Optimal Stochastic Pine Stands Harvest Rotation Policies Evaluations

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Abstract

The simple and multiple optimal rotation harvests pine stands models under Brownian price diffusion and Logistic or Gompertz wood stock processes for the deterministic and stochastic case, are formulated and solve for a Chilean company pine stands harvests. The applications of these models show that the company optimal cut policy for radiate pine stands validate the simple rotations stochastic models. The Logistical wood stock diffusion model optimal cuts underestimate the company actual cut 2.3% and the saturation volume in 4.5%. The Gompertz diffusion model overestimates the actual cut in a 1.2% and the saturation volume in 0.5%. Obviously the company actual cut policy does not follow the theoretical multiple rotations optimal policies: The Faustmann stochastic models underestimate the company optimal cut by 40.4% in the Logistic case and by 35.8% in the Gompertz model case. These discrepancies can be explained by the fact that the company considers just simple rotation planning horizons to evaluate the harvesting time of the pine stands.

Key words*: Optimal tree cutting, Stands optimal harvest rotation policies*

JEL codes: Q23; G32

1 Introduction to the harvest rotation stands problems

The optimal rotation forest harvest models development is marked by the emergence of two controversial issues. The first issue was formulated in its early beginning by Faustmann in 1849. He formulated the multiple rotation versions considering the dynamic effect that the future renovations had in the rotation period of the cut and plant of the trees. This issue was not considered relevant by researcher and forester managers, who preferred a simple rotation model. The model was finally rediscovered by American Nobel Economic price winner Samuelson in 1976. Samuelson validated Faustmann's deterministic formula as the correct one, since it was the only one to consider land's rent. An increasing

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number of researchers continued extending this model. Brazeer, 2001, Chang 2001, Amacher, Brazee, Deeger, 2011, presented their works in three International Faustmann Symposiums, held's in 1999, 2005 and 2009.

The second issue was also strongly formulated by Samuelson, 1976, who called to replace the "simple notion of stationary equilibrium by the notion of perpetual Brownian motion of wood price". Many researcher followed his recommendation, (see Newman, 2002), and applied the real option methodology to this problem. Clark & Reed, 1989, Thomson, 1992, Platinga et al., 1998, considered the single rotation problem as an America call considering the wood price as geometric Brown diffusion. Others, as Morck & Schwartz, 1989, and Insely, 2002, also considered wood stochastic diffusion as a geometric Brown diffusion. Alvarez & Koskela, 2007 and Navarrete, 2012, considered it as a logistic diffusion in agreement with sigmoid characteristic required by of most of the deterministic tree growing models (see Garcia, 2005). Willassen, 1998, was the first one to extend uncertainty to the Faustmann model, 1995, incorporating the forest growth as a stochastic Markov diffusion processes and characterizing the properties of the optimal solutions. Sodal, 2002, simplified Willassen approach in a closed-form rotation formula for the same state stochastic variables, and Insely & Rollins, 2005, proposed a numeric algorithm to solve its Hamilton-Jacobi-Bellman solution. Amacher, Ollikainen, Koskela, 2009, summarized these extensions in theirs seminal book. Navarrete & Bustos, 2013, extended the Faustmann model version under wood stock sigmoid, Logistical and Gompertz diffusion and price geometric Brown diffusion for risky agent, transforming the model in an equivalent optimal stopping problem with a single ITO wood stock diffusion solvable with a time parameterized Hamilton-Jacobi-Bellman equation. The optimal original cut volume was obtained by intersecting the volume diffusion equation with the different optimal parameterized solutions of the Hamilton-Jacobi-Bellman equation.

The objective of the present paper is to validate the optimal cut policies for the multiple and single rotation stands harvest under geometric Brown price and Logistical or Gompertz wood stock diffusions, with the practical optimized cut policy of a Chilean forest company.

