Optimizing Modular Expansions in an Industrial Setting Using Real Options

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Abstract

The optimization of a modular expansion strategy, while extremely relevant in the industrial setting, requires sophisticated numerical modeling for the valuation of even simple scenarios. In this work, we develop both a numerical model and a model based on Monte Carlo simulation utilizing real options, to provide a methodology for optimizing a plant expansion strategy. Our case study is associated with a wastewater treatment plant expansion; however, the methodologies developed here can be extended to many industrial settings, including mining, oil and gas, and manufacturing. The value of the Monte Carlo simulation is that it is much more easily understood by practitioners and more versatile in that it can be used to model non-standard processes. The results of both of our models match consistently, essentially validating the Monte Carlo technique.

Introduction

In industry, managers are often faced with the task of deciding on the capacity of a given facility before its construction Generally, the optimal size is driven by valuation models and methods that use real options are known to be superior to standard methods relying on discounted cash-flow (DCF) methods (Trigeorgis 1996). However, optimizing a modular expansion strategy, while extremely relevant in the industrial setting, requires sophisticated numerical modeling for the valuation of even simple scenarios. For example, consider the case of a new mining operation where the managers have the option to build a modular processing facility with the potential for future expansions. The managers are faced with questions regarding overall facility size, the size of modular units and the timing of future expansion, all in the context of both market (commodity prices) and technical (richness of the mine site) uncertainties. Generally, the decision makers faced with such problems have engineering and / or accounting backgrounds and shy away from models based on complex financial mathematics. However, it appears that simpler approaches based on Monte Carlo simulation are accepted (Benedetti, Bixio and Vanrolleghem 2005) (Ferrer, et al. 2008).

In this paper, the problem on hand is the expansion of a wastewater treatment plant. As one of the most capital-intensive industrial sectors in North America, the municipal water and wastewater sector shares the difficulties commonly faced by traditional public infrastructure projects – the need for capital expenditure for drinking water and wastewater treatment projects is substantial. According to the Federation of Canadian Municipalities, in 2007, a total of $31 billion was needed for refurbishment of existing systems and $57 billion was needed for replacing existing systems and constructing new ones.

The Conference Board of Canada reported that the average capital expenditures in municipal water and wastewater have reached $1.5 billion annually from 1998-2006, with the capital expenditures in 2006 being close to $2.4 billion for the whole country (PPP CANADA 2013). Although some advanced water treatment technologies have been developed and put into use in municipal systems, capital and research expenditures are being held back in the water sector due to restricted margins and regulated pricing leading to suboptimal returns (Global Cleantech Center 2013). Better capital asset allocation strategies in the wastewater sector can significantly improve capital expenditure efficiency and allow more capital to be invested in the research and development of better treatment technologies.

A single stage "real option" approach that copes with future demand uncertainties for wastewater treatment plant expansion was first developed by (Lawryshyn and Jaimungal 2010), who modeled the growth in demand for a residential wastewater treatment as a geometric Brownian motion (GBM) that is partially correlated to an appropriate traded market index. Their model transformed expected future penalty costs for wastewater connections as an Asian option with the expansion size being the strike price. The finite difference method was used to numerically solve for the capital needed to fund the single stage expansion. Jaimungal and Lawryshyn further developed a closed form approximation to the modular expansion problem. The modular expansion model allows a second-time expansion of the plant at a pre-defined future date. Their results showed that a modular expansion requires significantly less up-front capital investment, and the overall expected expenditures were reduced compared to the single-stage expansion model (Lawryshyn and Jaimungal 2014). However, the modular expansion model developed by Jaimungal and Lawryshyn can only account for a single expansion at a pre-fixed future date, which limits the applicability of the model in practice.

In the model developed here, the pre-fixed expansion date is relaxed – i.e. the second stage expansion is modeled in such a way that it can be carried out at any point after initial expansion. This is accomplished by utilizing a combination of the finite difference method and the analytical approximation developed by (Jaimungal & Lawryshyn, 2010). The result is that a more realistic future expansion strategy is modeled. However, the resulting model may still be considered somewhat mathematical in nature and too complicated for practicing engineers / decision makers. Thus, a practical, and easy to understand Monte-Carlo simulation approach was also developed. An important advantage of the Monte-Carlo model is its flexibility to model other stochastic processes regarding the plant demand. Results obtained by using both methods are compared and found to be similar to each other.

Methodology

In this section, we present the methodology utilized in our models. First, we present the project background, which has its roots from the work of Jaimungal and Lawryshyn (see (Lawryshyn and Jaimungal 2010) and (Lawryshyn and Jaimungal 2014)). Next, we formulate our model utilizing a partial differential equation (PDE) coupled with an analytical approximation to develop the valuation. In the last section we present our model using Monte Carlo simulation. While the simulation relies on least squares

Monte Carlo (LSMC) techniques, the model is much more intuitive and will likely be more readily accepted within the industry.

