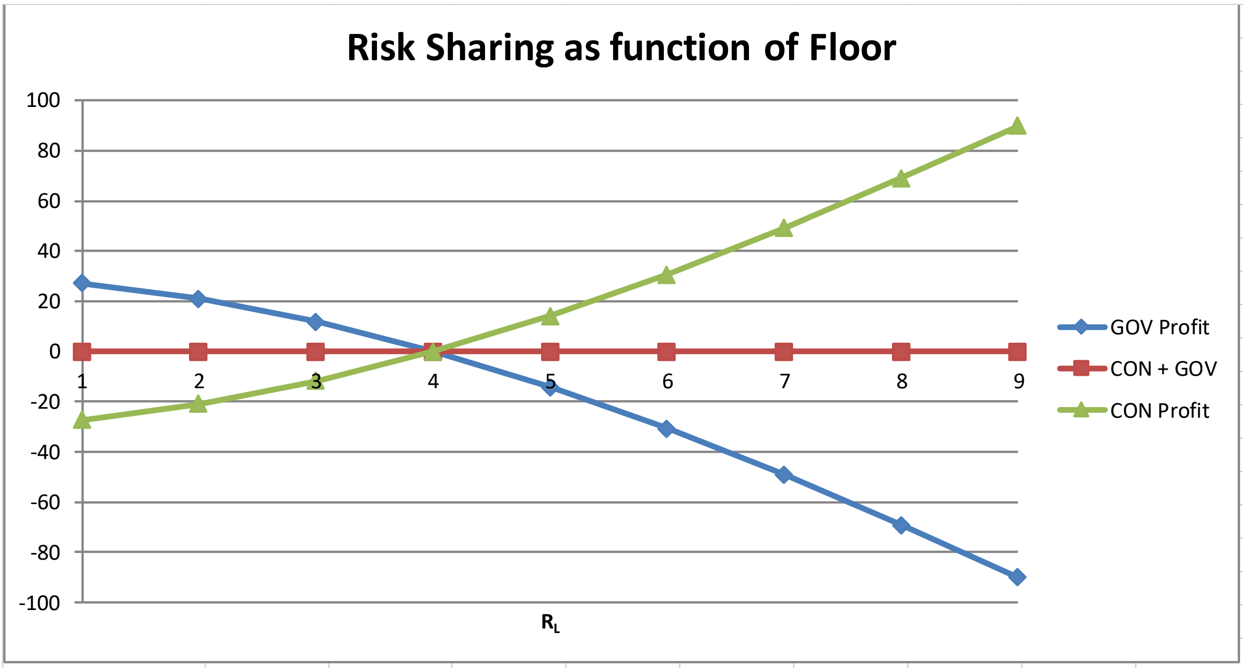
Assumes
Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when R=6.
ACTIVE infrastructure is sold to Concessionaire (CON) at fair value R/δ when R=6 and
Government (GOV) guarantees a minimum R of 4 and receives all R over 15.60, interest rate is 4%.
At R=6 and the other parameter values, -CALL=PUT for a "costless collar",
so the combined "profit" over the fair value of the CON and GOV is 0.

Figure 8



Assumes
Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when R=6.
ACTIVE infrastructure is sold to Concessionaire (CON) at fair value R/δ when R=6 and δ is 4%.
Government (GOV) guarantees a minimum R of 4 and receives all R over 15.60, interest rate is 4%.
At R=6 and the other parameter values, -CALL=PUT for a "costless collar",
so the combined "profit" over the fair value of the CON and GOV is 0.

Figure 9



Assumes

Assumes the GOV has ceded control over a valuable monopoly, so GOV profit deducts the PV when R=6.

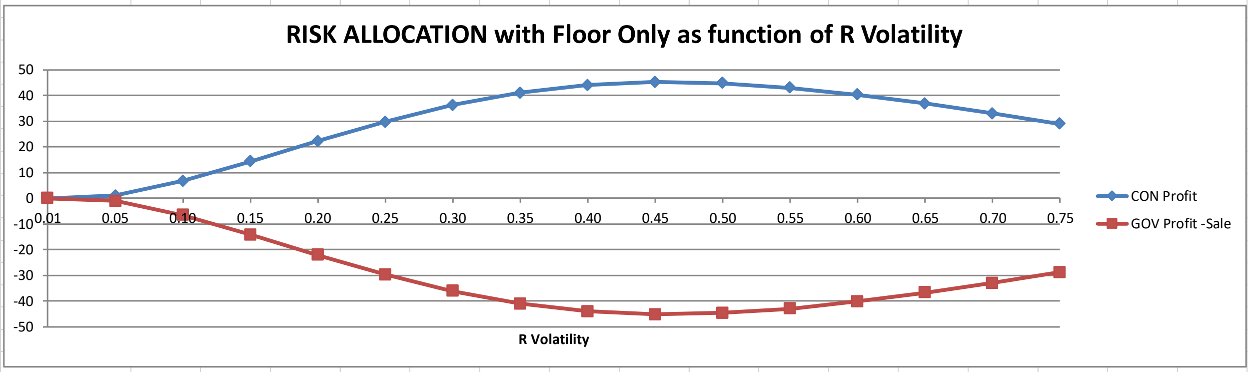
ACTIVE infrastructure is sold to Concessionaire (CON) at fair value R/δ when R=6 and δ is 4%. Base case is

GOV guarantees a minimum R of 4 and receives all R over 15.60, interest rate is 4%.

At R=6 and the other parameter values, -CALL=PUT for a "costless collar",

so the combined "profit" over the fair value of the CON and GOV is 0.

Figure 10



Interpretation

At σ=.01, the CALL and PUT for both the CON and GOV would have been of little value, when R=6 when there is a floor only.

