Uncertainty and Competition in the Adoption of

Complementary Technologies

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Abstract

We study the combined effects of uncertainty, competition and "technological complementarity" on firms' investment behaviour in a leader/follower pre-emption investment game. Our results contradict the conventional wisdom which says that "when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously". We found that when competition and uncertainty are considered, this is very unlikely to be the case for the leader and mixed strategies are possible for the follower. Some of the illustrated results show nonlinear and complex investment criteria and significant differences between the leader's and the follower's investment behavior.

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Key Words: Real Options, Uncertainty, Pre-emption, Duopoly Games, Technological Complementarity.

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1. Introduction

Since the pioneering work of Smets (1993), the effect of uncertainty and competition on investment behavior in a duopoly has been extensively studied in the real options literature[2], but the influence of the degree of complementarity between the inputs of an investment on firms' investment decisions has been neglected. However, firms often use inputs whose qualities are complements, such as computer and modem, equipment and structure, train and track, and transmitter and receiver. In such cases, investment decisions on upgrades or replacements must consider the degree of complementarity between investments. In this paper, "complementarity" exists if the adoption of one technology increases the marginal or incremental return to other technology in terms of cost savings. More generally, in the context of industrial organization, complementarity exists if the implementation of one practice increases the marginal return to other practice (Carree et al., 2010). When the implementation of a technology/practice decreases the marginal return to the other technology/practice, there is "substitutability" (or subadditivity)[3].

The concept of complementarity has been used to study economic decisions in many contexts. In the context of a country, it is used to set innovation policies, for instance, the optimization of the balance between technology imports and in-house R&D (Braga and Wilmore, 1991) and (Cassiman and Veugelers, 2004), the allocation of financial resources to industries (Mohnen and Roller, 2000), to enhance innovation and/or to favor clustering (Anderson and Schmittlein, 1984), and to define production policies, for instance, the coordination between product and process innovation (Miravete and Pernías, 1998).

R&D is an area where the concept of complementarity plays an important role, since when planning their R&D activities, firms make strategic decisions regarding the degree of complementarity (sometimes called compatibility) between the new products they aim to launch in the future and the complement products that are already available in the market and those they conjecture will be launched by their opponents in the future, in the sense that the diffusion of an innovation depends, to some extent, on the diffusion of complement innovations which amplify its value[4]. It has been

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[2] Dixit and Pindyck (1994), chapter 9, Grenadier (1996), Lambrecht and Perraudin (1997), Huisman (2001), Weeds (2002) and Paxson and Pinto (2005), Pawlina and Kort (2006) address such problems.

[3] See Carree et al. (2010) for further details on this issue.

[4] Note that, in R&D contexts, firms who do not have a dominant market position and want to growth quickly tend to guide their R&D efforts in order to launch new products that are compatible with those from their opponents who have a dominant market positions; firms who have dominant market positions tend to guide their R&D efforts in order to launch new products that are, as much as possible, not complements (compatible) with rivals. An example of the later strategy is the nine-year battle between the European Union (EU) commission and Microsoft that culminated in October 2007 with a fine of €497 million due to its supposed misconduct in developing software that does not allow open-source software developers access to

also argued that the pace of modernization of an industry is quite often influenced by the degree of technological complementarity between the new technologies adopted in that industry[5].

The concept of complementarity between economic activities (sometimes referred in the literature as synergy) plays also an important role in mergers and acquisitions since these are guided by the level of complementarity between firms' businesses processes, technologies, IT applications, clients, geographic location, etc. The merger between Air France and KLM and the acquisition of Abbey by Santander, in 2004, are two good examples of the importance of the complementary concept. In the former case, both firms justified the merger on the strong complementarity between their businesses in terms of the optimization of networks based on two powerful hubs, the possibility of using a more effective redeployment of passenger and cargo activities and expanding the offer of aircraft maintenance services, and the existence of cost savings in purchasing, sales distribution and IT applications; in the latter, Santander justified the acquisition of Abbey based on similar arguments and emphasizing the fact that, apart from other important business complementarities, the existence of a strong complementarity between the two banks IT applications was very important in the outcome of its decision given that it facilitates the integration of the two banks businesses[6].

Examples of relevant contributions to the literature around the concept of "complementarity" and its use in economic analyses are Milgrom and Roberts (1990, 1995), who use the theories of supermodular optimization and games as a framework for the analysis of systems marked by complementarity; Milgrom and Roberts (1994), who study the Japanese economy between 1940 and 1995 to interpret the characteristic features of Japanese economic organization in terms of the complementarity between some of the most important elements of its economic structure; and Colombo and Mosconi (1995), who examine the diffusion of flexible automation production and design/engineering technologies in the Italian metalworking industry, giving particular attention to the role of the technological complementarity and the learning effects associated with the experience of previously available technologies.

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inter-operability information for work-group servers used by businesses and other big organizations (see Etro (2007), p. 221, and Financial Times, October 23, 2007, p. 1).

[5] Smith and Weil (2005) investigated how changes in retailing and manufacturing industries, together, affected the diffusion of new information technologies in the U.S. apparel industry between 1988 and 1992, and suggest that there is a significant effect of the complementarity between new technologies on the pace of modernization of interlinked industries.

[6] For detailed information about this and other merger and acquisitions in EU see the "European Foundation for the Improvement of Living and Working Conditions" website: http://www.eurofound.europa.eu/.

Conventional wisdom says that "when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously", i.e., when raising the quality of one input it should upgrade its complements at the same time (Javanovic and Stolyarov, 2000, p. 15). From Milgrom and Robert (1990, 1995) models, we infer that it is relatively unprofitable to adopt only one part of the modern manufacturing strategy. In Milgrom and Roberts (1990, p. 524), it is suggested that "we should not see an extended period of time during which there are substantial volumes of both highly flexible and highly specialized (i.e., non-complementary) equipment being used side-by-side". Cho and McCardle (2009) show that the economic dependence that inherently defines cost relationships inside the firm can significantly influence the timing of adoption, by expediting or delaying the adoption of an improved technology.

However, the conclusions above have been made for contexts where uncertainty and competition are ignored. We study the effect of the complementarity between two technologies on their optimal time of adoption, considering competition between (two) firms and uncertainty about revenues and investment costs. Smith (2005) studies a similar problem but neglects competition.

Our initial intuition is that when uncertainty or drift differences about the investment cost of the technologies is considered, the conventional wisdom stated above may not hold, since due to technological progress the cost of a technology can decline rapidly. When firms anticipate that the cost of technologies may not fall at the same rate, it may pay to adopt first the technology whose cost is falling more slowly and wait to adopt the technologies whose cost is falling more rapidly. The manufacturing industry is by nature a sector where the concept of technological (or performance) complementarity applies to and where some of our results can be empirically tested. Azevedo and Paxson (2008) use empirical evidence from two firms from the Portuguese textile industry, whose production activities (units) have strong efficiency complementarity, to show some of the results highlighted in this research.

In our model, the word "complementarity" between the two technologies means the degree to which two technologies are better off when operating together rather than operating alone; γ_{12} in inequality $\gamma_{12} > \gamma_1 + \gamma_2$, is the parameter that represents the degree of complementarity between the two technologies, where, γ_1 and γ_2 are defined as the proportion of the firm's revenues that are expected to be saved if *tech 1* and *tech 2*, respectively, are adopted alone (i.e., firms operate with one technology, *tech 1* or *tech 2*), and γ_{12} is the proportion of the firm's revenues that are expected to be saved if both technologies are adopted together (i.e., firms operate with the two technologies at the same time).

There are econometric techniques to test/estimate complementarity between industrial organization practices, namely the “adoption” or “correlation” approach and the “production function” approach (Carree et al., 2010). Detailed descriptions of the techniques and empirical examples of the concept of “complementarity” can be found in Arora and Gambardella (1990), who suggest a test for complementarity, and Arora (1996), Athey and Stern (1998), and Miravete and Pernías (1998, 2010).

We use a real options methodology to derive, for a duopoly market with a first-mover advantage, analytical expressions for the value functions of the leader and the follower and their respective investment threshold values. We assume that the market is composed of two idle firms[7]; at the beginning of the investment game there are two new (complementary) technologies available, *tech 1* and *tech 2*; firms are allowed to invest twice; firms’ cost savings are a proportion of the firms’ revenues; and both the revenues and the cost of each technology are uncertain, following independent, and possible correlated, geometric Brownian motion (gBm) processes.

The rest of this paper is organized as follows. In section 2, we outline the model assumptions and define the duopoly investment game. In section 3, we derive the firms’ value functions and their investment threshold values. Section 4 presents the results. Section 5 concludes and offers some guidelines for possible extensions of this research.

[7] In this paper, an idle firm means a firm which is inactive or that it is active but operating without the most recent technology. For instance, a firm operating with an old rail train with old tracks is idle in not yet adopting high-speed trains and new tracks, if available.

2. The Investment Game

In Figure 1 we represent the investment game using an extensive-form representation[8].

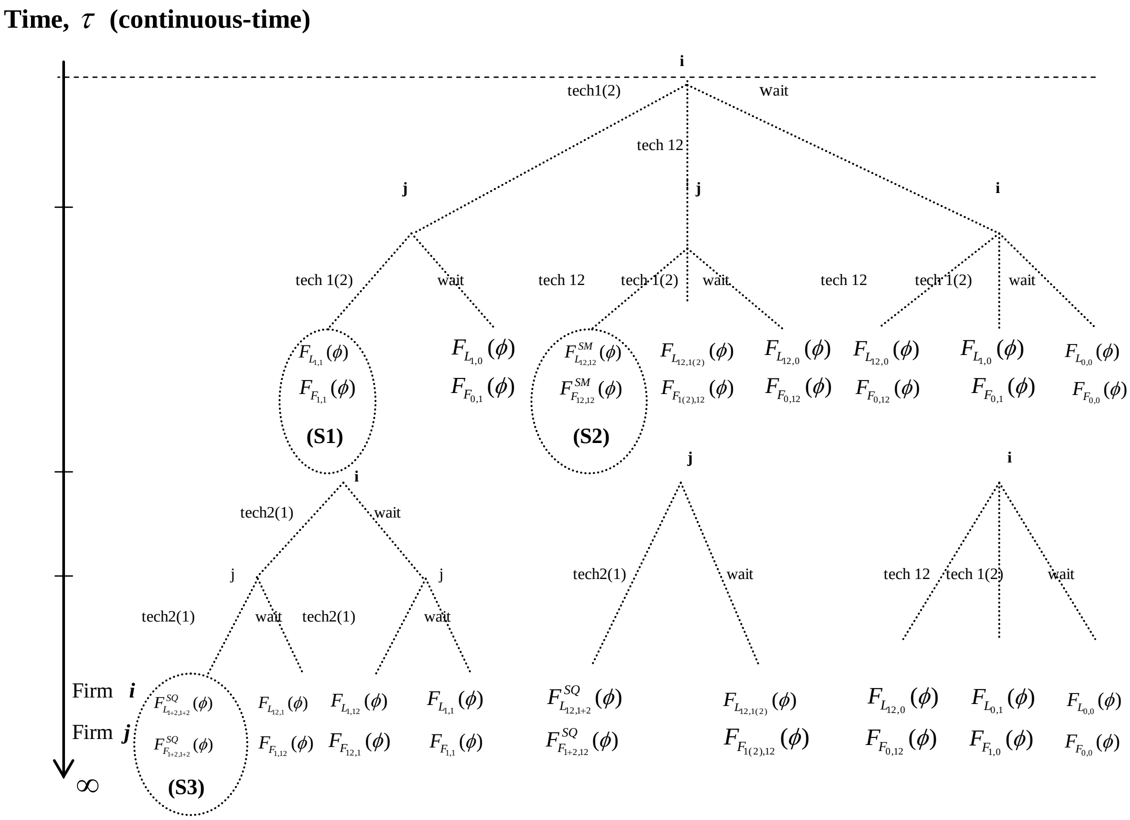

Figure 1 - Extensive-form Representation of a Continuous-time Real Option Game (CTROG) with Two Firms and Two Complementary Technologies.

Regarding the notation above, $F_{i_{k,k}}(\phi)$, represents the firm's value function for a particular investment game scenario, where $i=\{L,F\}$, with "L" and "F" meaning "leader" and "follower", respectively, $k=\{1,2,12\}$, with "1", "2" and "12" representing, respectively, the case where the firm operates with "technology 1 alone", "technology 2 alone" or with "technology 1 and 2 at the same time"; ϕ_k is the ratio "market revenue" (X) over the cost (I) of technology k, $\phi_k = X / I_k$ [9]; the superscripts "SM" and "SQ" on some of the value functions mean "simultaneous investment"

[8] For a detailed description of this type of game representation see Gibbons (1992). For an extensive literature review on real option games, with detailed descriptions about the game theory concepts and how to combine them with the real option framework, see Azevedo and Paxson (2010).
[9] In the game-tree we drop the subscript *k* for simplicity of notation.

(i.e., investment on tech 1 and tech 2 at the same time) and "sequential investment" (i.e., investment on tech 1 and tech 2 sequentially), respectively[10].

In section 3 we derive the firms' value functions and investment thresholds for the scenarios identified in Figure 1 as (S1), (S2) and (S3). Below we characterize these investment scenarios.