Project Background

The objective of the model presented in this paper is to determine the optimal sizing of a wastewater treatment plant for a municipality that is undergoing high but uncertain growth. Currently, the existing plant has limited capacity and a decision is required regarding the size of the current expansion and the maximum total potential capacity after a second, future expansion, is made. Plant construction is expected to take just under 2 years and the total plant life is estimated to be 25 years. An undersized plant will lead to significant overflow penalty costs while an oversized plant may show poor performance and will also require significant up front capital costs that may be difficult to recoup if significant population expansion is not realized. Clearly, a staged design of the plant expansion can help the municipality to reduce the cost of an initial over-design yet still allow for the option to expand, if significant population growth is realized.

There are four phases involved in the two-stage expansion of a WWTP project (see Figure 1):

- Phase 1: a decision on the first module size will be made at time t_0 and construction will take place for the first modular plant from time t_0 to time t_1.
- Phase 2: the operation of the first modular plant starts from time t_1; after time t_1, the municipality has the option to expand the wastewater treatment plant at any time t_2 if necessary.
- Phase 3: if necessary, construction of the second module will begin at t_2, ending at t_3.
- Phase 4: the operation of the second module will take place from time t_3 to T.

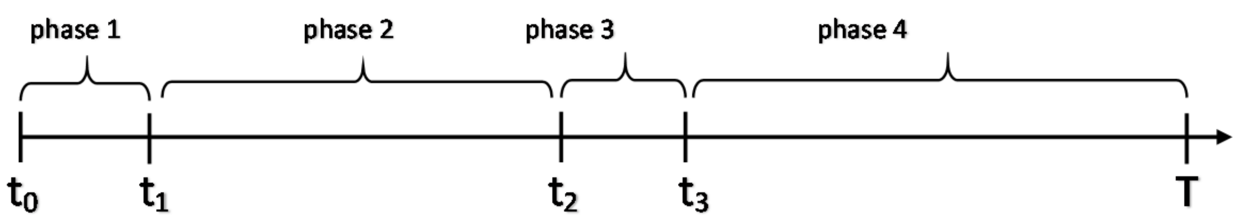


Figure 1. Depiction of the timelines for the plant expansion.

Following (Lawryshyn and Jaimungal 2010) and (Lawryshyn and Jaimungal 2014), we assume that the expansion rate follows a GBM, that is correlated to a traded security (market index). The price of the security also follows a GBM process,

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_t \quad (1)$$

where μ_s and σ_s are constants representing the drift and volatility, respectively and W_t is a Wiener process. Similarly, the connection rate (expansion rate) is modeled as

$$dX_t = \mu_X X_t dt + \sigma_X X_t \left(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp}\right) \quad (2)$$

where μ_X is the drift and σ_X is the volatility, ρ is the correlation and $W_t^{\perp}$ is another Wiener process, independent of W_t. Under the risk-neutral measure, the connection rate becomes

$$dX_t = \bar{r} X_t dt + \sigma_X X_t \left(\rho d\widetilde{W}_t + \sqrt{1-\rho^2} dW_t^{\perp}\right) \quad (3)$$

where

$$\bar{r} = \mu_X - \frac{\rho \sigma_X}{\sigma_S}(\mu_S - r), \tag{4}$$

r is the risk-free rate and $\widetilde{W}_t$ is an equivalent process to W_t under the risk-neutral measure. The market parameters are presented in Table 1.

Table 1. Market parameters.

Parameter	***Value***
Market drift, μ_S	10%
Market volatility, σ_S	16%
Connection rate drift, μ_X	8%
Connection rate volatility, σ_X	5%
Correlation factor, ρ	0.5
Risk free rate, *r*	2%

The total number of connections at time *t* is given by

$$N_t = N_0 + \int_0^t X_u du, \tag{5}$$

and defining the penalty cost associated with a lack of capacity as

$$PC_t = \max(0, (N_t - K) \cdot PC_0), \tag{6}$$

leads to the following present value of the penalty cost incurred from time *t* to *T*:

$$\widetilde{\mathbb{E}}\left[PC_{t,T;K}^{PV}\right] = \int_t^T PC_0 e^{-ru} \cdot \widetilde{\mathbb{E}}\left[\left(N_0 + \int_0^u X_s ds - K\right)_+\right] du, \tag{7}$$

where PC_0 is the penalty cost rate associated with insufficient plant capacity per connection, and *K* is the size of the plant expansion. (Lawryshyn and Jaimungal 2014) provided an analytical approximation for equation (7) as

$$\tilde{\mathrm{E}}\left[\mathrm{PC}_{\mathrm{t,T;K}}^{\mathrm{PV}}\right] \sim \int_{\mathrm{t}}^{\mathrm{T}} \mathrm{PC}_0 \mathrm{e}^{-(\mathrm{r})\mathrm{u}} \cdot \mathrm{e}^{\tilde{\mu}_{0,\mathrm{u}}} \left(\mathrm{X}_0 \Phi(\mathrm{d}_{0,\mathrm{u};+}) - (\mathrm{K} - \mathrm{N}_0)\mathrm{e}^{-\tilde{\mu}_{0,\mathrm{u}}} \Phi(\mathrm{d}_{0,\mathrm{u};-})\right) \mathrm{du} \tag{8}$$

where Φ is the standard normal cumulative distribution function, $\mathrm{d}_{\mathrm{t,T};\pm} = \dfrac{\ln(\mathrm{X_t} / \mathrm{K}'_\mathrm{t}) + \tilde{\mu}_{\mathrm{t,T}} \pm \frac{1}{2}\tilde{\sigma}_{\mathrm{t,T}}^2}{\tilde{\sigma}_{\mathrm{t,T}}}$,

