CONTRACT CALL OPTIONS: WHEN ARE THEY A WIN-WIN SITUATION?

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Abstract

An agent with a short position in a call option typically assumes a contingent liability where the optimal exercise of this option by the buyer entails a loss for this agent. However, we show that there may be situations where the optimal exercise by the agent who is long on the options is also optimal for the agent with the short position, which apparently violates traditional practice.

In this article we analyze the reasons for this apparent discrepancy and illustrate with a real case of an asset in the oil sector under the real options approach. This apparent violation of law of one price, which is a cornerstone of modern financial theory, can be justified in the case of customized real assets, which may have a different value for each party. Unlike freely traded financial assets, real assets can exhibit this behavior because of incomplete information, patent protection, asymmetrical synergies or by force contractual clauses. In this case, unlike traditional behavior where the agent who is short the option assumes a contingent liability, the optimal exercise of the option by the agent who is long the option is also optimal for the agent who is short the option as this exercise represents additional project cash flows at the end of the original contractual period.

Keywords

Real Options, opportunity costs, contractual call option, valuation, FPSO.

1 Introduction

A financial option provides the buyer with the right, but not the obligation, to buy or sell an asset at a predetermined price at a future date. The option will be exercised if the exercise results in a benefit for the buyer, who is long the option. The seller of the option, who holds a short position, on the other hand is obligated to bear any costs of the buyer's decision to exercise the option.

In the case of an European call option, at maturity this relationship can be expressed as $c = \max\left(S - X;\ 0\right)$ where S is the value of the underlying asset at maturity, X is the strike price of the option and c is the gain for the agent who is long the option, consequently, the cost to the agent who is on the short position. If $S > X$ at expiration, the agent who is short the option incurs a loss of $S - X$ if $S \leq X$, then the option will expire without any loss for the seller. It is clear then, that is in the interest of the seller that the option expire without being exercised.

Although initially developed for the financial asset markets, options soon found a wide range of applications in the real asset and services markets. Contracts for the provision of services, for example, may contain clauses which create managerial flexibility for the parties involved, such as the option to abandon, to extend the term, to temporary interrupt services, etc. Because they have option like characteristics, these contractual flexibilities can only be valued through option pricing methods, such as the real option approach.

Contractual options are typically included in order to protect one of the parties from certain risks of a project. In private equity ventures, for example, it is common to include clauses that protect the investor such as "drag along" and "tag along" options, or abandonment option in venture capital funds.

In the oil industry, it is common to charter oil rigs under fixed term contracts, where these contracts typically contain clauses that give the E&P firm the option to extend the service term beyond the original contractual period. This is equivalent to giving the E&P firm an option to purchase an additional years of service. An increase in oil prices creates incentives for the maintenance of the current oil field and the exploration of new ones, which increases the demand for rigs and, in turn, also raises both their market price and the cost of chartering. In this scenario, it is optimal for the E&P firm to exercise the option to extend the term of the contract in order to guarantee the current lease price, while the charterer, which is in a short position on this option, will incur in an opportunity cost for lost revenue potential that could be earned by chartering out the rig to another firm in the market for a higher price. Nonetheless, we will see that in this case it is also optimal for the chartering firm that the extension option be exercised by the E&P firm.

In this paper we analyze the reasons for this apparent discrepancy and develop a model to determine the value of a charter contract where there are term extension options and apply this model to a typical FPSO charter contract, from the point of both agents. Given that this managerial flexibility is not captured by traditional project valuation methods, we use the real options approach to price these options.

This paper is organized as follows: after this introduction we present a brief literature review, following by a discussion on the basic concepts of the offshore exploration and production industry and FPSOs. In Section 4 we develop our model and illustrate with the case of a FPSO charter contract which has term extension options. In section 5 we present the results and finally we conclude.

2 Literature Review

The real options approach is a natural evolution of the application of financial option pricing originally developed by Black and Scholes (1973) and Merton (1973) to real assets. Some of the early representative work in this area are those of Tourinho (1979), Titman (1985), Brennan and Schwartz (1985), McDonald and Siegel (1986), and Majd and Pindyck (1987). More recently, Dixit and Pindyck (1994), Trigeorgis (1996b) and Copeland and Antikarov (2001) expanded the applications by providing the first textbooks on the subject.

In a FPSO charter contract, the charterer is responsible for the operation and is the owner of the vessel, and the contractor is the E&P firm which hires the services for a fixed term. These contracts also have characteristics of assets that pay fixed, and not proportional dividends, because the contract cash flow remain constant as the value of the asset (the vessel) depreciates over time.

Trigeorgis (1996a) analyzed the value of the options embedded in operating leases through Contingent Claims. Geske and Johnson (1984) derive an analytical formula for pricing American options with fixed dividends payments and without the payment of dividends. When options on stocks that pay fixed dividends, as opposed to a fixed dividend rate, however, the efficiency of the binomial model deteriorates (Schroder, 1988). Roll (1977) derived an analytical formula for the pricing of US stock options with fixed dividends before expiration by showing that the payoff of the option can be replicated by a portfolio of two European options and one compound option.

