Pollutant Abatement Investment under Ambiguity in a Two-Period Model*

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Abstract

This paper investigates a pollutant abatement investment under ambiguity in a two-period setting. We consider there are representative consumer and firm in an economy and formulate the social welfare maximization problem. Then we numerically derive the optimal level of abatement investment. Furthermore, we analyze the comparative static effects of the model's parameters and find an increase in the degree of ambiguity encourages pollutant abatement investment.

Keywords: pollutant abatement; ambiguity

1 Introduction

An important factor in environmental policy's decision making is the treatment of uncertainty. For example, there exist several kinds of uncertainty. One of these is the scientific uncertainty concerning the relationship between green house gas (GHG) emissions and temperature rise. See, for example, **?** and **?**. This uncertainty raises from the immense complexity of our climate system. There is also uncertainty regarding the potential impact of climate change on our lives. Thus, when we develop a model that evaluates a environmental policy like mitigating GHGs, we need to take this uncertainty into account.

In this paper we use the concept of *ambiguity* or *Knightian uncertainty*. **?** defines two kinds of uncertainty: risk, under which the probability of an outcome is uniquely determined; and uncertainty, under which it is not. The latter is termed Knightian uncertainty or deep uncertainty. In this paper, following **?**, we term Knightian uncertainty ambiguity. For a survey of decision making under uncertainty, see, for example, **?**, **?** and **?**.

We examine a general and simple pollutant abatement investment under ambiguity in a two-period setting. We extend the model of **?** which examine capital investment under ambiguity in a two-period setting by including the investments in pollutant abatement capital. **?** extends the model of **?**, which investigates optimal consumption under ambiguity in a two-period setting. Tsujimura analyzed a production economy and derived optimal capital investment in a general

*This research was supported in part by a Grant-in-Aid for Scientific Research (No. 24510213) from the Japan Society for the Promotion of Science.

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equilibrium setting. We consider a two-period production economy as in **?**. For analytical simplicity, the number of consumers is equal to that of firms, and consumers own the firms. This enables us to consider a representative consumer and firm. The representative consumer is risk averse and has a constant absolute risk aversion utility function. Because there is ambiguity in future income, the representative consumer considers a set of probability distributions. Then, we formulate the utility function as the multiple-priors expected utility of **?**.

The firm produces output by using production capital. The production process generates, however, pollutant emissions that are proportional to output. The pollutant emissions cause damage to the consumer. Then, the firm has to invest in the pollutant abatement capital to reduce pollutant emissions. To solve the problem of consumer and firm, we formulate their problems as the central planner's social welfare maximizing problem and derive the optimal production and pollutant abatement capital investment and consumption. Furthermore, we analyze the comparative static effects of the model's parameters. We find that the production and abatement capital are increasing in the degree of ambiguity and volatility of income. These results are consistent with precautionary principle.

The rest of the paper is organized as follows. In Section 2, we describe the setup of the economy and formulate the central planner's problem. In Section 3, we solve the central planner's problem. In Section 4, we conduct a numerical analysis. Section **??** concludes the paper.

2 The Model

We consider a two-period production economy as in **?**. There are a large number of identical consumers and firms. The number of consumers is equal to that of firms. The firms are owned by the consumers and produce identical outputs. Then, we consider hereafter a representative consumer and firm.

The representative consumer receives an endowment Y_t in each period t $(t = 1, 2)$. This endowment is a random variable on $(\Omega, \mathcal{F}, \mathbb{P})$. The consumer receives utility from consumption C_t in each period. The utility function $u(C_t)$ is assumed to be given by:

$$u(C_t) = -\frac{1}{\theta}\mathrm{e}^{-\theta C_t}, \tag{2.1}$$

where the coefficient $\theta > 0$ is the degree of absolute risk aversion. The consumer also suffers from pollutant emissions P and the damage is measured by the following disutility/damage function:

$$D(P) = \mathrm{e}^{\theta_p P}, \tag{2.2}$$

where $\theta_p > 0$ is the damage coefficient. Then, the consumer's welfare is assumed to be multiplying by the utility and disutility:

$$w(C_t, P) = u(C_t)D(P) = -\frac{1}{\theta}\mathrm{e}^{-\theta C_t}\mathrm{e}^{\theta_p P}. \tag{2.3}$$

