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Title: Copula Models in a Mexican Natural Gas Pipeline Real Option Valuation

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Abstract:

One of the main methodological challenges of the real option valuation is to recognize and account for multiple underlying assets and their impact on the volatility of the project. Even though, notable contributions are found in order to develop a "rainbow real option" approach, its use is somewhat limited. This work proposes a Copula-GARCH methodology to be used within the real option valuation. The main objective is to exploit the advantages of the volatility treatment, through GARCH models, and the dependence structure determination, through copula modeling, and apply them in the real option valuation. The methodology is applied to the valuation of the Mexican natural gas pipeline expansion project "Los Ramones", with the USD-MXN exchange rate and the natural gas price as underlying assets. We use individual TGARCH models to estimate the volatility and terminal value of two assets, then use copula modeling to determine a measure of association between them in order to define their joint volatility, here six copulas are proposed the Normal, Student's t, Clayton, Gumbel, Frank and Tawn and their pertinence is discussed. Finally, the information obtained in the previous steps is used as input in the real option context for the valuation of the project. Our findings suggest the project should be taken, as with four of the copulas (Normal, Student's t, Clayton and Frank) the value for the real option is positive.

Keywords: Copula, Heteroskedasticity, TGARCH, Natural Gas, Expansion Real Option

JEL: H43, H54, C14, C58, C32, G31

1. Introduction

When addressing the valuation of investment projects, there is a generalized trend to apply traditional approaches based on the discount cash flow (DCF hereafter) such as net present value. Even though these techniques have been criticized because their lack of adaptation to the existent business environment[1], they are still the most used tools of investment project valuation. The real options analysis (ROA hereafter) provides a framework of how the valuation of investment projects can be managed so to address the business conditions, and improve the results presented in the traditional valuation techniques; particularly, the treatment of uncertainty and strategic thinking. Mun (2006) points out that the ROA is a technique that systematically incorporates financial theory, economic analysis, management science, decision science, statistics, and econometric modeling into the application of the option pricing theory as a valuation tool of real assets.

In general terms, the main advantage of the ROA follows two directions: how it overcomes the obstacles presented under the traditional DCF techniques, and its capacity to quantify strategic implications into the valuation process. Even though the benefits of the ROA–as a superior investment decision valuation approach in the presence of uncertainty and irreversibility–are clearly stated in works such as Leslie and Michaels (1997), Luehrman (1998, 1998a), Amram and Kulatilaka (1999, 1999a), Dixit and Pindyck (1994, 1995), Copeland and Keenan (1998, 1998a), there is still a general reluctance of its use in the practitioner world. This can be explained either by its complexity, as not all managers master the mathematical tools needed in the approach, or because there is an accentuated use in the commodity markets such as gold, gas, and oil. What most of the critics of this approach argue is that the examples and assumptions used in research and applications lack real life characteristics, as they are seen more as an academic exercise rather than a business decision tool.

As an example of an over simplification of real situations, most of the ROA research and application focused on the presence of one underlying asset. However, options rarely

[1] Myers (1984), Trigeorgis and Mason (1987), Kulatilaka (1995a), Ross (1995)

arise in isolation, most of the times investment projects depend on more than one underlying asset or even other projects. The biggest challenge in real-life project valuation is to correctly identify the collection of multiple real options and underlying assets involved, as well as measuring the interaction between them. As a measure of interaction one can talk about the joint volatility of underlying assets, and then the problem arises when increasing the number of them: how to address the inclusion of two or more volatilities into the model. Hence, when considering projects with two or more underlying assets, the valuation process needs to focus on their volatility its impact to the value and volatility of the project.

Conceptually, the volatility concept illustrates the uncertainty factors that do not dissolve during the projects' lifetime. Following Amram and Kulatilaka (1999), it is treated as a constant in the ROA approach, as they noted that, in most of the cases, real options are virtually unaffected by unexpected changes in the volatility during the life of the project. They even state that including the stochastic nature of the volatility often leads to more errors in the final valuation result rather than major improvements in it. Another common practical error is to use the terms risk and uncertainty interchangeably. As described by Hung and So (2011), most of the valuation inaccuracies come directly from it. In order to perform this distinction, they propose a method to filter the risk of the project without the influence of uncertainty, using an adjusted Black-Scholes pricing formula. One tool that has gained acceptance for the treatment of multiple sources of uncertainty is the copula modeling.

Literature on the copula founding concepts, statistical properties, and financial applications has developed rapidly. Joe (1997) and Nelsen (1999) are excellent and highly technical introductory texts, while Frees and Valdez (1998) provide an introduction to the statistical properties of copulas and their applications to the actuarial world. Bouyé, Durrleman, Nikeghbali, Riboulet and Roncalli (2000), along with Cherubini, Luciano and Vecchiato (2004), cover relevant material on copula application in financial econometrics. In the last fifteen years, there has been a notable expansion of academic literature regarding the application of copula modeling in the bivariate and multivariate financial context, and an important effort has been made to expand its application into ROA. A noteworthy effort to enhance this trend is found in Cherubini and Luciano (2002, 2002a); who make a comprehensive description on how to price bivariate and multivariate digital options trough copula modeling. This work led to an interesting development, focused on rainbow options, presented in Cherubini, Luciano and Vecchiato (2004). Knox and Ouwehand (2006) apply copula modeling in rainbow option pricing, they estimated the marginal risk-neutral asset returns distributions of two South Africa's market indexes.

The application of copula models into the real options theory and decision-making under uncertainty, in the context of new investments in power generation technologies, is gaining popularity in current research directions, as exposed by Westner and Madlener (2010). The main reason is that energy derivatives tend to present non-linear dependencies derived from an increasingly intertwined commodity markets. Armstrong, Galli, Bailey and Couët (2004) used an Archimedean copula base model to include technical uncertainty in the valuation of expansion projects in the oil industry. Grégoire, Genest and Gendron (2008) studied the dependence structure between prices for futures on crude oil and natural gas using a copula approach and discussed an appropriate copula family selection for these markets.

Denault, Dupuis and Couture-Cardinal (2009) used a copula model to analyze the diversification effect of energetic generation plants when considering a combination of inflows. They determined that the risk value of a project which considers a mixed hydro-and-wind generator is lower that when considering an all-hydro project. Valizadeh, et al. (2010) developed a copula approach to study the planning and operation characteristics of renewable energy generation in Iran. Even though Fleten and Näsäkkälä (2010) did not worked within the copula modeling environment, they develop an interesting model to determine thresholds for energetic prices in which it will be optimal to make an investment decision in gas-fired power plants under the ROA context.