Schroder (1988) used an adaptation to the binomial model to price of assets with fixed dividend payments. In this model, the author argues that according to the assumption that the stock price before the dividend distribution follows a lognormal distribution, the price of a dividend paying stock can be separated into two parts. The first part is riskless and can be represented by the present value of all future dividends paid during the option exercise period. The second risky part represents the uncertain net residual value of the firm. Duarte (2012) analyzed the pricing of FPSO, rigs and vessel charter contracts from the point of view of the E&P firm using option pricing models similar to the ones used for fixed dividend paying assets.

Nonetheless, we did not find in the literature a discussion of the situation we analyze in this paper.

3 The E&P Process

The exploration and production of oil (E&P) involves the discovery, drilling and production of oil with the help of extraction and processing rigs. In the case of deep water offshore reservoirs, floating platforms known as FPSO (Floating Production Storage and Offloading) that can extract, process and store hydrocarbons until they are transferred to a tank vessel, eliminating the need for a local infrastructure pipelines.

E&P firms can hire an FPSO either by purchase or by charter. In the first case, the firm contracts for the construction or upgrading of the FPSO and is responsible for all maintenance and the operation of the vessel. In the second model, after determining the characteristics and specifications of the reservoir, the oil, the production capacity and the storage needs, the E&P firm conducts a competitive bidding process in which the winning company will be responsible for building, operating and maintaining the vessel over the life of the contract. In this model, the ownership of the vessel's belongs to the charterer.

The choice of charterer firm depends on several factors, the most important being the daily charter rate offered in the competitive bidding process, which is usually calculated based on projected future cash flows that will be generated by the construction, operation and maintenance of the FPSO. In this model, all the capital investment, plus the operating and maintenance costs of the FPSO over the life of the contract are included, and the Net Present Value (NPV) and Internal Rate of Return (IRR) are determined.

Most FPSO charter contracts allow the E&P firm to extend the contract for additional periods at the same daily charter rate. From a practical perspective, this flexibility means that the charterer grants the E&P firm extension options which are equivalent to European Calls.

An important feature of the FPSO, and one that also differentiates these assets from traditional oil and gas production drills, is that FPSOs are customized to meet the specific characteristics of each field in which they operate. According to Catherine (2011), the main features that differentiate a FPSO from other types of production platform is that FPSOs can be anchored at different depths and have large tanks where the oil can be stored until it can be offloaded by a support vessel. This allows for efficient exploration of fields that are far away from the shore. Furthermore, FPSOs have a high capacity to carry loads and production modules as their ample deck provides more space than other types of platform, allowing greater flexibility in the production process. Once production in a given field is depleted, the FPSO can be relocated to other fields, but this requires a costly customization to the characteristics of the new field, which may turn out to be unfeasible.

The hiring process in the charter model begins with the specification of the main technical characteristics of the vessel such as production capacity, type of oil, equipment, life, etc. by the E&P firm. The revenue for charterer is determined by the daily rate, which consists of the operating rate and the charter rate (bareboat), multiplied by the days of operation of the vessel in the contract. The next step is a competitive bid where the charterer that offers the best technical and economic proposal is declared the winner and shall be responsible for operating the FPSO over a pre-defined period. Since the components of a FPSO are manufactured by the same suppliers, and that the chartering firms have similar expertise and experience, there is little difference between the technical proposals of the various competitors. Thus, the key criterion for selecting the winning firm is the daily rate offered.

Typically, one or more time extension options may be embedded in a FPSO charter contract, where this option can be exercised at the sole discretion of the E&P firm depending on current oil price, production volume still remaining in the field and other factors. Since the relocation and customization of the FPSO to another site to the end of contract requires a significant investment, the opportunity for new contracts tends to be reduced. For this reason, the industry practice is to assume that the life of the vessel is equal to the contractual term and that the FPSO will be fully depreciated during this period with a zero residual value.

However, unlike traditional behavior where the agent who is in a short position on the option assumes a contingent liability, in this case the optimal exercise of the option by the

E&P business is also optimal for the charterer as it aggregates additional cash flows to the project beyond the end of the original contractual period. Thus, the exercise of this option by the E&P firm is also of interest to the charterer, which apparently conflicts with standard behavior assumed in financial theory. In addition, this feature imposes greater complexity in setting the daily rate, since the charterer's point of view, the contractual term becomes uncertain.

