Optimal IPO Timing in an Exchange Economy*

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Abstract

We model the IPO decision of an entrepreneur in an exchange economy. The entrepreneur holds a Lucas Tree, and when the IPO occurs, the market converges to a Two Trees economy built on Cochrane, Longstaff, and Santa-Clara (2008). We solve the optimal timing problem and study the diversification effects over the firm's value and entrepreneur's consumption. The model predicts that IPOs should be correlated with the firm's size and explains why firms with lower betas are expected to IPO first.

Keywords: IPO, equilibrium, Lucas trees, diversification, real options.

JEL Classification: D51, G11, G12, G34.

1 Introduction

Market conditions have been considered, by most empirical studies, as one of the most important factors that could trigger an Initial Public Offerings (IPO) decision by an unlisted firm. Empirical evidence, such as the so called "IPO Waves", suggests that entrepreneurs wait for certain conditions in the market to IPO their companies (e.g. Ritter 1984 and Pastor and Veronesi 2005). IPO waves are also characterized by the presence of firms from similar sectors overweighting particular industries in the market portfolio. IPO decisions should not only be determined by market conditions, but also by the characteristics shared within firms from similar industries.

IPO studies have mostly been focused in developed and mature economies. These economies are generally well balanced, and comparatively big in relation to a newly listed firm implying that the introduction of the new firm has no impact in its industrial composition. However, this is not true for developing economies, where markets are immature and the industrial composition of the economy is not well represented in the market portfolio. In these economies, large firms or sectors that IPO may deeply impact the composition and characteristics of the market portfolio. The IPO decision is inevitably influenced by its effect on the overall economy.

Benninga, Helmantel, and Sarig (2005) summarizes several motives that might encourage an entrepreneur to IPO: financial support for investment opportunities, a possible increase in the company's valuation due to factors such as external monitoring and liquidity of shares (see also Holmstrom and Tirole 1993), and differences in the way owners and buyers price the company (undiversified versus diversified entrepreneurs respectively). Nevertheless, these factors have some problems in determining a generalized theory of initial public offerings. Financial support can be obtained through private equity and bank loans, and it has been documented that investments in firms actually decline after an IPO (e.g. Pagano, Panetta, and Zingales (1998) and Benninga, Helmantel, and Sarig (2005)). Factors such as external monitoring may increase firm's value, but as shown in Datta, Iskandar-Datta, and Patel (1999), some monitoring costs can be lowered using bank debt (which reduces public debt borrowing costs). Differences in valuation due to entrepreneurs diversification may be diminished by credibility issues due to information asymmetries(Courteau (1995)).

Many authors have argued that diversification may play a central role in IPOs. Bodnaruk, Kandel, Massa, and Simonov (2008) finds empirical evidence that relates portfolio diversification with IPOs: *less diversified shareholders are more eager to take their company public.* This effect is more important when individuals are more risk-averse. Courteau (1995) argues that after the IPO the owner can diversify some firm-specific risk (non systematic risk) and get a better balanced investment portfolio. Astudillo, Braun, and Castaneda (2008) study the empirical process in which economies complete. In incomplete markets, few industries are well represented, and low beta sectors are overweighted in the market composition. This means that the first companies expected to IPO have lower betas, which are probably the ones with higher non-systematic risk.

Benninga, Helmantel, and Sarig (2005) clearly states that IPOs are not single-shot opportunities: entrepreneurs have the ability to time their IPOs. If we believe that IPO decisions are irreversible (an entrepreneur is not planning to reacquire the company, at least not soon enough to consider it in the decision), real option analysis seems like a natural candidate to model this kind of problems. Several authors have already attempted alternative real option approaches with different conclusions. Pastor and Veronesi (2005) and Pastor, Taylor, and Veronesi (2009) consider the IPO edcision as an option, analyzing the trade-off between the benefits of waiting for a better discount rate and the abnormal returns from new technologies. Draho (2000) elaborates a real option model where the firm waits for high prices in listed companies from the same industry. Benninga, Helmantel, and Sarig (2005) uses a binomial option-pricing model in which diversified investors are willing to pay higher prices for the risky cash flows. The counterpart of the IPO loses private benefits of control.

All of these are partial equilibrium models where the entrepreneur considers the trade-off between waiting for better conditions in the market to IPO versus costs associated to the action. The IPO decision is influenced by the market, but the market stays unaffected by the new listed company. These models may be applied to economies comparatively big and complete, but not for developing economies, in which an IPO may change market condition, its industrial compositions, diversification scenarios, risks and expected returns, within others. To consider effects like this one in an IPO decision, a general equilibrium framework must be used. A new company or group of companies going public may affect the market composition and the entrepreneur knows this before making his firm publicly listed.

We propose a simple model in an exchange economy, in which the benefits from an IPO comes purely from diversification, eliminating any other incentive that might trigger an IPO (as the ones mentioned above). The company is represented by an asset that pays a dividend stream that has some firm-specific characteristics. Why modeling the IPO decision in an exchange economy? As is well known, an exchange economy has as a main advantage its simplicity. We can obtain nice and intuitive solutions without caring on how goods and services are produced. This helps us to concentrate exclusively in the diversification benefits perceived by the entrepreneur.

We assume that a transaction cost is payed by the entrepreneur to IPO his firm. This cost is a constant fraction of the overall economy. This assumption is based on empirical facts documented by Ritter (1987) who documents the existence of fixed and variable costs, that are a constant fraction of the economy and IPO gross proceeds respectively.[1]

In our model, the IPO decision depends on the dividend of the unlisted firm relative to the aggregate consumption in the economy, i.e. the *dividend ratio*. Before the IPO, the entrepreneur perceives a higher price-dividend ratio because his firm has an embedded option related to the IPO decision, i.e. the IPO timing option. At the IPO time the option value is zero. The diversification

[1] Ritter (1987) estimates, using information from IPOs in the U.S. in 1977-82, that direct costs of going public have a fixed part (250.000 $USD) plus a variable component (7% of the gross proceeds.)

effect is measured through the entrepreneur's consumption stream. The dividend ratio required by the entrepreneur to IPO his firm is lower when the potential benefits from diversification effect are higher. In line with recent empirical findings, our model predicts that firms with lower expected betas at the IPO time are expected to IPO first. Our model predicts that IPOs from the same industry (similar dividend streams) should be positively correlated with the company's size. We also allow for possible discontinuities in the dividend streams possibly due to economic disasters or technological changes.

Our paper is related to a recent literature that studies the asset pricing implications in exchange economies. Cochrane, Longstaff, and Santa-Clara (2008) builds on Lucas (1978) and study the effect of having a second Lucas trees in the economy. Martin (2009) generalizes the utility function, dividend processes, and number of trees in the economy (a Lucas orchard). Other asset-pricing studies that consider multiple trees are Longstaff (2009), Parlour, Stanton, and Walden (2009a) and Parlour, Stanton, and Walden (2009b), among others.

This paper is structured as follows. Section 2 develops the framework in which the model is constructed, and solves the optimal control problem associated with the IPO decision. In Section 3, we analyze some results derived from the model, studying sensibility of optimal states to parameters, and after versus before IPO scenarios. Finally, Section 4 summarizes and concludes.

2 The Model

This section develops the model and its assumptions. The section is structured as follows. First, we define the assets and agents that interact in the economy. Then, we present the decision that the entrepreneur faces and set the optimal control optimization problem, which solution is summarized in Proposition 1

2.1 Basic assumptions

Let Z_t be a 2-dimensional brownian motion defined in the filtered probability space $(\Omega, \mathcal{F}^Z, (\mathcal{F}^Z_t)_t, \mathbb{P})$. Let N be an l-dimensional Poisson random measure defined in $(\Omega, \mathcal{F}^N, (\mathcal{F}^N_t)_t, \mathbb{P})$. We define the general filtered probability space as $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, \mathbb{P}) = \big(\Omega, \mathcal{F}^Z \otimes \mathcal{F}^N, (\mathcal{F}^Z \otimes \mathcal{F}^N_t)_t, \mathbb{P}\big)$.

Suppose an exchange economy with two assets, each of them represented by a Lucas trees with positive dividends following Lévy jump diffusion processes. One tree, P_1, represents a company that is going public at some time in the future, while the other one, P_2, corresponds to the tradable market before the IPO of P_1. The assets pay dividend streams D_{1t} and D_{2t}, respectively. We model the dividends as $D_{it} = e^{y_{it}}$, where $y_t = (y_{1t}, y_{2t})^\top$ is defined by the process

$$y_t = y_0 + \mu t + \sigma Z_t + \int_{\mathbb{R}} g(z)\, N(dt, dz) \tag{1}$$

Here $g(z)$ is an $2 \times l$ matrix function, μ is a 2-dimensional constant and σ an 2×2 matrix. Using Ito's differential formula for jump diffusion we can write the dividend processes as

$$\frac{dD_{it}}{D_{it}} = \mu_{D_i} dt + \sigma_i dZ_t + \sum_{k=1}^{l} \int_{\mathbb{R}} (e^{g_{ik}(z)} - 1)\, N^{(k)}(dt, dz_k) \tag{2}$$

where $\mu_{D_i} = \left[\mu_i + \frac{1}{2}\sigma_i \sigma_i^{\top}\right]$, with σ_i being the $i^t h$ row of σ.

We define the dividend ratio $s : \Omega \times \mathbb{R} \to [0, 1]$ as the contribution of the company's dividends to the aggregate consumption, i.e. $s_t = \frac{D_{1t}}{D_{1t}+D_{2t}}$. In some cases, it will be easier to work with a monotonic transformation of s_t given by $u_t = \log\left(\frac{1-s_t}{s_t}\right)$.

There are two representative agents in the economy. Agent 1 is the entrepreneur that initially owns the unlisted firm, P_1, and agent 2 represents the investors that holds the market portfolio P_2. Each of the agents has a constant relative risk aversion (CRRA) utility function of the form

$$U_t(C) = \mathbb{E}_t \left[\int_t^{\infty} e^{-\beta s} u_i(C_s)\, ds \right] \tag{3}$$

with

$$u(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{4}$$

2.2 The IPO decision

We now describe the conditions the entrepreneur faces when making the decision about when to IPO his company. Let $(C_{it})_t$ be the process of consumption of the agents $i = (1, 2)$ and τ be the optimal time chosen by the entrepreneur to IPO his firm. Before the IPO the economy is segmented and the agents can only consume the dividend payoffs of their respective assets. The entrepreneur has the option to decide when to make his company public. We must first determine conditions over entrepreneur's wealth at τ.

We assume that the entrepreneur faces transaction costs CT at the IPO time. These costs include any costs perceived by the entrepreneur, e.g. administrative costs, losing private control benefits Benninga, Helmantel, and Sarig (2005), legal costs, etc. His interchangeable wealth is given by the price the market is willing to pay for P_1 minus the cost CT. We assume that CT is proportional to the economy's overall wealth, i.e. $CT = \alpha(P_1+P_2)$. This simple assumption is based in empirical facts documented in Ritter (1987). Fixed costs can be assumed as a fraction of overall wealth, so transaction costs are function of P_1 and P_2 (homogeneous of degree 1). We can easily extend the model to consider a more general transaction cost structure (i.e. $CT = \alpha_1 P_1 + \alpha_2 P_2$ with $\alpha_1 \neq \alpha_2$), but we prefer to keep the model simple.

Let $(W_{it})_t$ be agent's i wealth process. The entrepreneur wealth at the IPO time τ is:

$$W_{1\tau} = P_{1\tau} - \alpha(P_{1\tau} + P_{2\tau}) \tag{5}$$

At τ, the asset is incorporated to the market portfolio. Once the IPO is made, the market completes and the economy prices the stocks as in Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2009).[2] At the IPO time both agents rebalance their positions and end up holding the new market portfolio.

