Dixit-Pindyck and Arrow-Fisher-Hanemann-Henry Option Concepts in a Finance Framework

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Abstract

We look at the debate on the equivalence of the Dixit-Pindyck (DP) and Arrow-Fisher-Hanemann-Henry (AFHH) option values. Casting the problem into a financial framework allows to disentangle the discussion without unnecessarily introducing new definitions. Instead, the option values can be easily translated to meaningful terms of financial option pricing. We find that the DP option value can easily be described as time-value of an American plain-vanilla option, while the AFHH option value is an exotic chooser option. Although the option values can be numerically equal, they differ for interesting, i.e. non-trivial investment decisions and benefit-cost analyses. We find that for applied work, compared to the Dixit-Pindyck value, the AFHH concept has only limited use.

Keywords: Benefit cost analysis, irreversibility, option, quasi-option value, real option, uncertainty.

Abbreviations: i.e.: id est; e.g.: exemplum gratia.

1 Introduction

In the recent decades, the real options[1] concept gained a large foothold in the strategy and investment literature, even more so with Dixit & Pindyck (1994)'s (DP) seminal volume on investment under uncertainty. In environmental economics, the importance of irreversibility has already been known since the contributions of Arrow & Fisher (1974), Henry (1974), and Hanemann (1989) (AFHH) on quasi-options. As Fisher (2000) notes, a unification of the two option concepts would have the benefit of applying the vast results of real option pricing in environmental issues. In his

[1]The term *real option* has been coined by Myers (1977).

analysis, he finds that the two concepts are indeed equal and gives out a *license* to employ real option results in environmental settings. However, Mensink & Requate (2005) show that his results are based partly on a faulty inequality relation and that the two concepts are equal only in special cases, as is conceded by Fisher (2005). Traeger (2014) discusses the difference between the concepts and analyzes when they differ in a numerical example, showing the relevance of the concepts' difference for decision-making. His term *full value of sophistication* emphasizes Hanemann's (1989) point that option values are calculated in closed-loop control. That is, at each decision stage before the last, it is acknowledged that information is to be revealed later.

While the results of Mensink & Requate (2005) and Traeger (2014) are valid, they introduce several new terms and concepts, like e.g. the *pure postponement value* or the (actually, not so simple) *simple option value*. Outside of the narrow scope of the setting, these terms have only limited meaning. As we show, the same concepts are already widely employed in the financial options literature under different names. Given the vast literature on financial option pricing and the inroads that real option pricing could make into managerial decision making, the efforts seem indeed warranted to unify the environmental concept of *quasi-option value* (QOV) not only with the *Dixit-Pindyck option value* (DPOV), but also with the background of the latter, financial option valuation. Hanemann (1989) explained the concept of the quasi-option indeed by reference to the financial option, albeit only in a footnote[2]. However, we believe this quick reference does not do justice to the difficulties and rather hinders analysis by suggesting that price and value of an option are generally different.[3]

Investment decisions are rarely carried out in monopolistic situations. Real options literature has therefore increased its scope to include game-theoretic analysis (Real option games), see Smit & Trigeorgies (2006) for an overview. The quasi-

[2] Journal of Environmental Economics and Management 16, p. 28, footnote 7.

[3] What we believe that Hanemann (1989) refers to as an option's *price* is rather its intrinsic value.

option has so far not been used in competitive settings, with the exception of Fuji & Ishikawa (2012), who state that the QOV concept arrives at theoretical difficulties in competitive settings. Before attempts are made to overcome these and develop quasi-option concepts parallel to real option games, we think it is advisable to further clarify the relation of quasi-options to both real options and financial options, and analyze whether any concept is superior for economic analysis.

The remainder is organized as follows: in section 2, we state a 2-period decision problem analogous to the previous literature. Using finance concepts, we can readily relate the expected net benefits of actions to terms of standard options pricing. In section 3, we then show that the AFHH and DP values correspond one-to-one to terms already employed in the financial option literature. Using these results, we find the conditions under which the concepts coincide and state whether the coincidence is meaningful or not and analyze a numerical example. In section 4, we assess which concept is more amenable to applied work. In section 5 we conclude.