In terms of considering multiple underlying assets as determinants for the valuation of an investment project, Herath, Kumar and Amershi (2011) applied the copula methodology to price refinery crack[2] spread options. This is used as a base for risk management in the volatile commodity markets as they allow refiners to hedge their operating margins while letting them to participate in any future widening of their refining margins. They concluded that a Clayton copula model is more appropriate to describe this particular spread option. A similar approach is presented by Benth and Kettler (2006) as they developed a non-symmetric copula to model the spark[3] spread options following a bivariate non-Gaussian autoregressive process. Similarly, Westner and Madlener (2010) applied a specific spread[4] copula-based real options approach in order to determine if an investment project of a power generation plant should work without heat utilization technology or should it be a plant with combined heat-and-power (CHP) generation. They showed that power plants with CHP generation present a lower real option value than those without heat utilization.

As noted, the approaches in copula modeling instead of working with the underlying assets, they consider the combination of financial options and/or real options. On the other hand, the described option valuation approaches commonly use the spread to describe the dependence structure of the underlying assets. The present work describes a methodology to deal with the volatility of two or more underlying assets in the context of the ROA. First a TGARCH model is applied to each individual asset to get their volatility structure and then a copula is fitted to the filtered residuals to account for the association among the assets. The results are then applied to valuation of the most ambitious investment project in the last 40 years in the Mexican energetic sector. The project is known as “Los Ramones Natural Gas Pipeline” and is designed to transport natural gas from the U.S.A.-Mexico border (between Texas and Tamaulipas) to Aguascalientes, Querétaro and Guanajuato. The Mexican energetic State company, *Petróleos Mexicanos* (PEMEX), is in charge of the project and expects this project to materialize the potential of the new Mexican energy sector legal framework: supplying more energy and reducing its

[2] Also known as a refinery spread, refers to the purchase (sale) of crude oil against the purchase (sale) of refined petroleum products.
[3] It refers to the comparison between electricity and natural gas prices.
[4] Is the difference between the price of the output (electrical power) and the costs of the input factors (e.g. fuels), that is, the contribution margin that a plant operator earns for converting fuels into power.

cost[5]. To the best of our knowledge, there has been yet no application of Copula-GARCH methodology into the ROA context considering the effect that two (or more) underlying assets have over the value of a project. Therefore, the intended contribution to the abovementioned literature is to illustrate the application of the proposed methodology for an expansion real option project in Mexico.

The work is organized as follows, Section 2 revises some facts on real options, Section 3 presents basic theory and important results on copulas, Section 4 elaborates on the treatment of volatility; the methodology is presented in Section 5, the PEMEX project los Ramones is described in Section 6, results and concluding remarks are presented in Section 7 and Section 8 respectively.

2. Real Option Analysis

As mentioned in the Introduction, Mun (2006) states that ROA is a technique that systematically incorporates financial theory, economic analysis, management science, decision science, statistics, and econometric modeling into the application of the option pricing theory as a valuation tool of real assets.

He developed an eight-step framework for its implementation under a dynamic business environment characterized by uncertainty, flexibility, and strategic investment decisions. The main goal is to internalize the business conditions into the investment project valuation process. It drops the rigid assumptions made on traditional DCF approaches, and provides a sound statistical tool that identifies multiple decision pathways and optimally selects one. Being able to adapt decisions as new information presents itself helps the manager to reduce the risk in which the company is incurring by continuing the investment project.

The following advantages represent the arguments to think that the ROA has the potential to narrow the breach between strategic management and capital market theory:

- Obeys the law of one price; *eliminates arbitrage possibilities*.

[5] PEMEX (2015)

- Uses market information[6] as inputs in order to provide quantitative analysis for sensibility, uncertainty, and volatility; *eliminates estimation problems*.
- Combines the value of the financial and real options along with the manager's skills and the company's strategy; *promotes a comprehensive management*.
- Values and makes a strategic distinction between the initial investment opportunities and the additional embedded in it; *eliminates the pre commitment notion of investments*.
- Flexibility, presented in multistage investments, is explicitly taken into account as it maps out the relevant courses the project can follow and identifies the optimal path to follow in each period; *enhances the scenario perspective*.
- Accounts for different levels of risk incurrence in the cash flow evolution and *avoids discretionary risk selection of the real assets*.

Trigeorgis (1993a), in order to take advantage of the simplicity of the NPV as a valuation tool and enhance its properties by the ROA benefits, introduces the concept of Expanded Net Present Value (ENPV hereafter) defined as the addition of the traditional NPV along with the value added by the active (dynamic) management of the product.

$$ENPV = NPV + Real\ Option\ Premium$$

The ENPV quantifies the value of options resulting from active (dynamic) management and can be understood as a collection of real options (call or put) that take the gross project value of the DCF technique as an underlying asset. For this approach to work, two conditions are needed. First, the expansion has to be viewed as an option, that is, the NPV has to be compared to an optional ENPV accounting for the additional market value that comes from the flexibility. Secondly, in order to determine the option premium, an analogy between financial and real options has to be established; along with a proper clarification of assumptions that will be used. In other words, real options can be perceived as financial call options due to the fact that companies decide whether to undertake the investment today or in the future. As most investment projects and industry conditions are unique, there is no fixed methodology to find a financial analogy.

[6] Such as future prices, the standard deviation of the return rate of an underlying asset′ risk-free interest rates and equivalent probabilities.

The best solution for this is to construct one from the available information at the financial markets. Therefore, the treatment of the current value of the underlying asset, cost, time for decision, risk free rate and volatility is fundamental for the ROA to work properly, this and the characteristics of the project under consideration define the type of real option to be considered, Table 1 presents a summary of the characteristics, intrinsic value and notable works on some basic types of real options. The application we consider in Section 5 is an expansion real option with two underlying assets. In the financial context, an option that incorporates two or more underlying variables is known as a rainbow option. A similar approach has been undertaken in the ROA context by defining it as the real option whose payoff depends on several underlying assets.

In this perspective, the behavior of the involved underlying assets is compared, and the value of the project depends on a particular decision rule. The beginning of the rainbow option analysis can be traced back to the work of Margrabe (1978) in which he evaluated a European option to exchange one asset for another. His idea was developed by the work of Stulz (1982) where analytical formulas are presented for pricing a put and call European option when considering the maximum or the minimum of two risky assets. He transformed the double integral of the bivariate density function into a cumulative bivariate normal distribution. These results showed that a call option on the minimum of two risky assets, considering zero as its exercise price, can be evaluated with the same formula used to price an option to exchange one asset for another. Those results were extended by Johnson (1987) in order to define a solution for the general case of an option on several assets through an intuitive approach founded on the Black-Scholes formula. The inclusion of these ideas in the ROA context has led to an expansive research trend. Sødal, Koekebakker and Aadland (2008) modeled, under the valuation of a switching option context, the price spread as a mean-reverting process between the co-integrated dry and wet bulk markets for a combination carrier. Pimentel, Azevedo-Pereira and Couto (2008) used a high speed rail project to develop a partial differential equation model to address the impact multiple sources of uncertainty have over the optimal investment decision; hence, over the valuation process.

