Large and small firms: strategic investment decisions on uncertain existing markets*

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Abstract

This paper studies asymmetric firms that consider an innovative investment in an established market. Conjoining capacity choice, asymmetry among firms, and an innovative market, brings up a new, more profound, model to evaluate firms' investing behavior in a competitive, uncertain dynamic duopoly setting. An investment comprises both optimal timing and setting capacity.

We show that capacity choice induces some new results. In great contrast to models where capacity size is given, we find that larger firms have more incentives to invest and innovate. Larger firms moreover underinvest in order to obtain a temporary monopoly position. This leads to the general result that large firms lead innovation, but take the smallest stake on new products on established markets.

1 Introduction

The investment problem has been a topic on the global research agenda for many years. Great contributions have been made on the basis of real options theory. Real options theory considers an irreversible investment

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opportunity as an option. Options are to be exercised at the moment the difference between the option value and the production value is no longer positively evaluated. Most models, however, consider investment decisions on new markets. Under real options theory, only recently more research has been done on investment decisions involving existing markets. The research in this thesis comprises existing markets.

In previous literature, fundamental work is performed by Eisner and Strotz (1963) analyzing firm growth and by Spence (1979) and Reynolds (1987) both studying dynamic strategic competition between firms. Further work, based on the latter two papers, by e.g. Jun and Vives (2004) led to meaningful insights on intertemporal strategic effects. Different from these settings, Dawid et al. (2010b, 2011) considered a model in which one or both incumbents in a duopolistic market have the option to produce a new differentiated product in addition to the existing one, being different from the work in this thesis as the dynamic adjustment of capacities was not explicitly modeled.

The analysis of irreversible investments under uncertainty by strategic real options theory has some early contribution by Smets (1991). Strategic real options theory is the part of real options theory where two entrants have the option to invest in the same market. Different from the NPV approach, real options take the value of waiting for more information before making an irreversible investment into account. Existing papers either model the optimal timing, given the scale of investment (Dixit and Pindyck (1994) and Trigeorgis (1996) give a nice overview) or model investment size in a market without competition (Dangl (1999), Bar-Ilan and Strange (1999) and Hagspiel et al. (2011)).

In this paper, two asymmetric firms are considered. Both firms are presently involved in a production process under a different capacity. They face an additional investment opportunity. The investment brings a new product on the same market. This product comes on the market as a substitute. As a result, demand is partially shifted. As a firm is free in choosing its new capacity, it needs to weigh the positive and negative consequences in order to decide upon the best investment decision. Investment ought to increase profitability and it therefore should outweigh two negative effects: the decline in demand for the first product and the investment cost. The firms have two instruments to do so. In the first place the firm can choose the amount it invests in. Secondly, to a certain extent, it is flexible in choosing the moment to invest. The latter is a key concept in real options theory. In a oligopoly market firms have to weigh an additional factor. When one firm evaluates the investment problem, it should take into account to what extent its profits are going to be affected when another firm enters the market. Choosing capacity then brings a strategic aspect in: when choosing it sufficiently large, other firms are deterred from entering. In the other situation, choosing a smaller amount, it accommodates other firms. In the model presented in this thesis, two firms are considered. Exogenously it will be determined which of the two firms shall invest first, this firm is then called the leader. The succeeding firm shall be referred to as the follower. The leader can choose between deterring and accommodating. The follower's decisions are only considered with the capacity choice and the moment of investment.

These models are an extension of the models considered by Huisman & Kort (2013). Their models consider duopoly firms for a new market, whilst this paper considers established markets. The main point of interest for this thesis is the change in outcome for when adding a market. This contributes in two manners to the existing literature. It not only adds the old market, it also entails the additional capacity choice. The latter is not covered in traditional models (see e.g. Dixit and Pindyck (1994)).

The main findings can be summarized as follows. The existing market plays a significant role in the determinance of the strategy space. We find that the investment order is fully dependent on the firm's capacities on the old market. Large firms always preempt smaller firms in becoming leader. To do so they have to take a smaller stake than the follower does when the follower invests. Moreover, for future investment opportunities, it is shown that, due to the notion of capacity choice, simultaneous investment is never an equilibrium.

The combination of free choice of capacity, the introduction of the old market and the choice of inverse demand functions resulted in a model, being evaluated by means of real options theory, that is new to the literature. The results in this paper have shown that the existence of an old market is an important notion. Therefore, this extension to the literature is an important one.

This paper is constructed in the following way. Section 2 presents the model that is elaborated in Section 3. More specificly, Section 3.1 looks at exogenous firms roles, 3.2 elaborates the situation in which firms compete in becoming leader and finally section 3.3 describes all equilibria. Section 4 compares our model to some benchmark models to clarify the differences and to show the significance of our work. The paper is concluded by Section 5.