2 Investment under Uncertainty

2.1 Valuation by Expected Utility

We employ the structure as in Mensink & Requate (2005): An irreversible investment project of say, cutting down a forest can be carried out now (in $t = 1$) or can be postponed to $t = 2$.

The decision maker can also decide to not carry it out at all. The variable $x_{t=1,2}$ indicates whether the project has been carried out or not. Due to irreversibility, there are three possible sequences of development.

- No development: $x_1 = x_2 = 0$.
- Late development: $x_1 = 0, x_2 = 1$.
- Instant development: $x_1 = x_2 = 1$.

The net benefits of developing or not in $t = 2$ are uncertain and depend on a state $\tilde{\theta}$ that is revealed in $t = 2$ to be high ($\overline{\theta}$) or low ($\underline{\theta}$). Of course, to make the problem interesting, in at least one realization, the investment must not be justified.

The investor's welfare from investing is

$$u_1(x_1) + u_2(x_1, x_2, \tilde{\theta}), \tag{1}$$

with $u_1(x_1 = 1)$ and $u_1(x_1 = 0)$ the utility derived from the net payoff from investing and not investing in $t = 1$, respectively. $u_2(x_1, x_2, \tilde{\theta})$ is the utility derived in $t = 2$, dependent on the actions x_1, x_2 and the state $\tilde{\theta}$ realized. Discounting of benefits of period $t = 2$ is done via the utility function u_2. The model implicitly allows for later periods $t = 3, \ldots$. With uncertainty revealed and actions carried out in earlier periods, contingent payoffs from period $t = 3$ on can be subsumed to $t = 2$, e.g. as we do later by calculation of present values through a discount factor β.

The present value (1), which the investor tries to maximize, depends on his sophistication, availability of information and technical, legal constraints, etc. Traeger (2014) names the three important cases *learning*, *postponement*, and *now-or-never*.

Under learning, a sophisticated investor can invest either in $t = 1$, or she can wait until $t = 2$ to observe the realization of $\tilde{\theta}$ and decide then, wether investment is justified. In stochastic control analysis this is called a closed-loop control, see Mensink & Requate (2005). When the investor does not invest in $t = 1$ and has the ability to decide on the project after the realization of $\tilde{\theta}$ is revealed to her, her position is

$$V^l(0) = u(0) + E(\max\{u_2(0, x_2, \tilde{\theta})\}). \tag{2}$$

Under postponement, the decision maker can wait as well until $t = 2$ to invest, but she does not receive any information on the outcome of $\tilde{\theta}$. Thus, her position is equal to already committing in $t = 1$ on wether the investment is carried out in

$t = 2$ or not. This is called an open-loop control, see Mensink & Requate (2005).[4] If she does not invest immediately, her position is

$$V^p(0) = u(0) + maxEu_2(0, x_2, \tilde{\theta}). \tag{3}$$

Given the convexity of the maximum operator and Jensen's inequality, $V^p(0) \leq V^l(0)$ must hold.

When the decision cannot be postponed, an ability to wait is not given. Traeger (2014) calls this the now-or-never situation. If the investor can only invest *now or never*, and she does not invest in $t = 1$ (i.e. she invests never), the position is

$$V^n(0) = u(0) + Eu_2(0, 0, \tilde{\theta}). \tag{4}$$

If the decision maker decides to invest in $t = 1$, for valuation it is not of relevance what the alternatives would be, so $V^l(1) = V^p(1) = V^n(1)$ and

$$V^{l,p,n}(1) = u(1) + Eu_2(1, 1, \tilde{\theta}). \tag{5}$$

As in Traeger (2014), the option values can then be defined as

$$DPOV = max\{V^l(0), V^l(1)\} - max\{V^n(1), V^n(0)\} \tag{6}$$

and

$$QOV = \{V^l(0) - V^n(0)\} - \{V^p(0) - V^n(0)\}. \tag{7}$$

2.2 Translation to Financial Option Pricing

With only few and light additional assumptions over previous studies, these terms can be readily related to standard financial options pricing.

