The Investment Game under Uncertainty: An Analysis of Equilibrium Values in the Presence of First or Second Mover Advantage. *

Junichi Imai†and Takahiro Watanabe‡

September 23, 2006

Abstract

In this paper we develop a valuation model when there exist two competitive firms that face irreversible investment decisions under the demand uncertainty. We propose a numerical procedure to derive both project values and equilibrium strategies in the duopolistic environment. In numerical examples we consider two different economic conditions, which are labelled first mover advantage and second mover advantage, and examine the effects of these conditions on the equilibrium strategies as well as the project values. We show that these conditions cause significant changes in the equilibrium strategies of both firms.

JEL: G31,C61,C73

Keywords Real option, Investment game, Equilibrium strategy, Project Valuation

1 Introduction

The real option approach valuing real assets or projects has been playing an important role in the field of corporate finance and financial economics. It has been proposed as a useful concept to analyze a strategic investment. The usual real option analysis is implicitly based on the assumption that the underlying risk is exogenous and that management cannot affect the underlying stochastic process. This assumption is appropriate if a firm is nearly in a perfectly competitive market or it has monopolistic power over the market of the project. However, management in the real world should consider rival firms' behaviors in an imperfectly competitive market because one firm's action affects other firms' decisions and vice versa. Therefore, we should consider strategic interaction among competitive firms. Several studies recently focus on integrating real option analysis with game theory to reflect the interaction.

In this paper we develop a valuation model that can derive the project values and the equilibrium strategies when there exist two competitive firms that face irreversible investment decisions under uncertainty. Suppose each firm operates an existing project that generates a cash flow stream.

*Both authors acknowledge a research support from Grant-in-Aid for Scientific Research in Japan Society for the Promotion of Science 17510128.

†Graduate School of Economics and Management, Tohoku University, 27-1, Kawauchi, Aoba-ku, Sendai Miyagi, 980-8576, JAPAN. Tel:+81-22-795-4798,Facsimile:+81-22-795-4798,Email: jimai@econ.tohoku.ac.jp

‡Faculty of Urban Liberal Arts, Tokyo Metropolitan University, 1-1 Minami-Osawa Hachioji, Tokyo, JAPAN. Email: rogath3@nabenavi.net

Each firm has an option to reinvest in the project so that the marginal cash flow of the project is increased. The cash flow stream from the project depends on the level of demand that follows a continuous-time stochastic process, and the investment decisions of both firms. Each firm makes an investment decision to maximize its project value. We approximate the underlying continuous-time process with the discrete-time lattice process and propose a numerical procedure to derive the project values in equilibrium strategies. In numerical examples we derive the project values and analyze the strategic behavior of the competitive firms in the equilibrium under different economic situations.

Recent researches that integrate real option approach with game theory can be classified into two categories. The models in the first category assume that the underlying variable of the project follows a one-period or two-period process. Smit and Ankum (1993), Kulatilaka and Perotti (1998), Smit and Trigeorgis (2001) are in the first category. These studies give us intuitive ideas about the effect of the competition under uncertainty and clarify the strategic behaviors of the firms under competition. However, several strong assumptions are contained in the models. Especially, the underlying process is too simple and the number of decision opportunities is strictly restricted. As a result, they are not suitable for a quantitative analysis.

On the other hand, the models in the second category often assume that the underlying variable follows a continuous-time stochastic process. Smets (1991) and Dixit and Pindyck (1994) are examples of the earliest studies in this category. The models are constructed by extending a deterministic model proposed by Fudenberg and Tirole (1985) to a stochastic dynamic process. Grenadier (1996) applies this model to an analysis of land development projects while Huisman (2001) refines it to analyze the strategic behaviors of the firms. Other studies in this category are Hoppe (2000), Lambrecht (2000), Grenadier (2002), Huisman and Kort (2003) and Huisman and Kort (2004).

Although the models in the second category can derive valuable implication about strategic investments under uncertainty and competition, several assumptions in these models seem to be too naive to apply to a practical use of evaluating real projects. First, the assumption that the project has an infinite horizon is not always realistic and the models does not capture the effect of the maturity on the project value. Secondly, most researches assume that the underlying variable follows a geometric Brownian motion[1]. These assumptions are required to derive a closed-form optimal strategy. If we assume the different underlying process or loosen the infinite horizon assumption it is not easy to derive a closed-form solution of the problem. Accordingly, a numerical approach is valuable in this case. The third assumption is about the continuity of the decision opportunities. Most studies implicitly assume that management can make investment decisions continuously. It is observed, however, that these decision opportunities do not always come continuously even if the underlying risk would change continuously. They come once in a week or in a month, sometimes in a quarter in many actual investment projects. Therefore, it is necessary to construct a model to distinguishes the continuity of the underlying process from the discrete decision opportunities.

The purposes of this paper is: (1) to derive a numerical procedure to evaluate an investment project under the demand uncertainty and competition; and (2) to analyze the project values and the equilibrium strategies under two conditions of competitive environments, which is called First Mover Advantage (FMA) and Second Mover Advantage(SMA).

For the first purpose we develop a discrete-time lattice model and integrate real option approach with game theory. It is well known that the lattice process can approximate the continuous-time stochastic process if the parameter values are adequately adjusted. Furthermore, this model can be easily extended to deal with the finite number of decision opportunities under a wider class

[1]Kijima and Shibata (2002) consider the stochastic volatility process.

of continuous-time underlying processes. Our model is located between the two categories stated above; namely, our model can assume a continuous-time underlying process with finite number of decision opportunities, which enables us to derive more realistic results and implication. Although our numerical approach cannot obtain analytical solutions, we can obtain the current project value that depends on both the current demand and time to the project maturity, which have not been analyzed by the past studies.

Imai and Watanabe (2006b) propose a similar numerical procedure. In their model, however, two firms are assumed to be asymmetric where the two firms make an investment decision sequentially; one firm can make a decision first at each period and the other firm does after observing the rival's decision. This corresponds to a problem of a perfect information in game theory. The game is simple because a multiple equilibria problem never occurs and it can be solved by a simple backward induction.

The research with regard to symmetric and simultaneous decisions is also necessary because it is a more fundamental setting from a theoretical viewpoint. It also enables us to compare the results of the existing researches such as Smets (1991), Dixit and Pindyck (1994), Grenadier (1996) and Huisman (2001). Therefore, in our model, both firms make investment decisions simultaneously at each decision opportunity.

Since multiple equilibria could emerge in the simultaneous decision case we introduce the following two criteria to select an equilibrium. In the first criterion, we distinguish the two competitive firms before starting the game, which are labelled firm L and firm F in the paper. Then we assume that firm L has a competitive advantage over firm F in the sense that one equilibrium is selected so that the project payoff of the firm L is maximized if there exist multiple equilibria. We call it *firm L advantage criterion* (LAC) for convenience in the paper. Although LAC looks rather ad hoc Imai and Watanabe (2006a) point out that LAC is consistent with a theory of equilibrium selection in game theory developed by Harsanyi and Selten (1988) if we suppose that the cash flow of firm L is infinitesimally greater than that of firm F after both firms' investment.

In the second criterion, we assume that one equilibrium is selected with an equal probability when there exist multiple equilibria. This criterion is consistent with the previous studies of Grenadier (1996) and Huisman (2001) that examine a preemption game under the assumption of FMA. The criterion is exogenously assumed in our paper, but Fudenberg and Tirole (1985) derive the same criterion endogenously with a more sophisticated model. In this paper we call it *50% criterion*.

The second purpose of this paper is motivated as follows. Recent studies concerning the integration of real option approach with game theory mainly focus on investments of two competitive firms under the condition of FMA. FMA is a condition in which the increment of the marginal cash flow of a firm that invests earlier is greater than that of the other firm that invests later. In this paper a firm which invests in the project earlier is labelled the *first mover* while a firm which invests after the first mover is labelled the *second mover*. It is known that the condition of FMA causes a preemption game because each firm wants to invest earlier than the rival firm in order to prevent the rival firm's investment and to enjoy the monopolistic revenue from the investment. In other words, both firms want to become the first mover of the game.