The representative firm produces output by using capital K. The firm's production function $f(k)$ is expressed as:

$$f(K) = AK^{\alpha}, \tag{2.4}$$

where $A > 0$ reflects the level of technology and $\alpha > 0$ is the output elasticity of capital. This production generates pollutant emissions P proportional to output. Pollutant emissions P is given by:

$$P = \gamma f(K) g(K_a), \tag{2.5}$$

where $\gamma > 0$ is a constant conversion factor between output and pollutant emission. g is the pollutant abatement function and K_a is the abatement capital. Following **?**, we assume that $g(K_a) > 0$, $g'(K_a) < 0$, and $g''(K_a) > 0$. We specify g as:

$$g(K_a) = bK_a^{-\lambda}, \tag{2.6}$$

where $b > 0$ is the emission abatement coefficient and $\lambda > 0$ is the emission abatement elasticity of abatement capital.

The representative consumer maximizes the welfare w subject to the following intertemporal budget constraint:

$$C_1 + K + K_a = Y_1, \tag{2.7}$$

$$C_2 = Y_2 + (1 - \delta)(K + K_a) + f(K), \tag{2.8}$$

where $\delta \in (0, 1)$ is the depreciation rate of capital.

Suppose that the representative consumer does not uniquely determine the probability distribution of future endowments but instead considers a set of probability distributions. Then, we formulate the representative consumer's welfare function as the multiple-priors expected utility of **?**:

$$W(C_1, C_2, P) = w(C_1) + \beta \min_{\mathbb{Q} \in \mathcal{P}} \mathbb{E}_{\mathbb{Q}}[w(C_2, P)], \tag{2.9}$$

where $\beta \in (0, 1)$ is a discount factor and $\mathcal{P}$ is a set of priors over $(\Omega, \mathcal{F})$. Following **?** and **?**, we define $\mathcal{P}$ as:

$$\mathcal{P}(\mathbb{P}, \phi) = \left\{ \mathbb{Q} \in \mathcal{M}(\Omega); \mathbb{E}_{\mathbb{Q}} \left[\ln \left(\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}} \right) \right] \leq \phi^2 \right\}, \tag{2.10}$$

where $\mathcal{M}(\Omega)$ is the set of probability measures on Ω, $\mathrm{d}\mathbb{Q}/\mathrm{d}\mathbb{P}$ is the Radon–Nikodym derivative and $\mathbb{E}_{\mathbb{Q}}[\ln(\mathrm{d}\mathbb{Q}/\mathrm{d}\mathbb{P})]$ is the relative entropy index.[1] This specification is based on robust control theory.[2] As ϕ increases, the condition of the set of priors $\mathcal{P}$ is eased and becomes larger. This implies that the decision-maker will accept wider range of ambiguity. Then, the parameter $\phi > 0$ represents the degree of ambiguity. ϕ indicates how the decision-maker is ambiguity averse. See, for example, **?**.

We assume that $\mathbb{P}$ is the probability measure of the normal distribution with mean μ and variance σ^2. All probability measures in $\mathcal{P}(\mathbb{P}, \phi)$ have normal distributions. $\mathbb{Q}$ is the probability measure of the normal distribution with mean $\mu - h$ and variance σ^2, where $h > 0$ represents the mean distortion chosen by the decision maker. Then, the relative entropy of $\mathbb{P}$ and $\mathbb{Q}$ is given by:

$$\mathbb{E}_{\mathbb{Q}} \left[\ln \left(\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}} \right) \right] = \frac{h^2}{2\sigma^2}. \tag{2.11}$$

The derivation of (2.11) is in Appendix A.

The representative firm maximizes profits, given prices and technology. Then, we formulate the central planner's problem as:

$$\max_{\{C_1, C_2, K, K_a\}} W(C_1, C_2, P), \tag{2.12}$$

[1] This is also termed the Kullback–Leibler divergence.

[2] See, for example, **?**.

$$\text{s.t. } (2.7) \text{ and } (2.8).$$

Rewriting the central planner's problem yields:

$$\max_{\{K,K_a\}}\left\{-\frac{1}{\theta}\mathrm{e}^{-\theta(Y_1-K-K_a)}+\beta\min_{\mathbb{Q}\in\mathcal{P}}\mathbb{E}_{\mathbb{Q}}\left[-\frac{1}{\theta}\mathrm{e}^{-\theta[Y_2+(1-\delta)(K+K_a)+AK^\alpha]}\mathrm{e}^{\theta_p(BK^\alpha K_a^{-\lambda})}\right]\right\}, \tag{2.13}$$

where $B := \gamma Ab$. In the next section, we solve the problem (2.13) and derive the optimal level of production capital and abatement capital investments.

