Economic Capacity Withholding: Effects of Power Plant Operational Characteristics on Optimal Dispatch Decisions

Nihat MISIR*

Department of Economics, Copenhagen Business School (CBS)

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Abstract In this paper we study the effects of operational characteristics of power plants on optimal dispatch decisions for a monopolist. The first aim of this paper is to give a mathematical model to show that the operational characteristics and uncertainty result in economic withholding. In this regard, we'll be able to identify effects of these operational characteristics on the market price. Operational characteristics include: total maximum production capacity, minimum operation level, start-up and shut-down costs. Firms in the industry adjust their production or take start-up/shut-down decisions for their power plants according to realization of industry-wide exogenous demand shocks. We show that in the case of ownership of multiple generation technologies, optimal dispatch decisions will cause *economic* withholding at peakload level. Therefore, more costly generation unit will start when market price is well above its marginal cost. This result also holds for corresponding social planner's problem. Hence, we can distinguish between the effects of market power and optimal dispatch decisions on market prices.

Keywords: Uncertainty, real options, electricity

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*Address: Porcelænshaven 16A, 3.36 DK-2000, DENMARK, telephone: +45-3815-2340, e-mail: *nm.eco@cbs.dk*.

1 Introduction and Literature Review

In this paper we study the effects of operational characteristics of a power plant on optimal dispatch decisions for a monopolist. In the relevant literature, we don't see a theoretical model showing the effects of operational characteristics of the generators on capacity withholding decisions. The first aim of this paper is to give a mathematical model to show those operational characteristics and uncertainty result in economic withholding. Therefore we'll be able to identify effects of these operational characteristics on the market price. These operational characteristics include: total maximum production capacity, minimum operation level, start-up cost and shut-down cost. Firms in the industry adjust their production or take start-up/shut-down decisions for their power plants according to realization of industry-wide exogenous demand shock.

Strategic actions in the electricity industry has been long lasting focus of researchers and policy makers alike. In general, studies of strategic bidding and capacity withholding behaviors are focused on the determination of the shortcomings of current market designs. Mainly focused on California electricity market crisis in 2001, an important number of studies including Wolfram (1998), Harvey and Hogan (2001), Joskow and Kahn (2002), Borenstein, Bushnell and Wolak (2002) and Wolak (2003) were published to understand what went wrong during the crisis and how we can overcome those problems in the future.

Some of the previous studies focus on the capacity withholding behavior in the electricity market. As Twomey, Green, Neuhoff and Newberry (2005): *there are two types of withholding- economic withholding where output is reduced because it is bid into the market above competitive prices, and physical withholding, where output is not bid into the market at all.* Hence in the case of physical withholding, market supply curve shifts to the left and total maximum market supply decreases by the amount of capacity withheld. On the other hand, in the case of economic withholding, market supply curve (partially) shifts upwards and total maximum market supply is unchanged.

Focusing on the economic withholding, Harvey and Hogan (2001) point out FERC's economic withholding criterion as "*during periods of high demand and high market prices, all generation capacity whose incremental costs do not exceed the market price would be either producing energy or supplying operating reserves*". They further discuss the effects of start-up and minimum load costs by saying: "*conclusions regarding the exercise of market power cannot be drawn based on a* *comparison of prices and incremental costs of off-line units*". As a result, they indicate that even if price levels are above marginal cost of production, a generator might not be dispatched because of the existence of start-up costs. This is simply because, marginal costs don't reflect the total incremental costs that are incurred by the firm to start the production.

Similarly Brennan (2003), on the construction of competitive supply curve, points out: "*prices will have to exceed not just average variable costs, but produce enough revenue to cover start-up and shut-down costs before a generator will go online*". He also mentions the demand uncertainty faced by generators must also be taken into account.

In the relevant literature, including Harvey&Hogan (2001), we don't see a theoretical model showing that operational characteristics of the generators will result in economic withholding. Hence, the first aim of this paper is to give a mathematical model to show that operational characteristics indeed result in economic withholding. Furthermore, by including demand uncertainty into the model, we find the thresholds where generators dispatch electricity(or economically withhold capacity). This way, we argue that uncertainty is also a reason for economic withholding. As suggested in the Møllgaard&Nielsen(2004) we use real options analysis to determine optimal dispatch decisions of generators facing start up and shut down costs under uncertainty.

At this point, it would be good to have a look at the literature where valuation and optimal operation of generation technologies were studied using real options methods. Gardner and Zhuang (2000) study a short-term and discrete-time real options model for power plant valuation under some operational constraints, including start-up costs and minimum generation level. By using New England power pool data they calculate that operating constraints, specifically, minimum generation levels can have a significant effect on power plant valuation.