2 METHODOLOGY

2.1 Stochastic Rotations Models

The basic model considers ITO diffusion for the wood stock and a geometric Brownian diffusion for the wood price respectively, given by equations (1) and (2).

$$dV = \mu(V)dt + \sigma(V)dW, \quad (1)$$

$$dPt = \alpha Ptdt + \beta PtdW \quad (2)$$

Under the assumption of a weak solution (V, t) for the diffusion equations (1, 2) and initial conditions ($V_0 \geq 0$, $P_0 \geq 0$), the simple and multiple rotation functional objective are given in (3) and (4).

$$F^S(V,t) = \frac{\sup}{\forall(t \geq t_o)}[E^R(e^{-it}P_tV_t - C] \quad (3)$$

$$F^M(V,t) = \frac{\sup}{\forall(t \geq 0)}(\frac{E^R(e^{-it}P_tV_t) - C}{1 - e^{-it}}) \quad (4)$$

With:

V = Wood stock variable

μ(V) = Wood stock diffusion drift rate parameter

σ(V) = Wood stock volatility parameter

P_t = Wood stumpage spot price at time t

P_o = Initial stumpage wood price

α = Wood price diffusion drift rate

β = Wood price volatility

W = Wiener diffusion

C = Stand regeneration cost

c	$= C/P_o$
R, Q	= Probabilistic metrics
F,Z	= Functional Objective
i	= Risky rate of return
r_t	$= i/(1-e^{-it})$ Capitalized rate of return

2.2 Reformulation of the Simple Harvest Rotation Problem

The stochastic model (1, 2, and 3) is difficult to solve. The Girsanov, *Theorem 1,* see appendice 1, reduces this model to a one dimensional stopping problem that is more amenable, see Navarrete (2012).

$$Z^S(V) = \frac{\sup}{\forall(t \geq 0)} E^Q(e^{-(i-\alpha)t} V_t) \tag{5}$$

Furthermore, under the metric Q, the process V follows the diffusion (6)

$$dV = \{\mu(V) + \beta\sigma(V)\}dt + \sigma(V)d\overline{W} \tag{6}$$

Transforming this problem in an equivalent one dimensional diffusion optimal stopping problem, its solution is given by the Hamilton Jacobi Bellmann equation (7) see, Navarrete (2012).

$$\underset{V \geq 0}{\text{Max}}[\ \tfrac{1}{2}\sigma^2 V^2 F''(V) + [\mu(V) + \beta\,\sigma(V)]\ F'(V) - (i-\alpha)\,F(V)\ ,\ V - F(V)\ ] = 0 \tag{7}$$

Under the assumption of the existence of a frontier V* that divides the zone in two, a continuation (no-cutting), and stopping (immediate-cutting), the solution to the equation HJB is finally given, for the continuation Zone $V<V^*$:

$$\tfrac{1}{2}\sigma^2 V^2 F''(v) + [\mu(V) + \beta\,\sigma(V)]\ F'(V) - (i-\alpha)\,F(V) = 0 \tag{8}$$

and for the stopping region $V \geq V^*$,

$$V\text{-}F\,(V) = 0 \tag{9}$$

A solution of (8) is given by (10) (see Johnson T.C, 2006).

$$F^{V}(V) = \begin{cases} A\Psi(V) + B\Phi(V) & V < V^* \\ V & V \geq V^* \end{cases} \tag{10}$$

Where Ψ (resp.,Φ) is strictly increasing (resp., decreasing), (functions since the payoff function are bounded and small) and V is positive and should remain bounded and positive as $V \rightarrow 0$, necessarily then $B \rightarrow 0$. The solution must also fulfil the so called "smooth–pasting" condition at the free boundary point V*, so that

$$A\ \Psi(V^*) = V^* \quad \text{and} \quad A\,\Psi'(v^*) = 1 \tag{11}$$

$$A = v^*/\ \Psi(V^*) = 1/\Psi'(V^*) \tag{12}$$

and V* must fulfil the following smooth pasting equation :

$$\Psi(V^*) = V^*\ \Psi'(V^*)\,. \tag{13}$$

2.3 Reformulation of the Multiple Harvest Rotation Problems

The condition of the application of Girsanov, theorem 1 also applies to the Faustmann objective functional (4), see Appendices 1, and the problem is reduced to the equivalent following Optimal stopping problem under metric Q.

$$F^{M}(V,t) = \frac{\sup}{\forall(t \geq 0)} \Big(\frac{E^{Q} P_0 (e^{-(i-\alpha)t} V_t) - C}{(1 - e^{-it})} \tag{14}$$

And dividing by the constant $P_{0,}$ we transform $Z^M = F^M / P_0$

$$Z^{M}(V) = \frac{\sup}{\forall(t \geq 0)} \frac{E^{Q} (e^{-(i-\alpha)t} V_t - c}{(1 - e^{-it})} \tag{15}$$