3 Optimal Capital Investment

In this section, we derive optimal production capital and abatement capital investments.

Since the welfare function w is strictly concave, the first order conditions of optimality are sufficient. Then, putting $\delta = 1$ and taking the logarithm of the first order conditions, we obtain the following equations:

$$\begin{aligned}-\theta(Y_1-K-K_a) =& \ln\left(\beta\left\{[\alpha AK^{\alpha-1}]-\frac{\theta_p}{\theta}\alpha BK^{\alpha-1}K_a^{-\lambda}\right\}\right)\\&-\theta AK^\alpha+\theta_p BK^\alpha K_a^{-\lambda}+\ln\left(\max_{\mathbb{Q}\in\mathcal{P}}\mathbb{E}_{\mathbb{Q}}[\mathrm{e}^{-\theta Y_2}]\right),\end{aligned} \tag{3.1}$$

$$\begin{aligned}-\theta(Y_1-K-K_a) =& \ln\left(\beta\left\{\frac{\theta_p}{\theta}\lambda BK^\alpha K_a^{-\lambda-1}\right\}\right)\\&-\theta AK^\alpha+\theta_p BK^\alpha K_a^{-\lambda}+\ln\left(\max_{\mathbb{Q}\in\mathcal{P}}\mathbb{E}_{\mathbb{Q}}[\mathrm{e}^{-\theta Y_2}]\right).\end{aligned} \tag{3.2}$$

It follows from (3.1) and (3.2) that we have the relationship between the production capital and the abatement capital:

$$\hat{K} := K(K_a) = \frac{\theta}{\theta_p}\frac{\alpha}{\lambda}\frac{1}{\gamma b}K_a^{\lambda+1}-\frac{\alpha}{\lambda}K_a \tag{3.3}$$

Now we are in a position to obtain the optimal consumption of period 1, C_1^*, production capital K^*, and abatement capital, K_a^*.

Proposition 3.1. *Suppose that the representative consumer's welfare function is given by (2.3) under uncertain endowment. Assume that the the abatement capital K_a satisfies with the following inequality:*

$$K_a > \left(\frac{\theta_p}{\theta}\gamma b\right)^{1/\lambda}. \tag{3.4}$$

Then, the optimal consumption of period 1, C_1^* and abatement capital, K_a^* are numerically calculated from the following equation:

$$\begin{aligned}-\theta(Y_1-\hat{K}-K_a) =& \ln\left(\beta\left\{\alpha A\hat{K}^{\alpha-1}-\frac{\theta_p}{\theta}\alpha B\hat{K}^{\alpha-1}K_a^{-\lambda}\right\}\right)\\&-\theta A\hat{K}^\alpha+\theta_p B\hat{K}^\alpha K_a^{-\lambda}-\theta(\mu-\sqrt{2}\sigma\phi)+\frac{\theta^2\sigma^2}{2}.\end{aligned} \tag{3.5}$$

and the budget constraint of period 1, (2.7). After K_a^* and C_1^* are calculated, it follows from (2.7) or (3.3) that we obtain the optimal production capital K^*.

Proof. Substituting (3.3) into (3.1) yields:

$$
\begin{aligned}
-\theta(Y_1 - \hat{K} - K_a) = &\ln\left(\beta\left\{\alpha A\hat{K}^{\alpha-1} - \frac{\theta_p}{\theta}\alpha B\hat{K}^{\alpha-1}K_a^{-\lambda}\right\}\right) \\
&- \theta A\hat{K}^{\alpha} + \theta_p B\hat{K}^{\alpha}K_a^{-\lambda} + \ln\left(\max_{\mathbb{Q}\in\mathcal{P}}\mathbb{E}_{\mathbb{Q}}[\mathrm{e}^{-\theta Y_2}]\right)
\end{aligned}
\tag{3.6}
$$