Deng and Oren (2003) use a real options based valuation for power plants incorporating start-up and shut-down costs. They also use price and cost uncertainties in their model. They conclude that the start-up costs reduce the "option value" of a power plant. Furthermore, they show that, under the mean-reversion models for prices, ignoring the start-up cost alone can explain a sizeable portion of the overstated capacity value of a power plant.

Thompson, Davison and Rasmussen (2003) also study valuation and optimal operation of hydroelectric and thermal power generators. For thermal power plants they focus on variable start-up costs and minimum generating levels. As mentioned in Thompson and Rasmussen (2003), related literature on valuation and optimal operation of generation technologies, it is common to have price taking firms in the market. Furthermore, relevant literature also focus on the ownership of a single generation technology. In contrast, we look into a monopolist having two different generation technologies facing start up, shut down costs and minimum operating levels as the operational characteristics of the generators.

On the irreversibility of investments, Dixit&Pindyck (1994 p.249): "Investment is partially or totally irreversible when some or all of its costs are sunk." Pindyck (2008) also discusses sunk costs as: "... a *prospective* sunk cost is quite relevant for the firms decisions, which is why sunk costs play an important role in antitrust analysis. A firm might find it uneconomical to enter a market, for example, if entry involves a large prospective sunk cost". In our analysis, by taking start up and shut-down costs as "prospective sunk costs", we use an optimal switching decisions under uncertainty model (Dixit&Pindyck, Chapter 7).[1]

The rest of the paper is organized as follows: In section 2, we give the formal model and initial derivations. In section 3, we adopt the model to the corresponding social planner's problem to set a benchmark to distinguish the effects of market power. In section 4, we give an example with a specific inverse demand function. In section 5, we give a discussion on economic capacity withholding.

2 The model

As mentioned in the previous section, we use a continuous time "optimal switching decisions under uncertainty model" to show the effects of start-up and shut down costs (alongside minimum operating levels and uncertainty) on the optimal operation of the generators. We are not concerned with either initial investment problem for the existing generators or investment in new generators. A main property of our model is having a hypothetical monopolist using two different generation technologies. We made this assumption/simplification because in the electricity industries, ownership of multiple generation technologies is not uncommon. One can also refer this situation as having electricity generation portfolio.[2]

[1]Harvey and Hogan 2001, also identify start up costs as sunk costs in real time.

[2]Another reason we have two generation technologies is if one of the generators temporarily shut downs, the other generator will still be operating. Therefore, there won't be blackouts when one of the generators shut downs.

In our model, we will combine Dixit&Pindyck(Chapter 7)'s optimal switching model with Hagspiel, Huisman and Kort (2010)'s flexible production under uncertainty model. In this paper, each generator will be able to have flexible production capabilities and optimal switching will mean utilizing one of the generators or switching on the other to utilize both of them. In other words, switching on the inactive generator doesn't mean shutting down the other.

2.1 Assumptions and Setup

The industry consists of a monopolist producing electricity. The monopolist have two types of electricity generation technologies in its disposal. For simplification, we assume that the monopolist only have one *baseload*(B) and one *peaker*(P) generation unit. Each generation unit is characterized by $(K_i, I_i, c_i, \overline{q}_i)$ for $i = B, P$. Where each generation unit has initial capacity K_i, start-up cost I_i and constant marginal cost of production c_i.[3] In addition, $\overline{q}_i$ shows the minimum possible operation level of the generator i. Therefore, at each instant in time, production level q_i of the active generator i satisfies: $0 < \overline{q}_i \leq q_i(t) \leq K_i$.[4] Furthermore, the peaker have shut-down cost E_P.

Generators have infinite lifetime and they are differentiated such that $I_B > I_P$ and $c_B < c_P$. This assumption is in line with literature where peaker generators have higher marginal cost and lower start-up cost. We further assume that there are no transmission costs and generators can adjust their output without a cost.

Our initial problem is to solve for the optimal operation of generation technologies. In general, firms operate their baseload generators almost throughout the year and start peaker generators when price is high enough. For simplification, we'll assume that the monopolist always keeps baseload generator operational[5] and minimum operation level for the baseload generator $(\overline{q}_B)$ is equal to zero. So, our problem boils down to optimal operation of the peaker generator.

At time t, the monopolist produces $Q(t) = q_B(t) + q_P(t)$ units of output. Since generators are capacity constrained, we'll have: $K_B + K_P \geq Q(t) \geq 0$ for all t. Price of electricity fluctuates

[3] In reality, start-up costs may depend on the capacity, technology and the total off-time of the generator. But since we have predetermined capacities and technologies, we'll also treat start-up costs as constant.

[4] $\overline{q}_i$ is purely a technological constraint and could be as small as possible.