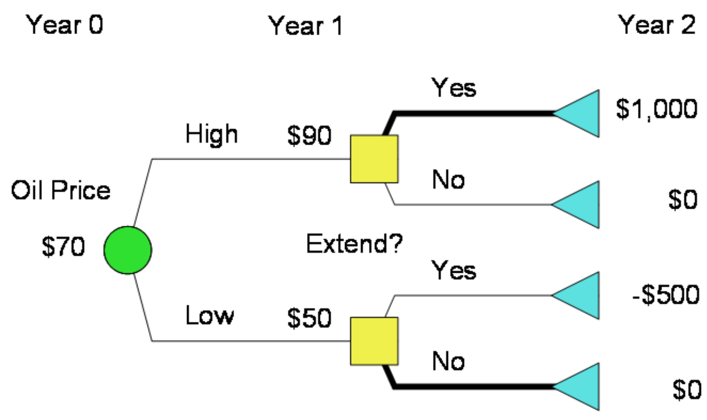
4 Model

We consider a FPSO charter contract where there are one or more term extension options, where this option may be exercised at the discretion of the E&P firm as a function of the oil price at the time of exercise. This assumption stems from the fact that oil production costs grow as the field is depleted, and there are critical values below which production is no longer economically viable, although there may still be a quantity of oil remaining in the reservoir. On the other hand, if oil prices are sufficiently high, continuation of production may be justified, even at higher costs.

Given the low opportunity cost of transferring the FPSO to other fields, the exercise of term extension option by the E&P firm is also of interest to the charterer who granted it this option, since it implies receiving cash flows for additional periods.

Figure 1 illustrates this mechanism, where in a simplified way we assume that the charter firm has a one-year contract with the E&P firm, which has an option to extend the contract for an additional year in exchange for a compensation of $1,000. We further assume that the minimum acceptable oil price for the E&P firm required to exercise the option is $75 per barrel, since below this value the revenue from oil sales would be insufficient to cover operating costs of the field, and that the opportunity cost for alternate uses of the FPSO is zero.

Figure 1– Example of Extension Option Exercise



If the E&P firm exercises the option to extend the contract, the charterer will receive an additional year of cash flows; otherwise the contract is terminated immediately (Eq.(1))

$$\begin{aligned} &\max\left(\text{exercise; non exercise}\right) \\ &\max\left(\phi(\delta)-\phi(\delta_{\min});0\right) \end{aligned} \quad (1)$$

where:

δ = Oil price on the date of exercise

δ_{min} = Lowest oil price required by the E&P firm to extend the contract.

$\phi(\cdot)$ = E&P project cash flows as a function of oil prices.

The value of the contract to the charterer can be expressed as the sum of the cash flows received plus the value of the extension options (Eq.(2)).

$$V(\gamma) = -I + \int_{t=1}^{n} f(\gamma)e^{-kt}dt + \sum_{1}^{j} c(\delta_{n+j})e^{-(k+n)} \quad (2)$$

where:

I = FPSO CAPEX

γ = contract daily rate

$f(\cdot)$ = cash flow to charterer as a function of the daily rate

n = contract term

k = risk adjusted discount rate

j = number of annual extension options

c = value of the option to extend in year $n+j$

δ_{n+j} = oil price in year $n+j$

When the contract is celebrated, there is no guarantee that the contract will be extended beyond its original term. Therefore, the charterer will consider only the original term length when determining the value of the contract and the charter rate that will be offered. However, the inclusion of contract extension options imply that there exists a non-zero probability that the actual duration of the contract will be greater than its original period, which is not captured by traditional asset valuation methods. Since this flexibility has option like characteristics, option pricing methods can be used to determine the value of these possible contract extensions.

The first two terms in Eq.(2) can be determined by standard DCF methods. The pricing of the options will depend on the stochastic process chosen to model the behavior of the uncertain variable, which are the future oil prices. One of the most widely used processes for modeling asset prices is the Geometric Brownian Motion (GBM), in which prices follow a lognormal diffusion process where the variance increases linearly with time. For commodity price modeling, where prices tend to converge to a long-term equilibrium level, Mean Reverting Models (MRM) can be used. Dias (2005) summarizes the alternatives of stochastic processes for modeling oil prices into three categories, as shown in Table 1:

Table 1 – Oil price processes for real options models

Type of Stochastic Model	Name of the Model	References
Unpredictable Model	Geometric Brownian Motion (GBM)	Paddock, Siegel & Smith (1988)
Predictable Model	Pure Mean Reversin (MRM)	Schwartz (1997, model 1)
More Realistic Models	Two and Three Fator Models	Gibson & Schwartz (1990) and Schwartz (models 2 e 3)
	MRM with Stochastic Equilibrium Level	Pyndyck (1999) and Baker, Mayfield & Parsons (1998)
	MRM with Jumps	Dias & Rocha (1998)

Dias (2005)

The continuous time GBM can be described as follows (Eq(3)):

$$dS = \mu S dt + \sigma S dz \tag{3}$$

where:

S = Asset price
μ = Expected rate of return
σ = Asset volatility
dt = Time step
dz = $\varepsilon\sqrt{dt}, \quad \varepsilon \approx N(0,1)$

The first term, μSdt, represents the proportional growth at a rate μ. The second term, σSdz, represents the proportional random growth factor considering a normal distribution with a standard deviation of σ.