With the following modified wood stock diffusion under the metric Q

$$dV = \{\mu(V) + \beta\sigma(V)\}dt + \sigma(V)d\overline{W} \quad (16)$$

The formulation of the Hamilton-Jacobi-Bellman equation for this problem is given by equation (17) for the capitalized risky rate of $r_t = i/(1-e^{-it})$.

$$\underset{\forall(V)}{Max}\,[½\sigma^2V^2F''(V)+[\mu(V)+\beta\sigma(V)]F'(V)-[r_t-\alpha]F(V)-c\, r_t,(V-c)/(1-e^{-rt})-F(V)]=0 \quad (17)$$

In this case the differential equation for the continuation region ($V \le V^*$) is given by the non homogenous differential equation (18).

$$½\sigma^2V^2F''(V)+[\mu(V)+\beta\sigma(V)]F'(V) - (r_t-\alpha)F(V) - c\, r_t = 0 \text{ with } F(0) = -(r_t/r)c. \quad (18)$$

And by equation (19) for the stopping zone ($V > V^*$)

$$(V-c)/(1-e^{-rt})-F(V) = 0. \quad (19)$$

The solution to this ordinary differential equation under the initial condition for a given capitalized risky rate of return r_t is given in (20), with $\psi(V)$ the positive increasing solution of the homogenous part and $[r_t/r]c$ the particular solution of equation (18).

$$F(V,r_t) = \begin{cases} A\Psi(V) - [r_t/r]c & V < V^* \\ (V-c)/(1-e^{-rt}) & V \ge V^* \end{cases} \quad (20)$$

In this case the smooth pasting condition for each parameter r_t is given by:

$$A\Psi(V^*)-(r_t/r)c = (V^*-c)/(1-e^{-rt}) = (r_t/r)(V^*-c) \quad \text{and } A\psi'(V^*) = r_t/r.$$

So V^*_t must fulfill smooth-pasting condition (21) for each parameter r_t.

$$\Psi(V^*) = V^*\psi'(V^*) \quad (21)$$

2.4 Wood stock sigmoid diffusion equations

The basic requirement of a pine stand growing diffusion is its sigmoid pattern (Garcia, 2005). The logistic diffusion, equation (22) is a special case of the sigmoid model given by $\mu(V) = \mu V(1-\gamma V)$ and $\sigma(V) = \sigma V$, where μ and γ are the drift and saturation parameters and σ is the volatility parameter**.**

$$dV = \mu V (1- \gamma V) dt + \sigma Vdw \qquad (22)$$

The integration of the value of V is given by equation (23) (Kloeden& Platen, 1991, page 125) and its expected value is given by equation (24)

$$V_t = \frac{V_0 Exp[(\mu - \frac{\sigma^2}{2})t + \sigma W}{1 + \mu\gamma V_0 \int_{t_m}^{t} Exp[(\mu - \frac{\sigma^2}{2})s + \sigma W]ds} \qquad (23)$$

$$E(V_t) = \frac{1/\gamma}{1 + \frac{1-\gamma V_0}{\gamma V_0} e^{-\mu t}} \qquad (24)$$

Another important sigmoid diffusion is the Gompertz geometrical diffusion, which is given by the following equation

$$dV=kV[\theta-\ln(V)]dt + \sigma VdW \qquad (25)$$

This equation is integrated to the following expression,

$$V(t)= \exp[\ln(V_0)e^{-kt} + \{(k\theta-\sigma^2/2)/k\}(1-e^{-kt}) + \sigma e^{-kt} \int dW] \qquad (26)$$

And the expected value takes the following expression, (see Gutierrez, 2009),

$$E[V(t)]= \exp[\ln(V_0)e^{-kt} + \{(\theta-\sigma^2/(2k)\}(1-e^{-kt}) + (\sigma^2/(4k))(1-e^{-2kt})] \qquad (27)$$