FMA is an appropriate condition to describe preempt competition under uncertainty such as land developments and oil refinery projects. However, some competitive situations cannot be captured by the context of FMA. For example, consider an investment opportunity to enter a new product market. The leader firm, which intends to launch a new product, often needs to invest for the research and development. The marketing cost is also necessary for the promotion of the new product to create a new market. On the other hand, the follower firm, which intends to enter the market after the leader, could learn from the leader's experience. The follower can observe the market development and decide to invest after the market is matured. Furthermore, the follower has to pay less money for the market creation. To describe this situation SMA is a more appropriate condition where the increment of cash flow of the second mover is larger than that of the first mover. In this paper, we examine the equilibrium strategy of the competitive firms under the condition of SMA as well as that of FMA.

In the numerical examples we analyze comparative statics under the conditions of both FMA and SMA with respect to the initial demand and examine the effect of these two conditions on the equilibrium strategies as well as the project values. We observe that a preemption game emerges under the condition of FMA that leads to multiple equilibria. As a result, an asymmetric equilibrium where only firm L invests first, is chosen in case of LAC. In that case the project value of firm L, which is the first mover, is larger than that of firm F. However, we can conclude that the project value of firm L in that case is not always larger than the project values accomplished by both firms in coordination. This is because the firms tends to invest earlier than the optimal timing of the firm that is assigned to be the first mover exogenously. The equilibrium strategies are always symmetric under the condition of SMA. Both firms can make the best use of the flexibility to defer the investment even in the presence of competition. As a result the project values of both firms are equivalent to the coordinated project value.

This paper is organized as follows. In section 2 we develop the valuation model. Numerical analyses are done in section 3. Finally, concluding remarks are in section 4.

2 Model Description

2.1 A Basic Setting

This section provides a valuation model of the project under demand uncertainty and competition. The model is based on Dixit and Pindyck (1994), Grenadier (1996) and Huisman (2001) which can be regarded as one of the typical models for the project valuation under uncertainty and competition. Two firms are introduced, denoted by firm L and firm F. Each firm operates a project, which has an option to reinvest in the project that increases the firm's cash flow by paying the investment cost I.

In the model we assume that the future demand is uncertain and follows a continuous-time stochastic process. Let $Y(t)$ denote the realized demand at time t and it follows

$$dY(t) = \mu\left(t, Y(t)\right) dt + \sigma\left(t, Y(t)\right) dz^{P}, \tag{1}$$

where $\mu\left(t, Y(t)\right)$ and $\sigma\left(t, Y(t)\right)$ are the instantaneous drift and volatility, respectively, and dz^{P} is the increment of the Wiener process under the probability measure P. As explained in the introduction, our model is established on the lattice process that can converge to the corresponding continuous-time process as the number of periods tends to infinity. In addition, the lattice model can be also interpreted as many but finite decision opportunities and it enable us to distinguish decision opportunities from the timings of changing demand. Imai and Watanabe (2006b) analyze the effect of the discrete decision opportunities thoroughly. In this paper we do not focus on this aspect and simply assume that the decision opportunities come continuously.

We assume that the projects of both firms continue within finite horizon and end up at some future time T which are called a maturity of the project. Thus, each firm can choose a time of the investment under uncertainty of the demand until the maturity of the project. To construct a lattice model, we divided the time interval $[0, T]$ into M periods of length Δt; $\Delta t = \frac{T}{M}$. The demand $Y(t)$

changes at time $t = m\Delta t$ for $m = 0, \cdots, M$ and we simply denote the demand $Y(m\Delta t)$ at period m by $Y(m)$. Note that M is assumed to be sufficiently large for the approximation.

The chance of the investment in the project for each firm is at most once in the $M + 1$ decision opportunities and the firm can never invest again after the investment. Thus, there are two states for each firm with regard to the firm's decision. Let $x_i(m)$ denote the state of firm $i(i = L, F)$ at each period $m = 0, \cdots, M$. $x_i(m) = 0$ represents the state where firm i has not invested in the project until period m, while $x_i(m) = 1$ represents the state where firm i has already invested. Note that $x_i(m)$ stands for the state of firm i *after* its decision of the investment at period m. There are totally four possible pairs of states, which are denoted by $(x_L(m), x_F(m))$.

At the m-th decision opportunity for each $m = 0, \cdots, M$ when both firms have already invested (i.e., $(x_L(m-1), x_F(m-1)) = (1, 1)$), both firms do not make any decision. When one firm has already invested but the other firm has not yet (i.e., $(x_L(m-1), x_F(m-1)) = (0, 1)$ or $(1, 0)$), only the firm that has not invested makes a decision. Finally, when neither firm has invested (i.e., $(x_L(m-1), x_F(m-1)) = (0, 0)$), both firms make their investment decisions. In this case, we assume that both firms determine their strategies simultaneously.

The marginal cash flow obtained by each firm at each period depends on the following two variables. The first one is $Y(m)$, the realized demand at period m. The second variable is a pair of the states of both firms about the investment $(x_L(m), x_F(m))$. The cash flow per unit of demand and unit of period is illustrated in *Figure 1*. Let $D_{x_i(m)x_j(m)}$ denote the cash flow per unit of demand of firm i where j is the rival firm of firm i.

insert Figure 1

In the usual real option analysis no-arbitrage principle is often assumed for the valuation. It is difficult, however, to apply this principle to our model since the demand of the merchandise cannot be observed in the market. Alternatively, Cox and Ross (1976), Constantinides (1978), and McDonald and Siegel (1984) propose the equilibrium approach for the option pricing. In this paper, we apply the equilibrium approach and assume that the demand risk can be considered private risk or unsystematic risk that is independent of the market risk. Since an investor pays no risk premium with respect to the unsystematic risk in equilibrium, we can assume that firms are risk neutral in the valuation model and let r denote the risk-free rate.

Now, we develop the valuation model from a game theoretical perspective. First, we define a strategy of each firm. A decision whether each firm has invested or not until each period, denoted by 1 or 0, respectively, is called an action of the firm. We apply the Markov subgame perfect equilibrium to a solution concept. This means that each firm's action depends only on the current demand and the pair of states instead of the history of their actions and the demands. The firm i's strategy s_i for $i = L, F$ is defined as a list of the actions at each period for any current demand and any state of both firms. $s_i(m, x_j(m-1), Y(m))$ is the firm i's strategy at period m when the realized demand is $Y(m)$ and the previous state of the rival firm j is $x_j(m-1)$. $s_i(m, x_j(m-1), Y(m)) = 1$ indicates that firm i finishes investing until period m while $s_i(m, x_j(m-1), Y(m)) = 0$ represents that firm i has not invested yet.

Now we define the present value of the project on the condition that the future demand is given at period m. We introduce additional notation to define it. Let $\boldsymbol{Y}_m = (Y(m+1), \cdots, Y(M))$ be the sequence of random variables for future demands after period m. Suppose that s_L, s_F and $\boldsymbol{Y}_m$ is determined for a given period m. Then, the state of each firm i for any period $l \geq m$ is specified, denoted by $\hat{x}_i(l, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m)$. In addition, whether firm i invests in the project exactly at period l for $l \geq m$ is given by indicator functions $\mathbf{1}_i$, which is formally written as:

$$\begin{aligned}
&\mathbf{1}_i(l, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m) \\
&= \begin{cases} 1 & \hat{x}_i(l-1, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m) = 0 \\ & \text{and } \hat{x}_i(l, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m) = 1, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

Now, let $u_i(m, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m)$ be the present value of the project of firm i. We set that the cash flow obtained by each firm i at maturity M is always equal to zero; $u_i(M, s_i, s_j, x_i(M-1), x_j(M-1), \boldsymbol{Y}_M) \equiv 0$. The project value at period m ($0 \leq m \leq M-1$) is defined by

$$\begin{aligned}
&u_i(m, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m) \\
&= \sum_{l=m}^{M-1} e^{-r\Delta t(l-m)} \{D_{\hat{x}_i \hat{x}_j} Y(l)\Delta t - I\mathbf{1}_i(l, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m)\},
\end{aligned}$$

where $\hat{x}_i$ and $\hat{x}_j$ are precisely written as

$$\begin{aligned}
\hat{x}_i &\equiv \hat{x}_i(l, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m), \\
\hat{x}_j &\equiv \hat{x}_j(l, s_j, s_i, x_j(m-1), x_i(m-1), \boldsymbol{Y}_m).
\end{aligned}$$

Next, we define the expected value of the project at m-th decision opportunity which is the expected cash flow after period m for a given strategy profile. The firm i's expected value at m-th decision opportunity for each $m = 0, \cdots, M$, denoted by $U_i(m, s_i, s_j, x_i(m-1), x_j(m-1), y)$, is given by

$$\begin{aligned}
&U_i(m, s_i, s_j, x_i(m-1), x_j(m-1), y) \\
&= E^Q[u_i(m, s_i, s_j, x_i(m-1), x_j(m-1), \boldsymbol{Y}_m) | Y(m) = y].
\end{aligned}$$

where $E^Q[\cdot]$ represents an expectation under the risk neutral probability measure Q.