2 Model

Currently production is present. Product 1 is referred to be the associated output, which will be maintained throughout the existence of the company. The latter is assumed to be infinite. Expansion of the market is done by introducing a second product, product 2. This product is both horizontally and vertically differentiated from the first product. Therefore the new product serves as a substitute for the old product. One firm will invest first in the second product and then naturally, the other firm is to make a decision subsequently. The first firm is called the leader, the latter firm is called the follower. By investing in the second product, the inverse demand functions change. Before investment, the inverse demand function is defined as

$$P(t) = X(t)(1 - \eta Q^1(t)),$$

where η is a fixed parameter, $X(t)$ is the value of an exogenous shock process and where $Q^1(t)$ is the aggregate output for the first product,

$$Q^1(t) = q_{1,L}(t) + q_{1,F}(t).$$

In this setting $q_{1,L}$ signifies the amount of product 1 produced by the firm taking the role of being leader for the second product. That means that in the period both firms were competing for the first product, roles could have been reversed. It is assumed that one always produces up to full capacity and Q^1 does therefore not depend on time. The process $X(t)$ is assumed to be a geometric Brownian Motion, i.e.

$$\mathrm{d}X(t) = \alpha X(t)\mathrm{d}t + \sigma X \mathrm{d}z(t).$$

Once the second product has been launched, the demand changes. The new demand functions for product one and two then respectively become

$$P_1(t) = X(t)(1 - \eta Q^1(t) - \omega Q^2(t)),$$
$$P_2(t) = X(t)(\nu - \eta Q^2(t) - \omega Q^1(t)),$$

where $\nu > 1$ is the vertical differentiation parameter, $\omega \leq 1$ the horizontal differentiation parameter, η a positive constant and where $Q^2(t)$ is the total market output of the second product,

$$Q^2(t) = q_{2,L}(t) + q_{2,F}(t).$$

These demand functions are in a more specificly defined form than the general ones from e.g. Dixit & Pindyck (1994, Chapter 9). A linear relation between demand and output is also utilized by, e.g., Pindyck (1988), He and Pindyck (1992), Aguerrevere (2003) and Wu (2007). Note that we keep the setup equivalent to Huisman & Kort (2013). As of now, the subscript t shall be omitted unless necessary.

Firms will have to make two decisions. In the first place it has to determine the moment of investing. This is done by using the methods of real options theory. Standard real options theory (e.g. Dixit and Pindyck (1994)) compares the option value function with a function reflecting payoffs as if it would start producing now. As long as the option creates a larger value than the actual production, the firm waits. Investment is done at the moment waiting no longer yields a larger value than starting production. The second decision involves the capacity. When the firm makes the investment, it is to decide upon the scales.

Discounting takes place under a fixed discount $r > \alpha$. Investment costs are assumed to be a linear function of the production amount, with marginal cost parameter δ.

3 Capacity choice

This section aims to elaborate the proposed model. Thereon this section starts by examining the firms' decisions when their consecutive roles are given, that is, it is exogenously given in which order the firms invest. For the leader may choose its capacity first, it has a strategic mechanism in hands. The moment the leader invests, it is to decide upon two strategies. By setting the production capacity relatively small, sufficient room is left on the market for the second firm to follow the leader. However, the leader may choose to set the production level relatively high, so that the follower is deterred from investing. These two strategies are called the accommodation and deterrence strategy. This mechanism is studies to a larger extent in the first part on exogenous firm roles. The second part on endogenous firm roles brings in competition, letting both firms compete in becoming leader on the new market. This section is concluded by a study on all possible equilibria.

3.1 Exogenous firm roles

The follower makes its choices on the basis of the information made available at the investment of the leader. The leader is therefore aware of the consequences of its choices. For this reason, this game should be solved backwards, i.e. preceding the elaboration of the leader's problem, the problem of the follower is solved.

Follower's decision

Assume $X(0)$ is sufficiently large for the follower in order to invest. The value function consists of the production of both products,

$$V_F(X,Q) = \mathbb{E}\left[\int_0^\infty (q_{1,F}P_1(t) + q_{2,F}P_2(t))e^{-rt}\mathrm{d}t - \delta q_{2,F}\,\middle|\, X(0) = X\right] \tag{1}$$

$$= \left[q_{1,F}(1-\eta Q^1 - \omega Q^2) + q_{2,F}(\nu - \eta Q^2 - \omega Q^1)\right]\frac{X}{r-\alpha} - \delta q_{2,F}, \tag{2}$$

where $r > \alpha$ reflects the risk free rate. This equation consists of three terms. The first two terms are the accumulated payoffs resulting from the first and second product. The third term reflects the investment costs which had to be made in order to start producing. The optimal capacity $q_{2,F}$ for the follower becomes,

$$q^*_{2,F}(X,Q) = \frac{1}{2\eta}\left(\nu - \frac{\delta(r-\alpha)}{X} - \omega(q_{1,L} + 2q_{1,F})\right) - \frac{1}{2}q_{2,L}.$$

The optimal threshold value $X^*_F(Q)$, i.e. the optimal moment for the follower to invest, is the value for which the option value $F_F(X,Q)$ no longer yields a larger value than the payoff function (2). Following standard real options analysis (e.g. Dixit and Pindyck (1994)) leads to the following option value function,

$$F_F(X,Q) = A_F(Q)X^\beta + \frac{X}{r-\alpha}(1-\eta Q^1 - \omega q_{2,L})q_{1,F},$$

where

$$\beta = \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}}$$

All elaborations can be found in the appendix.

