SUBSIDIES FOR RENEWABLE ENERGY FACILITIES UNDER UNCERTAINTY

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Abstract

We derive the optimal investment timing and real option value for a renewable energy facility with price and quantity uncertainty, where there might be a government subsidy proportional to the quantity of production. We also consider the possibility that the subsidy is retracted sometime subsequent to the investment. The easiest case is where the subsidy is proportional to the multiplication of the joint products (price and quantity), so the dimensionality can be reduced. Then quasi-analytical solutions are provided for different subsidy arrangements: a permanent subsidy proportional to the quantity of production; a retractable subsidy; a sudden permanent subsidy; and finally a sudden retractable subsidy. Policy is considered certain only in the first case of a permanent constant subsidy. Whether policy uncertainty acts as a disincentive for early investment, and thereby offsets the advantages of any subsidy, depends on the type of subsidy arrangement.

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1 Introduction

Do permanent or retractable government subsidies such as direct payments per unit revenue or per quantity produced, or specified feed-in-tariffs, or a renewable energy certificate or freedom from taxation, encourage early investment in renewable energy facilities? Does the size of the possible government subsidy reduce the price threshold that justifies investment significantly, when both unit prices and the units of production are stochastic, if the subsidy might be retracted?

The issue of the effect of government subsidies or charges on investment timing, when output prices are stochastic, is the original consideration in the first real option model of Tourinho (1979). Tourinho poses the dilemma that without a holding cost being imposed on the owner of an option to extract natural resources, the owner would never have a sufficient incentive to commit an irreversible investment to produce the resource. Other incentives to encourage early investment are the imposition (or presence) of an escalating investment cost, or as in Adkins and Paxson (2011b) the existence of a convenience (or similar) yield for future prices of the underlying resource.

There are numerous examples of government subsidies provided to encourage early investments in renewable energy, see Wohlgemuth and Madlener (2000), Menanteau et al. (2003), Blyth et al. (2009), Kettunen et al.(2011), Borenstein (2012), and Lapan and Moschini (2012). There are several authors who examine the macro-economic effects of uncertainty in the costs and benefits of taxing pollution, or subsidizing pollution reduction, and also the likely impact on the production and consumption of electricity, see Bajona and Kelly (2012). Several authors have studied separate price and quantity uncertainty, and correlation in environmental problems, see Stavins (1996). Pindyck (2012) notes that Pindyck (2002) and Pindyck (2007) use single factor real option models in addressing similar problems in environmental economics. But there appear to be no models solving the investment equation for separate price and quantity uncertainty, or for subsidies on one factor rather than the other.

Wind farms in Spain and Portugal have received different types of government subsidies including specified feed-in tariffs and investment tax credits. In January 2012, some of these subsidies were retracted in Spain. The Troika second review for Portugal in November 2011 raised the issue of retracting similar subsidies, see EU (2011)[1]. Domestically produced ethanol received both a direct subsidy in the US, benefitted from a tariff on imported ethanol, and also from EPA requirements regarding minimum quantities of ethanol in the gasoline mix. Finally, governments in both Norway and Sweden have considered various types of subsidies for hydro-facilities, see Linnerud et al. (2011).

We use a Poisson (jump) process to model sudden provision or permanent or alternatively retractable subsidies. Several authors have incorporated jump processes into real investment theory. Dixit and Pindyck (1994) discuss Poisson jump processes, and apply upward jumps to the expected capital gain from the possible implementation of an investment tax credit. Brach and Paxson (2003) consider Merton-style jumps in accounting for gene discovery and drug

[1] EU (2011) required Portugal to "review in a report the efficiency of support schemes for renewable(s), covering their rational, their levels, and other design elements [January 2012]…For existing contracts, assess in a report the possibility of agreeing renegotiating of the contracts in view of a lower feed-in tariff [Q4-2011]…For new contracts in renewable(s), revise downward the feed-in-tariffs. [Q3-2012]", p. 118.

development failures and successes. Martzoukos (2003) models exogenous learning as random information arrival of rare events (jumps resulting from technological, competitive, regulatory or political risk shocks) that follow a Poisson process.

We consider that the instantaneous cash flow from a facility is the respective commodity price of the output times the quantity produced, and either there is no operating cost, or there is a fixed operating cost that can be incorporated into the investment cost. There are no other options embedded in the facility such as expansion, contraction, suspension or abandonment. Further assumptions are that the lifetime of the facility is infinite and there are no taxes. Moreover, the typical assumptions of real options theory apply, with drifts, interest rates, convenience yields, volatilities and correlation constant over time. Many of these strong assumptions may be required for an analytical solution. Relaxation of some of these assumptions may lead to greater realism, but may then require much more complex analytical solutions or numerical solutions with possibly less transparency.

The next section considers some characteristic subsidies for such facilities, first where the subsidy is proportional to price times quantity, which is solved by simply scaling P*Q (Model I); then assuming there is a permanent subsidy proportional to the quantity generated (Model II); then assuming there is a retractable subsidy proportional to the quantity generated (Model III); then assuming there is the possibility of a permanent subsidy proportional to Q (Model IV); and finally assuming there is the possibility of a retractable subsidy proportional to Q (Model V). The third section compares the price thresholds and real option values using comparable base parameter values, and illustrates the sensitivity of these models to changes in some important variables such as quantity volatility, price and quantity correlation, the subsidy rate, and the intensities of possible sudden permanent or retractable subsidies. The final section concludes.

2 Models

2.1 Model I Stochastic Price and Quantity

We consider a perpetual opportunity to construct a renewable energy facility, such as a hydro-electric plant or a wind farm or another renewable energy process, at a fixed investment cost K. This investment cost is treated as irreversible or irrecoverable once incurred. The value of this investment opportunity, denoted by F_1, depends on the amount of electricity sold per unit of time, denoted by Q, and the price per unit of electricity, denoted by P. Both of these variables are assumed to be stochastic and are assumed to follow geometric Brownian motion processes:

$$\mathrm{d}X = \alpha_X X\mathrm{d}t + \sigma_X X\mathrm{d}Z \tag{1}$$

for $X \in \{P, Q\}$, where α denotes the instantaneous drift parameter, σ the instantaneous volatility, and $\mathrm{d}Z$ the standard Wiener process. Potential correlation between the two variables is represented by ρ. It may be reasonable to assume the price per unit of electricity follows such a stochastic process if it is a traded commodity, while treating the amount of electricity generated per unit of time as stochastic may reflect the random nature of demand or supply.

Assuming risk neutrality and applying Ito's lemma, the partial differential equation (PDE) representing the value to invest is:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_1}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_1}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_1}{\partial P \partial Q} + \theta_P P \frac{\partial F_1}{\partial P} + \theta_Q Q \frac{\partial F_1}{\partial Q} - rF_1 = 0. \tag{2}$$

where θ_X denote the risk-neutral drift rates and r the risk-free rate, (θ=r-α). Following McDonald and Siegel (1986) and Adkins and Paxson (2011a), the solution to (2) is:

$$F_1 = A_1 P^{\beta_1} Q^{\eta_1} . \tag{3}$$

β_1 and η_1 are the power parameters for this option value function. Since there is an incentive to invest when both P and Q are sufficiently high but a disincentive when either are sufficiently low, we would expect both power parameter values to be positive. Also, the parameters are linked through the characteristic root equation found by substituting (3) in (2):

$$Q(\beta_1, \eta_1) = \tfrac{1}{2}\sigma_P^2 \beta_1 (\beta_1 - 1) + \tfrac{1}{2}\sigma_Q^2 \eta_1 (\eta_1 - 1) + \rho\sigma_P\sigma_Q\beta_1\eta_1 + \theta_P\beta_1 + \theta_Q\eta_1 - r = 0 . \tag{4}$$

We assume that there is no operational flexibility once the investment to construct the plant has been made. After the investment, the plant generates revenue equaling (1+τ)* PQ, where τ is the permanent subsidy proportional to the electricity revenue sold (τ=0 indicates no possible subsidy). So from (2), the valuation relationship for the operational state is:

$$\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_1}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_1}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_1}{\partial P \partial Q} + \theta_P P \frac{\partial F_1}{\partial P} + \theta_Q Q \frac{\partial F_1}{\partial Q} + (1+\tau)PQ - rF_1 = 0 , \tag{5}$$

where we ignore the operating cost, which is assumed to be mainly fixed and treated as a constant[2]. The solution to (5) is:

$$\frac{(1+\tau)PQ}{r - \mu_{PQ}} ,$$

where $\mu_{PQ} = \rho\sigma_P\sigma_Q + \theta_P + \theta_Q$, see Paxson and Pinto (2005). The investment is made when the two variables attain their respective thresholds. If we denote the threshold levels for P and Q by

[2] Fixed costs that are constant can be absorbed within the investment cost.