Finally, we define a concept of an equilibrium. A couple of strategy (s_L^*, s_F^*) is said to be an equilibrium if and only if for any firm $i = L, F$, any $m = 0, \cdots M$, any current demand y, and any states of both firms at previous period $m-1$, $x_i(m-1)$ and $x_j(m-1)$, s_i^* satisfies

$$U_i(m, s_i^*, s_j^*, x_i(m-1), x_j(m-1), y) \geq U_i(m, s_i, s_j^*, x_i(m-1), x_j(m-1), y) \tag{2}$$

for any strategy s_i.

2.2 Derivation of the Project Values

One of our main interests is to derive *the project values* of the both firms in equilibrium. We define an equilibrium in the previous subsection by equation (2). This definition, however, is not written as a tractable form so that we cannot derive the equilibrium strategies with the definition. In this subsection we develop a dynamic programming procedure to obtain the project values of the both firms in equilibrium strategies. The recursive procedure is constructed by induction.

Let $V_L^{(x_L(m-1), x_F(m-1))}(m, y)$ and $V_F^{(x_L(m-1), x_F(m-1))}(m, y)$ denote the project values of firm L and firm F, respectively. The value functions depend on period m , the current demand $Y(m) = y$, and the states of both firms $x_L(m-1)$ and $x_F(m-1)$ at period $m-1$, assuming that the both firms follow equilibrium strategies after period m. Let s_L^* and s_F^* denote the equilibrium strategies of both firms. Then, the project values are formally written by

$$\begin{aligned}
V_L^{(x_L(m-1), x_F(m-1))}(m, y) &= U_L(m, s_L^*, s_F^*, x_L(m-1), x_F(m-1), y) \\
&= E^Q[u_L(m, s_L^*, s_F^*, x_L(m-1), x_F(m-1), \boldsymbol{Y}_m) | Y(m) = y], \\
V_F^{(x_L(m-1), x_F(m-1))}(m, y) &= U_F(m, s_F^*, s_L^*, x_F(m-1), x_L(m-1), y) \\
&= E^Q[u_F(m, s_F^*, s_L^*, x_F(m-1), x_L(m-1), \boldsymbol{Y}_m) | Y(m) = y].
\end{aligned}$$

We define that the cash flows obtained by both firms at maturity period M are supposed to be zero, i.e., $V_L^{(x_L(M-1),x_F(M-1))}(M,y) = V_F^{(x_L(M-1),x_F(M-1))}(M,y) = 0$ for any demand y and any states $(x_L(M-1), x_F(M-1))$.

$V_L^{(x_L(m-1),x_F(m-1))}(m,y)$ and $V_F^{(x_L(m-1),x_F(m-1))}(m,y)$ are obtained by the following recursive procedure for $m = 0, \cdots, M-1$. Let $v_i^{(x_L(m),x_F(m))}(m,y)$ denote the project values (not including the investment cost) after the decisions of both firms at period m, assuming that both firms follow equilibrium strategies after period $m+1$.

First, we consider the values of $V_L^{(1,1)}(m,y)$ and $V_F^{(1,1)}(m,y)$ which are easy to compute because both of the firms have already invested and there are no decision to make at period m. Thus, the project values can be derived by adding the discounted expected values of the next period to the current cash flow;

$$V_i^{(1,1)}(m,y) = v_i^{(1,1)}(m,y) \tag{3}$$

for any $i = L, F$ where

$$v_i^{(1,1)}(m,y) = D_{11} y \Delta t + e^{-r\Delta t} E^Q \left[V_i^{(1,1)}(m+1, Y(m+1)) | Y(m) = y \right].$$

Next, consider the project value of $V_F^{(1,0)}(m,y)$. In this case firm L has already invested in the project and firm F can make an investment decision solely. If firm F decides to invest in the project at period m the project value can be written as $v_F^{(1,1)}(m,y) - I$. On the other hand, if firm F decides not to invest at period m the project value is equal to $v_F^{(1,0)}(m,y)$ where

$$v_F^{(1,0)}(m,y) = D_{01} y \Delta t + e^{-r\Delta t} E^Q \left[V_F^{(1,0)}(m+1, Y(m+1)) | Y(m) = y \right].$$

Firm F makes the optimal decision to maximize its project value at period m. Thus, the following equation is satisfied.

$$\begin{aligned} &V_F^{(1,0)}(m,y) \\ &\quad = \begin{cases} v_F^{(1,1)}(m,y) - I & v_F^{(1,1)}(m,y) - I > v_F^{(1,0)}(m,y), \\ v_F^{(1,0)}(m,y) & \text{otherwise.} \end{cases} \end{aligned} \tag{4}$$

Next, we consider the project value of $V_L^{(1,0)}(m,y)$. Firm L which has already invested in the project has no decision to make at period m but the project value is dependent on the firm F's decision. Thus, the following equations hold.

$$\begin{aligned} &V_L^{(1,0)}(m,y) \\ &\quad = \begin{cases} v_L^{(1,1)}(m,y) & v_F^{(1,1)}(m,y) - I > v_F^{(1,0)}(m,y), \\ v_L^{(1,0)}(m,y) & \text{otherwise,} \end{cases} \end{aligned} \tag{5}$$

where $v_L^{(1,0)}(m,y)$ is given by

$$v_L^{(1,0)}(m,y) = D_{10} y \Delta t + e^{-r\Delta t} E^Q \left[V_L^{(1,0)}(m+1, Y(m+1)) | Y(m) = y \right].$$

Due to a symmetric aspect of the competition between firm L and firm F, the project values at state of $(0,1)$ are derived correspondingly;

$$\begin{aligned} &V_L^{(0,1)}(m,y) \\ &\quad = \begin{cases} v_L^{(1,1)}(m,y) - I & v_L^{(1,1)}(m,y) - I > v_L^{(0,1)}(m,y), \\ v_L^{(0,1)}(m,y) & \text{otherwise,} \end{cases} \end{aligned} \tag{6}$$

where

$$v_L^{(0,1)}(m,y) = D_{01} y\Delta t + e^{-r\Delta t} E^Q\left[V_L^{(0,1)}(m+1, Y(m+1))|Y(m)=y\right],$$

and

$$\begin{aligned} &V_F^{(0,1)}(m,y) \\ &= \begin{cases} v_F^{(1,1)}(m,y) & v_L^{(1,1)}(m,y) - I > v_L^{(0,1)}(m,y), \\ v_F^{(0,1)}(m,y) & \text{otherwise,} \end{cases} \end{aligned} \tag{7}$$

where

$$v_F^{(0,1)}(m,y) = D_{10} y\Delta t + e^{-r\Delta t} E^Q\left[V_F^{(0,1)}(m+1, Y(m+1))|Y(m)=y\right].$$

Finally, the project values at state of $(0,0)$ are considered when both firms have options to invest in the project. In this case actions for both firms in the equilibrium strategies can be derived from a Nash equilibrium of one shot game described by the payoff matrix illustrated in *Figure 2*.

insert Figure 2

In the figure, $v_L^{(0,0)}(m,y)$ is defined by

$$v_L^{(0,0)}(m,y) = D_{00} y\Delta t + e^{-r\Delta t} E^Q\left[V_L^{(0,0)}(m+1, Y(m+1))|Y(m)=y\right].$$

There could be multiple Nash equilibria in the payoff matrix in *Figure 2*, hence we need to define additional criteria to choose the unique equilibrium. As explained in the introduction, we introduce two criteria which are called LAC and 50% criterion.