$\mathrm{K}'_\mathrm{t} = \mathrm{K} - \mathrm{N}_\mathrm{t}$, $\tilde{\mu}_{\mathrm{t,T}} \equiv \tilde{\mu} = \ln\left(\mathrm{e}^{\bar{\mathrm{r}}(\mathrm{T-t})} - 1\right) - \ln \bar{\mathrm{r}}$, and

$\tilde{\sigma}_{\mathrm{t,T}} \equiv \tilde{\sigma} = \sqrt{\ln\left(\dfrac{2}{\bar{\mathrm{r}} + \sigma_\mathrm{X}^2}\right) + \ln\left(\dfrac{\mathrm{e}^{\left(2\bar{\mathrm{r}} + \sigma_\mathrm{X}^2\right)(\mathrm{T-t})} - 1}{2\bar{\mathrm{r}} + \sigma_\mathrm{X}^2} - \dfrac{\mathrm{e}^{\bar{\mathrm{r}}(\mathrm{T-t})} - 1}{\bar{\mathrm{r}}}\right) + 2\ln\bar{\mathrm{r}} - 2\ln\left(\mathrm{e}^{\bar{\mathrm{r}}(\mathrm{T-t})} - 1\right)}$, or, in the case where we are interested in a filtration, $\mathscr{F}_\tau$, where $\tau \le \mathrm{t}$, Equation (8) can be written as

$$\tilde{\mathrm{E}}\left[\mathrm{PC}_{\mathrm{t,T;K}}^{\mathrm{PV}} \middle| \mathscr{F}_\tau\right] \sim \int_{\mathrm{t}}^{\mathrm{T}} \mathrm{PC}_\tau \mathrm{e}^{-(\mathrm{r} - \mathrm{r_{cpi}})\mathrm{u}} \cdot \mathrm{e}^{\tilde{\mu}_{\tau,\mathrm{u}}} \left(\mathrm{X}_\tau \Phi(\mathrm{d}_{\tau,\mathrm{u};+}) - (\mathrm{K} - \mathrm{N}_\tau)\mathrm{e}^{-\tilde{\mu}_{\tau,\mathrm{u}}} \Phi(\mathrm{d}_{\tau,\mathrm{u};-})\right) \mathrm{du}. \tag{9}$$

(Lawryshyn and Jaimungal 2014) modeled the expansion costs with fixed as variable components as

$$C_1(K_1, K_{2max}) = \alpha_1 + \gamma_1 K_1 + \gamma_{12} K_{2max} \tag{10}$$

for the first expansion and

$$C_2(K_2 \le K_{2max}) = \alpha_2 + \gamma_2 K_2 \tag{11}$$

for the second, where K_1 is the size of the first expansion, K_2 is the size of the second expansion, K_{2max} is the maximum size for the second expansion and α_1, α_2 are positive coefficients associated with fixed construction cost, and $\gamma_1, \gamma_2, \gamma_{12}$are positive coefficients associated with variable construction cost. The cost parameter values are presented in Table 2. Throughout the models, X_0 is assumed to be 81 connections per year and N_0 is assumed to be -200 connections (i.e. the current plant capacity is for 200 connections).

Table 2. Construction cost parameter values.

Parameter	***Value***
Module 1 fixed cost, α_1	\$3,500,000
Module 2 fixed cost, α_2	\$525,000
Module 1 variable cost 1, γ_f	\$860/connection
Module 1 variable cost 2, γ_{12}	\$172/connection
Module 2 variable cost, γ_2	\$757/connection

(Lawryshyn and Jaimungal 2014) showed that at some time t^* the optimal second state expansion size can be determined by

$$\mathrm{K}_2^{\mathrm{Opt}} = \min_{\mathrm{K}_2}\left(\mathrm{C}_2(\mathrm{K}_2) + \tilde{\mathrm{E}}\left[\mathrm{PC}^{\mathrm{PV}}_{t^*+t_{\mathrm{const}},\mathrm{T};\mathrm{K}_1+\min(\mathrm{K}_2,\mathrm{K}_{2\max})} \middle| \mathscr{F}_{t^*}\right]\mathrm{e}^{-\mathrm{r}t^*}\right) \tag{12}$$

where t_{const} is the time required for construction of the expansion. Equation (12) will be used extensively in the numerical model for determining the optimal strategy at each decision point. It should be noted that equation (12) could also be used in the Monte Carlo simulation to reduce computational time, in view of the fact that one of the objectives was to develop a model purely based on Monte Carlo simulation, a technique much more readily accepted by practitioners, a different optimization algorithm was utilized. In the following subsection, the numerical model methodology is presented and in the subsequent one, the Monte Carlo model is developed.