Given the assumptions about the probability measures $\mathbb{P}$ and $\mathbb{Q}$, we obtain

$$
\begin{aligned}
\ln\left(\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}}\right) &= \ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \left(\frac{(y-(\mu-h))^2}{2\sigma^2}\right) - \ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) + \left(\frac{(y-\mu)^2}{2\sigma^2}\right) \\
&= \frac{-2h(y-\mu) - h^2}{2\sigma^2}.
\end{aligned}
\tag{3.7}
$$

Then, relative entropy is

$$
\mathbb{E}_{\mathbb{Q}}\left[\ln\left(\frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}}\right)\right] = \frac{-2h(\mathbb{E}_{\mathbb{Q}}[y]-\mu) - h^2}{2\sigma^2} = \frac{h^2}{2\sigma^2}.
\tag{3.8}
$$

Then, from the relative entropy expression, (2.11), we obtain:

$$
\begin{aligned}
\ln\left(\max_{\mathbb{Q}\in\mathcal{P}}\mathbb{E}_{\mathbb{Q}}[\mathrm{e}^{-\theta Y_2}]\right) &= \ln\left(\max_{h}[\mathrm{e}^{-\theta(\mu-h)+\theta^2\sigma^2/2}]\right) \\
&= -\theta(\mu - \sqrt{2}\sigma\phi) + \frac{\theta^2\sigma^2}{2}.
\end{aligned}
\tag{3.9}
$$

Substituting (3.9) into (3.1), we obtain (3.5). Since natural logarithm of negative number is undefined, we obtain the lower bound of abatement capital K_a, (3.4). Then, the optimal abatement capital K_a^* and consumption in period 1 C_1^* are numerically calculated from (3.5) and the budget constraint of period 1, (2.7). The proof is completed. □

Furthermore, it follow the above argument that we obtain the optimal social welfare.

Proposition 3.2. *Suppose that C_1^*, K^* and K_a^* are numerically calculated. From (2.3), (2.5), (2.9) and (3.9) we obtain the following social welfare function:*

$$
\begin{aligned}
W(C_1, K, K_a) &= w^1(C_1) + \beta w^2(C_2(K), P(K, K_a)) \\
&= -\frac{1}{\theta}\mathrm{e}^{-\theta C_1} - \frac{\beta}{\theta}\mathrm{e}^{-\theta(\mu-\sqrt{2}\sigma\phi)+\frac{\theta^2\sigma^2}{2}-\theta AK^{\alpha}}\mathrm{e}^{\theta_p BK^{\alpha}K_a^{-\lambda}},
\end{aligned}
\tag{3.10}
$$

where w^t is the welfare function at period t. Substituting C_1^*, K^* and K_a^* into (3.10), we obtain the optimal social welfare $W^* := W(C_1^*, K^*, K_a^*)$.

Notice that it follows from (3.3) and (3.4) that we obtain the production capital is positive: $K > 0$.

In the next section, we numerically derive optimal production, abatement capital and consumption in period 1.

4 Numerical Examples

In this section, we numerically calculate the optimal production capital, K^*, and abatement capital, K_a^*, and consumption of period 1, C_1^*. Furthermore, we investigate their response to parameter changes. The basic parameter values are as follows: $Y_1 = 10$; $A = 1$; $\alpha = 0.5$; $\gamma = 0.1$; $b = 1$; $\lambda = 0.5$ $\beta = 0.95$; $\mu = 10$; $\sigma = 2$; $\theta = 1$; $\theta_p = 1$; $\phi = 0.5$. Given these values, it follows from Proposition 3.1 that optimal production capital investment K^* is 1.2005, and optimal abatement capital investment is 0.2798 and optimal consumption in period 1 C_1^* is 8.5197. It follows from Proposition 3.2 the optimal social welfare W^* is -0.00073. Furthermore, the optimal pollutant emissions P^* is 0.2071 from (2.5).