$\hat{P}_1$ and $\hat{Q}_1$, respectively, and since value conservation requires the investment option value to be exactly balanced by the net value rendered by the investment, then the value matching relationship is specified by:

$$A\hat{P}_1^{\beta_1}\hat{Q}_1^{\eta_1} = \frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}} - K\,. \tag{6}$$

Optimality is characterized by the two smooth pasting conditions associated with (6) for P and Q, respectively:

$$\beta_1 A\hat{P}_1^{\beta_1}\hat{Q}_1^{\eta_1} = \frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}}\,, \tag{7}$$

$$\eta_1 A\hat{P}_1^{\beta_1}\hat{Q}_1^{\eta_1} = \frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}}\,. \tag{8}$$

From (7) and (8), our conjecture that the parameter values are positive is corroborated because of the non-negativity of the investment option value. Moreover, the parameters are equal, $\beta_1 = \eta_1$. This establishes that for determining the optimal investment policy, the two factors can be simply represented by their product PQ, the revenue from generating electricity per unit of time. This substitution is originally proposed by Paxson and Pinto (2005), who apply the principle of similarity for reducing the dimension of (5) to one in order to obtain a closed-form solution. It follows that:

$$\frac{(1+\tau)\hat{P}_1\hat{Q}_1}{r-\mu_{PQ}} = \frac{\beta_1}{\beta_1 - 1}K\,, \tag{9}$$

where β_1 is determined from $Q\left(\beta_1, \beta_1\right) = 0$, (4). Also

$$F_1 = \begin{cases} A_1 P_1^{\beta_1} Q_1^{\eta_1} & \text{for } PQ < \hat{P}_1 \hat{Q}_1, \\ \dfrac{(1+\tau) P_1 Q_1}{r - \mu_{PQ}} - K & \text{for } PQ \geq \hat{P}_1 \hat{Q}_1. \end{cases} \tag{10}$$

with:

$$A_1 = \frac{(1+\tau) \hat{P}_1^{1-\beta_1} \hat{Q}_1^{1-\eta_1}}{\eta_1 (r - \mu_{PQ})}.$$

2.2 Model II

Stochastic Price and Quantity with a Permanent Subsidy on Quantity

We now modify the analysis to consider the impact on the investment decision of a government subsidy, denoted by τ, whose value is proportional to the amount of electricity Q sold per unit of time. In the Appendix we also show the equivalent model and results based on revenue, for this and the next Q based model. In the presence of the subsidy, the generating plant is effectively producing two distinct outputs: (i) the revenue per unit of time generated by the plant PQ, and (ii) the subsidy revenue received from the government or electricity customers τQ. As before, the investment option value denoted by F_2 depends on the two factors P and Q. The risk neutral valuation relationship for F_2 takes a similar form as (2), so the valuation function is given by (3) except for the change in subscript, that is $F_2 = A_2 P^{\beta_2} Q^{\eta_2}$. Also, its characteristic root equation is $Q(\beta_2, \eta_2) = 0$, (4).

In the absence of any flexibility after incurring the investment, the present value of the operating revenue for the plant is:

$$\frac{PQ}{r-\mu_{PQ}}+\frac{\tau Q}{r-\theta_Q}.$$

The operating revenue is the present value of the operating revenue plus the government subsidy. If the two threshold levels signaling optimal investment are denoted by $\hat{P}_2$ and $\hat{Q}_2$ for P and Q, respectively, then the value matching relationship for this subsidized production model is:

$$A_2\hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}=\frac{\hat{P}_2\hat{Q}_2}{r-\mu_{PQ}}+\frac{\tau\hat{Q}_2}{r-\theta_Q}-K\,. \tag{11}$$

It is observable from (11) that the principle of similarity is no longer available, since the factors P and Q occurring in the relationship cannot be construed as a product PQ, even if $\beta_2=\eta_2$. The two smooth pasting conditions associated with (11) are:

$$\beta_2A_2\hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}=\frac{\hat{P}_2\hat{Q}_2}{r-\mu_{PQ}}, \tag{12}$$

$$\eta_2A_2\hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}=\frac{\hat{P}_2\hat{Q}_2}{r-\mu_{PQ}}+\frac{\tau\hat{Q}_2}{r-\theta_Q}. \tag{13}$$

These conditions, (12) and (13), reveal that both β_2 and η_2 are positive, otherwise the option value at investment $A_2\hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}$ would be negative. Moreover, by simplifying we have:

$$\frac{\hat{P}_2}{r-\mu_{PQ}}=\frac{\beta_2}{\eta_2-\beta_2}\frac{\tau}{r-\theta_Q},$$

which establishes that η_2 exceeds β_2 provided the subsidy rate τ is positive. We obtain reduced form value matching relationships by substituting (12) and (13) in (11), respectively:

$$\frac{\hat{P}_2\hat{Q}_2}{r-\mu_{PQ}}=\frac{\beta_2}{\beta_2-1}\left(K-\frac{\tau\hat{Q}_2}{r-\theta_Q}\right), \tag{14}$$

$$\frac{\hat{P}_2\hat{Q}_2}{r-\mu_{PQ}}=\frac{\eta_2}{\eta_2-1}K-\frac{\tau\hat{Q}_2}{r-\theta_Q}. \tag{15}$$

In these reduced forms, the government subsidy effectively reduces the investment cost of the plant with the economic consequence that the optimal revenue threshold justifying the investment is lower than without it.

The investment threshold that signals the amount of electricity sold per unit of time Q and the price per unit of electricity P economically justifying an optimal investment is specified by (i) and (ii) the two reduced form value matching relationships, (14) and (15), and (iii) the characteristic root equation $Q(\beta_2,\eta_2)=0$, (4). In principle, the boundary relationship is obtainable by eliminating β_2 and η_2 from the three constituent equations, but as no purely analytical solution exists, we resort to obtaining the boundary numerically.

2.3 Model III

Stochastic Price and Quantity with a Retractable Subsidy on Quantity

Subsidies are normally offered by governments in order to induce entrepreneurs to accelerate the timing of their investment in facilities, when otherwise they would defer making their commitment. As soon as the subsidy has activated sufficient plant investment, the government may decide to withdraw the subsidy, often without any advance warning. We now explore the financial consequences on the investment decision for a subsidy that can be withdrawn at any time and to determine its effects on the threshold levels for P and Q. We assume that once the subsidy is withdrawn, it will never again be provided.

We denote the value of the investment option in the presence of a subsidy, but when there is a possibility of an immediate withdrawal, by F_3, and in the absence of a subsidy by F_1, as before. We assume that the subsidy withdrawal is well explained by a Poisson process with a constant intensity factor, denoted by λ. The change in the option value conditional on the subsidy withdrawal occurring is $F_1(P,Q)-F_3(P,Q)$, so the expected change is given by:

$$\{F_1(P,Q)-F_3(P,Q)\}\lambda \mathrm{d}t+\{0\}(1-\lambda \mathrm{d}t)=\lambda\{F_1(P,Q)-F_3(P,Q)\}\mathrm{d}t\,.$$

From (2), it follows that the risk-neutral valuation relationship for F_3 is:

$$\begin{aligned}\frac{1}{2}\sigma_P^2P^2\frac{\partial^2F_3}{\partial P^2}+\frac{1}{2}\sigma_Q^2Q^2\frac{\partial^2F_3}{\partial Q^2}+PQ\rho\sigma_P\sigma_Q\frac{\partial^2F_3}{\partial P\partial Q}\\ +\theta_PP\frac{\partial F_3}{\partial P}+\theta_QQ\frac{\partial F_3}{\partial Q}+\lambda F_1-(r+\lambda)F_3=0.\end{aligned} \tag{16}$$

The solution to (16) adopts the form:

$$F_3=A_3P^{\beta_3}Q^{\eta_3}+A_1P^{\beta_1}Q^{\eta_1}\,, \tag{17}$$

where the parameters β_1 and η_1 are specified by $Q(\beta_1,\eta_1)=0$, (4), with $\beta_1=\eta_1$(with τ=0), while β_3 and η_3 are related through the characteristic root equation:

$$\begin{aligned}Q_3(\beta_3,\eta_3)=\tfrac{1}{2}\sigma_P^2\beta_3(\beta_3-1)+\tfrac{1}{2}\sigma_Q^2\eta_3(\eta_3-1)+\rho\sigma_P\sigma_Q\beta_3\eta_3\\ +\theta_P\beta_3+\theta_Q\eta_3-(r+\lambda)=0.\end{aligned} \tag{18}$$

For any feasible values of P and Q, the valuation function F_3 exceeds F_1 because the coefficient A_3 is positive. This implies that the option value to invest is always greater in the presence of a government subsidy that may be withdrawn unexpectedly than in its absence, which suggests that a subsidy, even one having an unexpected withdrawal, comparatively hastens the investment commitment, while it is comparatively deferred in its absence.

