The Timing of Green Investments under Regime Switching and Ambiguity

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Abstract:

The economic success of green investments does not only depend on the uncertain economic development but also on future regime switches in the relevant legislation. As a result of political decision-making the latter are assumed to be rather ambiguous than uncertain. Based on the example of biologic fuel we develop a real options model that takes into account economic uncertainty as well as political ambiguity. We calculate the option value of the green investment and derive the optimal investment-timing strategy. Furthermore, we analyze both the sole as well as the combined influence of economic uncertainty and political ambiguity on these topics.

Keywords: Optimal Investment Timing; Real Options; Green Investments; Regime Switching; Political Ambiguity; Ultra Long Investments
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1. Introduction

In today's vast changing global economic and business environment investments are exposed to multiple uncertainties. Hence, the design of an optimal investment policy and the optimal timing of investments are among the most crucial decision problems in corporate finance and capital budgeting, respectively. This holds especially true for ultra-long investments, i.e. investments where the investment appraisal's planning horizon is considerable short compared not only to the potential period the firm can extract economic benefits or rents from the project but also compared to the overall temporal consequences arising out of those projects in the future. For instance, financial resource commitments related to eMobility infrastructure or genetic engineering projects, as well as the abandonment of nuclear power plants can be considered as ultra-long (dis-)investments. Likewise, green investments, i.e. investments in renewable energies, in the production of biologic fuel, in the reduction of waste, emissions or pollution and in the increase of energy efficiency require substantial financial resources which are bound for a long time. Given this context, ultra long investments are affected by multiple uncertainties e.g. political and environmental risks, technology and demand shocks as well as price and foreign exchange rate uncertainties to name but a few.

Over the last decades, academic research has acknowledged that the maintenance of flexibility and their fair economic valuation, respectively, is of central importance when designing optimal investment policies. This is a direct result of the irreversible nature of these investments, i.e. once an investment is made the incurred sunk costs cannot be recovered should the project be abandoned at a later stage. Thus, in analogy to the financial flexibility financial options provide real options have been introduced to the finance and management literature which express the managerial flexibility assigned to a real investment decision, i.e. for example the decision to postpone or to abandon an investment without being obliged to. [1] Consequently, option-based investment appraisals have been proposed as analytical tools to address these issues and the finance and economic literature has provided various examples that give guidance on how to optimally time an investment under uncertainty. For instance, projects that take considerable "time-to-build" have been analyzed by Majd and Pindyck (1987), Milne and Whalley (2001), Friedl (2002), and Mölls and Schild (2012) while investments in research and development (R&D) have been analyzed by Brach and Paxson (2001), Childs and Triantis (1999), Koussis et al. (2007) and Schwartz (2004). Generally, the key insight is that such examples of ultra-long investments should be staged optimally when uncertainty is high in order to retain the full flexibility should the future unfold different from what was expected.[2]

While this strain of literature dealing with the optimal sequential nature of investment policies has continuously developed new ways how to cope with the complexity and magnitude of a broad spectrum of investment-specific uncertainty, predominantly by considering more complex Itô processes which map the projects' value uncertainty, they all fall short with respect to one crucial point. All stochastic processes assume that a probability measure exists and the agent's beliefs are identical to this probability law. Thus, decision makers are considered to be expected-value maximizer, which discount the future with a proper discount

[1] For example see Dixit and Pindyck (1994) or Trigeorgis (1999) for a comprehensive overview.
[2] Other examples for staged investments are found in e.g. Triantis and Hodder (1990), Schwartz and Zozaya-Gorostiza (2003), Gilroy and Lukas (2006) and Kort et al. (2010).

rate. With respect to ultra-long investments, however, it is particularly difficult to assign probabilities to events affecting the value dynamics of project cash flows, e.g. how likely is it that consumers will demand electric vehicles or how likely is the abandonment of the EURO as the lead currency in Europe. Consequently, ultra-long investment decisions are not only made under uncertainty but under ambiguity, too.[3]

With respect to green investments, for the moment green technologies are usually more expensive and thus governmental force and/or support is needed to make companies invest in green projects (see Kumbaroğlu et al., 2008). Hence, the economic success of green investments crucially depends on the relevant legislation which is simply the result of political processes that are basically ambiguous. Nevertheless, it can be argued that political decisions are driven by other predictable variables. In this regard, Pindyck (2002) and Lin et al. (2007) set up real options models that determine the optimal environmental policy (from a whole-society level) in dependence of a technological and an environmental variable that both evolve stochastically over time. Though, in both articles it is assumed that the politicians always act in the best interest of society and that they have all the relevant information. [4]

So far there exists already an extensive literature strain that deals with the optimal timing of green investments under various sources of uncertainty. Yun and Baker (2009) and Patiño-Echeverri et al. (2007) deal with the investment into new power plants if carbon emissions are costly. Similarly, Insley (2003), Abadie and Chamorro (2008) and Lukas and Welling (2013a) apply the real options

[3] We will refer to ambiguity, or Knightian uncertainty as risk that is immeasurable, i.e. we cannot assign probabilities to the possible states of nature of an uncertain variable.

[4] However, the environmental development is usually seen to be ambiguous itself (see Hallegatte et al., 2012).

methodology on investments that reduce carbon emissions. Kumbaroğlu et al. (2008) set light on the diffusion of renewable energies under uncertainty. Cortazar et al. (2013) determine when it is optimal to invest in emission reducing technologies if the emission of pollutants is costly or restricted. Bastian-Pinto et al. (2009) and Pederson and Zou (2009) deal with investments in the production of biofuel, whereby input prices and sales prices are evolving stochastically over time. However, the relevant legislation is always assumed to be exogenously given and thus the potential influence of political ambiguity is omitted.

Though not in the context of green investments, ambiguity has recently gained more and more attention in the area of corporate finance. In particular, its effect on optimal timing is of interest. Nishimura and Ozaki (2007) investigate the effect ambiguity has on acquiring a patent and thereby subsequently investing in a Greenfield site to produce and sell the patented products. The traditional Dixit & Pindyck real option pricing model serves as a starting point with the distinct difference that uncertainty, as measured by the standard deviation, is now defined as risk and furthermore that it is the worst element in the set of the probability measures that counts when computing the expected values. The results show that an increase in Knightian uncertainty lowers the value of irreversible investment opportunities while risk -as is known- increases option value. However, both agents that follow the underlying *maximin*-criterion and individuals that have perfect confidence in the standard real option model find it profitable to postpone the investment when risk and uncertainty respectable, increase.

Alike, Trojanowska and Kort (2010) analyze the optimal timing of an investment in the high-tech sector. Ambiguity is captured by a spectrum of drift rates and the project life is assumed to be finite. Again the worst-case scenario (uncertainty aversion) is applied. Because increasing ambiguity erodes the drift the optimal investment threshold decreases indicating that waiting becomes less valuable which is again in contrast to the impact risk, as measured by volatility, has on the traditional investment threshold. Interestingly, the authors find that investment timing is in general equivocal with respect to the level of ambiguity. Consequently, whether an increase in ambiguity leads to a higher or lower optimal investment threshold depends on whether the option (time value) or the NPV (intrinsic value) effect dominates. However, even under ambiguity it still holds that it generally pays to wait and that immediate exercise as implied by the classical net present value (NPV) is not a viable alternative. However, it also matters how the investment rewards the investors. As Miao and Wang (2011) show, if the project value is modeled as a one-time lump sum payoff and uncertainty is completely resolved after investment ambiguity accelerates investment which is in contrast to situations above, i.e. the net reward upon investment depends on the spread between a discounted sum of uncertain profit flows in the future and the irreversible investment outlay. Notably, the findings also reveal that the myopic NPV rule can be optimal for individuals with extreme ambiguity aversion, i.e. waiting for new information is of no value.

In this paper we set up a real options model of a company that has the possibility to adjust a fuel-consuming asset in a way that it also tolerates biologic fuel. Furthermore, we determine the value of this option to invest as well as the optimal timing threshold that triggers investment. In this regard, it should be noted that due to the modulation of the economic uncertainty as a mean-reverting process the setting can easily been transformed to other green investments like carbon-emissions-reduction, production of renewable energies or the increase of energy efficiency. As opposed to the previous literature on green investments we consider economic uncertainty as well as political ambiguity. In contrast to the above mentioned literature that deals with ambiguity in our model economic uncertainty and political ambiguity can occur independently. Thus, it is possible to analyze both their sole and their combined influence on investment-timing and the option value. Another important difference is that in our model ambiguity is not resolved with investment but resolves at a pre-specified time, regardless if the company has invested before.

Our findings reveal that economic uncertainty as well as political ambiguity have a crucial influence on the value of the option to invest into the project and on the investment-timing. While economic uncertainty is always increasing the option value, the influence of political ambiguity depends on the decision rule that is used by the company and/or the company's ambiguity-aversion. Interestingly, also slightly ambiguity-averse companies can profit from a higher political ambiguity due to the flexibility-value that rises from the possibility to wait with the investment until the ambiguity is resolved. Furthermore, ambiguity always delays investment due to the flexibility-effect described above. The influence of economic uncertainty on investment-timing is found to be equivocal. In particular, its sign depends on the degree of political ambiguity.

The rest of the paper is organized as follows. Section two presents the model and characterizes its mathematical solution. Section three illustrates numerically the impact of economic uncertainty and political ambiguity on the option value and on the timing of the investment. Finally, section four concludes and lays out directions for future research.

