

The Firm as a Three-Layer Cake -- Optimal Investment and Financial Structure

Thomas E. Copeland

Senior Lecturer MIT Sloan School of Business¹

Abstract

Herein the firm is modeled as an underlying asset portfolio (without flexibility) upon which are contingent a portfolio of real (European) call options to expand, a real (American) put to abandon, and wherein the equity is an (American) call on the flexible firm with an exercise price set equal to the face value of zero-coupon debt. The model results in an endogenously and simultaneously determined optimal investment structure and capital structure. Optimal investment structure results from the tradeoff between the benefits of increasing modularity vis-à-vis diseconomies of scale, and on the firm's optimal capital structure. Optimal capital structure results from the increasing cost of suboptimal exercise of the firm's real options versus the benefit of tax shields as the percent of capital supplied by debt increases, and on the firm's optimal investment structure.

¹ The author would like to thank Chris Cha for programming assistance.

There is surprising little literature in recent times about the valuation of firms, although publications of early models can be traced from the work of Gordon [1959], Miller and Modigliani [1961], and Malkiel [1963].² These models were all founded on the principle of discounting expected free cash flows at a risk-adjusted rate, and often included the explicit assumption that operating (or unlevered) cash flows were unaffected by the firm's choice of financing. We shall call this a model of the *primitive firm* – because, although it consciously adjusts for risk, it does not capture or model the flexibility of future state-contingent decisions that are available to managers.

A dozen years after Miller and Modigliani, Black and Scholes [1973] published a theory of the firm that viewed equity as a call option on the value of the (primitive) unlevered firm with an exercise price equal to the face value of zero coupon debt. Merton [1974] also valued corporate debt using option pricing. Put-call parity arguments similar to those of Stoll [1969] and the now famous Black-Scholes formula for pricing European call options were used to value the equity. In a taxless world, the value of the firm was independent of its capital structure, regardless of the approach that was taken. This work represents a theory of the firm with financial but without real options.

Up to that point in time little attention was given to the underlying asset – the firm itself. During the 1980s and 1990s, however, an extensive literature investigated the problems of valuing real options. Myers [1977] coined the term “real options” to describe the state-contingent decisions that management has to right but not the obligation to make. In what follows, this paper will refer to growth and abandonment options as the real options layer of decision making. Dixit and Pindyck [1994] provide a theory of investment that values the firm with real options, but no financial options.

This paper extends Black-Scholes and Dixit-Pindyck, by viewing the firm as a “three-layer cake”. At its base is the primitive, inflexible firm of the Miller-Modigliani type. Next, is a layer of real options composed of two types – a portfolio of European growth options to expand, and an American put – a liquidation or abandonment option. The third layer is financial options – equity as a call option and risky debt as a portfolio of a default-free zero-coupon bond and an American put option. Dixit and Pindyck [1994] and Schwartz and Moon [1999] have modeled an all-equity firm as a two-layer cake composed of the primitive firm and a real single real option (abandonment), and Black-Scholes also modeled a two-layer cake but with the primitive firm and financial options, namely debt and equity. Leland and Toft [1996] modeled the capital structure decision as a tradeoff between exogenously given business disruption costs and the tax benefit of interest deductibility, but had no real options.

The contribution of this paper is to explore the implications of a three-layer cake, namely the interactions between real and financial options. We show that flexibility can be provided by both real and by financial options, and that they are substitutes. There is an optimal real options (i.e. investment) structure, that can be expressed in terms of optimal capacity utilization. Simultaneously, an optimal capital (i.e. financial) structure --

² See Kaplan and Ruback [1995], Copeland, Koller and Murrin [2000], Damodaran [2006], and Cornell [1993].

determined as a tradeoff between the tax benefit and the flexibility cost of debt, namely the suboptimal exercise of real options.

Graham and Harvey [2001] reported in a survey of chief financial officers in the U.S., 58% of them said that the most important consideration in their financial structure decision was “flexibility.” In a more recent paper, Brounen, de Jong, and Koedjik [2004] also report that financial flexibility (defined as restricting debt to have enough internal funds to pursue new projects when they come along) was the top concern in the U.S. (59% of CFOs), the UK (50%), the Netherlands (51%), Germany (48%), and France (37%). This paper shows that flexibility provided by the value of real options affects the value of the firm’s financial options and vice versa. Operating decisions and financial decisions are not independent as Modigliani and Miller [1958] explicitly assumed.

This paper presents the simultaneous determination of optimal investment and financial structure. We demonstrate that the exercise price of the firm’s abandonment option (i.e. the collateral value) affects the timing of the firm’s exercise of its growth options. Furthermore, a firm that has greater operating flexibility can carry more debt and less equity, because the amount of debt affects the timing of the exercise of the real options. Thus, in a world with corporate taxes only, one is led to the conclusion that there is a tradeoff between the tax benefit of greater debt and a cost of debt associated with the suboptimal exercise of the firm’s real options, namely expansion and abandonment. Thus, this paper proposes a theory of the firm that suggests a rational explanation for why so many CFOs believe that operating and financial flexibility to be so important, why there are significant cross-sectional differences in capital structure, and suggests what to look for as cross-sectional differences in investment structure.

The organization of the paper is as follows. We first review the valuation of the primitive firm – no real or financial options and no taxes. Next we develop the valuation of the firm with real options (abandonment and growth) only – no financial options or taxes. We show that the real options can interact with each other. For example, as the exercise price of the abandonment put increases, the expansion calls increase in value and may be exercised earlier and more often. Consequently, we describe an “optimal real options or investment structure” of the firm. This is followed with analysis of the firm with debt and equity as financial options on the primitive firm (i.e. no real options). Without bankruptcy costs and taxes, the classic Modigliani and Miller results obtain. Finally, we extend the theory to describe a firm with growth and abandonment options (real options) and with debt and taxes (financial options). This “three layer cake” exhibits optimal investment and optimal capital structure. There is an optimal capital structure because there is a tradeoff between the tax benefit of debt and a marginal cost that can best be described as the value lost due to changes in the exercise of the firm’s real options. An optimal investment structure is characterized in terms of the cost of investment per unit of capacity, modularity, and collateral value. The last section postulates the empirical implications and summarizes.

1. Valuation of the Primitive Firm (risk but no flexibility)

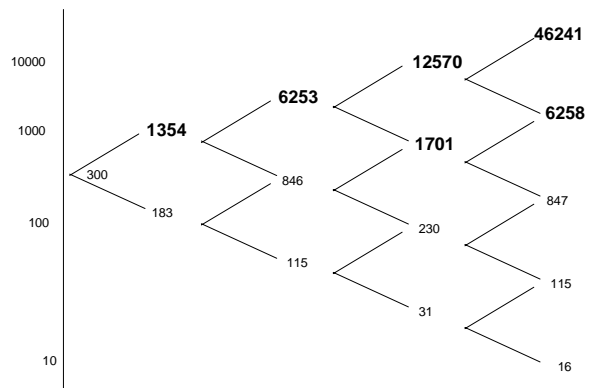
Standard discounted cash flow techniques discount expected free cash flows, $E(FCF_t)$, to the entity at the weighted average cost of capital, w , assuming that only systematic risk is

priced.³ The single source of uncertainty is the quantity of output demanded. The primitive firm in this section of the paper is financed with equity, and has no real options of any kind. It may be risky but it is totally inflexible. In other words, it pre-commits to an expansion policy that is not conditional upon the state of nature in a given time period. We designate V_0 as the present value, $E(V_t)$ as the expected future value at time t , and w as the cost of capital. The value of the primitive firm at period t can be written as:⁴

$$E(V_t) = \sum_{i=t}^N \frac{E(FCF_{i+1})}{(1+w)^{i-t}} \quad t = 0, 1, \dots, N \quad (1)$$

Unusual for a theory paper, we make use of a numerical example throughout with the hope it will make it easier for us to demonstrate our results. Exhibit 1 shows the uncertain evolution of the quantity demanded over a four year period. It starts at 300 units per year then with probability $p=.3115$ moves up by a multiplicative factor, u , and with probability $(1-p)$ moves down by $d=1/u$, forming a recombining binomial tree. Expected demand is found at the bottom of the exhibit. Our base case (arguably naïve) assumption is that the firm constructs capacity to come as close to expected demand as possible, given that each discrete addition to capacity (each new plant) adds 300 units. If so, the firm starts with 1 plant in place then commits to building 1 plant in the first year, 1 in the second, none in the third and one in the fourth. Exhibit 1 also shows expected capacity each year. Given this assumption set, there arises an implicit capacity cap. The states of nature where the quantity exceeds the capacity of the firm are written in larger bold typeface. In these states of nature expected demand is greater than expected supply.

Exhibit 1 -- Uncertain Demand



E(Demand)	300 units	548	1024	833	1240
E(capacity)	300 unitss	600	900	900	1200
E(output)		313	505	320	395

Note: Numbers in bold are demand in states of nature where demand is higher than expected

³ We assume that w is calculated from capital market equilibrium.

⁴ Equation 1 maps the value branch illustrated in Exhibit 3.

Assuming that the operating cash flow (i.e. revenues net of costs) is constant at \$1 per unit, that there is no depreciation or interest cost, that the firm already has a facility that produces 300 units per year, that new capacity will be added to provide expected supply as close as possible to expected demand, and that new investment in physical capital costs \$0.75 per unit of capacity; we have the cash flows of Exhibit 2. At a 7.8% cost of capital the present value of the company is \$2,001. This result is based five years of explicit cash flows plus a continuing (or terminal) value component, CV, that represents all cash flows in the fifth year and beyond.⁵

The expected cash flows do not follow any regular stochastic process. However, this fact presents no analytical difficulties because the value of the firm at time 0 can be viewed as a portfolio of futures contracts, one for the expected free cash flow each period. Samuelson [1965] proved that so long as expectations are rational and unbiased, the price of each futures contract will fluctuate randomly. Since the present value of the firm is a portfolio of futures contracts on annual expected cash flows, and the price of each is random, then, so too, the value of the firm will be random (even though the cash flows are not). It is important for our example to

Exhibit 2 -- Present Value Spreadsheet for the Primitive Firm

<u>Year(t)</u>	<u>E(CF)</u>	<u>Investment</u>	<u>PV@7.8%</u>	<u>PV@yr.0</u>	<u>PV @ t</u>	<u>PV ex-dividend</u>	<u>Wealth Relative</u>	<u>Return</u>
0		-225		- 225.0	2,001.0	2,226.0	NA	
1	312	-225	.928	80.7	2,228.3	2,141.3	2,400.0	7.8%
2	505	0	.861	434.5	2,146.5	1,711.9	2,587.0	7.8%
3	319	-225	.798	74.8	1,711.9	1,636.1	2,789.0	7.8%
4	395	0	.740	292.2	1,636.1	1,343.8	3,006.5	7.8%
CV ₄	1816	0	.740	1,343.8	1,636.1			
Sum				2,001.0				

include annual cash flows because their time pattern has implications for the expansion of the firm and its choice of debt policy. One can think of them as dividends paid out to (or received from) shareholders.