The GBM can be modeled in discrete time with the Cox, Ross, and Rubinstein (1979) (CRR) binomial model. The binomial lattice of CRR is built by multiplying the asset price (*S*) by the factors *u* and *d*, which determines respectively the upper and lower nodes of the next step of the tree with an up probability of *p*. In the CRR binomial lattice, *u*, *d* and *p* are defined in such a way so that at the limit, when ($\Delta t \rightarrow 0$), the distribution of the asset values at any time *t* is lognormal. Accordingly,

$$\mu = e^{\sigma\sqrt{\Delta t}} \quad (4)$$

$$d = e^{-\sigma\sqrt{\Delta t}} = 1/u \quad (5)$$

$$p^{+} = \frac{e^{r} - d}{u - d} \quad (6)$$

where:
σ = Asset price volatility
Δt = Time interval
r = risk free rate of return

The MRM is a Markov process where the direction and intensity of the deviations are a function of the current price, which tends to converge to a long term market equilibrium price. (Dixit & Pindyck, 1994) showed that the price of some commodities revert to their long term marginal cost of production, despite short term price variations. The simplest MRM model is the Ornstein-Uhlenbeck arithmetic model, which is defined by the following Eq.(7)

$$dx_t = \eta(\bar{x} - x_t)dt + \sigma dz_t \quad (7)$$

where:
x_t = the price of the asset
η = reversion speed
$\bar{x}$ = long term average to which x_t reverts to
σ = volatility of the process
dz = wiener increment = $\varepsilon\sqrt{dt}$

Dixit and Pindyck (1994) showed that the variable x_t has a normal distribution, and its mean an variance can the described as $E[x_t] = \bar{x} + (x_0 - \bar{x})e^{-nt}$ and $Var[x_t] = \frac{\sigma^2}{2\eta}(1 - e^{-2nt})$, respectively. The estimation of the model parameters can be determined from the discretization of the process $x_t = \bar{x}(1-e^{-\eta\Delta t}) + e^{-\eta\Delta t} x_{t-1}$.

$$x_t = \bar{x} + (x_{t-1} - \bar{x})e^{-\eta\Delta t}$$
$$x_t - x_{t-1} = \bar{x}(1-e^{-\eta\Delta t}) + (e^{-\eta\Delta t} - 1)x_{t-1}$$

which can be expressed as:

$$x_t - x_{t-1} = a + (b-1)x_{t-1} + \varepsilon_t \qquad (8)$$

where ε_t represents the error of the series. The parameters can be estimated by regressing the series x_t. From the estimators obtained from the linear regression, we can determine the parameters using Eq. (8): $b - 1 = e^{-\eta\Delta t} - 1$, $\eta = -\ln(b)/\Delta t$ e $\bar{x} = -\dfrac{a}{(b-1)}$.

The volatility parameter $\boldsymbol{\sigma}$ can be determined from the variance of the regression errors σ_ε^2, which is given by the expression $\sigma_\varepsilon^2 = \dfrac{\sigma^2}{2\eta}\left(1-e^{-2\eta\Delta t}\right)$, derived from the equation of the variance of the process. Rewriting and using the relationship $b^2 = e^{-2\eta\Delta t}$, we obtain:

$$\sigma = \sigma_\varepsilon \sqrt{\frac{2\ln b}{(b^2-1)\Delta t}}$$

The continuous MRM can be modeled in discrete time with the censored model of Nelson and Ramaswamy (1990). This model uses a binomial lattice with *n* periods of duration Δ*t*, and with a time horizon $T : T = n\Delta t$, which allows a recombining lattice to be built. The general equation for the stochastic process is given by:

$$dx = \mu(x,t)dt + \sigma(x,t)dz$$

The main parameters necessary to build the binomial lattice can be determined from the following equations:

(up movement) $x_t^+ \equiv x + \sqrt{\Delta t}\sigma(x,t)$ (9)

(down movement) $x_t^- \equiv x - \sqrt{\Delta t}\sigma(x,t)$ (10)

(up probability) $p_t \equiv 1/2 + 1/2\sqrt{\Delta t}\dfrac{\alpha(x,t)}{\sigma(x,t)}$ (11)

(down probability) $1 - p_t$ (12)

As the probability p_t can take on values that are negative or greater than 1 under Eq. (11), the authors suggest that these values be censored in these cases, as follows:

if $p_t \leq 0$ p_t is censored

if $p_t \geq 1$ p_t is censored

As suggested by Hahn (2005), if we compare the equation with Eq.(7) , we have $\alpha(x,t) = \eta(\bar{x} - x_t)$, and $\sigma(x,t) = \sigma$. One can still have negative or greater than 1 values in the following cases, and the probabilities will be censored:

if $(\bar{x} - x_t)\sqrt{\Delta t} > \sigma$, then $p_{xt} > 1$
if $(\bar{x} - x_t)\sqrt{\Delta t} < -\sigma$, then $p_{xt} < 0$

$$p_{xt} = \max\left(0, \min\left(1, \frac{1}{2} + \frac{1}{2}\frac{\eta(\bar{x} - x_t)}{\sigma}\sqrt{\Delta t}\right)\right) \tag{13}$$