2. The Model

We consider a risk-neutral company that discounts with a risk-free interest rate $r > 0$. It owns an asset that at time $t_0 = 0$ has a remaining finite life-time of $\tau \geq 0$ and that consumes $x > 0$ units of fuel per time unit. The price $p(t)$ of (standard) fuel evolves stochastically over time, i.e. we assume that it follows the mean-reverting process

$$dp(t) = \kappa\big(\theta - \ln\big(p(t)\big)\big)p(t)dt + \sigma p(t)dW(t), \qquad p(0) = p_0 \geq 0, \tag{1}$$

whereby $\kappa > 0$ is the mean reversion speed, $\theta > 0$ is the mean reversion level, $\sigma > 0$ is the uncertainty parameter and $dW(t)$ is the increment of a Wiener process with zero mean and variance equal to one. During the life-time of the asset the company has at any time the opportunity to adjust its asset in a way that it can also tolerate biologic fuel. Mainly driven by subsidies and lower taxes the price $p_B(t)$ of biologic fuel is cheaper than the price of standard fuel. In particular, we assume that

$$p_B(t) = (1 - \xi)p(t), \tag{2}$$

whereby $0 \leq \xi \leq 1$.[5] Thus, after investment the company saves

$$x\big(p(t) - p_B(t)\big) = x\xi p(t) \tag{3}$$

in every time unit. As we can derive from equation (1)

$$p(t) = e^{e^{-\kappa t}\ln(p_0) + \left(\theta - \frac{\sigma^2}{2\kappa}\right)(1 - e^{-\kappa t}) + \sigma e^{-\kappa t}\int_0^t e^{\kappa s} dW(s)} \tag{4}$$

and thus

$$\mathbb{E}\big(p(t_2)|p(t_1) = p_{t_1}\big) = e^{e^{-\kappa(t_2 - t_1)}\, ln(p_{t_1}) + \left(\theta - \frac{\sigma^2}{2\kappa}\right)\left(1 - e^{-\kappa(t_2 - t_1)}\right) + \frac{\sigma^2\left(1 - e^{-2\kappa(t_2 - t_1)}\right)}{2\kappa}} \tag{5}$$

[5] This assumption can be economically justified because the largest proportion of biologic fuel is blended to standard fuels in lower concentration (E10, etc.) and therefore the prices of biological fuel track the standard fuel price (see Tao and Aden, 2009). Furthermore, we assume, that the slightly worse efficiency of biologic fuel compared to standard fuel is already considered in ξ.

for every $t_2 \geq t_1 \geq 0$. If the company invests at a time $t \geq 0$, therefore, it expects discounted savings of

$$\begin{aligned} V(p(t),t) &= \mathbb{E}\int_t^\tau x\big(p(s)-p_B(s)\big)e^{-r(s-t)}\,ds \\ &= x\xi\int_t^\tau \mathbb{E}\big(p(s)|p(t)\big)e^{-r(s-t)}ds. \end{aligned} \tag{6}$$

Due to technical progress the necessary investment costs of the adjustment are expected to decline over time, i.e. we assume that at time t the investment costs equal

$$I(t) = I_f + I_v e^{-\eta t} \tag{7}$$

with $I_f \geq 0,\ I_v > 0$ and $\eta \geq 0$. Thus, investing at time t generates an expected profit of

$$\pi(p(t),t) = V(p(t),t) - I(t). \tag{8}$$

Following Dixit and Pindyck (1994) the possibility to invest can be regarded as a real option. Hence, the company should not invest immediately but wait with the investment until the price of standard fuel reaches the time-depending optimal threshold $p^*(t)$. Hence, the optimal investment time is determined by

$$t^* = inf\{t \geq t_0 | p(t) \geq p^*(t)\}. \tag{9}$$

Using Ito's Lemma it can easily be deduced from equation (1) that the value $F(p(t),t)$ of the option to invest is the solution of the differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F(p,t)}{\partial^2 p} + \big(\kappa(\theta - \ln(p))\big)p\frac{\partial F(p,t)}{\partial p} + \frac{\partial F(p,t)}{\partial t} = rF(p,t) \tag{10}$$

which also meets the following four conditions: Firstly, as zero is an absorbing barrier of the price process, we have

$$\lim_{p(t)\to 0} F(p(t),t) = 0\ \forall t \geq 0, \tag{11}$$

secondly, the investment opportunity has no value if the asset is no longer in use, hence

$$F(p(t),t) = 0\ \forall t \geq \tau, \tag{12}$$

thirdly, the continuity-condition

$$F(p^*(t), t) = \pi(p^*(t), t) \tag{13}$$

ensures that the value of the option equals its intrinsic value at the optimal exercise time and finally the smooth-pasting condition

$$\frac{\partial F(p^*(t), t)}{\partial p} = \frac{\partial \pi(p^*(t), t)}{\partial p} \tag{14}$$

guarantees that this transition is smooth which is a necessary condition for optimality of the exercise time (see Dixit and Pindyck, 1994).

2.1 Integration of a regime switch

So far, we have assumed a constant price advantage $p(t) - p_B(t)$ of biologic fuel. However, as this advantage is mainly driven by political decisions, it can be expected that changing political majorities may have an impact on its size. While some parties might claim to reduce the price advantage other parties might claim to increase the advantage of biologic fuel and for some other parties the issue might be of no relevance. Hence, the exact amount of the advantage of biologic fuel in the future depends on public opinion, future election results and the results of coalition negotiations. Therefore, in contrast to the economic price uncertainty the political uncertainty is an example of Knightian uncertainty or ambiguity, i.e. its probability distribution is unknown (Knight, 1971). In the following we will integrate this political uncertainty into the model. In particular, we assume that the price advantage may switch at a known point of time $T \geq 0$ (the election day) [6] and that the price of biologic fuel equals

[6] To assure the tractability of the model we simplify the political process by assuming that coalition negotiations and the legislative process do not require time and that laws come into force immediately. Furthermore, we assume that the topic of biologic fuel subsidies will only be of interest in this election (and not in following ones) and that the election date is fixed.

$$p_B(t) = \begin{cases} (1-\xi)p(t) & t_0 \le t \le T \\ \omega(1-\xi)p(t) & T \le t \le \tau \end{cases}. \tag{15}$$

Hereby, $0 \le \omega \le \frac{1}{1-\xi}$ indicates the result of the political process that will be revealed to the company at T. The higher ω the lower is the price advantage of biologic fuel after the election date, i.e. $\omega = 0$ would imply that biologic fuel can be bought for free while $\omega = \frac{1}{1-\xi}$ implies that there is no advantage of biologic fuel at all. If $\omega = 1$ the price advantage of biologic fuel does not change. Thus, we obtain for the possible savings

$$x\big(p(t) - p_B(t)\big) = \begin{cases} x\xi p(t) & t_0 \le t \le T \\ x(1-\omega+\omega\xi)p(t) & T \le t \le \tau \end{cases}. \tag{16}$$

For a short moment let us assume that the company already knows the value of ω in t_0. Then, if it invests at time $t \ge 0$ it expects discounted savings of

$$\begin{aligned} V_\omega(p(t),t) &= \mathbb{E}\int_t^\tau x\big(p(s)-p_B(s)\big)e^{-r(s-t)}\,ds \\ &= \mathbb{E}\int_t^{\max(t,T)} x\xi p(s)e^{-r(s-t)}\,ds + \mathbb{E}\int_{\max(t,T)}^\tau x(1-\omega+\omega\xi)p(s)e^{-r(s-t)}\,ds \\ &= \mathbb{E}\int_t^\tau x\xi p(s)e^{-r(s-t)}\,ds + \mathbb{E}\int_{\max(t,T)}^\tau x(\omega-1)(\xi-1)p(s)e^{-r(s-t)}\,ds \\ &= x\xi\int_t^\tau \frac{\mathbb{E}\big(p(s)|p(t)\big)}{e^{r(s-t)}}\,ds \\ &\quad + x(\omega-1)(\xi-1)\int_0^\infty \psi_p(y,p(t),\max(t,T),t)\int_{max(t,T)}^\tau \frac{\mathbb{E}(p(s)|y)}{e^{r(s-t)}}\,ds\,dy. \end{aligned} \tag{17}$$

Hereby,

$$\psi_p\big(y,p_{t_1},\mathrm{t}_2,t_1\big) := \frac{\partial\mathbb{P}\big(p(t_2)\le y|p(t_1)=p_{t_1}\big)}{\partial y} \tag{18}$$

denotes the transition density function of the price process p. Thus, investing at time t generates an expected profit of

$$\pi_\omega(p(t),t) = V_\omega(p(t),t) - I(t). \tag{19}$$

The value $F_\omega(p(t), t)$ of the option to invest and the optimal investment threshold $p^*_\omega(t)$ can be calculated as described above, i.e. the value $F_\omega(p(t), t)$ is the solution of the partial differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_\omega(p,t)}{\partial^2 p} + \big(\kappa(\theta - \ln(p))\big)p\frac{\partial F_\omega(p,t)}{\partial p} + \frac{\partial F_\omega(p,t)}{\partial t} = rF_\omega(p,t) \qquad (20)$$

under the constraints

$$\lim_{p(t)\to 0} F_\omega(p(t), t) = 0 \;\forall t \geq 0, \qquad (21)$$

$$F_\omega(p(t), t) = 0 \;\forall t \geq \tau, \qquad (22)$$

$$F_\omega(p^*_\omega(t), t) = \pi_\omega(p^*_\omega(t), t) \qquad (23)$$

and

$$\frac{\partial F_\omega(p^*_\omega(t), t)}{\partial p} = \frac{\partial \pi_\omega(p^*_\omega(t), t)}{\partial p}. \qquad (24)$$

The optimal investment time equals

$$t^*_\omega = inf\{t \geq t_0 | p(t) \geq p^*_\omega(t)\}. \qquad (25)$$

However, in reality the company does not know the value of ω before the election date T. As it even does not know the corresponding probability distribution it only has two different possibilities. It can either evaluate its investment opportunity based on its optimism regarding the political development or it can without information of the contrary assume that all possible values of ω will be equally likely. The first of these two approaches (called Hurwicz-rule) is discussed in the next subsection while the second approach (called Laplace-rule) is described in the subsequent subsection.

