

Optimal Strategy for Equipment Replacement : An application to R&D equipment case in Korea

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Abstract— Optimal equipment replacement is one of the important problems in engineering economy. Traditionally the economic life of equipment which can be calculated on the basis of discounted cash flow methodology has been regarded as a fundamental solution concept of the optimal equipment replacement problems. However it has been frequently pointed out that the concept of the economic life of equipment based on the discounted cash flow has a critical weakness such that it cannot consider uncertain factors which would affect the decision making of equipment replacement. The present paper proposes a new model of optimal equipment replacement under uncertainty by which the economic life of equipment considering technological and economic uncertainties can be calculated using real option approach. Furthermore we applied the model to determine the economic life of the public medium and large sized research equipment in Korea. In this empirical application, the real option methodology is used to consider the uncertainty of R&D projects and the models considered the discontinuous advancement of research equipment technologies in finite spans. According to the results of empirical analysis, it is found that if the uncertainty of R&D projects is high, the life cycle of research equipment will vary greatly, which indicates that our models are suitable for highly uncertain R&D environments. These models and analysis results seem to be helpful to institutions that need to calculate replacement periods in relation to the deterioration of research equipment.

Index Terms—*Equipment Replacement; Economic Life; Research Equipment; Real Option; Uncertainty*

I. INTRODUCTION

Recently, as scientific technologies have been modernized and upgraded, medium and large sized research equipment has been regarded as a prerequisite condition for the maximization of the outcomes of R&D [1]. The governments of many countries also perceived the importance of research equipment as such and have been making massive investments for research equipment. According to the ‘report of the 2011 survey of the actual conditions of operation and management of national research facilities and equipment (2013)’ published by the National Science & Technology Commission, the Korean government invested approximately 535.3 billion USD in national R&D projects over the last five years from 2007 and out of the amount, approximately 4.5 billion USD was invested in research equipment. This amount corresponds to 11.9% of the entire R&D project cost. In addition, yearly investments in research equipment have been maintained at a certain percentage of R&D project costs indicating that investments in research equipment have come into the stage of stabilization. Given the importance of research equipment, efforts for systematic construction and efficient management of research equipment can be said to be required at this time point. Because if systematic research equipment management measures are not prepared, investments cannot be effective no matter how large investments are made in research facilities and equipment. However, inefficiency is still shown in terms of the utilization of research equipment. Therefore, many relation studies are considered.

In particular, in the case of the replacement of research equipment that is made with investments in the public sector, reasonable grounds should be presented to stakeholders because large investments should be made in a lump unlike repairs or maintenance. That is, scientific grasping of the present conditions and the establishment of efficient policies are necessary for administration for the replacement of research equipment. However, history data or assessment techniques necessary for the calculation of replacement periods in relation to the deterioration of research equipment have not been clearly presented and thus related administrative burdens are complained of in the first lines. Furthermore, since research institutes or supervisory institutions simply use existing cases or legal service life as grounds for life for replacement and thus related problems are being pointed out. If the practice to simply determine the time of replacement deterministically

when research equipment replacement plans are established is continued due to the lack of systematic methodologies, errors cannot but be made in future establishment and execution of long-term replacement plans later because of distorted estimation of replacement costs.

The life of equipment varies with the condition of operation of the asset and how the asset is seen. However, if the necessity of use of equipment exists technically and there is no great risk against safety or the equipment can be continuously repaired, the economic life will be generally the criterion for the replacement of the equipment. Therefore, to analyze the issue of reasonable replacement of obsolescence equipment, knowing the economic life of each equipment asset is important. As practical studies on equipment policies and equipment replacement began after a study conducted by [2], Bellman proposed dynamic programming that can be used to solve equipment replacement problems under general technical changes for the first time in the middle of the 20th century [3]. In a this study, a series of decision making issues of 'maintaining' and 'replacing' were quantified in an attempt to calculate optimum replacement periods of assets held in determinative situations [4].