Numerical Model

Defining v_t as the total cost of the plant and recognizing that $v(t, X_t, N_t)$, applying Ito's lemma and standard arguments leads to the following PDE:

$$\frac{dv}{dt} + \bar{r}x\frac{dv}{dx} + x\frac{dv}{dy} + \frac{1}{2}\,\sigma_x^2\,x^2\frac{d^2v}{dx^2} = rv, \tag{13}$$

where the dummy variables x and y were used to replace X_t and N_t, respectively. We note that the same PDE was developed in (Lawryshyn and Jaimungal 2010), however, the boundary conditions in this case are different. Specifically, we assume constant slope as follows,

$$\left.\frac{\partial v}{\partial x}\right|_{x=x_{max}} = constant \tag{14}$$

$$\left.\frac{\partial v}{\partial y}\right|_{y=y_{max}} = constant \tag{15}$$

$$\left.\frac{\partial v}{\partial y}\right|_{y=y_{min}} = constant \tag{16}$$

and

$$v(t,0,y) = PC_0e^{-ru}\tilde{E}[max(N_t - K, 0)\quad] * (T-t). \tag{17}$$

As discussed previously, we assume that the managers have the option to expand the plant at any time after initial construction is completed during the life of the plant and decisions are made on an annual basis only. However, since the construction time of the second expansion is just under two years, it makes no sense to expand at year 24[1]. Thus, a decision regarding the size of the modular expansion is required at times 2 years to 23 years. Thus, at $t = 23\ y$, we utilize equation (12) to determine the optimal expansion size, $K_2^{Opt,t=23}$. Next, we need to compare the cost of constructing and incurring the new (reduced) penalty cost where the plant size is now $K_1 + K_2^{Opt,t=23}$ versus not constructing and incurring the penalty cost given the plant size of K_1. Utilizing equations (9) and (11) we can calculate the expansion strategy cost at $t = 23$ as

$$C_{t=23}^{ES}(X_t, N_t) = C_2\left(K_2^{Opt,t=23}\right) + \widetilde{\mathbb{E}}\left[PC_{t=23,T;K_1+K_2^{Opt,t=23}}^{PV}\right]. \quad (18)$$

The no construction cost at $t = 23$ is simply

$$C_{t=23}^{NC}(X_t, N_t) = \widetilde{\mathbb{E}}\left[PC_{t=23,T;K_1}^{PV}\right], \quad (19)$$

and therefore for every the boundary condition for v becomes

$$v(t = 23, X_t, N_t) = \min(C_{t=23}^{NC}(X_t, N_t), C_{t=23}^{ES}(X_t, N_t)). \quad (20)$$

The next decision time point is at year 22, thus we utilize equation (13) to determine $v(t = 22, X_t, N_t)$. At this point, we compare the expansion strategy cost to v as of this time. Thus, we have,

$$C_{t=22}^{ES}(X_t, N_t) = C_2\left(K_2^{Opt,t=22}\right) + \widetilde{\mathbb{E}}\left[PC_{t=22,T;K_1+K_2^{Opt,t=22}}^{PV}\right], \quad (21)$$

and

$$C_{t=22}^{NC}(X_t, N_t) = v(t = 22, X_t, N_t), \quad (22)$$

and v is updated as

$$v(t = 22, X_t, N_t) = \min(C_{t=22}^{NC}(X_t, N_t), C_{t=22}^{ES}(X_t, N_t)). \quad (23)$$

The methodology is repeated until $t = 2\ y$, after which equation (13) is utilized to find the cost at $t = 0$. This procedure was run with different K_1 and K_{2max} values to determine the optimal initial construction size and maximum modular expansion size. A depiction of the numerical model is presented in Figure 1.

[1] One could argue that there is no reason to construct at year 23, however, we make the assumption that the construction time for the modular expansion is just under 2 years so there is a small time, dt, where one could operate the plant in the expanded phase.

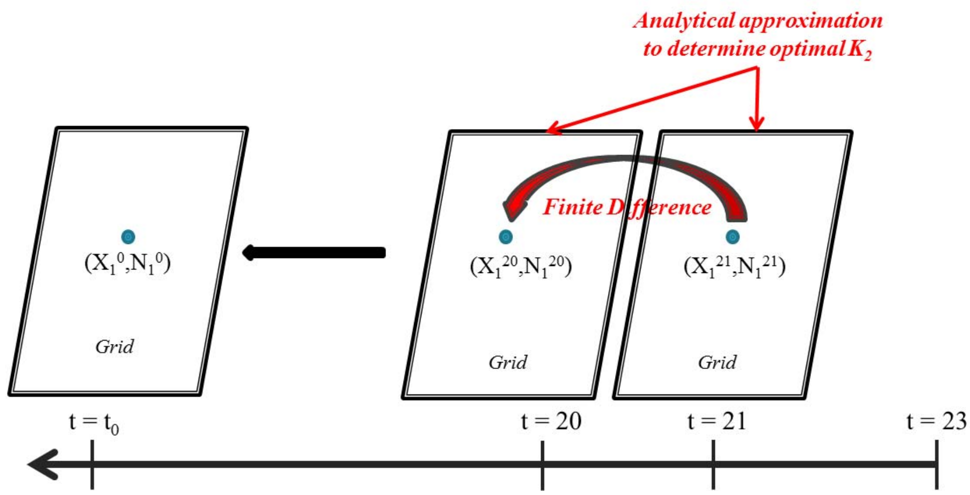


Figure 2. Depiction of the numerical model.