2.2 Valuation based on optimism

Of all the political party's demands before the election date we denote ω_{opt} as the highest demanded promotion of biologic fuel while we denote ω_{pes} as the lowest demanded promotion of biologic fuel. Obviously it is $0 \leq \omega_{opt} \leq \omega_{pes} \leq \frac{1}{1-\xi}$ and if ω is the actual value of ω after the election date it is $\omega_{opt} \leq \omega \leq \omega_{pes}$. A very optimistic decision-maker expects that after the election date ω_{opt} comes into effect. He therefore thinks that investing at time $t_0 \leq t \leq T$ would generate an expected profit of $\pi_{\omega_{opt}}(p(t), t)$ and that the option to invest has a value of $F_{\omega_{opt}}(p(t), t)$. He invests at time

$$t_{opt}^* = inf\{t \geq t_0 | p(t) \geq p_{opt}^*(t)\}, \tag{26}$$

i.e. as soon as the price of standard fuel reaches the time-depending optimal threshold $p_{opt}^*(t)$. Likewise, a very pessimistic decision-maker expects that after the election date ω_{pes} comes into effect. Therefore, he thinks that investing at time $t_0 \leq t \leq T$ would generate an expected profit of $\pi_{\omega_{pes}}(p(t), t)$, that the option to invest has a value of $F_{\omega_{pes}}(p(t), t)$ and that he should invest at

$$t_{pes}^* = inf\{t \geq t_0 | p(t) \geq p_{pes}^*(t)\}. \tag{27}$$

The very pessimistic decision-maker decides according to the Maximin decision rule as described by Wald (1950) while the very optimistic decision maker decides according to the Maximax decision rule. Hurwicz (1951) combines these two decision rules by introducing an optimism parameter $0 \leq \lambda \leq 1$. A decision maker following this decision rule thinks that investing at time $t_0 \leq t \leq T$ would generate an expected profit of

$$\pi_{H,\lambda}(p(t), t) = \lambda \pi_{\omega_{opt}}(p(t), t) + (1 - \lambda)\pi_{\omega_{pes}}(p(t), t). \tag{28}$$

Furthermore, he assumes that not investing until T will generate an option value at time T of

$$F_{H,\lambda}(p(T),T) = \lambda F_{\omega_{opt}}(p(T),T) + (1-\lambda)F_{\omega_{pes}}(p(T),T). \tag{29}$$

Thus, a decision maker with $\lambda = 0$ equals the very pessimistic decision maker and decides according to the Maximin rule while a decision maker with $\lambda = 1$ equals the very optimistic decision maker and decides according to the Maximax rule. In general optimism is increasing with λ. For $0 \leq t \leq T$ the value $F_{H,\lambda}(p(t),t)$ of the option to invest as well as the optimal investment threshold $p^*_{H,\lambda}(t)$ can be determined by solving the partial differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_{H,\lambda}(p,t)}{\partial^2 p} + \big(\kappa(\theta - \ln(p))\big)p\frac{\partial F_{H,\lambda}(p,t)}{\partial p} + \frac{\partial F_{H,\lambda}(p,t)}{\partial t} = rF_{H,\lambda}(p,t) \tag{30}$$

under the constraints

$$\lim_{p(t)\to 0} F_{H,\lambda}(p(t),t) = 0 \; \forall t \geq 0, \tag{31}$$

$$F_{H,\lambda}(p(T),T) = \lambda F_{\omega_{opt}}(p(T),T) + (1-\lambda)F_{\omega_{pes}}(p(T),T)\,, \tag{32}$$

$$F_{H,\lambda}\big(p^*_{H,\lambda}(t),t\big) = \pi_{H,\lambda}\big(p^*_{H,\lambda}(t),t\big) \tag{33}$$

and

$$\frac{\partial F_{H,\lambda}\big(p^*_{H,\lambda}(t),t\big)}{\partial p} = \frac{\partial \pi_{H,\lambda}\big(p^*_{H,\lambda}(t),t\big)}{\partial p}. \tag{34}$$

The optimal investment time equals

$$t^*_{H,\lambda} = inf\{T \geq t \geq t_0 | p(t) \geq p^*_{H,\lambda}(t)\}. \tag{35}$$

However, it should be noted that after the election date the company knows the actual value of ω regardless its former expectations and optimism. Thus, if the company has not invested before T it decides after the election date according to subsection 3.1 though it may be surprised by the amount of ω.

2.3 Valuation based on assumed equal distribution

Following Laplace a decision maker should assume equal distribution of a stochastic variable if he has no contrary information (see e.g. Bamberg and Coenenberg, 2006, p.133). Hence, a decision maker that follows this approach assumes ω to be equally distributed on the interval $[\omega_{opt}, \omega_{pes}]$, i.e. the probability density function of ω is

$$f_\omega(z) := \frac{\partial \mathbb{P}(\omega \le z)}{\partial z} = \begin{cases} \dfrac{1}{\omega_{pes} - \omega_{opt}} & \omega_{opt} \le z \le \omega_{pes} \\ 0 & else \end{cases}. \tag{36}$$

The expected value of ω is $\frac{\omega_{pes}+\omega_{opt}}{2}$ and the variance is $\frac{1}{12}\left(\omega_{pes} - \omega_{opt}\right)^2$. Assuming equal distribution the company expects discounted savings of

$$\begin{aligned} V_L(p(t),t) &= x\xi \int_t^\tau \frac{\mathbb{E}\big(p(s)|p(t)\big)}{e^{r(s-t)}} ds \\ &+ x(\xi - 1) \int_{\omega_{opt}}^{\omega_{pes}} \frac{(z-1)}{\omega_{pes} - \omega_{opt}} \int_0^\infty \psi_p(y, p(t), \max(t,T), t) \int_{\max(t,T)}^\tau \frac{\mathbb{E}(p(s)|y)}{e^{r(s-t)}} ds\, dy\, dz \\ &= x\xi \int_t^\tau \frac{\mathbb{E}\big(p(s)|p(t)\big)}{e^{r(s-t)}} ds \\ &+ x(\xi - 1)\left(\frac{\omega_{pes} + \omega_{opt}}{2} - 1\right) \int_0^\infty \psi_p(y, p(t), \max(t,T), t) \int_{max(t,T)}^\tau \frac{\mathbb{E}(p(s)|y)}{e^{r(s-t)}} ds\, dy\, dz \end{aligned} \tag{37}$$

if it invests at time $T \ge t \ge 0$. Thus, investing at time t generates an expected profit of

$$\pi_L(p(t),t) = V_{\frac{\omega_{pes}+\omega_{opt}}{2}}(p(t),t) - I(t). \tag{38}$$

Furthermore, he assumes that not investing until T will generate an option value at time T of

$$F_L(p(T),T) = \int_{\omega_{opt}}^{\omega_{pes}} \frac{1}{\omega_{pes} - \omega_{opt}} F_z(p(T),T) dz \tag{39}$$

Again for $0 \leq t \leq T$ the value $F_L(p(t), t)$ of the option to invest as well as the optimal investment threshold $p_L^*(t)$ can be determined by solving the partial differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_L(p,t)}{\partial^2 p} + \big(\kappa(\theta - \ln(p))\big)p \frac{\partial F_L(p,t)}{\partial p} + \frac{\partial F_L(p,t)}{\partial t} = rF_L(p,t) \tag{40}$$

under the constraints

$$\lim_{p(t)\to 0} F_L(p(t), t) = 0 \; \forall t \geq 0, \tag{41}$$

$$F_L(p(T), T) = \int_{\omega_{opt}}^{\omega_{pes}} \frac{1}{\omega_{pes} - \omega_{opt}} F_z(p(T), T) dz, \tag{42}$$

$$F_L(p_L^*(t), t) = \pi_L(p_L^*(t), t) \tag{43}$$

and

$$\frac{\partial F_L(p_L^*(t), t)}{\partial p} = \frac{\partial \pi_L(p_L^*(t), t)}{\partial p}. \tag{44}$$

The optimal investment time equals

$$t_L^* = inf\{T \geq t \geq t_0 | p(t) \geq p_L^*(t)\}. \tag{45}$$

Like in the previous subsection if it has not invested until T the company will decide after the election date according to subsection 3.1.