Thereafter, economic life analyses have been conducted in relation to equipment in diverse areas such as fork lifts [5], cars or buses [6, 7, 8], medical equipment [9, 10], escalators [11], aircraft [12], and computation equipment [13]. However, no study has been conducted yet in relation to the replacement or economic life of research equipment. In this respect, the present study is intended to develop models for analysis of the economic life of research equipment considering the characteristics of research equipment. Most of previous studies analyze the economic life of equipment in determinative situations [4, 9, 14, 15]. However, since diverse variable considered for economic life sometimes have uncertainty, optimum solutions for determinative problems may not be appropriate [16]. To overcome this problem, measures to consider the uncertainty of variables are necessary and one of representative measures to that end is real option methodology. In the present study, binomial models will be used among such real options to analyze the economic life of research equipment. After introduced the concept of real options for the first time in explanations regarding strategic plans and financing activities [17], the option theory has been applied to many areas such as the assessment of diverse financial securities and businesses based on the theory. Recently, movements to apply the real option methodology to the issue of equipment replacement have been in progress to consider the uncertainty of some variables.

Recently conducted studies on real options and related equipment replacements have been mostly centered on cost oriented continuous models [18, 19, 20]. To review related recent studies, a study conducted by [19] used a continuous real option model to show general analytical solutions and certain numerical solutions based on partial differential equations. To this end, two-factor models were made in terms of costs for equipment using Brown process. Reference [20] used the real option methodology to consider the uncertainty of lead time considering the characteristics of the area of heavy mobile equipment. Unlike the foregoing, [21] considered uncertainty resulting from technical advancement using continuous models. Unlike earlier studies, this study assumed that both the aspect of costs and that of benefit could bring about uncertainty and thus used multi-factor real option models. In addition, [10] used binomial models instead of continuous models. Economic life analysis models using binomial models are easily understandable and can be conveniently revised and applied to fit purposes. In addition, binomial models have an advantage that they enable obtaining diverse kinds of information to support decision making. To this end, costs of future equipment were assumed as a basic asset that indicates uncertainty and penalty costs were considered as a means to quantify losses due to equipment operation stops. However, no paper that has studied binomial models of real options to consider both cost and benefit elements has been seen yet.

The present study is intended to develop analysis models for the economic life of research equipment that are applicable to research equipment, consider the economic elements of both benefit and costs, and use real options to apply environmental factors that comprise the uncertainty of R&D project and the advancement of technologies. To this end, in chapter 2, the operating and maintenance costs of medium and large sized research equipment are estimated through investigations of cases of government supported research institutes in Korea and questionnaire surveys. In chapter 3, economic life analysis models are developed using real options to consider the uncertainty of R&D projects and in chapter 4, numerical examples using the models are presented. Finally, in chapter 5, conclusions are discussed.

II. ESTIMATION OF OPERATING AND MAINTENANCE COSTS

Reference [22] divides costs of replacement studies largely into acquisition costs, operating costs, and maintenance costs and classifies these costs into 31 cost elements. Whereas equipment acquisition costs are incurred only once at the beginning, operation and maintenance costs are incurred continuously as long as the operation of equipment is continued. Operating and maintenance costs include costs to supply articles necessary for operating and maintenance works, extra expenses, repairing expenses, insurance premiums, taxes, and indirect costs that correspond to current expenses and the amount may be quite large to the extent that it may be similar to initial costs in some cases. However, operating and maintenance costs are different from initial costs in that they are continuously incurred as long as the equipment is continuously operated. In addition, depending on the characteristics of the subjects of analysis, among cost elements, unnecessary elements may be disregarded and special elements may be added in some cases.

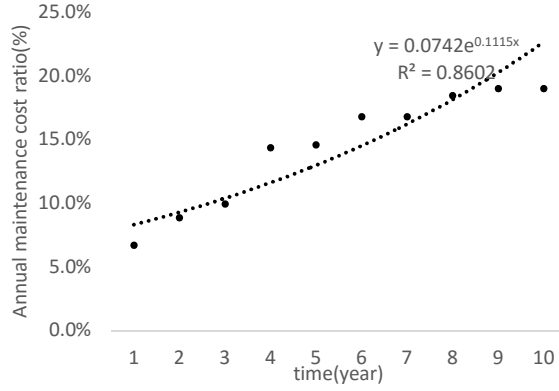


Figure 1. Annual maintenance cost ratio of research equipment

According to the results of investigations, as the scale of research equipment increased, equipment maintenance costs increased compared to personnel expenses or facility maintenance costs. In the present study, medium and large sized research equipment units are the subjects of analysis. Considering this fact, research equipment operating and maintenance costs were assumed to be equipment maintenance costs and to secure basic data for analysis of maintenance costs, questionnaire surveys were conducted on persons in charge of equipment in a total of 10 institutions such as the Korea Institute of Science and Technology and Korea Basic Science Institute. The questionnaire asked yearly ratios of maintenance costs to equipment acquisition costs from ideal viewpoints and research equipment maintenance cost ratio functions were determined through the questionnaire. R^2 was identified to be relatively significant when research equipment maintenance costs were assumed to exponentially increase over time and maintenance cost ratios were estimated in the form of exponential functions and the resultant coefficient values are as follows.