After some math, we obtain the following relation for the entrepreneur's consumption at τ:

$$C_{1\tau} = \omega_\tau \left(D_{1\tau} + D_{2\tau}\right) \tag{6}$$

where

$$\omega_\tau = \frac{P_{1\tau} - \alpha W_\tau}{W_\tau} \tag{7}$$

Here ω_τ represents the fraction of aggregate consumption consumed by the entrepreneur at τ and $W_t \equiv W_{1t} + W_{2t}$ is the aggregate wealth of the economy. Equations (B5) and (B4) in Appendix B.1 show that at any time $t > \tau$, the agents consume the same constant fraction of the aggregate consumption as in τ, i.e.

$$C_{1t} = \omega_\tau \left(D_{1t} + D_{2t}\right), \qquad t > \tau \tag{8}$$

$$C_{2t} = \left(1 - \omega_\tau\right)\left(D_{1t} + D_{2t}\right), \qquad t > \tau \tag{9}$$

The consumption streams before and after the IPO are a function depending on the processes $(D_{is})_s$ for $i = 1, 2$. Before the IPO, the entrepreneur's utility is given by

$$\begin{aligned} J^\tau(t, D_{1t}, D_{2t}) &= \mathbb{E}_t\left[\int_0^\infty e^{-\beta s} u(C_{1s})\, ds\right] && (10)\\ &= \mathbb{E}_t\left[\int_0^\tau e^{-\beta s} u(D_{1s})\, ds + e^{-\beta\tau}\int_\tau^\infty e^{-\beta(s-\tau)} u(C_{1s})\, ds\right] && (11)\\ &= \mathbb{E}_t\left[\int_0^\infty e^{-\beta s} u(D_{1s})\, ds + j_E(\tau, D_{1t}, D_{2t})\right] && (12) \end{aligned}$$

where $j_E(t, D_{1t}, D_{2t}) = e^{-\beta t}\mathbb{E}_t\left[\int_t^\infty e^{-\beta(s-t)}\left(u(C_{1s}) - u(D_{1s})\right) ds\right]$. The first term in the right side of equation (12) is the expected utility of the entrepreneur if he never makes the IPO. The second term is the extra indirect entrepreneur's utility associated to the option to IPO his firm. Appendix A shows that j_E can be obtained in closed form, and that it is given by an expression of the form $j_E(t, x, y) = e^{-\beta t} x (xy)^{-\gamma/2} j(s, \omega_\tau)$. The optimal IPO decision the entrepreneur faces is represented

[2] See Appendix B for further details in this point.

by an optimal stopping problem where he seeks to maximize the value function:

$$V(t,x,y) = \sup_{\tau} j_E(\tau,x,y) = j_E(\tau^*,x,y) \tag{13}$$

We use dynamic programming to solve this problem (see Oksendal and Sulem (2005)). The next proposition summarizes the solution of the value function.

Proposition 1 *The value function of the entrepreneur is given by a function $\phi(t,x,y)$ and an stopping time τ_D satisfying the following conditions. Suppose a function $H:[0,1]\to\mathbb{R}$ and constants $0\leq s_1^*<s_2^*\leq 1$ such that:*

$H(s)\in C^1((0,1))\cup C([0,1])\cup C^2((0,1)\setminus\partial D)$ *with* $D=\{s\in(0,1)\backslash H(s)>j(s)\}$ *and*

$$H(s)=\begin{cases} h_1(s) & \textit{if} \quad s<s_1^* \\ j(s,\omega_\tau) & \textit{if} \quad s_1^*<s<s_2^* \\ h_2(s) & \textit{if} \quad s>s_2^* \end{cases} \tag{14}$$

With

$$h_i(s)=c_i\left(\frac{1-s}{s}\right)^{\lambda_i} \quad i=1,2 \tag{15}$$

For some constants $\lambda_1<0<\lambda_2$ such that D is given by $D=[0,1]\setminus[s_1^*,s_2^*]$. Let $\tau_D := inf\{t>0\backslash s\notin D\}$. Then $\phi(t,x,y)=e^{-\beta t}x(xy)^{-\gamma/2}H(s)$ and $\tau^*=\tau_D$ (under some assumptions over the processes).

Proof The proof is based in solving the homogeneus PDE associated with the generator of the process $Z_s=(t+s,D_{1s},D_{2s})^\top$, with $Z_0=(t,x,y)$, verifying then optimality conditions. For further details see Appendix A. $\square$

This proposition shows that the optimal IPO strategy is determined by two IPO triggers, the *small-cap IPO trigger* s_1^* and the *large-cap IPO trigger* s_2^*. It may exist the case where $s_1^*=0$ or/and $s_2^*=1$. If the dividend ratio is too low (or too big), the entrepreneurs will decide to keep his firm unlisted. As soon as the dividend ratio reaches one of the triggers, the IPO decision will be made. The so called *value matching* and *smooth pasting* conditions are implicitly given by the proposition ($H(s)$ must be $C^1((0,1))\cup C([0,1])$). The following corollary gives us a direct method for obtaining the optimal triggers s_i^* and the constants c_i for $(i=1,2)$.

Corollary 1 *The optimal states s_i^* can be obtained by solving the following equations:*

$$s_i^*(1-s_i^*)j'(s_i^*,\omega(s_i^*))+\lambda_i j(s_i^*,\omega(s_i^*))=0 \quad i=1,2 \tag{16}$$

and c_i is given explicitly by the expression

$$c_i = \frac{j(s_i^*, \omega(s_i^*))}{\left(\frac{1-s_i^*}{s_i^*}\right)^{\lambda_i}} \quad i = 1, 2 \tag{17}$$

3 Results

In the following section we study the main implications of our model. Although the theoretical model accounts for jumps in the dividend processes, we consider only the diffusion component for most of our analysis. A simpler model is desirable to highlight the main contributions of the paper and the economic intuition behind the results. First, we analyze the behavior and sensibility of optimal IPO decision to the dividends correlation and transaction costs. The expected IPO time and its sensibility to the initial dividend ratio is also studied. Then, we analyze the diversification effect on the entrepreneur wealth and consumption. In addition we study the relations between optimal IPO triggers and Betas of the newly listed firm. Finally we consider the effect of jumps in dividends and analyze their effect on the IPO triggers.

3.1 Optimal IPO triggers

In this section we study in detail the market conditions at which the entrepreneur IPO his firm. As mentioned above, the optimal IPO strategy is determined by a closed interval between dividend ratios s_1^* and s_2^*: the *small-cap IPO trigger* and the *large-cap IPO trigger*.

When private company has a small share of the economy, i.e. $s < s_1^*$, it will be optimal to postpone the decision. Although the IPO diversifies the entrepreneur's consumption, transaction costs are too big. The optimal IPO strategy is to wait until the company owns a bigger share of the economy, which implies greater diversification benefits.[3]

In the unusual case in which the company owns a large share of the economy, i.e. $s > s_2^*$, the entrepreneur is not tempted to IPO its company: the potential market portfolio after the IPO will be too correlated with the company. Diversification benefits are small comparing with transaction costs (for values of s near 0 or 1 diversification is minimal). The entrepreneur will keep its company private until its share in the economy diminishes (produced by negative shocks to the company's dividends, or positive shocks to the market's dividends). This case is absurd if we only think of develop economies, but it may have occurred in an early stage of a less develop economy some decades ago (when few firms were listed in the stock market).

[3] For the symmetric case, Cochrane, Longstaff, and Santa-Clara (2008) shows that diversification finds its maximum for middle values of the dividend ratio, s.

Figure 1 presents the IPO triggers for different correlations between the dividend streams D_1 and D_2 and for different values of risk aversion of the agents. Figure 1(a) shows that the *small-cap IPO trigger* increases with the correlation of the dividend streams and decreases with risk aversion. Diversification benefits of the IPO diminishes if the company is correlated with the market, which means the entrepreneur will wait for a bigger share of the economy, increasing the optimal IPO trigger. For higher risk aversion, the agents value more the diversification benefits, implying that the entrepreneur IPOs in a lower share.

A symmetric effect is reflected in Figure 1(b). The *large-cap IPO trigger* decreases with the correlation and increases with risk aversion. If agent 1 holds a large fraction of the economy (i.e. $s > s_2^*$), a higher correlation with the market decreases its diversification benefits. Therefore, the entrepreneur will delay the IPO decision until these benefits are enough. This occurrs for lower IPO triggers. Higher risk aversion increases the IPO trigger because diversification is more desirable.

Further analysis will be focused only in the small-cap IPO trigger, being the most common case faced in economies. Figure 2 plots this trigger against the fraction of the economy that represents the transaction costs. Results support the intuition: the higher the transaction costs, the larger the requirements for the IPO. For lower risk aversion, the model is highly sensitive to transaction costs, and IPO trigger grows fast when facing large transaction costs.

3.2 Expected hitting time

In this section we study the expected time until the IPO option is exercised. In some cases, this analysis is more intuitive than studying the optimal triggers directly. The expected hitting time is calculated from that the monotonic transformation of s_t equals $u_t = y_{2t} - y_{1t}$. The random variable $u_t : \Omega \to \mathbb{R}$ distributes the same as the variables y_{it}, i.e. it follows a jump diffusion process.

It can be proved that expected IPO time when processes y_{it} have the same drift is infinite (see Shreve (2008)). An intuitive explanation might be that because the expected growth company equals market's growth, the firm is never expected to reach the optimal relative size to IPO. To calibrate the variables, we chose different drifts for dividends. We assume that the firm's dividends grow faster than the market dividend payoffs.

For obtaining the distribution of the stopping time variable $\tau_D = \inf\{t > 0 \backslash \text{The company stays unlisted}\}$ we use the results obtained by Coutin and Dorobantu (2009). Knowing the distribution, expectations can be easily computed.

Figure 3 shows the behavior of expected IPO time for different initial sizes (i.e. dividend ratio). For companies with lower s, expected hitting times are huge. This expectation lowers when the firm's size approaches the optimal s to IPO. This means that the smaller the IPO trigger, the lower is the expected time a company has to wait to IPO. As before, we find that entrepreneurs with lower risk aversion degrees are expected to wait more time to IPO their firms.

Assuming that firms within an industry have similar dividend streams, the model predicts that IPO occurrences will be positively correlated with firms' size. This means that the first IPOs will be triggered by larger companies in an industry, and then smaller firms will follow.

3.3 Welfare analysis

This section studies the effect of diversification contrasting scenarios before and after the IPO. First we analyze the perceived effect of the timing option over the firm's value. The second part studies how entrepreneur's consumption changes with the IPO.

We first compare the price-dividend ratio of the market portfolio with the one perceived by the entrepreneur. The embedded IPO option adds value to the entrepreneur. Our purpose is to transfer this value to the firm's price.

Before the IPO the asset is not open for trade in the market. Therefore, we cannot assume complete markets in its valuation (The uniqueness of the pricing kernel is not assured). Entrepreneur might value his company in a different way that the market does, mainly because there is an option involved in the IPO timing. We use a utility indifference approach to value the unlisted firm from the entrepreneur's point of view.