2.4 Absence of economic uncertainty

In this subsection we analyze the investment problem in absence of economic uncertainty, i.e. $\sigma = 0$. This allows us to separate the effects of economic uncertainty and political ambiguity. As the ambiguity is the higher the higher the difference of ω_{pes} and ω_{opt} and as it totally diminishes if $\omega_{pes} = \omega_{opt}$ we will measure the amount of political ambiguity in the following by

$$\omega_\Delta \coloneqq \omega_{pes} - \omega_{opt}. \tag{46}$$

From equation (4) we get that in absence of economic uncertainty the price of standard fuel at time $t \geq 0$ equals

$$p(t) = e^{e^{-\kappa t}\ln(p_0)+\theta(1-e^{-\kappa t})}. \tag{47}$$

It is easy to see that $p(t)$ is strictly monotonically increasing if and only if $\theta > \ln(p_0)$, that it is strictly monotonically decreasing if and only if $\theta < \ln(p_0)$ and that it is constant if and only if $\theta = \ln(p_0)$. In all cases we have that $\lim_{t\to\infty} p(t) = e^{\theta}$. If the company has not invested until the election date investment at $t \geq T$ would create a secure discounted cash flows of

$$\bar{V}_\omega(t,p_0) = x(1-\omega+\omega\xi)\int_t^\tau e^{e^{-\kappa t}\ln(p_0)+\theta(1-e^{-\kappa t})}e^{-r(s-t)}ds. \tag{48}$$

This corresponds to a secure gain of

$$\bar{\pi}_\omega(t,p_0) = -I(t) + \bar{V}_\omega(t,p_0) \tag{49}$$

Hence, for a company that has not invested before T the optimal investment time $\hat{t}$ is defined by

$$\bar{\pi}_\omega(\hat{t},p_0)e^{-r(\hat{t}-T)} = \max_{t\in[T,\infty]} \bar{\pi}_\omega(t,p_0)e^{-r(t-T)} \tag{50}$$

Hereby, $\hat{t} = \infty$ means that it is optimal not to invest at any time. Thus, at time T the value of the option to invest equals

$$\bar{F}_\omega(T,p_0) = \bar{\pi}_\omega(\hat{t},p_0)e^{-r(\hat{t}-T)}. \tag{51}$$

If the company is using the Hurwicz-rule, has an optimism parameter λ and invests at a time $0 \leq t < T$ it assumes to get

$$\begin{aligned}\bar{\pi}_{H,\lambda}(t,p_0) = -I(t) + x\xi\int_t^T e^{e^{-\kappa t}\ln(p_0)+\theta(1-e^{-\kappa t})}e^{-r(s-t)}ds \\ + e^{-r(T-t)}\left(\lambda\bar{V}_{\omega_{opt}}(T,p_0) + (1-\lambda)\bar{V}_{\omega_{pes}}(T,p_0)\right).\end{aligned} \tag{52}$$

If it uses the Laplace-rule instead it assumes to get

$$\bar{\pi}_L(t,p_0) = -I(t) + x\xi \int_t^T e^{e^{-\kappa t}\ln(p_0)+\theta(1-e^{-\kappa t})} e^{-r(s-t)} ds + e^{-r(T-t)} \int_{\omega_{opt}}^{\omega_{pes}} \frac{1}{\omega_{pes}-\omega_{opt}} \bar{V}_z(T,p_0)dz. \quad (53)$$

Let $\check{t}_{H,\lambda}$ be defined by the equation

$$\bar{\pi}_\omega(\check{t}_{H,\lambda},p_0)e^{-r\check{t}_{H,\lambda}} = \max_{t\in[0,T]} \bar{\pi}_\omega(t,p_0)e^{-rt} \quad (54)$$

then according to the Hurwicz-rule the optimal investment time for a company with optimism parameter λ is

$$\bar{t}^*_{H,\lambda} = \begin{cases} \check{t}_{H,\lambda} & \bar{\pi}_\omega(\check{t}_{H,\lambda},p_0)e^{-r\check{t}_{H,\lambda}} \geq \left(\lambda \bar{F}_{\omega_{opt}}(T,p_0) + (1-\lambda)\bar{F}_{\omega_{pes}}(T,p_0)\right) e^{-rT} \\ \hat{t} & else \end{cases}. \quad (55)$$

Consequently, the value of the option to invest at time $t_0 = 0$ can be calculated by

$$\bar{F}_{H,\lambda}(0,p_0) = max\left\{\bar{\pi}_\omega(\check{t}_{H,\lambda},p_0)e^{-r\check{t}_{H,\lambda}}, \left(\lambda \bar{F}_{\omega_{opt}}(T,p_0) + (1-\lambda)\bar{F}_{\omega_{pes}}(T,p_0)\right) e^{-rT}\right\}. \quad (56)$$

Similarly, if $\check{t}_L$ is be defined by

$$\bar{\pi}_\omega(\check{t}_L,p_0)e^{-r\check{t}_L} = \max_{t\in[0,T]} \bar{\pi}_\omega(t,p_0)e^{-rt} \quad (57)$$

the optimal investment time according to the Laplace-rule is

$$\bar{t}^*_L = \begin{cases} \check{t}_L & \bar{\pi}_\omega(\check{t}_L,p_0)e^{-r\check{t}_L} \geq \left(\int_{\omega_{opt}}^{\omega_{pes}} \frac{1}{\omega_{pes}-\omega_{opt}} \bar{F}_z(T,p_0)dz\right) e^{-rT} \\ \hat{t} & else \end{cases}. \quad (58)$$

The option value of the option to invest at time $t_0 = 0$ equals

$$\bar{F}_\lambda(0,p_0) = max\left\{\bar{\pi}_\omega(\check{t}_\lambda,p_0)e^{-r\check{t}_\lambda}, \left(\int_{\omega_{opt}}^{\omega_{pes}} \frac{1}{\omega_{pes}-\omega_{opt}} \bar{F}_z(T,p_0)dz\right) e^{-rT}\right\}. \quad (59)$$

3. Comparative-static analysis

In this section the model is solved numerically. In particular, the partial differential equations are solved by means of explicit finite differences, whereby the transition density function $\psi_p(y, p_{t_1}, \mathrm{t}_2, t_1)$ of the price process p is obtained via a Monte-Carlo-simulation. If not stated otherwise we assume the following values: $r = 0.1; \tau = 10;\ \kappa = 0.5; \theta = \ln(1.5)\,;\ \sigma = 0.2; \eta = 0.5; \lambda = 0.5;\ T = 3;$ $\omega_{opt} = 0.75; \omega_{pes} = 1.25;\ \xi = 0.3;\ I_f = 175; I_v = 100; p_0 = 1.5; x = 100\,.$ In the absence of uncertainty or ambiguity, i.e. $\sigma = 0$ and $\omega_{opt} = \omega_{pes} = 1$, we get deterministically that $t^* = 0.44$ and $F(t^*) = 9.98$.

In the following we analyze the influence of economic uncertainty (subsection 3.1), political ambiguity (subsection 3.2) or a combination of both (subsection 3.3) on the value of the option to invest and on the optimal investment timing. Finally we discuss in subsection 3.4 the influence of some other parameters, i.e. the election date T, the speed η of the technical progress and the remaining life-time τ of the asset.

3.1 The sole influence of economic uncertainty

First, we will discuss the base case without a regime switch, i.e. $\omega = 1$. As can be seen in Figure 1 the optimal investment threshold $p^*(t)$ shows a U-shaped pattern. If the remaining life-time of the asset is quite long the optimal investment threshold is reducing over time while it increases over time if the life-time of the asset approaches its end. In particular, four different effects have an influence on $p^*(t)$: Under uncertainty the company has an incentive to wait with its

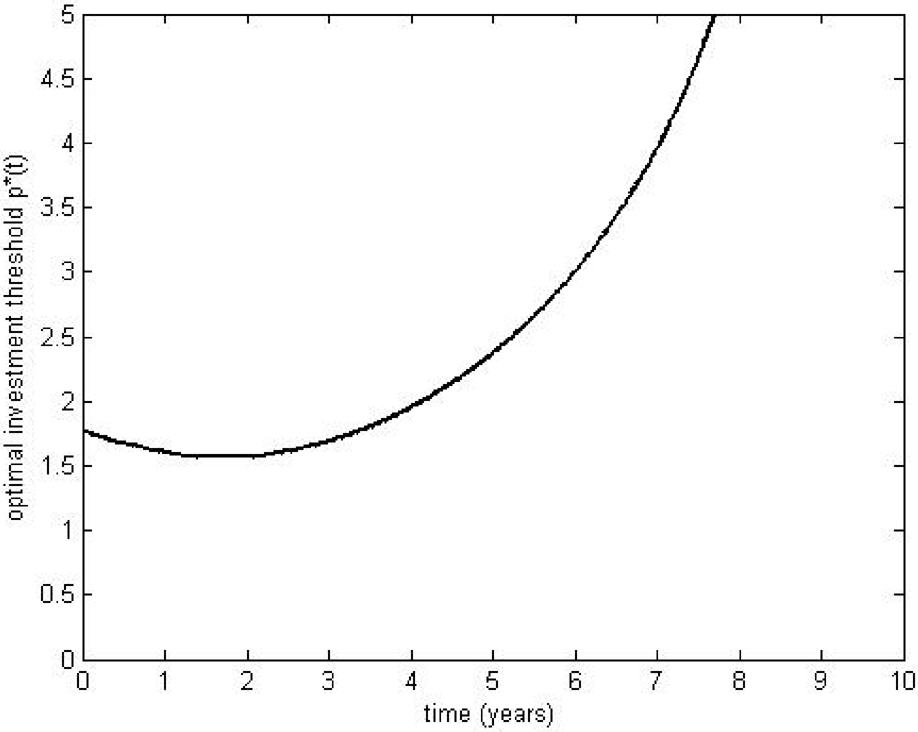

Figure 1: The optimal investment threshold in dependence of time.

investment. As the investment costs are sunk the possibility to wait with the investment while new information about the value of the future cash-flows of the investment possibility is becoming available has a value to the company. Contrarily, later investment also comes with later earned cash-flows and thereby with higher discounting. This effect is forcing the company to invest earlier. These two effects are well-known from the standard real options literature, but as long as they are time-independent they only have an influence on the level of $p^*(t)$ but not on its shape. The U-shape pattern is instead influenced by the effect of decreasing investment costs and the effect of a decreasing remaining life-time of the asset. Obviously, the company has an incentive to invest later if the investment costs decrease over time, i.e. the investment threshold $p^*(t)$ should be the higher the larger the reduction speed of these sunk costs (see also Kumbaroğlu et al., 2008). As the investment costs are assumed to decrease exponentially (see equation (7)) this effect is diminishing over time. The remaining life-time of the asset is shortening over time. Consequently, the company's future cash flows of the investment are decreasing and thus the investment opportunity is getting less worthwhile. Hence, the company is forced to invest earlier, i.e. the investment threshold increases over time, whereby this effect is the stronger the shorter the remaining life-time of the asset.