Figures **??**–**??** illustrate the results of the comparative statics analysis for optimal consumption in period 1, C_1^*, production capital investment K^* and abatement capital K_a^*, pollutant emissions, and social welfare. Figure **??** shows the optimal abatement capital, K_a^* is slightly increasing in the coefficient of absolute risk aversion, θ, and the optimal production capital K^* is also increasing in θ. Thus, optimal consumption C_1^* is decreasing in θ. Figure **?**.c) shows $\partial P/\partial\theta > 0$ and it yields:

$$\frac{K_\theta}{K_{a,\theta}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.1}$$

where $K_\theta = \partial K/\partial\theta$ and $K_{a,\theta} = \partial K_a/\partial\theta$. The left-hand side of (4.1) is the ratio of the production capital change to the abatement capital change to change in θ. The right-hand side of (4.1) is the marginal rate of technical substitution (MRTS) in the pollutant emissions. This implies that the increase rate of investing in the production capital is larger than that of the abatement capital, pollutant emissions are increasing in θ. From these impacts of θ, we naturally predict the optimal social welfare W^* is decreasing in θ. However, it follows from our numerical calculation that we obtain $\partial W^*/\partial\theta > 0$. Then, W^* is increasing in θ. This result is consistent with the standard comparative static analysis of CARA utility function with respect to the coefficient of risk aversion[3].

An increase in the damage coefficient, θ_p, causes more damage to the representative consumer. Then, Figure **??** shows the optimal abatement capital, K_a^* is increasing in the damage coefficient θ_p, while the optimal production capital K^* and the optimal consumption in period 1, C_1^* are decreasing in θ_p. The results of the both capitals change in θ_p yield the pollutant emissions are decreasing in θ_p and it yields:

$$\frac{K_{\theta_p}}{K_{a,\theta_p}} < \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.2}$$

where $K_{\theta_p} = \partial K/\partial\theta_p$ and $K_{a,\theta_p} = \partial K_a/\partial\theta_p$. The left-hand side of (4.2) is negative, while the right-hand side of (4.1), MRTS, is positive. Overall the optimal social welfare W^* is decreasing in θ_p.

Higher volatility σ means the possibility of having less wealth in period 2. Then, the risk-averse central planner reduces the consumption in period 1 and invests in production capital more. Figure **??** shows the behavior, i.e., the optimal production capital, K^* is increasing in σ, while the optimal consumption in period 1, C_1^* are decreasing in σ. Furthermore, the abatement capital is increasing in σ. An increase in K^* means more pollutant emissions. To reduce damage

[3]If a utility function u is of form a constant absolute risk aversion (CARA) utility given by $u = -(1/\theta)e^{-\theta c}$, then we have $\partial u/\partial\theta > 0$.

from pollutant emissions, the central planner invests more in the abatement capital. Figure **??**.c) shows that the pollutant emissions are increasing in σ. Then we have:

$$\frac{K_\sigma}{K_{a,\sigma}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.3}$$

where $K_\sigma = \partial K/\partial\sigma$ and $K_{a,\sigma} = \partial K_a/\partial\sigma$. This implies that the increase rate of investing in the production capital is larger than that of the abatement capital, pollutant emissions are increasing in σ. It follows from these impacts of σ that the optimal social welfare W^* is decreasing in σ.

It follow from Figure **??** that the results of comparative-static analysis on the degree of ambiguity, ϕ, are similar to σ. This result is consistent with precautionary principle. The ambiguity-averse central planner reduces the consumption in period 1 and invests in production capital more. Furthermore, she invests in abatement capital more in order to reduce pollutant emissions. The pollutant emissions are, however, increasing in ϕ and it yields:

$$\frac{K_\phi}{K_{a,\phi}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.4}$$

where $K_\phi = \partial K/\partial\phi$ and $K_{a,\phi} = \partial K_a/\partial\phi$. This implies that the increase rate of investing in the production capital is larger than that of the abatement capital, pollutant emissions are increasing in ϕ. Overall the optimal social welfare W^* is decreasing in ϕ.

Figure **??** shows that although K^* is initially increasing in the coefficient of the level of technology, A, once A has reached a certain level, K^* is decreasing in A. An increasing in A produces the opposite results for C_1^*. On the other hand, K_a^* is slightly increasing in A. Despite the result of K^*, Figure **??**.c) shows that the pollutant emissions are increasing in A. Then we have:

$$\frac{1}{A}\frac{K}{\alpha}\frac{1}{K_{a,A}} + \frac{K_A}{K_{a,A}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.5}$$

where $K_A = \partial K/\partial A$ and $K_{a,A} = \partial K_a/\partial A$. Overall the optimal social welfare W^* is increasing in ϕ.

