Valuing a firm's capital structure using profit caps, floors and bond default options

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Abstract

Despite many past papers concerning a firm's capital structure, the valuation of debt and equity and cost of capital, there are few that explicitly codify contingent sharing rules for the firm's cashflow over time. We motivate equity and debt valuation by modeling tax and distress costs using cap and floor technology as well as a default option at maturity. This approach sheds light on theoretical valuation issues, optimal capital structure choice as well as a firm's component costs of capital. JEL: G13, G33, G35.

1 Introduction

The capital structure of firms has been studied for at least fifty years (since before Modigliani and Miller (1958) [1]) yet the debate as to proper valuation techniques still seems to be current. A recent paper even suggested that tax shield valuation was not possible (see Fernandez (2004) [2] and Cooper and Nyborg (2006) [3] for a critique).

Evolving alongside net present value techniques, capital structure theory has traditionally modelled the NPV consequences of different financing decisions over time (see amongst others Miles and Ezzell (1985) [4], Ruback (2002) [5] and the work in numerous corporate finance text books). Most often these papers treat the expectation of future cashflows as certain, then discount them as if they were uncontingent using a risk adjusted rate (an input to the model). This is not an adequate approach when costs of capital are dynamic.

Exceptions to this rule follow from the introduction of contingent claims pricing (Black and Scholes (1973) [6] pricing and subsequent risk–neutral valuation Cox, Ross, Rubinstein (1979) [7]) and of course most notably the idea of Merton (1974) [8] that the capital structure itself has a default option on debt embedded which can be valued using option technology. Thus the idea of NPVs expanded to include options was brought to bear on corporate finance and project valuation (amongst others see Trigeorgis (1996) [9] for the contribution of real options, flexibility, and strategy to corporate finance).

The strand of papers using contingent claim and real options technology to value corporate securities has become increasingly influential in the corporate finance literature as models such as Ingersoll (1977) [10], Leland (1994) [11], Leland and Toft (1996) [12] (as well as practitioner versions such as KMV Moody's) gain popularity and credence.

There has also been substantial empirical work evaluating the cost–benefit trade–off to leverage including, most recently, papers by Graham (2000) [13], Kemsley and Nissim (2002) [14], Vassalou and Xing (2004) [15] and Bris, Welch and Zhu (2006) [16] as well as others[1] including comparisons of competing models (Eom, Helwege and Huang (2004) [17].

We do not question that finance can value tax, tax shields, costs of distress and default. Rather we question those papers that seek to do so without

[1] A great many other papers, many of considerable note, have been omitted from this paper which cannot attempt to fully review all past literature.

specifying the sharing rules between the competing firm claimants. Only by specifying these sharing rules (as boundary conditions of different claims) can proper valuation be conducted within a contingent claims framework. Moreover since it is the different boundary conditions which distinguish, say the tax from the equity, it is not possible to make conclusions as to the appropriate discount rates for such claims unless their sharing rules are made explicit.

2 This paper

In this paper, we make the firm's flows to competing claimants explicit. We do this by describing equity and later tax claims as caps on firm profit/cashflow over the bond's life at a level of the coupon rate, while bond holders are short this cap.

Equity and bond holders also exchange a terminal bond default option and are therefore also respectively long and short a European call on the value of assets at bond maturity (at which time it must be refinanced for another round).

The only drawback of this approach is that we have to take a stylized approach to the issue of financial distress before the bond's principal is repaid. We assume that the firm can fail to pay complete coupons before maturity without triggering default (which can occur at maturity only) and without giving the bond holders recourse to claim these later. However, if it does fail to completely pay coupons, we presume that it will incur costs of distress which we model as a function of a floor on the profit rate at a level of the coupon rate. Thus distress before maturity is separated from default at maturity which also has its own deadweight cost associated with the bankruptcy, legal and other fees that must be borne when attempting to refinance a firm in default. Distress and default costs are modelled using floors on profit and terminal puts on firm value.

Most closely in spirit, we are modelling something akin to the non–cumulative Income Bonds described (on page 791) in Weston and Copeland (1986) [18], where coupons are paid either fully or to the extent that current profit will allow, without future recourse from later profit. Although once popular (see McConnell and Schlarbaum (1981) [19] in particular for railroad financing) these have disappeared as a source of finance[2] we can still analyze

[2] There are potential agency and accounting manipulation problems associated with this

more realistic bond structures if we assume that the pricing of actual bonds is *similar* to Income Bond pricing.