Exhibit 3 shows what may be called a value branch – the expected value of the firm at time t. It also introduces the concept of wealth relative that plays an important role. While the present value of cash flows from time t to infinity is the present value of the primitive firm at time t, it is not the accumulated wealth of shareholders except at t=0. What we call wealth relative, W_t , is the value at time t of all accumulated cash flows from

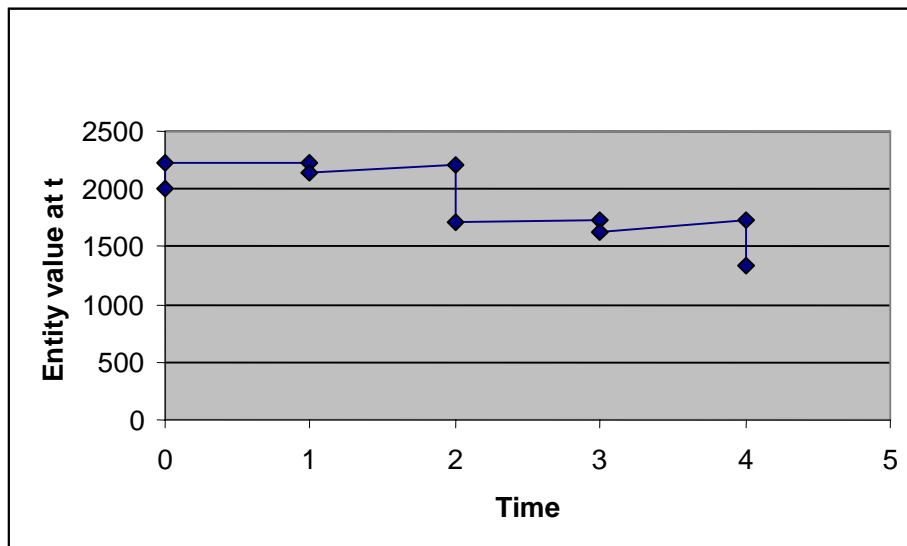
⁵ The firm is assumed to start with one plant in place and to pre-commit to adding (at a cost of \$225 each) a 300 unit plant in years 0, 1, and 3. Continuing value equals 4.6 times expected cash flow in year 4.

time 0 (the initial valuation) until t , plus the present value of future cash flows from time t to infinity. At time 0, we know that $V_0=W_0$. Cash flows generated by the firm accumulate at the cost of capital so that the rate of change in wealth relative, when averaged across all states of nature, always equals the cost of capital, independent of the path taken to reach that state of nature. At each point of time in the future, the wealth of shareholders is the value of the cumulative cash flows from time zero to time t , plus the present value of expected free cash flows from t to infinity,

$$W_t = \sum_{i=0}^t (1+w)^{t-i} FCF_i + E(V_t) \quad (2)$$

However, it is the second term that is the appropriate maximand for decisions at t because the first term, the value of cumulative cash flows (assumed to be reinvested) is unaffected. Thus, there are two (equivalent) views of the value of the primitive firm. The first is the traditional value at time zero, and the other is the expected value at time t . Note that the wealth relative (Equation 2) is expected to grow at rate w , but that the value of the firm declines (or increases) by the amount of cash paid out (or in) each period (an “ex dividend” effect).

Exhibit 3 – Primitive Firm Value Branch



Next, we explicitly build uncertainty into the picture by transforming the value branch of Exhibit 3 into a value tree – a recombining binomial lattice – given an estimate of the up and down movement per time period as derived by Cox, Ross and Rubinstein [1979]. If the annual standard deviation of the rate of return is 100% then the annual up and down movements will be a geometric process:

$$u = e^{\sigma\sqrt{T}} = e^{1.0\sqrt{1}} = 2.718 \quad (3)$$

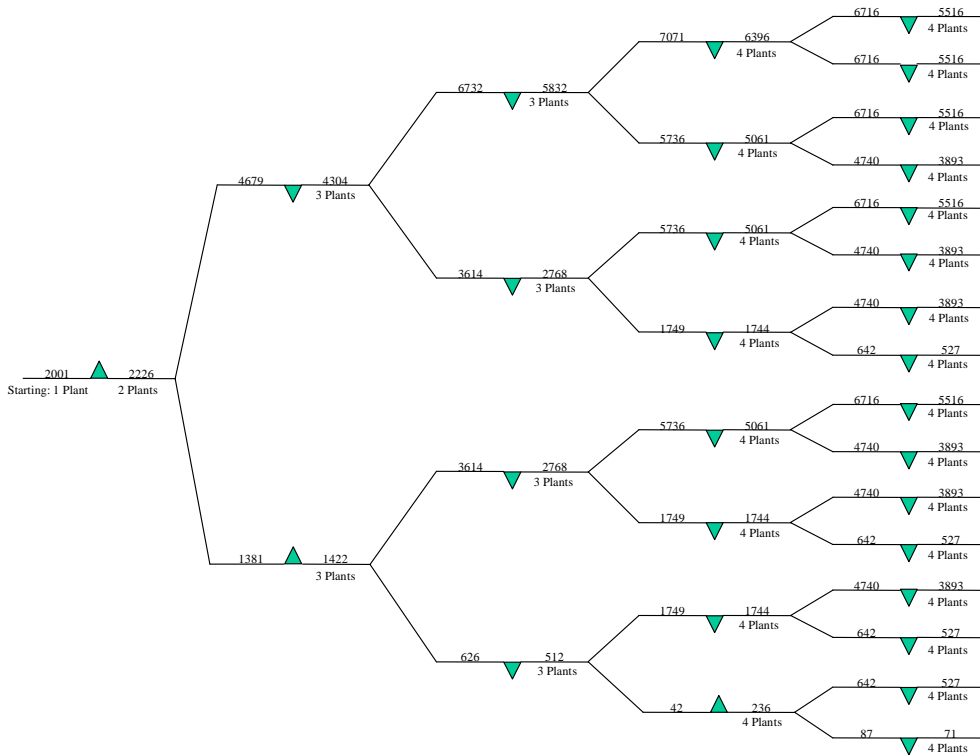
$$d = 1/u = 0.368$$

We can now construct the value tree of Exhibit 4.⁶ Two more realistic assumptions are added. First, physical capital costs \$0.75 per unit of capacity, and second, the firm plans to start with 300 units of capacity and to expand (in modules of 300) enough each year so that it can supply *expected* demand. Exhibit 4 represents the present value at each node of the expected cash flows from that point forward in time. Mathematically, the value tree is a recombining binomial tree with the payment at each node s in time period t

$$E(V_{t,s}) = \sum_{i=0}^N \binom{t}{i} p^{t-i} (1-p)^i u^{t-i} d^i V_0 \quad t = 0, \dots, N \quad (4)$$

We might also consider the fact that there are $2^n=16$ paths in the tree. For simplicity, we assume that each up or down movement is equally likely (i.e. $p=.5$), then order the paths according to the cash flows along each. First, note that we assume that dividends are strictly proportional to value, and that all up values are greater than their mirroring down

Exhibit 4 – Primitive Firm Value Tree



state values while preserving symmetry in the lattice, as well as the expected cash flow. Therefore, for a given value branch designated as {u, d, u, d} for example, the wealth of

⁶ Sigma is a function of the volatility of demand but not equal to it.

the firm may be written as:

$$W_t = \sum_{i=0}^t FCF_i (1+w)^{t-i} + u^{N-t} d^{t-N} V_0 \quad (5)$$

Note that if there are N time periods, then according to Pascal's triangle there will be N subsets of equal order. Also, expected wealth cum dividend is the same as expected wealth ex dividend. In Exhibit 4 the value of the firm is \$2,001 because capital costs \$0.75 per unit, and is implicitly capped at expected demand in Exhibit 4. This brings us to the realization that setting capacity equal to expected demand implicitly imposes a capacity restriction that is suboptimal. What then, is the optimal investment policy?

Investment assumptions of the primitive firm:

The primitive firm pre-commits to building capacity needed to supply *expected* demand, shown at the bottom of Exhibit 4. Given pre-commitment to invest there is no optionality, i.e. no flexibility, in the solution that is illustrated in Exhibit 4. Thus, Exhibit 4 is a traditional solution to the firm's investment strategy, namely that the firm will invest enough capital in the future to have capacity that is sufficient to meet *expected* demand.

So far, the analysis resembles a typical corporate plan for expansion, with the implied assumption that capacity will be generated via investment designed to meet expected demand. However, we find this analysis to be unrealistic, and in the next section, we analyze what happens if the firm has the ability to decide on its investment in capacity given the state of nature that it attains in the future.

2. Investment Structure given flexibility— It is obvious that the investment assumptions of the classical primitive firm are unrealistic. No firm actually pre-commits to a capital investment schedule for very long. Instead, firms wait to see how demand evolves and then take action to expand by making further capital expenditures, to shrink by allowing existing capacity to depreciate, or to sell some or all of it. We have simplified these choices somewhat by reducing the firm's investment choices to expansion or abandonment -- a European call and an American put. The first of these two real options is discussed here and the second later on.