Figure **??** shows that K^* is increasing in the output elasticity of capital, α, while C_1^* is decreasing in α. K_a^* is decreasing in α. These results are not expected. Because output increases in the output elasticity of capital. Both capital changes generate more pollutant emissions and we have:

$$\frac{1}{\alpha}\frac{K}{K_{a,\alpha}}\ln K + \frac{K_\alpha}{K_{a,\alpha}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.6}$$

where $K_\alpha = \partial K/\partial\alpha$ and $K_{a,\alpha} = \partial K_a/\partial\alpha$. Overall the optimal social welfare W^* is initially decreasing in α, once α has reached a certain level, W^* is increasing in α.

Figure **??** shows that K^* is slightly decreasing in the conversion factor between output and pollutant emission γ and K_a^* is increasing in γ, while C_1^* is decreasing in γ. An increasing in γ means the output generates more pollutant emissions. Figure **??**.c) shows the relationship between γ and pollutant emissions. It follows from the result of pollutant emissions that we obtain:

$$\frac{1}{\gamma\alpha}\frac{K}{K_{a,\gamma}} + \frac{K_\gamma}{K_{a,\gamma}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.7}$$

where $K_\gamma = \partial K/\partial\gamma$ and $K_{a,\gamma} = \partial K_a/\partial\gamma$. Overall the optimal social welfare W^* is decreasing in γ.

Figure **??** shows that an increasing in the emission abatement coefficient, b, results the similar impacts of γ: C_1^* and K^* is decreasing in b, while K_a^* is increasing in b. An increasing in b generates more pollutant emissions as in Figure **??**.c). Then we obtain:

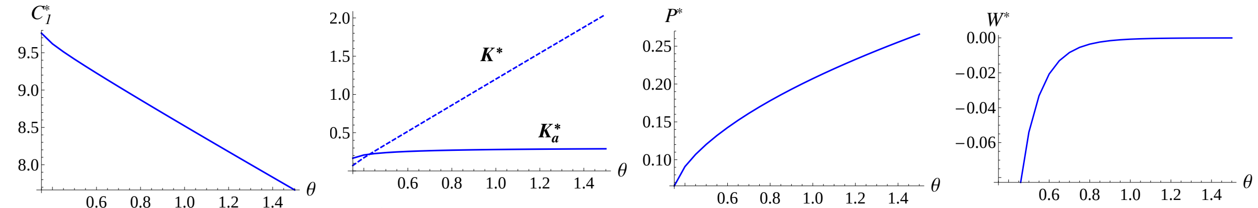
$$\frac{1}{b\alpha}\frac{K}{K_{a,b}} + \frac{K_b}{K_{a,b}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.8}$$

where $K_b = \partial K/\partial b$ and $K_{a,b} = \partial K_a/\partial b$. Overall the optimal social welfare W^* is decreasing in b.

Finally, Figure **??** shows that K^* and C_1^* are decreasing in the emission abatement elasticity of abatement capital, λ, while K_a^* is increasing in λ. It follows from (2.5) and (2.6) that an increasing in λ generates more pollutant emissions if the amount of both capacities does not change. Notice that K_a^* is less than 1 in our numerical examples. Figure **??**.c) shows, however, P^* is initially increasing in λ. Once λ has reached a certain level, P^* is decreasing in λ. Then we have:

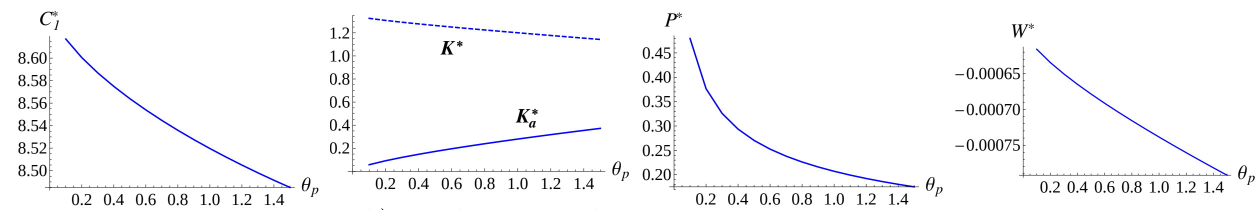
$$-\frac{1}{\alpha}\frac{K}{K_{a,\lambda}}\ln K_a + \frac{K_\lambda}{K_{a,\lambda}} > \frac{\lambda}{\alpha}\frac{K}{K_a}, \tag{4.9}$$

where $K_\lambda = \partial K/\partial\lambda$ and $K_{a,\lambda} = \partial K_a/\partial\lambda$. Overall the optimal social welfare W^* is decreasing in λ.