Proposition 1 *Let the current value of the stochastic demand process be denoted by X, the production capacity of product one and two be denoted by $q_{1,L}$ and $q_{2,L}$ respectively for the leader and $q_{1,F}$ and $q_{2,F}$ respectively for the follower. Then the optimal capacity level for the follower $q^*_{2,F}$ is to be*

$$q^*_{2,F}(X,Q) = \frac{1}{2\eta}\left(\nu - \frac{\delta(r-\alpha)}{X} - \omega(q_{1,L} + 2q_{1,F})\right) - \frac{1}{2}q_{2,L}. \tag{3}$$

The follower's value function is given by

$$V_F^*(X,Q) = \begin{cases} A_F(Q)X^\beta + \frac{X}{r-\alpha} q_{1,F}(1-\eta Q^1 - \omega q_{2,L}) & \text{if } X < X_F^*(Q), \\ \left[q_{1,F}(1-\eta Q^1 - \omega Q_2^*(X)) + q_{2,F}^*(X)(\nu - \eta Q_2^*(X) - \omega Q^1)\right] \frac{X}{r-\alpha} - \delta q_{2,F}^*(X) & \text{if } X \geq X_F^*(Q), \end{cases}$$

where

$$X_F^*(Q) = \frac{\beta+1}{\beta-1} \frac{\delta(r-\alpha)}{\nu - \omega Q^1 - \omega q_{1,F} - \eta q_{2,L}}, \tag{4}$$

$$A_F(Q) = \frac{\delta^2(r-\alpha)}{\eta(\beta-1)^2} \left(\frac{\nu - \eta q_{2,L} - \omega q_{1,F} - \omega Q^1}{\delta(r-\alpha)} \frac{\beta-1}{\beta+1} \right)^{\beta+1}. \tag{5}$$

Then also,

$$q_{2,F}^*(X_F^*(Q), Q) = \frac{\nu - \omega Q^1 - \omega q_{1,F} - \eta q_{2,L}}{\eta(\beta+1)}. \tag{6}$$

Note that all of these formulas are similar to the ones as obtained in Huisman & Kort (2013).

Two scenarios have to be distinguished. In the first scenario the leader uses a deterrence strategy by which it makes the follower wait. Once the shock process reaches the value of X_F^* the follower invests as well, setting the capacity equal to the value as given by equation (6). However, in the scenario where the second firm immediately follows, so when $X_F^* \leq X$, the follower chooses a different amount for its capacity, namely the value as given by equation (3).

Leader's decision

The leading firm was defined as the firm which was assigned to invest first. The leader has the advantage that it can anticipate on the follower's strategy, as the follower's strategy depends on the leader's decisions. If the leader chooses a larger capacity, X_F^* will increase as well. Hence, the leader may deter the second firm from entering for a longer expected period of time whenever choosing a larger capacity. This strategy is called the entry deterrence strategy. So, from equation (4) it follows that the follower is prevented from investing when choosing $q_{2,L} > \hat{q}_{2,L}(X,Q)$, where

$$\hat{q}_{2,L}(X,Q) = \frac{1}{\eta} \left[\nu - 2\omega q_{1,F} - \omega q_{1,L} - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X} \right]. \tag{7}$$

If the leader chooses the capacity in such a way that $q_{2,L} \leq \hat{q}_{2,L}$ the second firm is allowed to follow. This is then called the accommodation strategy. Let us first analyse the deterrence strategy and then take a look at the accommodation strategy.

For the value function, one must notice that the leader discriminates between two periods. In the first period $[0,T)$ the leader is the only firm offering the second product on the market, $T \geq 0$. After investment of the second firm at time $t = T$, both firms are actively producing. One then obtains,

$$
\begin{aligned}
V_L^{det}(X,Q) &= \mathbb{E}\Bigg\{ \int_0^T \left[q_{1,L}X(t)(1-\eta Q^1-\omega q_{2,L}) + q_{2,L}X(t)(\nu-\eta q_{2,L}-\omega Q^1)\right] e^{-rt}\mathrm{d}t \\
&\quad + \int_T^\infty \left[q_{1,L}X(t)(1-\eta Q^1-\omega Q^2) + q_{2,L}X(t)(\nu-\eta Q^2-\omega Q^1)\right] e^{-rt}\mathrm{d}t \Big| X \Bigg\} - \delta q_{2,L} \\
&= \frac{X}{r-\alpha}\left[q_{1L}(1-\eta Q^1-\omega q_{2L}) + q_{2L}(\nu-\eta q_{2L}-\omega Q^1)\right] - \frac{\delta}{\eta(\beta-1)}(\omega q_{1L}+\eta q_{2L})\left(\frac{X}{X_F^*}\right)^\beta - \delta q_{2L}.
\end{aligned}
$$

The value function for the leader looks similar to the value function of the follower. However, the former one contains one extra term. The first two terms reflect the value resulting from production in which solely the leader produces product two. The third term serves as a correction term for when the follower enters the market in future for the second product. Notice that $\left(\frac{X}{X_F^*}\right)^\beta = \mathbb{E}\left[e^{-rT}\right]$ can be seen as a discount factor.