[4] There are differing degrees regarding how myopically decisions are carried out. Under so-called open-loop with feedback control, in $t = 1$ the investor decides as if she would not learn the true state of θ in $t = 2$. When arriving in $t = 2$ and learning of θ, she might however decide to revise her plans. Nevertheless, since the revision in $t = 2$ does not affect the decision in $t = 1$, whether the open loop is with or without feedback is not relevant, at least in settings where uncertainty lasts only one period, see Hanemann (1989).

As in Fisher (2000), Mensink & Requate (2005) and Traeger (2014) we assume that the revelation of information in $t = 2$ is independent of the action in $t = 1$ and that pricing is effectively done by forming risk-neutral expectations E^Q. Also, discounting from $t = 2$ to $t = 1$ is done through an explicit discount factor $\beta = \frac{1}{1+r}$, not through the utility function u_2. We assume that all uncertainty in $t = 2$ stems from uncertain benefits $S_{2,\tilde{\theta}}$, while costs K are constant. It would be straightforward to allow costs to be uncertain, with benefits constant. As will be more lucid later, with a change of measure as in Margrabe (1978), it is possible to allow both of them to be uncertain. Since this would not yield new insights but only increase complexity, we only allow benefits to be uncertain, however.

Consider that investment occurs immediately in $t = 1$. Then, the investor bears costs K and receives benefits of

$$S_1 = s_1 + \beta E^Q(S_{2,\tilde{\theta}}), \tag{8}$$

where s_1 is the immediate (and certain) benefit, which can be captured by investing now and S_2 the uncertain future benefits. S_1 is the equivalent of the cum dividend (s_1) spot price of a stock S. Then, the value of immediate investment can simply be stated as

$$V^l(1) = V^p(1) = V^n(1) \simeq S_1 - K. \tag{9}$$

If, however, investment occurs never, there are neither any (immediate or future) benefits nor costs. It must therefore be the case that

$$V^n(0) \simeq 0. \tag{10}$$

$V^n(0)$ serves as the base case, measuring the value of the situation in which there exists no opportunity to carry out the project at any time.[5]

[5]Note that we do not require that $u_2(0, 0, \overline{\theta}) = u_2(0, 0, \underline{\theta}) = 0$. The investor's other assets may have exposure to $\tilde{\theta}$. Since $V^n(0)$, the investment project that is not carried out does not alter her position in any time or state, its value must be zero, however.

If the investor decides in $t = 1$ to carry out the investment in $t = 2$, there are two contrary effects. On the one side, she foregoes the immediate benefits s_1. On the other side, costs K are incurred one period later. Her position is

$$\beta E^Q(S_{2,\tilde{\theta}}) - \beta K \tag{11}$$

In the forward market, at $t = 1$ investors agree on a price F, to be paid at $t = 2$ at which a stock shall be exchanged in $t = 2$. This *forward price* is $F = E^Q_{t=1}(S_{2,\tilde{\theta}})$. In $t = 1$, the position of the investor is thus $\beta(F - K)$. Under postponement, the position of an investor who does not invest immediately, i.e. in $t = 1$ is then

$$V^p(0) \simeq max\{\beta(F - K), 0\}. \tag{12}$$

Under *learning*, when investment is not carried out in $t = 1$, the investor will carry out the project in $t = 2$ if profitable, that is, if $S_2 - K > 0$, so

$$V^l(0) \simeq \beta E^Q(max\{S_{2,\tilde{\theta}} - K, 0\}). \tag{13}$$

(13) is the equivalent of a European-style call option maturing in $t = 2$, see Duffie (2001, p.119.) European-style call (put) options give their owners the right, but importantly, not the commitment to buy (sell) an asset at a pre-specified price at a certain single future date, the *maturity date*. See Hull (2005) for an introduction to option pricing and Baxter & Rennie (1996) for an introduction to option pricing emphasizing the risk-neutral measure Q.