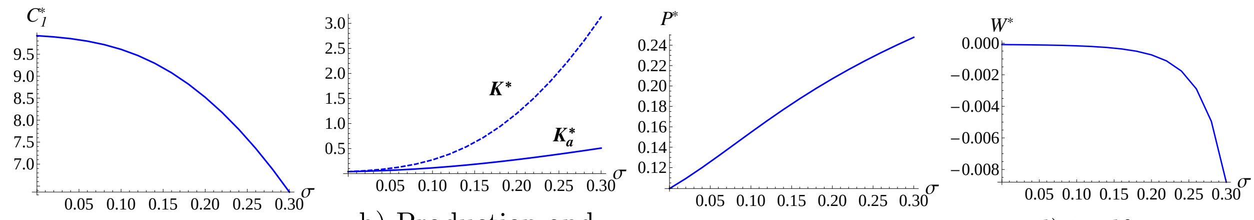
a) Consumption b) Production and Abatement Capitals c) Pollutant Emissions d) Welfare

Figure 1: Comparative static effects of θ



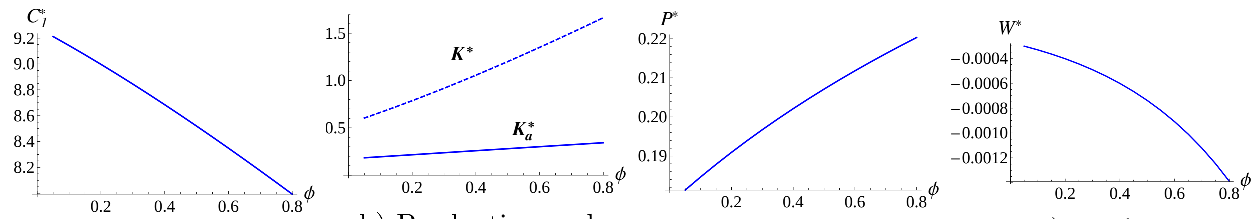
a) Consumption b) Production and Abatement Capitals c) Pollutant Emissions d) Welfare

Figure 2: Comparative static effects of θ_p



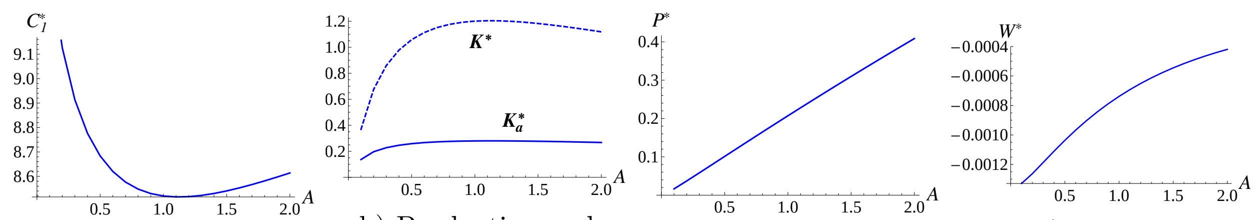
a) Consumption b) Production and Abatement Capitals c) Pollutant Emissions d) Welfare

Figure 3: Comparative static effects of σ



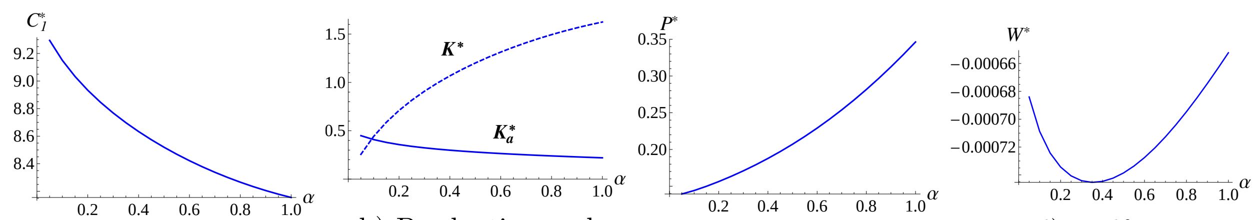
a) Consumption b) Production and Abatement Capitals c) Pollutant Emissions d) Welfare