Let us define the company's price as the minimum value at which the entrepreneur is willing to IPO his company. At this price the entrepreneur's utility before and after the IPO must be the same. We summarize this approach by the following statement. Suppose P^* is the price at which the entrepreneur is indifferent to IPO his firm. For $s \in [s_1^*, s_2^*]$ we have that $P^* = P$. Outside this region (therefore before de the IPO), we can isolate the price from the following relations:

$$P^* \text{ solves } \begin{cases} h_1(s) = j(s, \frac{P^* - \alpha W_t}{W_t}) & \text{if} \quad s < s_1^* \\ h_2(s) = j(s, \frac{P^* - \alpha W_t}{W_t}) & \text{if} \quad s > s_2^* \end{cases} \tag{18}$$

where the functions $h_i(\cdot)$ are defined in Proposition 1.

Before the IPO, the price-dividend ratio is calculated using the utility indifference price. Once the IPO is done, the asset becomes tradable in a complete market and there is a unique pricing kernel.

Figure 4 shows the IPO timing option in terms of price-dividend ratios. Before the IPO, the company has an extra value given by the option to postpone the IPO for better market conditions. This value is added to the price at which the entrepreneur is willing to sell his company. From the entrepreneur's point of view, the company has a bigger price-dividend ratio before the optimal IPO timing. Because the market is willing to pay a lower price that the one asked by the company owner, the IPO is not triggered. When the firm's relative size approaches the IPO trigger, the option value vanishes and the price-dividend ratios before and after the IPO equate.

For a better understanding of the entrepreneur's motives to IPO, we study the consumption streams, which becomes the core process of the utility function. Before the IPO the entrepreneur only consumes the dividends generated by its company. After the IPO, as stated in equations (8) and (9), the entrepreneur owns a fully-diversified portfolio. The diversification effect is reflected immediately in the consumption streams. As shown by Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2009), the effect of portfolio diversification in the consumption processes is significant. The consumption volatility achieves its minimum for intermediate values of s as seen below:

$$\frac{dC_{it}}{C_{it}} = [s_t\mu_{D_1} + (1-s_t)\mu_{D_2}]\, dt + s_t\sigma_1 dZ_t + (1-s_t)\sigma_2 dZ_t \tag{19}$$

Then

$$\mathbb{E}_t\left[\frac{dC_{it}}{C_{it}}\right] = [s_t\mu_{D_1} + (1-s_t)\mu_{D_2}]\, dt \tag{20}$$

$$\mathbb{V}ar_t\left[\frac{dC_{it}}{C_{it}}\right] = \left[s_t^2\sigma_1\sigma_1^\top + (1-s_t)^2\sigma_2\sigma_2^\top + s_t(1-s_t)\sigma_1\sigma_2^\top\right] dt \tag{21}$$

Figure 5 shows how the entrepreneur's consumption process changes before versus after the IPO. Before the IPO, the entrepreneur's consumption volatility is the same as the constant dividend volatility represented by the dashed line. After the IPO, the entrepreneur's wealth is given by a fraction of the economy's aggregate wealth, and his consumption is a constant fraction ω_τ of the aggregate consumption. For intermediate values of s, volatility reaches its minimum. At the IPO time, consumption volatility falls enough to compensate for the fixed transaction costs.

Figure 6 shows consumption-wealth ratio for the entrepreneur. Before the IPO, he consumes a lower fraction of his wealth comparing to the diversified case.[4]. After the IPO, marked with the red dashed line, the entrepreneur's consumption jumps to the solid line. The IPO diversification effect is reflected in today's consumption. A decrease in the consumption volatility due to the IPO decision reduces the need for precautionary savings. This implies that the entrepreneur is willing to consume a larger amount of his total wealth.

3.4 Relation Between Betas and IPO Triggers

The following section studies the betas of firms at the optimal IPO time. We wish to analyze the relation between the size at which a firm IPOs and its betas. For conducting our purpose, we define

[4]The values of s where the non-diversified consumption surpasses the diversified case occurs because if the entrepreneur IPOs for those values of s, the transaction costs are higher than the firm's value, in which case wealth and consumption are negative

the beta in terms of excess returns from the CAPM [5], i.e.

$$\mathbb{E}_t\left[r_{1t}\right] - r_t^f = \beta\left(\mathbb{E}_t\left[r_{Mt}\right] - r_t^f\right) \tag{22}$$

The expected return, $\mathbb{E}_t\left[r_{1t}\right]$ and the riskless interest rate, r_t^f, are calculated as in Martin (2009).[6] The expected return of the market after the IPO, $\mathbb{E}_t\left[r_{Mt}\right]$, is the weighted expected return of both assets. The beta is obtained from equation (22).

We wish to consider companies from several industries. Different industries will be characterized by the correlation of its dividends with the market. This way, within an industry we may have many companies of different sizes, but whose dividend streams follow the same process.

Figure 7 shows the relation of the beta at which a company optimally triggers the IPO against correlation of the dividend streams. We see that the higher the correlation of the industry with the market, the higher the Beta at which the company triggers the IPO. This information can be added to the information from Figure 1, and we can obtain the relation showed in next figure.

Figure 8 shows the relation between the optimal size at which a company IPOs versus its Beta in that moment. Industries with lower correlation with the market IPOs with lower betas, and companies within this industries will IPO with lower sizes. We observe that there is a direct relation between optimal s triggers and the betas. Considering that beta measures how much company's returns follow the market, the first companies expected to IPO will be from industries which returns do not follow the market.

The model predicts the dynamics of the economy over time. Remember Figure 3: the smaller the IPO trigger, the lower is the expected time a company has to wait to IPO. Because industries that IPO with lower betas has smaller optimal triggers, the first companies to IPO are expected to come from this industries. This predictions is in line with the empirical work of Astudillo, Braun, and Castaneda (2008) that compares the country's beta from economies in different stages of development. Astudillo, Braun, and Castaneda (2008) concludes that at early stages of the economy the first firms to IPO comes from industries that show lower betas, overweighting these sectors in the market. As the economy develops, firms from higher Beta industries IPO, raising the markets's Beta, until the market reaches a mature point in which al sectors in the economy are well represented.

3.5 Dividends subject to jumps: Idiosyncratic and Systematic Shocks

In this final section of the paper, we analyze the effect of dividend jumps over IPO decisions. For this purpose, we compare 4 scenarios: no jumps, Positive Jumps, Negative Jumps, and Economical

[5]Note that the CAPM is valid because at the IPO the asset is part of the complete market.
[6]See Appendix B for details.

Crisis. In Positive Jumps we consider normal jumps affecting the unlisted firm, with mean 0.05, variance 0.1^2 and an occurrence rate of 1/5 (1 every 5 years). Negative Jumps are the same, but with mean -0.05. Economical Crisis are characterized by jumps affecting both assets (Company and Market) with mean -0.1, variance 0.1^2 and occurrence rate 1/20.

Figure 9 shows how the IPO trigger is affected by the jump scenarios. Under the presence of positive jumps, the firm has a higher valuation for the same values of s (because of higher expected dividends). This higher valuation increases the diversification benefits obtained through the IPO, lowering the optimal trigger. Negative jumps produce the contrary effect: the company loses value, and diversification effect at the IPO diminishes, which increases the optimal IPO trigger. In an scenario with economical crisis, there are two opposing effects. On one hand, the company loses value because expected dividend streams decrease. On the other hand, the market loses value because of the negative jumps, which increases the diversification effects in the IPO. The second effect is greater, and the entrepreneur chooses to IPO at a lower share.

4 Conclusions

We summarize the basic procedure conducted in this study. First we built up a general equilibrium model in an endowment economy to analyze an IPO scenario. The model is solved using dynamic programming and asset pricing results from the works of Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2009). We analyzed the random variables and states affecting the decision, studying expected optimal hitting times and diversification effects over the entrepreneur.

Eliminating any extra benefit, portfolio diversification is strong enough to trigger an IPO: consumption streams are less volatile, which increases the entrepreneur's utility through time, overcoming any transaction costs. Within an industry, the relative size of the firm to the market is the main characteristic that triggers the IPO.

Studying expected hitting times, the model predicts that within an industry bigger firms are expected to IPO first (greater diversification benefits). In our model, each industry defines a dividend stream, and each dividend stream generates an optimal IPO trigger. As a firm approaches this dividend share, the time expected to IPO approaches to zero.

Optimal IPO triggers are higher when dividends face high correlation with the market, an so are the CAPM Betas in the moment of the IPO. Merging this two effects, we realize that IPO trigger and the Beta of the newly public firms move in the same direction. This means (considering that the lower IPO trigger the lower the expected time to IPO) that the first firms expected to IPO should have lower Betas. This effect might explain why most public firms in developing economies and emerging markets are concentrated in low Betas sectors. As time passes, the model predicts that firms with higher Betas will be incorporated in the market until the market completes.

It is left for future research to fit the model to the data. It will probably require the incorporation of more trees in the economy, fitting the dividend streams (i.e. drift, diffusion, and jumps), and more specific transaction costs. The model, beside complementing the IPO theory, might be used as a guide for any entrepreneur analyzing IPO timing decision, considering portfolio diversification benefits.

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Appendix

A Solution for the entrepreneur's value function

A.1 Value function

We want to compute the value of the following expression

$$\begin{aligned} j_E(t, D_{1t}, D_{2t}) &= e^{-\beta t}\mathbb{E}_t\left[\int_t^{\infty} e^{-\beta(s-t)}\left(\frac{C_{1s}^{1-\gamma}-1}{1-\gamma} - \frac{D_{1s}^{1-\gamma}-1}{1-\gamma}\right) ds\right] \\ &= \frac{e^{-\beta t}}{1-\gamma}\left(\underbrace{\int_t^{\infty} e^{-\beta(s-t)}\mathbb{E}_t\left[C_{1s}^{1-\gamma}\right] ds}_{(Et1)} - \underbrace{\int_t^{\infty} e^{-\beta(s-t)}\mathbb{E}_t\left[(D_{1s})^{1-\gamma}\right] ds}_{(Et2)}\right) \end{aligned} \tag{A1}$$

We notice from equation (B21) that $(Et1) = W_{1t}/C_{1t}^{\gamma}$. W_{1t} is solved below in Appendix B.2. Using this result we get

$$(Et1) = \frac{C_{it}^{1-\gamma}}{\sqrt{s_t^{\gamma}(1-s_t)^{\gamma}}}\int_{-\infty}^{\infty}\left(\frac{1-s_t}{s_t}\right)^{iv}\left(\frac{s_t}{\beta - c_1(v)} + \frac{1-s_t}{\beta - c_2(v)}\right)\Psi_{\gamma}(v)\, dv \tag{A2}$$

Using equations (8) and (9) and after some algebraic manipulations

$$(Et1) = \omega_{\tau}^{1-\gamma} D_{1t}\left(D_{1t}D_{2t}\right)^{-\gamma/2}\int_{-\infty}^{\infty}\left(\frac{1-s_t}{s_t}\right)^{iv}\left(\frac{1}{\beta - c_1(v)} + \frac{(1-s_t)/s_t}{\beta - c_2(v)}\right)\Psi_{\gamma}(v)\, dv \tag{A3}$$

$(Et2)$ is obtained after solving the conditional expectation:

$$\mathbb{E}_t\left[D_{1s}^{1-\gamma}\right] = \mathbb{E}_t\left[e^{(1-\gamma)(y_{1t}+\tilde{y}_{1(s-t)})}\right] = e^{(1-\gamma)y_{1t}}e^{(s-t)\mathbf{c}((1-\gamma),0)} \tag{A4}$$