In the LAC, we assume that firm L has a competitive advantage in the case of the multiple equilibria. In the payoff matrix shown in *Figure 2*, equations $v_L^{(1,1)} = v_F^{(1,1)}, v_L^{(1,0)} = v_F^{(0,1)}$ and $v_L^{(0,1)} = v_F^{(1,0)}$ are satisfied at any period. Hence, we can determine the equilibrium strategies using these equations. *Table 1* summarizes the equilibrium strategies that can occur under LAC. By using above equations, all possible cases are classified into nine cases. The second column labelled "conditions of equilibria" in *Table 1* corresponds to the conditions which determine equilibria. The resulting equilibria are shown in the third column denoted by the pair of the states, $(s_L(m, x_L(m-1), Y(m)), s_F(m, x_F(m-1), Y(m)))$. For example, $(1,0)$ represents the state where only firm L finishes the investment. Note that the third and forth cases correspond to symmetric equilibria while the fifth and sixth cases correspond to asymmetric equilibria. If there exist multiple equilibria, one equilibrium strategy is selected by LAC, which is reflected as an inequality in the fourth column labelled "condition of equilibrium selection". In the fifth column, the selected equilibrium by LAC is shown. The last two columns correspond to the project values of firm L and firm F in the equilibrium, respectively.

insert Table 1

It is important to note that in the case of symmetric equilibria, third and forth cases, LAC is equivalent to another criterion for equilibrium selection called payoff dominance. The payoff dominance criterion can choose the unique equilibrium if all players' payoffs in the equilibrium are greater than those of the other equilibrium. After Harsanyi and Selten (1988) apply this criterion to their theory of equilibrium selection it is often used in game theory. Fudenberg and Tirole (1985) and Huisman (2001) also use this criterion to select an equilibrium. The payoff of firm L in an equilibrium selected by LAC is greater than that of the other equilibrium in the case of symmetric equilibria. In the case of symmetric equilibria this as well as firm F's payoffs. Hence, an equilibrium selected by LAC in symmetric equilibria is consistent to the equilibria choosen by payoff dominance.

In applying the 50% criterion, there are four possible cases which are summarized in equation (8). By symmetric definition of our model $v_L^{(x_L,x_F)} = v_F^{(x_L,x_F)}$ and $V_L^{(x_L,x_F)} = V_F^{(x_L,x_F)}$ hold for any state of (x_L, x_F). Consequently, the project values of $V_L^{(0,0)}$ and $V_F^{(0,0)}$ are calculated by:

$$
\begin{aligned}
&V_L^{(0,0)}(m,y) = V_F^{(0,0)}(m,y) \\
&= \begin{cases}
v_L^{(1,1)}(m,y) - I & v_L^{(1,1)}(m,y) - I > v_L^{(0,1)}(m,y) \\
 & \text{and } v_L^{(1,0)}(m,y) - I > v_L^{(0,0)}(m,y), \\
\frac{1}{2}(v_L^{(1,1)}(m,y) - I + v_L^{(0,0)}) & v_L^{(1,1)}(m,y) - I > v_L^{(0,1)}(m,y) \\
 & \text{and } v_L^{(1,0)}(m,y) - I < v_L^{(0,0)}(m,y), \\
\frac{1}{2}(v_L^{(1,0)}(m,y) - I + v_L^{(0,1)}) & v_L^{(1,1)}(m,y) - I < v_L^{(0,1)}(m,y) \\
 & \text{and } v_L^{(1,0)}(m,y) - I > v_L^{(0,0)}(m,y), \\
v_L^{(0,0)}(m,y) & v_L^{(1,1)}(m,y) - I < v_L^{(0,1)}(m,y) \\
 & \text{and } v_L^{(1,0)}(m,y) - I < v_L^{(0,0)}(m,y).
\end{cases}
\end{aligned}
\tag{8}
$$

The first case indicates that both firms choose to invest at period m when the previous state is $(0,0)$ and in the fourth case neither firm invests in the project at period m. There exists the unique equilibrium in these cases On the other hand, the second and third case corresponds to multiple equilibria where the state of $(1,0)$ and $(0,1)$ satisfy the equilibrium condition. In these cases the project value can be given by the average of the two project values in the equilibria under 50% criterion.

3 Numerical experiences and economical implication

3.1 Settings of parameters

In this section we analyze a relation between competition and flexibility under uncertainty. Especially, we focus on the effects of FMA and SMA on the project values as well as the equilibrium strategies. In the numerical examples, both LAC and 50% criterion are applied in case of the multiple equilibria, which enables us to compare our model with the existing researches. In addition, we examine economical implication of each criterion by comparing the results.

We assume in our numerical examples that the underlying demand process follows a geometric Brownian motion with constant drift $r = 3\%$ and a constant volatility $\sigma = 30\%$ in the risk neutral world[2]. The demand process is approximated by the trinomial lattice model whose number of periods are equal to $M = 1000$ when the project horizon T is equal to one. To describe the economic condition between two firms we assume

$$D_{10} > D_{11} > D_{00} > D_{01}. \tag{9}$$

These inequalities imply that the investment can increase the marginal cash flow for both firms. In addition, we classify the conditions of FMA and SMA by comparing $D_{10} - D_{00}$ with $D_{11} - D_{01}$. Note $D_{10} - D_{00}$ indicates the increment of the marginal cash flow of the first mover while $D_{11} - D_{01}$ indicates that of the second mover. For brevity we define the condition of FMA as $D_{10} - D_{00} > D_{11} - D_{01}$, while SMA is defined as $D_{10} - D_{00} < D_{11} - D_{01}$. Note that the definition of FMA is the same as in Huisman (2001). The parameter values of $D_{ij}, i, j = 0, 1$ are given in each numerical example.

[2]Imai and Watanabe (2006b) analyze a similar model under a non-Gaussian stochastic process.

3.2 Project values under the condition of FMA

Figure 3 illustrates the project values of the firms with regard to the initial demand $Y(0)$ under the condition of FMA. Parameter values are given by $D_{10} = 8, D_{00} = 3, D_{01} = 0$ and $D_{11} = 4$, which satisfy the condition of FMA. In this case Imai and Watanabe (2006b) show that a chicken game could emerge; there exist two asymmetric equilibria in which one firm's strategy is different from the others'.

Our main interest is to analyze the project values of firm L and firm F in the equilibrium on condition that neither firm invests before period zero, which are given by $V_L^{(0,0)}(0, y)$ and $V_F^{(0,0)}(0, y)$, respectively. To analyze the equilibrium strategies we also derive project values of under different conditions before time zero; $V_L^{(1,1)}(0, y) - I$, $V_L^{(0,1)}(0, y)$ and $V_L^{(1,0)}(0, y) - I$.

Consider the firms' actions at period zero when the initial demand is $Y(0) = y$. The value of $V_L^{(1,1)}(0, y) - I$ indicates the project value of firm L at period zero on the condition that both firms invest at period zero . Note $V_L^{(1,1)}(0, y)$ does not include the investment cost. It is apparent that $V_L^{(1,1)}(0, y) - I$ is linear with respect to the initial demand since the project has no option to decide, which corresponds to the net present value of the project. $V_L^{(0,1)}(0, y)$ indicates the firm L's project value on the condition that firm F has already invested in the project. Since only firm L has an option to defer the investment, the shape of $V_L^{(0,1)}(0, y)$ with respect to the initial demand is similar to that of standard real option value to defer the investment without competition. According to the figure the value of $V_L^{(0,1)}(0, y)$ is relatively small but not zero when $y < Y_B$. This is because firm L defers the investment at period zero and the possibility of the future investment is small but positive. The value of $V_L^{(0,1)}(0, y)$ contacts with that of $V_L^{(1,1)}(0, y) - I$ at $y = Y_B$ where the optimal strategy for firm L has been changed into investing in the project at the beginning of the project. $V_L^{(1,0)}(0, y) - I$ indicates the firm L's project value on the condition that firm L invests at period zero. Although firm L has no decision to make its marginal cash flow depends on the firm F's action that changes the firm L's project value as well as the firm F's. It is of interest to note that the project value $V_L^{(1,0)}(0, y) - I$ peaks around $y = 47$. Increasing the initial demand makes a rise of the total cash flow for a given marginal cash flow, but it also increases a possibility for earlier investment of firm F. This leads to a decrease of the marginal cash flow of firm L, from D_{10} to D_{11}, and $V_L^{(1,0)}(0, y) - I$ decreases when y becomes greater than 47. $V_L^{(1,0)}(0, y) - I$ eventually becomes equal to $V_L^{(1,1)}(0, y) - I$ and $V_L^{(0,1)}(0, y)$ at $y = Y_B$ as the initial demand increases.