Figure 4: Comparative static effects of ϕ



a) Consumption b) Production and Abatement Capitals c) Pollutant Emissions d) Welfare

Figure 5: Comparative static effects of A



a) Consumption b) Production and Abatement Capitals c) Pollutant Emissions d) Welfare

Figure 6: Comparative static effects of α

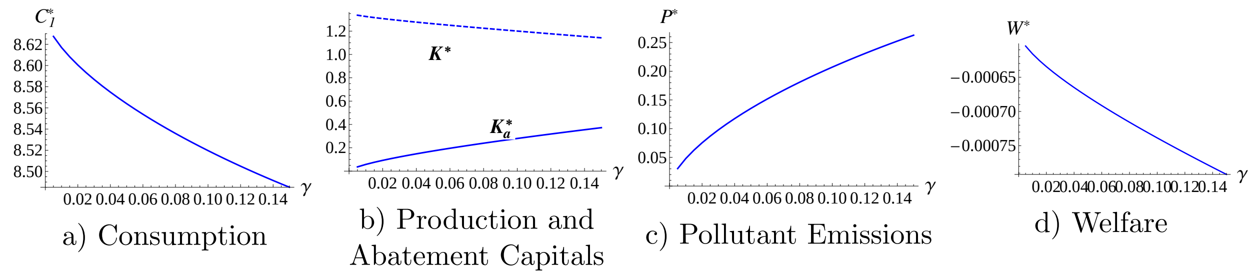
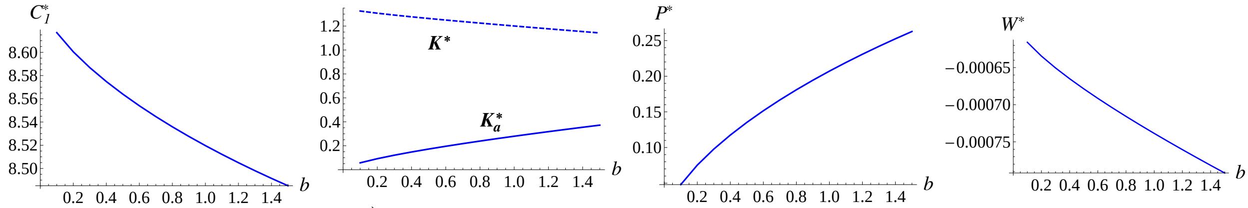
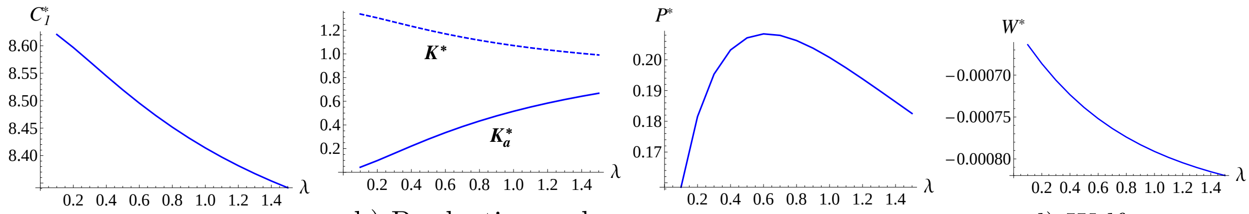


Figure 7: Comparative static effects of *gamma*



a) Consumption   b) Production and Abatement Capitals   c) Pollutant Emissions   d) Welfare

Figure 8: Comparative static effects of b



a) Consumption   b) Production and Abatement Capitals   c) Pollutant Emissions   d) Welfare

Figure 9: Comparative static effects of λ

5 Conclusion

In this paper, we analyzed pollutant abatement investment under ambiguity in a two-period setting. We solved the social welfare maximizing problem and numerically derived the optimal level of production and pollutant abatement capital investments. Comparative statics analysis revealed that an increase in the degree of ambiguity and volatility of income encourage the both capital investments.

There are several ways to extend this paper. First, we can extend the model to a multi-period setting by incorporating the dynamics of income or other economic parameters. Next, productivity is matter for environmental policy makers. Then, we incorporate uncertainty about technological progress as a next phase of research. Such uncertainty could be formulated by using the Poisson distribution. These important topics are left to future research.

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