When the firm invests it chooses its capacity in such a way that it maximizes the value function. So from the first order conditions one can find the optimal investment amount $q_L^{det}(X,Q)$. Notice that the value of $q_{2,L}$ is contrained by the condition that $q_{2,L} > \hat{q}_{2,L}(X,Q)$. Hence,

$$q_L^{det}(X,Q) = \operatorname{argmax}_{q_{2,L}}\{V_L^{det}(X,Q) \mid q_{2,L} > \hat{q}_{2,L}(X,Q)\}.$$[1]

When considering the deterrence strategy, one optimizes $V_L^{det}(X,Q)$ with respect to $q_{2,L}$. This value will depend on the value of X. Based on the value of $q_{2,L}$ firm 2 chooses to invest, $X_F^* \leq X$, or not, $X_F^* > X$. For large values of $q_{2,L}$ the leader does not leave much room on the market for the follower. However, it might be the case that for values of X larger that some threshold X_2^{det} it is no longer optimal for the leader to deter the second firm. There is an adequate amount of demand on the market (X is positively related to the demand on the market) so that the leader would loose too much value when deterring in stead of accommodating the other firm.

Deterrence does not apply if $q_{2,L} \leq \hat{q}_{2,L}(X,Q)$. So for such a threshold X_2^{det} to exist it must hold that for all $X \geq X_2^{det}$ we have that $q_{2,L} \leq \hat{q}_{2,L}(X,Q)$ is optimal. That is, in that case $X_F^*(q_{2,L}(X,Q)) \leq X$. As a result, the value of X_2^{det} can be found by solving

$$X_F^*(q_{2,L}(X,Q)) = X.$$

It shall only be profitable for the leader to invest whenever the optimal investment amount takes a positive value. Deterrence shall thus be considered for all X such that $q_{2,L}(X) \geq 0$. In this model there exists a value X_1^{det} such that for all $X < X_1^{det}$ deterrence shall not take place, as in that case $q_{2,L}(X) < 0$.

Proposition 2 *The deterrence strategy is considered by the leader for all values of X such that $X \in (X_1^{det}, X_2^{det})$, where X_1^{det} is defined as the solution of*

$$\frac{X}{r-\alpha}(\nu-\omega Q^1-\omega q_L^1) - \frac{\delta}{\beta-1}\left(\frac{X(\beta-1)(\nu-\omega Q^1-\omega q_F^1)}{\delta(r-\alpha)(\beta+1)}\right)^\beta\left[1-\frac{\omega\beta q_L^1}{\nu-\omega Q^1-\omega q_F^1}\right] = \delta,$$

[1]One might take notice that it is not necessary to define q_L^{det} as the supremum rather than the maximum as when the maximum should be attained at $\hat{q}_{2,L}(X,Q)$ one can already conclude that the deterrence strategy is not the optimal strategy.

and where

$$X_2^{det} = \frac{\beta+1}{\beta-1}\frac{2\delta(r-\alpha)}{\nu-(3+\beta)\omega q_F^1}.$$

If the leader decides to invest using the deterrence strategy, optimal capacity level $q_L^{det}(X,Q)$ *for a given level of* X *is chosen to satisfy*

$$\frac{X}{r-\alpha}(\nu-\omega Q^1-\omega q_L^1-2\eta q_L^2)-\frac{X}{r-\alpha}\left(\frac{X}{X_F^*(q_L^2)}\right)^{\beta-1}\frac{1}{\beta+1}\left[\nu-2\omega Q^1-\omega q_L^1-3\eta q_L^2\right]=\delta. \tag{8}$$

As a result, the value function for the leader's deterrence strategy is given by,

$$V_L^{det}(X,Q)=\frac{X}{r-\alpha}[q_L^1(1-\eta Q^1-\omega q_L^2)+q_L^2(\nu-\eta q_L^2-\omega Q^1)]-\frac{\delta}{\eta(\beta-1)}(q_L^1\omega+q_L^2\eta)\left(\frac{X}{X_F^*}\right)^{\beta}-\delta q_L^2.$$

For low values of X*, that is* $X<X_L^{det}$*, the leader will invest when* X *reaches investment threshold value* X_L^{det}*. The value of the associated capacity level* q_L^{det} *is implicitely defined as the solution of equation (8) when substituting* $X=X_L^{det}(q_L^2)$*, where*

$$X_L^{det}=\frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{\nu-\omega Q^1-\omega q_L^1-\eta q_L^2}.$$

The investment threshold value X_L^{det} *is then defined as* $X_L^{det}(q_L^{det})$*. The corresponding option value function is defined as*

$$F_L^{det}(X,Q)=A_L^{det}X^{\beta}+\frac{X}{r-\alpha}q_L^1(1-\eta Q^1),$$

where $A_L^{det}=(X_L^{det})^{-\beta}\frac{\delta q_L^{det}}{\beta-1}-\frac{\delta}{\eta(\beta-1)}\left(X_F^*\right)^{-\beta}\left(\omega q_L^1+\eta q_L^{det}\right)$.