***Scenario 1* (S1):** firm *i* adopts first *tech 1(2)* and becomes the leader, firm *j* adopts later *tech 1(2)*, and becomes the follower. The payoffs for firm *i* and *j* are given, respectively, by $F_{L_{1,1}}(\phi)$ and $F_{F_{1,1}}(\phi)$. ***Scenario 2* (S2):** firm *i* adopts first *tech 1* and *tech 2* (*tech 12*) simultaneously, and firm *j* does the same later. Firm *i* becomes the leader and firm *j* the follower and their payoffs are, respectively, $F_{L_{12,12}}^{SM}(\phi)$ and $F_{F_{12,12}}^{SM}(\phi)$. ***Scenario 3* (S3):** in the first two rounds of the game, firms *i* and *j* adopt *tech 1* or *tech 2* (*tech 1(2)*). Firm *i* adopts first (first round) and becomes the leader, firm *j* adopts second (second round) and becomes the follower. Then, at the third and fourth rounds of the game, both firms adopt the remaining technology available *tech 2(1)*, again, one after the other, firm *i* first and firm *j* second, and the firms' payoffs are given by $F_{L_{12,12}}^{SQ}(\phi)$ and $F_{F_{12,12}}^{SQ}(\phi)$, respectively for firm *i* and *j*.

In the next section we derive analytical expressions for the firms' value functions marked in Figure 1 with an ellipse (S1, S2 and S3). Figure 2 below is an illustration of the investment scenarios denoted in Figure 1 by (S1) and (S3), i.e., timelines for the investment thresholds of the leader and the follower, for the cases where the two technologies are adopted sequentially, first tech 1(2), (S1 in the game-tree), and then tech 2(1), (S3 in the game-tree).

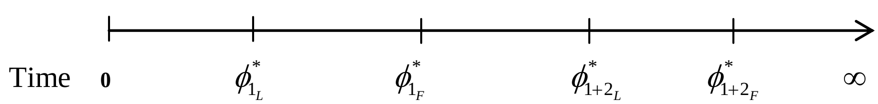

Figure 2 – Firms' Investment Thresholds when the Two Technologies are Adopted Sequentially.

[10] For instance, $F_{L_{1,1}}(\phi)$ and $F_{F_{1,1}}(\phi)$ represent, respectively, the value functions of the leader (L) and the follower (F) for the scenario where both firms operate with *tech 1*; $F_{L_{12,12}}^{SM}(\phi)$ and $F_{F_{12,12}}^{SM}(\phi)$ represent, respectively, the value function of the leader and the follower for the scenario where both firms adopted tech 1 and tech 2 simultaneously; $F_{L_{1+2,1+2}}^{SQ}(\phi)$ and $F_{F_{1+2,1+2}}^{SQ}(\phi)$ represent, respectively, the value function of the leader and the follower for the scenario where both firms adopted tech 1 and tech 2 sequentially, first tech 1 and then tech 2. Similar rationale applies to the notation used for the rest of the value functions in the game-tree.

$\phi^*_{1_L}$ represents the leader's investment threshold to adopt *tech 1*, given that none of the technologies have been adopted; $\phi^*_{1_F}$ denotes the follower's investment threshold to adopt *tech 1*, when the leader is operating with *tech 1* and the follower is not yet in the market; $\phi^*_{1+2_L}$ is the leader's investment threshold to adopt *tech 2* given that *tech 1* is in place; and $\phi^*_{1+2_F}$ is the follower's investment threshold to adopt *tech 2* given that it has adopted *tech 1* and the leader is already operating with both *tech 1* and *tech 2*.

Figure 3 represents the firms' investment threshold for the scenario where at the beginning of the investment game none of the technologies have been adopted and the two firms, one after the other, adopt the two technologies simultaneously (S2 in the game-tree); $\phi^*_{12_L}$ and $\phi^*_{12_F}$ represent, respectively, the leader's and the follower's investment thresholds.

Figure 3 – Firms' Investment Thresholds when the Two Technologies are Adopted Simultaneously.

Table 1 is a summary of the investment thresholds.

Firms' Investment Trigger Values	**The Adoption* of *Tech 1* or *Tech 2* alone**		**Sequential Adoption** (*tech 1*/*tech 2*)**	**Simultaneous Adoption (*tech 1* + *tech 2*)**
Leader	$\phi^*_{1_L}$ Equation (28) (**S1**)	$\phi^*_{2_L}$ Equation (28) (**S1**)	$\phi^*_{1+2_L}$ Equation (16) (**S3**)	$\phi^*_{12_L}$ Equation (32) (**S2**)
Follower	$\phi^*_{1_F}$ Equation (24) (**S1**)	$\phi^*_{2_F}$ Equation (24) (**S1**)	$\phi^*_{1+2_F}$ Equation (11) (**S3**)	$\phi^*_{12_F}$ Equation (30) (**S2**)

***** The mathematical expressions for the firms' investment threshold to adopt *tech 1* and *tech 2* alone are exactly the same, only the subscripts (1, 2) change.
****** In the derivation of these expressions we assumed firms adopt *tech 1* first and afterwards *tech 2.* However, nothing would change if we had assumed the other way round, apart from reversing the subscripts (1, 2).
Note: the terms S1, S2 and S3 above represent the investment game scenarios identified in the game-tree, p. 6.

Table 1 - Investment Thresholds for the Scenarios where Firms Adopt the Two Technologies, Sequentially and Simultaneously.

Due to the high number of the investment scenarios available, to avoid unnecessary complexity and without any lost of insight, we focus our derivation and analyses only on the scenarios marked in Figure 1 with an ellipse, i.e., (S1), (S2) and (S3). In addition, we assume that firms are not allowed to invest at the same time, i.e., if that occurs one of the firms will become the leader by flipping a coin. Nevertheless, these constrains in our analyses do not impose any lost of insight because, the framework and the nature of the methodology used to derive the firms' value functions and respective investment thresholds for scenarios (S1), (S2) and (S3) are informative enough to infer the results for the other investment scenarios. Additional information about another investment game scenario is provided in the Appendix A, section 5.

2.1 The Pre-emption Game

In games of timing the adoption of new technologies, the potential advantage of being the first to adopt may introduce an incentive for pre-empting the rival, speeding up the first adoption. Fudenberg and Tirole (1985) studied the adoption of a new technology and illustrate the effects of pre-emption in games of timing. We use their concept of pre-emption to derive the firms' value functions and investment thresholds.

3. The Model

In a risk-neutral world, at the beginning of the investment game, there are two new (complementary) technologies available, *tech 1* and *tech 2*, and two idle firms, *i* and *j*, which are considering the adoption of the two technologies, one after the other or both simultaneously depending on which one of these choices is the best.

The firms' cost savings flow is given by the following expression:

$$\gamma_k X(t)\left[ds_{k_i k_j}\right] \tag{1}$$

where, γ_k represents the proportion of a firm's revenues that is expected to be saved through the adoption of technology *k*, with $k = \{0,1,2,12\}$, where 0 means that firm is not yet active and 1, 2 and 12 mean that firm operates with *tech 1* only, *tech 2* only or *tech 1* and *tech 2* at the same time, respectively; $X(t)$ is the total market revenue flow; $\left[ds_{k_i k_j}\right]$ is a competition (deterministic) factor that ensures a first-mover market share (revenue) advantage, with $i, j = \{L, F\}$, where *L* means "leader" and *F* "follower", and represents the "proportion of the total market revenues" [11] that is held by each firm for each investment scenario. The relationship between these competition factors is governed by inequality (2).

[11] Suppose that by adopting a new technology a firm can get a 10% reduction in its operating costs per unit. Hence, within a certain production range, the more it produces/sales the more it saves due to the adoption of the technology.

The intuition used to justify the first-mover "market share advantage" is similar to that used by Dixit and Pindyck (1994), following Smets (1993). Implicitly we also assume that firms are symmetric in their ability to operate with the new technologies and that spillover information is not allowed, i.e., firms' "first-mover market share advantage" holds forever. In addition, the exit strategy is no allowed.

Consequently, for the leader, inequality (2) holds:

$$\left[ds_{12_L 0_F} = ds_{1_L 0_F} = ds_{2_L 0_F}\right] > ds_{12_L 1_F} > ds_{12_L 12_F} > \left[ds_{1_L 1_F} = ds_{2_L 2_F}\right] \tag{2}$$

As in a duopoly, the market share of the follower is a complement of the leader's, i.e., $ds_{k_F k_L} = 1 - ds_{k_L k_F}$, with $k = \{1, 2, 12\}$, inequality (3) holds for the follower:

$$\left[ds_{1_F 1_L} = ds_{2_F 2_L}\right] > ds_{12_F 12_L} > ds_{1_F 12_L} > \left[ds_{0_F 12_L} = ds_{0_F 1_L} = ds_{0_F 2_L}\right] \tag{3}$$

The economic interpretation for inequality (2) is the following: for firm *L* (the leader), the best investment scenario, in terms of market share, is when it is active with either *tech 1* or *tech 2*, alone, or with both technologies at the same time, and its rival firm *F* (the follower), is inactive $\left[ds_{12_L 0_F} = ds_{1_L 0_F} = ds_{2_L 0_F}\right]$[12]; its second best investment scenario is when it adopts both technologies first, and its rival adopts later only *tech 1* ($ds_{12_L 1_F}$); its third best investment scenario is when both firms adopt both technologies but the leader does so earlier ($ds_{12_L 12_F}$); its fourth best investment scenario is when both firms adopt one technology, *tech 1* or *tech 2*, but the leader does so earlier $\left[ds_{1_L 1_F} = ds_{2_L 2_F}\right]$. It is implicitly assumed that ($ds_{0_L 0_F} = ds_{0_L 0_F} = 0$), i.e., when both firms are inactive their payoff is zero[13]. Similar rational applies to the follower's inequality. A practical illustration about how these factors work in practice and their influence on the determination of the firms' value functions and the equilibrium of the game is given in Appendix B, sections 3 and 4.

[12] We assume that tech 1 and tech 2 are symmetric. Hence, $ds_{1_L 0_F} = ds_{2_L 0_F}$; $ds_{2_L 2_F} = ds_{1_L 1_F}$, i.e., the leader's first-mover market share advantage is the same regardless of the technology chosen. Our framework allows however the use of different assumptions in this regard.

[13] Note that, by assumption, the leader is the firm who adopts first. Hence, the scenario in which the follower is active and the leader is inactive is not considered.

In addition, we assume that total market revenues, $X(t)$, follow a geometric Brownian motion given by the following equation:

$$dX = \mu_X Xdt + \sigma_X Xdz_X \tag{4}$$

where, μ_X is the trend rate of growth of market revenues, σ_X is the volatility of the market revenues and dz_X is the increment of a standard Wiener process.

We consider that *tech 1* alone provides a net cost savings, S_1, that is a fraction, γ_1, of the firm's market revenues, $X\left[ds_{k_i k_j}\right]$:

$$S_1 = \gamma_1 X\left[ds_{k_i k_j}\right] \tag{5}$$

Since the firms' cost savings are proportional to revenues and revenues follow a gBm process, so firms' cost savings also follows a gBm process.

Similarly, the use of *tech 2* alone provides a cost savings equal to:

$$S_2 = \gamma_2 X\left[ds_{k_i k_j}\right] \tag{6}$$

And the simultaneous use of both technologies yields cost savings equal to:

$$S = \gamma_{12} X\left[ds_{k_i k_j}\right] \tag{7}$$

The technological complementarity between the two technologies is given by the following inequality:

$$\gamma_{12} > \gamma_1 + \gamma_2 \tag{8}$$

In practice, γ_1 and γ_2 are technology/product-specific, independent, and not necessarily correlated. For instance, technologies used to produce multi products/services may have different degrees of complementarity regarding each of the product/service. This fact explains why firms with huge fixed assets, manufacturing sector for instance, tend to guide their R&D policies to take advantage of the assets (technologies) in place.

Furthermore, we assume that the costs of adopting *tech 1* and *tech 2*, respectively, I_1 and I_2, follow gBm processes as well, given by:

$$dI_1 = \mu_{I_1} I_1 dt + \sigma_{I_1} I_1 dz_{I_1} \tag{9}$$

and

$$dI_2 = \mu_{I_2} I_2 dt + \sigma_{I_2} I_2 dz_{I_2} \tag{10}$$

where, μ_{I_1} and μ_{I_2} are the trend rates of growth of the cost of *tech 1* and *tech 2*, respectively; σ_{I_1} and σ_{I_2} are the volatility of the cost of *tech 1* and *tech 2*, respectively; and dz_{I_1} and dz_{I_2} are the increments of the standard Wiener processes for the costs *tech 1* and *tech 2*, respectively. For convergence reasons $r - \mu_X - \mu_k > 0$ holds.

In some cases, correlation between "revenues" and "cost of tech 1(2)" and between "cost of tech 1" and "cost of tech 2" are possible. This fact is considered in our model, Equations A2, A6, A9, 17, 18 and 22.

3.1 Technology 1 is in place

3.1.1 The Follower's Value Function

In this section we derive the follower's value function, $F^{SQ}_{F_{1+2,1+2}}(\phi_2)$, and investment threshold, $\phi^*_{1+2_F}$, to adopt *tech 2* assuming that *tech 1* is in place. Below are the results. See Appendix A, section 1, pp. 27-30, for a detailed derivation.