III. ECONOMIC LIFE ANALYSIS MODEL USING REAL OPTIONS

A. Benefit of Research Equipment

Research equipment is a means of R&D projects. To consider the benefit of research equipment that is as means, the economic value of R&D projects that are the purposes should be considered. R&D benefit can be evaluated as economic value, and if the benefit of research equipment is regarded as only part of the economic value, it can be calculated by multiplying the economic value of R&D projects by the contribution ratio of the equipment. In addition, it is assumed that if an R&D project is successfully implemented, the economic value of the R&D project will be increased at a certain rate. A binomial tree considering it is as follows.

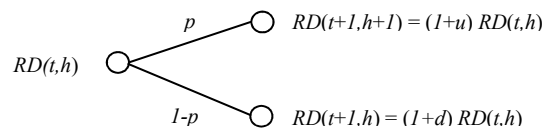


Figure 2. Binomial tree for R&D

where, in $RD(t, h)$ refers to the outcome of R&D in year t . h refers to the number of times of success of R&D after the initial year. The outcome of R&D in year $t + 1$ can be regarded as increasing by u if the R&D project is successful during the unit period and decreasing by d if the R&D project fails. $RD(0,0)$ refers to the outcome of the R&D in the initial year and the unit period of the outcome of R&D was set as one year.

If the benefit of research equipment is determined by the economic value of R&D projects, the contribution ratio of research equipment can be considered as the ratio of the economic value of R&D projects to the benefit of research equipment. Although the contribution ratios of research equipment units with the same performance to R&D projects can be generally considered to be the same, if the performance of a research equipment unit is changed, the contribution ratio

of the research equipment unit will be also changed. If discontinuous technical advancement is assumed, the contribution ratios of research equipment units can be indicated.

$$r_t = (1 + \alpha)^{t-j} r_j \quad (1)$$

where, r_t is the contribution ratio of the research equipment units purchased at time t to the outcome of R&D. Equipment units' contribution ratio is increased by annual increasing rate of contribution rate α of the contribution ratio r_j of the equipment units held. This model assumes that technical advancement will definitely appear. Such an assumption involves an assumption that the contribution ratio of new research equipment units will be higher than that of existing equipment units.

B. Cost of research equipment

If the components of the cost of research equipment are assumed to be initial investment costs and maintenance costs, the following costs of research equipment can be considered. Although the period of construction of research equipment is necessary in general cases, in the present study, it is assumed that there is no construction period for convenience of calculations. That is, if a decision is made to replace research equipment, the equipment can be immediately replaced by new research equipment.

First, the initial cost of equipment may increase [23, 24, 25] or decrease [26, 15] due to technical advancement. The initial cost of new equipment P_t can be modeled as follows in relation to technical advancement.

$$P_t = (1 + \beta)^{t-j} P_j \quad (2)$$

where, P_j refers to the initial investment cost of the research equipment held in j year and t refers to the period of replacement by new equipment. Equation (2) assumes that the investment cost of new equipment will be increased by annual change rate β of the initial investment cost.

In addition, under the assumption that the maintenance cost of research equipment will increase exponentially over time, the maintenance cost ratio function to the initial investment cost can be estimated as follows.

$$r_{OM}(k) = ae^{bk} + c \quad (3)$$

where, a , b , and c are arbitrary constants and indicates the period of use of the equipment held. In the case of this assumption, since technical advancement increases the initial price of research equipment, maintenance costs are also increased relatively.

C. Salvage value

In the case of issues of replacement of research equipment, costs that are incurred at the time of replacement should be considered. In the present study, the salvage value of research equipment is considered and we assumed to be the book value by declining balance depreciation method. In addition, other costs required for replacement of research equipment is assumed to be zero.

D. Analysis of decision making

In the present study, the uncertainty of the outcomes of R&D is considered in relation to the benefit of research equipment. In cases where research equipment is used for R&D, the problem of replacement of research equipment should consider all of the benefit, cost, salvage value that can be obtained through the research equipment, which can be considered as follows. As such, the analysis model for the economic life of research equipment presented in the present study is a binomial option pricing model that uses the economic value of R&D as its underlying asset.