Table 1: Types of Real Options

Real Option	Characteristics	Intrinsic Value	Notable works
Option to defer	The investment opportunity can wait a specific amount of time in order to see if future conditions are favorable enough to undertake the project later on.	$c_d(S_T, T; K) = \max(S_T - K, 0)$	Ingersoll and Ross (1992), Paddock, Siegel and Smith (1988) and McDonald and Siegel (1986)
Option to switch inputs or outputs	The company has the opportunity to either use the same factors of production in order to produce another good or service, or produce the same good or service but with different factors of production.	$c_s(S_T, T) = \max(S_{2T} - S_{1T} - K, 0)$	Kulatilaka and Trigeorgis (1994) and Margrabe (1978)
Option to abandon	Market or economic conditions are not favorable anymore and, in order to stop the loss, the company has the opportunity to sale their assets in a secondhand market.	$c_a(S_T, T) = \max(S_T, V_T)$	Myers and Majd (1990)
Option to alter operating scale	The company has the opportunity to enlarge or reduce the size of the project in order to fit the new environment.	**Expansion** $c_e(S_T, T; \alpha, K) = \max(\alpha S_T - K, S_T)$ **Contraction** $c_c(S_T, T; \beta, N) = \max(\beta S_T + N, S_T)$ **Temporary shutdown** $c_x(S_T, T; X_T, C, a) = \max(S_T - X_T - a, S_T - C - a)$	McDonald and Siegel (1985), Trigeorgis and Mason (1987), Brennan and Schwartz (1985) and Pindyck (1988)
Time-to-built option	Also known as compound option or staged investment, is applied when a series of investments are planned and they could be manage and evaluated individually while its subsequent value depends on previous stages.	$c_{com}(S_{T_1}) = \max\left(K_1, c(S_{T_1}, T_2 - T_1; K_2)\right)$	Carr (1988), Majd and Pindyck (1987) and Trigeorgis (1993)
Growth option	Also known as interproject compound option, opportunity for the company to modify the original project structure in order to generate new products or processes from it.	Generation of new processes or products	Pindyck (1988) and Kester (1984, 1993)
Multiple interaction options	Opportunity to combine several of the abovementioned real options.	Combination	Brennan and Schwartz (1985), Kulatilaka (1995) and Trigeorgis (1993)

Initial approaches on the presence of multiple underlying assets only considered the European option case. Still, in the last twenty years, interesting advances have been offered for the American Option case, as Fu et al. (2001) expose. Most of the research considers pricing methods of options with a finite number of exercise opportunities, known as Bermudan options, as an approximation to the Americans. Tan and Vetzal (1995) analyzed how elements such as the nature, time to mature, correlation, and volatility of assets, modify the exercise region for American options on the maximum and minimum of multiple underlying assets. Barraquand and Martineau (1995) developed a numerical method that combined Monte Carlo simulation with a partitioning method for the underlying assets' space called Stratified State Aggregation. By doing so, an approximation of the prices of American securities with multiple underlying assets can be calculated; determining the exercise strategy. Longstaff and Schwartz (2001) developed a simulation model in order to have an approximation to the value of American options with multiple factors. They used the least squares Monte Carlo approach to estimate the conditional expected payoff of the option holder from cross-sectional information found on the market. Broadie and Glasserman (2004) introduced a stochastic mesh method for pricing American options whose value depends on multiple assets, providing bounds and confidence intervals for their results. García (2003) presents an extensive and detailed description of the different numerical methods in the American option pricing theory. Ibáñez and Zapatero (2004) introduced a Monte Carlo simulation method for pricing multidimensional American options, on the maximum of up to five underlying assets, based on the computation of their optimal exercise frontier.

In his doctoral work, Dockendorf (2010) developed two sequential rainbow option models, one for the best of two stochastic assets, and the other on the mean-reverting spread between two co-integrated assets. Dockendorf and Paxson (2010) incorporated two sources of uncertainty into the ROA valuation by working on the spread of two co-integrated variables into a continuous rainbow option model. Despite their work, few efforts have been made in order to develop a "rainbow real option" approach.

Recognizing and accounting for multiple underlying assets is only the first step to introducing a realistic perspective into the ROA analysis. For that to be introduced in the valuation model, the key determinant must be the treatment of the relationship that exists in the multiple underlying assets, either as a dependence structure or as a measure of association. Therefore, is required to include the effect that the interaction has over the valuation of investment projects. A critical element to address this requirement is to understand, and measure, the dynamic of the interaction of the underlying assets through the analysis of the co-movements of their processes.

Typically the multivariate normal distribution is used to describe this interaction. Yet, it restricts the association measure between margins, the covariance and correlation, to be linear. That is far from being a realistic characterization. When working with derivatives, three main problems appear: the analysis has to move away from the assumptions of normality and market completeness, along with the presence of credit risk. In other words, the traditional tool does not fit in the typical characteristics of the financial world: uncorrelated but dependent returns, heavy tailed and asymmetric distributions, volatility effects, along with the presence of clusters. The following section will describe a sophisticated, yet accurate, tool that will help to deal with these caveats: copula modeling.

3. Copula Modeling

The association between random variables has been one of the most studied concepts in statistics, probability, and therefore, in the financial context. In order to characterize the nature of the dependence structure between financial time series, measures of association are needed. This is usually done through the Pearson coefficient widely known as the linear correlation measure. Its characteristics and assumptions fail to be applied in the financial environment, especially the ones regarding nonlinearity and non-normality properties of the series. Embrechts, McNeil and Straumann (2002) provide an eloquent and detailed coverage of the dependence concept and its treatment trough copulas. As for its application in the financial context, Chen, Fan, and Patton (2004) provided two simple goodness-of-fit tests so as to apply copula models into multivariate financial time series. Copula modeling represents an appropriate alternative as they offer a non-parametric, scale-invariant measure, which is independent from the margins' distributions. It becomes clear that, in trying to maintain the intention of the ROA, the inclusion of other association measures is needed. Due to its characteristics, copulas constitute an alternate measure of stochastic dependence, which addresses the limitations of the correlation as a dependence measure.

a) Definition

The copula concept was introduced by Sklar (1959) as a tool to model the dependence between random variables, he states that if $F_j(x_j)$ is the CDF of a univariate continuous random variable X_j then $H(x_j) = C(F_1(x_1), \dots, F_n(x_n))$ is an n-variate distribution for $X = (X_1, \dots, X_n)$ with marginal distributions F_j for $j = 1, \dots, n$. Conversely, if F is a continuous n-variate CDF with univariate marginal $F_1, \dots, F_n$ CDFs then there exists a unique n-variate copula C such that $F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n))$. That means that any choice for the marginal distributions will be consistent with the copula approach, but also, that the resulting function will provide a separated description of the margins and their dependence structure. This conclusion represents an attractive feature that directs practitioners to its application in finance.