Observations of the project values under the condition of FMA: According to *Figure 3*, $V_L^{(0,0)}(0, y)$ and $V_F^{(0,0)}(0, y)$ are nearly equal but could not be identical when the initial demand is less than Y_A. In this interval, the project value takes a maximum around $y = 26$, which results from the similar effect to those of $V_L^{(1,0)}(0, y) - I$ around $y = 47$. Examining both firms' project values in detail reveals the fact that the firm F's project value is sometimes greater than the firm L's project value around $y < Y_A$ in spite of the advantage of firm L. This phenomena is called flexibility trap discussed by Imai and Watanabe (2004) and Imai and Watanabe (2006a). In the interval of $y \in (Y_A, Y_B)$ the project value of firm L is larger than that of firm F under LAC. Note $V_L^{(0,0)}(0, y)$ is equal to $V_L^{(1,0)}(0, y) - I$ while $V_F^{(0,0)}(0, y)$ is equal to $V_L^{(0,1)}(0, y)$ in the interval. This implies that only firm L invests in the project at period zero while firm F defers the investment and waits until the optimal investment timing. Both project values become identical when $y \geq Y_B$, which means that it is optimal to invest in the project at period zero.

Effect of the criteria on equilibrium selection: Finally, let $V_{50\%}^{(0,0)}(0,y)$ denote both firms' project values in the equilibrium at period zero when we apply 50% criterion to the case of multiple equilibria. Both project values are identical under this criterion because both firms are completely symmetric by definition. The comparison among $V_L^{(0,0)}(0,y)$, $V_F^{(0,0)}(0,y)$ and $V_{50\%}^{(0,0)}(0,y)$ enables us to understand how the criterion of equilibrium selection affect the project values. We focus on the comparison in the interval of $y \in (Y_A, Y_B)$ since all the values are nearly identical otherwise. In this interval we can observe that the value of $V_{50\%}^{(0,0)}(0,y)$ is exactly the same as an average of $V_L^{(0,0)}(0,y)$ and $V_F^{(0,0)}(0,y)$; $V_{50\%}^{(0,0)}(0,y) = \frac{1}{2}\left\{V_L^{(0,0)}(0,y) + V_F^{(0,0)}(0,y)\right\}$. Under the LAC one of the competitive firms, firm L, is predetermined to have a competitive advantaged in advance. Therefore, we can interpret that the value of $\frac{1}{2}\left\{V_L^{(0,0)}(0,y) + V_F^{(0,0)}(0,y)\right\}$ represents the project value of each firm when one of the two firms are selected as firm L with a equal probability before the game. On the other hand under the 50% criterion every time multiple equilibria emerges one of them are equally selected. Note 50% criterion does not select a advantaged firm before the game. Consequently, the result indicates that 50% criterion is equivalent to the criterion of selecting a advantaged firm as firm L with an equal probability in advance[3].

insert Figure 3

3.3 Project values under the condition of SMA

Figure 4 shows the project values of both firms under the condition of SMA. The parameter values of the marginal cash flows are set as $D_{10} = 8, D_{00} = 3, D_{01} = 0$ and $D_{11} = 6$. Note the value of D_{11} is changed so that the condition of the SMA holds. Although multiple equilibria could emerge as in the previous example, the type of equilibria is different. Under the condition of SMA there could be symmetric equilibria which consist of the state of $(1,1)$ and $(0,0)$. If we apply LAC to the equilibrium selection problem, the unique symmetric equilibrium is always selected, which leads to the identical project values of firm L and firm F. Furthermore, LAC is consistent with the payoff dominance criterion stated in the previous section.

Observations of the project values under the condition of SMA: The shapes of the values of $V_L^{(1,1)}(0,y) - I$, $V_L^{(1,0)}(0,y) - I$,and $V_L^{(0,1)}(0,y)$ depicted in *Figure 4* are similar to those in *Figure 3* while the shapes of the project values of $V_L^{(0,0)}(0,y)$ and $V_F^{(0,0)}(0,y)$ are quite different. The most important difference in this figure is that $V_L^{(0,0)}(0,y)$ is equivalent to $V_F^{(0,0)}(0,y)$ regardless of the initial demand. By examining the equilibrium strategies of both firms, we find that when $y \le Y_D$ both firm L and firm F defer the investment at period zero. These values contact with $V_L^{(1,1)}(0,y) - I$ at $y = Y_D$ where the optimal strategies of both firms are changed into investing at period zero. We can confirm from the figure that asymmetric equilibrium never emerges under the condition of SMA even if we apply LAC. With a comparison of $V_L^{(0,0)}(0,y)$ (or $V_F^{(0,0)}(0,y)$) with $V_L^{(1,0)}(0,y) - I$ we can conclude neither firm has an incentive to invest earlier to become the first mover under the condition of SMA.

Effect of the criteria on equilibrium selection: The comparison of the two criteria in the equilibrium selection is also examined under the condition of SMA. In *Figure 4* it is shown that the project value at period zero under 50% criterion, $V_{50\%}^{(0,0)}(0,y)$, is less than or equal to $V_L^{(0,0)}(0,y)$ (and $V_F^{(0,0)}(0,y)$). The relation between the two criteria is not clear under the condition of SMA. Since applying 50% criterion is inconsistent with the payoff dominance criterion, 50% criterion is not proper under the condition of SMA.

[3]This observation does not hold when $y < Y_A$. In this interval flexibility trap is observed that breaks the equality.

insert Figure 4

3.4 Discontinuity between FMA and SMA

Figure 5 depicts the values of both firms with respect to the initial demand under the condition of FMA and SMA to understand the diffrence of the project values between two conditions. The common parameter values are set as $D_{00} = 3, D_{01} = 0$ and $D_{11} = 6$ and $D_{10} = 10$ is used for representing the condition of FMA while $D_{10} = 8$ in the SMA setting. Note that values of $V_L^{(1,1)}(0, y) - I$ and $V_L^{(0,1)}(0, y)$ are common under both conditions.

As above mentioned in the *Figure 4* both project values under the condition of SMA are identical; both firms defer the investment at period zero until the initial demand becomes $y = Y_D$. Although we assume that firm L and firm F are competitors the result implies that the equilibrium strategies of the competitive firms are equivalent to those of the firms in cooperation. It is called a *coordination* in game theory, which means that one firm is in cooperation with the other to accomplish their common interest. The firms in coordination choose identical strategies in this case. Consequently, under the condition of SMA the equilibrium strategies of the competitive firms are equivalent to those of the coordinated firms. In other words, both firms act as if they were one firm that obtains the marginal cash flow of D_{00} before the investment and that of D_{11} after the investment. For that reason we refer the value of $V_L^{(0,0)}(0, y)$ $(= V_F^{(0,0)}(0, y))$ as *coordinated value*. The coordinated value can be accomplished if a firm can coordinate with the other firm. It is important to point out that the coordination is not an assumption but the result in the equilibrium under SMA.

Under the condition of FMA, on the other hand, asymmetric equilibria emerge in the interval of $y \in (Y_F, Y_I)$ and firm L's project value is larger than firm F's in the interval. Especially, firm L's project value is larger than the coordinated value in the interval of $y \in (Y_G, Y_H)$. This implies that it is better for firm L to take advantage of the LAC and invest in the project solely at period zero as the first mover when the initial value is in $y \in (Y_G, Y_H)$. In the interval of $y \in (Y_E, Y_G)$ or $y \in \left(Y_H, Y_J\right)$, however, the value of $V_L^{(0,0)}(0, y)$ and $V_F^{(0,0)}(0, y)$ are smaller than the coordinated value. It must be noted that in these intervals the strategies that correspond to the coordinated value are not in the equilibrium even though they are feasible. In other words, neither firm can accomplish the coordinated value under the condition of FMA in the interval of $y \in \left(Y_E, Y_J\right)$. This can by explained by a fear of preemption. Both firms know that they accomplish the coordinated value by waiting to invest until the optimal stopping time if they can cooperate with each other. In the presence of the preemption, however, they have no choice but invest in the project earlier due to a fear of preemption, which decreases the project values of both firms. It can also be interpreted as an example of prisoner's dilemma; without a coordination both players must make less attractive decisions.

insert Figure 5

Figure 6 also shows the difference of the equilibrium strategy between FMA and SMA. The figure illustrates the project values at period zero under LAC with respect to the marginal cash flow D_{11}[4]. The parameter values are common in the first two figures; $D_{10} = 8, D_{00} = 3, D_{01} = 0$. The initial demand is set as $Y(0) = 45$ and let D_{11} be in the interval $D_{11} \in [3, 8]$ where inequality (9) is always

[4]We remove the value of $V_{50\%}^{(0,0)}(0, Y(0))$ from the figure. As discussed in this section, it is hard to compare the results of FMA and SMA in the 50% criteria because the condition of FMA could create asymmetric equilibria while that of SMA could create symmetric equilibria.

satisfied. Note $D_{11} \in [3,5)$ satisfies the condition of FMA while $D_{11} \in (5,8]$ satisfies that of SMA. $D_B = 5$ represents the boundary value between FMA and SMA.