The follower's investment threshold, $\phi^*_{1+2_F}$, is given by equation (11):

$$\phi^*_{1+2_F} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_2})}{(\gamma_{12} - \gamma_1)\left[ds_{12_F 12_L}\right] - \gamma_1\left[ds_{1_F 12_L}\right]} \tag{11}$$

With A_{1+2} given by equation (12):

$$A_{1+2} = \frac{\left(\phi^*_{1+2_F}\right)^{-\beta_1}}{\beta_1 - 1} \frac{(\gamma_{12} - \gamma_1)\left[ds_{12_F 12_L}\right] - \gamma_1\left[ds_{1_F 12_L}\right]}{r - \mu_X - \mu_{I_2}} \tag{12}$$

And the follower's value function is given by equation (13):

$$F^{SQ}_{F_{1+2,1+2}}(\phi_2) = \begin{cases} \dfrac{\gamma_1 X\left[ds_{1_F 12_L}\right]}{r - \mu_X} + A_{1+2}\left(\dfrac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} I_2 & \phi_2 < \phi^*_{1+2_F} \\ \dfrac{\gamma_{12} X\left[ds_{12_F 12_L}\right]}{r - \mu_X} - I^*_{2_F} & \phi_2 \geq \phi^*_{1+2_F} \end{cases} \tag{13}$$

Scenario (S3) in the game-tree, p. 6.

Equation (13) tells us that for the follower, before $\phi^*_{1+2_F}$ is reached, its value, when it adopts the two technologies sequentially, is given by the value of operating with *tech 1* forever, $\frac{\gamma_1 X\left[ds_{1_F 12_L}\right]}{r - \mu_X}$, plus its option to adopt *tech 2*, $A_{1+2}\left(\frac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} I_2$; as soon as $\phi^*_{1+2_F}$ is reached and it adopts tech 2, its value is equal to the present value, in perpetuity, of the cost savings obtained from operating with both technologies from $\phi^*_{1+2_F}$ until infinity, $\frac{\gamma_{12} X\left[ds_{12_F 12_L}\right]}{r - \mu_X} - I^*_{2_F}$.

3.1.2 The Leader's Value Function

Assuming that both firms are operating with *tech 1* and that the follower will adopt *tech 2* at $\phi^*_{1+2_F}$ (derived above), the leader's value function is described by the following expression:

$$E\left[\int_{t=0}^{(T_{2_F} - T_{2_L})} \gamma_{12} X_\tau \left[ds_{12_L 1_F}\right] e^{-r\tau} d\tau - I^*_{2_L} e^{-rT_{2_L}} + \int_{T_{2_F}}^{\infty} \gamma_{12} X_\tau \left[ds_{12_L 12_F}\right] e^{-r\tau} d\tau\right] \tag{14}$$

where, the first integral represents the leader's cost savings in the period where it operates with the two technologies and the follower operates with *tech 1*; the second integral represents the leader's cost savings for the period where both firms are operating with the two technologies, *tech 1* and *tech 2*; $I^*_{2_L}$ is the cost of *tech 2* at the leader's adoption time.

Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the following expression for the leader's value function:

$$F_{L_{1+2,1+2}}^{SQ}(\phi_2)=\begin{cases}\dfrac{\gamma_{12}X\left[ds_{12_L1_F}\right]}{r-\mu_X}+\dfrac{\beta_1}{\beta_1-1}\left(\dfrac{\phi_2}{\phi_{1+2_F}^*}\right)^{\beta_1}\gamma_{12}\left[ds_{12_L12_F}-ds_{12_L1_F}\right]I_2 & \phi_2<\phi_{1+2_F}^* \\ \dfrac{\gamma_{12}X\left[ds_{12_L12_F}\right]}{r-\mu_X}-I_{2_L}^* & \phi_2\geq\phi_{1+2_F}^*\end{cases} \tag{15}$$

Scenario (S3) in the game-tree, p. 6.

Expression $\dfrac{\gamma_{12}X\left[ds_{12_L1_F}\right]}{r-\mu_X}$ corresponds to the leader's total payoff if it operates alone with the two technologies forever; $\dfrac{\beta_1}{\beta_1-1}\left(\dfrac{\phi_2}{\phi_{1+2_F}^*}\right)^{\beta_1}\gamma_{12}\left[ds_{12_L12_F}-ds_{12_L1_F}\right]I_2$ is negative, given that $\left[ds_{12_L12_F}-ds_{12_L1_F}\right]<0$ (inequality 2, p. 10), and corresponds to the correction factor that incorporates the fact that in the future if $\phi_{1+2_F}^*$ is reached the follower will adopt *tech 2* and the leader's profits will be reduced. $\dfrac{\gamma_{12}X\left[ds_{12_L12_F}\right]}{r-\mu_X}-I_{2_L}^*$ is the leader's total payoff if it operates with the follower, both with the two technologies from $\phi_{1+2_F}^*$ until infinity.

There is no closed-form solution for the leader's investment threshold value. However, numerical methods can be used to solve to equation (16) for $\phi_{1+2_L}^*$. Equation (16) is derived by equalizing the value functions of the leader and the follower, for $\phi_2<\phi_{1+2_F}^*$.

$$\frac{\gamma_{12}X\left[ds_{12_L1_F}\right]}{r-\mu_X}-I_{2_L}^*+\frac{\beta_1}{\beta_1-1}\left(\frac{\phi_2}{\phi_{1+2_F}^*}\right)^{\beta_1}\gamma_{12}\left[ds_{12_L12_F}-ds_{12_L1_F}\right]I_2-\frac{\gamma_1X\left[ds_{1_F12_L}\right]}{r-\mu_X}+I_{1_F}^*-A_{12}\left(\frac{\phi_2}{\phi_{1+2_F}^*}\right)^{\beta_1}I_2=0 \tag{16}$$

3.2 None of the Technologies have been adopted

Now that we have the value of the implicit option on *tech 2* if *tech 1* has been adopted, we can analyse the first-stage decision to adopt *tech 1*. Similarly as we have done for the scenario where we assume that *tech 1* is in place, here we derive the firms' value functions and investment trigger values for the scenario where neither of the technologies has been adopted.

3.2.1 The Follower's Value Function

Let $F(X, I_1, I_2)$ be the value of the option to adopt either one or both technologies. Setting the return on the option rF equal to the expected capital gain on the option and using Ito's lemma, we obtain this differential equation for the region in which the firm waits to invest:

$$0 = \frac{1}{2}\left(\sigma_X^2 X^2 \frac{\partial^2 F}{\partial X^2} + \sigma_{I_1}^2 I_1^2 \frac{\partial^2 F}{\partial I_1^2} + \sigma_{I_2}^2 I_2^2 \frac{\partial^2 F}{\partial I_2^2} + 2\rho_{XI_1}\sigma_X \sigma_{I_1} X I_1 \frac{\partial^2 F}{\partial X \partial I_1} + 2\rho_{XI_2}\sigma_X \sigma_{I_2} X I_2 \frac{\partial^2 F}{\partial X \partial I_2} + \ldots \right.$$
$$\left. \ldots + 2\rho_{I_1 I_2}\sigma_{I_1}\sigma_{I_2} I_1 I_2 \frac{\partial^2 F}{\partial I_1 \partial I_2} \right) + \mu_X X \frac{\partial F}{\partial X} + \mu_{I_1} I_1 \frac{\partial F}{\partial I_1} + \mu_{I_2} I_2 \frac{\partial F}{\partial I_2} - rF \tag{17}$$

where, ρ_{XI_1} and ρ_{XI_2} are the correlation coefficients between market revenues and cost of *tech 1* and market revenues and cost of *tech 2*, respectively, and $\rho_{I_1 I_2}$ is the correlation coefficient between the cost of *tech 1* and the cost of *tech 2*.

In the region where the firm is waiting to adopt, this value can be separated into the value of the option to acquire *tech 1* plus the value of the option to acquire *tech 2* as well. Assuming first-order homogeneity, i.e., $F(X, I_1, I_2) = I_1 f_1(X/I_1) + I_2 f_{12}(X/I_2)$, the relevant partial derivatives yield:

$$0 = \left(\frac{1}{2}\sigma_{m_1}\phi_1^2 \frac{\partial^2 f_1(\phi_1)}{(\partial\phi_1)^2} + (\mu_X - \mu_{I_1})\frac{\partial f_1(\phi_1)}{\partial(\phi_1)} - (r - \mu_{I_1})\phi_1 f_1 \right) + \ldots$$
$$\ldots + \left(\frac{1}{2}\sigma_{m_2}\phi_2^2 \frac{\partial^2 f_{12}(\phi_2)}{(\partial\phi_2)^2} + (\mu_X - \mu_{I_2})\frac{\partial f_{12}(\phi_2)}{\partial(\phi_2)} - (r - \mu_{I_2})\phi_2 f_{12} \right) \tag{18}$$

where, $\sigma_{m_1}^2 = \sigma_X^2 + \sigma_{I_1}^2 - 2\rho_{XI_1}\sigma_X \sigma_{I_1}$ and $\sigma_{m_2}^2 = \sigma_X^2 + \sigma_{I_2}^2 - 2\rho_{XI_2}\sigma_X \sigma_{I_2}$.

In the region where the current value of the ratio "market revenues" over "cost of *tech 2*" is lower than the threshold to adopt *tech 2* if *tech 1* is already in place, i.e., in the region where $\phi_2 \le \phi_{1+2_F}^*$ (see Equation 11), the second bracketed expression is equal to zero, leaving this second-order linear differential equation equal to[14]:

[14] The rationale underlying this derivation is the following: the investment threshold to adopt *tech 2* if *tech 1* is in place is $\phi_{1+2_F}^*$, which, due to the effect of complementarity between *tech 1* and *tech 2* , is lower than the threshold to adopt *tech 2* alone, $\phi_{2_F}^*$. Hence, before $\phi_{1+2_F}^*$ is reached, it is not optimal to adopt *tech 2* alone, i.e., the option to adopt *tech 2* is worthless.

$$0 = \left(\frac{1}{2} \sigma_{m_1} \phi_1^{\ 2} \frac{\partial^2 f_1(\phi_1)}{(\partial \phi_1)^2} + (\mu_X - \mu_{I_1}) \frac{\partial f_1(\phi_1)}{\partial(\phi_1)} - (r - \mu_{I_1}) \phi_1 f_1 \right) \tag{19}$$

Therefore, the economically meaningful solution is:

$$f_1(\phi_1) = A_1(\phi_1)^{\beta_1} + B_1(\phi_1)^{\beta_2} \tag{20}$$

where, $\beta_{1(2)}$ is the characteristic quadratic function of the homogeneous part of equation (18), given by:

$$\frac{1}{2} \sigma_{m_1} \beta_1(\beta_1 - 1) + (\mu_X - \mu_{I_1}) \beta_1 - (r - \mu_{I_1}) = 0 \tag{21}$$

Solving the equation above for β_1 leads to:

$$\beta_1 = \frac{1}{2} - \frac{\mu_X - \mu_{I_1}}{\sigma_{m_1}^2} + \sqrt{\left(\frac{(\mu_X - \mu_{I_1})}{\sigma_{m_1}^2} - \frac{1}{2} \right)^2 + \frac{2(r - \mu_{I_1})}{\sigma_{m_1}^2}} \tag{22}$$

As the ratio revenues over the cost of tech 1, ϕ_1, approaches 0, the value of the option becomes worthless, so $B_1 = 0$. Using the "*value matching*" and the "*smooth pasting*" conditions at the threshold ratio, $\phi_{1_F}^*$, we obtain:

$$A_1 = \frac{\phi_{1_F}^{*\ -\beta_1}}{\beta_1 - 1} \frac{\left[ds_{1_F 1_L} \right] \gamma_1}{r - \mu_X - \mu_{I_1}} \tag{23}$$

$$\phi_{1_F}^* = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_1})}{\left[ds_{1_F 1_L} \right] \gamma_1} \tag{24}$$

$$F_{F_{1,1}}(\phi_1) = \begin{cases} \dfrac{I_1}{\beta_1 - 1} \left(\dfrac{\phi_1}{\phi_{1_F}^*} \right)^{\beta_1} \dfrac{\left[ds_{1_F 1_L} \right] \gamma_1}{r - \mu_X - \mu_{I_1}} & \phi_1 < \phi_{1_F}^* \\ \dfrac{\gamma_1 X \left[ds_{1_F 1_L} \right]}{r - \mu_X} - I_{1_F}^* & \phi_1 \geq \phi_{1_F}^* \end{cases} \tag{25}$$

Scenario (S1) in the game-tree, p. 6.

Since we did not differentiate the two technologies, the expressions for the case of *tech 2* are exactly the same as those derived above for the case of the adoption of *tech 1*. The only difference is the subscript used in the notation for the complementarity parameters and the competition factors, where the subscript "2" replaces "1".

Notice that $\phi^*_{1_F}$ is the follower's threshold for adopting *tech 1* alone; $\phi^*_{1+2_F}$ is the follower's threshold to adopt *tech 2* given that *tech 1* is in place. From Equations 24 and 11, which represent the thresholds above, respectively, we can see that when the two technologies are complements, the degree of complementarity does not affect the decision to adopt either technology by itself, but does reduce the threshold for adopting the other technology if one technology is adopted.