Between European calls and puts on the same underlying, same exercise price and same maturity, there exists an analytic relation, so-called put-call parity, derived by Stoll (1969):

$$C^{Eur} - P^{Eur} = \beta(F - K), \tag{14}$$

where the forward F has the same maturity date as the options.[6] Using put-call parity, equation (12) can be rewritten as

$$V^p(0) \simeq max\{0, C^{Eur} - P^{Eur}\}. \tag{15}$$

3 DP and QOV as Time Value and Exotic Option

3.1 Analytic Comparison

We analyze Traeger (2014)'s definition of the Dixit & Pindyck option value with financial terms:

$$max\{V^l(0), V^l(1)\} - max\{V^n(1), V^n(0)\}, \tag{16}$$

$V^l(0)$ is the value of a European option maturing at $t = 2$, but $V^l(1)$ is the value of exercising it earlier in $t = 1$. We therefore see that the first part of equation (16) is an American option. The second part is the maximum of exercising the option now or never, $max\{S_1 - K, 0\}$, which is the exact definition of the intrinsic value of an option, see Hull (2005). As the difference between option value and intrinsic value, the Dixit-Pindyck option value is thus the *time value* of an American[7] option and can be written as

$$max\{C^{Eur}, S_1 - K\} - max\{S_1 - K, 0\} \tag{17}$$

If we assess on the other side the AFHH quasi-option (QOV) value

$$QOV = \{V^l(0) - V^n(0)\} - \{V^p(0) - V^n(0)\} \tag{18}$$

the term in the first bracket is a European call option value, while the term in

[6] Here is a short proof for put-call parity for the model above: Consider building a portfolio of one put and one bond worth βF. In $t = 2, \overline{\theta}$, the portfolio will have a worth of $0 + S_{2,\overline{\theta}}$ and a value of $K - S_{2,\underline{\theta}} + S_{2,\underline{\theta}} = K$ in $t = 2, \underline{\theta}$. Next, consider building a portfolio of one call and one bond worth βK. In $t = 2, \overline{\theta}$, the portfolio will have a worth of $S_{2,\overline{\theta}} - K + K = S_{2,\overline{\theta}}$ and a value of K in $t = 2, \underline{\theta}$. Since the portfolio values are equal in every state, equation (14) must hold. See also Hull (2005).

[7] A call option is called American (-style) when it can be exercised, i.e. settled at $S_t - K$, at every instant $t \leq T$. Since we we have a discrete two-periods setting, this is the case in our example.

the second pair of brackets looks like an intrinsic value, but is adjusted for the fact that it is paid out only in $t = 2$. Using put-call parity (14) and rearranging

$$\begin{aligned} QOV &= C^{Eur} - max\{C^{Eur} - P^{Eur}, 0\} \\ &= -max\{-P^{Eur}, -C^{Eur}\} \\ &= min\{P^{Eur}, C^{Eur}\}, \end{aligned} \tag{19}$$

we see that the QOV is a rather exotic option, the minimum of a put and a call.

In the financial options pricing literature this is called a *dealer's choice chooser option*.[8] With a standard chooser option, at first maturity T_1, the option holder decides wether the option shall be converted to either a European put or a European call, which is then exercised at maturity $T_2 > T_1$. At T_1, the chooser's option price is $max\{P^{Eur}, C^{Eur}\}$. Chooser options are used by speculators and hedgers who expect strong volatility, but can not assess the direction of the volatility, i.e., whether prices go up or down. Usual chooser options are always at least as valuable as a single put or call. They show a u-shaped relation to the underlying's price S. This means the more the underlying's price has gone into either direction, the higher the chooser option's price. For a dealer's choice chooser option, the option holder's counterparty, the dealer (also called underwriter) can decide on wether it be a put or call, its value is $min\{P^{Eur}, C^{Eur}\}$, as is the case here. These options are most valuable, when the uncertainty on whether the price will be high or low is still not resolved. They show a tent-shaped price pattern with regard to the underlying's price, as can be observed in the upper panel of figure 1.