Then

$$(Et2) = \frac{D_{1t}^{1-\gamma}}{\beta - \mathbf{c}((\mathbf{1}-\gamma),\mathbf{0})} = \frac{D_{1t}\left(D_{1t}D_{2t}\right)^{-\gamma/2}\left(\frac{1-s_t}{s_t}\right)^{\gamma/2}}{\beta - \mathbf{c}((\mathbf{1}-\gamma),\mathbf{0})} \tag{A5}$$

for $\beta_1 - \mathbf{c}((\gamma - \mathbf{1}),\mathbf{0}) > 0$. The function $\mathbf{c}(\cdot,\cdot)$ is defined in Appendix A.3. Finally

$$\begin{aligned} j_E(t, D_{1t}, D_{2t}) &= \frac{e^{-\beta t}D_{1t}\left(D_{1t}D_{2t}\right)^{-\gamma/2}}{1-\gamma} \\ &\left[\omega_{\tau}^{1-\gamma}\int_{-\infty}^{\infty}\left(\frac{1-s_t}{s_t}\right)^{iv}\left(\frac{1}{\beta - c_1(v)} + \frac{(1-s_t)/s_t}{\beta - c_2(v)}\right)\Psi_{\gamma}(v)\, dv + \frac{\left(\frac{1-s_t}{s_t}\right)^{\gamma/2}}{\beta - \mathbf{c}((\mathbf{1}-\gamma),\mathbf{0})}\right] \end{aligned} \tag{A6}$$

A.2 Optimal Stopping States

At first, our state variables are the dividend processes D_{1t} and D_{2t} given by the following PDE:

$$\frac{dD_{it}}{D_{it}} = \mu_{D_i} dt + \sigma_i dZ_t + \sum_{k=1}^{l} \int_{\mathbb{R}} (e^{g_{ik}(z)} - 1)\, N^{(k)}(dt, dz_k) \tag{A7}$$

The optimal control problem consists in finding the optimal states in which $J^{\tau}(t, D_{1t}, D_{2t})$ reaches its maximum value, or equivalently, the optimal states in which $j_E(\tau, D_{1t}, D_{2t})$ is maximized. The problem is summarized in equations (13). We apply Theorem (2.2) from Oksendal and Sulem (2005) for finding the value function V:

Define the 3-dimensional state process

$$Z_s = \begin{bmatrix} t+s \\ D_{1s} \\ D_{2s} \end{bmatrix}, Y_0 = \begin{bmatrix} t \\ x \\ y \end{bmatrix} \tag{A8}$$

Define also $\widehat{S} = \mathbb{R}^3_+$, $\widehat{D} = \{z \in \widehat{S} \backslash \phi(z) > j_E(z)\}$, $\tau_{\widehat{S}} = \inf\{t \in \mathbb{R}^+ \backslash Z_t \notin \widehat{S}\}$ and $\mathcal{T} = \{\tau \text{ stopping time } \backslash \tau < \tau_{\widehat{S}}\}$.

We must find $\phi : \overline{S} \to \mathbb{R}$ that satisfies the following

(i) ∂D is a Lipschitz surface

(ii) $\phi \geq j_E$ on S

(iii) $\phi \in C^1(\widehat{S}) \cup C(\overline{\widehat{S}})$ and $\phi \in C^2(\widehat{S} \setminus \partial D)$ with locally bounded derivatives near ∂D

(iv) $\mathbb{E}\left[\int_t^{\tau_{\widehat{S}}} \mathbb{1}_{\partial D}(Y_s)\, ds\right] = 0$

(v) $Y_{\tau_{\widehat{S}}} \in \partial\widehat{S}$ a.s. on $\{\tau_{\widehat{S}} < \infty\}$ and $\lim_{s \to \tau_{\widehat{S}}^-} \phi(Y_s) = j_E(Y_{\tau_{\widehat{S}}}) \mathbb{1}_{\{\tau_{\widehat{S}} < \infty\}}$

(vi) $\mathbb{E}\left[|\phi(Y_\tau)| + \int_t^{\tau_{\widehat{S}}} \left(|A\phi(Y_s)| + \|\sigma^\top \nabla\phi(Y_s)\|^2 + \sum_{k=1}^{l} |\phi(Y_t e^{g_k(z_k)}) - \phi(Y_t)|^2\, d\nu_k(z_k)\right) ds\right] < \infty$ for all $\tau \in \mathcal{T}$

(vii) $A\phi = 0$ on D

(viii) $A\phi \leq 0$ on $S \setminus \partial D$

(ix) $\tau_D := \inf\{t \in \mathbb{R} \backslash Z_t \notin D\} < \infty$ a.s.

(x) $\{\phi(Y_\tau) \backslash \tau \in \mathcal{T}\}$ is uniformly integrable, for all $(t, x, y) \in \mathbb{R}^3_+$

We begin searching for a function $\phi(t, x, y)$ that fits the conditions above. The generator of the process Z_t is given by

$$\begin{aligned} A\phi(t,x,y) \quad = \quad & \frac{\partial\phi}{\partial t} + x\mu_{D_1}\frac{\partial\phi}{\partial x} + y\mu_{D_2}\frac{\partial\phi}{\partial y} + \frac{1}{2}A_1A_1^\top x^2\frac{\partial^2\phi}{\partial x^2} + A_1A_2^\top xy\frac{\partial^2\phi}{\partial x\partial y} + \frac{1}{2}A_1A_1^\top y^2\frac{\partial^2\phi}{\partial y^2} + \\ & \sum_{k=1}^{l}\int_{\mathbb{R}} \{\phi(t, x + x(e^{g_{1k}(z_k)} - 1), y + y(e^{g_{2k}(z_k)} - 1)) - \phi \\ & -\frac{\partial\phi}{\partial x}x(e^{g_{1k}(z_k)} - 1) - \frac{\partial\phi}{\partial y}y(e^{g_{2k}(z_k)} - 1)\}\, \nu^{(k)}(dz_k) \end{aligned} \tag{A9}$$

Condition (vii) requires that $A\phi = 0$. We suppose that $\phi(t,x,y)$ has the following structure:$\phi(t,x,y) = e^{-\beta_1 t} x(xy)^{-\gamma/2} h(u)$, with u given by $u_t \equiv \log \frac{1-s}{s} = \log y/x$. Replacing in equation above:

$$
\begin{aligned}
A\phi(t,x,y) = e^{-\beta_1 t} x(xy)^{-\gamma/2} (\theta_0 h(u) + \theta_1 h'(u) + \theta_2 h''(u) + \\
\sum_{k=1}^{l} \int_{\mathbb{R}} \{ e^{g_{1k}(z_k)} e^{-\gamma/2(g_{1k}(z_k)+g_{2k}(z_k))} h(u + g_{2k}(z_k) - g_{1k}(z_k)) + \\
h(u) \left[e^{g_{1k}(z_k)}(\gamma/2-1) + e^{g_{2k}(z_k)}\gamma/2 - \gamma \right] + h'(u) \left[e^{g_{1k}(z_k)} - e^{g_{2k}(z_k)} \right] \} \, d\nu^{(k)}(z_k))
\end{aligned}
\tag{A10}
$$

where
$\theta_0 = -\beta_1 + (1-\gamma/2)\mu_{D_1} - \gamma/2\mu_{D_2} + 1/2(\gamma^2/4 - \gamma/2)A_1 A_1^\top$
$+ (\gamma^2/4 - \gamma/2)A_1 A_2^\top + 1/2(\gamma^2/4 + \gamma/2)A_2 A_2^\top$
$\theta_1 = \mu_{D_2} - \mu_{D_1} + 1/2(\gamma - 1)A_1 A_1^\top + A_1 A_2^\top + 1/2(-\gamma - 1)A_2 A_2^\top$
$\theta_2 = 1/2 A_1 A_1^\top - A_1 A_2^\top + 1/2 A_2 A_2^\top$

We propose $h(u) = e^{\lambda u}$:

$$
\begin{aligned}
A\phi(t,x,y) = e^{-\beta_1 t} x(xy)^{-\gamma/2} e^{\lambda u} (\theta_0 + \theta_1 \lambda + \theta_2 \lambda^2 + \\
\sum_{k=1}^{l} \int_{\mathbb{R}} \left\{ e^{g_{1k}(z_k)(1-\lambda-\gamma/2) - g_{2k}(z_2)(-\lambda+\gamma/2)} + e^{g_{1k}(z_k)}(\gamma/2 + \lambda - 1) + e^{g_{2k}(z_k)}(\gamma/2 - \lambda) - \gamma \right\} d\nu^{(k)}(z_k))
\end{aligned}
\tag{A11}
$$

A.3 Cumulant-generating function

Cumulant-Generating function $\mathbf{c}(v)$ is defined as the logarithm of the moment-generating function

$$
\mathbf{c}(v) := \log \left(\mathbb{E} \left[e^{\mathbf{v}'(\tilde{y_{t+1}} - \tilde{y_t})} \right] \right) \tag{A12}
$$

In our case

$$
\mathbf{c}(\mathbf{v}) = \mathbf{v}'\mu + \frac{1}{2}\mathbf{v}'\Sigma\mathbf{v} + \omega \left(\mathbb{E} e^{\mathbf{v}' S_k} - 1 \right) \tag{A13}
$$

$$
\mathbf{c}(\mathbf{v}) = \mathbf{v}'\mu + \frac{1}{2}\mathbf{v}'\Sigma\mathbf{v} + \sum_{k=1}^{l} \int_{\mathbb{R}} \left(e^{\mathbf{v}' g^k(z_k)} - 1 \right) \nu^{(k)}(dz_k)^7 \tag{A14}
$$

where $\Sigma = AA'$. If ν is the Lévy measure of Poisson jumps $\backsim N(\mu_S, \Sigma_S)$, $\mathbf{c}(v)$ results in

$$
\mathbf{c}(v) = \mathbf{v}'\mu + \frac{1}{2}\mathbf{v}'\Sigma\mathbf{v} + \omega \left(e^{\mathbf{v}'\mu_S + 1/2\mathbf{v}'\Sigma_S \mathbf{v}} - 1 \right) \tag{A15}
$$

B Equilibrium after the IPO

B.1 Prices

First, suppose the IPO decision is irreversible, and done at some time τ. At the IPO time, the market completes and both agents rebalance their portfolio. Each of the agents can choose freely in t how much to consume and how much to invest in P_1 and P_2. After this happens, the market is in equilibrium, and the scenario resembles the *two Lucas trees* economy Cochrane, Longstaff, and Santa-Clara (2008).