If the subsidy is present, then the present value of the plant is $PQ/\left(r-\mu_{PQ}\right)+\tau Q/\left(r-\theta_Q\right)$, and if absent, then $PQ/\left(r-\mu_{PQ}\right)$, so the net present value following the investment commitment is:

$$\frac{PQ}{r-\mu_{PQ}}+\frac{(1-\lambda)\tau Q}{r-\theta_Q}.$$

The thresholds signaling investment for a subsidy with unexpected withdrawal are denoted by $\hat{P}_3$ and $\hat{Q}_3$ for P and Q, respectively. The value matching condition becomes:

$$A_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3}+A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1}=\frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}}+\frac{(1-\lambda)\tau\hat{Q}_3}{r-\theta_Q}-K. \tag{19}$$

The two associated smooth pasting conditions are, respectively:

$$\beta_3A_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3}+\beta_1A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1}=\frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}}, \tag{20}$$

$$\eta_3A_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3}+\eta_1A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1}=\frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}}+\frac{(1-\lambda)\tau\hat{Q}_3}{r-\theta_Q}. \tag{21}$$

The parameter values A_1, β_1 and η_1 are known from the solution to Model I with τ=0.

$$A_3=(-\beta_1A_1\hat{P}_3^{\beta_1}\hat{Q}_3^{\eta_1}+\frac{\hat{P}_3\hat{Q}_3}{r-\mu_{PQ}})/(\beta_3\hat{P}_3^{\beta_3}\hat{Q}_3^{\eta_3})$$

2.4 Model IV Stochastic Joint Products with Sudden Provision of a Permanent Subsidy on Quantities

We now explore the financial consequences on the investment decision for a subsidy that can be provided permanently at any time and to determine its effects on the threshold levels for P and Q. We consider only the case where the subsidy thereafter can never be withdrawn, and compare the case of building the facility without a possible subsidy with the cases of a permanent subsidy.

Since a sudden unexpected subsidy withdrawal makes an operating plant appear to be less economically attractive, it is likely that investment is hastened to capture the subsidy before it is withdrawn. In contrast, a sudden unexpected permanent subsidy introduction is expected to produce the opposite effect of investment deferral so that the subsidy income can be more fully captured.

In Model II, the revenue threshold that signals an economically justified investment in the presence of a subsidy is $\hat{R}_2 = \hat{P}_2\hat{Q}_2$. Before the investment is made, the threshold $\hat{R}_2$ creates either side separate domains over which the investment option value differs in form. The prevailing revenue is denoted by $R = PQ$. If the prevailing revenue R is less than the threshold $\hat{R}_2$, then a sudden unexpected subsidy announcement does not trigger an immediate investment and the investment is deferred until R attains $\hat{R}_2$. If, on the other hand, $R \geq \hat{R}_2$, then a sudden unexpected subsidy announcement automatically triggers an immediate investment in plant. This asymmetry around the threshold $\hat{R}_2$ means that the investigation of a sudden unexpected subsidy announcement has to treat the case where $R < \hat{R}_2$ differently from where $R \geq \hat{R}_2$.

The value for the investment option, denoted by F_4, is specified over the two domains:

$$F_4 = \begin{cases} F_{40} & \text{for } R < \hat{R}_2, \\ F_{41} & \text{for } R \geq \hat{R}_2. \end{cases} \tag{22}$$

We first consider the domain $R < \hat{R}_2$, which is considered to be out-of-the money because over this domain, investment in the presence of a subsidy is not economically justified. It is assumed that a subsidy introduction is well described by a Poisson process with intensity λ, and that once introduced, it cannot be withdrawn. The risk neutral valuation relationship then becomes:

$$\begin{aligned} &\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_{40}}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_{40}}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_{40}}{\partial P \partial Q} \\ &+ \theta_P P \frac{\partial F_{40}}{\partial P} + \theta_Q Q \frac{\partial F_{40}}{\partial Q} + \lambda F_2 - (r + \lambda) F_{40} = 0. \end{aligned} \tag{23}$$

The solution to (23) adopts the form:

$$F_{40} = A_{40} P^{\beta_{40}} Q^{\eta_{40}} + A_2 P^{\beta_2} Q^{\eta_2} \tag{24}$$

where the parameters β_2 and η_2 are specified by $Q(\beta_2, \eta_2) = 0$, (4), and β_{40} and η_{40} by $Q_3(\beta_{40}, \eta_{40}) = 0$, (18).

If there is no subsidy, then the present value of the plant is given by $PQ/(r - \mu_{PQ})$, while if there is an additional subsidy, then the present value is $PQ/(r - \mu_{PQ}) + \tau Q/(r - \theta_Q)$. The net present value for the investment is given by:

$$\frac{PQ}{r-\mu_{PQ}}+\frac{\lambda\tau Q}{r-\theta_Q}-K\ .$$

The thresholds signaling investment for a sudden unexpected subsidy introduction are denoted by $\hat{P}_{40}$ and $\hat{Q}_{40}$ for P and Q, respectively. The value matching condition becomes:

$$A_{40}\hat{P}_{40}^{\beta_{40}}\hat{Q}_{40}^{\eta_{40}}+A_2\hat{P}_{40}^{\beta_2}\hat{Q}_{40}^{\eta_2}=\frac{\hat{P}_{40}\hat{Q}_{40}}{r-\mu_{PQ}}+\frac{\lambda\tau\hat{Q}_{40}}{r-\theta_Q}-K\ . \tag{25}$$

The two associated smooth pasting conditions can be expressed as, respectively:

$$\beta_{40}A_{40}\hat{P}_{40}^{\beta_{40}}\hat{Q}_{40}^{\eta_{40}}+\beta_2A_2\hat{P}_{40}^{\beta_2}\hat{Q}_{40}^{\eta_2}=\frac{\hat{P}_{40}\hat{Q}_{40}}{r-\mu_{PQ}}, \tag{26}$$

$$\eta_{40}A_{40}\hat{P}_{40}^{\beta_{40}}\hat{Q}_{40}^{\eta_{40}}+\eta_2A_2\hat{P}_{40}^{\beta_2}\hat{Q}_{40}^{\eta_2}=\frac{\hat{P}_{40}\hat{Q}_{40}}{r-\mu_{PQ}}+\frac{\lambda\tau\hat{Q}_{40}}{r-\theta_Q}. \tag{27}$$

We now consider the domain $R\geq\hat{R}_2$, where investment is justified if the subsidy is introduced. The risk neutral valuation relationship for this domain is:

$$\begin{aligned}&\frac{1}{2}\sigma_P^2P^2\frac{\partial^2F_{41}}{\partial P^2}+\frac{1}{2}\sigma_Q^2Q^2\frac{\partial^2F_{41}}{\partial Q^2}+PQ\rho\sigma_P\sigma_Q\frac{\partial^2F_{41}}{\partial P\partial Q}\\&\quad+\theta_PP\frac{\partial F_{41}}{\partial P}+\theta_QQ\frac{\partial F_{41}}{\partial Q}+\lambda\hat{F}_2-\left(r+\lambda\right)F_{41}=0.\end{aligned} \tag{28}$$

When an unexpected subsidy is announced for $R\geq\hat{R}_2$, the option valuation function instantaneously changes from F_{41} into $\hat{F}_2=A_2\hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}$, which denotes the threshold option value for committing an investment in the presence of a subsidy. The solution to (28) is:

$$F_{41}=A_{41}P^{\beta_{41}}Q^{\eta_{41}}+\frac{\lambda}{r+\lambda}A_2\hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}, \tag{29}$$

where the parameters β_2 and η_2 are specified by $Q(\beta_2, \eta_2) = 0$, (4), and β_{41} and η_{41} by $Q_3(\beta_{41}, \eta_{41}) = 0$, (18).