From the fact that one concept is rather standard, while the other is more exotic, one should not yet favor one concept over the other. Despite being called exotic, European chooser options can be valued easily, while for American options usually only numerical approximations can be derived. We therefore defer the question,

[8] See Rubinstein & Reiner (1992) and Rubinstein (1991) for a detailed analysis of chooser options and their use.

which concept should be considered superior for applied work to section 4.

The values of the competing concepts of DP and AFHH's QOV are depicted for different prices of S in a Black-Scholes[9] case in Figure 1. As Traeger (2014) states, both the DP and the AFHH value are positive. But as long as there remains some uncertainty, only the AFHH value is strictly positive, while the DP option value is zero when immediate exercise is warranted.

To analyze when the two concepts yield identical values, we look at the difference

$$\Delta = QOV - DP = min\{P, C\} - (max\{C, S-K\} - max\{S-K, 0\}). \qquad (20)$$

All in all, there are $2 \times 3 = 6$ cases to be considered: $P < C$, $P \geq C$, and, after deleting the impossible case of $C < 0$, three possible relations between C, $S-K$, and 0:

I)	$C < P$	$\cap$	C	$\geq$	S-K	$\geq$	0	$\rightarrow \Delta_I$	$=$	$S - K \geq 0$
II)	$C < P$	$\cap$	S-K	$\geq$	C	$\geq$	0	$\rightarrow \Delta_{II}$	$=$	$C \geq 0$
III)	$C < P$	$\cap$	C	$\geq$	0	$\geq$	S-K	$\rightarrow \Delta_{III}$	$=$	$C - C = 0$
IV)	$C \geq P$	$\cap$	C	$\geq$	S-K	$\geq$	0	$\rightarrow \Delta_{IV}$	$=$	$P - C + (S - K)$
V)	$C \geq P$	$\cap$	S-K	$\geq$	C	$\geq$	0	$\rightarrow \Delta_V$	$=$	$P - 0 \geq 0$
VI)	$C \geq P$	$\cap$	C	$\geq$	0	$\geq$	S-K	$\rightarrow \Delta_{VI}$	$=$	$P - C \leq 0$

The first three relations are equivalent to the ones of formula (17) in Hanemann (1989, p. 29). The value of the quasi option is (weakly) higher than the DP value in cases I, II, and IV, and (weakly) lower in case VI.

Looking at IV) more closely, we see that the DP value is higher than the AFHH value only when $(C - P) = \beta(F - K) > (S - K)$, which we call (IVa). For this to hold, the forward price must be sufficiently higher than the spot price, $F > \frac{S-K}{\beta} + K$. This happens when the project is profitable now, but is expected to be even more

[9]The celebrated Black & Scholes (1973) formula allows for a closed-form solution of an option's price by imposing several assumptions. Most importantly, it is assumed that stocks can be traded continuously without transaction costs, that dividends and interest are paid out continuously, and that the underlying's price follows a geometric Brownian motion.