For the accommodation strategy, the leader leaves sufficient room on the market for the second firm to follow. The value function of the leader then no longer contains the correction factor as for accommodation the firms invest simultaneously,

$$\begin{aligned}V_L^{acc}(X,Q)&=\mathbb{E}\left\{\int_0^{\infty}\left[q_L^1X(t)(1-\eta Q^1-\omega Q^2)+q_L^2X(t)(\nu-\eta Q^2-\omega Q^1)\right]e^{-rt}\mathrm{d}t\right\}-\delta q_L^2\\&=\frac{X}{r-\alpha}\left[q_L^1(1-\eta Q^1-\omega Q^2)+q_L^2(\nu-\eta Q^2-\omega Q^1)\right]-\delta q_L^2.\end{aligned}$$

For accommodation to happen, the leader is restricted in choosing q_L^2, for it should be smaller than $\hat{q}_L^2(X,Q)$ (equation (7)). As in previous cases, the optimal amount q_2^{acc} is found by maximizing the firm value function $V_L^{acc}(X,Q)$. There exists a single point X for which the functions $q_L^{acc}(X)$ and $\hat{q}_L^2$ intersect. If this point is unique and both functions are continuous it is shown that accommodation only occurs for a single region. It is however not said that this point yields a positive capacity. Hence, accommodation is considered for $X\in(X_1^{acc},\infty)$, where X_1^{acc} is the maximum of the intersection point and the point where X yields zero capacity,

$$X_1^{acc}=\max\left\{\frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{\nu-4\omega q_F^1}\;,\;\frac{\delta(r-\alpha)}{\nu-2\omega q_L^1}\right\}.$$

Proposition 3 *The accommodation strategy is considered by the leader for all values of X such that $X \in (X_1^{acc}, \infty)$, where X_1^{acc} is defined as*

$$X_1^{acc} = \max \left\{ \frac{\beta + 3}{\beta - 1} \frac{\delta(r-\alpha)}{\nu - 4\omega q_F^1} \ , \ \frac{\delta(r-\alpha)}{\nu - 2\omega q_L^1} \right\}.$$

If the leader decides to invest using the accommodation strategy, optimal capacity level $q_L^{acc}(X,Q)$ for a given level of X is defined as

$$q_L^{acc}(X) = \frac{1}{2\eta} \left[\nu - 2\omega q_L^1 - \frac{\delta(r-\alpha)}{X} \right].$$

As a result, the value function for the leader's accommodation strategy is given by,

$$V_L^{acc}(X,Q) = \frac{X}{r-\alpha} \left[q_L^1(1 - \eta Q^1 - \omega Q^2) + q_L^2(\nu - \eta Q^2 - \omega Q^1) \right] - \delta q_L^2.$$

For low values of X, that is $X < X_L^{acc}$, the leader will invest when X reaches investment threshold value X_L^{acc}. The value of X_L^{acc} is found by solving for X when substituting $q_F^*(X,Q)$ and $q_L^{acc}(X,Q)$, where

$$X_L^{acc}(X,Q) = \frac{\delta\beta(r-\alpha)}{\beta - 1} \frac{\eta q_L^{acc} - \omega q_L^1}{\eta q_L^{acc}(\nu - 2\omega q_L^1 - \eta q_L^{acc}) - \omega q_L^1(\nu - 2\omega q_F^1 - \omega q_L^1))}.$$

The corresponding capacity equals $q_2^{acc}(X_L^{acc})$. The option value function is defined as

$$F_L^{acc}(X,Q) = A_L^{acc} X^\beta + \frac{X}{r-\alpha} q_L^1 (1 - \eta Q^1),$$

where

$$A_L^{acc} = (X_L^{acc})^{-\beta} \frac{\delta q_L^{acc}}{\beta - 1}.$$

3.2 Endogenous firm roles

If one assumes that the consequetive order of firms' investments is determined exogenously, competition is left out. However, in real life firms compete in taking the best shares. This section aims to explore the situation in which both firms want to become leader.

To that purpose, two firms are defined. Firm A is the firm having the largest stake on the old market, i.e. has the largest capacity for product one. Naturally, firm B is defined to be the firm with the smallest stake. It is assumed that both firms find it profitable to innovate and the assumption that $X(0)$ is sufficiently small remains for this section.

Preemptive equilibrium

NEW VERSION: Figure 1 shows the two curves for when a firm would become leader (solid) or follower (dashed) for different values of X. Clearly, for small values of X firms prefer to rather become follower, and invest at a larger value, than to invest at that particular moment and become leader. For larger values firms find it preferable to become leader. Following Huisman & Kort (2013), the intersection point is called the preemption point. Suppose the investment trigger for firm Y is smaller than the investment trigger of