The advantage of this approach is that we can make use of recent results in cap and floor valuation. In particular for geometric flows and values, Shackleton and Wojakowski (2007) [20] contains closed form expressions for the finite maturity caps and floors which are necessary to value the contingent coupons and dividends that an Income Bond financed firm contains.

Thus we are adopting an approach consistent with Modigliani and Miller's (1958) [1] idea that total valuation of cashflows should not depend on their separation or aggregation. Then we proceed using the risk–neutral valuation methods embedded in almost all real options literature to value the coupons and principal of an Income Bond financed firm together with the tax, distress and default costs and the equity residual. This is justified by the fact that the equity of the firm (and possibly the bond as well) will be well traded and as such represents a sufficient hedge asset to replicate all other firm claims such as tax, distress and default costs.

The rest of this paper contains sections that build the model stage by stage explaining the cap and floor notation and then sequentially adding the frictional terms.

3 Caps and floors

Consider assets A whose profit or cashflow rate[3] is driven by a continuous geometric Brownian motion P, i.e. generating profit or cash of Pdt in the instantaneous time dt. Their valuation is governed by risk–neutral dynamics (i.e. W_t is a standard Brownian motion under current risk–neutral expectations E_0^Q) with positive constants, r, δ, σ respectively representing the risk free, growth shortfall (asset yield) and volatility rates

$$\frac{dP_t}{P_t} = (r - \delta)\, dt + \sigma dW_t \Longleftrightarrow \frac{P_t}{P_0} = e^{\left(r-\delta-\frac{1}{2}\sigma^2\right)t+\sigma W_t}.$$

instrument, furthermore they have often been used to refinance failing firms prompting McConnell and Schlarbaum to say that they have "the smell of death"! We agree that their valuation is useful for firms in distress but also argue that the separability and tractablity implied by the non–cumulative nature of missed coupons is attractive for all risky firms, including start–ups as well as declining firms.

[3]By profit we mean income attributable to debt and equity capital holders. A more sophisticated model could take the difference between profit and actual cashflow into account, however here we make no distinction and set all accounting accruals to zero.

The growth shortfall in the profit process δ represents an asset yield on its present value because the expected present value of profits is given by

$$A_0 = \int_0^\infty e^{-rt} E^Q \left[P_t\right] dt = \int_0^\infty P_0 e^{-\delta t} dt = \frac{P_0}{\delta}.$$

Alternatively, the profit P_0 represents a constant cash yield (or fraction δ) on its present value $A_0 = P_0/\delta$. Thus δ is referred to as an asset yield on the underlying value of P_0 as well as a rate of return shortfall.[4]

Initially we will partition this into (income) bond and equity, then later firm frictions will be introduced. Assets A are driven by the same dynamics as P and although the firm frictions will change what fraction of can be recovered by capital holders, A remains exogenous to the capital structure decision. Thus A_0 represents the current total value of the firm (bond and equity) when there are no firm frictions or total of bond, equity and firm frictions if they are included.

Now suppose that the income bond financing requires service of a continuous coupon at a rate K, i.e. an amount Kdt over time dt. The key condition that determines whether or not this coupon rate is being serviced in full or not is given by $P_t - K \lesseqgtr 0$. If the profit rate exceeds the coupon rate then the latter is paid in full and the surplus is paid to equity holders as a continuous dividend. If not, then what profit is available is paid to the bond holders and the equity holders suspend their dividend stream.

Using notation $()^+$ to indicate the positive part, the profit P can be decomposed into floorlet and caplet payoffs which map to bond (coupon rate less floorlet) and equity (caplet)

$$\begin{array}{rcccc} P & = & K - (K-P)^+ & + & (P-K)^+ \\ \text{Profit Rate} & = & \text{Coupon Rate} - \text{Floorlet} & + & \text{Caplet} \\ & = & \text{Debt Flow} & + & \text{Equity Flow.} \end{array}$$

Thus in terms of caplet or floorlet payoffs on cashflow P struck at K, we can say that at all instants t, the equity holder's flow is a long a cap on P while the bond holders flow is long the rate P but short the cap (equivalently long K and short a floorlet on P at K). Either way it is understood that the

[4] This δ is not the dividend yield on equity, that is dynamic and will be derived later.

total flow P has been accounted for.