3.2.2 The Leader's Value Function

Focusing again on the adoption of *tech 1*, the leader's expected value is given by:

$$E\left[\int_{t=0}^{(T_{1_F}-T_{1_L})}\gamma_1 X_\tau\left[ds_{1_L0_F}\right]e^{-r\tau}d\tau - I^*_{1_L}e^{-rT_{1_L}} + \int_{T_{1_F}}^{\infty}\gamma_1 X_\tau\left[ds_{1_L1_F}\right]e^{-r\tau}d\tau\right] \tag{26}$$

The first integral represents the leader's payoff when alone in the market; the second integral represents the leader's payoff when operating with the follower, both with *tech 1*; $I^*_{1_L}$, is the cost of *tech 1* at the time of the adoption.

Applying similar procedures as those used in previous section (see pages 12 to 14 and appendix A, section 1), following the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the following expression for the leader's value function:

$$F_{L_{1,1}}(\phi_1) = \begin{cases} \dfrac{\gamma_1 X\left[ds_{1_L0_F}\right]}{r-\mu_X} + \dfrac{\beta_1}{\beta-1}\left(\dfrac{\phi_1}{\phi^*_{1_F}}\right)^{\beta_1}\left[ds_{1_L1_F} - ds_{1_L0_F}\right]I_1 & \phi_1 < \phi^*_{1_F} \\ \dfrac{\gamma_1 X\left[ds_{1_L1_F}\right]}{r-\mu_X} - I^*_{1_L} & \phi_1 \geq \phi^*_{1_F} \end{cases} \tag{27}$$

Scenario (S1) in the game-tree, p. 6.

Again, there is no closed-form solution for the leader's trigger value. However, numerical methods can be used to solve equation (28) for $\phi^*_{1_L}$. Equation (28) is obtained by equalizing the value functions of the leader and the follower, for $\phi_1 < \phi^*_{1_F}$.

$$\frac{\gamma_1 X\left[ds_{1_L 0_F}\right]}{r-\mu_X}+\frac{\beta_1}{\beta_1-1}\left(\frac{\phi_1}{\phi_{1_F}^*}\right)^{\beta_1}\left[ds_{1_L 1_F}-ds_{1_L 0_F}\right]I_1-\frac{I_1}{\beta_1-1}\left(\frac{\phi_1}{\phi_{1_F}^*}\right)^{\beta_1}\frac{\left[ds_{1_F 1_L}-ds_{0_F 1_L}\right]\gamma_1}{r-\mu_X-\mu_{I_1}}=0 \tag{28}$$

The procedure used to get this equation is the same as that used for Equation (16).

3.3 Simultaneous Adoption

Following similar procedures as those used in previous sections we get the expressions for the firms' value functions and investment threshold values for the case where the two technologies are adopted simultaneously. For simplicity of notation we use $I_1+I_2=I_{12}$.

3.3.1 The Follower's Value Function

$$F_{F_{12,12}}^{SM}(\phi_{12})=\begin{cases}\dfrac{I_{12}}{\beta_1-1}\left(\dfrac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1} & \phi_{12}<\phi_{12_F}^* \\ \dfrac{\gamma_{12}X\left[ds_{12_F 12_L}\right]}{r-\mu_X}-I_{12_F}^* & \phi_{12}\geq\phi_{12_F}^*\end{cases} \tag{29}$$

Scenario (S2) in the game-tree, p. 6.

Investment Trigger Value:

$$\phi_{12_F}^*=\frac{\beta_1}{\beta_1-1}\frac{(r-\mu_X-\mu_{I_{12}})}{\left[ds_{12_F 12_L}\right]\gamma_{12}} \tag{30}$$

3.3.2 The Leader's Value Function

$$F_{L_{12,12}}^{SM}(\phi_{12})=\begin{cases}\dfrac{\gamma_{12}X\left[ds_{12_L 0_F}\right]}{r-\mu_X}+\dfrac{\beta_1}{\beta_1-1}\left(\dfrac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1}\left[ds_{12_L 12_F}-ds_{12_L 0_F}\right]I_{12} & \phi_{12}<\phi_{12_F}^* \\ \dfrac{\gamma_{12}X\left[ds_{12_L 12_F}\right]}{r-\mu_X}-I_{12_L}^* & \phi_{12}\geq\phi_{12_F}^*\end{cases} \tag{31}$$

Scenario (S2) in the game-tree, p. 6.

Investment Trigger Value:

$$\frac{\gamma_{12}X\left[ds_{12_L 0_F}\right]}{r-\mu_X}-I_{12_L}^*+\frac{\beta_1}{\beta_1-1}\left(\frac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1}\left[ds_{12_L 12_F}-ds_{12_L 0_F}\right]I_{12}-\frac{I_{12}}{\beta_1-1}\left(\frac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1}=0 \tag{32}$$

3.4 Other Investment Scenarios

In the game-tree, Figure 1, p. 6, there are investment scenarios which we did not fully characterize in our derivations in section 3. Indeed, to avoid unnecessary complexity we focused only on the three investment scenarios marked with an ellipse. However, this does not lead to any lost of insight in our results/analyses, since the characterization of those three scenarios are informative enough to show that conventional wisdom does not always hold for contexts where uncertainty and competition hold. On the other hand, the characterization of the other investment scenarios can be easily done by following the same rationale and the technique used in section 3, as we exemplify in Appendix B, table B2, p. 35.

4. Results and Sensitivity Analysis

In this section we analyse the sensitivity of our real options model to changes in some of its most important parameters. To illustrate our analysis we use the following basic model inputs[15]: $X = 60$, $I_1 = 7.0$, $I_2 = 7.0$, $I^*_{1_L} = 6.0$, $I^*_{1_F} = 5.0$, $I^*_{2_L} = 6.0$, $I^*_{2_F} = 5.0$, $\mu_X = 0.05$, $\mu_{I_1} = -0.05$, $\mu_{I_2} = -0.10$, $\sigma_X = 0.4$, $\sigma_{I_1} = \sigma_{I_2} = \sigma_{I_{12}} = 0.20$, $r = 0.09$, $\rho_{XI_1} = \rho_{XI_2} = \rho_{XI_{12}} = 0$, $\gamma_1 = 0.10$, $\gamma_2 = 0.10$, $\gamma_{12} = 0.30$. The competition factors used are: $ds_{1_L 0_F} = ds_{2_L 0_F} = ds_{12_L 0_F} = 1.0$, $ds_{1_L 1_F} = ds_{2_L 2_F} = ds_{12_L 1_F} = 0.60$, $ds_{1_F 1_L} = ds_{2_F 2_L} = ds_{1_F 12_L} = 0.40$, $ds_{12_L 12_F} = 0.55$, $ds_{12_F 12_L} = 0.45$.

According to the inputs above *tech 1* and *tech 2* are symmetric except regarding their cost growth rates, the cost of *tech 1* is expected to fall at 5% per annum ($\mu_{I_1} = -0.05$) and the cost of *tech 2* is expected to fall at 10% per annum ($\mu_{I_2} = -0.10$).

Table 2 shows the results for investment scenarios S1, S2, S3. The variables $\Phi_1(t)$, $\Phi_2(t)$ and $\Phi_{12}(t)$ represent the current value of the ratios "revenues (X)/cost of tech 1 (I_1)", "revenues(X)/cost of tech 2(I_2)", and "revenues(X)/the sum of the costs of tech 1 and tech 2($I_1+I_2=I_{12}$)", respectively; Φ^*_{1L}, Φ^*_{2L} and Φ^*_{12L} are the leader's investment thresholds to adopt *tech 1* alone, *tech 2* alone and *tech 1 and tech 2* at the same time, respectively; and Φ^*_{1F}, Φ^*_{2F} and Φ^*_{12F} are the follower's investment thresholds to adopt *tech 1* alone, *tech 2* alone, and *tech 1* and *tech 2* at the same time, respectively.

[15] In our simulations we use inputs that are generous for the leader. This shows the features of the model in extreme conditions.

Firms’ investment thresholds depend on the evolution of two stochastic underlying variables, revenues (X) and investment cost (I_k, with k=1, 2 and 12). Therefore, these investment thresholds are defined by straight lines plotted in the space (X,I_k) whose slope is equal to the value of the respective threshold their represent, and where points on the straight lines or on the area above the straight lines represent possible combinations of the variables X and I_k that lead firms to adopt technology k, and points on the area below the straight lines represent possible combinations of the variables X and I_k that lead firms to defer the adoption of technology k. Hence, for each firm and investment scenario, the higher the investment threshold (i.e., the slope of the investment threshold line), the later is the adoption of technology k.

Current Values			Follower’s Thresholds					Leader’s Threshold				
			Idle Firm			Active Firm		Idle Firm			Active Firm	
						Tech 1 In place	Tech 2 In Place				Tech 1 In place	Tech 2 In Place
$\Phi_1(t)$	$\Phi_2(t)$	$\Phi_{12}(t)$	$\Phi^*_{1,F}$	$\Phi^*_{2,F}$	$\Phi^*_{12,F}$	$\Phi^*_{1+2,F}$	$\Phi^*_{2+1,F}$	$\Phi^*_{1,L}$	$\Phi^*_{2,L}$	$\Phi^*_{12,L}$	$\Phi^*_{1+2,L}$	$\Phi^*_{2+1,L}$
8.57	**8.57**	**4.29**	14.53	26.70	8.34	16.67	8.58	0.92	1.44	6.77	0.33	0.17
Investment decision:			**wait**	**wait**	**wait**	**wait**	**invest**	**Invest**	**invest**	**wait**	**invest**	**invest**

Table 2 – Firms’ Investment Thresholds

In Table 2 we can see that the investment thresholds for an idle leader and follower, for the scenarios where *tech 1* is adopted alone, *tech 2* is adopted alone and *tech 1* and *tech 2* are adopted at the same time, are, respectively, 0.92, 1.44 and 6.77, and, 14.53, 26.70 and 8.34. These results show that the “leader should adopt *tech 1* and *tech 2* sequentially”, first, *tech 1*, as soon as $\Phi^*_{1,L}$= 0.92 is reached, and second, *tech 2*, as soon as $\Phi^*_{2,L}$= 1.44 is crossed the first time; the “follower should adopt *tech 1* and *tech 2* simultaneously”, as soon as $\Phi^*_{12,F}$= 8.34 is reached. For all investment scenarios considered here, the leader adopts before the follower, as expected. In addition, the results also show that when sequential adoption is optimal, firms should adopt first the technology whose cost is decreasing more slowly, *tech 1* ($\mu_{I_1} = -0.05$), and, second, the technology whose cost is decreasing more rapidly, tech 2 ($\mu_{I_2} = -0.10$).

In Table 2 we have also results for the investment thresholds for an active leader and follower, i.e., for the case where firms are operating with one of the technologies. In these simulations we assume that at the beginning of the investment game firms are active (operating) with either *tech 1* or *tech 2*. If firms are active with *tech 1(2)*, firms have the option to adopt *tech 2(1)*[16]. Our results show that when the follower is active with *tech 1*, it should adopt *tech 2* as soon as $\Phi_2(t)$ reaches $\Phi^*_{1+2,F}$

[16] Note that as soon as one of the technologies, *tech 1* or *tech 2*, is adopted, the option to adopt both technologies at the same time is eliminated.

=16.67, and when active with *tech 2* it should adopt *tech 1* as soon as $\Phi_1(t)$ reaches $\Phi^*_{2+1,F} = 8.58$. When the leader is active with *tech 1*, it should adopt *tech 2* as soon as $\Phi_2(t)$ reaches $\Phi^*_{1+2,L} = 0.33$, and when active with *tech 2* it should adopt *tech 1* as soon as $\Phi_1(t)$ reaches $\Phi^*_{2+1,L}=0.17$. The asymmetry in firms' investment behavior regarding the adoption of tech 1/tech 2 is due to the use of different cost growth rates ($\mu_{I_1} = -0.05$, $\mu_{I_2} = -0.10$) and the asymmetry between the leader's and the follower's investment behavior is due to the first-mover market share advantage.

Consequently, both the leader and the follower should adopt *tech 1* and *tech 2* sequentially. Conventional wisdom says that "when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously". The results above show, however, that this view neglects the effects of competition and uncertainty on investment timing.

Figures 4 and 5 show the follower's investment thresholds as a function of the "difference between the cost growth rates of *tech 1* and *tech 2*", $[\mu_{I_1} - \mu_{I_2}]$, for two scenarios, respectively: (i) the adoption of *tech 1* alone, and (ii) the adoption of *tech 1* and *tech 2* simultaneously. In Figure 4 we simulate $\Phi^*_{1,F}$ using the following "cost growth rates of tech 1": $\mu_{I_1} \in \{0, -0.05, -0.10\}$. In Figure 5, we simulate $\Phi^*_{12,F}$ using the following degrees of "complementarity between tech 1 and tech 2", $[\gamma - (\gamma_1 + \gamma_2)] \in \{0.1, 0.2, 0.5\}$. The variables $\Phi_1(t)=8.57$ and $\Phi_{12}(t)=4.29$, in Figures 4 and 5, respectively, are the current value of the underlying variables of the investment on "tech 1 alone" and "tech 1 and tech 2 simultaneously". Note that these variables do not depend on $[\mu_{I_1} - \mu_{I_2}]$, so they are represented by horizontal straight lines[17].