b) Measures of association

A fundamental element in dependence structures within the copula construction is the bound concept proposed by Hoeffding (1940) and developed by Fréchet (1957). Consider a copula $C(u) = C(u_1, \dots, u_d)$, the Fréchet-Hoeffding bounds are defined by

$$\max\left\{\sum_{i=1}^{d} u_i + 1 - d\,, 0\right\} \leq C(u) \leq \min\{u_1, \dots, u_d\}$$

Therefore, according to this definition, every bivariate copula has to lie inside the surface given by the lower bound (counter monotonicity copula) $C(u_1, u_2) = \max(u_1 + u_2 - 1{,}0)$ and the upper bound $C(u_1, u_2) = \min(u_1, u_2)$. The reason for this is the presence of extreme cases of dependency. Dependence properties and measures of association are interrelated. The most known scale-invariant measures of association are the Kendall's tau and the Spearman's rho rank correlation, both measures of concordance. The concordance concept describes that the probability of having simultaneous large (small) values for X and Y, is high, while having an opposite value is low. Two observations (x_i, y_i) and (x_j, y_j) from a vector (X, Y) of continuous random variables are said to be concordant if $(x_i - x_j)(y_i - y_j) > 0$, and discordant if $(x_i - x_j)(y_i, - y_j) < 0$. Similarly, two-random vectors (X_i, Y_i) and (X_j, Y_j) are said to be concordant if $P[(X_i - X_j)(Y_i - Y_j) > 0] > P[(X_i - X_j)(Y_i - Y_j) < 0]$.

Nonparametric statistics concentrate on the ranks of given data rather than on the data itself. Therefore, working with rank correlation leads to scale-invariant estimates that allow fitting copula modeling into the obtained data. The two most known rank correlation measures are the Kendall's tau and Spearman's rho, they measure the degree of monotonic dependence within the non-elliptical context. Since both are not moment-based correlations, manipulation over the variance-covariance structure is not permitted. As expressed by Nelsen (1999), the Spearman's rho can be interpreted as a measure of "average" quadrant dependence[7] while the Kendall's tau can be as an "average" of likelihood ratio dependence. In the present work we focus on the latter.

The Kendall's tau is defined as the probability of concordance minus the probability of discordance:

$$\tau = \tau_{X,Y} = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0],$$

Nelsen (1999) showed that it could be expressed as

$$\tau_C = \tau_{X,Y} = 4 \iint C(u,v)dC(u,v) - 1$$

As noted, the abovementioned integral is the expected value of the random variable $C(U,V)$ where $U,V \sim U(0,1)$ with joint distribution function C, that is:

$$\tau_{X,Y} = 4E(C(U,V)) - 1.$$

[7] X And Y are said to be positive quadrant dependent if the probabilities that they are simultaneously small (large) is at least as great as it would be if they were independent. That is, their joint probability at each point must be not smaller than the independence one (product) $F(x,y) \geq F_1(x)F_2(y)$.

For the definition of the Spearman's rho, three independent random vectors with a common joint distribution function H, with margins F and G, are considered; say $(X_1, Y_1), (X_2, Y_2), (X_3, Y_3)$. It is defined to be proportional to the probability of concordance minus the probability of discordance for a pair of vectors with the same margins while the components of another are independent; (X_1, Y_1) and (X_2, Y_3) for example. The representation in this case is:

$$\rho_C = \rho_{X,Y} = 3(P[(X_1 - X_2)(Y_1 - Y_3) > 0] - P[(X_1 - X_2)(Y_1 - Y_3) < 0])$$

Nelsen (1999) shows that it can be expressed as

$$\rho_C = \rho_{X,Y} = 12 \iint C(u,v)dudv - 3$$

As noted by Nelsen (1999) Spearman's rho is often called the grade correlation coefficient (population analogy for rank) and for a pair of continuous random variables X and Y is identical to Pearson's product-moment correlation coefficient for the grades of X and Y; that is, the variables $U = F(X)$ and $V = G(Y)$.

$$\rho_C = \rho_{X,Y} = \frac{E(UV) - {}^1/_4}{{}^1/_{12}} = \frac{E(UV) - E(U)E(V)}{\sqrt{Var(U)}\sqrt{Var(V)}} = E(UV) - 1.$$

As expressed by Nelsen (1999), the Spearman's rho can be interpreted as a measure of "average" quadrant dependence[8] while the Kendall's tau can be as an "average" of likelihood ratio dependence. Following the definitions of the four tail monotonicity conditions[9], made by Esary and Proschan (1972), the positive quadrant dependence can be strengthened by adding a non-increasing (decreasing) property to the function v. As noted by Capéraà and Genest (1993), the most relevant consequence of this is that the bounds for the Kendall's tau and Spearman's rho can be narrowed when one random variable presents a left tail decreasing behavior while the other shows a right tail increasing one.

[8] X And Y are said to be positive quadrant dependent if the probabilities that they are simultaneously small (large) is at least as great as it would be if they were independent. That is, their joint probability at each point must be not smaller than the independence one (product) $F(x,y) \geq F_1(x)F_2(y)$.

[9] Left tail decreasing, left tail increasing, right tail increasing and right tail increasing. For a further explanation of this implication refer to Nelsen (1999).

c) Copula Families

There are several copula families, in this subsection the relevant families for the work are briefly described.

Product: Also known as the *independence copula.* Is the simplest copula that can be found and is typically used as a benchmark as it depicts independence between the underlying assets u_1, u_2. It has the form,

$$C(u_1, u_2) = u_1 u_2$$

Where u_1 and u_2 take values in the unit interval f of the real line.

Elliptical: This class of copulas considers that u_1 and u_2 present elliptical distributions. The main argument is that, as they share most of the tractable properties of the multivariate normal distribution, it makes possible to model other forms of non-normal dependences, however, the main drawbacks when applying this class of copulas in finance are that they do not have closed form expressions, only consider one type of distribution for the margins, and, as they are restricted to have radial symmetry[10], the tail dependence cannot be modeled with them. As they can be easily parameterized by the typical linear correlation matrix, the most characteristic elliptical copulas are the Gaussian (normal) and the Student´s t-copula. The normal copula takes the form,

$$C(u_1, u_2; \theta) = \Phi_G(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \theta)$$

It can also be expressed as,

$$\int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi(1-\theta^2)^{1/2}} \exp\left\{\frac{-(s^2 - 2\theta st + t^2)}{2(1-\theta^2)}\right\} dsdt$$

Where Φ is the cumulative distribution function (CDF) of the standard normal distribution, and $\Phi_G(u_1, u_2)$ is the standard bivariate normal distribution with correlation parameter θ restricted to the interval $[-1, 1]$. In this copula, θ represents the usual linear correlation coefficient of the corresponding bivariate normal distribution.