$$\begin{aligned}
\text{Bond cashflow over } dt &= \left(K - (K - P_t)^+\right) dt \\
&= \left(P_t - (P_t - K)^+\right) dt \\
\text{Equity cashflow over } dt &= (P_t - K)^+ \, dt.
\end{aligned}$$

Both caplet and floorlet payoffs are of infinitesimal duration dt and are uncountable in number but integrable. Valuation requires us to discount these payoffs to time 0 thus defining caplet $c\,(P, K)$ floorlet $f\,(P, K)$ (the term floorlet and f is used rather than put to avoid confusion with P the profit variable) values as risk–free discounted, risk–neutral expectations given by

$$\begin{aligned}
c\,(P, K, t) &= e^{-rt} E_0^Q \left[(P_t - K)^+ \right] \\
f\,(P, K, t) &= e^{-rt} E_0^Q \left[(K - P_t)^+ \right].
\end{aligned}$$

Under Black Scholes dynamics and valuation these expressions yield standard European option values (lower case expressions are used here to distinguish from the integrated caps and floors in the next paragraph)

$$\begin{aligned}
c\,(P_0, K, t) &= P_0 e^{-\delta t} N\,(d_1\,(t)) - K e^{-rt} N\,(d_0\,(t)) \\
f\,(P_0, K, t) &= K e^{-rt} N\,(-d_0\,(t)) - P_0 e^{-\delta t} N\,(-d_1\,(t)) \\
d_{\beta=1,0}\,(t) &= \frac{\ln P_0 - \ln K + \left(r - \delta + \left(\beta - \frac{1}{2}\right) \sigma^2\right) t}{\sigma \sqrt{t}}.
\end{aligned}$$

Here, cumulative normal integrals $N\,(d_\beta)$ are labelled with parameters $d_{1,0}$ (d_1 is common to the standard setting but the standard Black–Scholes d_2 is represented here as d_0). Furthermore, β has an interpretation as an elasticity for reasons that will also become clear later.

Each caplet and floorlet operates over dt only. Over the life of the bond T the caplet can be integrated up to retrieve a present value for the total cap[5] $C\,(P_0, K, T)$ on profits P_t over the life T (and floor $F\,(P_0, K, T)$ from floorlets)

$$\begin{aligned}
C\,(P_0, K, T) &= \int_0^T c\,(P, K, t)\, dt \\
F\,(P_0, K, T) &= \int_0^T f\,(P, K, t)\, dt.
\end{aligned}$$

[5]The cap has notation $C(P_0, K, T)$ indicating that it is of length T while the caplet $c(P_0, K, t)$ operates at t alone, both have current value conditioned on P_0. The former is a value the second a flow rate, while $c(P_0, K, t)dt$ is an infinitessimal cash sum.

Both of these integrals converge for perpetual bonds ($T \to \infty$) so long as the asset payout ratio δ and risk free r are positive (since these conditions ensure that $c(.,t)$, $f(.,t) \to 0$ as $T \to \infty$).

Moreover closed form expressions and sensitivities for these integrals are given in Shackleton and Wojakowski (2007) [20]. These will be presented later when comparative statics are discussed and valuation examples given. Although slightly more complex than simple Black Scholes European options (they contain four, not two d_β components) they are no more difficult to programme and offer very similar intuition.

The expected present values of the unconditional flows are easily derived

$$\begin{aligned}
\int_0^T e^{-rt} K dt &= \frac{K}{r}(1-e^{-rT}) \\
\int_0^T e^{-rt} E_0^Q [P_t] \, dt &= \frac{P_0}{\delta}(1-e^{-\delta T}) = A_0(1-e^{-\delta T}).
\end{aligned}$$

4 Costless default at T

Thus the decomposition of expected PV asset flow over the horizon $0, T$ between equity and bond is

$$\begin{aligned}
\int_0^T e^{-rt} P_t dt &= \int_0^T \left(e^{-rt}K - f(P,K,t)\right) dt + \int_0^T c(P,K,t) \, dt \\
\frac{P_0}{\delta}(1-e^{-\delta T}) &= \frac{K}{r}(1-e^{-rT}) - F(P_0,K,T) + C(P_0,K,T)
\end{aligned}$$

where C, F are subject to this cap–floor parity condition.