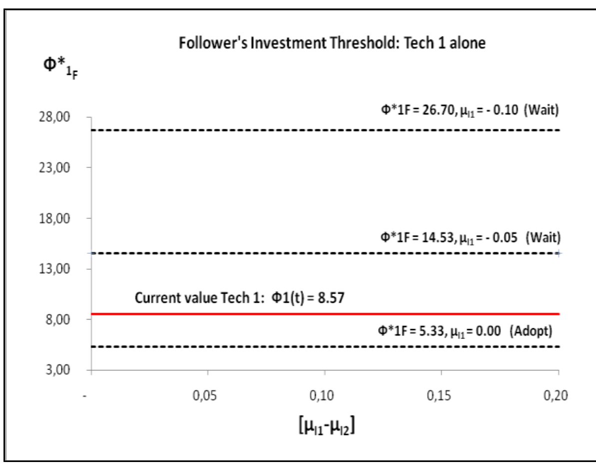

Figure 4

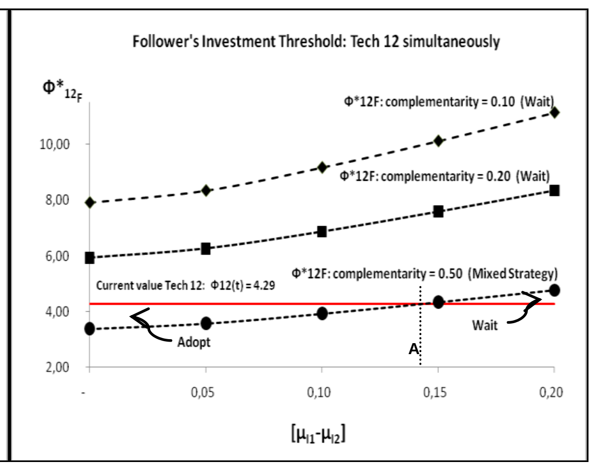

Figure 5

[17] In Figures 4, 5, 6 and 7 to compute $[\mu_{I_1} - \mu_{I_2}]$ we set $\mu_{I_1} = -0.05$, base case, and changed μ_{I_2} according to $\mu_{I_2} = \{-0.05, -0.10, -0.15, -0.20, -0.25\}$.

In Figure 4, the follower's threshold lines to adopt *tech 1* alone, $\Phi^*_{1,F}$, for each of the cost growth rates used, $\mu_{I_1} \in \{0, -0.05, -0.10\}$, do not depend on $[\mu_{I_1} - \mu_{I_2}]$, so they are horizontal straight lines, where the more negative the cost growth rate of *tech 1*, μ_{I_1}, the higher is the investment threshold (i.e., the later is the adoption).

In Figure 5, the follower's investment threshold lines to adopt *tech 1* and *tech 2* simultaneously, $\Phi^*_{12,F}$, depend on $[\mu_{I_1} - \mu_{I_2}]$. *Ceteris paribus*, the higher the $[\mu_{I_1} - \mu_{I_2}]$, the higher is the investment threshold (i.e., the later is the adoption of tech 1 and tech 2 simultaneously). The results also show that the higher the complementarity between *tech 1* and *tech 2*, the lower is the investment threshold (i.e., the sooner is the adoption of both technologies at the same time). When we set $\gamma_{12} - (\gamma_1 + \gamma_2) = 0.50$, the early adoption of both technologies at the same time is optimal for low values of $(\mu_{I_1} - \mu_{I_2})$. In this scenario, mixed strategies are possible for the follower, and point A is a strategic "switching point", where if $[\mu_{I_1} - \mu_{I_2}]$ decreases, it is optimal to adopt both technologies at the same time, and if $[\mu_{I_1} - \mu_{I_2}]$ increases crossing point A, it is optimal to defer such simultaneous investments.

The illustration in Figure 5 shows that the existence of high degrees of complementarity between two technologies, *tech 1* and *tech 2*, for instance [$\gamma_{12} - (\gamma_1 + \gamma_2) = 0.50$], in contexts of uncertainty and competition (first-mover advantage) does not necessarily mean that the adoption of both technologies at the same time is optimal. Notice that, high "complementarity between two technologies" is an incentive for the follower to adopt both technologies at the same time, but, a high "difference between the cost growth rates of the two technologies" $[\mu_{I_1} - \mu_{I_2}]$ is an incentive for the follower to adopt the two technologies sequentially, first, the technology whose cost is decreasing slowly and, second, the technology whose cost is decreasing rapidly. These two effects can offset each other.

Figures 6 and 7 illustrate the results for the leader, regarding the adoption of *tech 1* alone and the adoption of *tech 1* and *tech 2* at the same time, respectively. Figure 6 shows that the leader's current value of the adoption of *tech 1* alone, $\Phi_1(t)$=8.57, is significantly higher than the leader's investment thresholds lines for the scenarios analyzed, $\Phi^*_{1,L}$= 2.45 ($\mu_{I_1} = -0.20$) and $\Phi^*_{1,L}$= 1.94 ($\mu_{I_1} = -0.15$). Hence, it is optimal for the leader to adopt *tech 1* alone, even when high rates of decrease in the cost of *tech 1* hold. In Figure 7 the leader's current value of the adoption of *tech 1* and *tech 2* at the same time, $\Phi_{12}(t)=4.29$, is lower than the leader's investment threshold lines, $\Phi^{*}_{12,L}$. Therefore, the adoption of both technologies at the same time is not optimal for the leader, even when high degrees of complementarity between *tech 1* and *tech 2* (0.20 or 0.50) hold.

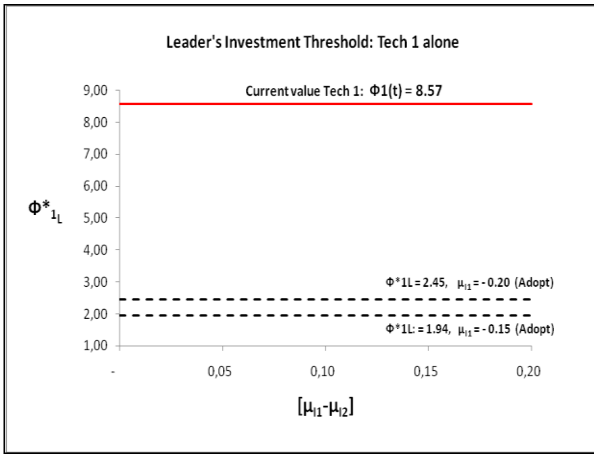

Figure 6

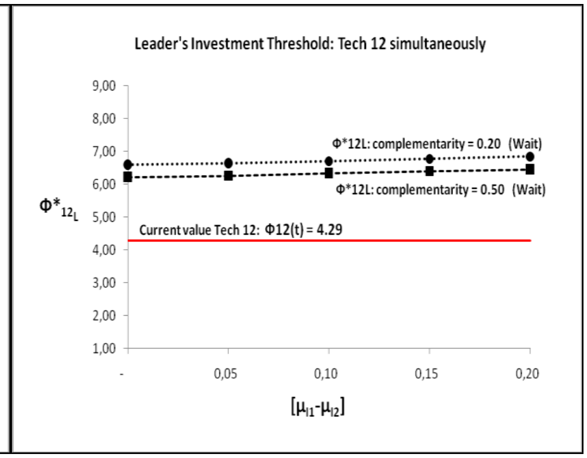

Figure 7

Comparing Figure 5 (follower's threshold lines to adopt tech 1 and tech 2 at the same time) with Figure 7 (leader's threshold lines to adopt tech 1 and tech 2 at the same time), we concude that the leader is much less sensitive to changes in the degree of complementarity between the two technologies than the follower (the leader's threshold curves, $\Phi^{*}_{12,L}$, in Figure 7, are much closer than the follower's threshold curves, $\Phi^{*}_{12,F}$, in Figure 5). Through particular cases, we show that "conventional (simultaneous adoption) investment behavior" is more likely for the follower than for the leader. This asymmetry in the leader's and the follower's investment behavior is due to the so called effect of "fear of pre-emption", which affects the leader and does not affect the follower. Given that in a leader/follower duopoly market, as soon as the leader invests the follower is in a monopoly-like position, so our results also show that "conventional investment behavior" regarding the adoption of complementary technologies is more likely to happen in markets where there is no competition.

The huge area between the straight lines $\Phi_1(t)=8.75$ and $\Phi^{*}_{1,L}=2.45$, Figure 6, and between the straight lines $\Phi_1(t)=4.29$ and the curve $\Phi^{*}_{12,L}$ for complementarity = 0.50, in Figure 11, is somewhat a "surprise", since it means that even when conditions are extremely in favour of the adoption of *tech 1* and *tech 2* at the same time, when compared to the adoption of *tech 1(2)* alone, "simultaneous adoption" is still unlikely to be justified for the leader[18]. This results show that, for

[18] Note that the inputs used, $\mu_{I_1} = -0.15$ and $\gamma_{12} - (\gamma_1 + \gamma_2) = 0.50$, can be considered extreme conditions favoring the adoption of *tech 1* and *tech 2* simultaneously, since higher complementarity between *tech 1* and *tech 2*

the leader, in a context of competition with first-mover advantage, the effect of the degree of complementarity between two technologies can be offset by the advantages from the leadership in the investment, and that in such cases the latter effect is likely to be the main driver of the leader's investment behavior. The same does not happen, however, for the follower, where the degree of complementarity plays a more important role in its investment behavior (for similar conditions, simultaneous adoption is optimal for the follower if the "difference between the cost growth rates of tech 1 and tech 2" is not very high).

Figures 8 and 9 shows the sensitivity of the firms' investment threshold to changes in the "complementarity between the technologies" and the "leader's market share advantage" [19].

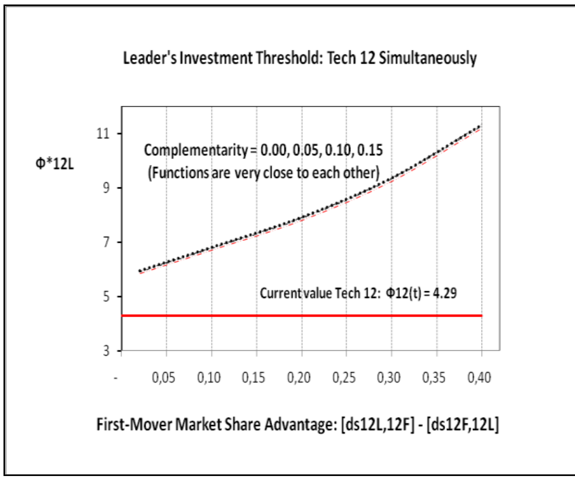

Figure 8

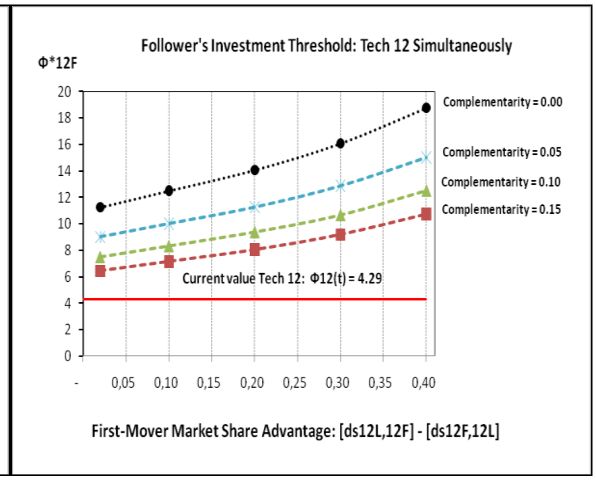

Figure 9

The results show that both firms should delay the investment for all range of leader's market share advantage and degree of complementarity ($\Phi_{12}(t)=4.29 < \Phi^*_{12,L}$ and $\Phi_{12}(t)=4.29 < \Phi^*_{12,F}$) used. In addition, we can also see that the complementarity between the technologies affects significantly the follower's investment threshold and has almost no effect on the leader's investment threshold, and that the leader's and the follower's investment threshold increases as the first-mover advantage increases.

Figures 10 and 11 show the sensitivity of firms' thresholds to adopt tech k (with $k=\{1,2,12\}$) to changes in the volatility of the investment cost (σ_{I_1}, σ_{I_2} and $\sigma_{I_{12}}$, respectively). The results show

favours "simultaneous adoption" and high rates of decrease in the cost of *tech 1* favours a delay in the adoption of *tech 1* alone, i.e., a non-sequential adoption.

[19] For a total market of 100, in Figures 12 and 13, a "first-mover market share advantage" equal to 0.20 means that after both firms invest the leader gets 60 and the follower 40, i.e., a first-mover advantage equal to 20 percent of the total market.

that the follower is much more sensitive to changes in the volatility of the cost of the technology(ies), than the leader and that for both the higher the volatility the higher are investment thresholds (i.e., the later is the adoption). The difference between the sensitivity of the leader and the follower to changes in the volatility of the cost of the technology(ies) is due to the pre-emption effect, which affects the leader and does not affect the follower.