The t-copula has the form,

$$C_{v,R}^{t}(u_1, u_2) = t_{v,R}\left(t_v^{-1}(u_1), t_v^{-1}(u_2)\right)$$

[10] The coefficient or degree of upper and lower tail dependence are equal.

Where R denotes the correlation of the margins and $t_{v,R}$ the CDF. For the bivariate case with two dependence parameters (θ_1, θ_2), ν degrees of freedom and correlation ρ, this expression can be written as,

$$C_{v,R}^{t}(u_1, u_2;\ \theta_1, \theta_2) = \int_{-\infty}^{t_{\theta_1}^{-1}(u_1)} \int_{-\infty}^{t_{\theta_2}^{-1}(u_2)} \frac{1}{2\pi(1-\theta_2^2)^{1/2}} \left\{1 + \frac{s^2 - 2\theta_2 st + t^2}{\nu(1-\theta_2^2)}\right\}^{-(\theta_1+2)/2} dsdt$$

Where $t_{\theta_1}^{-1}(u_1)$ denotes the inverse of the CDF of the standard univariate *t*-distribution with θ_1 degrees of freedom. This parameter controls the heaviness of the tails; noting that if $\theta_1 < 3$ variance does not exist and, with $\theta_1 < 5$, the fourth moment does not exist. The coefficient of upper tail dependence is increasing in θ_2 and decreasing in θ_1. Noteworthy is that, as $\theta_1 \to \infty$, the t-copula $C_{v,R}^{t}(u_1, u_2;\ \theta_1, \theta_2)$ approximates the Gaussian copula.

Archimedean*:* This type of copulas is one of the most used in financial applications mainly because they are easy to construct and allow working with a variety of dependence structures, because they are consistent with bivariate extreme value theory, they are inherently fitted to work with tail dependence; a key aspect in financial applications. Moreover, in contrast to elliptical copulas, they commonly present closed-form expressions. Because of this, in the present work three main one-parameter Archimedean copulas are used: the Frank, Clayton, and Gumbel copula families; with a particular emphasis on the last two as they exhibit asymmetric dependence. However, an extensive description of the entire set of this class can be found in Joe (1997) and Nelsen (1999).

The Frank (1979) copula takes the form,

$$C(u_1, u_2; \theta) = -\theta^{-1} \log\left\{1 + \frac{\left(e^{-\theta\, u_1} - 1\right)\left(e^{-\theta\, u_2} - 1\right)}{e^{-\theta} - 1}\right\}.$$

The dependence parameter θ may take any real value in $(-\infty, \infty)$. This copula is popular because it allows negative dependence between the margins, and θ is symmetric in both tails, akin to the Gaussian and Student-*t* copulas. Still, under this copula, the strongest dependence located in the middle of the distribution and, as pointed out by Embrechts, Lindskog and Mc Neil (2003), dependence in the tails tend to be weak in relation to the Gaussian copula. This suggests that the Frank copula is better suited for margins that exhibit weak tail dependence.

The Clayton (1978) copula takes the form,

$$C(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$$

In which the dependence parameter θ is restricted to $(0, \infty)$. As θ approaches zero, the margins become independent, that is, it gives the product copula. This copula family is not able to describe negative dependence. Because it exhibits strong left tail dependence (lower tail) and relatively weak right tail dependence, the Clayton copula is highly used in correlated risk studies.

Also known as the logistic copula, the Gumbel (1960) copula takes the form,

$$C(u_1, u_2; \theta) = \exp\left(-\left(\tilde{u}_1^\theta + \tilde{u}_2^\theta\right)^{1/\theta}\right)$$

Where $\tilde{u}_j = -\log u_j$ and θ being restricted to $[1, \infty)$. Similar to the Clayton copula, it describes positive association only. Yet, it presents strong right tail dependence (upper tail) and relatively weak left tail dependence. Consequently, it is an appropriate copula to be applied when u_1 and u_2 are expected to be strongly correlated at high values but less at low ones.

Tawn (1998) introduces an extension of the Gumbel copula, known as the asymmetric logistic copula or the Tawn copula, and has the following dependence function

$$A(t) = 1 - \beta + (\beta - \alpha)t + [\alpha^r t^r + \beta^r (1-t)^r]^{1/r},$$

where $0 \leq \alpha, \beta \leq 1$ and $r \geq 1$. This is not an archimedian copula, but rather an extreme-value one.

Among other characteristics, Tawn (1998) mentions the importance of this asymmetric copula for its application in risk management of investment portfolios and credit portfolios when extreme-values are considered, for example a financial loss due to meteorological events.

d) Estimation

One of the most challenging tasks in copula modeling is the correct method selection in order to fit observed market data. It is important to note that there is no defined rule for copula selection. When selecting a copula, it is very important to acknowledge that the nature of the dependence structure has to be the determinant argument in order to allocate a specific functional form to the relationship. As noted by Frees and Valdez (1998), identifying the appropriate copula family is not a trivial task. In most financial applications, the real challenge consists in finding a convenient distribution to fit some stylized facts expected for the underlying asset behavior. Due to the characteristics of the copula function, much of the classical statistical theory cannot be used as part of its estimation process. This is commonly developed in the bivariate iid context through asymptotic maximum likelihood estimation. The following methods, according to Cherubini, Luciano and Vecchiato (2004), are the most used in the literature: exact maximum likelihood, inference for the margins, canonical maximum likelihood and non-parametric. Therefore, the final selection is commonly derived from the analysis of several distribution functions and the comparison of which one yields the best fit according to the provided information.

Suppose that we observe n independent realizations from a multivariate distribution $\{(X_{i1}, \dots, X_{ip})^T \,|\, i = 1, \dots, n\}$, according to Sklar (1959) the distribution may be specified by p margins with probability distributions F_i, densities f_i and a copula with density C. Let β be the vector of marginal parameters and α the vector of copula parameters, according to Yan (2007) this can be estimated by one of the following procedures:

1. *Assume that the marginal distributions F_i are known*.

1.1 The vector of parameters to be estimated is $\theta = (\beta^T, \alpha^T)^T$ and the log likelihood function is

$$l(\theta) = \sum_{i=1}^{n} \log C(F_1(X_{i1}; \beta), \dots, F_p(X_{ip}; \beta); \alpha) + \sum_{i=1}^{n} \sum_{i=1}^{p} \log\, f_i(X_{ij}; \beta).$$

The caveat with this optimization problem is that as the dimension p gets large the computational effort required increases rapidly.