In each period, to replace the equipment held, the decision maker may select between replacement and maintenance. This case can be modeled (5).

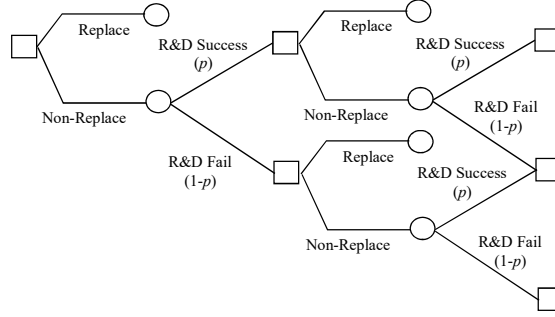


Figure 3. Decision tree of decision making

$$V(t, h, j, k) = \begin{matrix} R: RD(t, h)r_t - \{P_t + P_t \times r_{OM}(k)\} + S(j, k) + \gamma\{pV(t + 1, h + 1, t, 2) + (1 - p)V(t + 1, h, t, 2)\}; \\ K: RD(t, h)r_j - P_j \times r_{OM}(k) + \gamma\{pV(t + 1, h + 1, j, k + 1) + (1 - p)V(t + 1, h, j, k + 1)\}; \end{matrix} \quad (4)$$

where, $V(t, h, j, k)$ refers to the final option value, refers to the option value in case the existing equipment is replaced, and refers to the option value in case the existing equipment is maintained. In addition, t refers to the present period, h refers to the number of times of success of R&D, j refer to old equipment replacement period, k refers to research equipment use period, γ refers to cash discount factor, and p refers to the probability of success of the R&D. To have the option to replace with new equipment, the existing equipment should be held currently.

In cases where the existing equipment is replaced at the beginning of a year, the value of the option is determined considering the benefit and cost that may be obtained from and incurred due to the new equipment, the salvage value of the existing equipment, and the expected value of the cash flow that may occur one year later.

$$RD(t, h)r_t - \{P_t + P_t \times r_{OM}(k)\} + S(j, k) + \gamma\{pV(t + 1, h + 1, t, 2) + (1 - p)V(t + 1, h, t, 2)\} \quad (5)$$

On the other hand, in cases where the existing equipment is maintained at the beginning of a year, the value of the option is determined considering the benefit and cost that may be obtained from and incurred due to the equipment held and the expected value of the cash flow that may occur one year later. Given the option values set under (4), if the service of the equipment in initial year is j , the option value can be expressed as $V(t, h, j, k)$. Thereafter, if the initial equipment has been used 1 times in year $t+1$ when R&D has succeeded one time, the value of option of equipment replacement will be $V(t + 1, h + 1, j, k + 1)$ and if the initial equipment has been used 1 times in year $t+1$ when R&D has failed one time, the value of option of equipment replacement will be $V(t + 1, h, j, k + 1)$. Once the equipment has been replaced, the equipment cannot be replaced by new equipment any further and the option disappears. In the case of the project termination period, the final salvage value of the value of the option will be considered.

IV. NUMERICAL EXAMPLE

The assumptions considered in the present study for numerical examples are as follows. To identify the economic life and remaining life of the equipment held, the retention period was assumed to be 8 years. According to the results of analysis of government supported research institutes, the average equipment period of the relevant research institutes was 8.2 years and the service life of the oldest equipment was identified to be 23.9 years. In addition, the implementation periods of R&D projects for which prior feasibility studies were conducted after 2008 were approximately 7.2 years with a minimum project period of 2 years(mobile harbor based transportation system innovation project, 2010) and a maximum project period of 12 years(global frontier project, 2009). Considering these R&D project implementation periods, the analysis period was set to 10 years.

As for the discount rate and the period during which the equipment is considered not used, 5.5%, which is the social discount rate used for investments in the public sector and 13 years identified as the average unused period of equipment through investigations were applied respectively and as the coefficients of equipment maintenance cost functions, 0.0784, 0.0558, and -0.081 were applied respectively. The fixed amount method was used as the depreciation method.