In this scenario, the market clearing condition states that the aggregate consumption C equals the sum of the dividend payoffs, i.e.

$$C_t := C_{1t} + C_{2t} = D_{1t} + D_{2t} \tag{B1}$$

The first order conditions for the asset prices, derived by Lucas (1978), states that from the point of view of the entrepreneur

$$P_{it} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\beta(s-t)} \left(\frac{C_{1s}}{C_{1t}} \right)^{-\gamma} D_{is}\, ds \right] \tag{B2}$$

and from the point of view of the market

$$P_{it} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\beta(s-t)} \left(\frac{C_{2s}}{C_{2t}} \right)^{-\gamma} D_{is}\, ds \right] \tag{B3}$$

Since the market is complete, state-price deflators must be unique and prices form both points of view must be the same. It follows that

$$e^{-\beta(s-t)} \left(\frac{C_{1s}}{C_{1t}} \right)^{-\gamma} = e^{-\beta(s-t)} \left(\frac{C_{2s}}{C_{2t}} \right)^{-\gamma} = \xi \tag{B4}$$

Using (B1) we can isolate ξ in terms of the dividend processes:

$$\xi = e^{-\beta(s-t)} \left(\frac{D_{1s} + D_{2s}}{D_{1t} + D_{2t}} \right)^{-\gamma} \tag{B5}$$

Asset prices are given by the following proposition, in line with the results of Martin (2009)

Proposition B1 *Under the above conditions, the asset prices in equilibrium are given by*

$$P_{it} = \frac{D_{it}}{\sqrt{s_t^{\gamma}(1-s_t)^{\gamma}}} \int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t} \right)^{iv} \frac{\Psi_{\gamma}(v)}{\beta - c_i(v)}\, dv \tag{B6}$$

$$\tag{B7}$$

Where

$$\Psi_{\gamma}(v) = \frac{1}{2\pi} \frac{\Gamma(\gamma/2 - iv)\Gamma(\gamma/2 + iv)}{\Gamma(\gamma)} \tag{B8}$$

$$c_1(v) = \mathbf{c}(1 - \gamma/2 - iv, -\gamma/2 + iv) \tag{B9}$$

$$c_2(v) = \mathbf{c}(-\gamma/2 - iv, 1 - \gamma/2 + iv) \tag{B10}$$

and $\mathbf{c}(\theta)$ *the CGF of the process* $y_{t+1} - y_t$

Proof The method for obtaining asset prices in complete markets can be found in Martin (2009). We repeat the same procedure: it was previously stated that asset prices are given by (B2), (B3) and (B5). Using both, we get

$$P_{it} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\beta(s-t)} \left(\frac{D_{1s} + D_{2s}}{D_{1t} + D_{2t}} \right)^{-\gamma} D_{is}\, ds \right] \tag{B11}$$

$$= \int_t^{\infty} e^{-\beta(s-t)} (D_{1t} + D_{2t})^{\gamma}\, \mathbb{E}_t \left[\frac{D_{is}}{(D_{1s} + D_{2s})^{\gamma}} \right] ds \tag{B12}$$

For solving $\mathbb{E}_t\left[\frac{D_{is}}{(D_{1s}+D_{2s})^\gamma}\right]$, we use the fact that

$$\begin{aligned}
\mathbb{E}_t\left[\frac{D_{is}}{(D_{1s}+D_{2s})^\gamma}\right] &= e^{y_{it}}\mathbb{E}_t\left[\frac{e^{\tilde{y}_{i(s-t)}}}{\left(e^{y_{1t}+\tilde{y}_{1(s-t)}}+e^{y_{2t}+\tilde{y}_{2(s-t)}}\right)^\gamma}\right] \\
&= e^{y_{it}}e^{-\gamma/2(y_{1t}+y_{2t})}\mathbb{E}_t\left[\frac{e^{(\mathbb{1}_{\{i=1\}}-\gamma/2)\tilde{y}_{1(s-t)}+(\mathbb{1}_{\{i=2\}}-\gamma/2)\tilde{y}_{2(s-t)}}}{\left(2\cosh\frac{y_{2t}-y_{1t}+\tilde{y}_{2(s-t)}-\tilde{y}_{1(s-t)}}{2}\right)^\gamma}\right] \qquad \text{(B13)}
\end{aligned}$$

We now use de Fourier transform $\Psi_\gamma(v)$ of $1/[\cosh(u/2)]^\gamma$ for $\gamma>0$ given by

$$\frac{1}{[2\cosh(u/2)]^\gamma}=\int_{-\infty}^{\infty}e^{iuv}\Psi_\gamma(v)\,dv \qquad \text{(B14)}$$

Martin (2009) obtains a close expression for $\Psi_\gamma(v)$, which is

$$\Psi_\gamma(v)=\frac{1}{2\pi}\frac{\Gamma(\gamma/2-iv)\Gamma(\gamma/2+iv)}{\Gamma(\gamma)} \qquad \text{(B15)}$$

Replacing in (B13), the right part of the equation becomes

$$\begin{aligned}
&= D_{it}e^{-\gamma/2(y_{1t}+y_{2t})}\mathbb{E}_t\left[\int_{-\infty}^{\infty}e^{iv(y_{2t}-y_{1t}+\tilde{y}_{2(s-t)}-\tilde{y}_{1(s-t)})}e^{(\mathbb{1}_{\{i=1\}}-\gamma/2)\tilde{y}_{1(s-t)}+(\mathbb{1}_{\{i=2\}}-\gamma/2)\tilde{y}_{2(s-t)}}\Psi_\gamma(v)\,dv\right] \\
&= D_{it}\,e^{-\gamma/2(y_{1t}+y_{2t})}\int_{-\infty}^{\infty}\mathbb{E}\left[e^{(\mathbb{1}_{\{i=1\}}-\gamma/2-iv)\tilde{y}_{1(s-t)}+(\mathbb{1}_{\{i=2\}}-\gamma/2+iv)\tilde{y}_{2(s-t)}}\right]e^{iv(y_{2t}-y_{1t})}\Psi_\gamma(v)\,dv \qquad \text{(B16)}
\end{aligned}$$

Last equality stands because of the independent increments of $\tilde{y}_{is}$. Finally,

$$\mathbb{E}_t\left[\frac{D_{is}}{(D_{1s}+D_{2s})^\gamma}\right]=D_{it}e^{-\gamma/2(y_{1t}+y_{2t})}\int_{-\infty}^{\infty}e^{(s-t)\mathbf{c}(\mathbb{1}_{\{i=1\}}-\gamma/2-iv,\mathbb{1}_{\{i=2\}}-\gamma/2+iv)}e^{iv(y_{2t}-y_{1t})}\Psi_\gamma(v)\,dv \qquad \text{(B17)}$$

We now replace in our last expression for P_{it}

$$\begin{aligned}
P_{it} &= \int_t^{\infty}e^{-\beta(s-t)}\,(D_{1t}+D_{2t})^\gamma\,D_{it}\,e^{-\gamma/2(y_{1t}+y_{2t})} \\
&\quad \int_{-\infty}^{\infty}e^{(s-t)\mathbf{c}(\mathbb{1}_{\{i=1\}}-\gamma/2-iv,\mathbb{1}_{\{i=2\}}-\gamma/2+iv)}e^{iv(y_{2t}-y_{1t})}\Psi_\gamma(v)\,dv\,ds \qquad \text{(B18)} \\
&= D_{it}e^{-\gamma/2(y_{1t}+y_{2t})}\,(D_{1t}+D_{2t})^\gamma \\
&\quad \int_t^{\infty}e^{-\beta(s-t)}\int_{-\infty}^{\infty}e^{(s-t)c_i(v)}e^{iv(y_{2t}-y_{1t})}\Psi_\gamma(v)\,dv\,ds \qquad \text{(B19)}
\end{aligned}$$

Where $c_i(v):=\mathbf{c}(\mathbb{1}_{\{i=1\}}-\gamma/2-iv,\mathbb{1}_{\{i=2\}}-\gamma/2+iv)$ for simplifying notations. For conditions over the parameters given by $\beta-c_i(v)>0$ for $i=1,2$, the integrand above is absolutely integrable. Using Fubini's theorem

$$\begin{aligned}
P_{it} &= D_{1t}\,e^{-\gamma/2(y_{1t}+y_{2t})}\,(D_{1t}+D_{2t})^\gamma\int_{-\infty}^{\infty}e^{iv(y_{2t}-y_{1t})}\left(\int_t^{\infty}e^{-(s-t)(\beta-c_i(v))}\,ds\right)\Psi_\gamma(v)\,dv \\
&= D_{1t}\,e^{-\gamma/2(y_{1t}+y_{2t})}\,(D_{1t}+D_{2t})^\gamma\int_{-\infty}^{\infty}\frac{e^{iv(y_{2t}-y_{1t})}}{\beta-c_i(v)}\Psi_\gamma(v)\,dv \qquad \text{(B20)}
\end{aligned}$$

Replacing with the process $s_t=\frac{D_{it}}{D_{1t}+D_{2t}}$ finally proves Proposition B1.

□

B.2 Wealth

Let W_{it} $(i = 1, 2)$ be the wealth of the agents in equilibrium (entrepreneur and market resp.). In an exchange economy today's consumption must equal the discounted cash flows in the future, i.e.

$$W_{it} \quad = \quad \mathbb{E}_t \left[\int_t^{\infty} e^{-(s-t)\beta_1} \left(\frac{C_{is}}{C_{it}} \right)^{-\gamma} C_{is} \, ds \right] \tag{B21}$$

This leads us to the following proposition

Proposition B2 *Under equilibrium conditions, the economy agents' wealth will be given by*

$$\begin{aligned} W_{it} &= \frac{C_{it}}{\sqrt{s_t^{\gamma}(1-s_t)^{\gamma}}} \int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t} \right)^{iv} \left(\frac{s_t}{\beta - c_1(v)} + \frac{1-s_t}{\beta - c_2(v)} \right) \Psi_{\gamma}(v) \, dv & \text{(B22)} \\ &= \frac{C_{it}}{D_{1t} + D_{2t}} \left(P_{1t} + P_{2t} \right) & \text{(B23)} \end{aligned}$$

Where c_1, c_2, and Ψ_{γ} are as in (B8).

Proof For obtaining the relations between wealth and consumption, we solve (B21) using a similar procedure as we did with prices. First we discount the consumption streams with our state price deflator (B5):

$$W_{it} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\beta(s-t)} \left(\frac{D_{1s} + D_{2s}}{D_{1t} + D_{2t}} \right)^{-\gamma} C_{is} \, ds \right] \tag{B24}$$

From equations (B4) and (B5) we can isolate consumption streams in terms of consumptions in t and the dividend processes, i.e. $C_{is} = C_{it} \left(\frac{D_{1s}+D_{2s}}{D_{1t}+D_{2t}} \right)$. Replacing in last equation:

$$W_{it} = C_{it} \, (D_{1t} + D_{2t})^{1-\gamma} \int_t^{\infty} e^{-\beta(s-t)} \mathbb{E}_t \left[\frac{1}{(D_{1s} + D_{2s})^{1-\gamma}} \right] ds \tag{B25}$$

The conditional expectation is solved as we did with prices:

$$\begin{aligned} \mathbb{E}_t \left[\frac{1}{(D_{1s} + D_{2s})^{\gamma-1}} \right] &= \mathbb{E}_t \left[\frac{D_{1s}}{(D_{1s} + D_{2s})^{\gamma}} \right] + \mathbb{E}_t \left[\frac{D_{2s}}{(D_{1s} + D_{2s})^{\gamma}} \right] & \text{(B26)} \\ = D_{1t} \, e^{-\gamma/2(y_{1t}+y_{2t})} & \int_{-\infty}^{\infty} e^{(s-t)\mathbf{c}(1-\gamma/2-iv, -\gamma/2+iv)} e^{iv(y_{2t}-y_{1t})} \Psi_{\gamma}(v) \, dv & \\ +D_{2t} \, e^{-\gamma/2(y_{1t}+y_{2t})} & \int_{-\infty}^{\infty} e^{(s-t)\mathbf{c}(-\gamma/2-iv, 1-\gamma/2+iv)} e^{iv(y_{2t}-y_{1t})} \Psi_{\gamma}(v) \, dv & \text{(B27)} \\ = e^{-\gamma/2(y_{1t}+y_{2t})} & \int_{-\infty}^{\infty} \left(D_{1t} e^{(s-t)c_1(v)} + D_{2t} e^{(s-t)c_2(v)} \right) e^{iv(y_{2t}-y_{1t})} \Psi_{\gamma}(v) \, dv & \text{(B28)} \end{aligned}$$

We now use Fubini's theorem, imposing conditions of integrability given by $\beta - c_i(v) > 0$ for $i = 1, 2$:

$$\begin{aligned} W_{it} &= C_{1t} e^{-\gamma/2(y_{1t}+y_{2t})} \, (D_{1t} + D_{2t})^{1-\gamma} \int_{-\infty}^{\infty} e^{iv(y_{2t}-y_{1t})} \Psi_{\gamma}(v) \\ & \quad \int_t^{\infty} e^{-\beta(s-t)} \left(D_{1t} e^{(s-t)c_1(v)} + D_{2t} e^{(s-t)c_2(v)} \right) ds \, dv & \text{(B29)} \\ &= C_{1t} e^{-\gamma/2(y_{1t}+y_{2t})} \, (D_{1t} + D_{2t})^{1-\gamma} \\ & \quad \int_{-\infty}^{\infty} e^{iv(y_{2t}-y_{1t})} \left(\frac{D_{1t}}{\beta - c_1(v)} + \frac{D_{2t}}{\beta - c_2(v)} \right) \Psi_{\gamma}(v) \, dv & \text{(B30)} \end{aligned}$$

Finally, we replace with our state process $s_t = \frac{D_{1t}}{D_{1t}+D_{2t}}$. This concludes the proof to Proposition B2. $\square$

B.3 Asset returns

We will proceed with the study of asset's returns and risk free returns. This will permit us to analyze risk factors faced by the entrepreneur.