Each up probability p_{xt} will depend on x_t, and will generate a probability lattice. As option pricing requires that the risk neutral measure be used, an adjustment must be made to transform the risky MRM process into a risk neutral one. This is done by penalizing the long term mean $\bar{x}$, by the normalized risk premium of the process: $\bar{x} - \lambda_x / \eta$ (C. Bastian-Pinto & Brandão, 2007; Dixit & Pindyck, 1994),

$$p_{x_t} = \max\left(0, \min\left(1, \frac{1}{2} + \frac{1}{2}\frac{\eta\left[(\bar{x} - \lambda_x / \eta) - x\right]}{\sigma}\sqrt{\Delta t}\right)\right) \tag{14}$$

The choice of the most appropriate stochastic model for oil prices is controversial. Pindyck (1999) concluded that oil prices only follow a MRM if one considerers very long price series. A statistic test that can be used to verify if a historical price series has characteristics of a MGB or an MRM is the Augmented Dickey-Fuller unit root test. This test performs a regression on the equation $x_t = a + bx_{t-1} + \varepsilon_t$ to verify if the null hypothesis *b=1* is rejected. The most common way to write this equation is as shown in Eq (15):

$$x_t - x_{t-1} = a + (b-1)x_{t-1} + \varepsilon_t \tag{15}$$

The H_0 (null hypothesis) is that the series has a unit root. If the value found is greater than the critical value, the null hypothesis is rejected, which means that the series is non-stationary and shows characteristics of a MGB. As the test statistic does not follow the usual *t* distribution, since the series is assumed to be non-stationary, the series does not follow a standard distribution. As a result, the value found in the test should be compared to Table 2. (Dickey & Fuller, 1981):

Table 2 – Critical values for the Augmented Dickey-Fuller test

Level of Significance	10%	5%	1%
Critival Value without temporal tendency	-2,57	-2,86	-3,43
Critival Value with temporal tendency	-3,12	-3,41	-3,96

For more detailed discussion of the unit root rest see Brooks (2008).

5 Aplication

We apply the model to the case of a typical charter contract FPSO that has embedded extension options in order to verify is the exercise of the option by the E&P firm aggregates value to the charterer. We assume that the contract has the characteristics shown in Table 3:

Table 3 – Charter Contract Characteristics

Contract period:	13 years
Construction Period:	3 years
Operation Period:	10 years
Vessel CAPEX:	US$ 1.5 billions
Deductions from Revenue:	10% of Gross Revenue
Daily OPEX:	US$ 100,000.00.
Cost of Capital:	12% per year
Risk free rate:	5% per year
Income Tax:	10%
Depreciation rate:	6,7% per year
Projected life of the FPSO:	15 years
Residual value:	Book value of the vessel at end of contract
Extension Option:	Five annual options to extend term for an additional year each, starting on the 10^{th} year of operation
Price threshold for option exercise:	U$$ 34,00/barrel

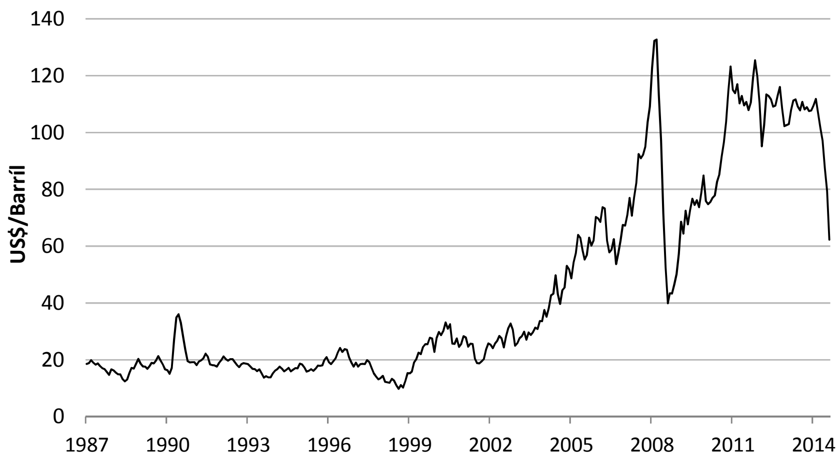
The minimum daily charter rate that offers the required return considering the project assumptions is the rate that provides a zero NPV. The value of US$ 991,000.00 is obtained using the "Goal Seek" tool in the spreadsheet. The project cash flows are shown in Table 4.