First we analyze the figure in the interval under FMA, i.e., $D_{11} \in [3,5)$. The project value of firm L, $V_L^{(0,0)}(0,y)$, is equivalent to $V_L^{(1,0)}(0,y) - I$ and the project value of firm F, $V_F^{(0,0)}(0,y)$, is equivalent to $V_L^{(0,1)}(0,y)$ when $D_{11} < D_A$. This indicates that only firm L invests at period zero while firm F defers the investment under LAC. In the interval $D_A \leq D_{11} < D_B$ the equilibrium strategy for both firms is to invest immediately at period zero and the both project values are on the line of $V_L^{(1,1)}(0,y) - I$. We can observe that under the condition of FMA, firm L that has a competitive advantage over firm F could have opportunities to make the best use of the competitive advantage and to become the first mover to maximize the project value.

The project values of the firms and the corresponding equilibrium strategies are discontinuously changed at the boundary between FMA and SMA at $D_{11} = D_B$. When $D_{11} > D_B$ both project values jump up and the corresponding equilibrium strategies are also changed from investing immediately into waiting to invest. We can figure out that both firms can obtain the real option values, which are given by $V_L^{(0,0)}(0,y) - \left\{V_L^{(1,1)}(0,y) - I\right\}$. This results from the absence of preemption; namely both firms can consequently coordinate the optimal timing of the investment without a fear of preemption. The real option values tend to decrease as D_{11} increases and finally vanishes at the point D_c where investing immediately becomes the equilibrium strategy. It is important to point out that both project values keep increasing as D_{11} increases under the conditions of SMA. Consequently, we can conclude again that under the condition of SMA, the existence of the real option to defer the investment always has a positive impact on the project values of both firms.

4 Concluding Remarks

In this paper we develop a valuation model when there exist two competitive firms under the demand uncertainty. We propose a numerical procedure to derive both project values and equilibrium strategies in the duopolistic environment. We apply two types of criteria, which we call firm L advantage criterion(LAC) and 50% criterion to choose the unique equilibrium under the simultaneous decision case.

Our numerical examples reveal the following. Under the condition of FMA there could emerge asymmetric equilibria in the LAC where firm L usually becomes the first mover and relishes the larger project value. However, the project values of both firms often become smaller than the coordinated project value because of the existence of a fear of preemption. On the other hand, the condition of SMA causes a completely different influence on the equilibrium strategies of the competitive firms. Both firms' project values are equal and the corresponding equilibrium strategies are always symmetric even if LAC is assumed. Furthermore, it is shown that both firms can make the best use of the flexibility to defer the investment in the presence of competition. As a result the project values of both firms are equivalent to the coordinated project value. In other words, the value of flexibility under uncertainty does not conflict with the existence of competition under the condition of SMA, which contrasts to the results under the condition of FMA.

References

Constantinides, G. M. (1978): "Market Risk Adjustment in Project Valuation," *Journal of Finance*, 33(2), 603–616.

Cox, J. C., and S. A. Ross (1976): "The Valuation of Options for Alternative Stochastic Processes," *Journal of Financial Economics*, 3(1-2), 145–166.

Dixit, A. K., and R. S. Pindyck (1994): *Investment under Uncertainty*. Princeton University Press.

Fudenberg, D., and J. Tirole (1985): "Pre-emption and Rent Equalisation in the Adoption of New Technology," *The Review of Economic Studies*, 52, 383–401.

Grenadier, S. R. (1996): "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets," *Journal of Finance*, 51(5), 1653–1679.

Grenadier, S. R. (2002): "Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms," *The Review of Financial Studies*, 15(3), 691–721.

Harsanyi, J. C., and R. Selten (1988): *A General Theory of Equilibrium Selection in Games*. MIT Press.

Hoppe, H. C. (2000): "Second-mover Advantages in the Strategic Adoption of New Technology under Uncertainty," *International Journal of Industrial Organization*, 18, 315–338.

Huisman, K. J. M. (2001): *Technology and Investment: A Game Theoretic Real Options Approach*. Kluwer Academic Publishers.

Huisman, K. J. M., and P. M. Kort (2003): "Strategic Investment in Technological Innovations," *European Journal of Operational Research*, 144, 209–223.

——— (2004): "Strategic technology adoption taking into account future technological improvements: A real options approach," *European journal of Operational Research*, 159, 705–728.

Imai, J., and T. Watanabe (2004): """A Two-stage Investment Game in Real Option Analysis," Discussion Paper.

——— (2006a): "Kyousou Jyoukyouka deno Real Option to Jyuunannsei no Wana," Discussion Paper.

——— (2006b): "A Numerical Approach for Real Option Values and Equilibrium Strategies in a Duopoly," Discussion Paper.

Kijima, M., and T. Shibata (2002): "Real Options in a Duopoly Market with General Volatility Structure," 2002.

Kulatilaka, N., and E. C. Perotti (1998): "Strategic Growth Options," *Management Science*, 44(8), 1021–1031.

Lambrecht, B. (2000): "Strategic Sequential Investments and Sleeping Patents," *Project Flexibility, Agency, and Competition*, pp. 297–323.

McDonald, R., and D. Siegel (1984): "Option Pricing When the Underlying Asset Earns A Below Equilibrium Rate of Return: A Note," *Journal of Finance*, 39(1), 261–265.

Smets, F. (1991): "Exporting versus FDI: The effect of uncertainty, irreversibilities and strategic interactions," Working Paper, Yale University.

Smit, H. T. J., and L. A. Ankum (1993): "A Real Options and Game-Theoretic Approach to Corporate Investment Strategy Under Competition," *Financial Management*, pp. 241–250.

Smit, H. T. J., and L. Trigeorgis (2001): "Flexibility and Commitment in Strategic Investment," *Real Options and Investment Under Uncertainty/ Classical Readings and Recent Contributions*, pp. 451–498.

		Firm F	
		Invest	Not invest
Firm L	Invest	(D_{11}, D_{11})	(D_{10}, D_{01})
	Not invest	(D_{01}, D_{10})	(D_{00}, D_{00})

Figure 1: **Marginal cash flow of the project**. When both firms have already invested in the project the cash flow obtained by each firm at each period is given by $D_{11}Y(m)\Delta t$. It is given by $D_{00}Y(m)\Delta t$ when neither firm has invested yet. The cash flow obtained by firm L is given by $D_{10}Y(m)\Delta t$ when only firm L finishes the investment. The cash flow obtained by firm F is given by $D_{01}Y(m)\Delta t$ in this case. On the other hand, when only firm F has invested in the project the cash flow obtained by firm L and firm F are given by $D_{01}Y(m)\Delta t$ and $D_{10}Y(m)\Delta t$, respectively.