At σ=.75, the CALL would be worth 46.93 for the GOV (and -46.93 for the CON), while the PUT would be worth -75.85 for the GOV.

So with these values, the CON would welcome R volatility especially around 45%.

Assumes the monopoly over which the GOV cedes control is of no value to the GOV.

ACTIVE infrastructure is sold to Concessionaire (CON) at fair value R/δ when

Table 2

Switch Option Coefficients for the With-Collar Model for Variations in Floor and Cap Levels

Panel A: A_{C11}

P_H	$P_L=0$	$P_L=1$	$P_L=2$	$P_L=3$	$P_L=4$
10	0.0000	8.2537	4.2116	2.6454	1.7862
20	0.0000	8.9944	4.9523	3.3862	2.5270
50	0.0000	9.5400	5.4979	3.9317	3.0726
100	0.0000	9.7663	5.7241	4.1580	3.2988
200	0.0000	9.9020	5.8599	4.2937	3.4346
500	0.0000	10.0020	5.9599	4.3937	3.5345
1000	0.0000	10.0435	6.0013	4.4352	3.5760
Infinity	0.0000	10.1057	6.0635	4.4974	3.6382

Panel B: A_{C21}

P_H	$P_L=0$	$P_L=1$	$P_L=2$	$P_L=3$	$P_L=4$
10	-1.8520	-1.8520	-1.8520	-1.8520	-1.8520
20	-1.1112	-1.1112	-1.1112	-1.1112	-1.1112
50	-0.5656	-0.5656	-0.5656	-0.5656	-0.5656
100	-0.3394	-0.3394	-0.3394	-0.3394	-0.3394
200	-0.2036	-0.2036	-0.2036	-0.2036	-0.2036
500	-0.1037	-0.1037	-0.1037	-0.1037	-0.1037
1000	-0.0622	-0.0622	-0.0622	-0.0622	-0.0622
Infinity	0.0000	0.0000	0.0000	0.0000	0.0000

Panel C: A_{C22}

P_H	$P_L=0$	$P_L=1$	$P_L=2$	$P_L=3$	$P_L=4$
10	0.000	10.106	33.685	68.123	112.280
20	0.000	10.106	33.685	68.123	112.280
50	0.000	10.106	33.685	68.123	112.280
100	0.000	10.106	33.685	68.123	112.280
200	0.000	10.106	33.685	68.123	112.280
500	0.000	10.106	33.685	68.123	112.280
1000	0.000	10.106	33.685	68.123	112.280
Infinity	0.000	10.106	33.685	68.123	112.280

Panel D: A_{C32}

P_H	$P_L=0$	$P_L=1$	$P_L=2$	$P_L=3$	$P_L=4$
10	-551	-541	-518	-483	-439
20	-1838	-1828	-1804	-1770	-1726
50	-9027	-9017	-8994	-8959	-8915
100	-30090	-30080	-30057	-30022	-29978
200	-100299	-100289	-100265	-100231	-100187
500	-492596	-492586	-492562	-492528	-492484
1000	-1641948	-1641938	-1641914	-1641880	-1641836
Infinity	-Infinity	-Infinity	-Infinity	-Infinity	-Infinity

Using the baseline data, the coefficients $A_{C11}, A_{C21}, A_{C22}, A_{C32}$ are evaluated from (15) for the various indicated P_L and P_H values.

Table 3

Option Threshold and Coefficient Values for the With-Collar Model for Variations in Floor and Cap Levels

Panel A: $\hat{P}_C$

P_H	$P_L = 0$	$P_L = 1$	$P_L = 2$	$P_L = 3$	$P_L = 4$
10	9.4279	9.1627	8.4930	7.3178	4.0000
20	9.4279	9.1627	8.4930	7.3178	4.0000
50	9.4279	9.1627	8.4930	7.3178	4.0000
100	9.4279	9.1627	8.4930	7.3178	4.0000
200	9.4279	9.1627	8.4930	7.3178	4.0000
500	9.4279	9.1627	8.4930	7.3178	4.0000
1000	9.4279	9.1627	8.4930	7.3178	4.0000
Infinity	9.4279	9.1627	8.4930	7.3178	4.0000

Panel B: A_{C0}

P_H	$P_L = 0$	$P_L = 1$	$P_L = 2$	$P_L = 3$	$P_L = 4$
10	0.9028	0.9434	1.0513	1.2581	1.7862
20	1.6435	1.6842	1.7920	1.9989	2.5270
50	2.1891	2.2298	2.3376	2.5444	3.0726
100	2.4153	2.4560	2.5638	2.7707	3.2988
200	2.5511	2.5918	2.6996	2.9064	3.4346
500	2.6511	2.6917	2.7996	3.0064	3.5345
1000	2.6925	2.7332	2.8410	3.0479	3.5760
Infinity	2.7547	2.7954	2.9032	3.1101	3.6382

Using the baseline data, the investment threshold and option coefficient are evaluated from (18) and (19), respectively, for the indicated P_L and P_H values. The solutions for the floor and cap models are obtainable from (18), and from (6), respectively.

Table 4

Illustrative Pairs of P_L and P_H for a Investment Opportunity "Costless" Collar

P_L	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
P_H	9430.5	1780.0	649.94	307.02	164.71	93.963	54.011	27.302
$\hat{P}_C$	9.350	9.163	8.879	8.493	7.986	7.318	6.375	4.000
A_{C0}	2.755	2.755	2.755	2.755	2.755	2.755	2.755	2.755
A_{C21}	-0.012	-0.041	-0.085	-0.148	-0.235	-0.355	-0.534	-0.883
A_{C22}	3.032	10.106	20.437	33.685	49.632	68.123	89.038	112.280