Table 4 – Project Expected Cash Flows (US$ Millions)

Project Cash Flows													
Year	1	2	3	4	5	6	7	8	9	10	11	12	13
Gross Revenus	**0**	**0**	**0**	**362**	**362**	**362**	**362**	**362**	**362**	**362**	**362**	**362**	**362**
Deductions	**0**	**0**	**0**	**(36)**	**(36)**	**(36)**	**(36)**	**(36)**	**(36)**	**(36)**	**(36)**	**(36)**	**(36)**
%	0	0	0	(36)	(36)	(36)	(36)	(36)	(36)	(36)	(36)	(36)	(36)
Net Revenues	**0**	**0**	**0**	**326**	**326**	**326**	**326**	**326**	**326**	**326**	**326**	**326**	**326**
Costs	**0**	**0**	**0**	**(137)**	**(137)**	**(137)**	**(137)**	**(137)**	**(137)**	**(137)**	**(137)**	**(137)**	**(137)**
Operational Costs	0	0	0	(37)	(37)	(37)	(37)	(37)	(37)	(37)	(37)	(37)	(37)
Depreciation	0	0	0	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)
EBIT	**0**	**0**	**0**	**189**	**189**	**189**	**189**	**189**	**189**	**189**	**189**	**189**	**189**
Taxes	**0**	**0**	**0**	**(19)**	**(19)**	**(19)**	**(19)**	**(19)**	**(19)**	**(19)**	**(19)**	**(19)**	**(19)**
Net Income	**0**	**0**	**0**	**170**	**170**	**170**	**170**	**170**	**170**	**170**	**170**	**170**	**170**
(-) Depreciation	0	0	0	100	100	100	100	100	100	100	100	100	100
(-) Capex	(500)	(500)	(500)	0	0	0	0	0	0	0	0	0	0
(+) Residual Value	0	0	0	0	0	0	0	0	0	0	0	0	500
Free Cash Flow	**(500)**	**(500)**	**(500)**	**270**	**270**	**270**	**270**	**270**	**270**	**270**	**270**	**270**	**770**

To define the stochastic process, we used the historic Brent crude price series from May 1987 to December 2014, as shown in Figure 2:

Figure 2 – Historic Prices of Brent crude oil (Europe)



Source: Bloomberg

The *Eviews* software was used for the unit root test. The results for the sample used can be seen in Table 5.

Table 5 – Unit Root Test (DF)

Null Hipothesis: Brant has unit root
Exogenous: Constant
Lag Lenght: 1 (Automatic - based on SIC, maxlag = 16)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.867270	0.3477
Test critical values:	1% level	-3.449977	
	5% level	-2.870084	
	10% level	-2.571391	

* MacKinnon (1996) on-side p-values.

When comparing the results with the critical values, we note that the null hypothesis that the series has a unit root was not rejected. This result confirms the conclusion of Pindyck (1999) that the GBM only be rejected when the analyzed series is longer than 120 years. On the other hand, the fact that the random walk (GBM) is not rejected does not guarantee the existence of some level of autoregression (mean reversion) in the variable under analysis (C. Bastian-Pinto & Brandão, 2007). For illustration purposes, in this paper we use both stochastic models for pricing options in order to compare the results.

Modeling oil prices with GBM

The historical volatility was determined considering the standard deviations of the log returns of the series between 1987 and 2014, with a result of 30.3% per year. Consequently, the up, down and probability parameters are respectively $u = 1.3539$, $d = 0.7386$ and $p = 0.5061$. The starting price for the *Brent* crude for the binomial lattice was US$ 62.34/barrel, which was the spot market price of the last day of negotiation of December, 2014.

Table 6 shows the binomial lattice for oil prices in the next 18 years.

Table 6 – GBM Oil price model

Simulated Oil Prices - GBM																		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
62	84	114	155	209	284	384	520	704	953	1.290	1.747	2.365	3.201	4.334	5.868	7.945	10.756	14.563
	46	62	84	114	155	209	284	384	520	704	953	1.290	1.747	2.365	3.201	4.334	5.868	7.945
		34	46	62	84	114	155	209	284	384	520	704	953	1.290	1.747	2.365	3.201	4.334
			25	34	46	62	84	114	155	209	284	384	520	704	953	1.290	1.747	2.365
				19	25	34	46	62	84	114	155	209	284	384	520	704	953	1.290
					14	19	25	34	46	62	84	114	155	209	284	384	520	704
						10	14	19	25	34	46	62	84	114	155	209	284	384
							7	10	14	19	25	34	46	62	84	114	155	209
								6	7	10	14	19	25	34	46	62	84	114
									4	6	7	10	14	19	25	34	46	62
										3	4	6	7	10	14	19	25	34
											2	3	4	6	7	10	14	19
												2	2	3	4	6	7	10
													1	2	2	3	4	6
														1	1	2	2	3
															1	1	1	2
																0	1	1
																	0	0
																		0