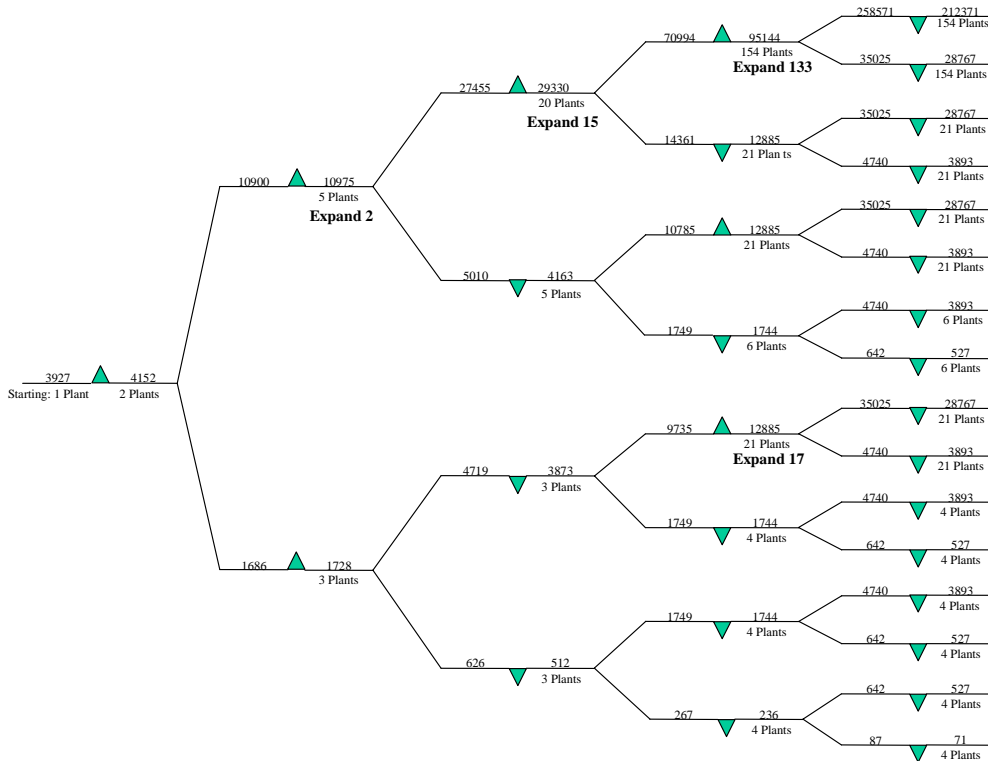
Portfolio of growth options -- Suppose the firm can invest in "modules" of capacity that expand the firm's output by 300 units each at a cost of \$0.75 per unit. The exercise price of this one-year European option is \$225, paid at the beginning of a period while the new capacity becomes available at the end of the period. We assume that any number of these options is available each year, and they make investment "lumpy". The firm can, at its discretion, expand capacity when the quantity demanded is growing and choose not to exercise its option(s) to expand when it has excess capacity. A portfolio of growth options is the next step in complexity. Exhibit 5 shows the result. It has a path-dependent solution with up to $N=4!(2)=48$ different branches. We assume that the price per unit of output is a constant. For an investment of $I=\$225$, the productive capacity of the firm is greater by "g=300" units of production at the end of the period. However, as the firm may expand in consecutive years, we stipulate that g is not multiplicative. For example if the capacity of the primitive firm were Q units in year one and it decided to expand in both

periods, the capacity given expansion would be $Q(1+g)$ for an exercise price, $\$I$, in year 1 and in year 2 it would be $(1+2g)Q$ for an additional $\$I$, thus physical capacity does not grow at a compound rate. Furthermore, capacity is assumed to be permanent (non-depreciating).

The problem is solved by starting at the back of the value tree, Exhibit 5, and solving for a number of modules that maximizes the present value of the cash payouts beginning of that period – a value that is obtained by using the cash flows from the underlying value tree in combination with a default-free zero coupon bond to form a replicating portfolio that has exactly the same payouts as the optimal growth option. Next, the problem is reworked assuming that $N-1$ modules are in place at the beginning of the period – until the optimum number is found. These two steps go backward and forward in the decision tree, working back one period at a time, until we reach the root of the tree at time 0. The solution is a path-dependent process. See Bellman [1957].

To the left of each triangle is the value of the firm cum dividend and to the right is the value of the firm ex dividend. An upward-facing triangle means that equity holders must provide cash to the firm, and a downward-facing triangle means that the firm pays a dividend.

Exhibit 5 – Firm Value Tree, Multiple Expansion Options (with a capital outlay of \$0.75/unit and Plant Scale of 300 units)



Looking at Exhibit 5 we see the optimal number of plants, each with 300 units of capacity, that the firm chooses to have at the end of each period. There are 154 plants in the (unlikely) uppermost branches of the tree in year four, 21 plants in the next lower branch pair, then six plants, and four plants in the lowest branches. Note that the Value of the

firm rises from \$2,001 to \$3,927 in this example. Recall that the firm's base case investment was to start at time zero with 1 plant in place, to invest in a second plant at time 0, a third in year 1, none in year 2, and a fourth in year 3. Notice that, given the right to invest in growth, there are a total of 5 plants in place along the upper branch of the tree in the first year, and three along the lower branches. One arrives at these totals as follows. The firm started with a base of one plant and added another precommitted plant at time 0, for a total of 2. In the up state of year 1 it added a third precommitted plant and two new discretionary plants for a total of 5. In the down state of year 1 it added the same third precommitted plant, but 0 discretionary plants for a total of 3.

In addition to a portfolio of one-year growth options there are other possibilities as well. For example, the firm might engineer larger two-year projects that are more efficient. Said efficiency can be expressed either as capital efficiency, namely the same g but investment less than $2I$, or as operating efficiency, expressed as greater growth, g , in value. In our model, combinations of one and two year projects are also possible, although not discussed here.

Abandonment option only -- An abandonment put option allows the firm to cease operations by liquidating all previously constructed plants each having a cash value, $\$A'$ per unit of capacity. The sum of total cash received is interpreted as the collateral value of the firm. Note that the collateral value is the path dependent exercise price of the abandonment put option, and is conditional upon the amount of investment along the path. These real options are evaluated at each node in the value tree, starting at the back of the tree and working in decreasing chronological order to the present, using a "backward-forward" dynamic programming algorithm.

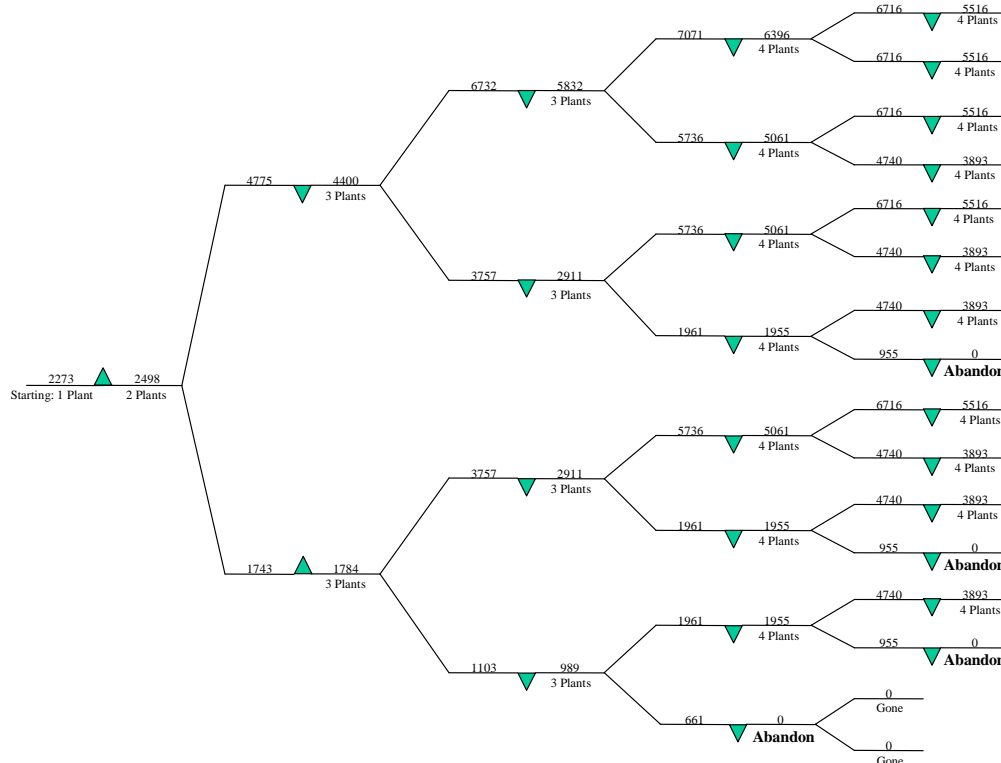
Exhibit 6 shows the valuation – assuming only an abandonment option with an exercise price, A' , of \$0.70 per unit of capacity (\$210 for each 300 unit plant). Note that the value has increased from \$2,001 (the primitive firm in Exhibit 4) to \$2,273 (the firm with abandonment, Exhibit 6).⁷ Abandonment occurs in the lowest state of nature in year 3, and in the second, third and fifth lowest states of nature in year 4.

Single abandonment and multiple growth options -- The combined effect of a single growth and abandonment option is seen in Exhibit 7. At each decision node the abandonment decision precedes the expansion decision. Note that their effects on the value of the primitive firm are not additive because their boundary conditions can (and in the example shown do) interact. Given the possibility of expansion there are often more plants to abandon than in the case without growth options. Abandonment given growth occurs in four additional states of nature in Exhibit 7 versus Exhibit 6. The call is exercised only at three nodes, given no abandonment (Exhibit 5). But with abandonment (Exhibit 7), there is exercise at both nodes in the first period, and at an extra second period node, because abandonment enriches the lower branches of the tree. Thus it is mathematically possible for the exercise conditions to overlap and to be interdependent. In isolation, the value of the primitive firm, which was \$2,001 without any real options, increased to \$2,275 with the option to abandon, and to \$3,927 with the portfolio of options to grow. Thus, the option to abandon increased the firm's value by \$274 and the

⁷ See the appendix for a description of the solution methodology.

expansion options increased it by \$1,926. The sum of these separate effects is \$2,200. In Exhibit 7, however, due to their interaction, the firm's value increases to \$4612, an increase of \$2611 – more than the simple addition of their separate effects.⁸

Exhibit 6 – Firm Value Tree with Abandonment Option (salvage value of \$0.70/unit)