Table 1: The value of the option to invest at $\boldsymbol{t_0 = 0}$ in dependence of uncertainty.

$F(p_0, 0)$	$\kappa = 0.25$	$\kappa = 0.5$	$\kappa = 0.75$	$\kappa = 1$
$\sigma = 0.1$	12,60	10,93	10,54	10,41
$\sigma = 0.2$	19,72	14,03	12,22	11,50
$\sigma = 0.3$	28,66	18,26	14,66	13,10

In contrast to the classical real option models that use a geometric-Brownian motion (see e.g. McDonald and Siegel, 1986; Dixit and Pindyck, 1994; Trigeorgis, 1999) under mean-reversion economic uncertainty has two dimensions. On one hand uncertainty is the higher the higher the impact of the stochastic error term $W(t)$ on the other hand uncertainty is the lower the higher the mean-reversion effect. Consequently, uncertainty is increasing with σ and decreasing with κ. Real options theory generally postulates that higher uncertainty is increasing the value of the option to invest due to a higher flexibility value. As can be seen in Table 1 this result still holds in our mean-reversion setting. The option value is increasing in σ and decreasing in κ and hence always increasing in uncertainty.

Table 2: The optimal investment threshold at $t_0 = 0$ in dependence of uncertainty.

$p^*(0)$	$\kappa = 0.25$	$\kappa = 0.5$	$\kappa = 0.75$	$\kappa = 1$
$\sigma = 0.1$	1,65	1,63	1,63	1,63
$\sigma = 0.2$	1,82	1,78	1,74	1,73
$\sigma = 0.3$	1,98	1,92	1,88	1,85

Likewise, higher uncertainty leads to a higher investment threshold (see Table 2), regardless if it is caused by an increase in σ or by a decrease in κ.

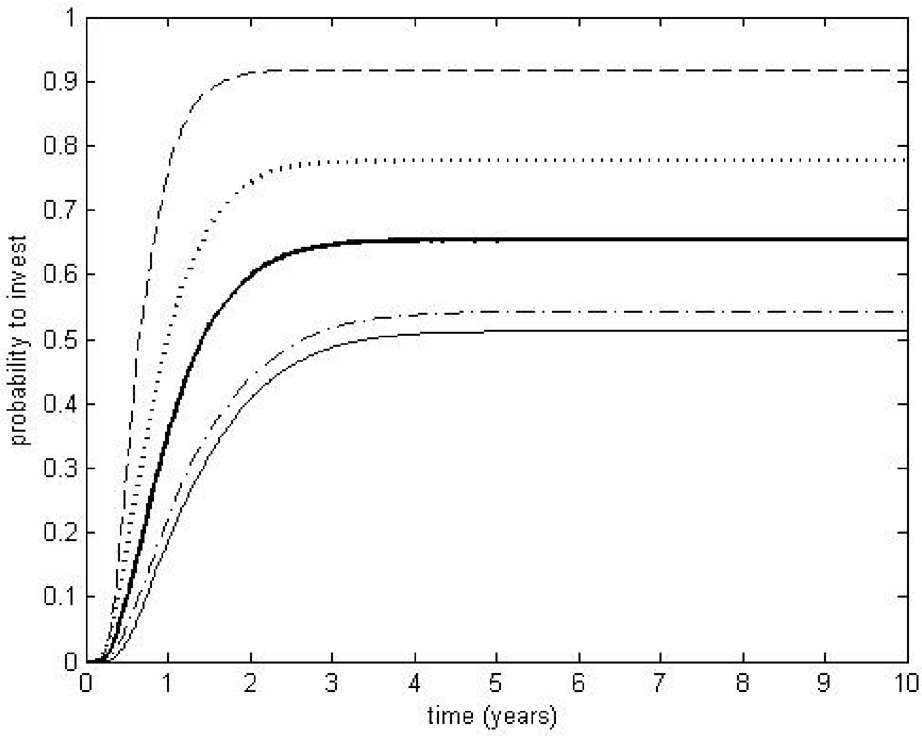

Figure 2: The probability to invest until a given time for varying economic uncertainty: Fat solid line: $\sigma = 0.2, \kappa = 0.5$; solid line: $\sigma = 0.2, \kappa = 0.25$; dash-dotted line: $\sigma = 0.3, \kappa = 0.5$; dotted line: $\sigma = 0.2, \kappa = 0.75$; dashed line: $\sigma = 0.1, \kappa = 0.5$.

However, it is noteworthy to state that a higher investment threshold does not always imply later investment if it is caused by a change in uncertainty, i.e. in σ or κ. In particular, σ and κ also have an influence on the probability that the price process $p(t)$ reaches a certain threshold in a given time. Thus, the influence of σ and κ on $\mathbb{P}(t^* \leq t)$ is generally equivocal (see also Sarkar, 2000; Lukas and Welling, 2013b). As can be seen in Figure 2 in our example higher uncertainty is always leading to a later investment regardless whether the higher uncertainty is caused by an increase in σ or by an decrease in κ. Furthermore, it can be seen that a higher uncertainty also lowers the probability that the investment takes place at any time, i.e. $\mathbb{P}(t^* \leq \tau)$ is decreasing with increasing uncertainty.

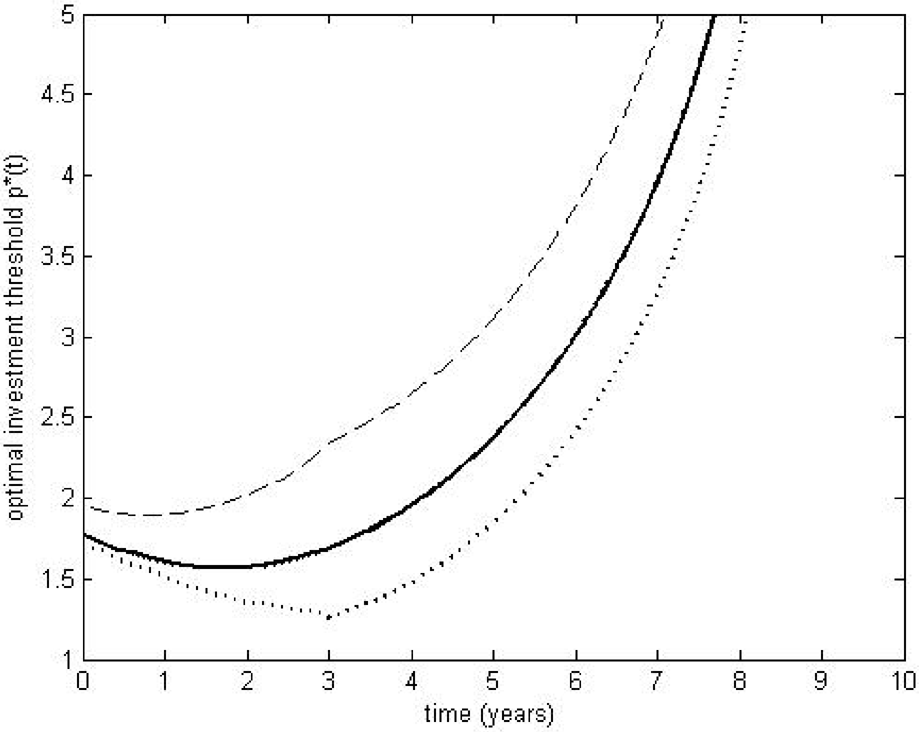

Figure 3: The optimal investment threshold in dependence of time for different values of ω: solid line: $\omega = 1$; dashed line: $\omega = 1.05$; dotted line: $\omega = 0.95$.

In absence of political ambiguity the amount of the regime switch is secure. A low value of ω implies higher future cash flows and therefore a higher option value and a greater incentive to invest. Consequently, a high value of ω implies a lower option value and a lower incentive to invest. For $\omega = 0.95$ we get $F_{0.95}(p_0, 0) = 32{,}4$ and for $\omega = 1.05$ we get $F_{0.95}(p_0, 0) = 2{,}23$. Figure 3 depicts the investment threshold in dependence of time for these values of ω and for the benchmark case $\omega = 1$. It can be seen that a higher value of ω indeed leads to a higher investment threshold and hence to later investment. Furthermore, Figure 3 depicts that the U-shape of the investment threshold sustains. However, though still continuous the investment threshold is no longer smooth in T because due to the decrease or increase of the cash flow waiting is differently expensive directly before and directly after the election date.

3.2 The sole influence of political ambiguity

In absence of economic uncertainty the optimal timing of the company is deterministic if it is optimal to invest before the election date T. Otherwise, it can only be stated that the investment takes place after the election date. In this case the exact investment time depends on the actual value of ω which the company does not to know until the election date. Table 3 depicts the optimal investment time in dependence of the degree of political ambiguity and the used decision rule.

Table 3: The optimal investment time in dependence of the degree of ambiguity and the used decision rule. Note: ω_Δ is increased by holding $\frac{\omega_{opt}+\omega_{pes}}{2} = 1$ constant.