The thresholds signaling investment for a sudden unexpected subsidy introduction are denoted by $\hat{P}_{41}$ and $\hat{Q}_{41}$ for P and Q, respectively. The value matching condition becomes:

$$A_{41}\hat{P}_{41}^{\beta_{41}}\hat{Q}_{41}^{\eta_{41}} + \frac{\lambda}{r+\lambda} A_2 \hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2} = \frac{\hat{P}_{41}\hat{Q}_{41}}{r-\mu_{PQ}} + \frac{\lambda\tau\hat{Q}_{41}}{r-\theta_Q} - K . \tag{30}$$

The two associated smooth pasting conditions can be expressed as, respectively:

$$\beta_{41}A_{41}\hat{P}_{41}^{\beta_{41}}\hat{Q}_{41}^{\eta_{41}} = \frac{\hat{P}_{41}\hat{Q}_{41}}{r-\mu_{PQ}}, \tag{31}$$

$$\eta_{41}A_{41}\hat{P}_{41}^{\beta_{41}}\hat{Q}_{41}^{\eta_{41}} = \frac{\hat{P}_{41}\hat{Q}_{41}}{r-\mu_{PQ}} + \frac{\lambda\tau\hat{Q}_{41}}{r-\theta_Q} . \tag{32}$$

The reduced form value matching relationships are obtained by substituting (31) and (32) in (30), respectively, to give:

$$\frac{\hat{P}_{41}\hat{Q}_{41}}{r-\mu_{PQ}} = \frac{\beta_{41}}{\beta_{41}-1}\left[K + \frac{\lambda}{r+\lambda} A_2 \hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2} - \frac{\lambda\tau\hat{Q}_{41}}{r-\theta_Q}\right], \tag{33}$$

$$\frac{\hat{P}_{41}\hat{Q}_{41}}{r-\mu_{PQ}} = \frac{\eta_{41}}{\eta_{41}-1}\left[K + \frac{\lambda}{r+\lambda} A_2 \hat{P}_2^{\beta_2}\hat{Q}_2^{\eta_2}\right] - \frac{\lambda\tau\hat{Q}_{41}}{r-\theta_Q} . \tag{34}$$

It is observed from (33) and (34) that the effect of an unexpected sudden subsidy introduction is to effectively raise the investment cost, by an amount equaling the option value for an economically justified investment in the presence of a subsidy, adjusted by the Poisson intensity parameter λ. For $\lambda = 0$, the solution simplifies to the case of no subsidy. As λ becomes increasingly large, the investment cost is raised by the amount equaling the option value.

2.5 Model V Stochastic Joint Products with Sudden Provision of a Retractable Subsidy

Finally, we consider the case where a government suddenly provides a retractable subsidy, but only for those facilities built after the announcement of the subsidy provision. Since a sudden unexpected subsidy withdrawal makes an operating plant appear to be less economically attractive, there is the incentive to capture the subsidy before it is withdrawn, but also the incentive to wait until the retractable subsidy is available.

In Model III, the revenue threshold that signals an economically justified investment in the presence of a retractable subsidy is $\hat{R}_3 = \hat{P}_3 \hat{Q}_3$. Before the investment is made, the threshold $\hat{R}_3$ creates either side separate domains over which the investment option value differs in form. The prevailing revenue is denoted by $R = PQ$. If the prevailing revenue R is less than the threshold $\hat{R}_3$, then a sudden unexpected subsidy announcement does not trigger an immediate investment and the investment is deferred until R attains $\hat{R}_3$. If, on the other hand, $R \geq \hat{R}_3$, then a sudden unexpected subsidy announcement automatically triggers an immediate investment in the plant. This asymmetry around the threshold $\hat{R}_3$ means that the investigation of a sudden unexpected subsidy announcement has to treat the case where $R < \hat{R}_3$ differently from where $R \geq \hat{R}_3$.

The value for the investment option, denoted by F_5, is specified over the two domains:

$$F_5 = \begin{cases} F_{50} & \text{for } R < \hat{R}_3, \\ F_{51} & \text{for } R \geq \hat{R}_3. \end{cases} \tag{35}$$

We first consider the domain $R < \hat{R}_3$, which is considered to be out-of-the money because over this domain, investment in the presence of a retractable subsidy is not economically justified. It is assumed that a subsidy introduction is well described by a Poisson process with intensity λ, and that once introduced, it is retractable. The risk neutral valuation relationship then becomes:

$$\begin{aligned} &\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_{50}}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_{50}}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_{50}}{\partial P \partial Q} \\ &\quad + \theta_P P \frac{\partial F_{50}}{\partial P} + \theta_Q Q \frac{\partial F_{50}}{\partial Q} + \lambda F_3 - (r+\lambda) F_{50} = 0. \end{aligned} \tag{36}$$

The solution to (36) adopts the form:

$$F_{50} = A_{50} P^{\beta_{50}} Q^{\eta_{50}} + A_3 P^{\beta_3} Q^{\eta_3} \tag{37}$$

where the parameters β_3 and η_3 are specified by $Q(\beta_3, \eta_3) = 0$, (18), and β_{50} and η_{50} by $Q_3(\beta_{50}, \eta_{50}) = 0$, (18).

The thresholds signaling investment for a sudden unexpected subsidy introduction are denoted by $\hat{P}_{50}$ and $\hat{Q}_{50}$ for P and Q, respectively. The value matching condition becomes:

$$A_{50} \hat{P}_{50}^{\beta_{50}} \hat{Q}_{50}^{\eta_{50}} + A_3 \hat{P}_{50}^{\beta_3} \hat{Q}_{50}^{\eta_3} = \frac{\hat{P}_{50}\hat{Q}_{50}}{r - \mu_{PQ}} + \frac{\lambda \tau \hat{Q}_{50}}{r - \theta_Q} - K . \tag{38}$$

The two associated smooth pasting conditions can be expressed as, respectively:

$$\beta_{50} A_{50} \hat{P}_{50}^{\beta_{50}} \hat{Q}_{50}^{\eta_{50}} + \beta_3 A_3 \hat{P}_{50}^{\beta_3} \hat{Q}_{50}^{\eta_3} = \frac{\hat{P}_{50}\hat{Q}_{50}}{r - \mu_{PQ}}, \tag{39}$$

$$\eta_{50} A_{50} \hat{P}_{50}^{\beta_{50}} \hat{Q}_{50}^{\eta_{50}} + \eta_3 A_3 \hat{P}_{50}^{\beta_3} \hat{Q}_{50}^{\eta_3} = \frac{\hat{P}_{50}\hat{Q}_{50}}{r - \mu_{PQ}} + \frac{\lambda \tau \hat{Q}_{50}}{r - \theta_Q}. \tag{40}$$

We now consider the domain $R \geq \hat{R}_3$, where investment is justified if the retractable subsidy is introduced. The risk neutral valuation relationship for this domain is:

$$\begin{aligned} &\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_{51}}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_{51}}{\partial Q^2} + PQ\rho\sigma_P\sigma_Q \frac{\partial^2 F_{51}}{\partial P \partial Q} \\ &+ \theta_P P \frac{\partial F_{51}}{\partial P} + \theta_Q Q \frac{\partial F_{51}}{\partial Q} + \lambda \hat{F}_3 - (r+\lambda) F_{51} = 0. \end{aligned} \tag{41}$$

When an unexpected subsidy is announced for $R \geq \hat{R}_3$, the option valuation function instantaneously changes from F_{51} into $\hat{F}_3 = A_3 \hat{P}_3^{\beta_3} \hat{Q}_3^{\eta_3}$, which denotes the threshold option value for committing an investment in the presence of a subsidy. The solution to (41) is:

$$F_{51} = A_{51} P^{\beta_{51}} Q^{\eta_{51}} + \frac{\lambda}{r+\lambda} A_3 \hat{P}_3^{\beta_3} \hat{Q}_3^{\eta_3}, \tag{42}$$

where the parameters β_3 and η_3 are specified by $Q(\beta_3, \eta_3) = 0$, (18), and β_{51} and η_{51} by $Q_3(\beta_{51}, \eta_{51}) = 0$, (18).

The thresholds signaling investment for a sudden unexpected withdrawal subsidy introduction are denoted by $\hat{P}_{51}$ and $\hat{Q}_{51}$ for P and Q, respectively. The value matching condition becomes:

$$A_{51} \hat{P}_{51}^{\beta_{51}} \hat{Q}_{51}^{\eta_{51}} + \frac{\lambda}{r+\lambda} A_3 \hat{P}_3^{\beta_3} \hat{Q}_3^{\eta_3} = \frac{\hat{P}_{51}\hat{Q}_{51}}{r-\mu_{PQ}} + \frac{\lambda\tau\hat{Q}_{51}}{r-\theta_Q} - K . \tag{43}$$

The two associated smooth pasting conditions can be expressed as, respectively:

$$\beta_{51} A_{51} \hat{P}_{51}^{\beta_{51}} \hat{Q}_{51}^{\eta_{51}} = \frac{\hat{P}_{51}\hat{Q}_{51}}{r-\mu_{PQ}}, \tag{44}$$

$$\eta_{51} A_{51} \hat{P}_{51}^{\beta_{51}} \hat{Q}_{51}^{\eta_{51}} = \frac{\hat{P}_{51}\hat{Q}_{51}}{r-\mu_{PQ}} + \frac{\lambda\tau\hat{Q}_{51}}{r-\theta_Q} . \tag{45}$$

There are several additional subsidy arrangements which could be modeled similarly such as proportional subsidies on P only, permanent, retractable, and a suddenly introduced permanent or retractable subsidy. Also there are combinations of P subsidies and separate Q subsidies, and some arrangements such as investment credits which reduce effective K, which are also amendable to quasi-analytical solutions[3]. Possibly these approaches can be utilized to model the consequences of tax (price) versus trading (quantities) in environmental abatement policies, see Pezzey and Jotzo (2012). Other arrangements such as guaranteed minimum prices for certain quantities, or guaranteed purchases for certain quantities at certain times, and finite facilities, may not be amenable to quasi-analytical solutions.