Monte Carlo Simulation

The Monte Carlo simulation methodology requires that we form a grid of starting values for X_t and N_t at each decision making time point – i.e., as before, $t = 2, 3, \dots, 23$ years. Starting at $t = 23\ y$, for each grid point (i, j) we simulate X_t from $t = 23y$ to $t = T$, for N paths and thus we arrive at a $M_{path} \times M_{time}$ matrix, where M_{path} is the number of paths (per grid point) and M_{time} is the number of time steps for the simulation (at $t = 23$ we would be simulating until $t = T = 25y$, so $M_{time} = (25 - 23)/dt$), $\boldsymbol{X}_{t=23,T}^{i,j}$. Integrating, it is then possible to determine the number of new connections, $\boldsymbol{N}_{t=23,T}^{i,j}$, where the k-th, l-th element of this matrix is given by

$$\boldsymbol{N}_{t=23,T}^{i,j}\big|_{k,l} = N_{t=23}^{i,j} + \sum_{m=1}^{l} \boldsymbol{X}_{t=23,T}^{i,j}\big|_{k,m} dt, \tag{24}$$

and where dt is the time step in the simulation. Thus, for any total plant size K the present value of the penalty cost for each simulation path is given by

$$\boldsymbol{PC}(K)_{t=23,T}^{i,j}\big|_{k} = \sum_{m=1}^{M_{time}} \mathrm{e}^{-\mathrm{r}(\mathrm{t_m}-t)} \max\left(\boldsymbol{N}_{t=23,T}^{i,j}\big|_{k,m} - K, 0\right) \cdot PC_0 dt \tag{25}$$

where $\boldsymbol{PC}(K)_{t=23,T}^{i,j}$ is a vector of length M_{path}. and the expected penalty cost is given by averaging the vector so that

$$PC(K)_{t=23,T}^{i,j} = \frac{1}{M_{path}} \sum_{k=1}^{M_{path}} \boldsymbol{PC}(K)_{t=23,T}^{i,j}\big|_{k}. \tag{26}$$

For the case of no construction, at $t = 23$, we have

$$C_{t=23}^{NC;i,j} = PC(K_1)_{t=23}^{i,j}. \tag{27}$$

Next, for the case of expansion, we simplify our problem slightly, by assuming a discrete number of possible modular expansion sizes are available, i.e. $K_2 \in \{K_2^1, K_2^2, K_2^3, \dots, K_2^n\}$, and calculate the expansion cost strategy as

$$C_{t=23}^{ES;i,j} = \min\left(PC(K_1 + K_2^1)_{t=23}^{i,j} + C_2(K_2^1), PC(K_1 + K_2^2)_{t=23}^{i,j} + C_2(K_2^2), \dots, PC(K_1 + K_2^n)_{t=23}^{i,j} + C_2(K_2^n)\right). \tag{28}$$

Thus, the value of the cost at each grid point is calculated as

$$v_{t=23}^{i,j} = \min\left(C_{t=23}^{NC;i,j}, C_{t=23}^{ES;i,j}\right). \tag{29}$$

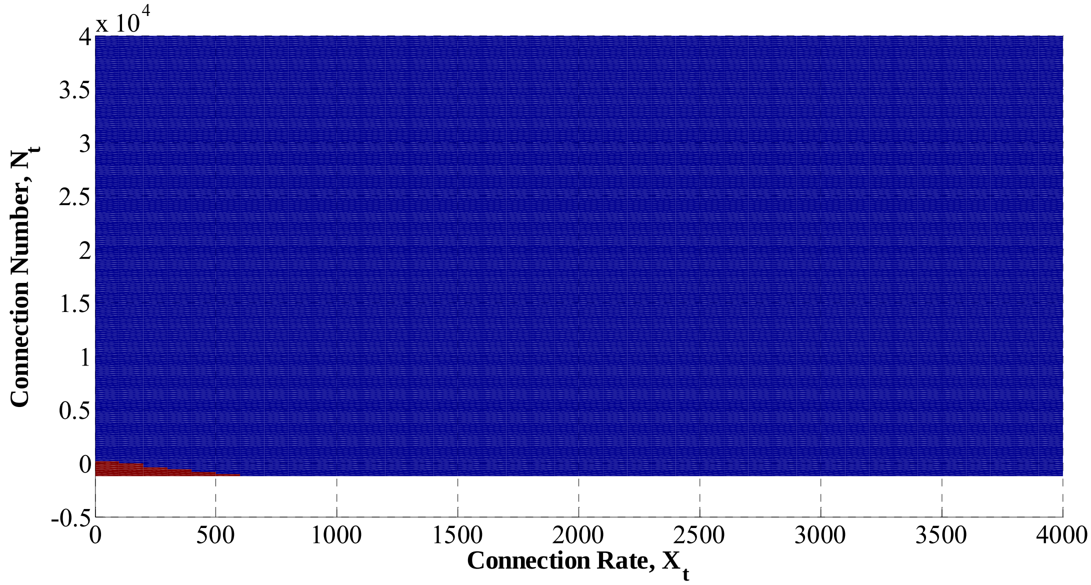
At the next expansion decision point, $t = 22$, we consider two scenarios: the case of no construction and the case of construction. In the case of no construction, we simulate X_t from $t = 22y$ to $t = 23y$ to get $\boldsymbol{X}_{t=22,23}^{i,j}$ and $\boldsymbol{N}_{t=22,23}^{i,j}$. We apply equation (25) to find $\boldsymbol{PC}(K)_{t=22,23}^{i,j}$. Then, for each end point $\boldsymbol{X}_{t=22,23}^{i,j}\big|_{k,M_{time}}$ and $\boldsymbol{N}_{t=22,23}^{i,j}\big|_{k,M_{time}}$, we use linear interpolation to determine the value of $v_{t=23}^{i,j}(\boldsymbol{X}_{t=22,23}^{i,j}\big|_{k,M_{time}}, \boldsymbol{N}_{t=22,23}^{i,j}\big|_{k,M_{time}})$. Thus, the cost for no construction becomes