Geometric Brown Price diffusion is given by

$$dP = \alpha Pdt+\beta PdW \qquad (28)$$

The equation integrate to

$$P_t = P_0\, e^{\alpha t} \exp\{\beta W_t - 1/2\beta^2 t) \qquad (29)$$

Being Mt= $\exp\{\beta W_t - 1/2\beta^2 t)$ a martingale

3 EXPERIMENTAL DATA AND PARAMETERS FITTING

3.1 Logistic Diffusion Fitting

The experimental data was provided by a Chilean forest company. These data belong to harvest volume stock of its pine stands between 1999 and 2005 and came from different sample plots, with site indexes between 30 and 35 meters, representing sites with high forest aptitude and a tree average initial volume of 32 m3/ha at the first 4 years after initial seed cultivation period. The additional 20 point for years 11 and 12 were taken from Alvarez et al., 2012. The business harvest cut data for a Logistic diffusion 95% confidence range is given by data point plotted in figure 3.1.

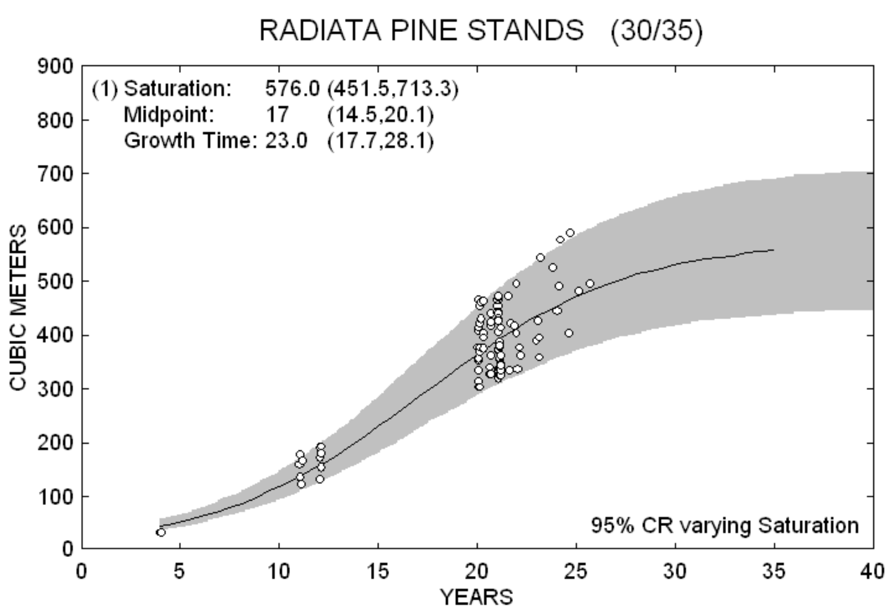

Figure 3.1 Wood Volume per hectare, (m3/ha) versus years, single plot

The logistic diffusion parameter cannot be adjusted by maximum verisimilitude (see Beskos et al., 2006), so it was fitted using a logistical nonlinear regression and a Monte Carlo/Bootstrap simulation sampling method, implemented by Meyer et al. (Loglet Lab.1 software, 1999).

Choosing $V_0 = 1/2\gamma$ = half saturation volume, $T_0 = T_m$ = time to achieve that volume equation (13) is transformed in the more conventional expression (27).

$$E(V_t) = \frac{1/\gamma}{1 + e^{-\mu(t - t_m)}} \tag{27}$$

With; $1/\gamma$= saturation volume, μ= growth rate parameter, V_s= Saturation volume and t_m = time to achieve the midpoint of the saturation volume. The standard deviation is constant and given by $Sd(\infty) = \sigma V_s$ = (95% saturation confidence interval)/(2 *1.96) , at the saturation zone.

Since the saturation volume V_s is also constant, sigma can easily be estimated by

$$\sigma = Sd(\infty)/V_s \,. \tag{29}$$

The summary of the parameter fitting is shown in table 3.1.