1.2 Joe and Xu (1996) proposed a two-stage method known as Inference Functions for Margins (IFM) that reduces the computational difficulty. In the first step, parameter β is estimated by $\hat{\beta}$, with the log likelihood function

$$l(\beta) = \sum_{i=1}^{n} \sum_{i=1}^{p} \log f_i\big(X_{ij}; \beta\big),$$

and then estimates the copula parameters

$$l(\alpha) = \sum_{i=1}^{n} \log C\big(F_1\big(X_{i1}; \hat{\beta}\big), \dots, F_p\big(X_{ip}; \hat{\beta}\big); \alpha\big).$$

2. *Not assuming any particular marginal distribution.*

This approach uses the empirical distribution function of each marginal distribution to transform the observations $\{(X_{i1}, \dots, X_{ip})^T \big| i = 1, \dots, n\}$ into pseudo-observations or ranks with uniform margins

$$\big(\widehat{U}_{i1}, \dots, \widehat{U}_{ip}\big) = \left(\frac{R_{i1}}{n+1}, \dots, \frac{R_{ip}}{n+1}\right),$$

where $R_{ij} = \sum_{k=1}^{n} I\big(X_{kj} \leq X_{ij}\big)$, I being the indicator function. The loglikelihood function in this case is

$$l(\alpha) = \sum_{i=1}^{n} \log C\big(U_{i1}, \dots, U_{ip}; \alpha\big).$$

Where the $U_{i1}, \dots, U_{ip}$ are obtained from the TGARCH models. As noted by Patton (2006, 2006a) and Chiou and Tsay (2008) this two-step perspective yields asymptotically efficient estimates. The present work uses procedure 2, when not assuming a particular marginal distribution, the details are described in Section 5.

4. Volatility

As mentioned before the main problem to address when dealing with two or more underlying assets in ROA is the volatility treatment. Volatility of the underlying asset, typically referred as σ, is the standard deviation of future cash inflows' growth rate associated with the project. It is used as a measurement of the risk incurrence inherent to the stochastic process of the underlying source of uncertainty. Davis (1998) formalizes some concepts for estimating a project's volatility and dividend yield when valuing options to invest or abandon a project. He establishes that the instantaneous rate of volatility of the project, σ_V^j, is directly linked to the one of the price of the project's output good, σ_S , via a positive elasticity term ε^j following the form: $\sigma_V^j = \varepsilon^j \sigma_S$. Noteworthy is the fact that, if there is not a correct treatment of it, managers can be tempted to manipulate the parameter σ_S in order to alter the value of the project.

Volatility treatment with multiple underlying assets is typically made through a spread perspective (σ_S), which is assessed by using a bivariate lognormal distribution with a constant correlation factor ρ, typically expressed as:

$$\sigma_S = \sqrt{\sigma_{f_1}^2 + \sigma_{f_2}^2 + 2\rho\sigma_{f_1}\sigma_{f_2}}$$

Where σ_{f_1} and σ_{f_2} represent the volatility measure of the individual underlying assets. Copeland and Antikarov (2003) noted that the standard deviation for each asset can be estimated from the residuals of the individual time series regression. However, they must be adjusted as confidence bands widen for out-of-sample forecasts. On the other hand, Mbanefo (1997) pointed out that, typically in spread option valuation models, the volatility and the co-movement structures are not treated adequately when analyzing two underlying assets. Most of the assumptions made in the spread models[11] are not adequate when applying them in practice, particularly in the energetic industry. Therefore, he suggests that a special treatment over these elements is required in order to have a better implementation of these models.

It is widely known that financial time series exhibit clustering and negative correlation with returns (leverage effect), to account for this Engle (1982) first introduced the autoregressive conditional heteroskedasticity model ARCH, and Bollerslev (1986) introduced the Generalized autoregressive conditional heteroskedasticity models GARCH. Instead of working with the basic GARCH model, in the present work we make use of the threshold GARCH (TGARCH hereafter) model. Acknowledged after the work of Zakoian (1994), but also developed by Glosten, Jagannathan, and Runkle (1993), it is commonly

[11] Such as considering that the difference of two correlated lognormal variables is also lognormal.

used to handle the leverage effect presented in financial time series. A $TGARCH\ (m, s)$ model assumes a similar ARMA structure than the GARCH model, but the process for the volatility takes the form:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} (\alpha_i + \gamma_i I_{t-i}) u_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2$$

Where I_{t-i} is an indicator for negative u_{t-i}, that is,

$$I_{t-i} \begin{cases} 1 & if\ u_{t-i} < 0 \\ 0 & if\ u_{t-i} > 0 \end{cases}$$

α_i, γ_i and β_i are non-negative parameters satisfying conditions similar to those of GARCH models. From the model, it is seen that a positive u_{t-i} contributes $\alpha_i u_{t-i}^2$ to σ_t^2, while negative u_{t-i} impacts in $(\alpha_i + \gamma_i) u_{t-i}^2$ with $\gamma_i > 0$. This way the indicator will capture the leverage effect of the financial series; a missing consideration in GARCH models. For the empirical application, both series considered a $TGARCH\ (1, 1)$ model following the form:

$$r_{it} = \mu_i + u_{it}$$

$$u_{it} = \sigma_{it} \varepsilon_{it}$$

$$\sigma_{it}^2 = \alpha_{i0} + \alpha_{i1} u_{it-1}^2 + \ \beta_i \sigma_{it-1}^2 + \gamma_i I(u_{it-1} < 0)$$

Where innovations ε_{it} are assumed to follow a standardized t-student distribution with v degrees of freedom. The joint volatility treatment will be discussed in detail in the next section, where the methodology is described.

5. Proposed Methodology

The objective of this work is to exploit the advantages of the volatility treatment, through GARCH models, and the dependence structure determination, through copula modeling, and apply them in the ROA context. The general idea of this methodology can be summarized in three steps. First, individual TGARCH models are use to determine the volatility and terminal value of the underlying assets. In a second step, their residuals are calculated and used to determine a measure of association between the underlying assets, be it the Kendall tau and the Spearman rho.

Finally, the information gathered in the two previous steps is taken as input for a model of Real Option Valuation. In order to do so, a last element is needed: the joint volatility. As mention before, the volatility treatment with multiple underlying assets is typically made through a spread perspective. Instead of working with the volatility of the spread of two variables, this work considers the volatility of the product of these two variables; being this, the main novel element in the Copula-GARCH real option literature. Its treatment then is made through:

$$Var(xy) = Cov(x^2, y^2) - 2\rho\, \sigma_x \sigma_y\, \mu_x\, \mu_y - \rho^2 \sigma_x^2 \sigma_y^2 + \sigma_x^2 \sigma_y^2 + \sigma_y^2\, \mu_x^2 + \sigma_x^2\, \mu_y^2$$

Where μ_x and μ_y represent the expected value of the log-return series, σ_x and σ_y the volatility measure of the individual underlying asset, obtained from the TGARH models, and ρ is the measure of association obtained through copula modeling (Spearman's rho). In practice, most real option problems must be solved using numerical methods. In terms of the valuation procedure, generally there are two types of numerical techniques that are used: the ones that directly approximate the stochastic process of the underlying asset, and those approximating the resultant partial differential equations (PDE's). The most representative solutions under the ROA perspective are: path-dependent simulation (typically Monte Carlo), closed-form models, PDE's, and binomial/multinomial approaches. The advantage of the solutions is that they not only provide a value for the project, but also illustrate the optimal strategy to follow in the investment opportunity. Their selection relies on the project's characteristics; the most straightforward solution can be found in the binomial/multinomial approach while the Monte Carlo simulation and PDE's are, somewhat, more complicated methods.

