The Combined Effect of Market, Technical and Technological Uncertainties on New Technology Adoptions

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Abstract

We derive a real options model in a game theoretic context where the optimal time to adopt a new technology is affected by three types of uncertainties: *market*, *technical* and *technological* uncertainty. Market uncertainty represents the uncertainty of changes in demand, price and competition; technical uncertainty concerns the "efficiency" of the new technology after adoption, which may be firm specific; and technological uncertainty is considered by assuming that there is a probability of a second, and more efficient, technology arriving in the market. Using a multi-factor model, we derive analytical expressions for the leader and the follower value functions, and their respective investment trigger values, in a game-choice setting considering the pre-emption effect. We show that high correlation between market and efficiency factors increases slightly the leader's value function relative to the follower and delays the adoption time for both. We find that a high probability of a second technology arrives in the market delays the adoption time of the leader and has no effect on the adoption time of the follower.

Keywords: Multi-factor model, technical and technological uncertainty, duopoly investment game.

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1. Introduction

In some circumstances the technical quality of a new technology becomes apparent only after adoption. Consequently, the assumption that a new technology, after adoption, will perform, technically, as the developer/adopter predicts is not appropriate for some investment decisions.

There are two main reasons for the existence of technical uncertainty in the adoption of a new technology. The first regards the difficulty of fully testing some technologies before launch. The second relates to the fact that the performance of some technologies be dependent, at least to some extent, on the firm's technical skills, a function of the quality of its human resources, organizational culture and management commitment to the adoption, which vary over time and differ among firms.

In this paper, we relax the assumption that after adoption the performance of the new technology will be technically perfect. As an illustration of the effect of the technical uncertainty on the value of the adoption of a new technology we cite two events recently described in the press: the delay in the construction of the new Airbus A380 and the appearance of cracks in the boiler pipes of the British Energy number 3 and number 4 nuclear power reactors. These are two good examples of the impact of technical uncertainty on the value of the adoption of a new technology, since, according to the information released, the delays of the Airbus A380 project, in late 2006, appear to be due to *technical* reasons (the great difficulty in integrating a huge number of new technologies) rather than due to any unexpected change in the market variables, and the technical problems with the British nuclear power reactors has forced the company to take the power stations out of service for several months, and, to avoid further cracks, the reactors must operate in the future at a 70 percent load[3].

[3] See Financial Times, November 18, 2006, p. 1.

In this paper, we define technical uncertainty as the uncertainty regarding the performance of a new technology that persists after it has been developed and adopted. Besides technical uncertainty we consider also market and technological uncertainties. Consequently, in this model we study the combined effect of three different types of uncertainty.

The model is derived for a duopoly market. It is assumed that at the beginning of the investment game there are two firms which can become active in the market by adopting one new technology, for which they have to spend a sunk cost *I*. There is one new technology currently available (*tech 1*) and the probability that a second and more efficient one (*tech 2*) arrives in the future. Firms are allowed to invest only once (a "*one-shot*" game) and the two technologies cost the same.

This model is an extension of Azevedo and Paxson (2007a) where we derive analytical expressions for the leader and the follower value functions under similar assumptions but where technological uncertainty is not considered. As in Azevedo and Paxson (2007a), market uncertainty is denoted by the volatility of the variable "revenues", and technical uncertainty is represented by the volatility of the variable "efficiency" of the new technology after adoption. The variables "revenues" and "efficiency of the new technology after adoption" are assumed to follow a geometric Brownian motion (gBm) process and the arrival of the second technology in the market is assumed to follow a Poisson distribution with parameter λ.

This paper also relates to Huisman (2001) model, where, under similar assumptions, the author derives a real option model, but not considering technical uncertainty. Huisman (2001) shows that the optimal investment timing for both firms is governed, to a large extent, by the magnitude of the probability that a second new technology (*tech* 2) becomes available within a given period of time.

Technological uncertainty is also considered in Grenadier and Weiss (1997), but, contrary to Huisman (2001), as a state variable that follows a gBm process[4]. According

[4] Though technological progress is typically due to random moves in one direction only, i.e., technological declines are not common.

this model, markets with higher levels of growth are assumed to have speedier innovation arrivals and markets with greater levels of volatility are assumed to have greater uncertainty over the arrival of future innovations.

Our model also has some similarities with Paxson and Pinto (2005), in the sense that both models use two underlying variables following gBm processes. However, in Paxson and Pinto (2005), technological uncertainty is not considered and both of the variables used, “price” and “quantities”, relates to market uncertainty, while in our model one variable, the “revenues”, relates to market uncertainty, and the other, the “efficiency of the new technology after adoption”, relates to the technical uncertainty underlying the adoption.

Finally, a recent paper by Murto (2006) also treats simultaneously revenues and technological uncertainties. For a monopoly market, the author derives a real options model to determine the optimal time to adopt a new technology under “technological” and “revenue-related” uncertainties, where he shows that technological uncertainty has no effect on the optimal investment policy when revenue uncertainty is absent, but when combined with revenue uncertainty an increase in technological uncertainty makes investment less attractive relative to waiting.

An extensive survey on literature about new technology adoption models can be seen in Hoppe (2002).

In this model market uncertainty represents the uncertainty of changes in demand, price and competition. For example, income, tastes, and the pricing decisions of competitors can change unpredictably, or a substitute product might arrive that makes the firm’s product suddenly obsolete.

Technical uncertainty captures the uncertainty regarding the performance (efficiency) of the new technology that persists after it has been developed and adopted. The efficiency of a technology can be quantified using the concept of “efficiency production frontier”

(EPF), from the theory of industrial organization (see Aigner et al. (1997) and Coelli et al. (1998))[5].

The concept of the EPF defines, for a current stage of the technological development, i.e., for a current *state-of-the-art*, a reference to which firms' current operational performance (efficiency) should be compared. Performances above the EPF frontier are not possible since points at the EPF are benchmarking points only possible to achieve in the ideal scenario where, after adoption, the technology operates without any technical imperfection and in a context of zero human inefficiencies (technological perfection). In such a scenario the efficiency of the technology is considered to be 100 percent. On the other hand, if after the adoption of the new technology the most catastrophic scenario occurs, i.e., the new technology fails completely, the firm will operate with an efficiency of zero percent. In intermediate, and more likely, scenarios, after adoption, the new technology neither will be technically perfect (100 percent efficiency) nor a complete failure (zero percent efficiency), but somewhere in between these two extreme cases. Therefore, the domain of the underlying variable defined here as "efficiency of the new technology after adoption" turns to be $E_t = [0,1]$, with both extremes being very unlikely to be reached[6].

Technological uncertainty is related to the evolution and the stage of the industry where technologies are developed. For instance, in the early stages of the development of a new industry, the number of innovations is usually huge and sometimes the direction in which the industry will develop is not clear. During such times, technological uncertainty reaches its maximum level. As the industry matures and the technology standardizes, the rate of innovation tends to decrease and so does the technological uncertainty.

The importance of each of the uncertainties described above, in the firms' decision to adopt a new technology, depends on the economic environment in which the investment

[5] Although other methods can also be used, see for instance Slack and Lewis (2002) and Todinov, M. (2005). As example, we show in Appendix A a dataset of records of daily efficiency of a textile production technology where Slack and Lewis (2002) method was used.

[6] For a description about basic reliability concepts, which are underlying some of our assumptions regarding variable E_t, and how the variable "efficiency of a new technology after adoption" can be measured in practice, we refer to Appendix A.

decision is made and on the type of technology involved. For instance, software programs and telecommunication technologies can be almost fully tested in a laboratory before launch and almost do not need operators to work efficiently. Consequently, technical uncertainty is very low. However, technological uncertainty is usually huge, due to the high innovation rate that characterizes both industries. On the other hand, firms operating in commodity industries may give great importance to the uncertainty regarding the output price and market share and little importance to the technical and technological uncertainties and firms operating in markets where market, technical and technological uncertainties hold simultaneously must take all of them into account.

This paper is organized as follows. In section 2, we characterize the investment game. In section 3, we describe the model and derive the leader and follower value functions and their respective investment trigger values. In section 4, we do some sensitivity analysis and comment on our most important results. In section 5 we conclude.

2. The Investment Game

The investment game is characterized as follows: there are two firms which can become active in the market by adopting a new technology for which they have to spend a sunk cost I. There is a new technology currently available (*tech 1)* and some probability that a second and more efficient one (*tech 2*) becomes available in the future. The time is considered to be continuous and the life of the technology infinite. Firms are allowed to invest only once (a "*one-shot*" game), the improvement in firms' profits can only be made at the cost of its opponent (a *zero-sum* game) and three types of uncertainties hold simultaneously: *market*, *technical* and *technological* uncertainty.

Market uncertainty is due to the random evolution of demand and competition on the output price, *technical uncertainty* exists because the performance of the new technology after adoption is uncertain, and *technological uncertainty* holds because it is likely that, at an unknown date, $\tilde{\tau}$, a second and more efficient technology (*tech 2*) arrives in the market.

In Figure 1 we represent this investment game in an extensive form. For a detailed description of this type of game representation see Gibbons (1992).