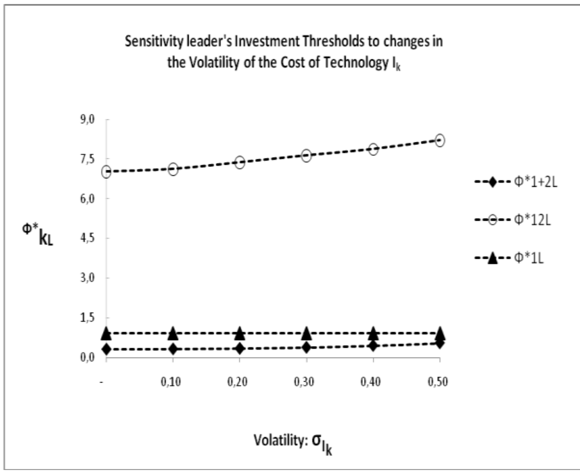

Figure 10

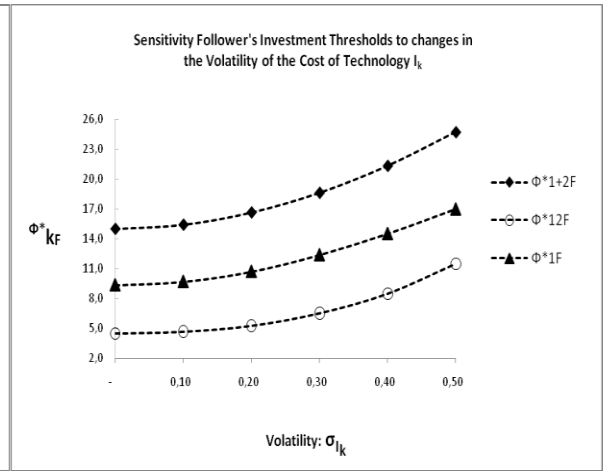

Figure 11

Similar results apply to the volatility of the revenues, given that both $X(t)$ and $I(t)$ follow similar stochastic processes. Other complementary sensitivity analyses are supplied in Appendix C, p. 37.

5. Conclusions and Further Research

Firms' investment thresholds to adopt each technology alone are not sensitive to changes in the degree of complementarity between the two technologies, since the option to adopt tech 1 is independent of the option to adopt tech 2 (i.e., γ_2 does not affect the firms' investment threshold to adopt tech 1 and γ_1 does not affect the firms' investment threshold to adopt tech 2, Equation 24, p. 16). In addition, the option to adopt tech 1 and tech 2 at the same time is independent of the options to adopt tech 1 alone and tech 2 alone, i.e., in our model the proportion of the market revenues that can be saved when tech 1 and tech 2 are adopted at the same time, γ_{12}, affects only the firms' investment threshold to adopt the both technologies at the same time, and not the optimal time to adopt any of the technologies alone (Equation 30, p. 18).

This research extends Huisman (2001, ch. 9) and Smith (2005). The former, studies the effect of competition and revenue uncertainty on timing the adoption of a technology for a context where there is one technology available and the possibility that a second and more efficient technology arrives in the future, at a not yet known date, and firms adopt/operate with one technology only; the latter, studies the adoption of two complementary technologies for a context of uncertainty, but neglects competition. We develop a real options model which considers the simultaneous effect of three key variables in the optimization of the adoption of new technologies: uncertainty, competition and technological complementarity. In our days very few monopoly markets remain, hence Smith (2005) model is very limited. For many industries, for instance manufacturing, software and telecommunications, the degree of complementarity between technologies is very important. Huisman (2001, ch. 9) model neglects this aspect. In addition, Huisman considers only the uncertainty about the revenues. We extend the uncertainty to the investment cost as well.

Our investment game setting is built under the assumption that there is a first-mover advantage (pre-emption game). An interesting extension of this research would be to derive a similar investment model for an economic context where a second-mover advantage (war of attrition game) holds. The extension of this model to oligopoly markets, although technically challenging, would also be an interesting complement of this research.

In addition, we also assume that firms have two technologies available which can be adopted at the same time or at different times. Given that it is quite common to find projects that have more than two inputs whose functions are a complement, an interesting research would be to extend this model to investments with more than two complementary inputs, as well as the incorporation of stochastic complementarity and technology cost drifts.

Acknowledgments

We thank Roger Adkins, Michael Flanagan, Peter Hammond, Wilson Koh, Helena Pinto, Lenos Trigeorgies, Mark Shackleton, Martin Walker, two anonymous referees and participants at the Portuguese Finance Network Conference Coimbra 2008, the Seminar at Centre for International Accounting and Finance Research HUBS 2010 and the Real Options Conference Rome 2010, for comments on earlier versions. Alcino Azevedo gratefully acknowledges support from the Fundação Para a Ciência e a Tecnologia.

Appendix A

1. Derivation of the Follower's Value Function and Investment threshold when Technology 1 is in place

In this section we derive the follower's option value to adopt *tech 2* assuming that *tech 1* is in place, $f_{12}(X, I_2)$. Once we have $f_{12}(X, I_2)$, we will derive the expression for the total value $F_{12}(X, I_2) = V_1 + f_{12}(X, I_2)$, where V_1 is the follower's expected value from operating with *tech 1* forever, and given by expression (13) [20]:

$$V_1 = \frac{\gamma_1 X \left[ds_{1_F 12_L} \right]}{r - \mu_X} \tag{A1}$$

Setting the returns on the option equal to the expected capital gain on the option and using Ito's lemma, we obtain this partial differential equation (PDE) for the value function of an active follower (i.e., a follower which is operating with *tech 1*) in the region in which it waits to adopt *tech 2*:

$$\frac{1}{2}\sigma_X{}^2 X^2 \frac{\partial^2 F_{12}}{\partial X^2} + \frac{1}{2}\sigma_{I_2}^2 I_2{}^2 \frac{\partial^2 F_{12}}{\partial I_2{}^2} + X I_2 \sigma_X \sigma_{I_2} \rho_{XI_2} \frac{\partial^2 F_{12}}{\partial X \partial I_2} + \mu_X X \frac{\partial F_{12}}{\partial X} + \mu_{I_2} I_2 \frac{\partial F_{12}}{\partial I_2} + \gamma_1 X \left(ds_{1_F k_L} \right) = r F_{12} \tag{A2}$$

where, ρ_{XI_2} is the correlation coefficient between the market revenues, *X*, and the cost of *tech 2* , I_2 and *r* is the riskless interest rate.

Equation (A2) must be subjected to two boundary conditions. The first is the *"value matching"* condition:

(i) There is a value of $F_{12}(X, I_2)$ at which the follower will invest and at that point in time the follower's value equals the present value of the cash flows minus the investment costs ($I_{2_F}^*$):

$$F_{12}(X, I_2) = \frac{(\gamma_{12} - \gamma_1) X^* \left[ds_{12_F 12_L} \right] - \gamma_1 X^* \left[ds_{1_F 12_L} \right]}{r - \mu_X} - I_{2_F}^* \tag{A3}$$

---

[20] Notice that in our framework the total market, $X(t)$, is equal to 100 percent and, at each instant of the investment game, each firm gets a proportion, $ds_{k_i k_j}$, of $X(t)$, which depends on whether it is the leader or the follower, active or inactive, and if active on whether it is operating with "tech 1 alone", "tech 2 alone" or with "tech 1 and tech 2 at the same time".

where, $(\gamma_{12}-\gamma_1)X^*\left[ds_{12_F12_L}\right]$ represents the follower's cost savings at the time it adopts *tech 2*; $\gamma_1 X^*\left[ds_{1_F12_L}\right]$ represents the follower's cost saving while operating with tech 1, which concurs in determining the "value of waiting"; $X^*\left[ds_{12_F12_L}\right]$ is the follower's revenues share at the time of adoption of tech 2; $X^*\left[ds_{1_F12_L}\right]$ is the follower's revenue share while operating with tech 1 only; $(\gamma_{12}-\gamma_1)$ is the proportion of the follower's revenues that is expected to be saved due to the adoption of *tech 2* when *tech 1* is in place; X^* and $I_{2_F}^*$ are, respectively, the total market revenue and the cost of *tech 2* at the follower's adoption time.

The second boundary condition comes from the *"smooth pasting"* conditions, for the value of both the idle and the active follower:

(ii) The first derivative, with respect to both stochastic variables, $X(t)$ and $I_2(t)$, of the value functions equals the present value of the cash flows, at $(X/I_2)^*$. Therefore, it holds that:

$$\frac{\partial F_{12}(X,I_2)^*}{\partial X^*}=\frac{(\gamma_{12}-\gamma_1)\left[ds_{12_F12_L}\right]-\gamma_1\left[ds_{1_F12_L}\right]}{r-\mu_X} \tag{A4}$$

$$\frac{\partial F_{12}(X,I_2)^*}{\partial I_{2_F}^*}=-1 \tag{A5}$$

In the present case, the natural homogeneity of the investment problem, i.e., $F_{12}(X,I_2)=I_2 f_{12}(X/I_2)$, where f_{12} is the variable to be determined, allows us to reduce it to one dimension. Using the following change in the variables $\phi_2 = X/I_2$ and substituting this relation in the PDE (A2) yields[21]:

$$\frac{1}{2}\sigma_{m_2}(\phi_2)^2\frac{\partial^2 f_{12}(\phi_2)}{\partial \phi_2{}^2}+\left(\mu_X-\mu_{I_2}\right)(\phi_2)\frac{\partial f_{12}(\phi_2)}{\partial \phi_2}-(r-\mu_{I_2})f_{12}(\phi_2)+\gamma_1 X\left(ds_{1_L1_F}\right)=0 \tag{A6}$$

where, $\sigma_{m_2}^2=\sigma_X^2+\sigma_{I_2}^2-2\rho_{XI_2}\sigma_X\sigma_{I_2}$.

[21] A detailed derivation of Equation (A6) is given in the Appendix C, p. 36.

Equation (A6) is a homogeneous second-order linear ordinary differential equation (ODE) whose general solution has the form[22]:

$$f_{1+2}(\phi_2) = A_{1+2}(\phi_2)^{\beta_1} + B_{1+2}(\phi_2)^{\beta_2} \tag{A7}$$

where, $\beta_{1(2)}$ is the characteristic quadratic function of the homogeneous part of equation (A6), given by:

$$\frac{1}{2}(\sigma_{m_2})^2 \beta_1(\beta_1 - 1) + (\mu_X - \mu_{I_2})\beta_1 - (r - \mu_{I_2}) = 0 \tag{A8}$$

Solving the equation above for β_1 leads to:

$$\beta_1 = \frac{1}{2} - \frac{\mu_X - \mu_{I_2}}{\sigma_{m_2}^2} + \sqrt{\left(\frac{(\mu_X - \mu_{I_2})}{\sigma_{m_2}^2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{I_2})}{\sigma_{m_2}^2}} \tag{A9}$$

Note that as the ratio of market revenues to cost of *tech 2,* ϕ_2, approaches 0, the value of the option to adopt *tech 2* becomes worthless; therefore, in Equation (A7) $B_{1+2} = 0$. Rewriting the boundary conditions we obtain the following "value-matching" condition:

$$f_{1+2}(\phi^*_{1+2_F}) = \frac{(\gamma_{12} - \gamma_1)\left[ds_{12_F 12_L}\right]\phi^*_{1+2_F} - \gamma_1\left[ds_{1_F 12_L}\right]\phi^*_{1+2_F}}{r - \mu_X - \mu_{I_2}} - 1 \tag{A10}$$

where, $\phi_2 = \phi^*_{1+2_F}$ is the follower's investment threshold to adopt *tech 2* given that *tech 1* is already in place, and the "smooth-pasting" condition:

$$\frac{\partial f_{1+2}(\phi^*_{1+2_F})}{\partial \phi^*_{1+2_F}} = \frac{(\gamma_{12} - \gamma_1)\left[ds_{12_F 12_L}\right] - \gamma_1\left[ds_{1_F 12_L}\right]}{r - \mu_X - \mu_{I_2}} \tag{A11}$$

Solving together equations (A7), (A10) and (A11) we get the following value for $\phi^*_{1+2_F}$, and the constant A_{12}:

$$\phi^*_{1+2_F} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_2})}{(\gamma_{12} - \gamma_1)\left[ds_{12_F 12_L}\right] - \gamma_1\left[ds_{1_F 12_L}\right]} \tag{A12}$$

[22] Proof that homogeneity of degree one exists is given in this appendix, section 2.

$$A_{1+2} = \frac{\left(\phi^*_{1+2_F}\right)^{-\beta_1}}{\beta_1 - 1} \frac{(\gamma_{12} - \gamma_1)\left[ds_{12_F 12_L}\right] - \gamma_1 \left[ds_{1_F 12_L}\right]}{r - \mu_X - \mu_{I_2}} \quad \text{(A13)}$$

where, $\phi^*_{1+2_F}$ is the follower's threshold for adopting *tech 2* if *tech 1* is in place.

Finally, using equations (A7), (A12) and (A13) we derive the follower's value function:

$$F^{SQ}_{F_{1+2,1+2}}(\phi_2) = \begin{cases} \dfrac{\gamma_1 X \left[ds_{1_F 12_L}\right]}{r - \mu_X} + A_{12}\left(\dfrac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} I_2 & \phi_2 < \phi^*_{1+2_F} \\ \dfrac{\gamma_{12} X \left[ds_{12_F 12_L}\right]}{r - \mu_X} - I^*_{2_F} & \phi_2 \geq \phi^*_{1+2_F} \end{cases} \quad \text{(A14)}$$

Scenario (S3) in the game-tree, p. 6.