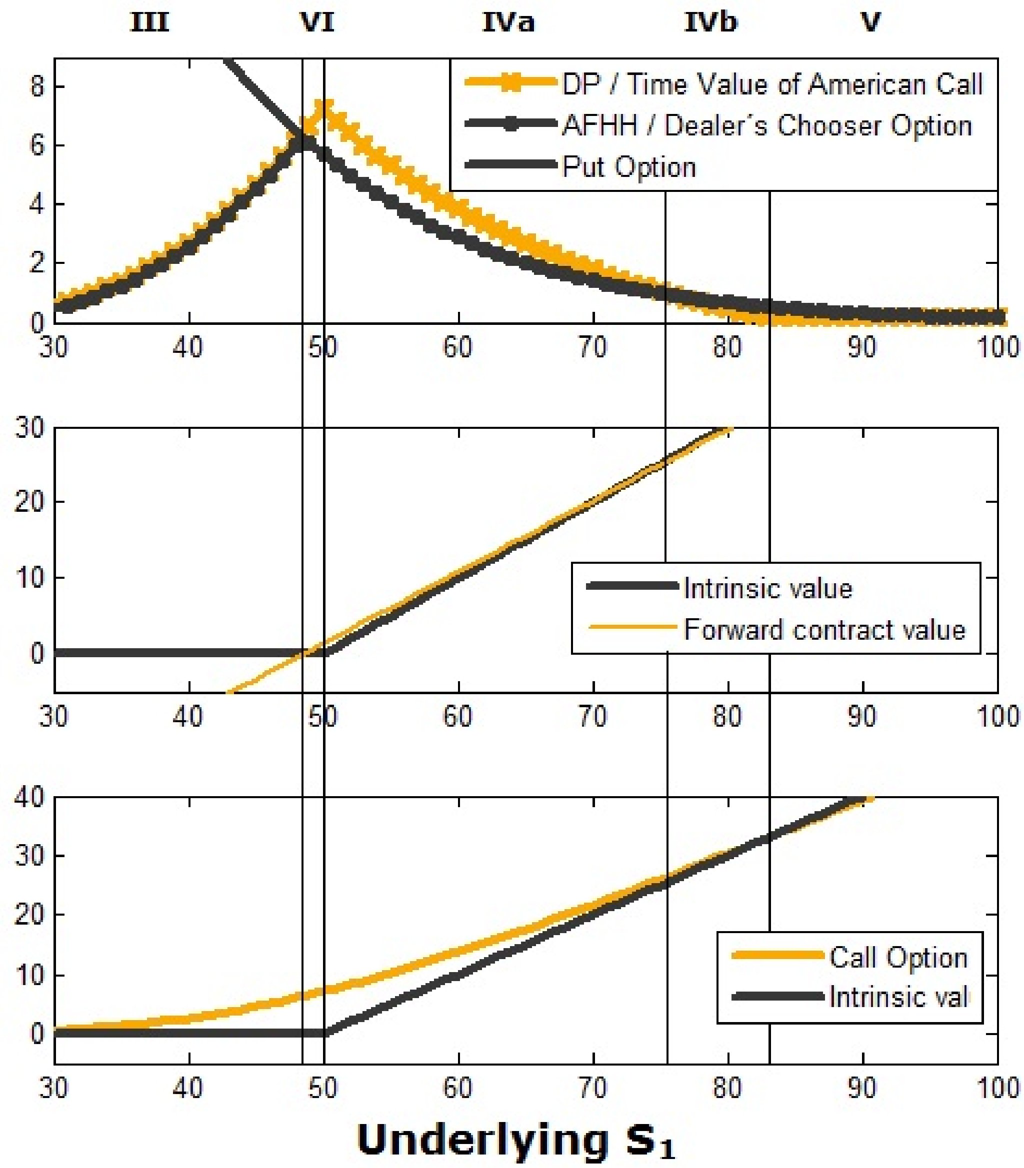

Figure 1: The upper panel depicts the AFHH and DP option values for a range of prices of the underlying S in $t = 1$. We calculate a call with strike price 50, time to maturity of 3 years, annual volatility of 20%, annual interest rate of 3%, dividend yield of 2%. The center panel depicts the intrinsic value $S_1 - K$ and the forward contract value $\beta(F - K)$. Their intersections separate case III from IV and case IVa from IVb. The lower panel depicts the European Call price C^{Eur} and its intrinsic value. Their intersection separates case IVb from V. The kink at 50 where the intrinsic value turns strictly positive separates case VI from IV. Cases I and II can only be shown in parameter settings where $y > r$.

profitable when carried out in a future period, so the forward contract value $F - K$ of the project is greater than the hurdle of $\frac{S-K}{\beta}$. In case VI), the project is not yet profitable ($S - K < 0$), but is (market-) expected to be be so in the future ($F - K > 0$). Then the DP value is unequivocally higher than the AFHH concept. Hanemann (1989) errs thus in stating that the quasi-option value is bound from below by the *economic value of perfect information* EVPI, which in his definition is equivalent to the DP option value.