In addition, outcome change rate when the R&D is successful; 5.5%, outcome change rate when the R&D has failed; 0%, R&D outcome in the initial year; 100 billion KRW, research equipment contribution ratio to the outcome of R&D (in the initial year); 5%, research equipment contribution ratio increase rate resulting from technical advancement; 55%,

cost of the research equipment held in the initial year; 40 billion KRW (large research equipment), and the rate of increase in the initial cost of research equipment in the case of technical advancement; 50% were set.

TABLE 1. Input values for analysis

Sign	Value	Description
u	5.5%	Outcome increase rate when the R&D is successful
d	0.0%	Outcome change rate when the R&D has failed
$R(0,0)$	10,000	R&D outcome in the initial year
α	2.5%	Annual increasing rate of contribution rate
r_0	5.0%	Initial contribution rate of the equipment purchased
β	4.0%	Annual change rate of initial cost in the case of technical advancement
P_0	2,500	Initial cost of the equipment
γ	0.948	Discount factor
p	75%	Probability of success of R&D
a	0.0784	Maintenance cost function(constant1)
b	0.0558	Maintenance cost function(constant2)
c	-0.081	Maintenance cost function(constant3)

We calculated the values of both equations in Eq. (4) and the results are illustrated in TABLE 2. Based on the results of TABLE 2, we can decide a better decision between ‘R’ and ‘NR’, where ‘R’ refers to replacing the existing research equipment and ‘NR’ refers to maintaining the research equipment, by comparing the magnitude of two values in each cell of TABLE 2. AS a result we can obtain the optimal decision lattice tree shown by TABLE 3. In TABLE 2 that corresponds to the results of analysis using the analysis model, the horizontal axis shows the flow of time and the vertical axis means the success or failure of R&D project. In this case, the horizontal movement from the left to the right means the failure of the R&D project and the ascending in the diagonal direction means the success of the R&D project.

In TABLE 3, once ‘R’ is chosen, the subsequent nodes are meaningless in reality because the replacement problem of a facility ends when it is replaced. Considering this facts, we can construct the optimal strategy for equipment replacement as the states will be unfolded which is illustrated by TABLE 4. It is shown in the TABLE 4 that the replacement of research equipment begins to occur from the 8th year after the initial use of the equipment. For instance, suppose that R&D project using the research facility is consequently successful for 7-year times. Then it is optimal to replace the equipment with new research equipment which has more advanced capability in the year 8 if the next year’s R&D would be successful. Otherwise it is better to continue to use the old equipment one more year. Thereafter if it is proven that the R&D performance of 9th year is successful, then the old facility should be replaced.

Based on the optimal strategy of equipment replacement obtained as such, it is possible to estimate the expected economic service life of the research equipment. In order to assess the expected economic service life, we calculated the probability and the number of paths that each state can be reached as shown by TABLE 5 & TABLE 6. Finally the it is found that the expected economic life is 11.62 year.