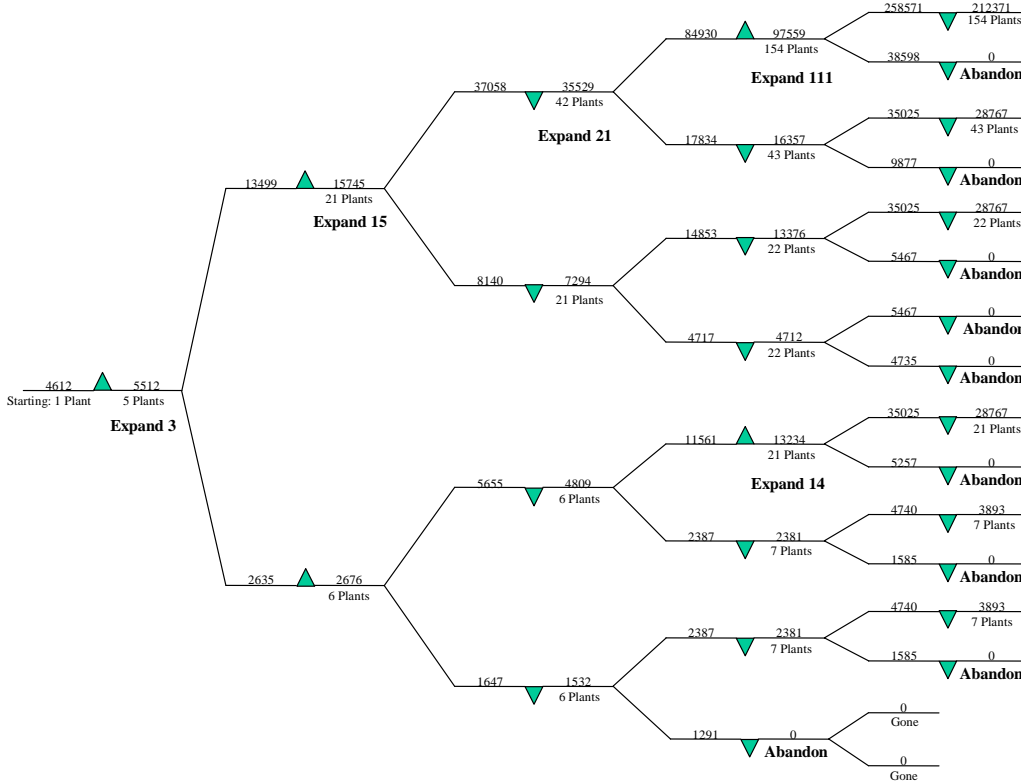
Optimal Investment Structure -- If we can conceive and debate the concept of optimal capital structure, then why not do the same for the structure of the firm's real options – its optimal investment structure? For example, a type of flexibility choice facing management is whether to build bigger, more efficient operating facilities less often, or to build smaller, less efficient facilities more often – a tradeoff between economies of scale and modularity. We can think of modularity as a measure of how inflexible that fixed costs are – what is sometimes called operating leverage. Another result is the firm's choice of excess capacity, measured as the difference between capacity to supply output and expected output. The real options to grow and abandon make excess capacity an increasing function of the volatility of demand, and of the abandonment value per unit of capacity; and a decreasing function of the exercise price per unit of capacity. To the best of our knowledge we know of no other theory that explains optimal excess capacity.

Later in the paper, when we combine real and financial options, we illustrate a case where debt policy causes suboptimal (delayed) exercise of the abandonment option and changes the timing of expansion as well. Thus, we relax the Modigliani-Miller assumption that operating decisions are independent of the way the firm is financed. Here, we maintain

⁸ Solution methodology described in appendix.

our assumption that the firm is all equity and ask whether the value of the firm is affected by its investment structure. It is. We also assume that there are no capital market constraints so that the firm’s investment policy is completely independent of its dividend or share repurchase policy.

Exhibit 7 – Firm Value Tree with Expansion and Abandonment



What are the choices that are described by “investment structure”? They are the parameters of the firm’s portfolio of European call options to expand, and the American put to abandon that management can influence. They include the cost of expansion, the increase in value that results from expansion, the number of years that it takes to complete the expansion – a time interval during which further expansion is prohibited in our model, and the abandonment value. We have demonstrated that growth and abandonment options are interrelated. Therefore, one can define the optimal investment structure as the value-maximizing combination modularity, efficiency, and abandonment policy.

Capacity utilization is a key feature of investment structure. In states of nature where demand exceeds the actual maximum output, capacity is said to be 100 percent utilized. In the remaining states of nature capacity exceeds demand. Expected capacity utilization in any given year is the probability-weighted average of utilization by state, given optimal expansion and abandonment. Appendix Exhibit A1 shows that, ceteris paribus, capacity utilization declines as the volatility of demand increases. Exhibit A2 shows that as abandonment value per unit of capacity increases, capacity utilization decreases. And Exhibit A3 shows that utilization increases as the cost per unit of expansion decreases.

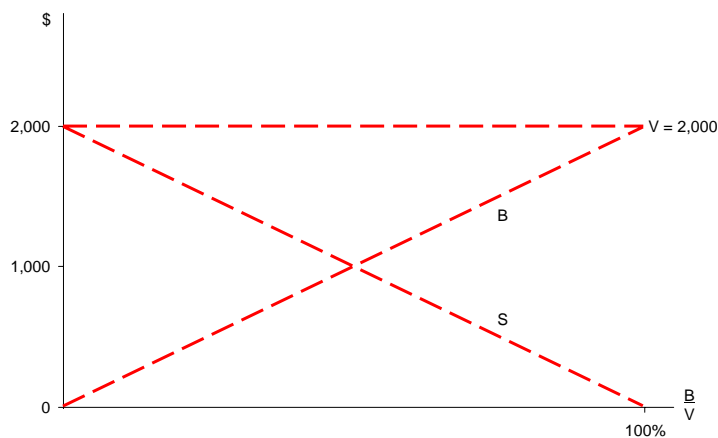
3. Valuation of a firm with only financial options

The first step for analyzing financial structure is to study financial options on the primitive firm (no real options). Of course this has been done earlier by Merton [1972] and Black and Scholes [1973]. A decision-maker, the entrepreneur, stands at time zero and chooses a capital structure policy that requires an initial commitment of equity capital, E , and an amount of borrowing, B , of zero coupon debt with fixed maturity (e.g. $T=4$ years) and face value D . Both the entrepreneur and the lenders are fully informed of all state contingent future cash flows, and there are no market frictions of any kind, including corporate taxes (that will be addressed later on). Let r be the T -year rate on default-free debt, and assume that lenders either are risk neutral, or that they treat default risk as idiosyncratic.⁹

The entrepreneur's problem is to maximize her wealth given ownership and control of the primitive firm. As there are no real options in this section of the paper, the only choice variable is debt policy, a decision that does not affect the value of the primitive firm. A marginal investment will create the same value regardless of the choice of debt policy given this set of assumptions. Exhibit 8 shows the value of the entity, and debt, and it then shows the value of equity as a call on the value of the primitive firm. With only the primitive firm as the underlying (that is with no real options), with no taxes, and with no business disruption costs, the MM propositions are valid and the value of the entity is unaffected by the mix of debt and equity.

Confidential

Exhibit 8 — Value with Debt but No Real Options (and no taxes)



ZKN-MFB-Stoll_Charts-5-18-05-TC

8

Copyright © 2005 Monitor Company Group, L.P. — Confidential — CAM

⁹ They can do so because they can diversify their portfolio across firms.

4. Financial Structure and Abandonment

The entrepreneur's problem becomes complicated because the exercise of the real options affects the exercise of the financial options and vice versa. For example, as the liquidation or collateral value of the firm, A , becomes high relative to the face value of debt, the probability of early exercise of the American put to abandon the firm also goes up, and this will result in an increase in the value of operating cash flows vis-à-vis the base case primitive firm. However, it will also increase the value of debt because the higher abandonment exercise price serves to provide greater value.

Flexibility Cost of debt – suboptimal exercise of real options:

If the entrepreneur exercises her abandonment option because $A > V$ in a state of nature, $t < T$, before the maturity of debt, and if the abandonment value is sufficiently high, the debt holders receive the face value of debt sooner than expected, thereby providing an additional benefit to them (if $A > e^{-rt}D$). The entrepreneur and the lenders both realize this, *ex ante*. Consequently, the entrepreneur will choose either to exercise the liquidation option later than would have been optimal were the firm all equity, or will choose to borrow less, *ab initio*. If the abandonment value of the firm is low relative to the face value of debt, then the creditors will force default at a point in time that antedates the time when the entrepreneur would voluntarily abandon. Either way, we have a situation wherein real and financial options interact to affect the value of the firm. The suboptimal exercise of the abandonment option can be seen as a cost of debt.

One might ask why the firm, when it seeks to liquidate, does not buy the bonds at market value rather than paying their full face value. Given symmetric information, the bond holders know that they can receive at least the abandonment value (or collateral value) and consequently the market value of the bonds will not fall below this limit. Given the abandonment option is exercised, we know that bondholders will receive D if $A > D > PV(D)$, and A if $A < D$. In some scenarios, when $A > PV(D)$ and bondholders receive $D > PV(D)$, they may actually benefit from abandonment. Exhibit 9 shows the six states of nature that are possible at a node. Conflict between the entrepreneur and debt holders arises when the face value of debt is greater than both the collateral value of the firm and the underlying value of the firm, and the bond holders seek Chapter 11 – but the entrepreneur wants to keep the firm alive because $V > A$. This conflict can be resolved *ex ante* if creditors set lending limits equal to the collateral value, i.e. if $A = D$. In fact, this seems to be common practice.¹⁰

Note that these potential wealth transfers from equity to debt (rows 3 and 4) are not an artifact of our choice to model the problem with zero coupon debt. The results are directionally the same for coupon debt, self-amortizing debt, and for zero coupon debt.

¹⁰ Tom Blake points out that collateral is often mainly short term assets such as inventory. If the company borrows when inventory builds due to slack demand, then realizes that it needs to reduce excess inventory, it is constrained by the collateral covenant – a sort of Catch 22.

Exhibit 9 – Abandonment-Debt Conflict

State	Action	Stock value	Debt value	Sum	Comment
V>A>D	Go	V-PV(D)	PV(D)	V	No exercise
V>D>A	Go	V-PV(D)	PV(D)	V	No exercise
A>V>D	Abandon	A-D	D*	A	Debt benefits
A>D>V	Abandon	A-D	D*	A	Debt benefits
D>A>V	CH 11	0	A	A	Debt loses
D>V>A	CH 11	0	V	V	Conflict

*Debt may benefit from abandonment if PV(D)<A.