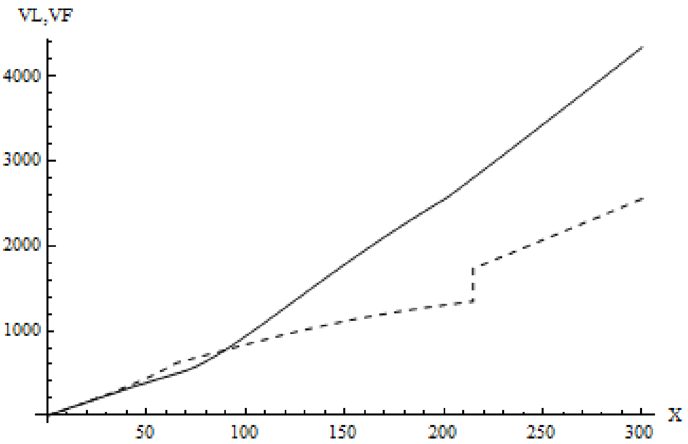
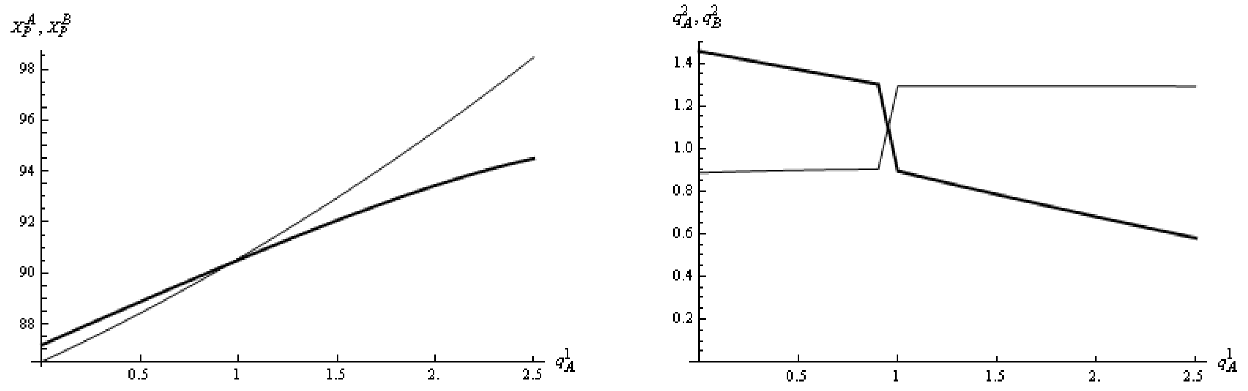


Figure 1: V_L^A and V_F^A (dashed) in case firm A becomes leader or follower.
$\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.2,\ \omega = 0.08,\ \nu = 1.5,\ d = 1000,\ q_{1,A} = 1.19,\ q_{1,B} = 0.90$

firm Z, but still larger than the preemption point for firm Z. In that case, if firm Y would invest at its investment trigger, firm Z would become follower and end up with an expected payoff that is lower than when it would have become leader itself. Therefore, investing just before the investment trigger of firm Y is a better strategy than waiting to become follower. A similar reasoning holds firm Y. Consequently, the firm with the smallest preemption point will be the leader and invest just before the preemption point of the other firm, for the other firm has no incentives to become leader for values of X that are smaller than its own preemption point. END

OLD VERSION: In order to evaluate firm A's situation one should compare the value functions for the cases in which firm A is leader and when firm A is follower on the new market. So one needs the leader curve in case firm A becomes leader and the follower curve in case firm B is leader. One then obtains a graph similar to Figure 1. Obviously there is a point X_P^A such that for $X < X_P^A$ firm A prefers to be follower. For $X \geq X_P^A$ firm A rather becomes leader. Similarly, for firm B there is an associated point X_P^B. Now suppose X_L^{det} exceeds X_P^i for both firms $i = A, B$, then the preemption strategy comes in. By investing earlier than X_L^{det} it is still better to become leader than follower and therefore it is better to preempt the other firm. However the other firm tries to preempt as well. Preemption stops when either of the firms reaches X_P^i. Then the firm no longer wants to become leader for smaller values of X. In this case one can calculate that for firm A $X_P^A = 91$ and $X_L^{det} = 130$, for firm B $X_P^B = 92$ and $X_L^{det} = 126$. Hence, firm A invests at $X = X_P^B$ and becomes leader. END

The parameter values associated with Figure 1, where $q_{1,A} = 1.19$ and $q_{1,B} = 0.90$ result into preemption values $X_{P,A} = 91$ and $X_{P,B} = 92$. Here, firm A invests first and becomes leader. The natural question arises whether this is always the case. Figure 2a shows the preemption points of both firm A, $X_{P,A}$, and B, $X_{P,B}$, for different values of $q_{1,A}$. Here it is assumed that firm A invested first on the old market, so that one knows that firm B invested at its threshold $X_F^*(q_{1,A})$, leading to a determinable capacity for firm B on



(a) Preemption investment triggers (b) Capacities

Figure 2: Preemption investment triggers for firms A $X_{P,A}$ (thick) and firm B $X_{P,B}$ and the resulting capacities $q_{2,A}$ (thick) and $q_{2,B}$ for different values of $q_{1,A}$.

$$\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.2,\ \omega = 0.08,\ \nu = 1.5,\ d = 1000,\ q_{1,B} = \frac{1-\eta q_{1,A}}{\eta(\beta+1)}$$

the old market. Again, for myopic investment the problem can be reduced to a problem with one variable. The curves of the preemption points intersect each other at one point. It can be verified that this is exactly the point where $q_{1,A} = q_{1,B}(q_{1,A})$. For $q_{1,A}$ smaller than this point one obtains that $q_{1,A} < q_{1,B}(q_{1,A})$ and in the same way, $q_{1,A} > q_{1,B}(q_{1,A})$ for larger values of $q_{1,A}$. One can conclude that the largest firm on the old market obtains the smallest preemption point.

Result 1 *Larger firms have more incentives to innovate.*

To explain this notion note that in case product 2 is offered there is a large drop in demand for the first product. The firm with the largest stake will be affected most and has therefore more to loose. The largest firm prefers to do the cannibalization itself and become monopolist for a period in time more than having eating demand and being deterred from the market by the smallest firm. Thus, the biggest firm wants to prevent the second firm from investing by entering the second market first. This inference would only be reasonable if this firm would underinvest, that is, since it invests at an early stage it would only be possible to make a small investment. Figure 2b shows the associated capacities on the new market for both firms. Clearly the leader on the new market, i.e. the largest firm on the old market, becomes the smallest firm for the second product.