Equation (A14) tells us that for the follower, before $\phi^*_{1+2_F}$ is reached, its value, when it adopts the two technologies sequentially, is given by the value of operating with *tech 1* forever, $\frac{\gamma_1 X \left[ds_{1_F 12_L}\right]}{r - \mu_X}$, plus its option to adopt *tech 2*, $A_{12}\left(\frac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} I_2$; as soon as $\phi^*_{1+2_F}$ is reached and it adopts tech 2, its value is equal to the present value, in perpetuity, of the cost savings obtained from operating with both technologies from $\phi^*_{1+2_F}$ until infinity, $\frac{\gamma_{12} X \left[ds_{12_F 12_L}\right]}{r - \mu_X} - I^*_{2_F}$.

2. Proof - Homogeneity of Degree One

If the value matching relationship can be expressed as the equality between the option value denoted by $F_{12}\left(\bar{X}, \bar{I}_2\right)$ and the difference between the two functions, $f_2(\bar{X})$ and $f_3(\bar{I}_2)$, representing the net value generated from exercising the option, where the vectors $\bar{X}$ and $\bar{I}_2$, of size n and m respectively are defined by $\bar{X} = \{X_1, X_2, \ldots, X_n\}$ and $\bar{I}_2 = \{I_2^1, I_2^2, ..., I_2^m\}$, then Euler's theorem on homogenous functions applies (see Sydsaeter and Hammond, 2006). The value matching relationship is:

$$F_{12}\left(\bar{X}, \bar{I}_2\right) = f_2(\bar{X}) - f_3(\bar{I}_2)$$

The associated smooth pasting conditions are:

$$\frac{\partial F_{12}}{\partial X_i} = \frac{\partial f_2}{\partial X_i} \ \forall i$$
$$\frac{\partial F_{12}}{\partial I_{2j}} = -\frac{\partial f_3}{\partial I_{2j}} \ \forall j$$

These conditions imply:

$$\sum_{i=1}^{n} X_i \frac{\partial F_{12}}{\partial X_i} + \sum_{j=1}^{m} I_{2j} \frac{\partial F_{12}}{\partial I_{2j}} = \sum_{i=1}^{n} X_i \frac{\partial f_2}{\partial X_i} - \sum_{j=1}^{m} I_{2j} \frac{\partial f_3}{\partial Y_j}$$

If the two functions, $f_2(\bar{X})$ and $f_3(\bar{I}_2)$, possess the homogeneity degree-one property, then by Euler's theorem:

$$\sum_{i=1}^{n} X_i \frac{\partial F_{12}}{\partial X_i} + \sum_{j=1}^{m} Y_j \frac{\partial F_{12}}{\partial I_{2j}} = f_2 - f_3 = F_{12}$$

which implies that F_{12} is a homogenous function of degree one. The assertion that the option value is represented by a homogenous degree-one function can be tested by the value matching relationship and its associated smooth pasting conditions. Examining the value "matching conditions" we can easily prove that homogeneity exists. Taking the "value matching" condition given by Equation A3, p. 29, reproduced here as Equation A15, we have:

$$F_{12}(X, I_2) = \frac{(\gamma_{12} - \gamma_1) X^* \left[ds_{12_F 12_L} \right] - \gamma_1 X^* \left[ds_{1_F 12_L} \right]}{r - \mu_X} - I_{2_F}^* \tag{A15}$$

Therefore, if the option value is $F_{12}(X, I_2)$ and the value after exercising the option is $\frac{(\gamma_{12} - \gamma_1) X^* \left[ds_{12_F 12_L} \right] - \gamma_1 X^* \left[ds_{1_F 12_L} \right]}{r - \mu_X} - I_{2_F}^*$, with both X (market revenues) and I_2 (investment cost) stochastic, then if $F_{12}(X, I_2) = \frac{(\gamma_{12} - \gamma_1) X^* \left[ds_{12_F 12_L} \right] - \gamma_1 X^* \left[ds_{1_F 12_L} \right]}{r - \mu_X} - I_{2_F}^*$ holds, doubling X^* and $I_{2_F}^*$ doubles $F_{12}(X, I_2)$, if so there is homogeneity of degree one. If the "value matching" relationship exhibits homogeneity of degree one, then the two variables (X, I_2) can be replaced by, in this case, the ratio $X / I_2 = \phi_2$. This can be easily proved empirically using the model inputs of section 4 with changes in the variables X^* and $I_{2_F}^*$. More specifically, in Table A1 below, we compute $F_{12}(X, I_2)$ for two scenarios from the "value matching condition" (A15); the difference between scenario 1 and 1 is that in "scenario 2" we double the values of X^* and $I_{2_F}^*$ in Equation A15 (ceteris paribus). If homogeneity exists, the value of $F_{12}(X, I_2)$ for scenario 2 is twice that of scenario 1. This is the case, as shown in Table A1. Hence, homogeneity is proved.

Value-matching Parameters (Equation B15)	γ_1	γ_{12}	r	μ_X	$ds_{12_F 12_L}$	$ds_{1_F 12_L}$	X^*	$I^*_{2_F}$	$F_{12}(X, I_2)$
Scenario 1	0.10	0.30	0.09	0.05	0.45	0.40	**60**	**5**	***70***
Scenario 2 (doubling X^* and $I^*_{2_F}$)	0.10	0.30	0.09	0.05	0.45	0.40	**120**	**10**	***140***

Table A1 –Homogeneity of Degree 1

3. The Competition Factors

In our framework the leader's first-mover market advantage, altogether with the assumption about the technological complementarity, is ensured by inequality (2), page 10, replicated below as inequality B16, where each of the deterministic factors represents the leader's market share for each investment scenario, given as a proportion of the total market.

$$\left[ds_{12_L 0_F} = ds_{1_L 0_F} = ds_{2_L 0_F}\right] > ds_{12_L 1_F} > ds_{12_L 12_F} > \left[ds_{1_L 1_F} = ds_{2_L 2_F}\right] \tag{A16}$$

For instance, for a market value of 10 million if we set $ds_{12_L 12_F} = 0.6$ this means that when both firms are active operating with tech 1 and tech 2 at the same time, the leader gets 60 percent of the market revenues (6 million) and the follower the remaining 40 percent (4 million). In a duopoly market the sum of the market share of the leader and the market share of the follower is equal to 100 percent, hence, $ds_{12_L 12_F} + ds_{12_F 12_L} = 1.0$, i.e., if $ds_{12_L 12_F} = 0.6$, so $ds_{12_F 12_L} = 1 - 0.60 = 0.4$.

In addition, inequality (A16) means that when the leader operates with tech 1 and tech 2 at the same time, its market share is higher if the follower is active operating with one technology alone than if the follower is active operating with both technologies at the same time (hence $ds_{12_L 1_F} > ds_{12_L 12_F}$). This is due to the fact that when the follower operates with one technology alone it does not benefit from the effect of the complementarity between the two technologies. Note that according to our assumptions, when the leader is alone in the market it gets 100 percent of the market revenues, regardless of which technology(ies) it has adopted, tech 1 alone, tech 2 alone, or tech 1 and tech 2 at the same time ($ds_{1_L 0_F} = ds_{2_L 0_F} = ds_{12_L 0_F} = 1.0$). Inequality (A16) also shows that the best scenario for the leader is when it is alone in the market, for obvious reasons.

Our investment model is set as a “zero-sum pre-emption game” with two firms competing for a percentage of the total market revenues. For each firm and investment scenario we deterministically assign a revenues market share. The relative market revenues advantage assigned to each strategy is guided by inequality (A16). Backed by inequality (A16), we can compare at each node of the investment game-tree (Figure 1, p. 6) the value functions of the leader and the follower (firms’ payoffs) for the investment strategies available and, consequently, determine their optimal decision. We derive the firms’ payoffs and their respective investment threshold values for some specific investment game scenarios (those marked in Figure 1, p. 6, with an ellipse), combining the real options theory with the Fudenberg and Tirole (1985, pp. 386-389) principle of rent equalization.

4. The Firms’ Payoffs

In our investment game there are two firms and two technologies available which can be adopted at the same time or at different times. Therefore, the number of investment scenarios grows substantially when compared with investment games with two firms but with only one technology or with the case where there are two technologies involved in the investment decision but they cannot be adopted at the same time. However, at each node of the game-tree, the use of the information underlying inequality (2), p. 10, simplifies substantially our work regarding the determination of the firms’ optimal strategy. Expression (B17) below replicates expression 1, p. 8, as the general expression for the firms’ value functions:

$$\gamma_k X(t)\left[ds_{k_i k_j}\right] \tag{A17}$$

where, $X(t)$ is the market revenue flow, γ_k represents the proportion of firm’s revenues that is expected to be saved through the adoption of technology *k*, with $k=\{0,1,2,12\}$, where 0 means that firm is not yet active and 1, 2 and 12 mean that firm operates with *tech 1* only, with *tech 2* only or with *tech 1* and *tech 2* and the same time, respectively; $ds_{k_i k_j}$ is a deterministic factor that ensures a first-mover revenue advantage, with $i, j=\{L,F\}$, where *L* means “leader” and *F* “follower”, and represents the proportion of the market revenues that is held by each firm (*i*, *j*) for each investment scenarios (see inequality 2, p. 10).

Taking *i* as the leader and *j* as the follower, $ds_{12_i 1_j} > ds_{12_i 12_j}$ turns into $ds_{12_L 1_F} > ds_{12_L 12_F}$. This means that the leader’s revenues market share is higher when it operates with tech 1 and tech 2 and the follower operates with tech 1 only ($ds_{12_L 1_F}$) than when the leader operates with tech 1 and tech 2 and the follower as well ($ds_{12_L 12_F}$). Using this logic at each node of the game-tree we determine the optimal investment strategy for the leader and the follower.

5. Investment Scenarios

<table>
<tr><th colspan="3">Investment Game Scenarios</th></tr>
<tr><td>Model Assumptions</td><td colspan="2">1. At each instant of the investment game, both firms are subjected to the same economic conditions (model parameters) except for the proportion of the “market share revenue”, $ds_{k_i k_j}$, which is asymmetric favoring the leader due to the first-mover advantage.
2. At the beginning of the investment game, both firms hold two “independent” options: (i) the option to adopt tech 1 and the option to adopt tech 2. These option values are “independent” because the threshold to adopt tech 1 does not depend on the evolution of the ratio revenue over cost of tech 2, ϕ_2, and vice-versa –see Equation 24, p.16.
3. Due to the first-mover advantage, the leader starts and ends (if that is the case) the game first. Hence, scenarios where the follower adopts both technologies before the leader are not possible.
4. Firms are not allowed to exercise their options at the same time. We assume that when that is the case the leader will be chosen by flipping a coin.
5. As soon as the leader invests in both technologies its game ends, and the follower is in a monopoly like thereafter.
6. In section 3, tech 1 and tech 2 are assumed to be symmetric, i.e., the economic benefit from operating with one or the other is the same. Hence, the expressions for tech 2 are the same as those for tech 1, only the subscript changes.</td></tr>
<tr><th>Modeled Scenarios</th><th>Firms’ thresholds (Section 3)</th><th>Comments</th></tr>
<tr><td>S1</td><td>$\phi^*_{1_L}$ --- $\phi^*_{1_F}$ <u>or</u> $\phi^*_{2_L}$ --- $\phi^*_{2_F}$</td><td><u>Characterized</u>: see derivation pp. 14-18 and appendix B, section 1.</td></tr>
<tr><td>S2</td><td>$\phi^*_{12_L}$ --- $\phi^*_{12_F}$</td><td><u>Characterized</u>: see derivation pp. 18-19 and appendix B, section 1.</td></tr>
<tr><td>S3</td><td>$\phi^*_{1_L}$ --- $\phi^*_{1_F}$ --- $\phi^*_{1+2_L}$ --- $\phi^*_{1+2_F}$
<u>or</u>
$\phi^*_{2_L}$ --- $\phi^*_{2_F}$ --- $\phi^*_{2+1_L}$ --- $\phi^*_{2+1_F}$</td><td><u>Characterized</u>: see derivation pp. 12-14 and Appendix B, section 1.</td></tr>
<tr><th>Another (not modeled) Scenario</th><th>Firms’ thresholds</th><th>Comments</th></tr>
<tr><td>S4</td><td>$\phi^*_{1_L}$ --- $\phi^*_{1+2_L}$ --- $\phi^*_{1_F}$ --- $\phi^*_{1+2_F}$
<u>or</u>
$\phi^*_{2_L}$ --- $\phi^*_{2+1_L}$ --- $\phi^*_{2_F}$ --- $\phi^*_{2+1_F}$</td><td>This scenario (S4) is not fully characterized in section 3. However, the expressions for $\phi^*_{1_L}$ and $\phi^*_{2_L}$, and, $\phi^*_{1+2_F}$ and $\phi^*_{2+1_F}$ are the same as those derived for (S3), given that the conditions are the same; and the leader’s thresholds to adopt tech 2 when tech 1 is in place, $\phi^*_{1+2_L}$, or to adopt tech 1 when tech 2 is in place, $\phi^*_{2+1_L}$, can be easily derived by following the rationale and the technique used in the derivations of the investment thresholds of scenario (S3).