Table 2. Binomial lattice of option values

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NR																					897
R																					870
NR																				1,702	833
R																				1,563	767
NR																			2,425	1,583	773
R																			2,355	1,372	668
NR																		3,072	2,256	1,469	716
R																		3,148	2,092	1,191	574
NR																	3,649	2,859	2,095	1,361	661
R																	3,884	2,824	1,841	1,018	485
NR																4,163	3,398	2,657	1,942	1,258	609
R																4,530	3,510	2,516	1,601	853	400
NR															4,618	3,878	3,159	2,463	1,796	1,160	560
R															5,072	4,117	3,155	2,222	1,374	696	319
NR														5,020	4,304	3,607	2,931	2,280	1,657	1,066	513
R														5,507	4,628	3,724	2,817	1,942	1,157	546	242
NR													5,374	4,681	4,005	3,348	2,714	2,104	1,524	977	468
R													5,837	5,040	4,206	3,350	2,494	1,676	950	404	168
NR												5,683	5,014	4,358	3,720	3,102	2,507	1,938	1,398	893	425
R												6,070	5,353	4,595	3,803	2,993	2,187	1,422	753	268	98
NR											5,952	5,305	4,671	4,051	3,449	2,868	2,310	1,779	1,278	812	384
R											6,242	5,574	4,892	4,171	3,420	2,654	1,895	1,180	566	139	31
NR										6,184	5,559	4,945	4,344	3,758	3,191	2,644	2,122	1,627	1,164	735	346
R										6,366	5,739	5,101	4,452	3,767	3,055	2,330	1,616	950	388	16	-32
NR									6,383	5,779	5,185	4,602	4,033	3,479	2,945	2,432	1,943	1,483	1,055	662	309
R									6,428	5,860	5,260	4,651	4,034	3,383	2,707	2,022	1,351	730	218	-101	-93
NR								6,552	5,969	5,394	4,829	4,275	3,736	3,214	2,710	2,229	1,773	1,346	951	592	273
R								6,446	5,923	5,379	4,803	4,223	3,635	3,017	2,376	1,729	1,099	522	56	-212	-151
NR							6,694	6,131	5,574	5,026	4,489	3,964	3,454	2,961	2,487	2,036	1,611	1,215	852	525	240
R							6,438	5,945	5,443	4,920	4,369	3,815	3,255	2,668	2,061	1,450	858	323	-99	-318	-205
NR						6,810	6,267	5,729	5,198	4,677	4,166	3,668	3,185	2,720	2,275	1,853	1,457	1,091	758	462	208
R						6,422	5,943	5,468	4,984	4,483	3,955	3,426	2,894	2,336	1,761	1,184	629	133	-245	-420	-258
NR					6,904	6,380	5,860	5,346	4,840	4,344	3,858	3,386	2,929	2,490	2,072	1,678	1,310	972	668	402	178
R					6,420	5,935	5,471	5,013	4,548	4,067	3,561	3,056	2,549	2,019	1,475	930	411	-47	-385	-516	-308
NR				6,978	6,473	5,970	5,473	4,982	4,499	4,026	3,565	3,117	2,685	2,272	1,880	1,511	1,170	859	583	344	149
R				6,456	5,943	5,471	5,022	4,580	4,133	3,671	3,186	2,703	2,221	1,718	1,202	689	203	-219	-518	-608	-355
NR			7,034	6,546	6,061	5,580	5,104	4,635	4,175	3,724	3,285	2,861	2,453	2,064	1,696	1,353	1,037	752	501	290	121
R			6,562	5,991	5,489	5,030	4,595	4,168	3,737	3,293	2,828	2,367	1,909	1,431	943	459	5	-383	-645	-695	-400
NR		7,074	6,603	6,135	5,670	5,209	4,753	4,305	3,865	3,436	3,020	2,617	2,232	1,865	1,521	1,201	910	649	424	238	95
R		6,774	6,109	5,548	5,057	4,610	4,187	3,775	3,360	2,934	2,488	2,047	1,612	1,158	696	241	-184	-538	-766	-778	-443
NR	4,599	6,645	6,193	5,743	5,297	4,855	4,419	3,990	3,571	3,162	2,766	2,385	2,021	1,677	1,355	1,058	789	552	350	188	70
R	4,599	6,335	5,678	5,126	4,645	4,209	3,800	3,401	3,001	2,592	2,164	1,743	1,328	898	461	32	-363	-687	-881	-858	-484

V. CONCLUSION

The research equipment's economic life analysis models presented in the present study consider the benefit of research equipment, technical advancement, and the uncertainty of R&D projects that cannot be considered in existing equivalent annual costs. As such, the present study tried to present models more suitable for the characteristics of research equipment and enabled the analysis of the economic life and remaining life of research equipment being used. In addition, the study models considered easiness in terms of application by applying binomial option models that enable easy applications of not only past data based uncertainty but also research equipment related experts' qualitative judgments. Since binomial option models have an advantage that they present strategic directions for individual nodes of decision making, they seem to be capable of helping diverse stakeholders in terms of utilization.

These results mean that economic life and remaining life can vary according to situations. There reason why these decisions are shown seems to be the fact that if the R&D project is successful, higher economic value can be obtained and sufficiently large benefits that can sufficiently cover the costs incurred when the research equipment is replaced by new equipment can be obtained from the research equipment. In addition, when the period of use has exceeded 16 years(8 years from the initial year), costs to maintain the research equipment retained from the initial year become higher than costs to replace the research equipment with new equipment and thus replacing the equipment was shown to be an optimum decision. Therefore, in terms of management of research equipment, researchers should examine the trend of outcomes of R&D projects and determine the time of replacement of research equipment considering the trend.

The results of the present study are expected to present valid grounds for judgment through the economic life methodology when research equipment is replaced and to be utilizable as basic data for reliable judgment when an investment in research equipment is made. To utilize the results of the present study more practically, empirical analysis of the economic value of R&D projects and the resultant level of contribution of research equipment and efforts to collect diverse cost data related to the research equipment are required.

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