As a summary, for the purpose of this work, an initial estimation of a TGARCH (1,1) model was performed on the individual log-return series of each underlying asset, the natural gas price and the USD-MXN exchange rate. Their resulting residuals series and individual volatilities were saved and use for the copula-fitting step. For it, the libraries rugarch, copula and vine-copula from R software were used to estimate six copulas:

Normal, Student's t, Clayton, Gumbel, Frank and Tawn. Finally, the information obtained in the previous steps is being used as inputs in the ROA context for the valuation of an expansion real option in order to determine its feasibility.

For the final step, as suggested by Brandão et al.(2005), the binomial option valuation model will be used to determine the value of the investment project. This discrete time lattice-based model was developed by Cox, Ross and Rubinstein (1979) with the intention to provide a simple representation of the evolution of the underlying asset value and how it generates a change in the option's value. Is noteworthy that this multiplicative binomial model of uncertainty, for European options without dividends, converges with the Black–Scholes formula values as the number of time steps increases. It is neither the scope of this section, nor this work, to compare the abovementioned tools, but to illustrate how the valuation process is taken under this perspective.

6. Project Description: Los Ramones Natural Gas Pipeline[12]

Pemex Gas y Petroquímica Básica (PGPB hereafter), a branch of *Petróleos Mexicanos* (PEMEX hereafter), through its subsidiary, Mex Gas International Enterprises, Ltd (MGI), is considering the investment project to expand the National Gas Pipeline System (SNG for its acronym in Spanish) by increasing the natural gas distribution in the country; mainly in the central-west area. The project is known as "Los Ramones Natural Gas Pipeline" and is designed to transport natural gas from the U.S.A.-Mexico border (between Texas and Tamaulipas) to Aguascalientes, Querétaro and Guanajuato. PEMEX estimates that the project will provide approximately 23% of the natural gas consumption of the Midwest region, encompassing the states of Aguascalientes, Colima, Guanajuato, Jalisco, Michoacán, Nayarit, Querétaro, San Luis Potosí and Zacatecas.

PEMEX is the 15th world's producer of natural gas however, one of its main obstacles, is the national distribution of this energetic. In order to reduce that problem, the investment project was designed in two phases. In a broad perspective, Los Ramones I

[12] For simplicity and data comparability, all the information presented in this section was taken from public documents provided by PEMEX as of October 2015.

focuses on the transportation of the natural gas imports from United States to the northern part of the country, while, Los Ramones II, on the distribution of both, national and foreign production, to the Midwest. For the construction of Los Ramones I, PGPB signed a long-term transport service contract with the company *Gasoductos del Noreste* to build a, 48 inches in diameter and 116.4 kilometers in length, pipeline running from Agua Dulce, Texas to Los Ramones, Nuevo León. The objective was to increase the transport capacity by 1.0 billion cubic feet per day (Bcfpd) and its operation started in December 2014. An additional construction of two compression stations was made in order to rise the capacity up to 2.1 Bcfpd. As of the present date, the construction of the compression stations is almost finished and they are scheduled to be operational by December 2015. PEMEX reported that total investment for the construction of the pipeline and the two compression stations is approximately $587 million USD.

The second segment of this investment project focuses on the supply of natural gas to Central and Western Mexico, running from Los Ramones, Nuevo León to Apaseo el Alto, Guanajuato. For construction, financing and operative purposes, the project has been divided into two sections: Los Ramones II North and Los Ramones II South. TAG Pipelines, an indirect subsidiary of PGPB, is in charge of the development of the second segment through joint ventures for each of these stages.

Los Ramones II North is estimated to run from Los Ramones, Nuevo León to San Luis Potosí, San Luis Potosí for a total of 452 km and a $1,287 million USD investment. Los Ramones II South is estimated to be a 291 kilometers pipeline that will run from San Luis Potosí, San Luis Potosí to Apaseo el Alto, Guanajuato. The expected investment requirement is $873 million USD and operations are estimated to initiate in 2016 for the northern section and 2107 for the southern.

The construction of both sections is scheduled to be somewhat simultaneous, eliminating the possibility of analyzing Los Ramones II North and Los Ramones II South as time-to-built real option. Therefore, the investment project is considered as a whole and its valuation is performed under the expansion real option context. Table 2 summarizes the three segments of Los Ramones investment project[13]. The project valuation is conducted considering the situation of Los Ramones II (North and South) compared to Los Ramones I. For that the expansion option will be performed using a factor of 1.3, as we are considering compression capacity as a measure of distribution power, and a cost of expansion of $2.16 billion USD.

Table 2: Los Ramones Natural Gas Pipeline Project Summary

Phase	Description	Estimated Cost	In-service by
Los Ramones I	**USA-Mexico Border – Los Ramones** 116 kilometers of 48-inch pipeline **Additional Compression** **Extra capacity: 2.1 Bcfpd**	$587 million USD (Combined)	Dec 2014: Pipeline Dec 2015: Compression Stations
Los Ramones II North	**Los Ramones – San Luis Potosí** 452 km of 42-inch pipeline **Additional Compression** **Extra capacity: 1.43 Bcfpd**	$1,287 million USD	2016
Los Ramones II South	**San Luis Potosí – Guanajuato** 291 km of 42 and 24-inch pipeline **Extra capacity: 1.353 Bcfpd**	$873 million USD	2017

Source: PEMEX

a) Data Description

The two underlying assets used in this work are the United States Dollar (USD)-Mexican Peso (MXN) exchange rate and the natural gas price (NGP hereafter). The choice of these

[13] PEMEX is considering additional investments after the completion Los Ramones II to be performed between 2018-2022. However, as no public information has been released describing them, for the purpose of the present work, those investments are not considered.

variables derives from their relevance in the Mexican energetic industry. The MXN-USD exchange rate is a key determinant in project evaluation in the country because, due to its geographic location and investment dynamic, most of the information used and presented is commonly expressed in USD rather than MXN. This work uses the indirect quotation (USD-MEX) of the monthly FIX average quote, from January 2001 to October 2015, in order to homogenize, in USD terms, the variables used in the valuation process.[14]

The value of the NGP considered in this work is the price of U.S. Natural Gas Pipeline Exports. It is being use as a proxy to the Mexican NGP because the gas industry in Mexico is not sufficiently developed in order to carry out the entire transformation process of it; as a consequence, most of the natural gas consumed in the country is being imported from Southern Texas. Therefore, Mexican NGP presents a close behavior with the movement of the U.S. NGP. In addition, as the Energy Ministry of Mexico does not keep a record of the evolution of the NGP prior to 2007, there is not a consistent time series for the Mexican case. The price used is a monthly publication by the US department of energy[15] and is expressed in USD per thousand cubic feet.