Result 2 *Larger firms underinvest.*

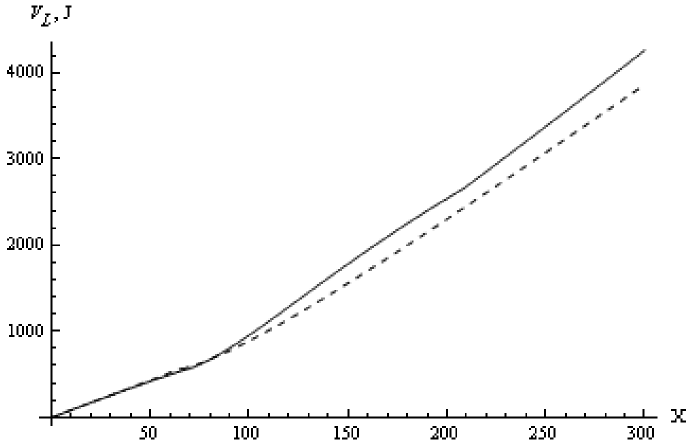


Figure 3: $V_{L,A}$ and $V_{J,A}^{Nash}$ (dashed) in case firm A becomes leader or invests jointly.
$\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.2,\ \omega = 0.08,\ \nu = 1.5,\ d = 1000,\ q_{1,A} = 1.19,\ q_{1,B} = 0.90$

Sequential equilibrium

Simultaneous equilibrium

Despite the competitive nature of the firms' incentives, there is a second class of strategies that lead to potential equilibria. Firms may tacitly colluse to wait and invest when the market is big enough in order to invest simultaneously. Investing together is an optimal strategy when it yields a larger waiting value than competitive investment at an earlier stage. See i.a. Boyer et al (2002) and Pawlina & Kort (2006). The models in the latter paper also consider asymmetric firms, but Pawlina & Kort assume fixed capacity. It is due to freedom of capacity choice that there are two types of equilibrium candidates.

The first type can be characterized as a Stackelberg type of equilibrium. Here, firms wait for the accommodation region to start and pick the optimal moment X_S^{Stack} such that profits are maximized. By asymmetry one obtains a different value for each firm. The firm with the smallest value invests first and becomes Stackelberg leader. However, the then assigned follower would be better off to preempt the Stackelberg leader and become leader itself. Naturally one ends up in the same situation as in the first type of equilibria and the firm with the smallest preemption point invests and becomes leader. This type of equilibria will never occur.

The second type of simultaneous investment is the Nash/Cournot version. Here firms agree to invest at the same time, alloting no leader or follower. Given the investment moment both firms simultaneously set capacities. The resulting value function V_S^{Nash} can be studied from Figure 3. It is obvious that for playing a joint investment strategy the leader always gives in. The value function for joint investment always gives a lower payoff than the value function for the leader. For the same reasoning as before, this type of equilibria will not be played.

Result 3 *As a result of freedom of capacity choice, simultaneous investment never yields an equilibrium.*

Proof.

$$V_L^{acc}(X,Q) = \frac{X}{r-\alpha}\left[q_{1,A}\left(1 - \frac{\eta^2-\omega^2}{\eta}Q^1 - \frac{\omega}{\eta}\left(\nu - \frac{\delta(r-\alpha)}{X}\right)\right) + \frac{1}{8\eta}\left(\nu - \frac{\delta(r-\alpha)}{X}\right)^2\right]$$
$$V_L^{join}(X,Q) = \frac{X}{r-\alpha}\left[q_{1,A}\left(1 - \frac{\eta^2-\omega^2}{\eta}Q^1 - \frac{\omega}{\eta}\left(\nu - \frac{\delta(r-\alpha)}{X}\right)\right) + \frac{1}{9\eta}\left(\nu - \frac{\delta(r-\alpha)}{X}\right)^2\right]$$

Hence, for $X \in [\hat{X}, \infty)$ it holds that $V_L^{acc}(X,Q) > V_L^{join}(X,Q)$. This is sufficient. ∎

4 Model significance

In the previous section different results are derived. As will be shown, these results contradict some other papers in the same field. This section aims to clarify the differences and aims to show the significance of the work presented in this paper.

4.1 The importance of capacity choice

From result 3 it already follows that freedom of capacity choice has a significant influence on the firms' behavior. To explore this enhancement a bit more, two models are presented to serve as a benchmark. The setup of the first model is identical to the setup in this paper, but here one assumes fixed capacity. The second section will compare the presented model with the model from Pawlina & Kort (2006) where they look at asymmetric firms without freedom of capacity choice.