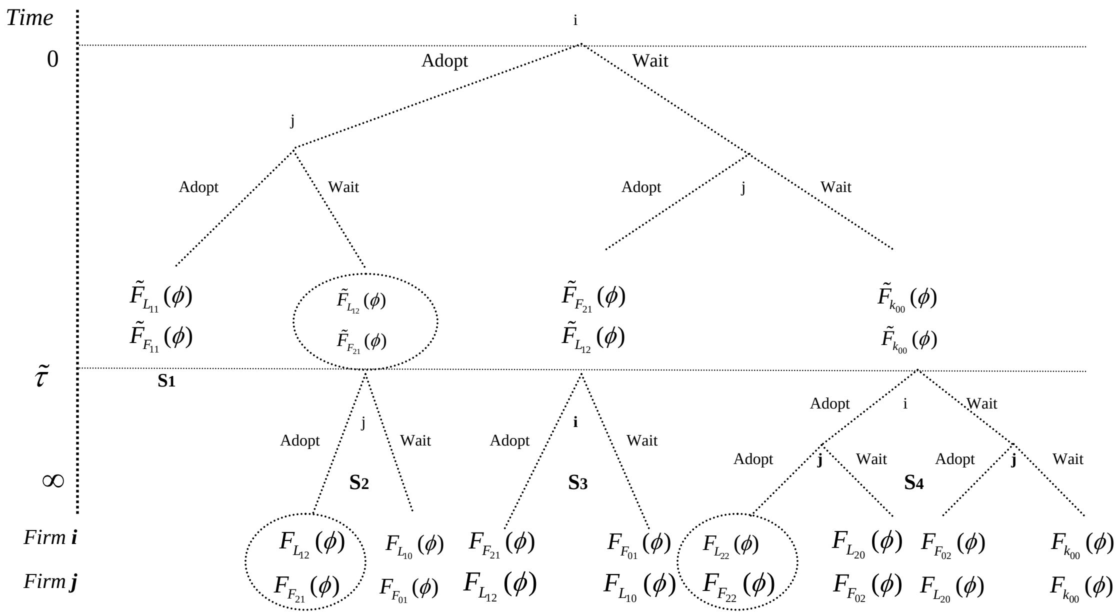

Figure 1 - Extensive-form representation of a Continuous Time Real Option Game (CTROG) for the cases where firms have one or two technologies available.

In Figure 1 we characterize the following four main game scenarios:

Scenario 1 (S1)*:* both firms adopt *tech 1* before the arrival date of *tech 2* ($\tilde{\tau}$). Firm *i* adopts first and becomes the leader, firm *j* adopts second and becomes the follower. The firms' payoffs, for firm *i* and *j* respectively, are given by the following functions $\tilde{F}_{L_{11}}(\phi)$ and $\tilde{F}_{F_{11}}(\phi)$.

Scenario 2 (S2)*:* In the first round of the game firm *i* adopts *tech 1*, and becomes the leader, and firm *j* waits for *tech 2*, after which it can adopt it or not. If firm *j* adopts *tech 2*, it becomes the follower, but now with a more efficient technology. The firms' payoffs in this case are given, respectively for firms *i* and *j*, by the functions $F_{L_{12}}(\phi)$ and $F_{F_{21}}(\phi)$. If firm *j* neither adopts *tech 1* nor *tech 2*, firm *i* will operate with *tech 1* in a monopoly and gets $F_{L_{10}}(\phi)$ as payoff, while firm *j* will get $F_{F_{01}}(\phi) = 0$.

Scenario 3 (S3)*:* In the first round of the game, firm *i* waits for *tech 2* and firm *j* adopts *tech 1*, before *tech 2* being available. This is the symmetric case of scenario 2. The payoffs from this scenario are the same as scenario 2, only the firms change their positions.

Scenario 4 (S4)*:* After *tech 2* has been released, the technological uncertainty is eliminated. Therefore, from time $\tilde{\tau}$ onward only the market and the technical uncertainties affect the firms' investment decision. In a *ceteris paribus* analysis, with the arrival of *tech 2* firms have an additional incentive to make the investment, considering the market and technical uncertainties constant. As it is assumed that no more than two technologies are available, the cost of the two technologies is the same and *tech 2* is more efficient than *tech 1*, so after the arrival of *tech 2* the adoption of *tech 1* is not any more optimal. Consequently, the investment game turns into a standard new technology investment game with two firms, one available technology and no technological uncertainty. However, in order to reach this scenario, there is one pre-condition on the firms' behaviour: to delay the adoption of *tech 1* until the arrival of *tech 2*.

Assuming that neither firm adopts *tech 1* before $\tilde{\tau}$, and that *tech 2* is now available, $t \geq \tilde{\tau}$, they still have the following four different investment scenarios:

(i) Firms *i* and *j* adopt *tech 2*, one after the other. Firm *i* becomes the leader, and firm *j* the follower with the following payoffs: $F_{L_{22}}(\phi)$ and $F_{F_{22}}(\phi)$, respectively;

(ii) Firm *i* adopts *tech 2* and becomes the leader while firm *j* waits, i.e., it is not in the market yet. The firms' payoffs are given, respectively, by $F_{L_{20}}(\phi)$ and $F_{F_{02}}(\phi)$;

(iii) Firm *j* adopts *tech 2* and becomes the leader while firm *i* waits. Firms *j* and *i* payoffs are given, respectively, by $F_{L_{20}}(\phi)$ and $F_{F_{02}}(\phi)$. This is the symmetric case of that described above;

(iv) In this scenario, even with *tech 2* available, firms have not adopted it yet. Therefore, the investment game continues to the next round. The firms' payoffs are null and given, by $F_{k_{00}}(\phi) = 0$. The leadership on the investment is still open.

To avoid complexity, we derive analytical expressions for the leader and the follower value functions only for the investment scenarios marked in Figure 1 with an ellipse. A summary of the firms' payoffs derived in the paper are in Table 1.

	$t < \tilde{\tau}$			$t \geq \tilde{\tau}$		
	Investment before $\tilde{\tau}$			**Investment before $\tilde{\tau}$**		
	None	One	Two	None	One	Two
Leader' Value Function	$\tilde{F}_{k_{00}}(\phi)$	$\tilde{F}_{L_{12}}(\varphi)$ **(Equation 27)**	$\tilde{F}_{L_{11}}(\varphi)$	$F_{L_{22}}(\varphi)$ **(Equation 8)**	$F_{L_{12}}(\varphi)$ **(Equation 15)**	$F_{L_{11}}(\varphi)$
Follower's Value Function	$\tilde{F}_{k_{00}}(\phi)$	$\tilde{F}_{F_{21}}(\varphi)$ **(Equation 20)**	$\tilde{F}_{F_{11}}(\varphi)$	$F_{F_{22}}(\varphi)$ **(Equation 8)**	$F_{F_{21}}(\varphi)$ **(Equation 12)**	$F_{F_{11}}(\varphi)$

Table 1

For a good review of some game theory concepts used in this section and examples of real option investment games see Smit and Trigeorgis (2004).

2.1 The Pre-emption game

In games of timing the adoptions of new technologies, the potential advantage from being the first to adopt may introduce an incentive for preempting the rival, speeding up the first adoption of the new technology. The first contribution on adoption timing under rivalry is the Reinganum (1981) game-theoretic approach. In this model, the adoption of one firm has a negative effect on the profits of the other firm and the increase in profits due to the adoption is greater for the leader than for follower. Fudenberg and Tirole (1985) studied the adoption of a new technology and illustrate the effects of preemption in games of time. We use Fudenberg and Tirole (1985) concept of preemption to derive the leader and the follower value functions.

3. The Model

Consider that at time t there are two idle firms, i and j, one new technology available (*tech 1*) and uncertainty regarding the evolution of the "revenues" and the "efficiency of the new technology after adoption" (reliability of the new etchnology). Assume also that the firm that moves first (the leader) gets, simultaneously, two types of advantages over the follower: a market advantage (higher market share) and an efficiency advantage (higher operational efficiency)[7].

Given the context above, the firms' revenues flow is given by

$$\varphi_t \left[de_{k_i k_j} \right] \tag{1}$$

where, φ_t are the revenues weighted with the technical uncertainty[8], and $de_{k_i k_j}$ is a deterministic factor that ensures the leader's advantage combining the effect of both "market share advantage" (*d*) and "operational efficiency advantage" (*e*)[9]; $k = \{1, 2\}$, with 1 and 2 describing the cases where firms operate with *tech 1* or *tech 2*, respectively; firms $i,\ j = \{L, F\}$, where *L* means "leader" and *F* "follower".

The intuition underlying the possibility of a first-mover "market" advantage is the same as that used in Dixit and Pindyck (1994) following Smets (1993). The intuition about the first-mover "efficiency" advantage is that as the leader adopts the new technology first it initiates earlier the correspondent learning process and, therefore, when the follower adopts an asymmetry in the firms' operational efficiency holds. As firms are assumed to be symmetric in their ability to learn and spillover information is not allowed, so the leader's initial advantage holds forever. Therefore, inequality (2.0) holds.

$$de_{1_i 0_j} > de_{1_i 1_j} > de_{0_i 0_j} > de_{0_i 1_j} \tag{2.0}$$

The relation above means that the best investment scenario, for firm *i* (*j*), from the *market* and the *technical* point of views, is when it adopts *tech 1* and its opponent does not ($de_{1_i 0_j}$); its second best investment scenario occurs when it adopts *tech 1* and its opponent also does, though a little later ($de_{1_i 1_j}$); its third best investment scenario is when

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[7] We refer to Azevedo and Paxson (2007a) for a detailed discussion on this setting.

[8] $\varphi_t = X_t * E_t$ (see differential equations (6) and (7)).

[9] See inequalities (2.0) and (2.1).

it does not adopt *tech 1* and so does its rival ($de_{0_i 0_j}$); and finally, firm *i* worst investment scenario is when it does not adopt *tech 1* and its opponent does, ($de_{0_i 1_j}$).