In order to capture the nature of the financial time series, the price of the underlying assets were computed as log-return rates, following the form:

$$r_{it} = \ln(P_{it}) - \ln(P_{it-1})$$

Where i represents the underlying asset and P_{it} its price in period t.

7. Results

Los Ramones II (North and South) natural gas pipeline investment project can be seen as an option to alter operating scale; particularly an expansion option. This work is considering the product of two underlying assets, therefore, the intrinsic value of this option can be seen as:

$$Max\big((Expansion * e_{USD-MXN} * P_{NG}) - Investment,\ Continue\big)$$

[14] The FIX average exchange rate is the market reference exchange rate in Mexico and is published by the Bank of Mexico (BANXICO).
[15] EIA (2015)

According to the information published by PEMEX, the expansion factor used in the valuation is 1.3 and the investment cost $2.16 billion USD. The valuation period considered in this analysis is from 2015 to 2020 with a market return rate of 12%[16].

Table 3: Summary of Estimated Results.

Copula	Normal	Student t	Clayton	Frank
Spearman's rho	0.046734	0.047175	0.069949	0.323768
Kendall's tau	0.029763	0.030044	0.033793	0.035937
Joint Volatility	0.002630	0.002630	0.002625	0.002461
ROA Value USD	$2,939,460.05	$2,940,030.58	$2,976,154.52	$4,690,431.15
ROA Value MXN[17]	$48,689,216.24	$48,698,666.51	$49,297,023.54	$77,692,301.52

Table 3 shows the results obtained by the proposed methodology for the Normal, Student's t, Clayton and Frank copulas, as mentioned in previous sections six copulas were fitted, as originally suggested by the optimization algorithms and functions already incorporated in R software: Normal, t, Clayton, Gumbel, Frank and Tawn; however, the Gumbel and the Tawn are not further reported as when performing the goodness of fit tests, they do not provide a good fit with the available information of the project. It is interesting to observe that, even though Normal and Student's t copulas do not capture tail dependence, the result is very similar to the one from the Clayton copula; used when variables exhibit lower tail dependence. Both cases yield a positive ROA valuation, indicating that the project should be accepted. The case of the Frank copula is also peculiar as in this type of copula the strongest dependence is located in the middle of the distribution and, therefore, is known to be better suited for margins that exhibit weak tail dependence. When using the results of this final copula, the ROA value is around 60% bigger than with the other three.

The original objective of this work was to compare the NPV resulting from PEMEX's procedure with the valuation result from this methodology. The expected result for this comparison was to illustrate that, by combining the advantages of the Copula-TGARCH

[16] Consider by PEMEX as the generally accepted value for energetic investment projects in Mexico.
[17] Considering a 14.5129 MXN per USD exchange rate, value at January 2015.

modeling into the ROA context, a higher value for the investment project will be obtained. Unfortunately, despite our efforts to obtain such information, at the time this project was finished the information was not available, as PEMEX argued that as it is an strategic ongoing project, no official information could be published in terms of the valuation results. Is in our best interest to perform this missing comparison but, as in real options, we have to defer that plan until information is available.

8. Concluding Remarks

The proposed methodology is an alternative process to value investment opportunities that seeks to harness the benefits of the ROA, T-GARCH and Copula models. The main argument is that the three components are best fitted to capture and describe the nature of the financial series. It has been established that the ROA perspective outperforms the traditional valuation techniques as it incorporates flexibility, uncertainty, irreversibility, discipline and strategic perspective into the valuation process. Special attention has been paid to the volatility treatment as it is a fundamental variable in the investment project valuation. For doing so, a T-GARCH model was used as it outperforms the traditional volatility models by incorporating the clustering and leverage effect into its value. On the other hand, copula modeling enables the establishment of a correlation structure for variables that are not normally distributed. It provides a flexible tool to analyze nonlinear and asymmetric dependence structure between markets and risk factors, preserving the specification of the individual marginal distributions and eliminating their influence in the joint structure. The bivariate Gumbel and Clayton copula are useful to work with variables that present tail dependence; a main focus of risk management.

The objective of this work is to exploit the advantages of the volatility treatment, through GARCH models, and the dependence structure determination, through copula modeling, and apply them in the ROA context. By implementing a Copula-TGARCH model, the treatment for the volatility and terminal value of the margins is made, in a first step, through TGARCH individual models. Afterwards, copula modeling is used to determine a measure of association between them in order to define their joint volatility. The proposed methodology suggests a third step that uses the previous information as inputs in the ROA context for the valuation of an expansion real option. For the empirical section, we applied the proposed methodology in the valuation of the Mexican natural gas pipeline expansion project “Los Ramones”. Our results suggest that the project should be taken as with four of the copulas (Normal, Student’s t, Clayton and Frank) the value for the real option is positive.

Even though notable contributions are found in order to develop a “rainbow real option” approach, to the best of our knowledge, there has been no application of a Copula-TGARCH methodology into ROA pricing context considering the effect of two underlying assets. Instead, most of the existing work uses the combined effect of two (or more) real options. Moreover, when working with volatility, the common treatment is to perform a spread analysis. The novel of this work is to consider the volatility of the product of two variables, in order incorporate their combined effect over the joint volatility in a linear and non-linear sense; consistent with the Copula-TGARCH model.

In order for this methodology to be improved, some final recommendations must be established. The use of high frequency data is consistent with the intention of this procedure. Working, for example, with daily information will enhance the properties of the Copula-TGARCH model. For this work, this type of information was not available for the Mexican Natural Gas Price due to the limited data infrastructure on the matter. We highly encourage future research to focus on this. After reviewing and comparing the energetic investment opportunities in the world, the Mexican energetic sector presents lags in terms of the development of projects with a real option perspective; expansion or contraction projects are predominant in the country. New types of real options should be considered in the sector to reinforce its strategic perspective.

Finally, two main expansions are suggested for this methodology. The methodology used for a bivariate case can be directed to develop multivariate Copula-GARCH models for real option analysis. By doing so, the number of relevant variables considered in the analysis increases. If done correctly, this clearly enlarges the possibility of capturing their effect in the value of the project. Also, the Copula-GARCH model (in any form) can be enriched by the addition of a discount rate model that adequately estimates and captures the nature of the energetic industry; particularly the Mexican. This will eliminate the arbitrary selection of a discount rate by the manager, increasing the possibilities of estimating a value for the project that completely, or at least in the maximum possible way, captures and reflects the characteristics of its financial environment.

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