For obtaining the instant risk-free return in equilibrium, we will base our procedure on Martin (2009). Suppose we are at a time t. First, we price a bond B_T that pays a unit in $T > t$. Then we calculate the dividend yield $R_{t\to T}$ using the fact that $B_T = e^{-R_{t\to T}(T-t)}$. Finally, we can obtain the instant risk-free return with the limit of the dividend yield as T tends to t, i.e. $r_t^f = \lim_{T\to t} R_{t\to T}$. Details of this procedure can be found on the Appendix. The result is shown below.

$$r_t^f = \frac{1}{\sqrt{s^\gamma(1+s)^\gamma}} \int_{-\infty}^{\infty} \left(\frac{1-s}{s}\right)^{iv} (\beta - \mathbf{c}(-\gamma/2 - iv, -\gamma/2 + iv))\, \Psi_\gamma(v)\, dv \tag{B31}$$

Procedure for obtaining expected asset returns can be found in Martin (2009). First, we proceed solving the expected capital gains, i.e. $\mathbb{E}_t\,[dP]$. We start with the expressions for prices obtained above:

$$P_{it} = (D_{1t} + D_{2t})^\gamma \int_{-\infty}^{\infty} h_i(v) e^{(\mathbb{1}_{\{i=1\}} - \gamma/2)\tilde{y}_{1t} + (\mathbb{1}_{\{i=2\}} - \gamma/2)\tilde{y}_{2t}}\, dv \tag{B32}$$

where

$$h_i(v) = \frac{\Psi_\gamma(v)}{\rho - c_i(v)} \tag{B33}$$

Using the Newton's generalized binomial theorem, we may expand $(D_{1t} + D_{2t})^\gamma$. It follows that

$$P_{it} = \sum_{n=0}^{\infty} \binom{\gamma}{n} \int_{-\infty}^{\infty} h_i(v) e^{w_{in}(v) y_t}\, dv \tag{B34}$$

with

$$w_{in} = (\mathbb{1}_{\{i=1\}} - \gamma/2 + n - iv, \mathbb{1}_{\{i=1\}} - \gamma/2 - n + iv) \tag{B35}$$

Let us call $X = e^{w_{in} y_t}$. Using Ito's differential formula applied to jump-diffusions we may obtain an expression for $\mathbb{E}_t\,[dX]$:

$$\begin{aligned}
\mathbb{E}_t\,[dX] &= e^{w_{in} y_t} \mathbb{E}_t \left[(w_{in}'\mu + w_{in}'\Sigma w_{in})dt + w_{in}' A dZ + \sum_{k=1}^{l} \int_{\mathbb{R}} (e^{w_{in} g^{(k)}(z_k)} - 1) N^{(k)}(dt, dz_k) \right] && \text{(B36)} \\
&= e^{w_{in} y_t} \left(w_{in}'\mu + w_{in}'\Sigma w_{in} + \sum_{k=1}^{l} \int_{\mathbb{R}} (e^{w_{in} g^{(k)}(z_k)} - 1)\, \nu^{(k)}(dz_k) \right) dt && \text{(B37)} \\
&= e^{w_{in} y_t} \mathbf{c}(w_{in}) dt && \text{(B38)}
\end{aligned}$$

Then $\mathbb{E}_t\,[dP_t]$ is given by

$$\mathbb{E}_t\,[dP_{it}] = \left(\sum_{n=0}^{\infty} \binom{\gamma}{n} \int_{-\infty}^{\infty} h_i(v) e^{w_{in}(v) y_t} \mathbf{c}(w_{in})\, dv \right) dt \tag{B39}$$

Replacing this result and expressions for prices (B34) in (B40), and after some algebraic manipulations, we get expression (B41) for returns.

B.4 Riskless rate

Let $(r_t^i)_t$ be the asset's return for $i = (1, 2)$. By definition, the asset's total return is given be the *capital gains* plus the *dividend yield*, i.e.

$$r_{it}\, dt = \frac{dP_{it}}{P_{it}} + \frac{D_{it}}{P_{it}} dt \tag{B40}$$

Following Martin (2009), we can find a close expression for the expected returns. Using expressions (B40), the *dividend yield* is given by the reciprocal of the $Price/Dividend$ obtained before. The only things left to calculate are the *capital gains*, specifically $\mathbb{E}_t\,[dP_t]$. The procedure is exactly the same as in Martin (2009). Finally

$$\mathbb{E}_t\,[r_{it}] = \frac{\sum_{n=0}^{\infty} \binom{\gamma}{n} \left(\frac{1-s_t}{s_t}\right)^{-n} \int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t}\right)^{iv} h_i(v)\mathbf{c}(w_{in})\, dv}{\sum_{n=0}^{\infty} \binom{\gamma}{n} \left(\frac{1-s_t}{s_t}\right)^{-n} \int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t}\right)^{iv} h_i(v)\, dv} + \frac{D_{it}}{P_{it}} \tag{B41}$$

Where

$$h_i(v) = \frac{\Psi_\gamma(v)}{\rho - c_i(v)} \tag{B42}$$

$$w_{1n} = (1 - \gamma/2 + n - iv, \gamma/2 - n + iv) \tag{B43}$$

$$w_{2n} = (\gamma/2 + n - iv, 1 + \gamma/2 - n + iv) \tag{B44}$$

First we need to price a bond B_T that pays a unit in period T. We use our state-price deflator (B5) to discount the payment:

$$B_T = \mathbb{E}_t \left[e^{-\beta(T-t)} \left(\frac{D_{1T} + D_{2T}}{D_{1s} + D_{2s}} \right)^{-\gamma} \right] \tag{B45}$$

$$= e^{-\beta(T-t)} \left(D_{1T} + D_{2T}\right)^{\gamma} \mathbb{E}_t \left[\frac{1}{\left(D_{1T} + D_{2T}\right)^{\gamma}} \right] \tag{B46}$$

Conditional expectation is solved analogously as we did with prices. Bond's price is then given by

$$\begin{aligned} B_T \;=\; & e^{-\beta(T-t)} \left(D_{1t} + D_{2t}\right)^{\gamma} \\ & e^{-\gamma/2(y_{1t}+y_{2t})} \int_{-\infty}^{\infty} e^{(T-t)\mathbf{c}(-\gamma/2-iv,-\gamma/2+iv)} e^{iv(y_{2t}-y_{1t})} \Psi_\gamma(v)\, dv \end{aligned} \tag{B47}$$

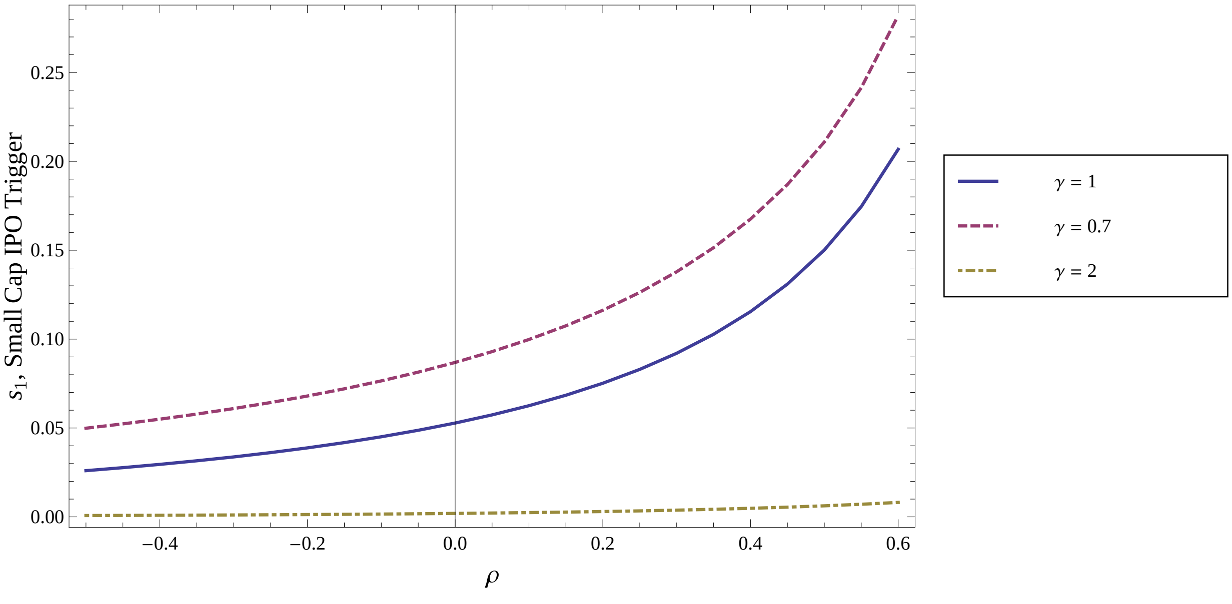
The interest yield $R_{t\to T}$ of a bond t that pays a unit in T is defined by the relation $B_T = e^{-R_{t\to T}(T-t)}$. Isolating the interest yield and replacing with our state process $s_t = \frac{D_{1t}}{D_{1t}+D_{2t}}$, it follows that

$$R_{t\to T} = \beta - \frac{1}{T-t} \log \left(\frac{1}{\sqrt{s_t^{\gamma}(1-s_t)^{\gamma}}} \int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t} \right)^{iv} e^{(T-t)\mathbf{c}(-\gamma/2-iv,-\gamma/2+iv)} \Psi_\gamma(v)\, dv \right) \tag{B48}$$