To model debt, we need to express both the cash flow each period and the value of the firm as well – a stock/flow problem. Because the cumulative cash flows along each path are different, the problem becomes a non-recombining binomial tree with 2^n branches. Define a value branch as the uniquely ordered set of up and down movements taken by the primitive underlying unlevered firm. Wealth relative (see Equation 2) along each branch at time t is the sum of two parts (1) cash flows accumulated at the cost of capital from the beginning of the branch up to time t and (2) plus the greater of the liquidation cash flows (contingent on liquidation), or the value of cash flows from that time forward. Each branch is path-dependent and is solved starting at the back of the tree and then working backward in time. We define this wealth relative as the “pace” of a path.

$$pace_p = \sum_{i=0}^t FCF_i (1+r)^{t-i} + CV_T \quad (6)$$

Dmin is defined as the greatest face value that can be accommodated without default along the minimum pace path. We order the “pace” from the lowest, pace(l), to the highest, pace(h). For a face value of debt, D_+ , that is greater than Dmin, a risk neutral lender will anticipate default in the lowest state of nature but can preserve its expected payoff by raising the face value in the other states of nature which have cash flows greater than pace(l), and by just enough that its expected payoff remains unchanged. If we designate p_{st} as the objective probability of state s in period t then, in our numerical example that follows,

$$D_+ = \sum_{s=1}^{16} p_{st} \text{Min}[D_{\min}, pace_{st}] = \sum_{s=2}^{16} p_{st} \text{Min}[D_+, pace_{st}] + p_{1t} pace(l) \quad (7)$$

On the left side of Equation 7, we know that $D_{\min} = \text{Min}(pace)$, so there is no default. On the right hand side $D_+ > D_{\min}$ because there is default in the lowest state of nature ($s=1$), where the payoff on debt would be $pace = D_{\min}$. For each end branch there is a unique face value of debt that represents the maximum face value that can be paid from the

cumulative cash flows from that state and all that are better. We assume that the entrepreneur wishes to commit the minimum amount of equity.

We check to see which branches have enough cash to pay off the face value of debt at maturity. If there is a shortfall in the lower branches of the tree, we assume that the face value of debt is increased (ex-ante by the lender who is assumed to be risk-neutral) enough in the solvent upper branches so that the *expected* payments are the same as before any probability of default.

The creditor's expectation of B_{\min} , i.e. the present value of debt along the branch with the lowest pace, is formed at time zero and takes into consideration optimal abandonment from the entrepreneur's perspective. If the entrepreneur wishes to take on more debt, she will default in the minimum cash flow state, but can provide the same expected payout to debtholders if the face value is sufficiently high in the other (higher payout) states of nature. A risk neutral lender will be indifferent between debt policies that, ex ante, have the same expected payouts, B_{\min} . It will therefore be indifferent between B_{\min} with face value D_{\min} and a greater amount of debt, B_+ , (given the firm's debt policy) with face value D_+ as long as the present values of the expected payouts are the same, i.e.

$$B_{\min} = PV[\text{Min}[\sum_{\forall p} \text{pace}_p]] = PV[\sum_{i=1}^P p_i \sum_{\forall p-1} (\text{pace}_p + ((D_+) - D_{\min})))] \quad (8)$$

There is an upper limit to the amount of debt that can be garnered from a given debt policy. As one increases the face value of debt that is borrowed, default occurs in more states of nature, and as one proceeds from the minimum cash flow state and moves up the event tree, the *probabilities* of higher cash flow states increase at first but soon decrease rapidly. When we reach a state of nature where the above indifference condition cannot be met, we establish an upper limit of the amount of borrowing.

For the set of cash flows presented in Exhibit 1 and the accompanying real options as given, the table in Exhibit 10 reviews the effect of debt policy. It shows the initial face value which is the entrepreneur's target percentage debt times the market value of the entity plus the target percentage of the present value of expansion costs, taken forward to their future value. Next, it provides the additional face value necessary in solvent states of nature to preserve the present value for a risk-neutral lender. Finally, it calculates the promised yield to maturity on the debt (a number that is not used elsewhere in the analysis).

Note that when the debt policy is conservative, e.g. 20%, the yield to maturity is lower with real options than without because the abandonment option exercise price is greater than the face value of debt, therefore debt holders receive full payment in all states of nature.

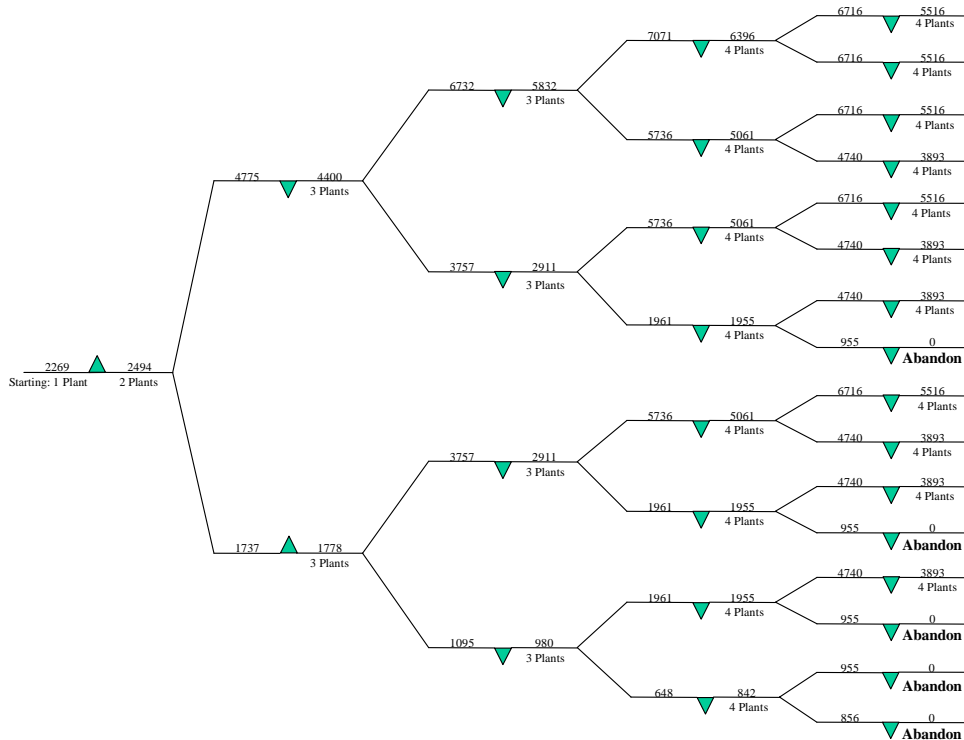
The preceding discussion of debt limits confronts the entrepreneur with a schedule of alternative debt policies, each with different residual cash flows paid to her, but with payouts that are a matter of indifference to her creditors. She can then choose among these debt policies the one that maximizes her gain in wealth, given the effect that the debt policy has on the exercise of the firm's real options, and on its tax shelter.

Exhibit 10 Example Face Values and Yields for Various Debt Policies (primitive firm value=\$2,001, A'=\$0.70 per unit, I=\$0.75 per unit, and modules of 300 units), without and with real options

Debt policy, B/V	Initial Face Value		Promised Face		Yield to maturity	
	without	with	without	with	without	with
20%	\$491	\$784	\$23	\$0	6.2%	4.8%
40	982	1,567	75	0	6.6	4.8
60	1,473	2,291	128	0	9.0	4.8
80	1,964	3,055	180	0	9.2	5.0
100	2,455	3,818	1,067	509	18.0	8.3

Comparing Exhibit 11 with Exhibit 6 we note that with the introduction of debt financing, abandonment (liquidation) no longer occurs in the lower state of nature at the end of period 3 when we assume that the firm incurs debt with face value \$746 and can liquidate for \$0.70 per unit. The existence of debt results in sub-optimal exercise of a real option – the right to liquidate – and consequently the value of the

Exhibit 11 – Firm Value Tree with Abandonment Option and Debt (salvage value of \$0.70/unit and 75% of investment paid by debt)



firm declines from \$2,273 in Exhibit 6 to \$2,269 in Exhibit 11. Intuition for suboptimal exercise is based on the fact that the “collateral value” provided by abandonment covers the present value of the loan and bondholders experience a wealth gain from what amounts to prepayment of their loan when the abandonment takes place. Equity holders, knowing that this will happen, decide to exercise one period later rather than transfer wealth to debt holders. To be more precise, start with the all equity firm with an abandonment option. To value the equity we use a replicating portfolio approach if the option is not exercised and compare it with the abandonment value if exercised as follows: Abandon if

$$A > S_0 = (S_u - S_d)(1 + r_f) - (dS_u - uS_d)/((u - d)(1 + r_f)) \quad (9)$$

When debt is included in the analysis, the value of equity is estimated by subtracting the value of debt from the value of the entity. However, if the abandonment value is high enough, then when abandonment takes place, the debt may be paid off at full face value prior to maturity. This changes the abandonment decision to become: Abandon if

$$A - D > S_0 = (S_u - S_d)(1 + r_f) - (dS_u - uS_d)/((u - d)(1 + r_f)) - B \quad (10)$$

With A=\$0.70 per unit and D=\$400 (debt policy is 75% debt), suboptimal execution can be seen at the node that is three down moves into the tree in Exhibit 11. At the same node in Exhibit 6, the all-equity case, the entity value (calculated via construction of a replicating portfolio) is \$617, which is less than A=\$630, therefore abandonment takes place. However, in Exhibit 11, with debt that has a face value of \$400, the equity value is S-B = \$236, while the abandonment value to equity holders is A-D=\$630-\$400=\$230. Hence, with debt, the abandonment decision is deferred one period in Exhibit 11. Thus, an opportunity cost of debt is the suboptimal exercise of the firm’s option to abandon. In our model, unlike MM, the amount of debt affects the operating decisions of the firm. In this case greater debt means that the firm gives up some of its flexibility to abandon. This lost flexibility is a direct cost of debt financing that is endogenous to the model.

The value of the firm increases when the exercise price of the abandonment put does. This was a result of the paper by Schwartz and Moon [1999] that added abandonment options (but not growth) to the value of a primitive firm. The new result in Exhibit 11 is that the interaction between abandonment, a real option, and debt, a financial option, results in changes in the actual operations of the firm. Thus, we extend Modigliani and Miller by relaxing their assumption that the operations of the firm are completely independent of the mix of financing. We have demonstrated that flexibility in the operating decisions of the firm is a substitute for flexibility in the financial mix of debt and equity, and we have done so by making both decisions endogenous. This result is completely independent of any assumption about business disruption costs or taxes.

The magnitude of what we shall coin “the flexibility cost of debt” appears to be relatively small. However, as the firm uses more debt and less equity, these real effects become a larger and larger percentage of the firm’s equity, and at the margin they are the same order of magnitude as the marginal tax benefit of debt.