In the model, the variable X_t denotes the revenues and the variable E_t represents the efficiency of the new technology (*tech 1* or *tech 2*) after adoption. Consequently, market uncertainty is expressed by the volatility of X_t and technical uncertainty is given by the volatility of E_t. We assume that both variables follow the gBm processes given by equations (3) and (4).

$$dX_t = \mu_X X_t dt + \sigma_X X_t dz_1 \tag{3}$$

$$dE_t = \mu_E E_t dt + \sigma_E E_t dz_2 \tag{4}$$

where, μ_X and μ_E are the instantaneous conditional expected percentage changes in X_t and E_t, respectively, per unit of time; σ_X and σ_E are the instantaneous conditional standard deviation per unit of time, in X_t and E_t respectively; dz_1 and dz_2 are the increment of a standard Wiener process, respectively, for the variables X_t and E_t. It is also assumed that $r > \left(\mu_X + \mu_E\right)$, where r is the riskless interest rate, and the two variables are possibly correlated. In further sections where no confusion is possible we will ignore the subscript *t*.

Given the context above, the value of the option to adopt *tech 1*, depends not only on the firms' expectations regarding the evolution of the market factors (changes in demand, prices, competition, etc), but also on firms' expectations about the reliability of the new technology. To introduce a little more complexity in the model we also add to the investment problem the technological uncertainty, i.e., the possibility that a second and more efficient technology (*tech 2*) arrives in the market.

The arrival date of the second technology is assumed to follows a Poisson distribution with parameter λ and mean $1/\lambda\ (>0)$. The Poisson probability distribution is a discrete distribution which expresses the probability of a number of events occurring in a fixed period of time if these events occur with known average rate, λ, and are independent of the time since the last event. Technological uncertainty is defined in this model by expression (5).

$$d\theta = \begin{cases} 1 & \text{with probability } \lambda \\ 0 & \text{with probability } 1\text{-}\lambda \end{cases} \tag{5}$$

where, $d\theta$ is the probability of arrival of tech 2.

Note that inequality (2.0) only characterizes firms' gains for scenarios where there is one new technology available (*tech 1*) and no technological uncertainty. Considering the possibility of a second technology (*tech 2*) arriving in the market, the following inequality holds:

$$de_{2_L 0_F} > de_{1_L 0_F} > de_{2_L 2_F} > de_{1_L 1_F} > de_{1_L 2_F} \tag{2.1}$$

The economic intuition used to characterize inequality (2.0) applies to inequality (2.1)[10]. The only difference is that inequality (2.1) allows for investment scenarios where one of the firms adopts *tech 2*.

3.1 Combining the two Underlying Variables

Let $F_F(X_t, E_t)$ be the follower's value function to adopt a new technology in a context where there is no technological uncertainty. According to our framework, this value function depends on two stochastic variables - "revenues" and "efficiency of the new technology after adoption". Given such circumstances, expression (6) is the partial differential equation that describes the evolution of the value function of an idle follower, and is subjected to the usual boundary conditions: *value-matching* and *smooth-pasting* conditions.

[10] In Appendix B we exemplify how to compute these parameters.

$$\frac{1}{2}\frac{\partial^2 F_F}{\partial X^2}\sigma_x{}^2 X^2 + \frac{1}{2}\frac{\partial^2 F_F}{\partial E^2}\sigma_E{}^2 E^2 + \frac{\partial^2 F_F}{\partial X\,\partial E} XE\sigma_x\sigma_E\rho_{XE} + \frac{\partial F_F}{\partial X}\mu_x X + \frac{\partial F_F}{\partial E}\mu_E E - rF_F = 0 \qquad (6)$$

An explicit closed-form solution can be derived by assuming: i) that $F_F(X_t, E_t)$ is *homogeneous of degree one*[11], which allows to reduce the dimensionality of the partial differential equation; and ii) the following changing in the variables, $(X_t * E_t) = \varphi_t$, has economic meaning. If both of this conditions hold, so, we can work, from this point onward, with just one variable, φ_t.

The multiplicative form assumed for the value of the adoption has an economic meaning[12], since we can think about the value of the adoption as the firms' expected "revenues", in a context of no technical uncertainty, i.e., in the ideal scenario where after adoption the new technology operates continuously at 100 percent efficiency, multiplied by the "efficiency" of the new technology that is expected to be achieved in a context of technical uncertainty.

Therefore, we assume that firms' revenues, X_t, are proportional to the efficiency of the new technology after adoption, E_t. The intuition is that in a *ceteris paribus* analysis, the higher the efficiency of the new technology after adoption the higher the number of units of output produced[13]. Therefore, by multiply "revenues", X_t, by "efficiency of the new technology after adoption", E_t, we get a new generic variable which we denote by

[11] A function $f(x_1, x_2, ..., x_n)$ is homogeneous of degree k if $\lambda^k f(x) = f(\lambda x)$, where $\lambda \geq 0$ and x is the vector $[x_1, x_2, ..., x_n]$.

[12] Which is also a pre-condition for changing the variables using the relation $(X_t * E_t) = \varphi_t$.

[13] Using an analogy of the adoption of a new technology in the oil industry, once the development of the oil well is completed the value of the project is roughly equal to the "revenues per barrel" multiplied by the "number of barrels" produced, and the number of barrels is positively correlated with the efficiency of the technology. Market uncertainty affects the former and technical uncertainty impacts primarily the latter.

"efficiency weighted revenues", φ_t [14]. For an example of how this changing in the variable works in a practice see example given in Appendix A3.

In the derivations below we assume that all sources of market and technical uncertainties are completely diversifiable. Therefore, no extra premium over the riskless interest rate is demanded as a result of the firm being exposed to those sources of risk.

Doing the respective substitutions in expression (6) we get the following second-order ordinary differential equation:

$$\frac{1}{2}\varphi^2\sigma_m{}^2 F_F\,''(\varphi) + \varphi\left(\sigma_X\sigma_E\rho_{XE} + \mu_X + \mu_E\right)F_F\,'(\varphi) - rF_F(\varphi) = 0 \tag{7}$$

where, $\sigma_m = \sqrt{\sigma_X{}^2 + \sigma_E{}^2 + 2\rho_{XE}\sigma_X\sigma_E}$

This ordinary differential equation has an analytical solution whose general form is given by:

$$F_F(X_t, E_t) = A\varphi^{\beta_1} + B\varphi^{\beta_2}$$

where, constants *A* and *B* are determined using the adequate boundary conditions (*value-matching* and *smooth-pasting*) and β_1 and β_2 are the square roots of a characteristic quadratic function of an Euler's type ordinary differential equation.

3.2 Technology 2 is Available, $t \geq \tilde{\tau}$

Consider the scenario where *tech 2* is available. According to our framework, when *tech 2* is released the technological progress stops, i.e., technological uncertainty disappears. On the other hand, as we assume that *tech 2* costs the same as *tech 1* and is more efficient, so after the arrival of *tech 2* it is no more optimal to adopt *tech 1*. Consequently, the firms' investment decision for this scenario is similar to that where we have two firms

[14] The variable X_t could equal unit net revenues multiplied by quantity produced or total gross revenues multiplied by a cost of production efficiency factor.

and just one technology available. This problem is treated, for the case of one-stochastic variable, in chapter 9 of Dixit and Pindyck (1994).

3.2.1 No Investment has been made

3.2.1.1 The Firms' Value Function

Following the Dixit and Pindyck (1994) methodology (see chapters 7 and 9), for one stochastic underlying variable, we derive the firms' value functions for two stochastic underlying variables. The expected value of each firm equals the follower's value given by expression (8).

$$F_{F_{22}}(\varphi) = \begin{cases} A_{22}\varphi^{\beta_1} & \varphi < \varphi^*_{F_{22}} \\ \dfrac{\varphi\left(de_{2_F 2_L}\right)}{r-\mu_X-\mu_E} - I & \varphi \geq \varphi^*_{F_{22}} \end{cases} \tag{8}$$

where,

$$\varphi^*_{F_{22}} = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu_X-\mu_E)}{\left(de_{2_F 2_L}\right)} I \tag{9}$$

$$A_{22} = \left(\varphi_{F_{22}}\right)^{-\beta_1}\left(\frac{\varphi_{F_{22}}\left(de_{2_F 2_L}\right)}{r-\mu_X-\mu_E} - I\right) \tag{10}$$

$$\beta_1 = \frac{1}{2} - \frac{(\delta_E - \delta_X)}{\sigma_m{}^2} + \sqrt{\frac{2\delta_E}{\sigma_m{}^2} + \left[\frac{(\delta_E - \delta_X)}{\sigma_m{}^2} - \frac{1}{2}\right]^2} \tag{11}$$

where, $\delta_X = r - \mu_X$ and $\delta_E = r - \mu_E$,

3.2.2 The Leader has Adopted Tech 1

3.2.2.1 The Follower's Value Function

If at the time *tech 2* becomes available one of the firms (the leader) has adopted *tech 1* already, then the problem for the firm that has not yet adopted (the follower) is like that of a monopoly investment decision with just one technology available and its value function is given by the following expression:

$$F_{F_{21}}(\varphi) = \begin{cases} A_{21}\varphi^{\beta_1} & \varphi < \varphi^*_{F_{21}} \\ \dfrac{\varphi\left(de_{2_F 1_L}\right)}{r-\mu_X-\mu_E} - I & \varphi \geq \varphi^*_{F_{21}} \end{cases} \tag{12}$$

where,

$$\varphi^*_{F_{21}} = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu_X-\mu_E)}{\left(de_{2_F 1_L}\right)} I \tag{13}$$

$$A_{21} = \left(\varphi^*_{F_{21}}\right)^{-\beta_1}\left(\frac{\varphi^*_{F_{21}}\left(de_{2_F 1_L}\right)}{r-\mu_X-\mu_E} - I\right) \tag{14}$$