Now terminal options can be added to reflect sharing at time T. If A_T exceeds X the face value of debt, then the equity holders get the residual value and the bond face value is paid in full. If refinanced some of this value may be used to buy the bond financing over the next time interval. If A_T does not exceed X the face value of debt, the equity holders get nothing and although the bond holders get the full face value of the firm, this is still less than that promised X.

Thus as well as exchanging a cap (or equivalently a floor) on the interest expense over T, equity and bond holders have also exchanged a terminal call on A_T. This terminal default call is a lump sum given by the same formulae for the caplet and floorlet[6] but applied to A, X and the final time T

$$\begin{aligned} c(A_0, X, T) &= A_0 e^{-\delta t} N(d_1(T)) - X e^{-rt} N(d_0(T)) \\ f(A_0, X, T) &= X e^{-rt} N(-d_0(T)) - A_0 e^{-\delta t} N(-d_1(T)) \\ d_{\beta=1,0}(T) &= \frac{\ln A_0 - \ln X + \left(r - \delta + \left(\beta - \frac{1}{2}\right)\sigma^2\right) T}{\sigma\sqrt{T}}. \end{aligned}$$

The equity holders are long and the bond holders short this call. This element is exactly the same as the Merton (1974) [8] setup, except that the firm is paying out total cash on A at a rate δ (to all claimants) whereas typically, Merton's model is applied to a zero payout firm.

The advantage of our approach is that we can allow for early distribution of cash in a meaningful and contingent fashion using the caps and floors above. Thus the firm's equity E_0 has two components, a continuous cap over $0, T$ on profit P at K as well as one final option on A at X to reflect all profitability beyond T.

$$\begin{aligned} \text{Equity value } E_0 &= \int_0^T e^{-rt} E_0^Q\left[(P_t - K)^+\right] dt + e^{-rT} E_0^Q\left[(A_T - X)^+\right] \\ &= \int_0^T c(P_0, K, T)\, dt + c(A_0, X, T) \\ &= C(P_0, K, T) + c(A_0, X, T) \end{aligned}$$

The bond value B_0 until time T is long the profit in the company less the profit cap, and also long the value of the firm operations from T onwards less the call on A at X,this can also be seen to be the total value of the firm A_0 (over infinite horizon) less the two caps (one until T the other at T alone).

$$\begin{aligned} \text{Debt value } B_0 &= \int_0^T e^{-rt} E_0^Q\left[P_t - (P_t - K)^+\right] dt + e^{-rT} E_0^Q\left[A_T - (A_T - X)^+\right] \\ &= A_0(1 - e^{-\delta T}) - \int_0^T c(P_0, K, T)\, dt + A_0 e^{-\delta T} - c(A_0, X, T) \\ &= A_0 - C(P_0, K, T) - c(A_0, X, T) \end{aligned}$$

The bond (and equity) value(s) can also be represented using floor and put technology. This representation is only possible due to the separation

[6] We label the corresponding terminal put f for consistency and to avoid confusion with the profit process.

of distress (unpaid coupons without recourse) and default (on bond repayment). Clearly the actual distress and default condition that firms face is more complex, almost certainly linking the two with some implicit early exercise of default a possibility of distress. However, this sort of contingent model is beyond the scope of this paper.

Note that without frictions, total value A_0 has been preserved. In the next section, caps, floors and default options will be used to motivate corporate tax, distress and default costs, in which case the value jointly attributable to bond and equity holders will no longer be invariant to the capital structure choices of K, X.

5 With frictions

5.1 Tax

Now suppose that the tax regime in which the firm operates is very simple. When the firm profit less interest expense is positive, a fraction τ is lost to the firm as governmental tax. This allows use to specify the tax claim cashflow and also its value T and redefine that to equity. In this situation the total firm residual remains the same only it is now shared between equity and tax claimants.

$$\begin{aligned}
\text{Tax } T_0 &= \int_0^T e^{-rt} E_0^Q \left[\tau \left(P_t - K\right)^+\right] dt = \tau C\left(P_0, K, T\right) \\
E_0 &= \left(1-\tau\right) C\left(P_0, K, T\right) + c\left(A_0, X, T\right)
\end{aligned}$$

Clearly this is simplistic and in fact this situation puts the tax claim in exactly the same valuation camp as the equity itself since their cashflow conditions are identical in scale and differ only by fractions $\tau, 1-\tau$. In order to introduce discount rates for tax that were different to those on equity, a more convoluted and realistic sharing rule would have to be defined and moreover one that had fundamentally different payoff characteristics to equity and tax claimants. Most likely, real tax regimes are path dependent and therefore this simple rule faces the same criticism as above where the requirement of addititative separability reduces the ability to match actual practice.