λ / ω_Δ	0	0.2	0.4	0.45	0.5	0.55	0.6	0.8	1	Laplace
0	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
0.05	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
0.1	$\geq T$	$\geq T$	$\geq T$	0.44	0.44	0.44	0.44	0.44	0.44	0.44
0.15	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	0.44	0.44
0.2	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	0.44	$\geq T$
0.25	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	0.44	$\geq T$
0.3	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	0.44	$\geq T$
0.35	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	$\geq T$
0.4	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	$\geq T$
0.45	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	$\geq T$
0.5	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	$\geq T$	0.44	$\geq T$

It can be seen that it is optimal to wait with the investment until the ambiguity is resolved if the degree of ambiguity is high while for a low degree of ambiguity it is optimal to invest before the election date, precisely at time $t^* = 0.44$. Remarkably, in the latter case the optimal investment time neither depends on the used decision rule nor on the degree of ambiguity. This is due to the fact that if the company slightly delays the investment it forgoes a little bit of secure cash flows while it profits from decreasing investment costs.[7] Thus, this tradeoff is neither influenced by political ambiguity nor by the used decision rule. The decision whether to invest at all before the election date, however, depends on the degree

[7] Additionally, discounting plays a minor role.

of ambiguity as well as on the used decision rule. The higher the ambiguity-aversion of the company, i.e. the lower its optimism parameter, the lower are its assumed future cash flows after the election date. Hence, investment is becoming less worthwhile at all the higher the company's ambiguity-aversion. If the company decides not to invest before the election date it has the flexibility to invest only if the realized value of ω makes the investment worthwhile. If the result of the political process turns out to be unfavorable, i.e. ω is high, the company can omit the investment project and save the investment costs. Thus, even if investment would already be worthwhile before the election date it pays to wait with the investment if the flexibility value is higher than the tradeoff of forgone cash flows and the reduced investment costs. As the flexibility value is generally increasing with the degree of ambiguity a higher degree of ambiguity reduces the company's incentive to invest before the election date.

Table 4: The value of the option to invest at time $t_0 = 0$ in dependence of the degree of ambiguity and the used decision rule. Note: ω_Δ is increased by holding $\frac{\omega_{opt}+\omega_{pes}}{2} = 1$ constant.

λ / ω_Δ	0	0.2	0.4	0.45	0.5	0.55	0.6	0.8	1	Laplace
0	9.98	9.98	9.98	9.98	9.98	9.98	9.98	9.98	9.98	9.98
0.05	0.19	4.11	8.02	9.00	9.98	10.96	11.94	15.86	19.77	9.98
0.1	0	3.42	6.84	8.02	9.98	11.94	13.90	21.73	29.56	9.98
0.15	0	5.38	10.76	12.10	13.45	14.79	16.13	27.60	39.35	9.98
0.2	0	7.34	14.67	16.51	18.34	20.17	22.01	33.48	49.14	9.98
0.25	0	9.29	18.59	20.91	23.24	25.56	27.88	50.90	58.93	11.03
0.3	0	11.25	22.50	25.32	28.13	30.94	33.76	58.13	68.72	13.48
0.35	0	13.21	26.42	29.72	33.02	36.33	39.63	65.39	78.51	15.92
0.4	0	15.17	30.34	34.13	37.92	41.71	45.50	72.66	88.30	18.36
0.45	0	17.13	34.25	38.53	42.81	47.10	51.38	79.96	98.09	20.81
0.5	0	19.08	38.17	42.94	47.71	52.48	57.25	87.29	107.88	23.26

Table 4 depicts the influence of political ambiguity and the used decision rule on the option value at time $t_0 = 0$. Generally, it can be seen that the more ambiguity-averse the company the lower is its option value. Furthermore, it can be deduced that the option value of ambiguity-seeking companies increases with the degree of ambiguity. In contrast, the option value of ambiguity-averse companies has a U-shape. The option value is decreasing with ambiguity if ambiguity is so small that it is optimal to invest before the election date while the option value is increasing with ambiguity if ambiguity is so high that it is optimal not to invest until the election date. This is due to the fact that also ambiguity-averse companies benefit from the higher flexibility value. Only totally ambiguity-averse companies, i.e. $\lambda = 0$, never profit from this flexibility value. Hence, their option value is always decreasing with ambiguity. This corresponds to the results of Nishimura and Ozaki (2007). The option value of an ambiguity-neutral company and a company using the Laplace-rule do not change with the degree of ambiguity (and thus are equal) if it is optimal to invest before the election date. This is due to the fact that they evaluate the increasing risks and increasing chances equally. However, if it is optimal not to invest before the election date in both cases the option value increases with ambiguity. In this case the option value of a ambiguity-averse company using the Hurwicz-rule is higher than the option value of a company using the Laplace-rule because an ambiguity-averse company using the Hurwicz-rule only considers the extreme values ω_{opt} and ω_{pes} as possible and hence assumes a higher flexibility value than a company using the Laplace-rule. In this regard it is noteworthy to state that under a great degree of ambiguity even ambiguity-averse companies, for example $\lambda = 0.4$, that use the Hurwicz-rule assume a higher value of the option to invest than a company that uses the Laplace-rule.

3.3 The combined influence of economic uncertainty and political ambiguity

So far, we have separately analyzed the influence of economic uncertainty and political ambiguity. In this subsection we will analyze their combined effect on the value of the option to invest as well as on the timing of the investment. Regarding the option value we have seen that in absence of political ambiguity economic uncertainty is increasing the option value while in absence of economic uncertainty the influence of political ambiguity on the option value depends on the used decision rule or the degree of ambiguity-aversion, respectively.

Table 5: The value of the option to invest at time $t_0 = 0$ in dependence of the degree of ambiguity and the degree of economic uncertainty for various decision rules: upper left cell: $\lambda = 0.4$ (ambiguity-averse company); upper right cell: $\lambda = 0.6$ (ambiguity-seeking company); lower left cell: $\lambda = 0.5$ (ambiguity-neutral company); lower right cell: Laplace-rule. Note: ω_Δ is increased by holding $\frac{\omega_{opt}+\omega_{pes}}{2} = 1$ constant, economic uncertainty is increased by a change in σ.

	$\sigma = 0$		$\sigma = 0.1$		$\sigma = 0.2$		$\sigma = 0.3$	
$\omega_\Delta = 0$	9.98	9.98	10.93	10.93	14.03	14.03	18.26	18.26
	9.98	9.98	10.93	10.93	14.03	14.03	18.26	18.26
$\omega_\Delta = 0.1$	6.84	13.90	8.91	15.20	12.23	18.27	16.37	22.29
	11.94	9.98	11.88	11.10	15.16	14.35	19.27	18.59
$\omega_\Delta = 0.2$	14.67	22.01	14.66	22.62	15.56	25.09	18.09	28.42
	18.34	9.98	18.43	12.01	20.05	15.44	23.05	19.57
$\omega_\Delta = 0.3$	22.50	33.76	22.48	33.73	22.49	34.48	23.29	36.64
	28.13	13.48	28.10	14.50	28.29	17.46	29.67	21.26

Table 5 depicts the combined influence of economic uncertainty and political ambiguity for the different decision rules. It can be seen that higher economic uncertainty and less ambiguity-aversion are generally increasing the value of the option to invest into the project. In accordance to previous results the influence of political ambiguity on the value of the option to invest depends on the used decision rule, especially on the degree of ambiguity-aversion. In particular, if the company is ambiguity-neutral, ambiguity-seeking or uses the Laplace-rule a higher degree of political ambiguity increases the option value. If the company is ambiguity-averse for small degrees of ambiguity an increase in ambiguity is decreasing the option value while for higher degrees of ambiguity an increase in ambiguity is increasing the option value. Interestingly, economic uncertainty seems to mitigate the influence of political ambiguity on the option value. Furthermore it also mitigates the difference between the Hurwicz-rule for ambiguity-neutral companies and the Laplace-rule. Finally Table 5 depicts that under economic uncertainty for ambiguity-neutral companies and companies using the Laplace-rule the option value is no longer constant in ambiguity for low ambiguity-levels.

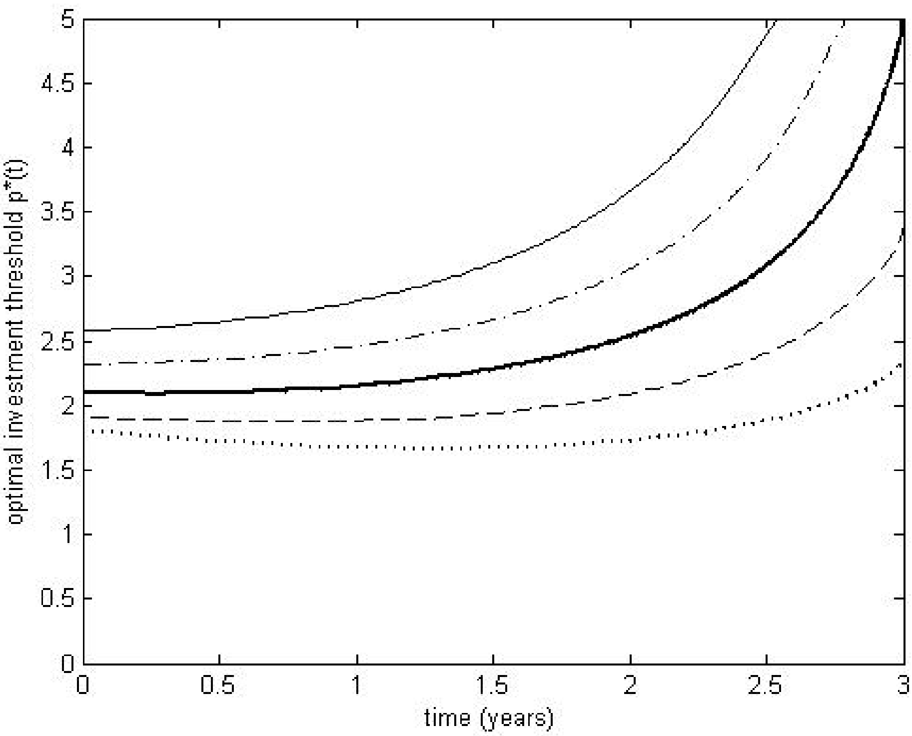

Figure 4: The optimal investment threshold in dependence of time for different degrees of ambiguity using the Laplace-rule: solid line: $\omega_\Delta = 0.5$; dash-dotted line: $\omega_\Delta = 0.4$; fat solid line: $\omega_\Delta = 0.3$; dashed line: $\omega_\Delta = 0.2$; dotted line: $\omega_\Delta = 0.1$. Note: ω_Δ is increased by holding $\frac{\omega_{opt}+\omega_{pes}}{2} = 1$ constant.