The two concepts are equivalent under two conditions: Firstly, when the prospects of the project are sufficiently unattractive, i.e. when $S < K$, so investment is not carried out in $t = 1$, and when $C < P \rightarrow F < K$, so also the forward contract value of the investment would be assessed as negative. In figure 1, where, because of continuous dividend and interest payments, forwards are calculated as $F = Se^{(r-y)T}$, the forward price is higher than the spot, so the two concepts are equal up to the spot $S = 50e^{-(0.01)3} = 48.52$.

Secondly, the two concepts are equal in the odd case where exactly $P - C = \beta(F - K) = S - K$. In the center panel of figure 1, this can be seen to be the case at $S = 73.9$. While this case does not bear any special meaning, it leads to a special parameter combination when indeed the two concepts are equal over the entire domain of S: Setting $\beta(F - K) = S - K$, we find that this can hold only when $\beta = 1$, i.e., when future payoffs are not discounted and when $S = F$, that is, when there is neither contango nor backwardation of forward prices of S. In the Black-Scholes case, this means that the convenience yield y is equal to the risk-less interest rate r. Thus, the two concepts are exactly equal, when $y = r = 0$ or when the uncertainty will be resolved very soon, so that discounting is negligible and the forward equals the spot, $T \rightarrow 0$.

In effect, for all interesting parameter settings, the two concepts are unequal. Also, the less trivial the investment decision, the more the two concepts differ numerically. This can again be observed in figure 1. In case III, where immediate exercise is not warranted and future exercise not expected, the decision against the project is easy. For the far right of V, AFHH's QOV converges to the DPOV, zero. With the investment project that profitable, the decision for immediate investment is again very easy.

3.2 A numerical example

We re-consider the widget factory example of Dixit & Pindyck (1994), as already Mensink & Requate (2003) did. A factory can produce one widget per period. The current price of widgets is 200, next period it will with equal probability either decrease to 100 or increase to 300 and then stay at this level forever. The investment outlays for the factory are -1600. The risk-free interest rate is 10% per period. Faced with a now-or-never decision, the investor would calculate the NPV as $-1600 + 200 + \frac{1}{0.1}\frac{100+300}{2} = 600$. She would therefore build the plant. Under postponement, decision in $t = 1$ to build the plant in $t = 2$ would give an NPV of $\frac{1}{1+0.1}(\frac{1+0.1}{0.1} \times 0.5(300 + 100) - 1600) = 545$, the investor would therefore opt for immediate investment.

Under learning, only in the good environment investment would occur, the value of the plant, similar to an American call would be $0.5 \times (\frac{300}{0.1} - \frac{1600}{1.1}) = 773$, as already found in Dixit & Pindyck (1994).

We can therefore determine the time value of the option, which is the DP value as $773 - 600 = 173$. To find the AFHH value, we have to find the minimum of a European call and a European Put. The quasi-option would therefore consider a counterfactual investment, where 1600 are received in $t = 2$, but then one widget per period has to be bought. Such an investment would only be carried out in the state where the price is 100. This investment's value would amount to $0.5 \times (\frac{-100}{0.1} + \frac{1600}{1.1}) = 227$. This value can also be found by simply invoking put-call parity, $773-545 = 227$, or by using formula (18) provided in Mensink & Requate (2003). The quasi-option value would then be $min(227, 773) = 227$.

To option concepts are equal in e.g. the case when the investment outlay K is at least 2200, making the intrinsic value exactly 0. The value of the call, and its time value as well is then $0.5 \times (\frac{300}{0.1} - \frac{2200}{1.1}) - 0 = 500$; the value of the put would be $0.5 \times (\frac{-100}{0.1} + \frac{2200}{1.1}) = 500$ as well, so $min(500, 500) = 500$ and the two concepts yield equal results.