$$C_{t=22}^{NC;i,j} = \frac{1}{M_{path}} \sum_{k=1}^{M_{path}} \left(\boldsymbol{PC}(K)_{t=22,23}^{i,j}\Big|_{k} + v_{t=23}^{i,j}\left(\boldsymbol{X}_{t=22,23}^{i,j}\Big|_{k,M_{time}}, \boldsymbol{N}_{t=22,23}^{i,j}\Big|_{k,M_{time}}\right) e^{-r(23-22)}\right). \tag{30}$$

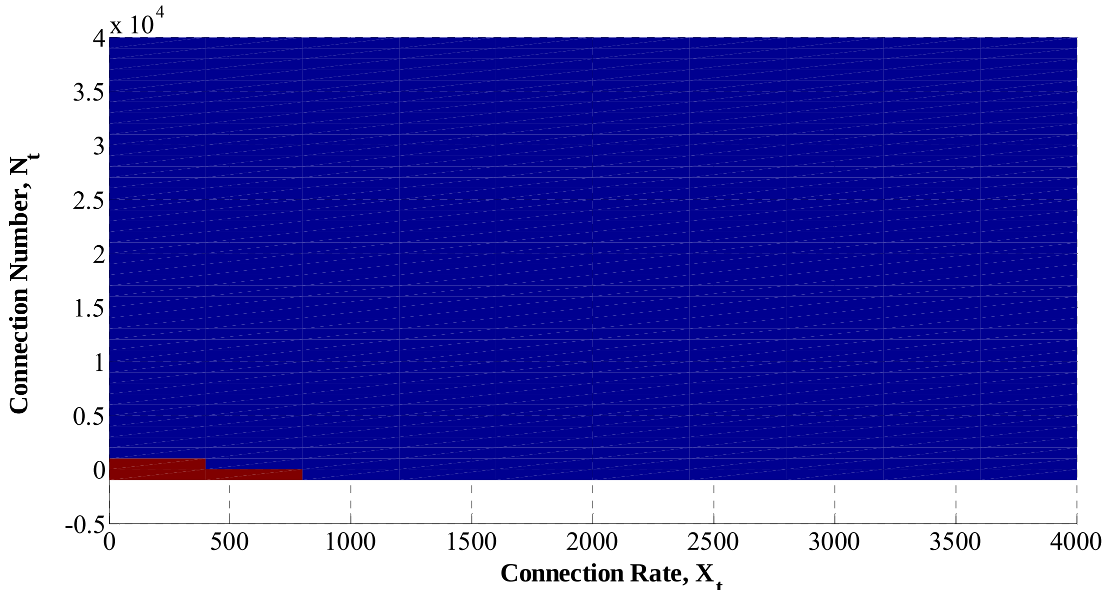
For the expansion strategy, we proceed as before, with the key distinction that we simulate X_t through the entire remaining time domain, such that we calculate $\boldsymbol{X}_{t=22,T}^{i,j}$ and $\boldsymbol{N}_{t=22,T}^{i,j}$ and utilize equation (28) to determine $C_{t=22}^{ES;i,j}$. Then, equation (29) is used to determine $v_{t=22}^{i,j}$. The methodology continues in a recursive fashion to $t = 2y$. For $t = 0$ to $2y$ we simply simulate X_t, determine N_t for each path, calculate the interpolated value of $v_{t=2}$ for each case, discount to $t = 0$ and calculate the average to find the final value.

Results

In this section we provide some of the key results of applying the two methods. In particular, we provide a comparison between the Numerical Model (NM) and the Monte Carlo Simulation (MCS). Figure 3 and Figure 4 show the construction boundaries for both the NM and MCS at years 22 and 2, respectively. As can be seen, the two methodologies provide similar results. For the case where $t = 2y$ (Figure 4), the MCS boundary looks slightly different because of the course discretization. In Figure 5 we plot the total cost value function as of $t = 0$. Again, we see that the results are very similar and an error analysis showed a difference of less than 5% for all calculated values between the NM and MCS. Finally, in Figure 6 we provide a plot of the total cost as a function of K_1 and K_{2max}. This plot can be used by the decision makers to optimize their plant expansion strategy.