	Invest	Not invest
Invest	$\left(v_L^{(1,1)} - I, v_F^{(1,1)} - I\right)$	$\left(v_L^{(1,0)} - I, v_F^{(1,0)}\right)$
Not invest	$\left(v_L^{(0,1)}, v_F^{(0,1)} - I\right)$	$\left(v_L^{(0,0)}, v_F^{(0,0)}\right)$

Figure 2: **The payoff matrix in case of simultaneous decision**

	conditions of equilibria	equilibria	condition of equilibrium selection	selected equilibrium	V_L	V_F
1	$v_L^{(1,1)} - I > v_L^{(0,1)}$, $v_L^{(1,0)} - I > v_L^{(0,0)}$,	$(1,1)$		$(1,1)$	$v_L^{(1,1)} - I$	$v_F^{(1,1)} - I$
2	$v_L^{(1,1)} - I > v_L^{(0,1)}$, $v_L^{(1,0)} - I < v_L^{(0,0)}$, $v_F^{(1,0)} - I > v_F^{(0,0)}$	$(1,1)$		$(1,1)$	$v_L^{(1,1)} - I$	$v_F^{(1,1)} - I$
3	$v_L^{(1,1)} - I > v_L^{(0,1)}$, $v_L^{(1,0)} - I < v_L^{(0,0)}$, $v_F^{(1,0)} - I < v_F^{(0,0)}$,	$(1,1),(0,0)$	$v_L^{(1,1)} - I > v_L^{(0,0)}$	$(1,1)$	$v_L^{(1,1)} - I$	$v_F^{(1,1)} - I$
4	$v_L^{(1,1)} - I > v_L^{(0,1)}$, $v_L^{(1,0)} - I < v_L^{(0,0)}$, $v_F^{(1,0)} - I < v_F^{(0,0)}$	$(1,1),(0,0)$	$v_L^{(1,1)} - I < v_L^{(0,0)}$	$(0,0)$	$v_L^{(0,0)}$	$v_F^{(0,0)}$
5	$v_L^{(1,1)} - I < v_L^{(0,1)}$, $v_L^{(1,0)} - I > v_L^{(0,0)}$, $v_F^{(1,0)} - I > v_F^{(0,0)}$	$(1,0),(0,1)$	$v_L^{(1,0)} - I > v_L^{(0,1)}$	$(1,0)$	$v_L^{(1,0)} - I$	$v_F^{(1,0)}$
6	$v_L^{(1,1)} - I < v_L^{(0,1)}$, $v_L^{(1,0)} - I > v_L^{(0,0)}$, $v_F^{(1,0)} - I > v_F^{(0,0)}$	$(1,0),(0,1)$	$v_L^{(1,0)} - I < v_L^{(0,1)}$	$(0,1)$	$v_L^{(0,1)}$	$v_F^{(0,1)} - I$
7	$v_L^{(1,1)} - I < v_L^{(0,1)}$, $v_L^{(1,0)} - I > v_L^{(0,0)}$, $v_F^{(1,0)} - I < v_F^{(0,0)}$	$(1,0)$		$(1,0)$	$v_L^{(1,0)} - I$	$v_F^{(1,0)}$
8	$v_L^{(1,1)} - I < v_L^{(0,1)}$, $v_L^{(1,0)} - I < v_L^{(0,0)}$, $v_F^{(1,0)} - I > v_F^{(0,0)}$	$(0,1)$		$(0,1)$	$v_L^{(0,1)}$	$v_F^{(0,1)} - I$
9	$v_L^{(1,1)} - I < v_L^{(0,1)}$, $v_L^{(1,0)} - I < v_L^{(0,0)}$, $v_F^{(1,0)} - I < v_F^{(0,0)}$	$(0,0)$		$(0,0)$	$v_L^{(0,0)}$	$v_F^{(0,0)}$

Table 1: Calculation of $V_L^{(0,0)}$ and $V_F^{(0,0)}$ under the second criterion of equilibrium selection

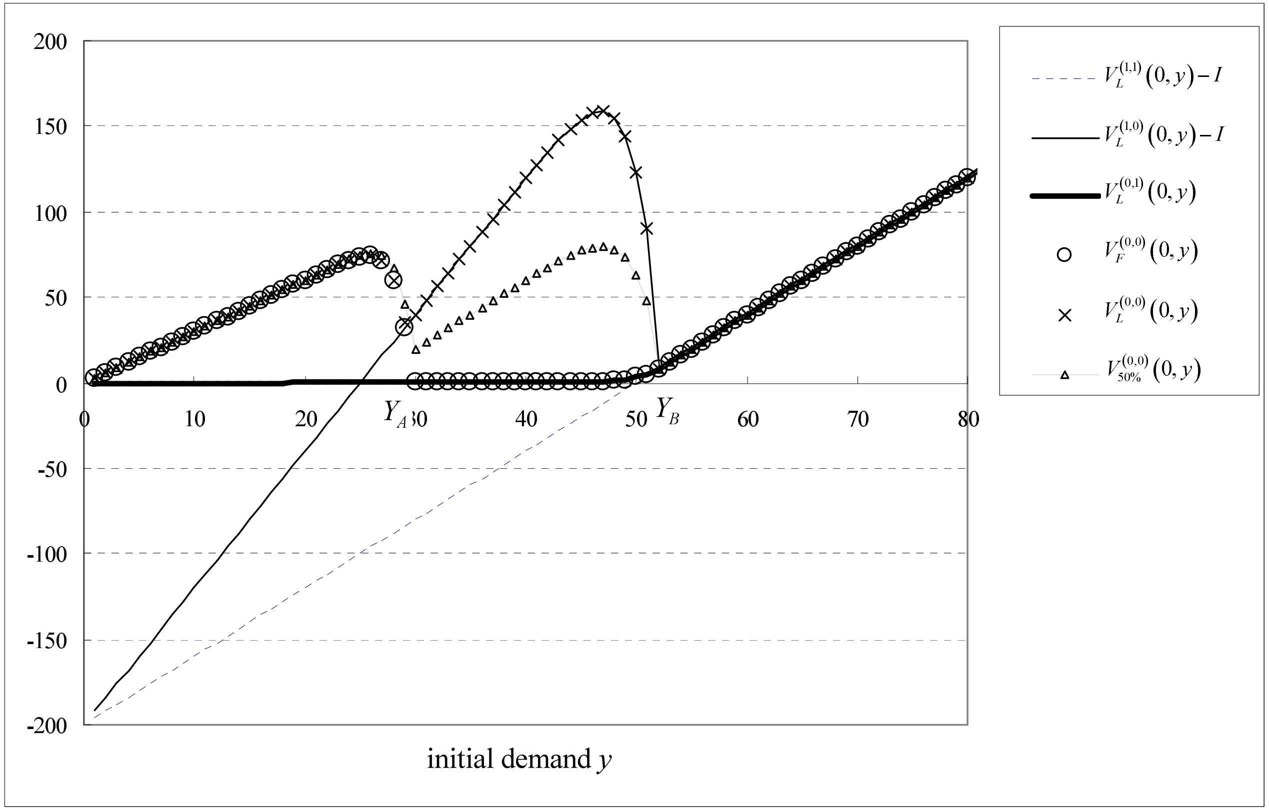

Figure 3: The project values of the firms with respect to the initial demand under FMA