3.2.2.2 The Leader's Value Function

The value of the leader is given by:

$$F_{L_{12}}(\varphi) = \begin{cases} B_{12}\varphi^{\beta_1} + \dfrac{\varphi\left(de_{1_L 0_F}\right)}{r-\mu_X-\mu_E} - I & \varphi < \varphi^*_{F_{21}} \\ \dfrac{\varphi\left(de_{1_L 2_F}\right)}{r-\mu_X-\mu_E} & \varphi \geq \varphi^*_{F_{21}} \end{cases} \tag{15}$$

The constant B_{12} is given by:

$$B_{12} = \left(\varphi^*_{F_{21}}\right)^{1-\beta_1}\frac{\left(de_{1_L 2_F} - de_{1_L 0_F}\right)}{r-\mu_X-\mu_E} \tag{16}$$

We did not find a closed-form solution for the leader's investment trigger value. However, a numerical solution can be found applying numerical method to the equation below, where $\varphi^*_{L_{12}}$ is the unknown variable.

$$B_{12}(\varphi^*_{L_{12}})^{\beta_1}\frac{\varphi^*_{L_{12}}\left(de_{1_L 0_F}\right)}{r-\mu_X-\mu_E} - A_{21}(\varphi^*_{L_{12}})^{\beta_1} - \frac{\varphi^*_{L_{12}}\left(de_{0_F 1_L}\right)}{r-\mu_X-\mu_E} = 0$$

3.3 Technology 2 is not Available, $t < \tilde{\tau}$

Now let consider the scenario where *tech 2* is not in the market yet but it is likely to be available in the future. In this case, three different types of uncertainty hold simultaneously: market, technical and technological uncertainty. Below we derive analytical expressions for the leader and the follower value functions for the scenario where the follower waits for *tech 2* and the leader adopts *tech 1* before the arrival date of *tech 2*. The arrival date of *tech 2* follows the Poisson distribution defined in expression (5).

3.3.1 The Follower Waits for Tech 2 and the Leader Adopts Tech 1

3.3.1.1 The Follower's Value Function

We start by deriving the follower value function, assuming that the leader adopts *tech 1* before the arrival date of *tech 2*. Using the second-order differential equation (7) and denoting the follower value function by $\tilde{F}_{F_{21}}(\varphi)$, we know that condition (17) is satisfied:

$$r\tilde{F}_{F_{21}}(\varphi) = \lim_{dt\to 0}\frac{1}{dt}\varepsilon\left[d\tilde{F}_{F_{21}}(\varphi)\right] \tag{17}$$

Considering technological uncertainty through the use of expression (5) and applying Ito's Lemma to equation (17) we get the following expression:

$$\varepsilon\left[d\tilde{F}_{F_{21}}(\varphi)\right] = (1-\lambda dt)\left(\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi^2}dt + \left(\sigma_X\sigma_X\rho + \mu_X + \mu_E\right)\varphi\frac{\partial\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi}dt\right) + \lambda dt\left(F_{F_{21}}(\varphi) - \tilde{F}_{F_{21}}(\varphi)\right) \tag{18}$$

Substituting expression (18) into equation (17) we obtain:

$$\frac{1}{2}\sigma_m^2\varphi^2\frac{\partial^2\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi^2} + \left(\sigma_X\sigma_E\rho + \mu_X + \mu_E\right)\varphi\frac{\partial\tilde{F}_{F_{21}}(\varphi)}{\partial\varphi} - (r-\lambda)\tilde{F}_{F_{21}}(\varphi) + \lambda F_{F_{21}}(\varphi) = 0 \tag{19}$$

Using the two possible expressions for $F_{F_{21}}(\varphi)$ (see equation (12)), we get the following solution:

$$\tilde{F}_{F_{21}}(\varphi) = \begin{cases} A_{21}\varphi^{\beta_1} + C_{21}\varphi^{\beta_3} & \varphi < \tilde{\varphi}^*_{F_{21}} \\ D_{21}\varphi^{\beta_4} + \dfrac{\varphi\left(de_{2_F 1_L}\right)}{(r-\mu_X-\mu_E)}\dfrac{\lambda}{(r-\mu_X-\mu_E)+\lambda} - \dfrac{\lambda I}{r+\lambda} & \varphi \geq \tilde{\varphi}^*_{F_{21}} \end{cases} \tag{20}$$

where, β_3 (β_4) is given by the following equation:

$$\beta_{3(4)} = \frac{1}{2} - \frac{(\delta_E - \delta_X)\lambda}{\sigma_m^2} + (-)\sqrt{\frac{2\delta_E + \lambda}{\sigma_m^2} + \left[\frac{(\delta_E - \delta_X)\lambda}{\sigma_m^2} - \frac{1}{2}\right]^2} \tag{21}$$

and

$$\tilde{\varphi}^*_{F_{21}} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_E)}{\left(de_{2_F 1_L}\right)} I \tag{22}$$

$$A_{21} = \left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_1} \left(\frac{\tilde{\varphi}^*_{F_{21}} \left(de_{2_F 1_L}\right)}{r - \mu_X - \mu_E} - I \right) \tag{23}$$

The expressions for C_{21} and D_{21} are derived by solving the continuity and differentiability conditions for $\tilde{F}_{F_{21}}(\varphi)$, at $\varphi = \tilde{\varphi}^*_{F_{21}}$, which gives:

$$C_{21} = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_3} \left[r(r - \mu_X - \mu_E)\beta_4 + (r - (\mu_X + \mu_E)\beta_1)\lambda\beta_4 - (r - \mu_X - \mu_E)(r + \lambda)\beta_1 \right] I}{(r + \lambda)(r + \lambda - \mu_X - \mu_E)(\beta_1 - 1)(\beta_3 - \beta_4)} \tag{24}$$

$$D_{21} = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{-\beta_4} \left[r(r - \mu_X - \mu_E)\beta_3 + (r - (\mu_X + \mu_E)\beta_1)\lambda\beta_3 - (r - \mu_X - \mu_E)(r + \lambda)\beta_1 \right] I}{(r + \lambda)(r + \lambda - \mu_X - \mu_E)(\beta_1 - 1)(\beta_3 - \beta_4)} \tag{25}$$

We can easily prove that $C_{21} < 0$ and $D_{21} > 0$.

3.3.1.2 The Leader's Value Function

Let assume that in equilibrium the leader adopts *tech 1* at $T_{L_{12}}$, with

$$T_{L_{12}} = \inf\left(t \mid \varphi_t \geq \tilde{\varphi}^*_{T_L}\right)$$

and, the follower adopts *tech 2* at $T_{F_{12}}$ with,

$$T_{F_{12}} = \inf\left(t \mid \varphi_t \geq \tilde{\varphi}^*_{T_F}\right)$$

If the follower waits for *tech 2* and the leader adopts *tech 1*, the leader's value equals:

$$\tilde{F}_{L_{12}}(\varphi) = \varepsilon\left[\int_0^{\tau}\varphi(t)\left(de_{1_L 0_F}\right)e^{-rt}dt + \left(F_{L_{12}}\right)e^{-r\tau} - I\,\Big|_{\varphi(0)=\varphi}\right] \tag{26}$$

Taking expectations and using the appropriate boundary conditions, we get the following expression:

$$\tilde{F}_{L_{12}}(\varphi) = \begin{cases} E_{12}\varphi^{\beta_3} + B_{12}\varphi^{\beta_1} + \dfrac{\varphi\left(de_{1_L 0_F}\right)}{(r-\mu_X-\mu_E)} - I & \varphi < \tilde{\varphi}^*_{F_{21}} \\ G_{12}\varphi^{\beta_4} + \dfrac{\varphi\left(de_{1_L 0_F}\right)}{(r-\mu_X-\mu_E+\lambda)} + \dfrac{\varphi\left(de_{1_L 2_F}\right)}{(r-\mu_X-\mu_E)}\dfrac{\lambda}{(r-\mu_X-\mu_E+\lambda)} & \varphi \geq \tilde{\varphi}^*_{F_{21}} \end{cases} \tag{27}$$

$$\tilde{\varphi}^*_{F_{21}} = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu_X-\mu_E)}{\left(de_{2_F 1_L}\right)} I \tag{28}$$

$$B_{12} = \left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_1}\frac{\left(de_{1_L 2_F} - de_{1_L 0_F}\right)}{r-\mu_X-\mu_E} \tag{29}$$

The expressions for E_{12} and G_{12} are derived by solving the continuity and differentiability conditions for $\tilde{F}_{L_{12}}(\varphi)$, at $\varphi = \tilde{\varphi}^*_{F_{21}}$, which gives:

$$E_{12} = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_3}\left[\left(r-\mu_X-\mu_E\right)\left(\beta_1-\beta_4\right)+\lambda\left(\beta_1-1\right)\right]\left(de_{1_L 0_F} - de_{1_L 2_F}\right)}{\left(r-\mu_X-\mu_E+\lambda\right)\left(r-\mu_X-\mu_E\right)\left(\beta_3-\beta_4\right)} \tag{30}$$

and

$$G_{12} = \frac{\left(\tilde{\varphi}^*_{F_{21}}\right)^{1-\beta_4}\left[\left(r-\mu_X-\mu_E\right)\left(\beta_1-\beta_3\right)+\lambda\left(\beta_1-1\right)\right]\left(de_{1_L 0_F} - de_{1_L 2_F}\right)}{\left(r-\mu_X-\mu_E+\lambda\right)\left(r-\mu_X-\mu_E\right)\left(\beta_3-\beta_4\right)} \tag{31}$$

Both E_{12} and G_{12} are positive.