Table 3.1 Logistic footing parameters

Models	Drift Parameter μ	Saturation Volume	Saturation Parameter γ	Volatility Parameter σ
Stochastic	0.191	576.0	0.00174	0.12
Deterministic	0.191	576.0	0.00174	

3.2 Gompertz Diffusion Fitting

The Gompertz model can be fitted by common statistic features, such as Maximum verisimilitude (see Gutierrez, et al, 2008), but the lack of even distribution of time data make it difficult. Therefore a quadratic fitting method was developed using the SSPS software. Taking natural logarithm and arranging it, we get

$$\ln E[V(t)] = A - Bx - Cx^2 \quad (30)$$

with

$A = \theta - \sigma^2/(4k)$ $B = \theta - \sigma^2/(2k) - \ln(V_0)$ $C = \sigma^2/(4k)$ and $x = e^{-kt}$

Given a value for k, a quadratic fitting for e^{-kt} and e^{-2kt} was done estimating the value of A, B and C until a common value for θ was obtained from A and B, determining the estimation for θ, k and σ. The deterministic parameter only requires a linear fitting with e^{-kt},. Both fittings were done for the initial value $V_0 = 32$ (m3/ha.) and the results are summarized in table 3.2.

Table 3.2 Gompertz Diffusions Parameters Estimations

Parameters	Initial	Saturation	Drift	Drift	Volatility	
Models	V_0	V_s	K	θ	σ	R^2
Gompertz Deterministic	32.00 32.00	653.3 653.3	0.102 0.102	6.538 6.538	0.151	0.992

3.4 Wood Price Diffusion Fitting

The stumpage stand price Brownian diffusion parameters were estimated by Navarrete, 2011. The summary of Brown diffusion parameters for the pulpwood and saw timber prices is given in Table 3.3.

Table 3.3 Stumpage Price Diffusion Parameters

Summary	Stumpage logs	Saw logs	Pulp logs
Percentage	100 %	83.9	16.1
Price drift α	2.9%	3.08	1.79
Price volatility β	15.9%	16.52	12.74
Average Actual Price	39.74		

See Navarrete 2012

3.5 Economic Cost

The regeneration costs of Radiata Pine Stands in 2009 are given in table 3.5

Table 3.5 Radiata Pine Stands cost of capital and Regeneration Cost

Risky rate of Capital (WAAC)	%	12%
Stands regeneration cost C	US$/ha	882
Actual stumpage log price P_T	US$/ha	39.74
Initial stumpage price P_0	US$/ha	21.43
$c=C/P_0$		41.16

Source: CEFOR-UACH

4 STOCHASTIC RADIATA PINE HARVESTING RESULTS

4.1 Logistic Wood stock and Brown Stumpage price diffusion

The Logistic wood stock diffusion is given by equation (22), with drift and volatility parameter given by $\mu(V) = \mu V\ (1- \gamma V)$ an $\sigma(V) = \sigma\ V$. The deterministic parameters of the growth models of the wood stock and the Stumpage price are $V' = \mu V\ (1- \gamma V)$ and $P_t = P_o e^{\alpha t}$, replacing these two equations in functional objectives of the simple and multiple rotations problems and maximizing this objective gives the following equations (31) for the optimal deterministic cut volume.

$$V^* = (\alpha+\mu-i)/(\gamma\mu) \qquad (31)$$

The optimal for the multiple rotation case is obtained intercepting condition (32) with equation (24), with To=4 and Vo= 32. (See Navarrete, 2013)

$$V^*_t = \{(\alpha+\mu-r_t) + \sqrt{[(\alpha+\mu-r_t)^2+4\mu\gamma c r_t e^{-\alpha t}]}\}/(2\mu\gamma) \qquad (32)$$

The stochastic positive increasing function ψ (V) for the Faustmann Logistical model is the solution fo the homogenous component (33) of the differential equation (10).

$$\tfrac{1}{2}\,\sigma^2 V^2 F''(v) + [\mu V(1-\gamma V) + \beta\ \sigma V]\ F'(V) - (r_t-\alpha)\ F\ (V) = 0 \qquad (33)$$

The solution of equation (33) is given by the Kummer´s confluent hyper geometric function, expression (34), (see Navarrete, 2012).

$$\psi(V) = V^{\theta}\ KummerM\ \{\frac{2\mu\gamma V}{\sigma^2},\ \theta,\ 2\theta + \frac{2(\mu+\beta\sigma)}{\sigma^2}\} \quad (34)$$

With θ the positive root is given by equation (35)

$$\theta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{\beta}{\sigma} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{\beta}{\sigma})^2 + \frac{2(r_t - \alpha)}{\sigma^2}} \quad (35)$$

The function ψ(V) in this case is parameterized by the capitalized interest rate r_t. The optimum is obtained by intersecting the smooth pasting condition with the corresponding expected value of the modify logistical diffusion for V under Q metric (20) for the different V^*_t solutions of equation (13).