Modeling Oil Prices with MRM

We follow the procedure proposed by C. L. Bastian-Pinto (2009), who used the censored mean reverting model of (Nelson & Ramaswamy, 1990) to model a OU MRM diffusion process as shown in Eq 7. As the MRM is an arithmetic model, the use of the Ornstein-Uhlenbeck model for simulating future price paths might result in negative values. A common way to avoid this is to consider that *x* is the logarithm of the price, rather than the actual price *S*, so that $x_t = \ln(S_t)$

The volatility is determined from Eq. (16):

$$\sigma = \sigma_\varepsilon \sqrt{\frac{2\log(b)}{(b^2-1)\Delta t}} \tag{16}$$

The remaining parameters are:

S_0 = initial value (at $t = 0$) of the stochastic variable S_t

$x_0 = \ln(S_0)$

$\bar{x}$ = long term mean to which $x_t = \ln(s_t)$ converges

η = reversion speed parameter of the process

Δt = time interval

To determine the remaining parameters, we use $b - 1 = e^{-\eta\Delta t} - 1$, $\eta = -\ln(b)/\Delta t$ and $\bar{x} = -\frac{a}{(b-1)}$. The parameters obtained from the historical series are σ=30.44%, η= 0.135, x_0= 62.34 and $\bar{x}$ = 68.35. The results of the simulation are showed in Table 7.

Table 7 – MRM Oil Price Model

Preço do Petróleo																		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
62	85	115	155	211	286	387	525	712	965	1.309	1.775	2.406	3.262	4.423	5.997	8.132	11.025	14.949
	46	62	85	115	155	211	286	387	525	712	965	1.309	1.775	2.406	3.262	4.423	5.997	8.132
		34	46	62	85	115	155	211	286	387	525	712	965	1.309	1.775	2.406	3.262	4.423
			25	34	46	62	85	115	155	211	286	387	525	712	965	1.309	1.775	2.406
				18	25	34	46	62	85	115	155	211	286	387	525	712	965	1.309
					14	18	25	34	46	62	85	115	155	211	286	387	525	712
						10	14	18	25	34	46	62	85	115	155	211	286	387
							7	10	14	18	25	34	46	62	85	115	155	211
								5	7	10	14	18	25	34	46	62	85	115
									4	5	7	10	14	18	25	34	46	62
										3	4	5	7	10	14	18	25	34
											2	3	4	5	7	10	14	18
												2	2	3	4	5	7	10
													1	2	2	3	4	5
														1	1	2	2	3
															1	1	1	2
																0	1	1
																	0	0
																		0

The extension options in the charter contracts give the E&P firm the right to use the FPSO and the services for an additional year, at the same daily chartering rate established in the contract. We analyze the value of option from the point of view of the chartering firm, considering that at the time of celebration of the contract, the E&P firm also acquired five extension options, that is, 5 calls against the charterer of the vessel. The minimum price required for the E&P firm to exercise the option depends on the characteristics and location of the field, and was arbitrated at US$ 34.00 / barrel.

The base year for the option exercise is the year 13 of the binomial lattice of oil prices, and represents the last year of operation of FPSO (3 years construction + 10 of operation) considering a contract with no options. Accordingly, the determination of the option value is carried out in reverse order. In the states where the price of oil is equal to or higher than US$ 34.00/barrel, the E&P firm will exercise the option to extend the contract and the charter firm receives an additional year of cash flows of US$ 270 million.

Moreover, as the residual value of the FPSO decreases in time, this amount must be deducted from the value of the option. If the contract is terminated by the end of the 13th year with no extensions, the firm will receive back a vessel that has a residual value of US$ 500 million. If the contract is extended for an additional year, the charterer will receive additional cash flows, but the residual value will decrease to US$ 400 million at the end of year 14.

In Table 8 we determine the present value of a one year contractual extension, by adding the additional cash flows the charter firm receives in this case.

Table 8 – Additional Cash Flows – One year option exercise - GBM

Additional Cash Flows in case of Option exercise - GBM - US$ Millions																		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
														270	0	0	0	0
														270	0	0	0	0
														270	0	0	0	0
														270	0	0	0	0
														270	0	0	0	0
														270	0	0	0	0
														270	0	0	0	0
														270	0	0	0	0
														0	0	0	0	0
														0	0	0	0	0
														0	0	0	0	0
														0	0	0	0	0
														0	0	0	0	0
														0	0	0	0	0
														0	0	0	0	0
															0	0	0	0
																0	0	0
																	0	0
																		0

Beginning with the additional cash flows, the option value is determined by rolling back the lattice using the parameters of the GBM process and discounting at the risk free rate (Table 9).