Tax benefit of debt:

Along each path-dependent branch of the real options tree (with expansion and abandonment) we calculate the tax benefit of debt as the product of the marginal tax rate and the present value of debt or zero if abandonment occurs, zero. The result, similar to Leland and Toft [1996], is the benefit function graphed in Exhibit 12.

5. Abandonment, Debt and Taxes – Endogenous Optimal Financial Structure

If we now combine the present value of the benefit of corporate taxes but with the added expected cost of suboptimal abandonment, the model of the firm has optimal capital structure that results from the tradeoff between the loss of operating flexibility that results from using more debt – an opportunity cost of debt – and the benefit of debt -- the tax deductibility of interest.

We assume a tax rate of 60% and it is paid when the firm is solvent, but no tax is paid in those states of nature when the firm is liquidated or bankrupt. Exhibit 14 shows the tax benefit as a function of the abandonment value. Note that it reaches a maximum for debt policies in the range of 70 to 90%. It is estimated by calculating the value of equity for a given abandonment price and various debt policies first without and then with taxes. Higher abandonment values increase the value of equity but also take away the tax shield of debt in more states of nature. As Exhibit 13 shows, the present value of the tax shield with an abandonment value of \$0.70 per unit of capacity lies everywhere below the cases where the abandonment value is either \$700 or \$1,100.

While the tax shield is a benefit from the use of debt capital, the loss of flexibility is an associated cost. We saw earlier, that with an all-equity firm, the value of optimal abandonment was \$2,273, and the value of growth was an additional \$1,339. If we focus on the relationship between abandonment and debt policy, ignoring growth for the moment, Exhibit 11 has already shown that debt can have the effect of suboptimal exercise of an abandonment option. Exhibit 12 shows the present value of the debt tax shield as a function of debt policy (i.e. B/V).

Exhibit 12 – PV of Tax Shield for Various Abandonment Values

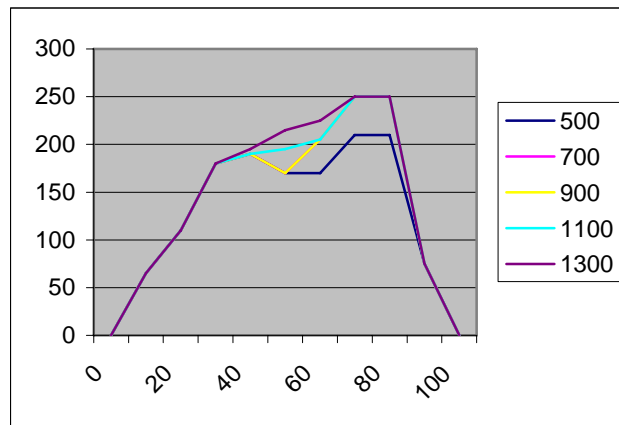
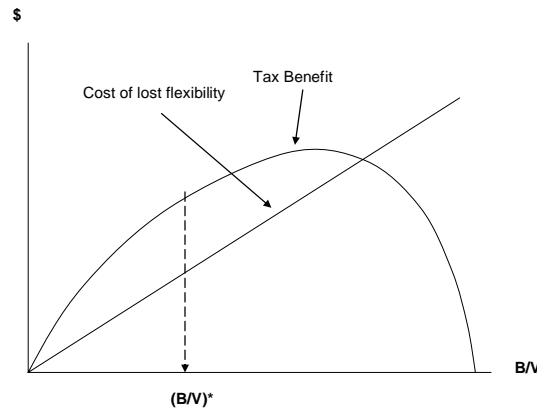


Exhibit 13 illustrates how an optimal capital structure is produced by the theory.

Exhibit 13 – Optimal Capital Structure



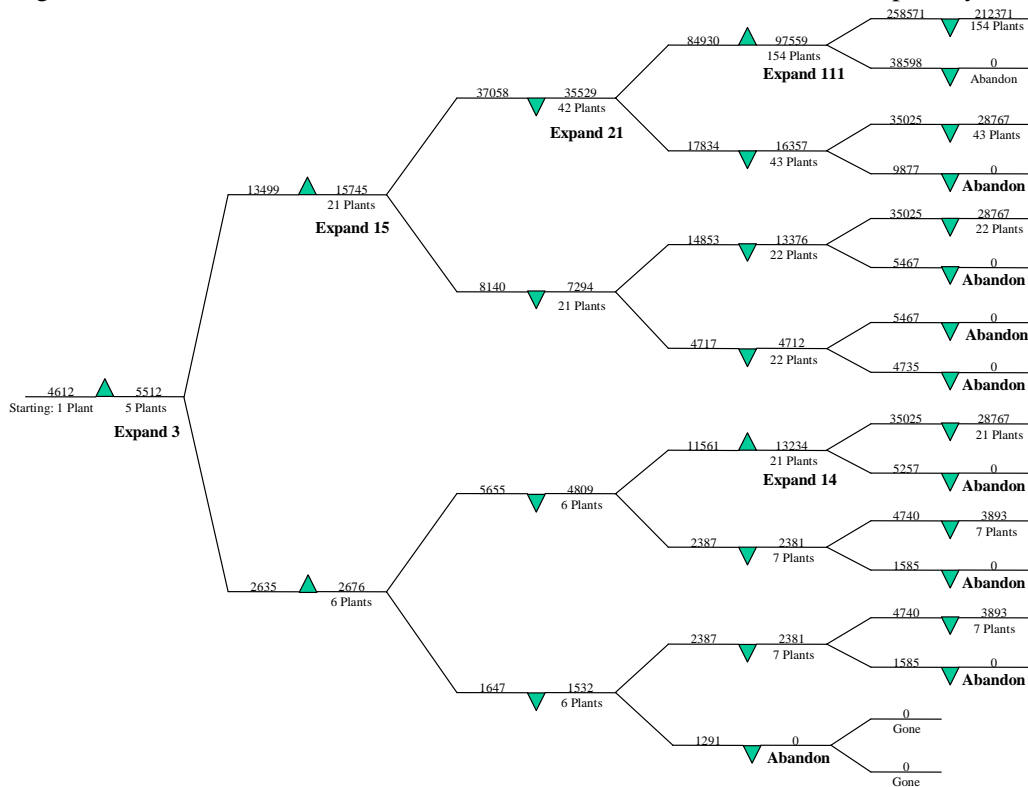
The present value of the tax benefit increases as debt substitutes for equity, but begins to decline as the firm begins to abandon or default in more states of nature. The cost of debt that results from suboptimal exercise of the firm’s real options appears (from our simulations) to be roughly linear. Therefore, the result is an endogenously determined optimal capital structure.

6. Real Options, Financial Options and Taxes -- Optimal Financial and Investment Structure

Thus far, we have not combined the firm’s option to invest in expansion with its ability to abandon and to finance with debt. What happens if we assume abandonment, expansion, and a debt policy such that $b\%$ of each amount invested will be financed with debt?¹¹ We now assume the firm has three layers – an underlying production function without any flexibility (similar to Ho and et. al. [2005]), a portfolio of one-year European growth options, a single American abandonment put option, and financial options (debt and equity). The entrepreneur is assumed to be the only equity holder and to be wealth maximizing with initial wealth of W . She is confronted with a production function with cash flows that are variable but have a time pattern that we shall call “pace”, and real options with predetermined exercise prices. She chooses an investment policy, an abandonment policy, and a debt policy simultaneously in order to maximize her wealth after corporate taxes. Initially we assume no taxes.

¹¹ The same policy is assumed to apply to the original investment as well as to each expansion, although these are decisions that could be separated. We also assume that the entrepreneur has an infinite amount of equity.

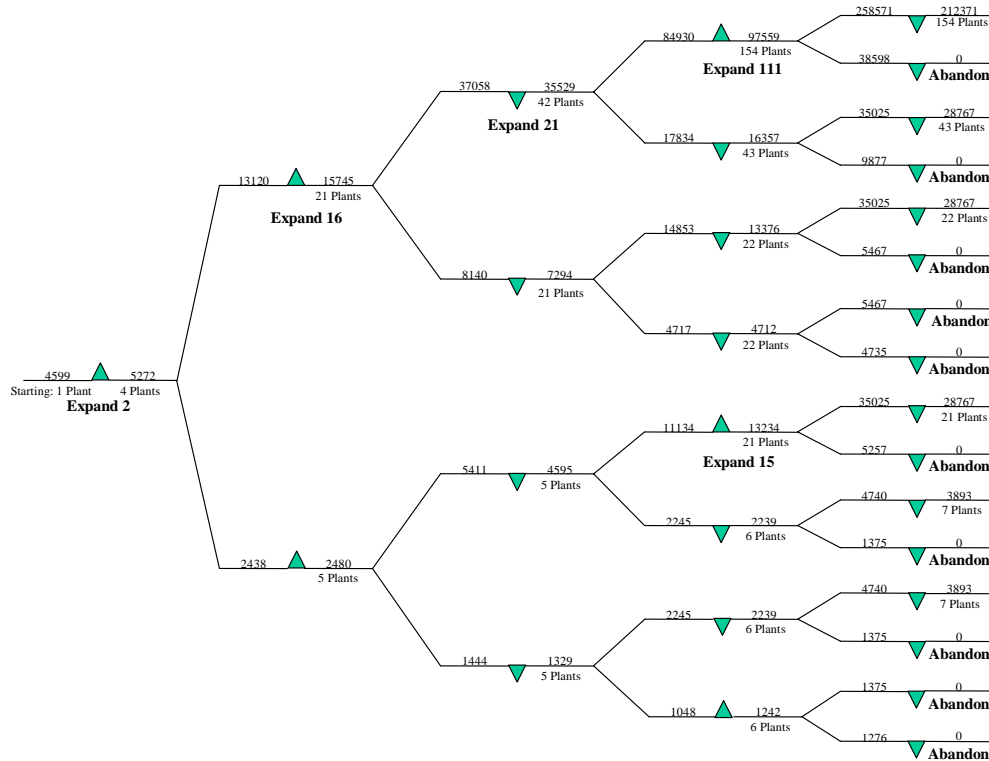
Exhibit 14 – Firm Value Tree with both Real Options and Debt (capital outlay of \$0.75/unit, salvage value of \$0.70/unit, and a Plant Scale of 300 units, 20% of investment paid by debt)



Exhibits 14 and 15 show the value of the entity using the parameters already discussed for our example. We have already established that the existence of debt causes suboptimal exercise of the abandonment decision. We now show that the expansion option may also be affected. In Exhibit 14 the debt policy is only 20% debt. Expansion occurs in both states of nature at the end of the first year – 21 plants in the favorable state and 6 in the unfavorable state. Abandonment occurs in the worst state of nature of year 3, and in states 2, 4, 6, 7, 8, 10, 12 and 14 of year 4. The entity value is \$4,612.