5.2 Distress

The next friction requires treatment of costs that the firm must bear continuously if the profit figure is insufficient to meet the coupon in full. In order to put off immediate default (at the bond holders instigation) the firm will have to engage in a court process (on behalf of the equity holders) to postpone creditor action. Thus we model a proportional flow of legal and other fees that increase with coupon shortfall as distress increases, this is easiest to represent using the floorlets and floor valuation notation

$$\text{Distress cost} = \int_0^T e^{-rt} E_0^Q \left[\theta \left(K - P_t \right)^+ \right] dt = \theta F \left(P_0, K, T \right).$$

This cost comes out of firm profits at a time when these are less than the coupon rate so the bond holders bear the cashflow loss immediately. At this time dividends are suspended and equity holders enjoying limited liability do not recapitalize the firm (we thus abstract from strategic debt service by equity holders). Thus the bond holder flow must be amended for a short option position (in addition to the profit cap transferred to the equity holders), that of court and legal fees incurred by the firm when the equity holders resist the bondholders taking immediate control and act to maintain a future claims on profits.

5.3 Default

Although the equity holders can apply firm cashflow (at the bond holders expense) to the postponement of default, they cannot put it off any longer that the time horizon T. At this time, the bankruptcy option crystallizes and distress becomes default. Legal fees can no longer be spent to protect the equity holders residual claim and their claim is limited to the ongoing value of the firm determined at that time alone rather than a claim determined at a forward date.

If default occurs, the current equity value becomes zero although potentially new equity holders will be able to then participate in the refinanced firm on new terms for another maturity T'. There are many possibilities for modelling the deadweight losses that the firm bears but we presume that this refinancing of the firm's continued activities will be more difficult the lower the value of total firm assets A_T at T. Thus we model the default cost as a fraction of the shortfall in asset value compared to face value

$$\text{Default cost} = e^{-rT} E_0^Q \left[\lambda \left(X - A_T \right)^+ \right] = \lambda f \left(A_0, X, T \right).$$

Again, ex–post, this is a cost that comes out of the bondholders cashflow but it will be anticipated ex–ante in both debt and equity valuation indicated that in expectation default costs (and taxes) are shared between both capital claimants.

Note however that there is the chance that although the firm had been in distress before time T it might have recovered by T such that bankruptcy was not an issue and refinancing a simple matter. This is potentially problematic if the cashflow or especially the profit of the firm (in the case where accruals are present) were not exogenous but subject to accounting opinion that the equity holders were able to influence! These are the exact problems that Weston and Copeland (1986) [18] and McConnell and Schlarbaum (1981) [19] refer to in their discussion of income bond usage.

We label the total deadweight losses due to distress and default as L these are the sum of distress and default costs

$$L_0 = \theta F \left(P_0, K, T \right) + \lambda f \left(A_0, X, T \right)$$

and now the total (invariant) firm value A has been decomposed into four parts of which only two B, E are insiders to the firm

$$\begin{aligned} A_0 &= L_0 + B_0 + E_0 + T_0 \\ V_0 &= B_0 + E_0. \end{aligned}$$

Whereas in our setup, the cost of capital for tax and equity are identical, this is not the case for any other claims. In particular although L, B share flows in the distress and default conditions, their cashflow boundary conditions are not simple fractions of each other and therefore there costs of capital will not generally be the same.

We label that part of total firm value that can be extracted by capital claimants as V. Furthermore V is no longer invariant to capital structure choices since the total third party costs (for given τ, θ, λ) can now be minimized as a function of K, X and potentially T

$$A_0 - V_0 = \tau C \left(P_0, K, T \right) + \theta F \left(P_0, K, T \right) + \lambda f \left(A_0, X, T \right).$$

This would ensure that the firm extracts maximum expected value over the debt financing cycle $0, T$.

6 Comparative statics

Since formulae for all components are available in closed form, much progress is possible in determining optimal debt structure. Having described the setup of the problem, this remains the main objective of the paper.

7 Conclusion

In order to separate distress from default, the total costs of bankruptcy can be modelled using non–cumulative income bond valuation and new cap and floor technologies. Coupled with some basic tax modelling, more progress than was previously the case can be made in determining optimal capital structures as well as determining costs of capital for firm components.

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