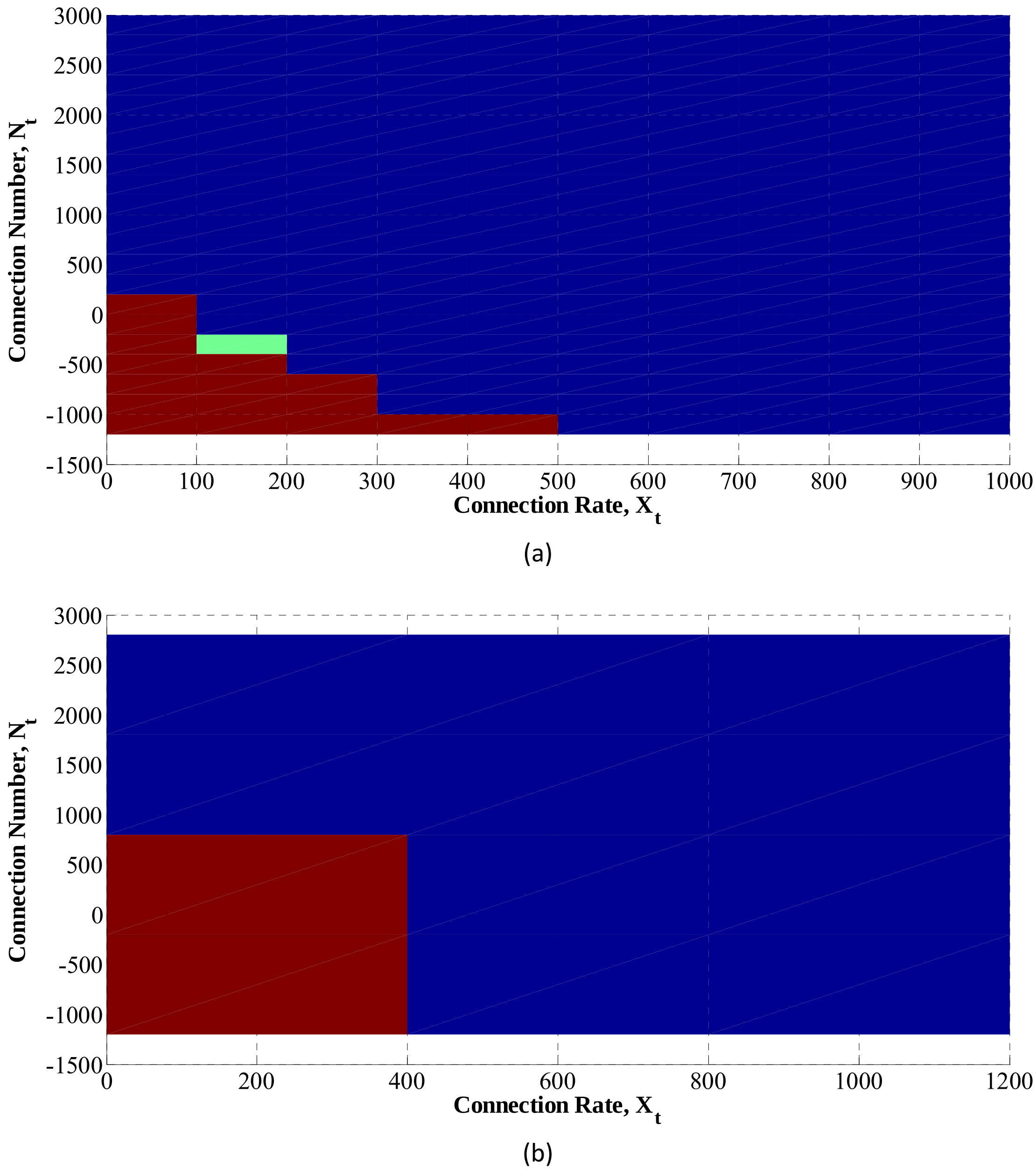


(a)



(b)