4 Which concept to adopt?

For optimal investment decisions and benefit-cost analysis, it is of high importance, which concept of option value should be employed.

Traeger (2014) analyzes this question by valuing an investment opportunity under both concepts. Using the DP concept, he finds that a sufficient condition for immediate investment is when the NPV ($S_1 - K$, when translated to real option terms.) of a project is greater than zero and the DP value exactly zero. In figure 1 this occurs in range V.

With the AFHH concept alone, it is however, not possible, to carry out a correct investment decision. Instead, the so-called *simple option value* (SOV) needs to be added to the QOV, which can then be compared to the investment's NPV. If $NPV > QOV + SOV$, investment should commence immediately. The SOV is defined as $V^p(0) - V^n(0)$. Translated to financial terms, it is thus $max(0, C^{Eur} - P^{Eur})$. Then, the use of the QOV concept is much reduced, since it cannot be used for investment analysis without the SOV correction. In effect, by adding SOV, investment analysis with the QOV goes one step back to arrive at the value of an European call.

$$QOV + SOV = min(C^{Eur}, P^{Eur}) + max(0, C^{Eur} - P^{Eur}) = C^{Eur}. \qquad (21)$$

Then, this value is compared to the the NPV, which equals immediate exercise $S_1 - K$. So corrected investment analysis by the QOV actually means making a decision on early exercise of an American option. Investment analysis by the QOV and SOV is the just a rose by an other name for investment analysis by the DP concept.

As we found earlier, when there are no interest rates and forward prices are equal to spot prices, the two concepts are equal. While indeed a case can be made for setting the risk-less rate at or near zero in environmental cost-benefit analyses, see Arrow et al. (2012), there can hardly be made the case that the forward price should always be equal to the spot price. In resource markets, forward prices are often found to be far below the spot (a situation known as backwardation in commodity trader jargon) or above the spot (known as contango), see Geman (2005). A zero dividend yield would imply that there are no benefits foregone by late investment. This might hold approximately for some problems, e.g. the amount of timber in the forest to be cut simply accumulates. But we think in most applications investing late implies foregoing benefits which can not be recovered.

The numerical example emphasizes the problems we find with the quasi-option value: Firstly, it yields results inapt for decision-making, which are starkly different from the applicable values. Secondly, it involves the often pointless valuation of a hypothetical investment, which in general does not bear any meaning in applications. Thirdly, the current benefits of 200 are absent in the calculation of the quasi-option value. The DP value is reduced one-for-one for every increase of current benefits; the quasi-option value, however, remains unchanged. It is thus a too languid measure to analyze immediate opportunities.

A further problem related to the European style of the QOV is that it has a set maturity T. Many problems, especially in environmental economics, are however such that there is no ex-ante specified exercise date, however far away. Rather, the option can be exercised anytime without a final exercise date. These *perpetual options* can readily be analyzed using the DP concept. The quasi-option concept, which needs a fixed maturity cannot be employed, however. With $T \to \infty$, in the limit QOV would go to zero.

In effect, we are more pessimistic than Mensink & Requate (2005) as to the usefulness of the *QOV* as a tool of investment or policy advice. On its own, it happens to give bad advice exactly when advice is needed most, that is, in investment analyses of interesting investment projects. Corrected, it ends up at the results of the Dixit-Pindyck option value.

5 Conclusion

In this article we translated the DP and AFHH option concept to financial options literature. The DP concept can be considered a standard concept, while the AFHH would be called an *exotic* concept. The two concepts lead to equal option valuations only for uninteresting parameter sets or comparably easy investment decisions, so which concept to employ is of importance. Given that the AFHH concept can not be used without a correction, its value in applied work appears rather limited. We therefore suggest using the Dixit-Pindyck option value.

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