In the previous subsection we have seen that in absence of economic uncertainty the influence of the degree of political ambiguity on the investment timing is merely to determine whether investment at a certain time before the election date, i.e. at $t^* = 0.44$, or investment after the election date is preferable. Hereby, the optimal investment time before the election date does neither depend on the degree of political ambiguity nor on the used decision rule. As can be seen in Figures 4 and 5 this changes if economic uncertainty prevails.

In particular, Figure 4 depicts that under economic uncertainty increasing political ambiguity leads to a higher investment threshold and hence to later investment. This corresponds to the results of Nishimura and Ozaki (2007) but differs from Trojanowska and Kort (2010) who find the contrary relation. Furthermore, it can be seen that political ambiguity is reducing the U-shape of the investment threshold. This is due to the fact that political ambiguity is increasing the flexibility value and thus gives an incentive to wait with the investment until the election date. Obviously this incentive is the stronger the less time remains until the election date. If the sum of this effect and the effect of the decreasing remaining life-time of the asset is greater than the effect of the decreasing investment costs the U-shape of the investment threshold diminishes. As can be seen in Figure 4 this is the case for $\omega_\Delta \geq 0.4$.

Figure 5 depicts that the investment threshold is also increasing with the ambiguity-aversion of the company. Hence, an ambiguity-averse company will invest later than an ambiguity-seeking company or in other words an optimist will invest earlier than a pessimist. Furthermore, it can be seen that the U-shape of the investment threshold is more pronounced for higher values of λ while it diminishes for lower values of λ.

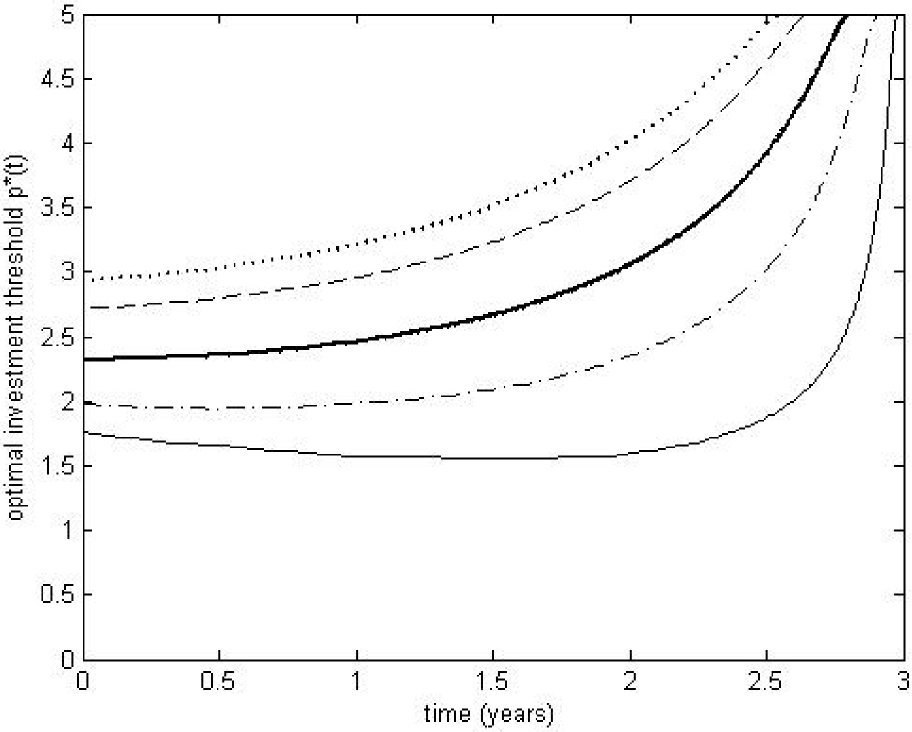

Figure 5: The optimal investment threshold in dependence of time for different values of the optimism parameter λ: dotted line: $\lambda = 0.2$; dashed line: $\lambda = 0.3$; fat solid line: $\lambda = 0.5$; dash-dotted line: $\lambda = 0.7$; solid line: $\lambda = 0.9$.

With the help of Figure 6 we can deduce some statements about the combined effect of economic uncertainty and political ambiguity on the investment threshold. Firstly, reflecting previous results we see that the investment threshold is the higher the higher the economic uncertainty and the higher the political ambiguity. Specifically, investment before the election date means to give up the flexibility value which originates from both economic uncertainty and political ambiguity. Secondly, we see that if the remaining time until the election date is only short the effect of the political ambiguity dominates the effect of economic uncertainty, i.e. curves depicting the same degree of political ambiguity converge as the time draws nearer to the election date. In particular, shortly before the

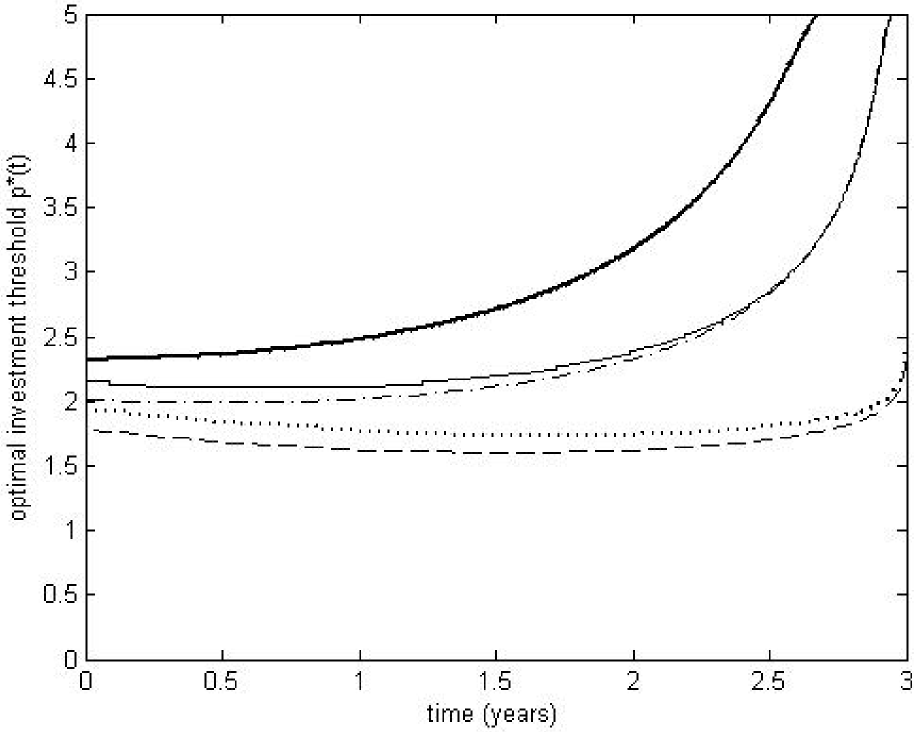

Figure 6: The optimal investment threshold in dependence of time for different degrees of uncertainty and ambiguity: fat solid line: $\sigma = 0.2$, $\omega_\Delta = 0.5$, Hurwicz-rule with $\lambda = 0.5$; solid line: $\sigma = 0.3$, $\omega_\Delta = 0.5$, Laplace-rule; dash-dotted line: $\sigma = 0.2$, $\omega_\Delta = 0.5$, Laplace-rule; dotted line: $\sigma = 0.3$, $\omega_\Delta = 0.1$, Laplace-rule; dashed line: $\sigma = 0.2$, $\omega_\Delta = 0.1$, Laplace-rule.

election date only few cash flows have to be given up in exchange for the possibility to invest with full information of ω. This also explains why the investment threshold is rising rapidly shortly before the election date. Obviously, this new information is the more valuable the higher ω_Δ, hence curves depicting the same degree of political ambiguity converge. Thirdly, the opposite holds if the remaining time until the election date is quite long. In this case the effect of uncertainty dominates the effect of political ambiguity, i.e. curves depicting the same degree of economic uncertainty converge as the remaining time until the election date is growing. If the remaining time until the election date is long the effect of political ambiguity described above becomes less important. If the time until the election date would approach infinity the effect would totally diminish.

Hence, the known effect of economic uncertainty becomes evident. Fourthly, we see that the U-shape pattern of the investment threshold depends on both economic uncertainty and political ambiguity. In particular, it gets the stronger the higher the economic uncertainty and the lower the political ambiguity. For high values of economic uncertainty ($\sigma = 0.3$) the U-shape exists regardless whether $\omega_\Delta = 0.1$ or $\omega_\Delta = 0.5$. Likewise, the U-shape also exists for a low degree of political ambiguity ($\omega_\Delta = 0.1$) regardless whether $\sigma = 0.2$ or $\sigma = 0.3$. However, if the economic uncertainty is low ($\sigma = 0.2$) and the political ambiguity is high ($\omega_\Delta = 0.5$) the optimal investment threshold is monotonically increasing over time. Finally, we can deduce from Figure 6 that the decision rule also has an important influence on the investment threshold. For an ambiguity-averse company the Laplace-rule suggests a much lower investment threshold than the Hurwicz-rule. Again this difference can be explained with the higher flexibility value of a company using the Hurwicz-rule.