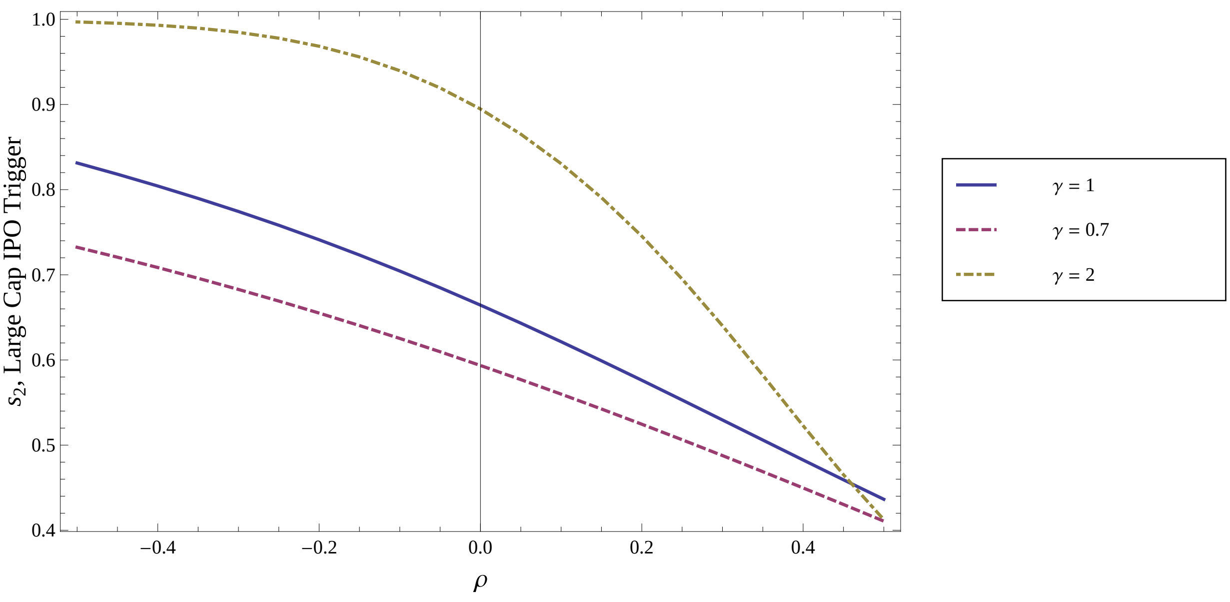
Instantaneous riskless rate $r_t = \lim_{T\to t} R_{t\to T}$ is obtained using L'Hopital rule:

$$r_t = \beta - \frac{\int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t}\right)^{iv} \mathbf{c}(-\gamma/2 - iv, -\gamma/2 + iv)\Psi_\gamma(v)\, dv}{\int_{-\infty}^{\infty} \left(\frac{1-s_t}{s_t}\right)^{iv} \Psi_\gamma(v)\, dv} \tag{B49}$$

By definition $(2\cosh(u/2))^{\gamma} \int_{-\infty}^{\infty} e^{iuv}\Psi_\gamma(v)\, dv = 1$. Using this fact and regrouping terms we finally obtain expression (B31).



(a) Small-cap IPO trigger versus ρ



(b) Large-cap IPO trigger versus ρ

Figure 1: Optimal IPO triggers with respect to the correlation between both dividend payoffs, ρ. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25\sqrt{1-\rho^2}$, $\sigma_{12} = 0.25\rho$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$ and $\alpha = 0.05$.

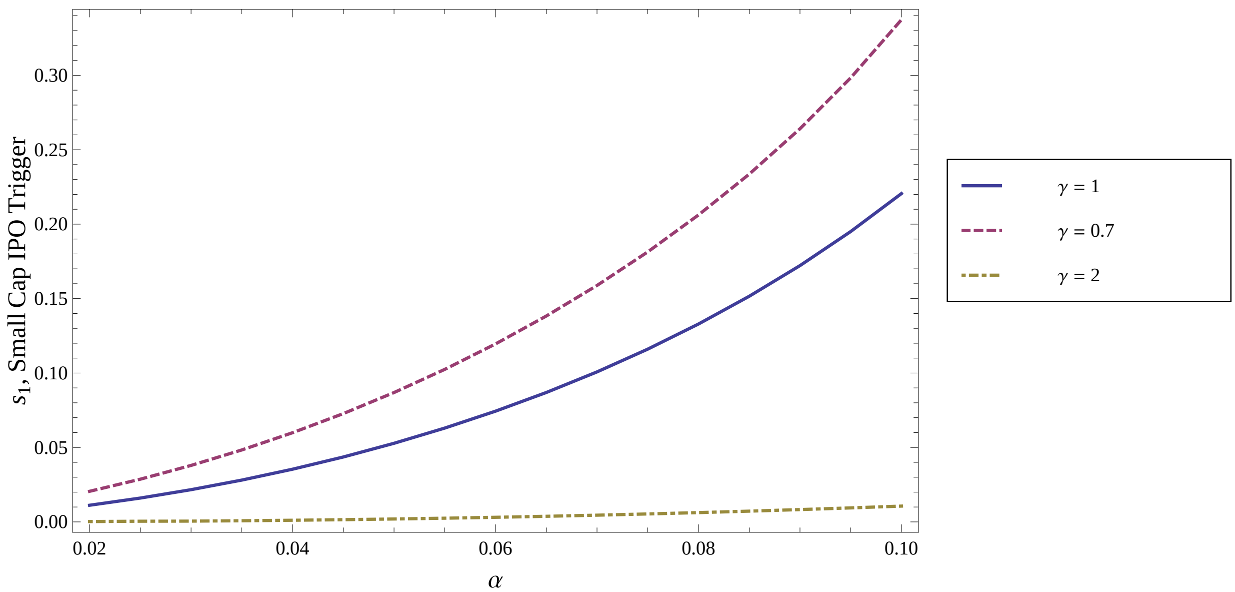


Figure 2: Small-cap trigger with respect to the fixed transaction costs α. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25$, $\sigma_{12} = 0$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$ and $\rho = 0$.

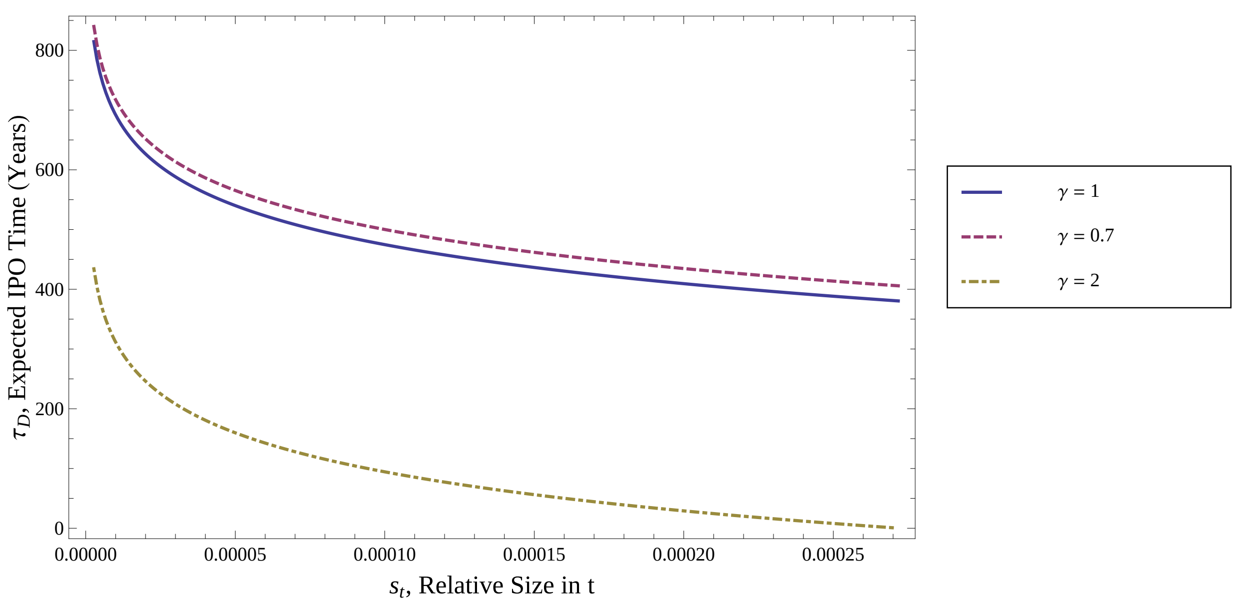


Figure 3: Expected IPO time as a function of the initial relative dividend. The parameters used are $\mu_{D_1} = 0.07$, $\mu_{D_2} = 0.04$, $\sigma_{11} = 0.25$, $\sigma_{12} = 0$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$, $\rho = 0$ and $\alpha = 0.05$.

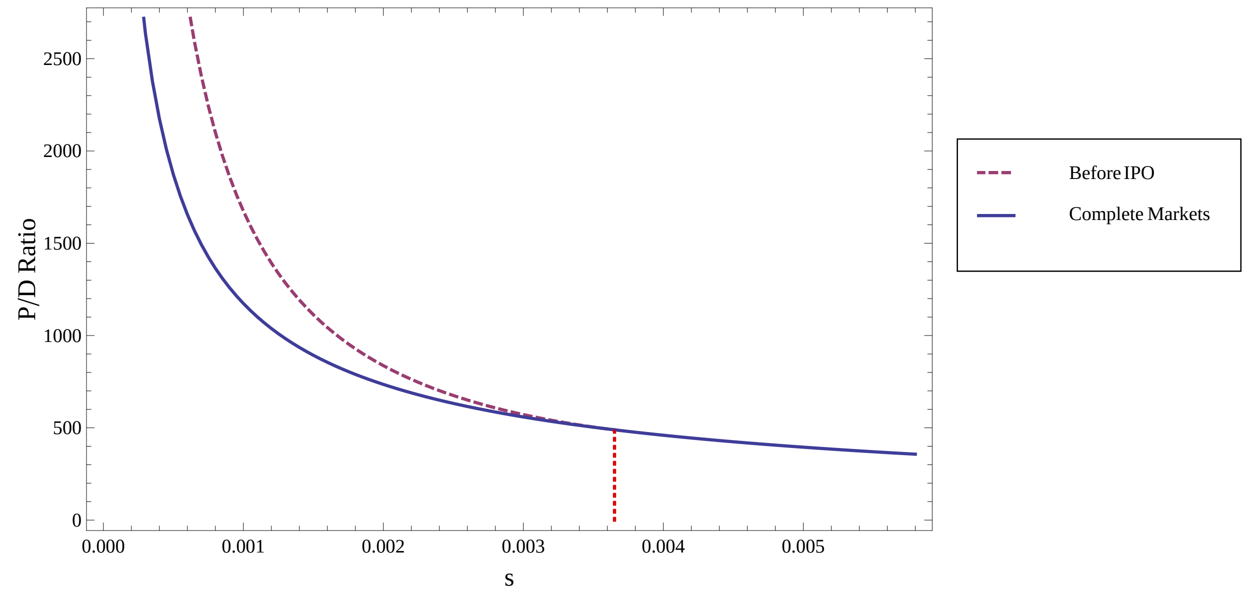


Figure 4: Price/dividend ratios for firm 1. The dashed line shows the price/dividend ratio before IPO, the solid line the ratio after IPO and the red dashed line shows the optimal IPO trigger. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25$, $\sigma_{12} = 0$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$, $\alpha = 0.05$, $\rho = 0$ and $\gamma = 2$.

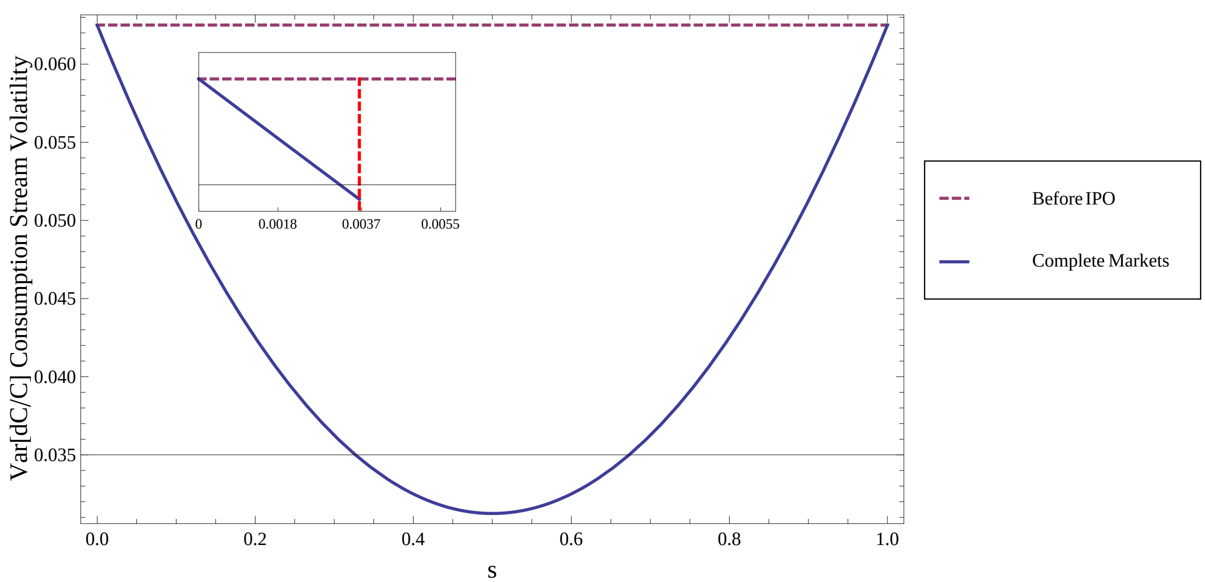


Figure 5: Consumption volatility as a function of the relative dividend. The dashed line shows the consumption volatility before IPO, the solid line the consumption volatility after IPO and the red dashed line shows the optimal IPO trigger. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25$, $\sigma_{12} = 0$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$, $\alpha = 0.05$, $\rho = 0$ and $\gamma = 2$.