3. Numerical Illustrations

It is interesting to compare the apparent effectiveness of different subsidy arrangements, and the possible sudden introduction or retraction of those subsidies on the real option value of those investment opportunities, and the price and quantity thresholds that justify commencing investments. Pairs of $\hat{P}$ and $\hat{C}$ could be generated by changing the solutions along a suitable Q range.

Since Model I $\hat{P}$ (τ=.20) is less than Model I $\hat{P}$ (τ=0), clearly a permanent subsidy makes a difference, with a 20% R subsidy reducing the price threshold by 16.6%, and increasing the ROV some 60%.

[3] Fisher and Newell (2004) show that a subsidy per unit output equal to the price of a green certificate is equivalent to $\tau=s/(1-\alpha)$ where s is the equilibrium value of the green certificate and α is the required proportion of fossil fuel generation that must be purchased. But if s is stochastic, then so is τ. Lesser and Su (2008) review several feed-in-tariff designs, noting that some US regulators have established gradually increasing annual minimum proportions of renewable energy that must be purchased or generated over time, but in Germany direct subsidies for renewable generation decrease over time.

For a comparable subsidy (at the price threshold) on the quantity generated, Model II, the permanent subsidy reduces the price threshold even more, and adds more than 16% to the ROV. R is more uncertain (34.6%) than Q due to the assumed volatilities and negative correlation.

Table I

Subsidy Incentive Effect under Different Models							
	τ	**P^Q^**	**P^ (Q^=7.8)**	**ROV**			
Model I	0.00	638.70	81.88	1022.72	NO SUBSIDY		
Model I	0.20	532.25	68.24	1631.49	PERMANENT SUBSIDY ON R		
Model II	13.65	486.07	62.32	1903.76	PERMANENT SUBSIDY ON Q		
Model III	13.65	461.72	59.19	1717.11	RETRACTABLE SUBSIDY ON Q		
Model IV0	13.65	718.97	92.18	1325.66	MAYBE PERMANENT SUBSIDY ON Q, R<R2^		
Model IV1	13.65	697.11	89.37	1913.12	MAYBE PERMANENT SUBSIDY ON Q, R>R2^		
Model V0	13.65	461.72	59.19	941.95	MAYBE RETRACTABLE SUBSIDY ON Q, R<R3^		
Model V1	13.65	584.77	74.97	1127.56	MAYBE RETRACTABLE SUBSIDY ON Q, R>R3^		
Q^=Q			7.80				
P			53.00				
R			413.4				
R Subsidy			106.45		Subsidy Value at R^		M I
Q Subsidy			106.47		Subsidy Value at P^Q^		M II

Model I is the solution to EQs 6-7-8 with ROV EQ 10, Model II is the solution to EQs 11-12-13 with ROV the LHS of EQ 11, Model III is the solution to EQs 19-20-21 with ROV EQ 17, Model IV is the solution to EQs 25-26-27 or 30-31-32 with ROV EQ 24 or 29, Model V is the solution to EQs 38-39-40 or 43-44-45 and ROV EQ 37 or 42, with the parameter values as follows: price P=€53, quantity Q=7.8 KWh, R subsidy τ=.20, Q subsidy 13.65, investment cost K=€4867 [4], price volatility σ_P=.20, quantity volatility σ_Q=.20, price and quantity correlation ρ=-.50, θ_P=.01, θ_Q=.01, and riskless interest rate r=.08. λ=.10 reflects the possibility of a subsidy being withdrawn, and both the possibility of a permanent subsidy and also a retractable subsidy.

[4] The P, Q and K parameter values are consistent with an Iberian wind farm. The subsidy rate .20 for R in Model I is comparable with the Q=13.65 subsidy in Model II at the P,Q which justifies exercise of the real option.

If a subsidy can be withdrawn, Model III versus Model II, the $\hat{P}$ decreases but ROV also decreases. Commence the project when the subsidy is available earlier if it might be withdrawn. A higher retractable λ results in $\hat{P}$ increasing and ROV decreasing, as shown in Figures 4 and 8 below.

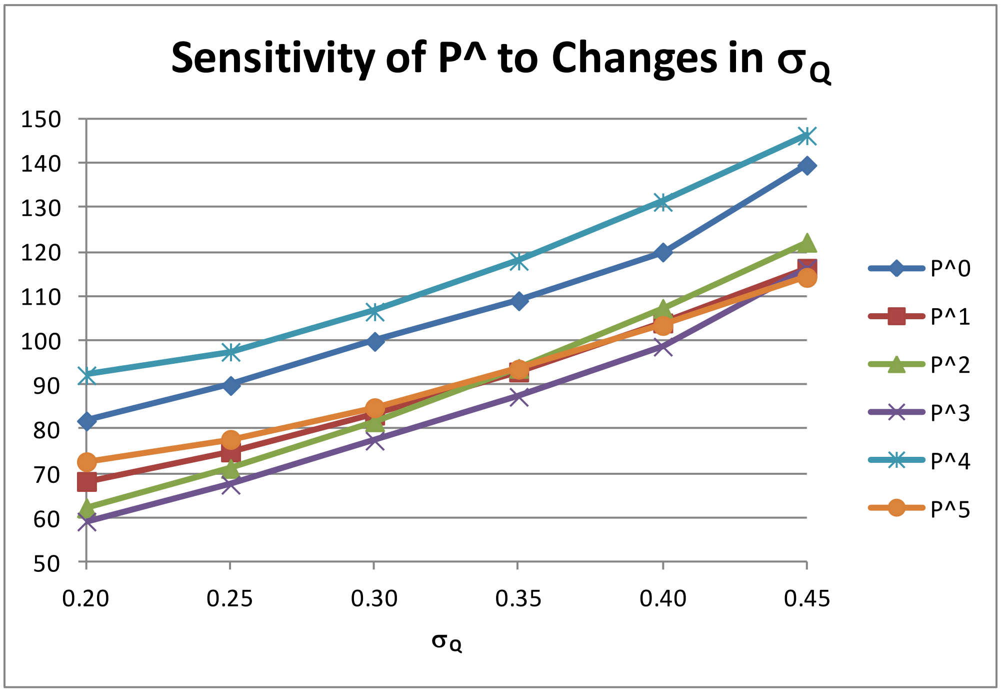
Comparing the out-of-the money Model IV0 with the out-of-the-money Model II, the P_{IV} price threshold exceeds P_{II}, naturally because a bird in the hand is worth more than the same bird in a bush (talk is cheap), and the ROV is lower. But for the in-the-money Model IV1, the $\hat{P}_{IV1}$ is lower than the Model IV0 and so is the ROV.

For sudden subsidies that might be withdrawn, if the current price is out-of-the money, $\hat{P}_{V0}$ is about the same as the $\hat{P}_{III}$, and the ROV is lower, naturally. If the current price is in-the-money, the $\hat{P}_{V1}$ is lower than for Model IV1, and the ROV is lower. So at these parameter values, either a possible or actual retractable subsidy is likely to encourage early investment. Sudden possible retractable subsidies are less valuable than possible permanent subsidies, as retractable subsidies are less valuable than permanent subsidies.

SENSITIVITIES

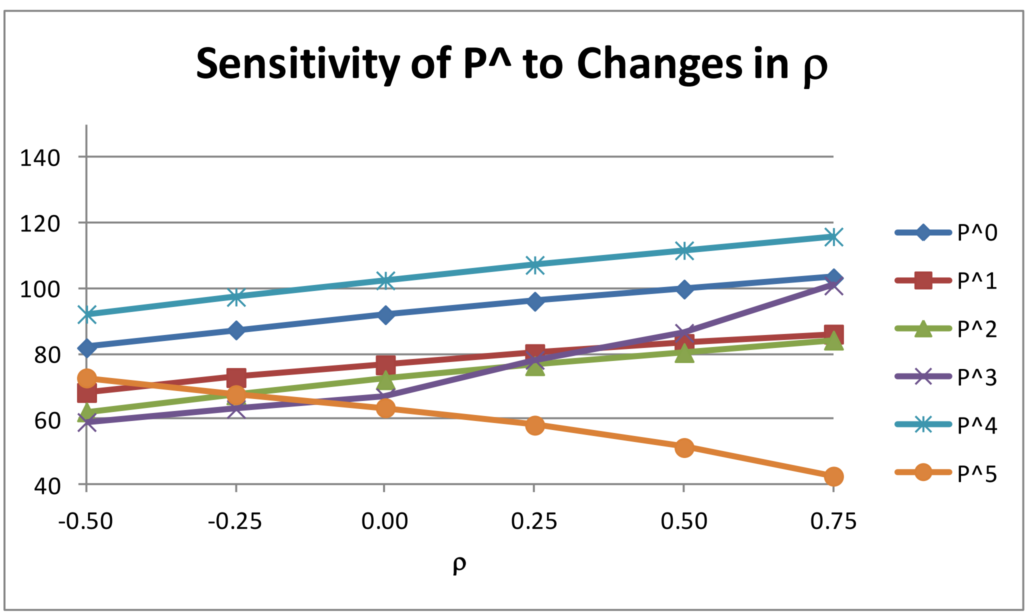
Our base parameters for the sensitivity of $\hat{P}$ and ROV to changes in parameter values are the same as for Table I, over a range of Q volatility 20% to 45%, correlation of P and Q from -.50 to .75, τ from .20 to .45 (and the comparables for Q), and λ from .10 to .225, both for retractable, and for possible permanent and possible retractable subsidies..