We do not get a closed-form solution for the leader's trigger value. However, a numerical solution can be determined applying numerical methods to the equation (32), where $\tilde{\varphi}^*_{L_{12}}$ is the unknown variable.

$$E_{12}(\tilde{\varphi}^*_{L_{12}})^{\beta_3} + B_{12}(\tilde{\varphi}^*_{L_{12}})^{\beta_1} + \frac{\tilde{\varphi}^*_{L_{12}}\left(de_{1_L 0_F}\right)}{r - \mu_X - \mu_E} - I - A_{21}(\tilde{\varphi}^*_{L_{12}})^{\beta_1} - C_{21}(\tilde{\varphi}^*_{L_{12}})^{\beta_3} = 0 \tag{32}$$

4. Sensitivity Analysis

In this section we do some sensitivity analysis to study the effect of the most important parameters of the model on the leader and follower value functions and their respective investment trigger values. Additionally, we also examine whether our multi-factor model replicates the results of Dixit and Pindyck (1994) one-factor model and the results of Huisman (2001) two-factor model, when some specific conditions hold.

4.1 Technology 2 is Available, $t \geq \tilde{\tau}$

As we described in Section 3, at the arrival time of *tech 2* we can have one of the following scenarios: *i*) neither of the firms has adopted tech 1; *ii*) one firm has adopted tech 1; or *iii*) both firms have adopted *tech 1*.

Note that as firms can invest only once so when *tech 2* arrives if one of them has already adopted *tech 1* the other, if have not adopted yet, has the monopoly over the decision to adopt *tech 2*. Therefore, we should expect that the results from our model, for such scenario, must be similar to those of Dixit and Pindyck (1994), chapter 7, with the exception that Dixit and Pindyck (1994) is a one-factor model while our model is a multi-factor model.

On the other hand, if at the arrival time of *tech 2* neither firms has adopted *tech 1*, as we assume that both technologies cost the same and *tech 2* is more efficient than *tech 1*, so it is no more optimal to adopt *tech 1*, and, therefore, both firms behave in this case as if they were in a duopoly market with only one technology available (*tech 2*) and consequently our results, in this circumstances must be similar to those of Dixit and Pindyck (1994), chapter 9, again with the exception that our model is a multi-factor model while Dixit and Pindyck (1994) model is a one-factor model.

In the scenario where both firms adopt *tech 1* before the arrival of *tech 2*, the existence of a second technology does not affect firms' investment decision due to the "*one-shot*" nature of the investment game.

4.2 Technology 2 is not Available, $t < \tilde{\tau}$

In this section we analyse the investment scenarios where technological uncertainty is considered, i.e., *tech 2* is not yet available ($t < \tilde{\tau}$) and one of the firms has not yet adopted *tech 1*. In this sensitivity analysis that follows we use the following parameters: $I = 1.0$, $\lambda = 0.5$, $\rho_{XE} = 0.55$, $\sigma_E = 0.2$, $\sigma_X = 0.3$, $\mu_X = 0.04$, $\mu_E = 0.02$, $r = 0.09$, $de_{1_L 0_F} = 0.15$ and $de_{1_L 2_F} = 0.04$.

Figure 2 shows the sensitivity of the leader and the follower value function to changes in the efficiency of the new technology after adoption. For this analysis we set the revenues equal to 4.0 millions and the remaining model parameters equal to the values mentioned above.

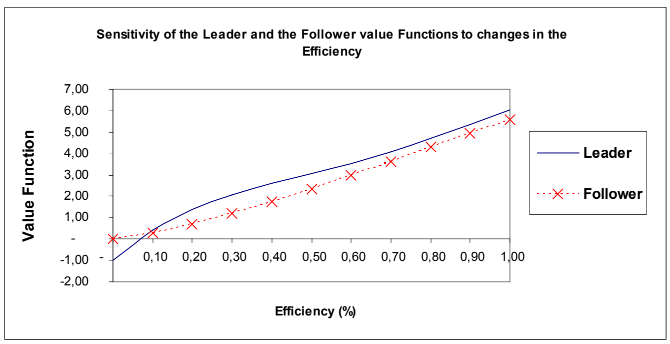
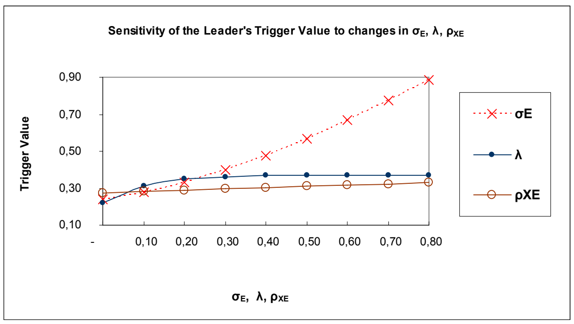


Figure 2

The value functions have the classic shape and therefore usual comments apply. One of the advantage of decomposing the revenues in two independent variables: "revenues" and "efficiency of the new technology after adoption", is that it allows us to simulate, for a given revenues, the level of the efficiency at which both firms would adopt, and *vice-versa*. After the follower's investment trigger value the functions would have reached if we had not assumed a permanent leader's advantage from the follower's investment trigger value onward. Note that in our framework, after the follower has adopted the leader's advantage over the follower is significantly reduced but not completely eliminated, which justify the fact that the two lines on Figure 2 get closer but did not match.

In Figures 3 and 4 we examine the effect of the volatility of the variable “efficiency of the new technology after adoption”, σ_E, the probability that *tech 2* arrives in the market, λ, and the correlation coefficient between the “revenues” and the “efficiency of the new technology after adoption”, ρ_{XE}, on the leader and follower investment trigger value, $\varphi^*_{L_{12}}$ and $\varphi^*_{F_{21}}$, respectively.



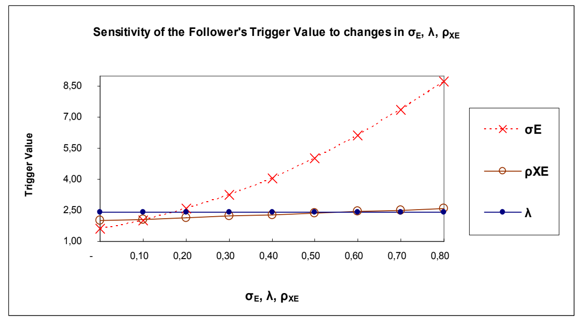
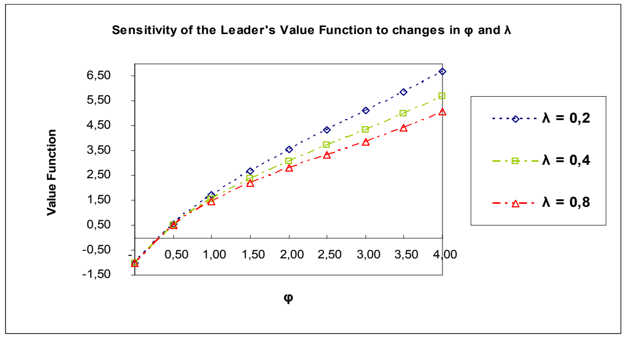


Figure 3 **Figure 4**

The results above show that an increase in σ_E leads to later adoptions for both firms, i.e., higher investment trigger value, $\varphi^*_{L_{12}}$ and $\varphi^*_{F_{21}}$, an increase in λ induces the leader to adopt later and has no effect on the follower adoption, and an increase in the correlation coefficient between the two underlying variable, ρ_{XE}, do not have any effect on adoption time of the leader and delays slightly the adoption time of the follower.

The effect of the volatility of the “revenues”, σ_X, on the leader and follower value functions is similar that of the volatility of the “efficiency of the new technology after adoption”, σ_E.

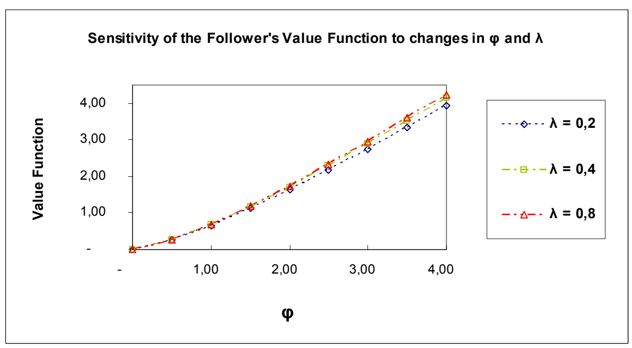
Figures 5 and 6 show the effect of φ and λ on the leader and the follower value functions, respectively.



λ=0,2	- 1,00	0,56	1,71	2,69	3,56	4,36	5,11	5,87	6,68
λ=0,4	- 1,00	0,51	1,55	2,38	3,09	3,73	4,34	4,98	5,67
λ=0,8	- 1,00	0,49	1,48	2,23	2,83	3,35	3,86	4,43	5,07
φ	**-**	**0,50**	**1,00**	**1,50**	**2,00**	**2,50**	**3,00**	**3,50**	**4,00**

Figure 5

Figure 5 shows that the leader's value function decreases as λ increases, being this tendency more notorious as φ gets higher.