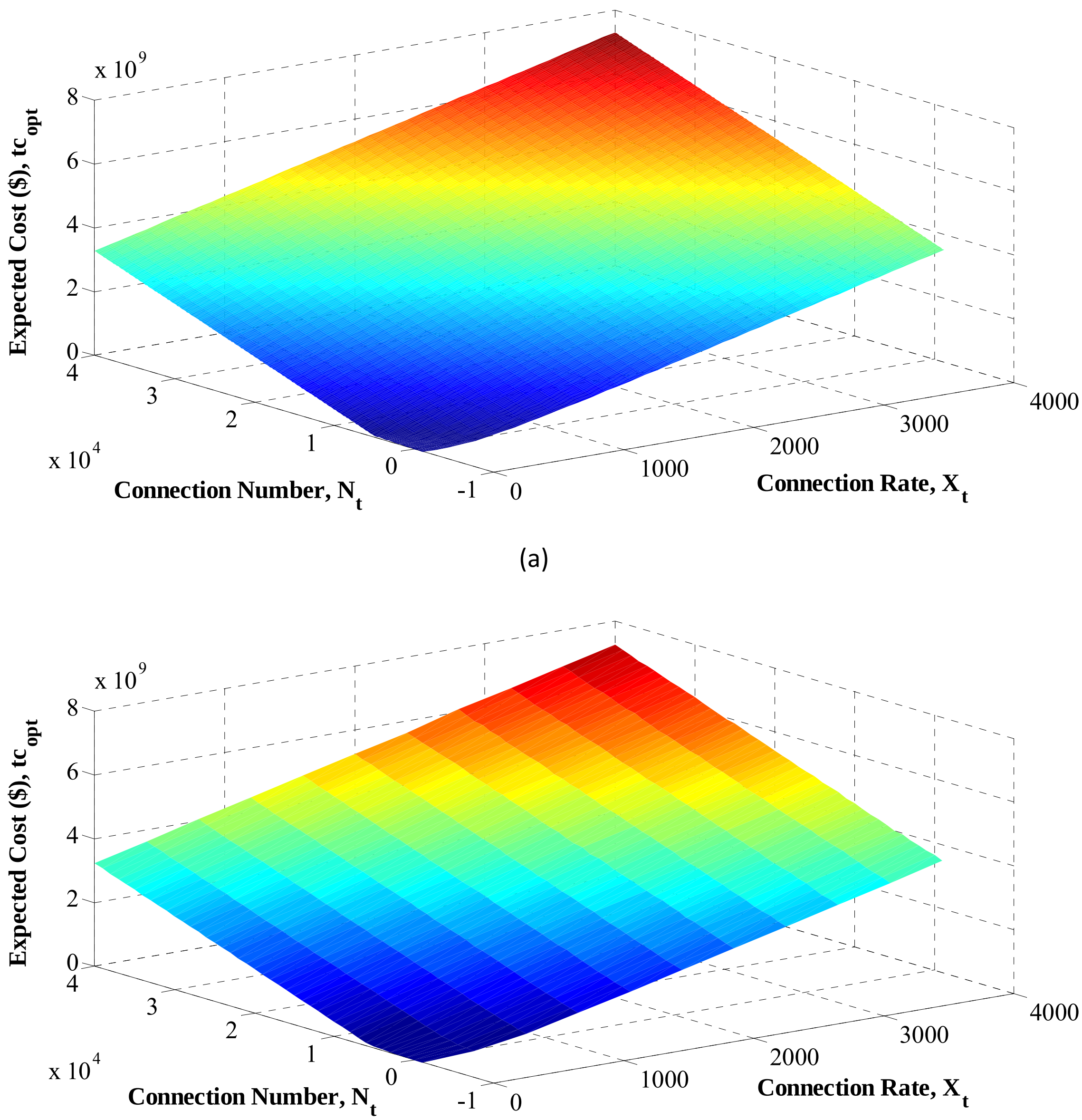
Figure 3.Construction boundary plots for the Numerical Model (a) and the Monte Carlo Simulation (b) at $t = 22y$. Blue designates construct.



(a)

(b)

Figure 4.Construction boundary plots for the Numerical Model (a) and the Monte Carlo Simulation (b) at $t = 2y$. Blue designates construct.



(a)

(b)

Figure 5.Total cost value plots for the Numerical Model (a) and the Monte Carlo Simulation (b) at $t = 0$.

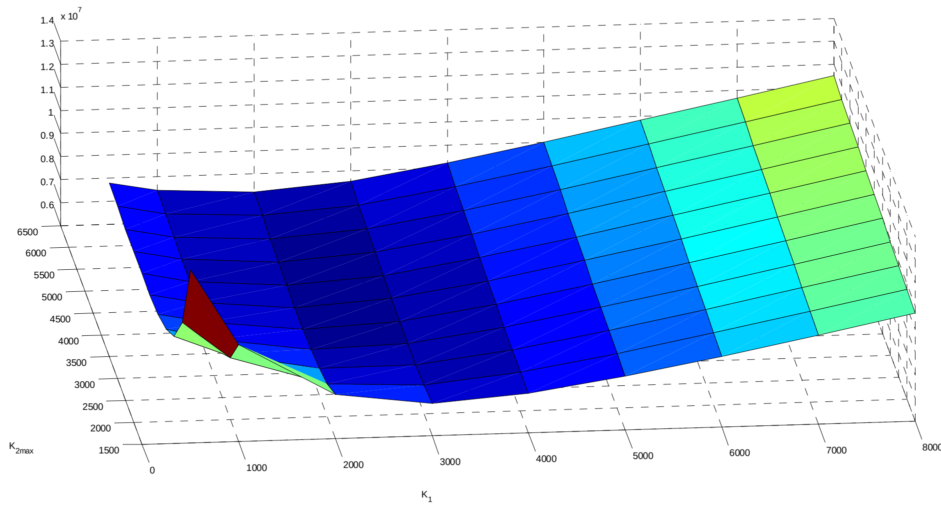


Figure 6. Total cost as a function of K_1 and K_{2max}.

Conclusions

We developed a framework to determine the optimal timing and sizing of a two-stage wastewater treatment plant expansion project using a real options approach. Two different models were developed: 1) Numerical Model (NM) and 2) Monte Carlo Simulation (MCS). The results of the two models showed very good agreement. The significant advantage of the MCS is that it is likely to be much more readily accepted by practitioners. Furthermore, the processes in the MCS can easily be changed, to simulate other, perhaps more realistic, processes for modeling connection rates. We believe the MCS modeled developed here is extremely relevant in the industrial setting. Our next challenge will be to develop a MCS methodology where multiple expansion decisions are required.

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