Figure 1



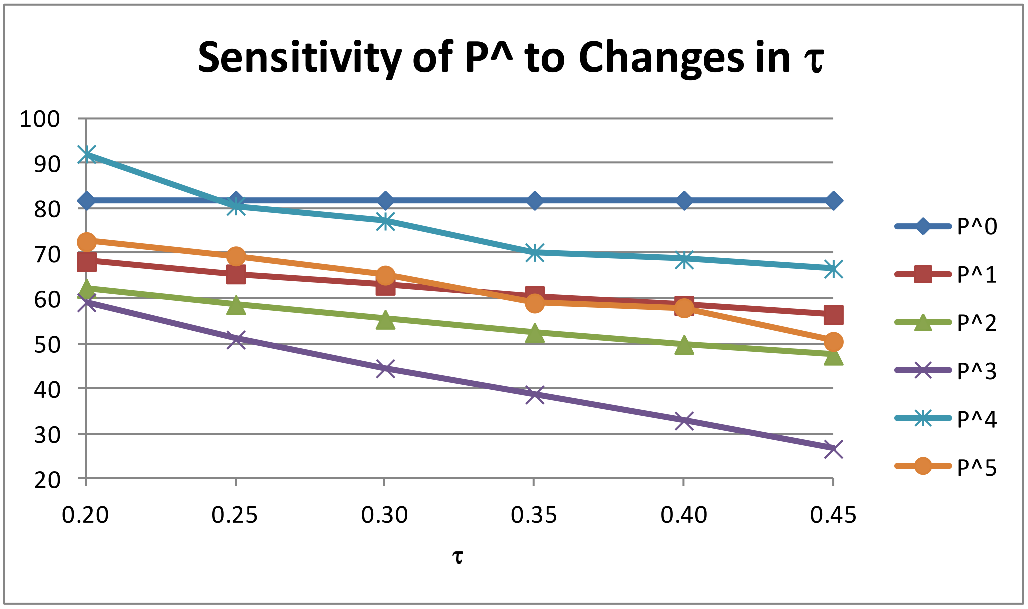
$\hat{P}_0$ is the solution to EQs 6-7-8 without a subsidy, and $\hat{P}_1$ with a subsidy, $\hat{P}_2$ is the solution to EQs 11-12-13, $\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 or 30-31-32, $\hat{P}_5$ is the solution to EQs 38-39-40 or 43-44-45 with the parameter values in Table I.

Figure 2



$\hat{P}_0$ is the solution to EQs 6-7-8 without a subsidy, and $\hat{P}_1$ with a subsidy, $\hat{P}_2$ is the solution to EQs 11-12-13, $\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 or 30-31-32, $\hat{P}_5$ is the solution to EQs 38-39-40 or 43-44-45 with the parameter values in Table I.

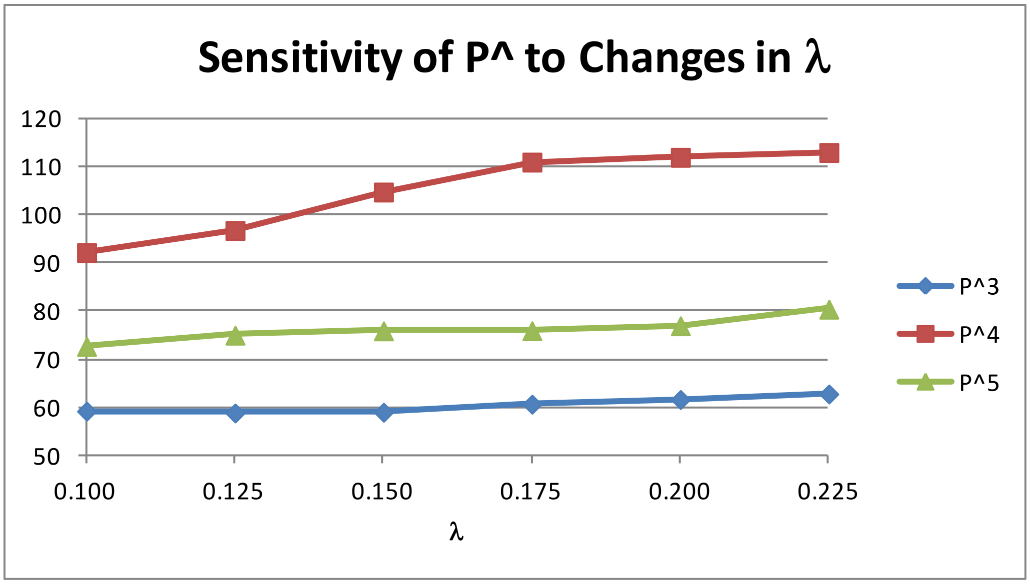
Figure 3



$\hat{P}_0$ is the solution to EQs 6-7-8 without a subsidy, and $\hat{P}_1$ with a subsidy, $\hat{P}_2$ is the solution to EQs 11-12-13, $\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 or 30-31-32, $\hat{P}_5$ is the solution to EQs 38-39-40 or 43-44-45 with the parameter values in Table I.

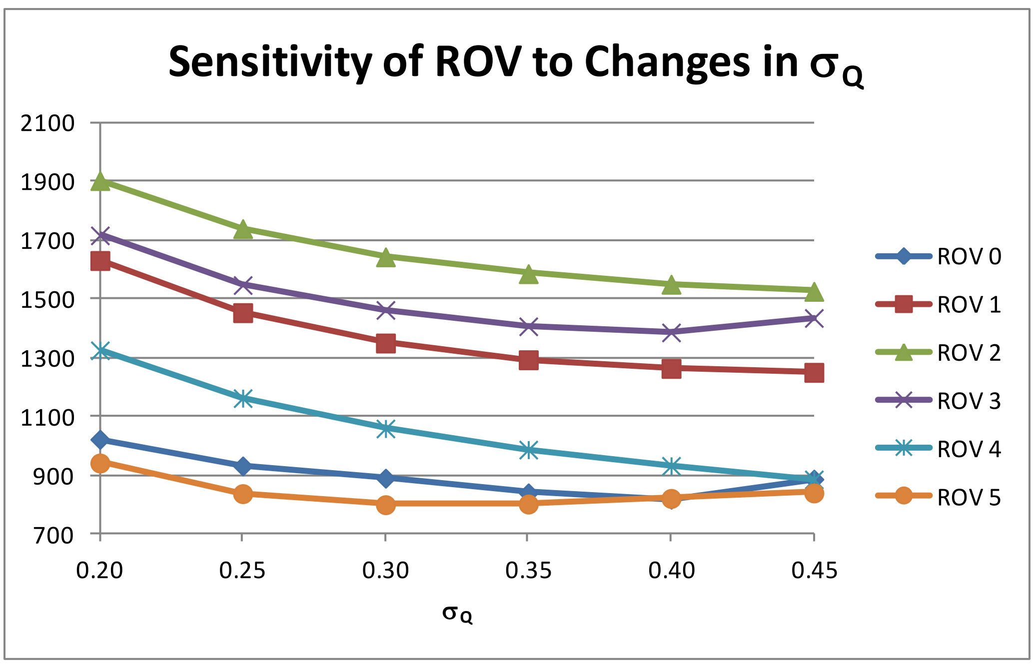
Price thresholds for all models increase with increases with quantity volatility, and decrease with the size of the subsidy. So either production volume floors or high subsidies of almost any type might encourage investment. Sensitivity to increases in the correlation and to possible retraction or introduction of subsidies is sometimes ambiguous. The λ for Model III ranges from .10 to .225, but the retractable Model III λ used for Model V is always .10, that is there is a .10 intensity of retraction, when the possibility of a retractable subsidy being introduced suddenly has an intensity ranging from .10 to .225.