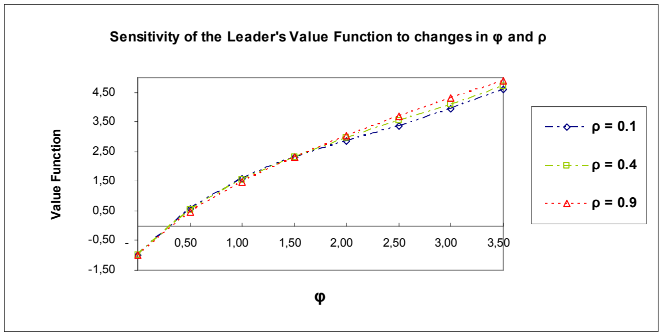


λ=0,2	-	0,27	0,66	1,13	1,64	2,18	2,76	3,34	3,93
λ=0,4	-	0,27	0,69	1,17	1,71	2,29	2,89	3,52	4,14
λ=0,8	-	0,28	0,69	1,19	1,74	2,33	2,96	3,60	4,24
φ	**-**	**0,50**	**1,00**	**1,50**	**2,00**	**2,50**	**3,00**	**3,50**	**4,00**

Figure 6

Regarding the follower value function, Figure 6 shows that the effect of λ in the follower's value function is almost inexistent, although slightly more notorious as φ increases.

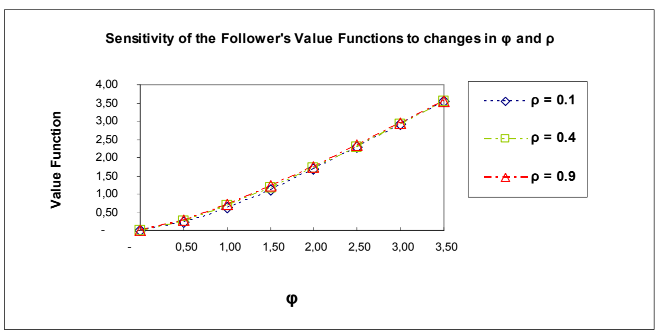
In Figures 7 and 8 we study the sensitivity of leader and the follower value functions to changes in the correlation coefficient between the two underlying variables, ρ_{XE}, respectively.



ρ = 0,1	- 1,00	0,58	1,58	2,30	2,87	3,38	3,94	4,60
ρ = 0,4	- 1,00	0,52	1,54	2,32	2,96	3,53	4,09	4,71
ρ = 0,9	- 1,00	0,45	1,47	2,31	3,03	3,68	4,30	4,91
φ	**-**	**0,50**	**1,00**	**1,50**	**2,00**	**2,50**	**3,00**	**3,50**

Figure 7

Figure 7 shows that an increase in ρ_{XE} leads to a slightly increases in the leader's value function, which becomes a little more notorious as φ gets higher.



ρ = 0,1	-	0,24	0,63	1,12	1,68	2,28	2,92	3,55
ρ = 0,4	-	0,26	0,67	1,16	1,71	2,29	2,92	3,55
ρ = 0,9	-	0,30	0,73	1,22	1,76	2,33	2,93	3,55
φ	**-**	**0,50**	**1,00**	**1,50**	**2,00**	**2,50**	**3,00**	**3,50**

Figure 8

Figure 8 shows that ρ_{XE} has no significant effect on the follower's value function.

4.3 Comparison with other models

In this section we compare our multi-factor model with the one-factor model of Dixit and Pindyck (1994), chapter 9, and the two-factor model of Huisman (2001), chapter 9. In fact, as it is mentioned in this paper introduction, our multi-factor model can be considered an extension of the two models above, in the sense that Huisman (2001) extends the work of Dixit and Pindyck (1994) by adding to the investment problem the technological uncertainty and our model extends the work of Huisman (2001) by adding to the investment problem the technical uncertainty.

Dixit and Pindyck (1994) considers the "revenues" as the unique source of uncertainty, Huiman (2001) considers the "revenues" and "technological progress" as the two sources of uncertainty and in this paper we consider the "revenues", the "technological progress" and the "efficiency of the technology after adoption" as the three sources of uncertainty. Consequently, for some specific conditions, the results of these three models must coincide, namely, when we set in our model $\sigma_E = 0,\ \mu_E = 0,\ \rho = 1$ and $\lambda = 0$, our model results must match those of chapter 9 of Dixit and Pindyck (1994), when we set $\sigma_E = 0,\ \mu_E = 0,\ \rho = 1$ and $\lambda \neq 0$ our model results must coincide with those of chapter 9 of Huisman (2001).

In Figures 9 and 10 we compare our model results with those of chapter 9 of Huisman (2001).

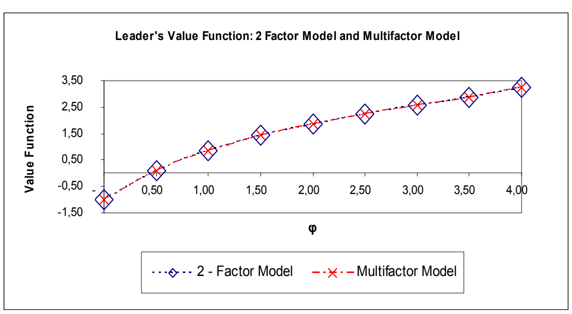


Figure 9

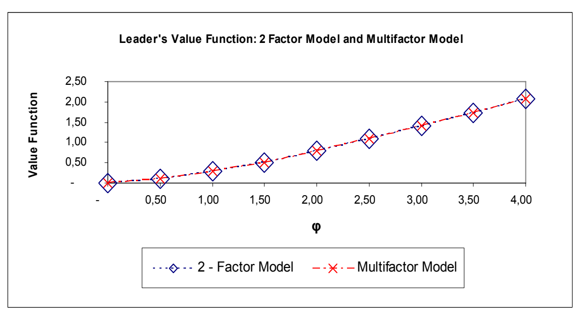


Figure 10

2-Factor Model			Multi-Factor Model					
μ_X	σ_X	λ	μ_X	σ_X	λ	μ_E	σ_E	ρ_{XE}
0,04	0,30	0,50	0.04	0.30	0.50	0.00	0.00	1.00

Table 2

As expected the results of both models coincide when we take the technical uncertainty out of our model.

5. Conclusions

In this paper we study the firms' optimal time to adopt a new technology, considering three different types of uncertainty: market, technical and technological uncertainty. The model is derived for a duopoly market. It is assumed that at the beginning of the investment game there are two idle firms in the market, one incumbent technology and the probability that a second and more efficient one arrives in the market. The firms are allowed to invest only once (a "*one-shot*" game), the two technologies cost the same and the incumbent technology assumed to be is less efficient that the newest one.

We get analytical solutions for the leader and the follower value functions as well as for the follower's investment trigger value. We did not find an analytical solution for the leader's investment trigger value but using simple numerical method a solution can be determined.

The results are in general intuitive. The higher the probability of a second technology arrives, λ, in the market the later the leader adopts the incumbent technology and the lower the value of being the leader with the incumbent technology. The effect of λ on the follower value function and its adoption time is very moderate. The correlation between "revenues" and "efficiency of the technology after adoption", ρ_{XE}, affects slightly the leader and the follower value functions and their respective investment trigger values, a higher correlation coefficient leads to later adoption for both firms.

The advantage of considering the joint effect of market, technical and technological uncertainties is not only that, in some cases, turns the model more realistic, but also that it can be used in cases where firms are symmetric in their overall uncertainty (market, technical and technological uncertainty altogether) and asymmetric in each type of uncertainty[15]. Note that in practice the only uncertainty that is likely to be symmetric for both firms is technological uncertainty, since it is exogenous to the firms. Market and technical uncertainties are very likely to be asymmetric, since they may be firm specific.

The model proves to be adequate to model duopoly investment decisions where market, technical and technological uncertainties exist simultaneously, and especially useful for investment scenarios where firms are asymmetric regarding at least one of these uncertainties.

In this model we use two underlying variables, revenues and efficiency of the technology after adoption, and assume that there is a first-move advantage regarding both. Therefore, by moving first firms can expect to gain an advantage in terms of revenues, due to a higher market share, and in terms of efficiency, due to higher operational efficiency. Furthermore, we also assume that the combined effect of the two advantages above favors always the leader, since spillover information is not allowed. However, it would be interesting to relax that assumption and study the effect of spillover information, specially, regarding the quality of the new technology, on the firms' adoption time. In a context of spillover information, the follower can get costless information about the technical quality (efficiency) of the new technology that the leader has adopted, and that would change the duopoly investment setting from a *preemption* game to an *attrition* game.

Other interesting extension of this research would be to consider the two technologies involved in the investment complementary, in a context where firms were allowed to invest twice and technological uncertainty was absent, i.e., considering that both

[15] Suppose that two firms, 1 and 2, are active in a duopoly market and that both intend to commercialize a new product for which they need to adopt a new technology. Assume that the technology is available but firms have different technical skill to operate it, for instance, firm 1 is technically more qualified to operate the technology than firm 2. Therefore, firm 1 faces less technical risks if it adopts the technology than firm 2. On the other hand, imagine that firm 2 has a market advantage due to its more efficient and extensive sales force. Hence, firm 2 faces lower market risk than firm 1. Given such circumstances, it is possible, at a certain point in time, the two firms are symmetric in their overall uncertainty, i.e., symmetric in the uncertainty that results from the combined effect of market and technical uncertainty, but asymmetric regarding each type of uncertainty.

technologies were already available. In such case, firms would have the option to adopt either one or two technologies at the same time or at different times, and the duopoly investment set could be consider for a *preemption* or an *attrition* game.