Model 1

The duopoly setting in the following model deviates from the model in the previous section in just one respect. In the previous section firms were free to choose their capacities. It shall now be assumed that investment can only be done under the assumption that the associated capacity equals the amount K for both the leader and the follower. The associated prices become,

$$P_1(t) = X(t)(1 - \eta Q^1 - 2\omega K),$$
$$P_2(t) = X(t)(\nu - 2\eta K - \omega Q^1),$$

Let firm A be the largest firm on the old market, then the associated leader and follower curves can be derived as,

$$
\begin{aligned}
V_{L,A}^{det}(X,Q) &= \frac{X}{r-\alpha}\left[q_{1,A}(1-\eta Q^1-\omega K)+K(\nu-\eta K-\omega Q^1)\right]-\frac{X_{F,B}^*}{r-\alpha}K(\omega q_{1,A}+\eta K)\left(\frac{X}{X_{F,B}^*}\right)^{\beta}-\delta K\\
X_{F,B}^* &= \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{\nu-\omega Q^1-\omega q_{1,B}-2\eta K}\\
V_{F,A}(X,Q) &= \frac{X}{r-\alpha}\left[q_{1,A}(1-\eta Q^1-2\omega K)+K(\nu-2\eta K-\omega Q^1)\right]-\delta K\\
F_{F,A}(X,Q) &= A_{F,A}^{det}X^{\beta}+\frac{X}{r-\alpha}q_{1,A}(1-\eta Q^1-\omega K)\\
A_{F,A}^{det} &= \frac{\delta K}{\beta-1}\left(X_{F,A}^*\right)^{-\beta}\\
X_{F,A}^* &= \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{\nu-\omega Q^1-\omega q_{1,A}-2\eta K}
\end{aligned}
$$

Most interesting are the preemption triggers for both firms in this model. When including capacity choice is was concluded that the biggest firm invests first, for it wants to prevent the other firm from eating its demand and being expelled on the market for a period in time. This would only be possible if the firm were to make an underinvestment. In this model firms have a fixed capacity on the new market. In that case one cannot make an underinvestment. Therefore, the smallest firm would now have most incentives to innovate, for it has more to gain. Figure 4 displays the different preemption points for both firms. The lines intersect at the point where both firms have equal stakes on the old market, making them identical. Recall, for smaller $q_{1,A}$ firm B is the largest firm on the old market, but would be the smallest firm for larger $q_{1,A}$. In contrast to earlier results it is found that the smallest firm would invest first in the new product. Hence, it is due to the freedom of capacity choice that bigger firms would invest first when applying the preemption strategy.

Model 2

There are a few differences between Pawlina & Kort (2006) and the model presented in this paper. First of all, it is assumed that the profits are identical to both firms. $D_{1,1}$ is denoted to be the payoff when both firms have invested. In our model this value is different for both firms. The same holds for the value of $D_{1,0}, D_{0,1}$ and $D_{0,0}$ denoting the payoffs when one firm invested and no firms invested. Another difference is that in our model the investment costs are dependent on the value of $X(t)$ and therefore dependent on the equilibrium. The paper does assume some payoffs when there is no investment, equivalent to the old market assumed in this paper. The asymmetry in Pawlina & Kort is formed by the difference in investment costs. Firm 1 faces costs I, whilst firm 2 has to pay κI where $\kappa > 1$.

It is found that, in case the preemptive equilibrium applies, the low-cost firm invests first. In our model bigger firms loose more on the old market, which would mean that smaller firms could be identified as low-

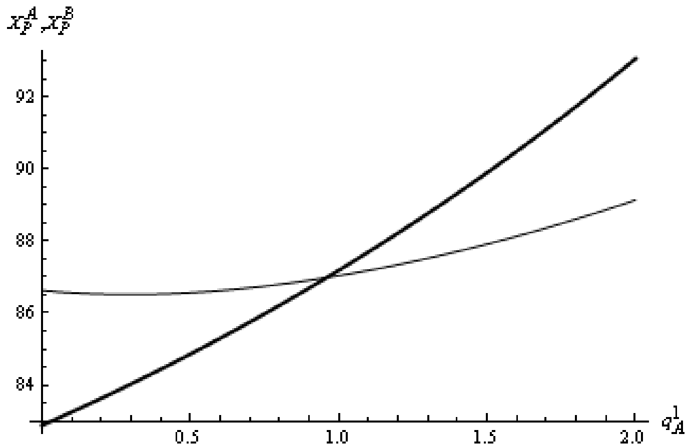


Figure 4: X_P^A (thick) and X_P^B for different values of q_A^1.
$\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.2,\ \omega = 0.08,\ \nu = 1.5,\ d = 1000,\ q_{1,A} = 1.19,\ q_{1,B} = 0.90$

cost firms. According to the paper of Pawlina & Kort (2006) that would mean that the smallest firm would preempt the biggest firm. As reasoned before, due to capacity choice the opposite is the case.

Hence, one can conclude that the enhancement of including freedom of capacity choice changes the investment order compared to the models where there is no freedom of capacity choice.

4.2 The importance of underinvestment

The model in this paper can be seen as an extension of the model by Huisman & Kort (2013) where they derive the same equations for an investment opportunity on an unestablished market. Inter alia, they concluded that in preemptive equilibria leaders make an underinvestment for the benefit of the monopoly advantage. However, in markets with high uncertainty investment is delayed. Delaying investment is equivalent to waiting for the market to expand. Along with that, investment quantities grow, eventually leading to a capacity that exceeds the follower's capacity. Thus, the leader only makes an underinvestment for markets where uncertainty is moderately low. This is not the case for the model treated in this paper. This can be explained by the presence of the old market, which was not present in the paper by Huisman & Kort (2013). The fact that the big firm has more to loose on the old market increases the incentives of the big firm to become leader on the new market. This effect outweighs the delayed investment effect on the capacities.

⇒ Picture

5 Conclusions

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