Figure 4



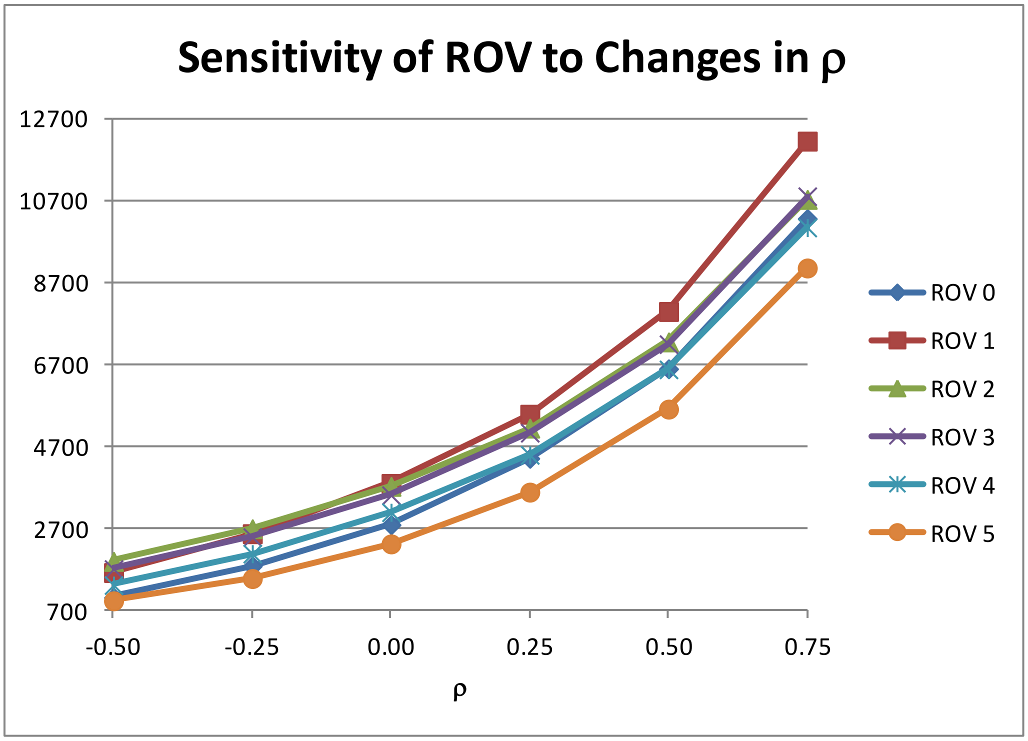
$\hat{P}_3$ is the solution to EQs 19-20-21, $\hat{P}_4$ is the solution to EQs 25-26-27 or 30-31-32, $\hat{P}_5$ is the solution to EQs 38-39-40 or 43-44-45 with the parameter values in Table I.

Figure 5



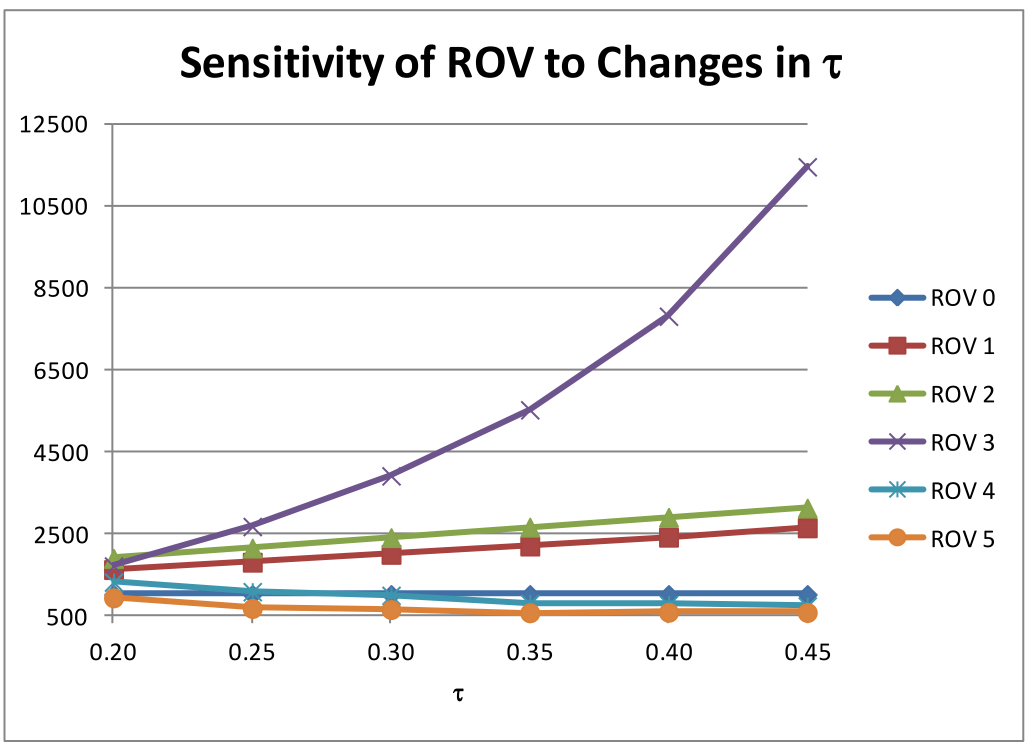
ROV_0 is the solution to EQ 10 without a subsidy, ROV_1 with a subsidy, ROV_2 the LHS of EQ 11, ROV_3 EQ 17, ROV_4 EQ 24 or 29, ROV_5 EQ 37 or 42, with the parameter values in Table I.

Figure 6



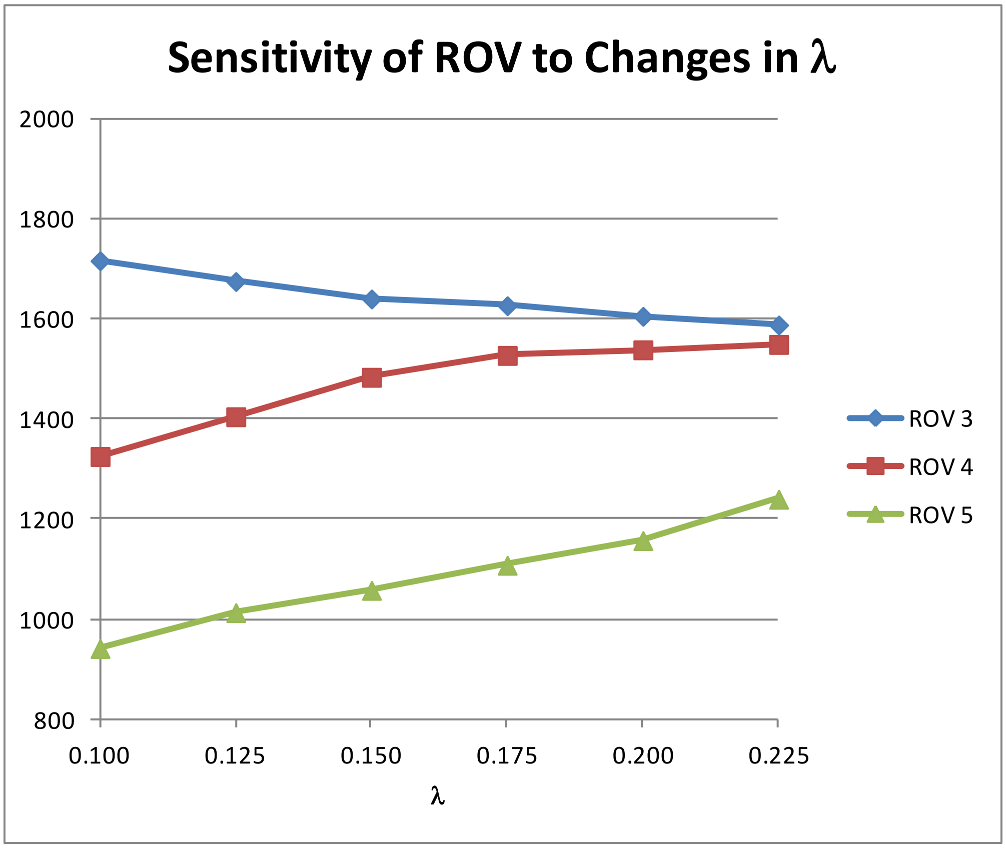
ROV_0 is the solution to EQ 10 without a subsidy, ROV_1 with a subsidy, ROV_2 the LHS of EQ 11, ROV_3 EQ 17, ROV_4 EQ 24 or 29, ROV_5 EQ 37 or 42, with the parameter values in Table 1.