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Appendix A

A1. Technological Reliability

According to commonly accepted definitions reliability is the "ability of an entity to perform a required function under given conditions for a given interval", and failure is the termination of the ability to perform the required function[16] (Todinov, M. (2005), p. 1). In the mathematical sense, reliability is measured by the probability that a system or a component will work without failure during a specific time interval (0, *t*), under given operating conditions and environment (see Figure A1).

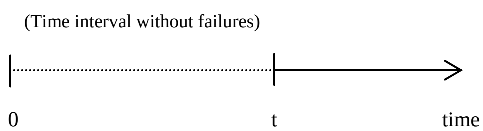

Figure A1

The probability that the time to failure, *T*, will be greater than a specified time *t*, $P(T > t)$, is given by the *reliability function* $R(t) = P(T > t)$, sometimes referred to as the *survival function*. The concept of technological reliability is ignored in most of the current theoretical models on new technology adoptions. The general assumption is that, after adoption, new technologies will perform exactly as expected (no technical uncertainty). Using the definition and the notation above, such assumption restricts the performance (reliability) of the new technology, after adoption, to a unique performance point, $R(t) = P(T = t) = 1$, i.e., the new technology can not be, neither more nor less reliable than what was expected at the adoption time (see Equations (A1) and (A2)).

$$R(t) = P(T > t) = 0 \qquad \text{(A1)}$$

$$R(t) = P(T < t) = 0 \qquad \text{(A2)}$$

When a technology is part of a production system, technological failures affect the rate of production, because their corrections take time and during such periods the technology is inactive.

[16] For a detailed discussion about the concept of reliability see Todinov, M. (2005), pp. 1-18.

The cost of a technological failure depends on the type of failure. The higher is the cost of a failure, the higher the demand on the reliability of the technology regarding that failure, fact that explains why in a technology small failures happen more often than big, and costly, failures.
In the group of big technological failures are those usually described as catastrophic events, i.e., failures capable of causing, for instance, the crash of an airplane or the total destruction of a production facility. In such cases, repairing is not possible and therefore a failure means the end of the investment. These extreme events are not considered in the paper.

The reliability (*R*) of a new technology is a monotonic non-increasing function, always close to unity at the start of the life, $R(0) \approx 1$, and approaching zero as life tends to infinity, $R(\infty) \approx 0$[17]. It is linked with a cumulative distribution function $F(t)$ of the time to failure by $R(t) = 1 - F(t)$, i.e., reliability is equal to 1 minus the probability of failure. If *T* is the time to failure, *F*(*t*) gives the probability that the time to failure *T* will be smaller than the specified time *t*, $P(T \leq t)$, i.e., the probability that the technology will fail before time *t*.

The probability density function of the time to failure is denoted by $f(t)$. It describes how the failure probability is spread over time. In the infinitesimal interval *t*, $t + dt$, the probability of failure is $f(t)dt$ and the probability of failure in any specified time interval $t_1 \leq T \leq t_2$ is

$$P(t_1 \leq T \leq t_2) = \int_{t_1}^{t_2} f(t)dt \qquad \text{(A3)}$$

Two basic properties of the probability density of the time to failure are

(i) *f*(*t*) is always non-negative and;

(ii) The total area beneath *f*(*t*) is always equal to one[18]: $f(t)dt = 1$.

The cumulative distribution function of the time to failure is related to the failure density function by

$$f(t) = dF(t)/dt \qquad \text{(A4)}$$

[17] For mathematical tractability this empirical evidence is ignored in the paper.
[18] *f*(*t*) is a probability distribution and therefore the probabilities of all possible outcomes for the time to failure must add up to unity.

The probability that the time to failure will be smaller than a specified value *t* is

$$F(t) = P(T \leq t) = \int_0^t f(v)dv \qquad \text{(A5)}$$

where v is a dummy integration variable; $F(\infty) = \int_0^\infty f(v)dv = 1$ and $F(0) = 0$.

Therefore, the assumption made in most of the literature on theoretic models on new technology adoption not only infer that, after adoption, a new technology will fail exactly the number of times that was initially predicted, but also that it will fail at the exact time that was initially predicted and that when such predicted failures occur the subsequent periods of inactivity of the technology as well as the failures repairing costs will be equal to those that were initially predicted. In this paper we relax such assumptions and allow for technical uncertainty after adoption which means that technologies can fail more, or less, times than what was initially predicted and that the periods of technologies inactivity due to technical problems may be more, or less, than what firms initially had predicted.

In practice, and making use of the notation above, our assumption regarding the technical uncertainty means that during the interval of time (0, *t*) failures can happen before *T*, which means that, during such interval of time, the *effective time available* that a new technology operates can differ from (0, *t*), the *theoretical time available* that was initially predicted on the assumption that there is no technical uncertainty, i.e., due to the assumption that during the life of a technology, technical failures are not totally predictable, either in terms of how often they will be as well as in terms of how long their repairing will take and cost, the time during which a technology is available to work can differ from the time during which it works effectively. So far in the literature on new technology adoptions models it has been assumed that the *theoretic time available* and the *effective time available* always coincide over the life of the technology.

Using Slack, *et al.* (2002) notation, we link the concept of technological reliability to the concept of production efficiency through the following equation,

$$Efficiency = \frac{ER}{EXR} = \frac{EPT}{APT} = \frac{EPC}{APC} \qquad \text{(A6)}$$

Where, ***ER*** **-** Effective Reliability;

EXR **-** Expected Reliability| given no failures before *T*.

EPT **-** Effective Production Time;

APT **-** Available Production Time| given no failures before *T*.

EPC **-** Effective Production Capacity;

APC **-** Available Production Capacity| given no failures before *T*.

We assume that the cumulative effect of technological failures over time makes the efficiency of the technology (production system) vary randomly. This phenomenon is more evident in cases where production systems are composed by a wide range of complementary technologies, whose failures are statistically independent but where the efficiency of the whole system depends on the efficiency of each one of its components (see Figure A2).

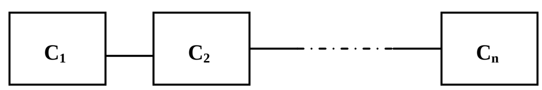

Figure A2

In such operating conditions, it is reasonable to assume that, after adoption, the efficiency of the production system follows a gBm process given by Equation (4).

Other aspect about the reliability of a production system that may worth to mention here is that the application of random events to production systems similar to that shown in Figure A2 leads to the following conclusion:

> If *S* denotes the event "*the system will be working*" and C_k denotes the event "*the kth component will be working*", for the series arrangement in Figure A2, event *S* is the intersection of all events C_k, $K = 1, 2, ..., n$, because the system will be working only if all the components work. Therefore, the probability that the system will be working is the product of the probabilities that the separate components will be working (see Equation A4).

$$P(S) = P(C_1)P(C_2)P(C_3) \ ... \ P(C_n) \qquad \text{(A7)}$$

Since $R = P(S)$ and $R_k = P(C_k)$, where *R* is the reliability of the system and R_k is the reliability of the *k*th component, the reliability of a series arrangement is

$$R = R_1 \text{ x } R_2 \text{ x } ... \text{ x } R_n \tag{A8}$$

Intuitively, two important conclusions can be made from (A8):

(i) The larger is the number of components, the lower the reliability of the production system.

(ii) The reliability of a production system in series is smaller than the reliability of R_k of the least reliable component *k*.

The last conclusion has important practical applications for the management of a production system, since it means that the reliability of a series arrangement cannot be improved unless the reliability of the least reliable component is improved. If a reliability improvement on a system level is to be made, the reliability efforts should be focused on improving the reliability of the least reliable component first, not on improving the reliability of the components with already high reliability.

A2. Efficiency Dataset

Figure (A3) shows a dataset of records of daily efficiency of a textile production technology collected during 1997 and 1998. According to Figure (A3) the production system operates during the first months with efficiency between 0.83 and 0.89 percent, after which it improves gradually over time by random moves stabilizing at the end of the series between 0.92 and 0.95 percent efficiency. It also shows that as the efficiency improves the daily efficiency volatility decreases, which may mean that the production system became more under control as time progressed. This is an example of data which can be used in our model.

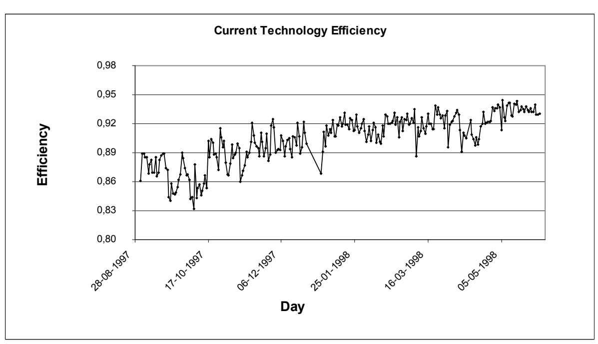

Figure A3

A3. The Effect of Efficiency on the Output Production

The firm considered in the dataset described above produces fabrics and its daily production is measured in "square meters". The quantity of square meters of fabrics produced each day depends on the technical specificities of the type of fabric produced, usually on the quantity of yarn per square meter of fabric. The higher the quantity of yarn per square meter, the lower the square meters produced per unit of time.