Notice that, as soon as the leader adopts both technologies (i.e., $\phi^*_{1+2_L}$ is crossed), its game ends, and the follower is in a monopoly-like thereafter. Hence, the follower’s threshold is given by: $\phi^*_{1_F} = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\mu_X-\mu_{I_1})}{\left[ds_{1_F 12_L}\right]\gamma_1}$. Compared with $\phi^*_{1_F}$ of scenario (S3) –see equation 24, p. 15, the competition factor in the expression above changes to $ds_{1_F 12_L}$ to reflect the fact that in this scenario (S4) when the follower adopts tech 1 the leader is operating with both technologies. Looking at inequality (3), p. 9, we conclude that this threshold is higher than that of (S3), since $ds_{1_F 12_L} < ds_{1_F 1_L}$. Similar rationale applies to the derivation of $\phi^*_{1+2_L}$ and $\phi^*_{2+1_L}$ and other uncharacterized investment scenarios.</td></tr>
</table>

Table A2 –Investment Game Scenarios: Characterization of Investment Thresholds

Appendix B

Derivation of the Ordinary Differential Equation (B6)

Equation (A2), p. 27, is written as:

$$\frac{1}{2}\frac{\partial^2 F_{12}}{\partial X^2}\sigma_X{}^2 X^2+\frac{1}{2}\frac{\partial^2 F_{12}}{\partial I_2{}^2}\sigma_{I_2}^2 I_2{}^2+\frac{\partial^2 F_{12}}{\partial X\partial I_2}XI_2\sigma_X\sigma_{I_2}\rho_{XI_2}+\frac{\partial F_{12}}{\partial X}\mu_X X+\frac{\partial F_{12}}{\partial I_2}\mu_{I_2}I_2-rF_{12}=0$$

In order to reduce the homogeneity of degree two in the underlying variables to homogeneity of degree one, similarity methods can be used. Let $\phi_2=\frac{X}{I_2}$, so:

$$F(X,I_2)=f\left(\frac{X}{I_2}\right)I_2=f(\phi_2)I_2$$

$$\frac{\partial F(X,I_2)}{\partial I_2}=f(\phi_2)-\frac{X}{I_2}\frac{\partial f(\phi_2)}{\partial \phi_2}$$

$$\frac{\partial F(X,I_2)}{\partial X}=\frac{\partial f(\phi_2)}{\partial \phi_2}$$

$$\frac{\partial^2 F(X,I_2)}{\partial I^2}=\frac{\partial^2 f(\phi_2)}{(\partial\phi)^2}\frac{X^2}{(I_2)^3}$$

$$\frac{\partial^2 F(X,I_2)}{\partial X^2}=\frac{\partial^2 f(\phi_2)}{\partial \phi_2{}^2}\frac{1}{I_2}$$

$$\frac{\partial^2 F(X,I_2)}{\partial X\ \partial I_2}=-\frac{\partial^2 f(\phi_2)}{\partial \phi_2{}^2}\frac{X}{(I_2)^2}$$

Substituting back to Equation (A2) we obtain Equation (A6):

$$\frac{1}{2}\sigma_{m_2}^2(\phi_2)^2\frac{\partial^2 f_{12}(\phi_2)}{\partial\phi_2{}^2}+\left(\mu_X-\mu_{I_2}\right)(\phi_2)\frac{\partial f_{12}(\phi_2)}{\partial \phi_2}+\gamma_1 X\left(ds_{1_L 1_F}\right)-(r-\mu_{I_2})f_{12}(\phi_2)=0$$

where, $\sigma_{m_2}^2=\sigma_X^2+\sigma_{I_2}^2-2\rho_{XI_2}\sigma_X\sigma_{I_2}$.

Appendix C

1. Complementary Results

Figures C1 and C2 show the sensitivity analysis of the impact of the degree of complementarity between *tech 1* and *tech 2* on the leader and the follower investment thresholds to adopt the two technologies at the same time.

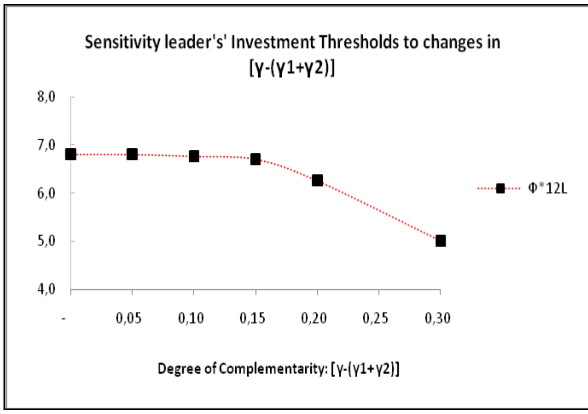

Figure C1

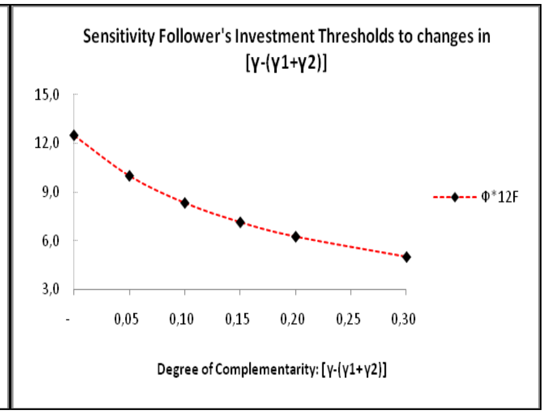

Figure C2

The results show that the higher is the complementarity between tech 1 and tech 2, $\gamma_{12} - (\gamma_1 + \gamma_2)$, the the lower are the leader and the follower investment thresholds, i.e., the earlier is the adoption of both technologies simultaneously, and that the leader's threshold is a convex function of the complementarity measure and the follower's threshold is a concave function of complementarity.

Figures C3 and C4 show the trade-offs between the "degree of complementarity" between *tech 1* and *tech 2* and the "difference between the rate of decrease in the cost of *tech 1* and *tech 2*", on the leader's and the follower's investment threshold.

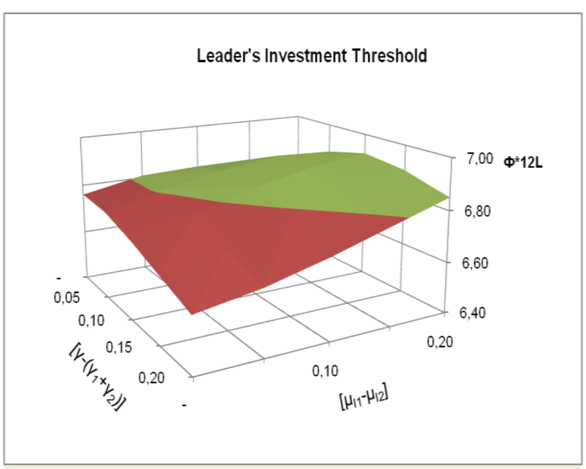

Figure C3

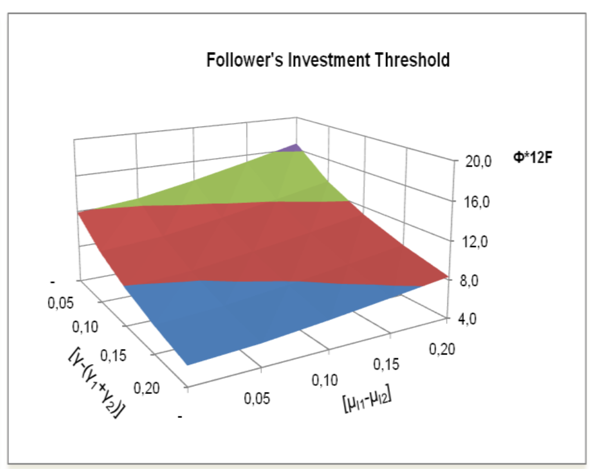

Figure C4

Although the scales are somewhat different for the leader and the follower, the leader's simultaneous thresholds appear to eventually be insensitive to lower complementarity and greater drift differences, while the follower's thresholds reach a peak at nil complementarity and large drift differences.

References

Anderson, E., Schmittlein, D., 1984. Integration of Sales Force: An Empirical Examination. Rand Journal of Economics 15, 385-395.

Arora, A., 1996. Testing for Complementarities in Reduced-Form Regressions: A Note. Economic Letters 50, 51-55.

Arora, A., Gambardella, A., 1990. Complementarity and External Linkages: the Strategies of the Large Firms in Biotechnology. Journal of Industrial Economics 38, 361-379.

Azevedo, A., Paxson, D., 2008. New Technology Adoption Games: An Aplication to the Textile Industry: Presented at the Real Options Conference 2008, Pontifícia Universidade Católica, Rio de Janeiro.

Azevedo, A., Paxson, D., 2010. Developing Real Option Game Models: Presented at the Real Options Conference 2010, Luiss Business School, University of Rome, Rome.

Braga, H., Wilmore, L., 1991. Technological Imports and Technological effort: An Analysis of their Determinants in Brazilian Firms. Journal of Industrial Economics 39, 421-432.

Carree, M., Lokshin, B., Belderbos, R., 2010. A Note on Testing for Complementarity and Substitutability in the Case of Multiple Practices. Journal of Productivity Analysis Published online, 20 July 2010.

Cassiman, B., Veugelers, E., 2004. In Search of Complementarity in Innovation Strategy: Internal R&D and External Knowledge Acquisition, Working Paper, University of Leuven.

Cho, Soo-Haeng and McCardle, K. (2009). The Adoption of Multiple Dependent Technologies. Operations Research 57, 157-169.

Colombo, M., Mosconi, R., 1995. Complementarity and Cumulative Learning Effects in the Early Diffusion of Multiple Technologies. Journal of Industrial Economics 43, 13-48.

Cooper, I., 2006. Asset Pricing Implications of Nonconvex Adjustment Costs and Irreversibility of Investment. Journal of Finance 61, 139-170

Dixit, A., Pindyck, R., 1994. Investments under Uncertainty. Princeton NJ, Princeton University Press.

Etro, F., 2007. Competition, Innovation, and Antitrust: A Theory of Market Leaders and its Policy Implication, Berlin, Springer.

Fudenberg, D., Tirole, J., 1985. Preemption and Rent Equalization in the Adoption of New Technology. Review of Economic Studies 52, 383-401.

Gibbons, R., 1992. A Primer in Game Theory, London, FT - Prentice Hall.

Grenadier, S., 1996. The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets. Journal of Finance 51, 1653-1679.

Huisman, K., 2001. Technology Investment: A Game Theoretical Options Approach, Boston, Kluwer.

Jovanovic, B., Stolyarov, D., 2000. Optimal Adoption of Complementary Technologies. American Economic Review 90, 15-29.

Lambrecht, B., Perraudin, W., 2003. Real Options and Preemption under Incomplete Information. Journal of Economic Dynamics and Control 27, 619-643.

Mason, R. and Weeds, H., 2010, Investment, Uncertainty and Preemption. Journal of Industrial Organization 28, 278-287.

Milgrom, P., Roberts, J., 1990. Economics of Modern Manufacturing: Technology, Strategy, and Organization. American Economic Review 80, 511-528.

Milgrom, P., Roberts, J., 1994. Complementarities and Systems: Understanding Japanese Economic Organization. Working Paper, Stanford University.

Milgrom, P., Roberts, J., 1995. Complementarities and Fit Strategy, Structure, and Organizational Change in Manufacturing. Journal of Accounting and Economics 19, 179-208.

Miravete, E., Pernías, J., 1998. Innovation Complementarity and Scale of Production. Working Paper, Center for Applied Economics, New York University.

Miravete, E., Pernías, J., 2010. Testing for Complementarity when Strategies are Dichotomous. Economic Letters 106, 28-31.

Mohnen, P., Roller, L., 2000. Complementarity in Innovation Policy. Working Paper, University of Quebec.

Morettto, M., 2008. Competition and Irreversible Investments under Uncertainty. Information Economics and Policy 20, 75-88.

Paxson, D., Pinto, H., 2005. Rivalry under Price and Quantity Uncertainty, Review of Financial Economics 14, 209-224.

Pawlina, G., Kort, P., 2006. Real options in an Asymmetric Duopoly: Who Benefits from your Competitive Advantage? Journal of Economics and Management Studies 15, 1-35.

Smets, F., 1993. Essays on Foreign Direct Investment. PhD thesis, Yale University.

Smith, M., 2005. Uncertainty and the Adoption of Complementary Technologies. Industrial and Corporate Change 14, 1-12.

Smith, M., Weil, D., 2005. Ratcheting Up: Linked Technology Adoption in Supply Chains. Industrial Relations 44, 490-508.

Susan, A., Stern, S., 1998. An Empirical Framework for Testing Theories about Complementarity in Organizational Design. Working Paper, MIT Sloan School of Management.

Sydsaeter, K., P. Hammond., 2006, Mathematics for Economic Analysis, Englewood Cliffs NJ, Prentice-Hall.

Weeds, H., 2002. Strategic Delay in a Real Options Model of R&D Competition. Review of Economic Studies. 69, 729-747.