Debt increases to 75% in Exhibit 15. The greater amount transfers enough wealth to debt holders in the lowest state of year 3 that shareholders elect to defer abandonment entirely to year 4 (states 2, 4, 6, 7, 8, 10, 12, 14, 15 and 16), and in so-doing the third expansion module in year 0 fails to be valuable enough to invest. Therefore, the firm has 5 plants in year two and year three instead of 6. In exhibit 15, at the lowest branch in year 3, the value of equity if the firm is abandoned is the abandonment value, \$1050, minus the face value of debt, which is \$811 – a total of \$239, given the debt policy of 75% and contingent on the fact that we are on that particular path. The alternative is the value of equity if there is no abandonment, namely the market value of the entity minus the market value of debt, i.e. \$1242-\$941=\$301. Because the value of equity is higher without abandonment, the decision is deferred. The extra debt causes a chain reaction that affects both abandonment and expansion, and the value of the entity falls to \$4599.

Exhibit 15 – Firm Value Tree with both Real Options and Debt (capital outlay of \$0.75/unit, salvage value of \$0.70/unit, and a Plant Scale of 300 units, 75% of investment paid by debt)



7. Comparative Statics

The results presented above show that optimal capital structure may result from the tradeoff between the flexibility lost when debt burden causes deferred abandonment or missed growth opportunities – a cost – and tax gain from leverage – a benefit. We have shown that abandonment and growth options can interact, and that debt will affect both types of real option, not abandonment alone. Intuition says that if the firm establishes a debt policy (e.g. 50% debt), and sticks to it when financing its growth options, then debt may restrict growth in the sense that it may become impossible to adhere to the stated policy and still optimally exercise all growth opportunities.

This section of the paper discusses the predicted relationships between optimal capital structure and variables that predict cross-sectional regularities in it, followed by the variables that one might expect would affect optimal investment structure. None of the following variables affect only one source of value, however, the variables that should primarily affect the firm’s choice of optimal financial structure are:

1. Abandonment value: As abandonment value (or collateral value) increases, so too does the value of the firm, but if debt is also introduced into the picture, then a cost of so-doing is that abandonment can benefit bond holders because the firm’s obligation to them comes due earlier than its contractual maturity date. This would transfer wealth from shareholders to debt holders. To avoid this unfavorable result, shareholders decide, ex ante, to exercise their right to abandon

later in time, thereby reducing the value of the firm today, but increasing their wealth.

2. Taxes: Higher tax rates diminish the value of the firm, but also increase the value of equity in those firms that use debt. This is called the tax benefit of debt. As tax rates rise, so does the tax benefit of debt. This tax benefit starts to decline as the firm takes too much debt because abandonment becomes more valuable and in states of nature where the firm is either bankrupt or abandoned, there is no tax and therefore no tax benefit.
3. Growth: Profitable growth occurs when either the exercise (i.e. investment) cost for expansion declines or the expansion factor (a multiple of the firm value that is related to operating efficiency and modularity) increases with a resulting increase in NPV. As the firm's debt policy becomes more aggressive, both abandonment and expansion are affected. Deferred exercise of abandonment decreases the value of the preceding branches of the value tree – enough to eliminate an expansion opportunity – not for lack of funding – but rather because the value of the firm sans expansion is higher.
4. Pace: Firms that have cash flows front-loaded have greater pace and lower requirements for external funding, therefore one might predict that the cost of debt resulting from the loss of flexibility would be relatively lower and consequently, they would use debt more aggressively.
5. Volatility: Greater volatility increases the value of both the real and the financial options of the firm. The net effect is that the cost of debt, namely the suboptimal exercise of the real options, increases and consequently the optimal capital structure is less debt.

The variables that are expected to primarily affect investment policy are:

6. Capital structure: The entrepreneur is not capital constrained in our model, but is a wealth-maximizer. The firm's set of real options is exercised suboptimally as debt is introduced but the amount of equity is lower and there is a tax benefit as well. Our model has an optimal capital structure that is solved simultaneously with optimal investment structure, therefore, optimal investment policy is determined by all of the aforementioned variables that affect the costs and benefits of debt as well as those variables that are discussed below.
7. Modularity: More flexibility to build capacity in response to changing demand is a good thing, *ceteris paribus*, and increases the firm's debt capacity
8. Abandonment (or collateral) value: Higher collateral value both increases the debt capacity of the firm and alters the exercise timing of the abandonment and growth options. Therefore, it plays a central (but not necessary) role in the determination of the investment structure of the firm. Higher collateral value makes the exercise of growth options more likely.

9. Capital to labor ratio: When interpreted as a measure of the efficiency of production the capital to labor ratio is a monotonic function of the growth factor that is used to model the benefit of expansion of the firm (we assume constant scale expansion). Ceteris paribus, greater efficiency increases value, but if it comes at the expense of reduced flexibility, it may not be of net benefit.

10. Volatility: see point 5 above.

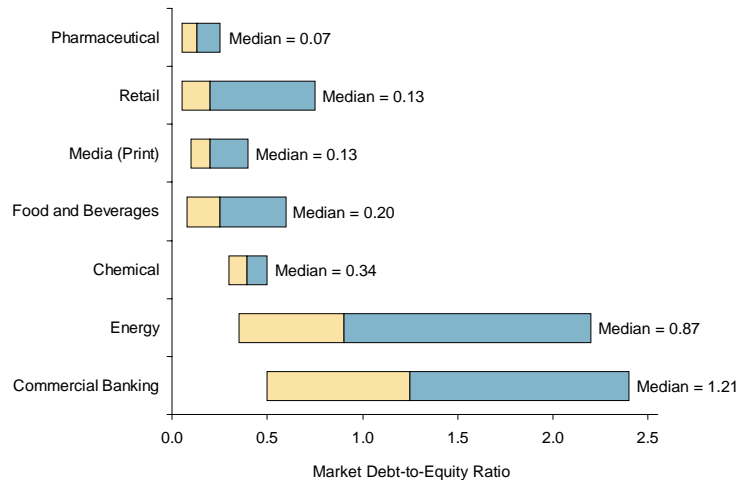
8. Empirical implications

The theory of the firm as a three-layer cake may possibly explain the cross-sectional regularities in capital structure. The value of flexibility of operations is a substitute for financial flexibility. Therefore, cross-sectional differences in flexibility by industry should explain differences in capital structure (and vice versa). One might argue that theories of capital structure founded on tradeoffs between the tax benefit of the deductibility of interest versus business disruption costs, generally fail to explain cross-sectional regularities in capital structure because tax rates and business disruption costs are relatively the same from company to company.

Exhibit 16 — Market Debt-to-Equity (2002)

Confidential

A-rated Companies



ZKN-MFB-Stoll_Charts-5-18-05-TC

13

Copyright © 2005 Monitor Company Group, L.P. — Confidential — CAM

Exhibit 16 shows industry average and median debt-to-equity ratios in market value terms and Exhibit 17 shows them in book value terms for seven industries in 2002. All companies in the sample have A-rated debt. Thus, holding the debt rating constant, there are clearly significant differences among industries. Casual empiricism, and the theory presented here might argue that pharmaceuticals have the least debt because, after deducting their research and development expenses from cash flow, the net is low. Furthermore, there is little flexibility in R&D spending because it is necessary to produce a pipeline of products, and when the pipeline fails there is no collateral value. These facts

point to a vacuum of flexibility in operations and therefore equity financing is necessary to provide flexibility.

At the opposite end of the spectrum are energy companies and commercial banks. Unlike pharmaceuticals, they have high collateral value, and we presume that their liquidation value would be high, allowing more flexibility and therefore more debt. One puzzle, however, is why the range of capital structures is so wide for these two industries. We conclude that firms that use more debt have:

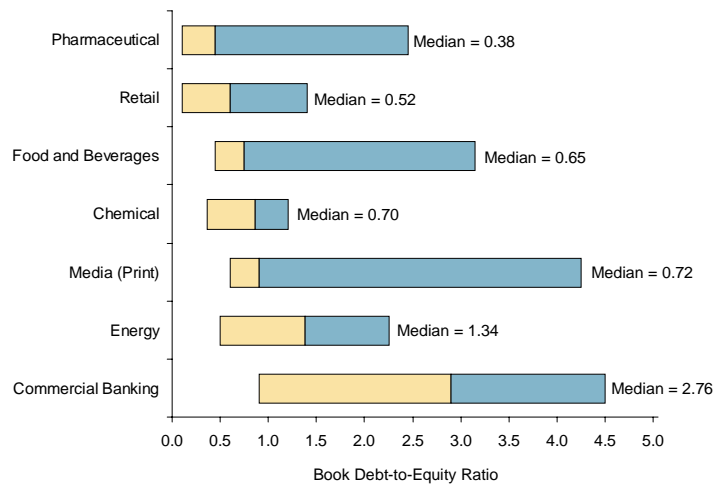
- greater flexibility in operations
- lower volatility of demand
- greater collateral value
- lower growth
- more profitable growth
- greater pace.

Net working capital (measured as the cumulative cash flows along a given branch of the value tree) should play a role because it affects the financing of growth options. Companies that generate negative working capital (e.g. Amazon.com) have more cash flow to finance growth.

Exhibit 17 — Book Debt-to-Equity (2002)

Confidential

A-rated Companies



Higher cash flows alleviate the cost of inflexibility that is associated with debt and the firm can use less debt, *ceteris paribus*.

The ratio of profitability (in cash flow terms) to capital needed for growth will affect the amount of debt. Firms that are expected to produce high cash flow per unit of investment will be able to have more debt, *ceteris paribus*.

There will be a relationship between the maturity structure of debt and the flexibility of a firm's operations.

Summary and Conclusions

We extend MM, Dixit and Pindyk, Black and Scholes, Leland and Toft, and Schwartz and Moon by modeling the firm as a “three-layer cake”. The primitive firm, the first layer, is valued without flexibility as the underlying asset for a second layer of real options to grow or abandon the firm. The third layer is made up of the financial options – debt and equity.

The resulting theory of the firm shows that there is an optimal investment structure as well as an optimal financial (i.e. capital) structure for the firm. Given an all-equity firm, the growth and abandonment options interact as a higher exercise price of the firm's abandonment put changes the timing of the exercise of the firm's growth option(s). Furthermore, optimal investment structure involves choices of modularity that are influenced by tradeoffs between flexibility and economies of scale.