The straight line in Figure A4 describes the relationship between the *efficiency* of the production system and the *units* produced (square meter of fabrics) in a daily basis. The conclusion is that when the efficiency of the technology is 1 (100 percent), the daily production, $Q_{E=1}$, is equal to 28,000 m^2 of fabrics and when the efficiency of the technology is zero, the daily production, $Q_{E=0}$, is equal to zero. Figure A4 also shows that the relationship between the *daily efficiency* of the technology and the *daily units* produced is positive, which means that the more efficient is the production system, the higher the units of output produced per unit of time.

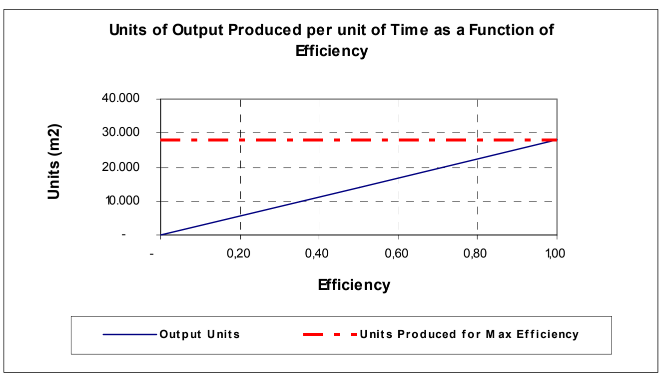

Figure A4

On the other hand, the intuition is that, for the same level of inputs, the more complex is the production of the output the lower the units produced per unit of time, i.e., the number of units produced per unit of time decreases as the complexity of the output increases. From the engineering and management point of views, the technical complexity of the production system is given in Figure A4 by the slope of the straight line.

Mathematically, the relation between efficiency of the production system and units of output produced is given by the following linear equation:

$$Q_T = C_{i_T} (E_T) \tag{A9}$$

where, Q_T is the number of units produced in the period (0, *T*); C_{i_T} is a constant that taken into account the technical specifications (complexity) of the output *i* relates the *efficiency* of the technology with number of *units* produced.

In our dataset, $T = day$ (24 hours), $C_{i_{day}}$ is equal to $28,000 \text{ m}^2$ and E_{day} is the daily efficiency of the production system. Substituting $C_{i_{day}} = 28,000 \text{ m}^2$ and expressing equation (A9) in continuous time we get,

$$dQ_t = 28,000\left(dE_t\right) \tag{A10}$$

where, dE_t follows equation (4).

A4. Monitoring the Efficiency State Variable

In the dataset described above $T = day$ and the efficiency of the production system, *E*, is computed using Equation (A6):

$$E = \frac{EPT}{APT}$$

Where, ***APT*** is the Available Production Time| given no failures before *T*,
EPT is the Effective Production Time, in the interval (0, *T*).

More specifically, the *APT* is the total time the technology would have worked if it had performed according to the initial expectations (no failure before/after *T*); *EPT*, is the effective time that the technology has worked during the interval (0, *T*).

Therefore, the efficiency of the new technology after adoption is defined in the interval $\left[0,1\right]$. In the case of our dataset "efficiency" is defined in a daily basis and *EPT* is always lower than *APT*, which implies that *E* is always lower than 1 (100 percent).

Appendix B

B1. The Deterministic Factors

In tables B1 and B2 below we exemplify how to arrive at the deterministic factors, whose general form is defined as $de_{k_i k_j}$, with $k = \{0,1,2\}$, where 0, 1 and 2 corresponds to the use of tech 0, tech 1 and tech 2, respectively; $i,\ j = \{L, F\}$, where L denotes the leader and F the follower (see inequalities (2) and (2.1)).

As in our model we use two underlying variables, revenues and efficiency of the technology after adoption, so the model setting for the leader's advantage differs a little from that used, for instance, in Dixit and Pindyck (1994), Huisman (2001), and Paxson and Pinto (2005), because, contrary to what happens in such models, in this model, the leader, by moving first, gets advantage not only in terms of the underlying variable, *X*, that relates to the market uncertainty and in our model denotes the variable revenues, but also regarding a second underlying variable, *E*, that relates to the technical uncertainty and in this model denotes the variable efficiency of the technology after adoption, which is absent from the models above.

As an example of how to compute the value of the deterministic factors $de_{k_i k_j}$, consider that firm *i* and *j* are competing for a total market revenue of 1 million dollars and that both consider the possibility to invest in a new technology. Assume that there is one technology available and the probability that a second and more efficient one to arrive in the future. Firm *i* decides to adopt *tech 1* and enters the market first getting both a market and technical advantages (higher market share - revenues - and higher operational efficiency). The follower, firm *j*, decides to wait. In Section 4, for this scenario, we use $de_{1_L 0_F} = 0.10$. To arrive at that value we assume that firm *i* adopts *tech 1* first and gets 54 percent of the total revenues (0.54 million dollars) and improves its operational efficiency from 0.85 to 90 percent, while the follower, firm *j*, gets the remaining 46 percent of the total revenues (0.46 million dollars) and continues to operate with an old technology at 85 percent efficiency. The procedure to compute $de_{1_L 0_F}$ for this scenario is the following:

Example 1

Leader:

Revenues (*d*): 0.54 millions

Efficiency (*e*): 0.90

Combining the leader's revenues and efficiency (*de*): 0.54 x 0.90 = 0.49 millions

Follower:

Revenues (*d*): 0.46 millions

Efficiency (*e*): 0.85

Combining the follower's revenues and efficiency (*de*): 0.46 x 0.85 = 0.39 millions

Leader's Advantage, $de_{1_L 0_F}$ *:*

The combined revenue and efficiency leader's advantage, $de_{1_L 0_F}$, is obtained by subtracting: 0.49 – 0.39 = 0.10. This value is used in Section 4 in the derivation of the leader and the follower value functions. A summary of the calculations above can be seen in Table B1, last row. There we also show other combinations of leader/follower revenues and efficiency values with their respective combined revenues and efficiency leader's advantage.

Total Market Revenues: 1 million dollars	**Leader**			**Follower**			**Leader's Advantage (million dollars)**		
Variables	*X*	*E*	φ	*X*	*E*	φ	*X*	*E*	*Leader/Follower*
Deterministic Factors	***d***	***e***	***de***	***d***	***e***	***de***	***d***	***e***	$de_{1_L 0_F} = de_{0_F 1_L}$
Before $\tilde{\tau}$	0.80	0.90	**0.72**	0.20	0.85	**0.17**	0.60	0.05	***0.53***
	0.70	0.90	**0.63**	0.30	0.85	**0.26**	0.40	0.05	***0.37***
	0.54	0.90	**0.49**	0.46	0.85	**0.39**	0.10	0.05	***0.10***

Table B1

It is important to note that, this framework allows the simulation of investment scenarios which using different leader's market and efficiency advantages lead to the same combined market and efficiency advantage, i.e., the same $de_{1_L 0_F} = 0.10$. That is possible as long as an increase in the leader’s revenues is compensated by a decrease, of the same economic value, in the leader’s efficiency, or vice-versa.

In Table B2 we define the deterministic factor $de_{1_L 2_F}$, used in Section 4 in the derivation of the leader and the follower value functions in a context of technological uncertainty. The subscript “2” refers to the fact that there is a second technology (tech 2) involved in the firms’ investment decision. The relation of $de_{1_L 2_F}$ with other deterministic factors, also use in Section 4, is expressed in inequality (2.1).

The economic intuition used to compute $de_{1_L 2_F}$ is the same as that used to compute $de_{1_L 0_F}$ in Table B1. However, in Table 2 we exemplify one scenario where the leader by moving first gets an efficiency disadvantage (see $e = -0.10$ in column 9). As was described before, $de_{1_L 2_F}$ characterizes investment scenarios where the leader adopts *tech 1* and the follower waits and adopts later *tech 2*. Consequently, as by assumption in this model *tech 2* is more efficient than *tech 1* so the leader is allowed to get an advantage only in terms of revenues. Nevertheless, as can be seen in Table B2, last column, rows 4 and 5, $de_{1_L 2_F}$ is still positive, equal to 0.13 and 0.04, respectively, because the advantage that the leader’s gets in terms of revenues more than compensate its efficiency disadvantage of 10 percent (see column 9). However, that does not occurs in the investment scenario described in the last row, where the leader’s revenues advantage is not enough to compensate its efficiency disadvantage, i.e., though by adopting first the leader gets a revenue advantage that advantage is not enough to compensate the efficiency disadvantage in which it incurs if *tech 2* arrives and the follower adopts it. This example illustrate an important characteristic of this model, which is absent from previous literature, that consists in allowing for a wider range of duopoly investment scenarios, namely that where moving first does not necessarily mean that the leader will improve all competitive aspects of its business.

In Section 4 we use $de_{1_L 2_F} = 0.04$. To analyse how we arrive at that value see Table B2 row 5 and follow the same procedure used for example 1.

Total Market Revenues: 1 million dollars	**Leader**			**Follower**			**Leader's Advantage**		
Variables	*X*	*E*	φ	*X*	*E*	φ	*X*	*E*	Leader/Follower
Deterministic Factors	***d***	***e***	***de***	***d***	***e***	***de***	***d***	***e***	$de_{1_L 2_F} = de_{2_F 1_L}$
After $\tilde{\tau}$	0,60	0,85	**0,51**	0,40	0,95	**0,38**	0,20	- 0,10	***0,13***
	0,55	*0,85*	**0,47**	*0,45*	*0,95*	**0,43**	*0,10*	- *0,10*	***0,04***
	0,51	0,85	**0,43**	0,49	0,95	**0,47**	0,02	- 0,10	***- 0,03***

Table B2