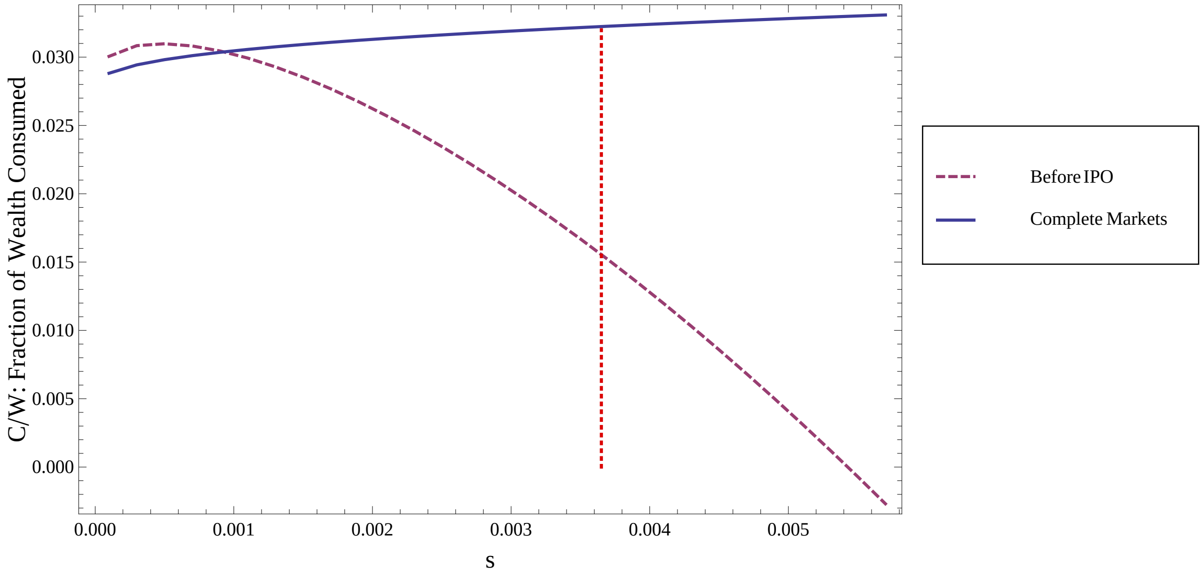


Figure 6: Consumption/wealth ratio for the entrepreneur as a function of the relative dividend. The dashed line shows the consumption/wealth ratio before IPO, the solid line the ratio after IPO and the red dashed line shows the optimal IPO trigger. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25$, $\sigma_{12} = 0$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$, $\alpha = 0.05$, $\rho = 0$ and $\gamma = 2$.

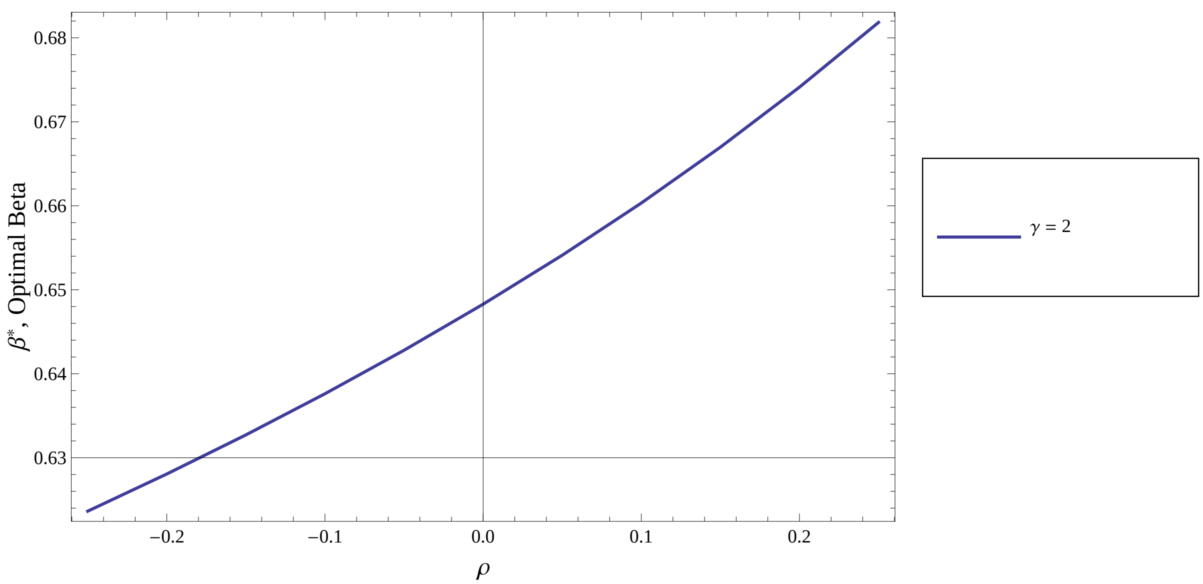


Figure 7: Beta of firm 1 at optimal IPO time against correlation between dividend payoffs. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25\sqrt{1-\rho^2}$, $\sigma_{12} = 0.25\rho$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$, $\alpha = 0.05$ and $\gamma = 2$.

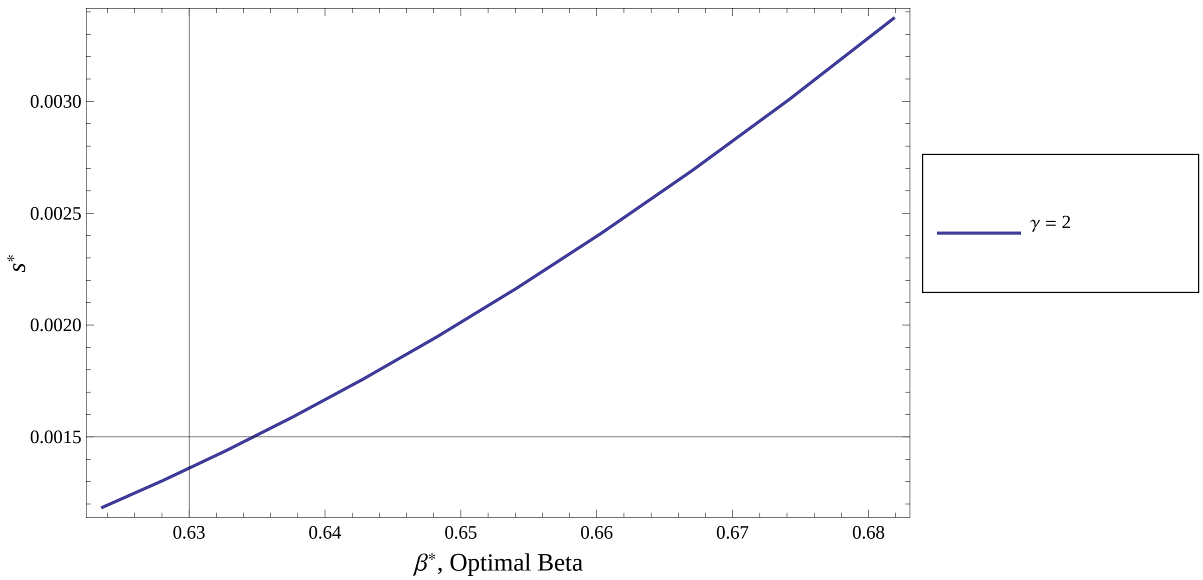


Figure 8: Small-cap trigger against the beta of firm 1. Both variables are monotonic functions of the correlation ρ. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25\sqrt{1-\rho^2}$, $\sigma_{12} = 0.25\rho$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$, $\alpha = 0.05$ and $\gamma = 2$.

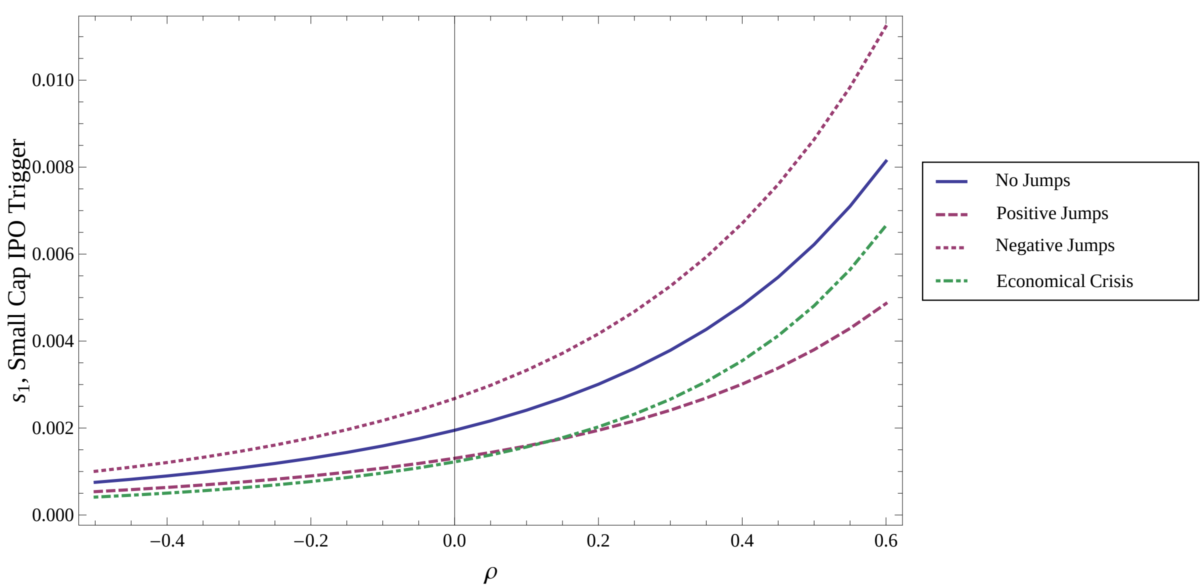


Figure 9: Small-cal IPO trigger with respect to the correlation between both dividend payoffs, ρ, when we allow for jumps in the dividend processes. The parameters used are $\mu_{D_1} = \mu_{D_2} = 0.05$, $\sigma_{11} = 0.25\sqrt{1-\rho^2}$, $\sigma_{12} = 0.25\rho$, $\sigma_{21} = 0$, $\sigma_{22} = 0.25$, $\beta = 0.04$ and $\alpha = 0.05$. The parameters of the jump processes are described in Section 3.5.