When debt is introduced, the equity of the firm becomes a call option on the value of the firm with its real options. We show that abandonment in particular (and also expansion) becomes exercised suboptimally (relative to an all equity case) as more debt is introduced because abandonment provides collateral and early payment of debt. Having identified suboptimal exercise of the firm's real options as an endogeneously determined cost of debt we conclude that the real and financial options of the firm interact. Introduction of a tax gain from leverage – a benefit of debt – results in optimal capital structure that is a tradeoff between the tax benefit of debt and its cost, namely, the loss of operating flexibility.

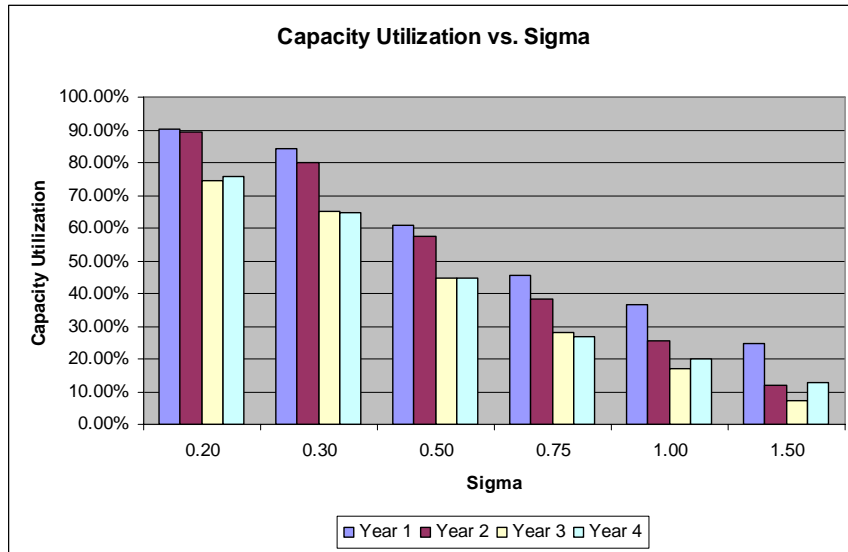
References

- Bellman, Richard E., 1957, *Dynamic Programming*, Princeton University Press, Princeton, N.J.
- Berk, J., 1999, "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, Vol. 54, 1553-1607.
- Biais, B., and C. Casamatta, 1999, "Optimal Leverage and Aggregate Investment," *Journal of Finance*, Vol. 54, No. 4, 1291-1323.
- Black, F. and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 637-659.
- Booth, L., V. Aivazian, A. Demirguc-Kunt, and V. Maksimovic, 2001, "Capital Structures in Developing Countries," Vol. 56, No. 1, 87-130.
- Brounen, D., de Jong, A., Koedjik, K., 2004, "Corporate Finance in Europe: Confronting Theory with Practice," *Financial Management*, Vol. 33, No. 4, 71-101
- Copeland, T., J. Murrin, and T. Koller, 2000, *Valuation: Measuring and Managing the Value of Companies*, 3rd edition, John Wiley & Sons, New York.
- Cornell, B., 1993, *Corporate Valuation*.
- Cox, J., S. Ross, and M. Rubinstein, 1979, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, 229-263.
- Damodaran, A., 2006, *Damodaran on Valuation*, John Wiley & Sons.
- Dixit, A. and R. Pindyk, 1994, *Investment Under Uncertainty*, Princeton University Press.
- Fama, E. and K. French, 1998, "Taxes, Financing Decisions, and Firm Value," *Journal of Finance*, Vol. 53, No. 3, 819-843.
- Gordon, M., "Dividends, Earnings, and Stock Prices, 1959," *Review of Economics and Statistics*, 99-105.
- Graham, J., and C. Harvey, 1996, "Market Timing Ability and Volatility Implied by the Investment Newsletters' Asset Allocation Recommendations," *Journal of Financial Economics*, 397-421.
- Kaplan, S. and R. Ruback, 1995, "The Valuation of Cash Flow Forecasts: An Empirical Analysis," *Journal of Finance*, Vol. 50, No. 4, 1059-1093.
- Leland, H., 1994, "Corporate Debt Value, Bond covenants, and Optimal Capital Structure," *Journal of Finance*, Vol. 49, No. 4, 1213-1252.
- Leland, H. and K. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 1996, Vol. 51, No. 3, 987-1019.

- Miao, Jianjun, 2005, "Optimal Capital Structure and Industry Dynamics," *Journal of Finance*, Vol. 60, No. 6, 2621-2659.
- Malkiel, B., 1963, "Equity Yields, growth and the Structure of the Share Price," *American Economic Review*, Vol. 53, 1004-1031.
- Merton, R., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 449-470.
- Miller, M. and F. Modigliani, 1961, "Dividend Policy, Growth and the Valuation of Shares," *Journal of Business*, 411-433.
- Myers, S., 1977, "The Determinants of Corporate Borrowing," *Journal of Financial Economics*, January 1977, 147-176.
- Myers, S., 1984, "The Capital Structure Puzzle," *Journal of Finance*, 575-592.
- Samuelson, P., 1965, "Proof that properly anticipated prices fluctuate randomly," *Industrial Management Review*, 1965, 41-49.
- Schwartz, E. and W. Moon, 1999, *Financial Analysts Journal*.
- Stoll, H, 1969, "The Relationship Between Put and Call Option Prices," *Journal of Finance*, 802-824.
- Titman, S. and R. Wessels, 1988, "Determinants of Capital Structure Choice," *Journal of Finance*, Vol. 43, No. 1, 1-19.
- Triantis, A., and J. Hodder, 1990, "Valuing Flexibility as a Complex Option," *Journal of Finance*, Vol. 45, No. 2, 549-565.

APPENDIX

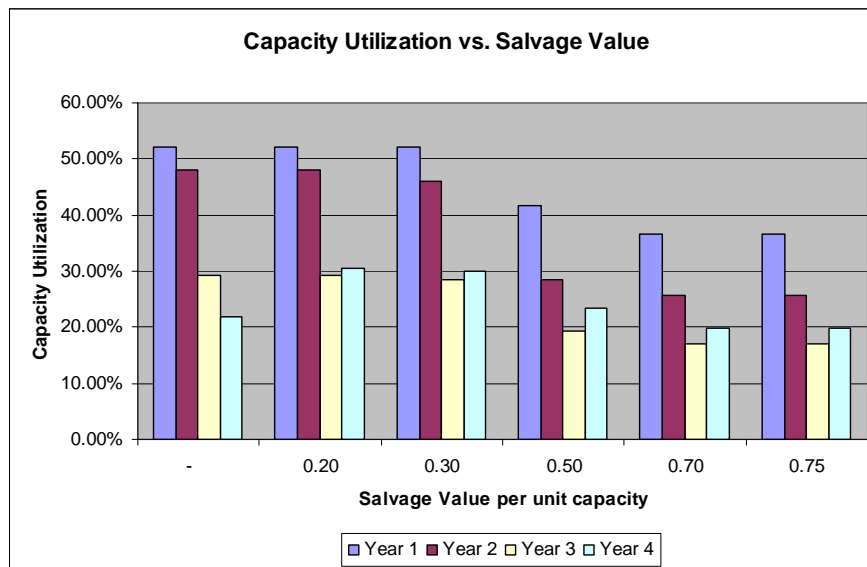
Appendix Exhibit A1 – Capacity Utilization versus Variance



Capacity utilization is the % of capacity used in a given state, averaged across all the possible states in a year. Here the Expansion Option is held constant, with a plant size of 300 units costing \$0.75/unit. The Salvage value is held constant at \$0.70/unit. Sigma varies from 0.2 to 1.5.

As variance rises, the demand in upper states of nature rises rapidly, so the firm expands more often. However, in down states the demand falls rapidly, leading to a lower capacity utilization overall.

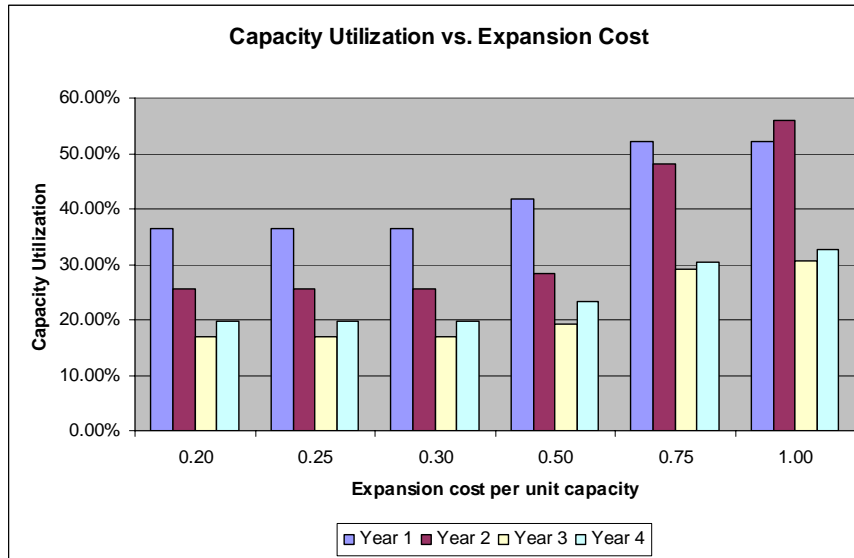
Appendix Exhibit A2 – Capacity Utilization versus Salvage Value



Capacity utilization is the % of capacity used in a given state, averaged across all the possible states in a year. Here sigma is held constant at 1 and the Expansion Option is held constant with a plant size of 300 units costing \$0.75/unit. The Salvage value varies from 0 to \$0.75/unit.

As the salvage value rises, the firm is able to expand earlier and more often. However, this earlier expansion leads to more down states of nature with significant overcapacity and thus lower capacity utilization.

Appendix Exhibit A3 – Capacity Utilization versus Expansion Cost



Capacity utilization is the % of capacity used in a given state, averaged across all the possible states in a year. Here sigma is held constant at 1 and the salvage value is held constant at \$0.20/unit. The the expansion cost varies from \$0.20/unit to \$1.00/unit with the same plant size of 300 units.

As the expansion cost rises, the firm expands less. This means the overcapacity in down states of nature is lowered and thus capacity utilization rises.