Figure 7



ROV_0 is the solution to EQ 10 without a subsidy, ROV_1 with a subsidy, ROV_2 the LHS of EQ 11, ROV_3 EQ 17, ROV_4 EQ 24 or 29, ROV_5 EQ 37 or 42, with the parameter values in Table I.

Figure 8



ROV_3 is the solution to EQ 17, ROV_4 EQ 24 or 29, ROV_5 EQ 37 or 42, with the parameter values in Table I.

The ROV for all models decrease with increases with quantity volatility, increase with the increase of correlation (which increases P*Q volatility) and (mostly) increase with the size of the subsidy. So while either production volume floors or high subsidies of any type might encourage investment, the value of a renewable energy concession will be dependent on expected volatilities, as well as the subsidy. Sensitivity of ROV to possible retraction or to the introduction of retractable subsidies is intuitive: the greater the possibility of retracting a subsidy, the less the ROV, but the greater the possibility of a retractable subsidy (rather than no subsidy) the lower the ROV. Of course, the greater the possibility of a permanent subsidy, the greater the ROV.

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4. CONCLUSION

We derive the optimal investment timing and real option value for a renewable energy facility with joint (and sometimes distinct) products of price and quantity of generation, particularly where there might be a government subsidy proportional to the quantity of generation. When the dimensionality cannot be reduced, the thresholds and real option values are derived as a simultaneous solution to a set of equations. Our base Model I shows that a permanent subsidy proportional to revenue lowers the investment threshold and raises the real option value substantially. In Model II, when the permanent subsidy is proportional to the quantity produced, the threshold is lower than the equivalent R threshold of Model I. In Model III, for a retractable subsidy the price threshold is even lower, showing the incentive of a bird in hand. Where there is the possibility of a permanent subsidy, for out-of-the-money investment options, Model IV, the price threshold is much higher, but for the possibility of a retractable subsidy, Model V, about the same as for an actual retractable subsidy. MIV0>M0>MI>MII>MIII>MV0, given that in all cases $P<\hat{P}$. Price thresholds for all models increase with increases with quantity volatility, and decrease with the size of the subsidy. So either production volume floors or high subsidies of almost any type might encourage investment.

The order of the ROV for each context is not exactly the same as for the price threshold. The ROV ranks by type of subsidy arrangement are MII>MIII>MI>MIV0>M0>MV0, given that in all cases $P<\hat{P}$. Model I shows that a permanent subsidy proportional to revenue lowers raises the real option value substantially. The highest ROV are the actual permanent subsidies on Q or the possibilities of such subsidies. The lowest ROV are the possibilities of a retractable subsidy. The ROV for all models decrease with increases with quantity volatility, increase with the increase of correlation (which increases P*Q volatility) and increase with the size of the subsidy. So while either production volume floors or high subsidies of almost any type might encourage investment, the value of a renewable energy concession will be dependent on expected volatilities, as well as the subsidy.

What are the apparent policy guidelines in using subsidies to encourage early investment in facilities with joint (and sometimes distinct) products? Subsidies matter, especially if regarded as permanent. But whether increasing a subsidy say from 0 to .35 per unit produced is worth reducing the threshold as indicated is questionable. Possibly less transparent incentives are price or quantity guarantees, which effectively reduce price and/or quantity volatility, with a significant impact on thresholds under all models.

Obvious areas for future research are other subsidy arrangements which could be modeled similarly, such as proportional subsidies on P only, permanent, retractable, and sudden permanent or retractable subsidies, along with some combinations with Q subsidy arrangement models. Also possibly some of the models herein might serve as comparisons for numerical analysis of more realistic, finite, investment opportunities.

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Appendix

In this Appendix, simplified solutions to three alternative models are obtained by assuming the subsidy depends on the generated revenue and not on one of its elements. By invoking the similarity principle, the value-matching relationship can be expressed as a one-factor formulation. If the proportional subsidy is represented by τ_M, then for a revenue $R = PQ$, the total cash inflow is specified by $R(1+\tau_M)$. The value for the investment opportunity is denoted by V, in order to differentiate between the original and simplified variants.

Model 0

The subsidy is set to equal zero in Model 0. If the threshold revenue signalling an optimal investment is denoted by $\hat{R}_1$, then:

$$\hat{R}_1 = \frac{\beta_1}{\beta_1 - 1} K \left(r - \mu_{PQ} \right). \tag{A1}$$

The value for the investment opportunity is defined by:

$$V_1 = \begin{cases} B_1 R^{\beta_1} & \text{for } R < \hat{R}_1, \\ \dfrac{R}{r - \mu_{PQ}} - K & \text{for } R \geq \hat{R}_1. \end{cases} \tag{A2}$$

where:

$$B_1 = \frac{\hat{R}_1^{1-\beta_1}}{\beta_1 (r - \mu_{PQ})}. \tag{A3}$$

Model I

For a positive proportional subsidy τ_M, the corresponding results are:

$$\hat{R}_2 = \frac{\beta_1}{\beta_1 - 1} K \frac{\left(r - \mu_{PQ} \right)}{\left(1 + \tau_M \right)}, \tag{A4}$$

$$V_2 = \begin{cases} B_2 R^{\beta_1} & \text{for } R < \hat{R}_2, \\ \dfrac{R\left(1+\tau_M\right)}{r - \mu_{PQ}} - K & \text{for } R \geq \hat{R}_2, \end{cases} \tag{A5}$$

$$B_2 = \frac{(1+\tau_M)\hat{R}_2^{1-\beta_1}}{\beta_1(r-\mu_{PQ})} \tag{A6}$$

Model II

The probability of a sudden unexpected withdrawal of the subsidy is denoted by λ. If the revenue threshold signalling an optimal investment is denoted by $\hat{R}_3$, then its solution is found implicitly from:

$$\hat{R}_3 = \frac{\beta_3}{\beta_3 - 1} K \frac{r-\mu_{PQ}}{1+\left(1-\lambda\right)\tau_M} + B_1 \hat{R}_3^{\beta_1} \frac{\beta_3 - \beta_1}{\beta_3 - 1} \tag{A7}$$

where B_1 is enumerated from (A3).. The value for the investment opportunity is specified by:

$$V_3 = \begin{cases} B_3 R^{\beta_3} + B_1 R^{\beta_1} \text{ for } R < \hat{R}_3, \\ \dfrac{R\left(1+\left(1-\lambda\right)\tau_M\right)}{r-\mu_{PQ}} - K \text{ for } R \geq \hat{R}_3, \end{cases} \tag{A8}$$

where:

$$B_3 = \frac{(1+\left(1-\lambda\right)\tau_M)\hat{R}_3^{1-\beta_3}}{\beta_3(r-\mu_{PQ})} - \frac{\beta_1}{\beta_3} B_1 \hat{R}_3^{\beta_1-\beta_3}. \tag{A9}$$

For $\lambda = 0$, when there is no likelihood of the subsidy being withdrawn unexpectedly, $\beta_3 = \beta_1$ and Model II simplifies to the Model I solution.

Model III

The probability of a sudden unexpected introduction of the subsidy is denoted by λ. If the revenue threshold signalling an optimal investment is denoted by $\hat{R}_4$, then:

$$\hat{R}_4 = \frac{\beta_3}{\beta_3 - 1} \times \frac{r - \mu_{PQ}}{1 + \lambda \tau_M} \left(K + \frac{\lambda}{r + \lambda} B_2 \hat{R}_2^{\beta_1} \right) \tag{A10}$$

where B_2 is enumerated from (A6). The value for the investment opportunity is specified by:

$$V_4 = \begin{cases} B_4 R^{\beta_3} + \dfrac{\lambda}{r+\lambda} B_2 \hat{R}_2^{\beta_1} & \text{for } R < \hat{R}_4, \\ \dfrac{R\left(1 + \lambda \tau_M\right)}{r - \mu_{PQ}} - K & \text{for } R \geq \hat{R}_4, \end{cases} \tag{A11}$$

where:

$$B_4 = \frac{(1 + \lambda \tau_M) \hat{R}_4^{1-\beta_3}}{\beta_3 (r - \mu_{PQ})}. \tag{A12}$$

For a zero likelihood of an unexpected introduction of a proportional subsidy, Model III simplifies to Model 0.

Numerical Evaluations

With the identical parameter values to those of Table I, the revenue thresholds are shown below.

Model	R^
R 0	638.702
R I	532.251
R II	504.277
R III	721.175

Model R 0 and Model R I results are identical to those shown in Table I, which are based on revenue, without and with a permanent subsidy on R. Where there is a permanent subsidy on Q rather than on R, Model R II shows a higher revenue threshold than Model II, indicating an incentive to defer investment. Where there is a retractable subsidy on Q rather than on R, Model R III shows a much higher revenue threshold than Model III, indicating a significant incentive to defer investment.