The simple stochastic solution is given by $r_t = i$ and ψ(V) must solve the homogenous equation

$$\frac{1}{2}\sigma^2 V^2 F''(v) + [\mu V(1-\gamma V) + \beta\ \sigma V]\ F'(V) - (i-\alpha)\ F(V) = 0 \quad (36)$$

and the optimum V* must only satisfy the smooth pasting condition.

The smooth pasting and optimum condition for these problems were programmed in Maple 15, using its *KummerM* function. The summary of all optimal cuts results for the aggregate 30/35 site index series of the multiple and simple rotation harvest or Faustmann formula is given in table 4.1.

Table 4.1 Multiple Harvest Rotation Optimal Results, Brown price Logistic Wood stock diffusion

Optimum Policy	Simple m^3/ha	%	%	Multiple m^3/ha	%	%
Company	392.9	100			100	
Deterministic	300.9	76.6	100	292.3	74.4	100
Stochastic	383.1	97.5	127.3	347.6	88.5	118.9

These results show that the simple geometric Brown price and Logistic wood stock rotation model is a better explanation of the company cut policy, since it only underestimate it by 2.5%. Both stochastic models increase the corresponding deterministic cut optimum between 27.3 % for the simple rotation and 18.9 % in the multiple rotation case.

4.3 Gompertz Wood Stock and Brown Stumpage price Deterministic case

In this case the parameters of the diffusion are: $\mu(V) = k\,V\,(\theta - \ln(V))$, and $\sigma(V) = \sigma\,V$. The deterministic optimums is obtained by replacing $P_t = P_0\,e^{\alpha t}$ and $V = \exp(\ln(V_0)e^{-kT} + \theta(1-e^{-kT})$ in functional objective equation of the simple and multiples rotation models and looking for a maximum. The optimal solution for the simple rotations is.

$$V^* = e^{(\alpha + k\,\theta - i)/k} \tag{37}$$

In the multiple rotation case the optimal is obtained by intercepting condition (38) with equation (22). (See Navarrete, 2013)

$$\ln V = (\alpha + k\theta - r)/k + (r_t c e^{-\alpha t})/(kV) \tag{38}$$

The stochastic increasing function $\psi(V)$ for the multiple rotation case is given by the solution of the homogenous part of the differential equation (10) equation (39)

$$\tfrac{1}{2}\sigma^2 V^2 F''(v) + [kV(\theta - \ln(V)) + \beta\,\sigma V]\,F'(V) - (r_t - \alpha)\,F(V) = 0 \tag{39}$$

Choosing $\theta' = \theta - \sigma^2/(2k) + \beta\sigma/k$ and $r' = r_t - \alpha$, and replacing it in equation (39) give the differential equation (40) of the Exponential Ornstein-Uhlembeck diffusion

$$\tfrac{1}{2}\sigma^2 V^2 F''(v) + \{k[\theta' - \ln(V)] + 1/2\,\sigma^2\}V\,F'(V) - r'\,F(V) = 0 \tag{40}$$

The positive increasing solution $\psi(V)$ of (40) is given by equation (41), (see Johnson, 2005).

$$\Psi(V) = \begin{cases} \{\Gamma(a+1-b)/\Gamma(1-b)\} KummerU\,(a,b,z) & V \le e^{\theta} \\ KummerM\,(a,b,z) & V \ge e^{\theta} \end{cases} \quad (41)$$

With the following modified parameters:

$a = (r_t - \alpha)/(2k)$ $b = 0.5$ and $z = (k/\sigma^2)\,[\theta - \sigma^2/(2k) + \beta\sigma/k - \ln(V)]^2$

The optimal solutions of the multiple rotation case must also satisfy the smooth pasting condition and the expected Gompertz diffusion to produce the optimal solution. The simple rotation $\psi(V)$ satisfies the homogenous equation (38) with $r_t = i$. The optimal condition in his case is given by the smooth pasting condition, programmed in Maple 15, using in this case the *KummerU* program function and its optimal value which is given in table 4.2. The results are summarized in table 4.2.