Table 9 – One year option exercise – GBM

Project PV with Expansion Option - GBM - US$ Millions																		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
85	104	123	141	157	171	182	192	202	212	222	233	245	257	0	0	0	0	0
	74	95	117	139	159	177	191	202	212	222	233	245	257	0	0	0	0	0
		61	82	106	132	158	181	199	212	222	233	245	257	0	0	0	0	0
			46	65	90	119	150	181	205	222	233	245	257	0	0	0	0	0
				30	46	69	99	135	174	209	233	245	257	0	0	0	0	0
					17	28	45	72	108	155	205	245	257	0	0	0	0	0
						7	13	23	41	71	119	185	257	0	0	0	0	0
							2	3	7	15	30	63	130	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0
													0	0	0	0	0	0
														0	0	0	0	0
															0	0	0	0
																0	0	0
																	0	0
																		0

(=) PV Option	84,90
(-) Loss in Residual Value	39,50
(=) NPV of Option	**45,40**

Next, the present value of the lost residual value increment is determined to be US$ 39,5 million, so the net benefit, or option value is US$ 45,40 million. The same calculation is performed for the remaining extension options of the following years, considering both a GBM and MRM. Table 10 presents a summary of the results:

Table 10 – Project Value Summary

Project Valuation Summary - US$ Millions					
	US$	GBM	MRM	Diference GBM/Base Case	Diference MRM/Base Case
Base Case NPV - Deterministic	148,04	148,04	148,04		
1st year Option Extension NPV	-	45,40	37,05	30,67%	25,0%
2nd year Option Extension NPV	-	59,09	11,61	39,92%	7,8%
3rd year Option Extension NPV	-	48,06	40,39	32,47%	27,3%
4th year Option Extension NPV	-	59,12	16,67	39,94%	11,3%
5th year Option Extension NPV	-	48,99	42,01	33,09%	28,4%
Total NPV	**148,04**	**408,70**	**295,76**	**276,08%**	**199,79%**

As shown in the Table 10, the extension options add significant value to the project, from the point of view of the charterer, even though the charterer is in a short position on these options.

Under a GBM diffusion process, the value was 276.1% higher than the base case value, and under MRM, the value was 199.8% higher. The chartering firm could use this information to offer a more competitive bid. The base case deterministic value assumed a daily charter rate of US$ 991,000.00, with 12.0% IRR per annum. Considering the value added by the five contract extension options, the IRR would increase to 16.24% pa and 14.2% pa respectively, for the case of the GBM and MRM. If the firm chooses to maintain the same rate of return, the daily fee can be reduced to US$ 795,000.00, an amount 19.8% lower than the original, in the case of MGB, and US$ 880,000.00 in the case of MRM, as shown in Table 11.

Table 11 – Daily Rate Summary

Dialy Rate Summary			
	Base Case Scenario	GBM Scenario	MRM Scenario
Dialy Rate	USD 990,66	USD 795,00	USD 880,00
Discount over base case rate		19,8%	11,2%

Given that the option has value, then it also has value to the E&P firm, as it is standard that option value is borne by the holder of the long position on the option. But how can the option add value to both agents? In this case, this can be explained by the fact that while the underlying asset for these options is the oil price, this affects each party differently due to the high refurbishing costs required to deploy the FPSO to another field or client, which essentially reduces this opportunity cost to zero. This implies that while the E&P firm will only exercise these options if this is optimal for it, this is also optimal for the chartering firm as it has no other profitable uses for the FPSO.

6 Conclusion

An agent who is short on a financial call option assumes a contingent liability that derives from the possible exercise by the agent who is long the option. In this article we show that for real options, there may be situations where an agent who is short may actually benefit by the optimal exercise by the agent who is in a long position on the option. This may occur due to depreciation of the asset, and if alternate uses of the underlying option are costly, which significantly reduces the opportunity cost of reallocating the asset for other uses. In this case the exercise of the option might be optimal for both parties, which is an unusual result.

To illustrate such a case we analyze a standard FPSO charter contract, where the residual value of the asset at the end of the lease term is typically small, and due to the high degree of customization required for deployment in other field, the opportunity cost may also be negligible. We show that the exercise of the option to extend the contract by the E&P firm may also be in the interest of the charterer, as they represent additional cash flows that otherwise would be forgone. Thus, the optimal exercise of extension option by the E & P firm also is optimal for the charterer, who is in a short position on this option.

In addition, we also show that by taking into account and adequately pricing these options, the charterer can make a more competitive bid offer, and thus increase the probability of wining the contract. The results suggest that contract value increases by 276.1% under a GBM model, and 199.8% for the MRM model, considering five one year extension options. This represents a discount of 19.8% (GBM) and 11.2% (MRM) on the daily rate, in relation to the base case.

The lack of an opportunity cost for alternate uses of the FPSO due to the high CAPEX expenditures required to customize the vessel to the particular characteristics of a new field, create a situation where it is also optimal for the charterer that the extension option be exercised, as this represents additional cash flows that otherwise would be zero. Thus, any term extension also creates value for the charterer.

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