As can be seen in Figure 7 the influence of economic uncertainty and political ambiguity on the investment threshold cannot simply be transferred to their influence on the probability to invest before a given time. In particular, we have seen that in absence of economic uncertainty investment at time $t^* = 0.44$ is optimal for a low degree of political ambiguity, i.e. $\omega_\Delta = 0.1$, while it is optimal to wait with the investment until the political ambiguity is resolved at the election date for a higher degree of political ambiguity, i.e. $\omega_\Delta = 0.5$. Hence, for $\omega_\Delta = 0.5$ increasing economic uncertainty can only increase the probability that the investment occurs before a given time $t \leq T$. In contrast, for $\omega_\Delta = 0.1$ increasing

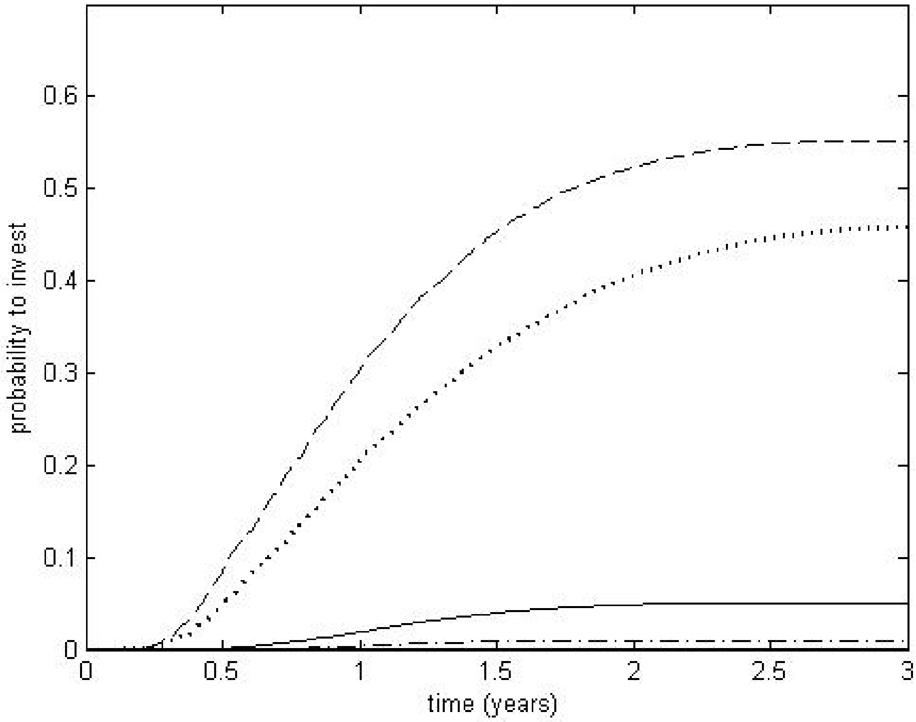

Figure 7: The probability to invest until a given time for different degrees of uncertainty and ambiguity: fat solid line: $\sigma = 0.2$, $\omega_\Delta = 0.5$, Hurwicz-rule with $\lambda = 0.5$; solid line: $\sigma = 0.3$, $\omega_\Delta = 0.5$, Laplace-rule; dash-dotted line: $\sigma = 0.2$, $\omega_\Delta = 0.5$, Laplace-rule; dotted line: $\sigma = 0.3$, $\omega_\Delta = 0.1$, Laplace-rule; dashed line: $\sigma = 0.2$, $\omega_\Delta = 0.1$, Laplace-rule.

uncertainty inevitably on one hand increases the probability that the investment occurs before a time $t < 0.44$ and on the other hand reduces the probability that the investment occurs before a time $t > 0.44$. Thus, we cannot generally state whether economic uncertainty accelerates investment. Rather it depends on the various model parameters, especially on the degree of political ambiguity. Here our result differs from Nishimura and Ozaki (2007) who always find that investment is postponed under higher uncertainty. Furthermore, we can see in Figure 7 that investments generally are getting less probable the shorter the remaining time until the election date. This result corresponds to the rapidly increasing investment thresholds observed in Figure 6. Finally, Figure 7 depicts that the actual investment timing also depends on the decision-rule used by the investing company. We can see that under political ambiguity a ambiguity-neutral company using the Hurwicz-rule ceteris paribus invests later than a company using the Laplace-rule.

3.4 The influence of various other parameters

The previous analysis of the investment decisions is governed by the underlying economic variables in the model. In this subsection we will modify these parameters to analyze their effect on the optimal investment threshold and on the option value. As can be seen in Figure 8 the threshold is influenced differently depending on which parameter change is considered. A lower life-time of the asset τ increases the investment threshold as the overall possible future cash flows decrease. Notably, the effect is so pronounced that U-shaped pattern diminishes even though the effect of the political ambiguity decreases. The decrease of the possible future cash flows and the increase in the optimal investment threshold additionally cause the value of the option to decline to $F_{\tau=8}(p_0, 0) = 0.08$ from the standard case of $F(p_0, 0) = 20.05$. Likewise, we can see that the decrease in θ which corresponds to lower expected future cash flows leads to an increase of the optimal investment threshold. In this case the required fuel price is higher at every point in time to make the investment profitable. The value of the option also decreases to $F_{\theta=\ln(1.4)}(p_0, 0) = 12.35$. Contrarily, the later the election time T the lower the investment threshold. In this case the effect of the political ambiguity decreases because the time during which cash flows can be earned after the election decreases. Furthermore, the sum of cash flows the company would have to forego to wait for the election increases and thus the option value decreases to $F_{T=4}(p_0, 0) = 16.07$. The effect of the increase of the factor η which with the investment costs decrease on the investment threshold is twofold. If the remaining asset life is high the investment threshold increases as it is profitable to wait for the reduction of the investment cost. However, for lower asset life times the optimal investment thresholds decreases because investments are lower and their absolute change is lower which leads to lower incentive to hold the investment. Furthermore, the option value increases to $F_{\eta=0.25}(p_0, 0) = 21.15$.

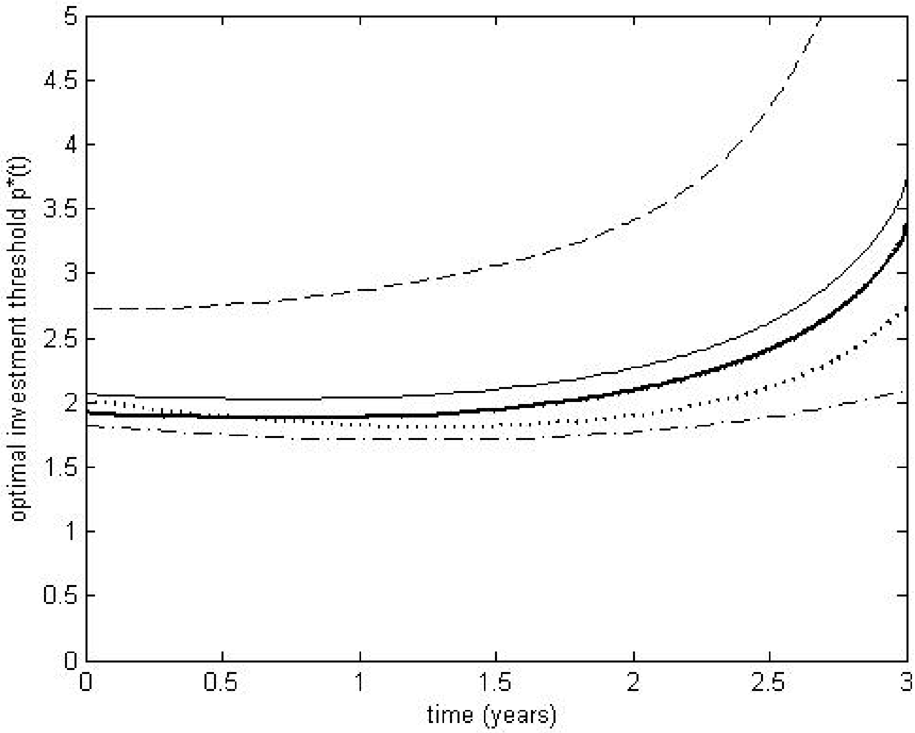

Figure 8: The optimal investment threshold in dependence of time with $\omega_\Delta = 0.2$ and $\lambda = 0.5$ for varying parameters changes: dashed line: $\tau = 8$; solid line: $\theta = \ln(1.4)$; fat solid line: standard case; dotted line: $\eta = 0.25$; dash-dotted line: $T = 4$.

4. Conclusion

In our paper we set up a model to analyze the option value and optimal timing of green investments under economic uncertainty and political ambiguity. Particularly, we consider an investment in the usage of biologic fuel whose price benefit follows a mean-reverting process and depends on election outcomes.

In accordance to real option theory higher economic uncertainty leads to higher option value. However, political ambiguity has no general effect on the option value but depends on the decision rule and degree of risk aversion of the investor. Generally, the option value is higher for investors with less ambiguity-aversion. However, if the investor is ambiguity-averse high levels of political ambiguity increase the value of the investment opportunity while low levels decrease the option value. Economic uncertainty seems to mitigate the influence of political ambiguity on option value.

The optimal investment threshold generally increases with economic uncertainty and political ambiguity while its shape depends on the combination of both. Under economic uncertainty and low political ambiguity the threshold has a U-shaped pattern which is more pronounced under higher economic uncertainty and vanishes with increasing political ambiguity. Furthermore, the effect of the political ambiguity is greater the shorter the remaining time until election. Contrarily, if the time until the election is long its effect on the optimal investment threshold decreases. As the time to election approaches infinity the effect of political ambiguity diminishes and the known effect of economic uncertainty prevails.

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