Table 4.2 Multiple Harvest Rotation Optimal Results, Brown price Gompertz Wood Stock diffusion

Optimum Policy	Simple m^3/ha	%	%	Multiple m^3/ha	%	%
Company	392.9	100			100	
Deterministic	283.2	72.1	100	262.8	66.9	100
Stochastic	397.6	101.2	140.4	356.8	90.8	135.8

These results show that the simple geometric Brown price and Gompertz wood stock rotation model is a better explanation of the company cut policy, since it only overestimates it in a 1.2%. Both stochastic models increase the corresponding deterministic cut optimum between a 40.4 % in the simple rotation and 35.8 % in the multiple rotation case.

4.5 Summary

Table 4.3 shows the summary of the cut policies evaluation in % of the forest company optimal policy.

Table 4.3. Cut Policies Validations Summary

Optimal policy	Price	Wood Stock	Simple %	Rotation %	Multiple %	Rotation %	Saturation Volume	%
Company			100	130.6	100		656.7	100
Deterministic	Brown	Logistical	76.6	100*	74.4	100*	576	87.7
	Brown	Gompertz	72.1	100**	59.5	100**	653.3	99.5
Stochastic	Brown	Logistical	97.5	127.3*	88.5	118.9*	576	87.7
	Brown	Gompertz	101.2	140.4**	90.8	135.8**	653.3	99.5

*Logistical % , ** Gompertz %

The optimal cut company policy validate the use of the simple stochastic rotations models, being the most accurate estimation, the Brown and Gompertz diffusions case which overestimate the company cut policy by 1.2% and underestimate the saturation volume in 0.5%. The Brown and Logistic diffusions case underestimate the cut company policy in 2.5% and the saturation volume in a significant 4.5%.

The deterministic policies underestimate the company optimal cut policy in significantly 23.4 % and 27.9% for the simple rotation and in 25.6% and 40.5% for the Multiple rotation for the Logistic and Gompertz wood stock diffusion case.

Finally the Faustmann optimal cut policies underestimate the company optimal cut in a significantly 12.5% and 10.8%, for the Logistic and Gompertz wood stock diffusion case.

5 CONCLUSIONS

1. The optimal cut policy of the forest company validates the use of the simple stochastic rotations models, being the most accurate the Brown & Gompertz diffusions case, which only overestimated the company cut policy by 1.2 % and underestimate the saturation volume in 5.0% .

2. The optimal model policies behaved as expected, being the simple rotation higher than the multiple rotation cut, and the stochastic behavior produced bigger cuts than the deterministic case. The deterministic models optimums in all cases significantly underestimated the company actual average cut. The stochastic optimum underestimate significantly the

company cut in the Faustmann or multiple rotations case, but gives the best estimation of the company optimal cut for the simple rotation case.

3. Obviously, the company cut policy did not agree with the theoretical correct multiple rotation optimal policies, such as the Faustmann models. This discrepancy can be explained by the lack of consideration that the company gave to the impact that the multiple rotation has on the harvest planting rotation cycle of the pine stands and to the forestry company preference given to the physical evaluation of the forest stands maturity over its stochastic diffusion in the determination of the harvesting decision.

5 APPENDICE 1

Theorem 1: A probabilistic measure Q exists and is equivalent to the actual metric P, such that it is proven (see, Jacco J.J. Thijssen, 2010)

$$W^V(V_0, P_0) = \frac{\sup}{\forall(t \geq t_o)} E^P(e^{-rt} P_t V_t)/(1-e^{-rt}) = P_0 \sup\{E^Q(e^{-(r-\alpha)t} V_t)/(1-e^{-rt})\}$$

(A1) Furthermore, under the metric Q, the process V_t follows the diffusion (A2)

$$dV_t = \{\mu V)_t + \beta\sigma(V)_t\}dt + \sigma(V)_t d\overline{W} \tag{A2}$$

Proof.

Replacing the integral solution of (2) in this last expression (A1), $P_t = P_0 e^{\alpha t}$ exp $\{\beta W_t - 1/2\beta^2 t]$, since M_t = exp $\{\beta W_t - 1/2\beta^2 t]$ is a martingale, a new metric Q ($dQ/dP = M_t$) can be defined via the Radon-Nikodym derivative. Considering that, in this case, β is positive, a straightforward application of Girsanov´s theorems I and II (Oksendal, 2000, pages155-157) yields the equivalent objective for metric Q, and the ITO diffusion (A2)

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