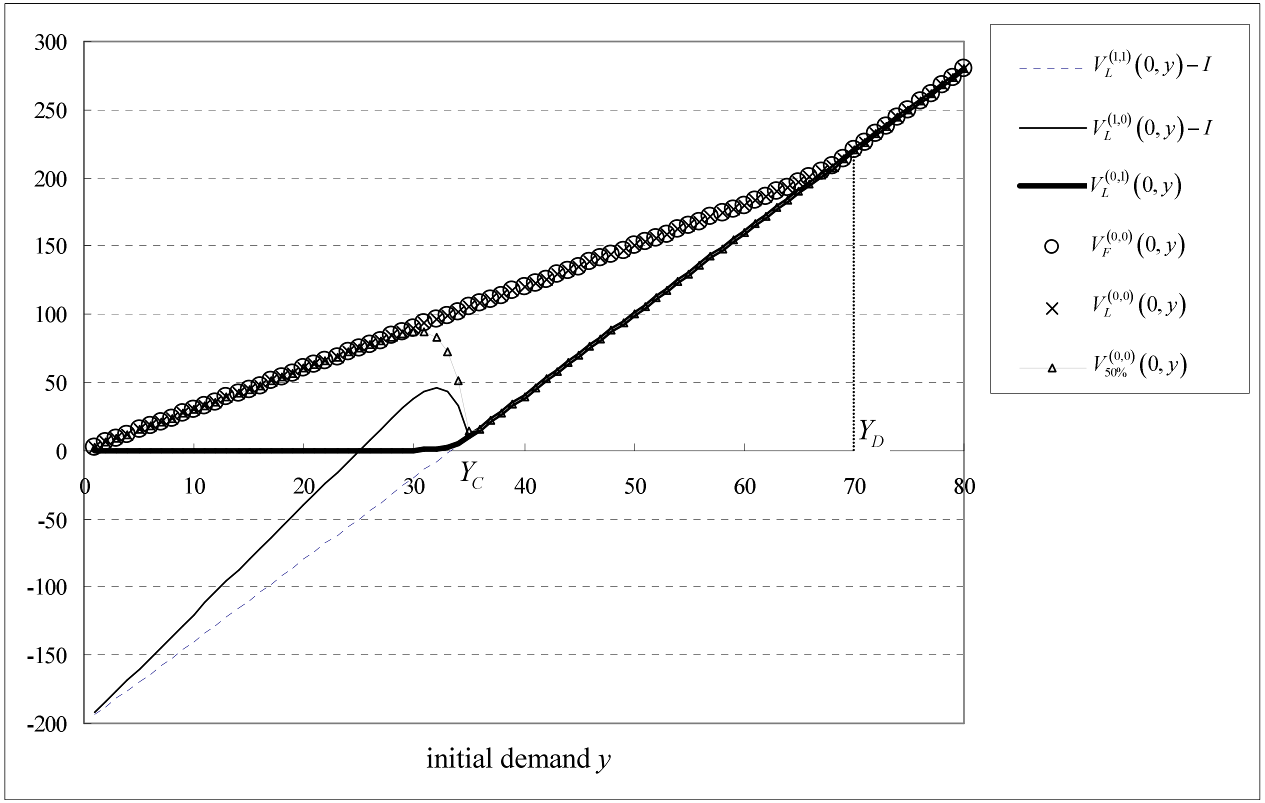

Figure 4: The project value of the firms with respect to the initial demand under SMA

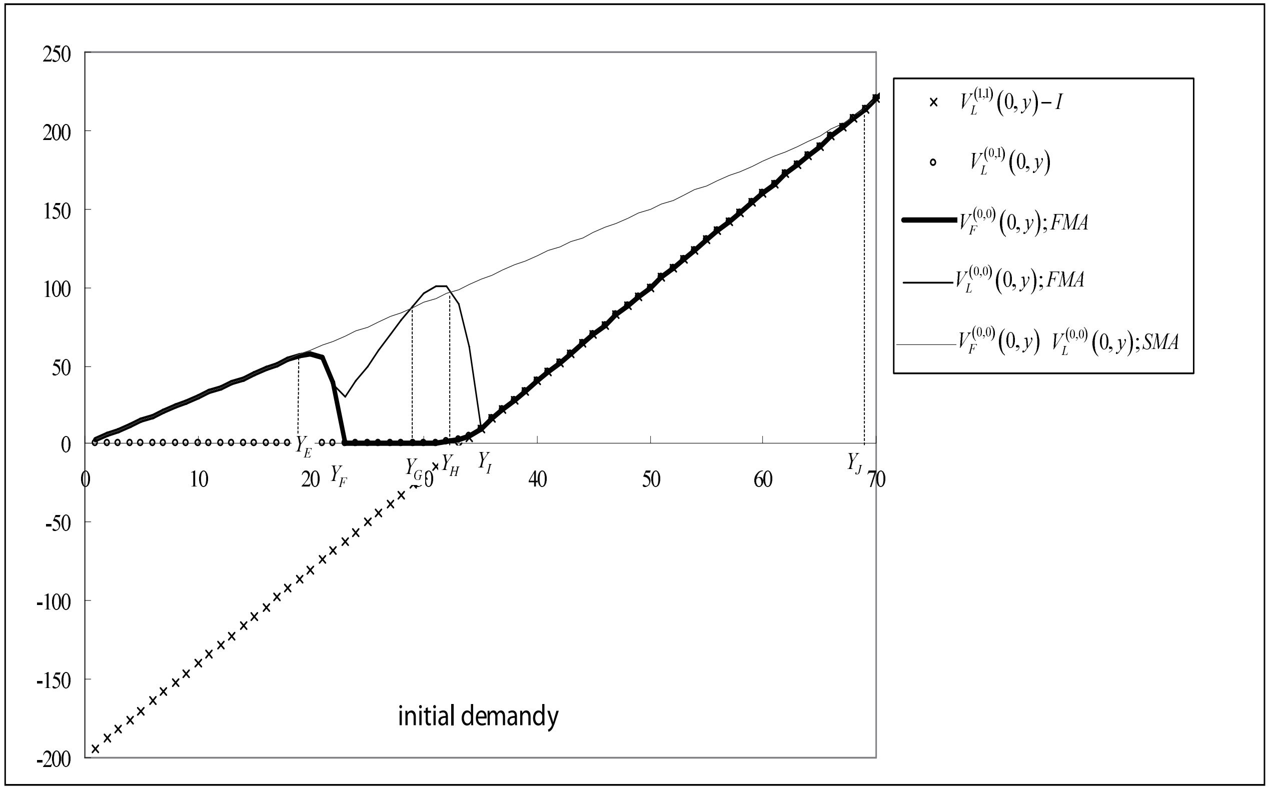

Figure 5: The comparison of the project values under FMA and SMA

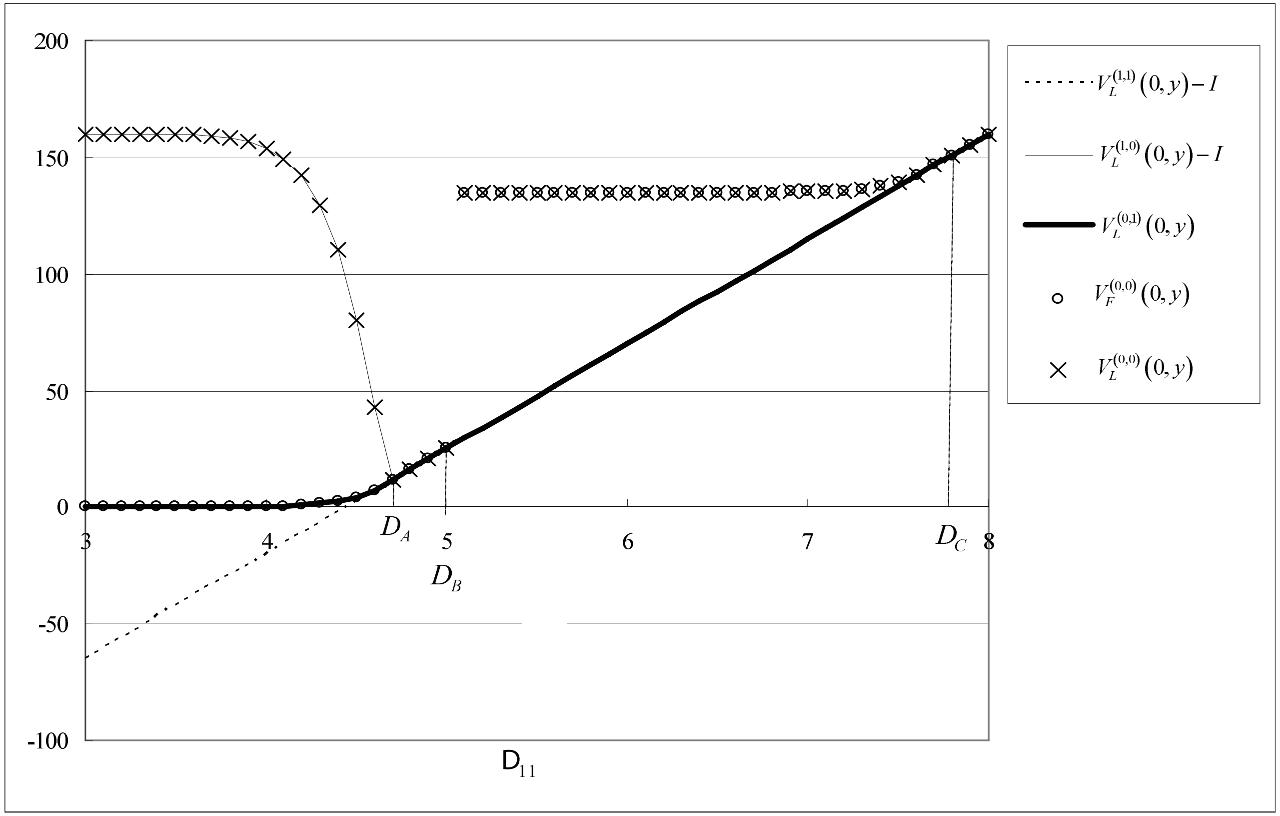

Figure 6: The comparison of the project values with respect to D_{11} under FMA and SMA