[5] This assumption can be justified by very high start up costs for baseload generators. If the firm shut downs the baseload generator, it'll have to incur a very high start up cost to restart the generator. Furthermore, very low marginal costs would also be in favor of this assumption.

stochastically according to:

$$P(t) = D[X(t), Q(t)] \tag{1}$$

where $D : \Theta \text{ x } R_{+} \rightarrow R$ is (twice) continuously differentiable inverse demand function with $\partial D/\partial X > 0$ and $\partial D/\partial Q < 0$. $X(t)$ is the demand shock following a Geometric Brownian Motion(GBM) on the filtered probability space $(\Omega, \mathscr{F}, P)$:

$$dX(t) = \alpha X(t)dt + \sigma X(t)dz \tag{2}$$

Since, we are concerned with the optimal operation of the peaker, we'll start to investigate the state of economy when the baseload is producing at full capacity (i.e, $q_B(t) = K_B$) and the peaker is idle. Therefore, producing an additional(or more precisely, producing at least $\overline{q}_P$) unit of output means making the peaker operational. In that case, the monopolist will incur the start-up cost I_P and start using the peaker. Afterwards, it can shut-down operation by incurring the shut-down cost E_P. In other words, the monopolist switches from operating only one of the generators to operating both of them. We'll be using a similar approach to Dixit&Pindyck(Chapter 7) where we'll determine thresholds X_L and X_H. The monopolist will start operating the peaker when $X(t) > X_H$ and shut down the operation when $X(t) < X_L$. (Therefore, we'll also be able to determine corresponding price thresholds for given demand functions.)

Proposition 1 *The monopolist is not going to start the peaker unless the baseload is operating at full capacity.*

Proof. This proposition is a straightforward result of the assumption $c_B < c_P$. Because of this assumption, when baseload generator is not operating at full capacity, producing an additional unit is more profitable using the baseload than the peaker. ■

In general, state of the industry is characterized by $[X(t), Q(t), \omega]$ where $\omega = 1$ when the peaker is operational, and $\omega = 0$ when the peaker is idle. So, in state $[X(t), Q(t), 0]$ the monopolist decides whether to start the operation of the peaker or not. In state $[X(t), Q(t), 1]$, the monopolist decides whether to shut down the operation of the peaker or not.

Let us denote $V^0[X(t), q_B^*(t)]$ as the expected net present value of the total investment when the peaker generator is in idle state with future optimal strategies. Similarly, $V^1[X(t), q_B^*(t), q_P^*(t)]$

is the expected net present value of the total investment when the peaker generator is in active state with future optimal strategies. Using standard real options techniques, $V^0[X(t), q_B^*(t)]$ will be the solution to Ordinary Differential Equation(ODE):

$$\frac{1}{2}\sigma^2 X(t)^2 V^0_{XX} + \alpha X(t) V^0_X - rV^0 + \Pi_B[X(t), q_B^*(t)] = 0 \tag{3}$$

Similarly, , $V^1[X(t), Q(t)]$ will be the solution to ODE:

$$\frac{1}{2}\sigma^2 X(t)^2 V^1_{XX} + \alpha X(t) V^1_X - rV^1 + \Pi_{B+P}[X(t), q_B^*(t), q_P^*(t)] = 0 \tag{4}$$

Profit function of the peaker consists of three parts. First, peaker is losing money but keeps operating at minimum level. Second, the peaker is making money and operating above minimum level. Third, peaker is operating at full capacity and it would even be profitable to produce more than that level. A thorough discussion on this issue can be found in the following section.

Therefore we'll have,

$$V^0 = \begin{cases} A_1 X^{\beta_1} + Z[X] & \text{if } X < \widehat{X} \\ B_1 X^{\beta_1} + B_2 X^{\beta_2} + Y[X] & \text{if } \widehat{X} \leq X < X_H \\ V^1 - I_P & \text{if } X \geq X_H \end{cases}$$

Similarly,

$$V^1 = \begin{cases} V^0 - E_P & \text{if } X < X_L \\ C_1 X^{\beta_1} + C_2 X^{\beta_2} + W[X] & \text{if } \underline{X} > X \geq X_L \\ D_1 X^{\beta_1} + D_2 X^{\beta_2} + U[X] & \text{if } \overline{X} > X \geq \underline{X} \\ F_2 X^{\beta_2} + T[X] & \text{if } X \geq \overline{X} \end{cases}$$

$E_P > 0$ is the (constant) cost of shutting down the peaker, $\widehat{X}$ is the level where baseload operates at the maximum level, $\underline{X}$ is the level where peaker operates at the minimum level, $\overline{q}_P$, and $\overline{X}$ is the level where peaker operates at the maximum level K_P. Furthermore $T[X]$, $U[X]$, $W[X]$ and $Z[X]$ are the solutions to the corresponding non-homogeneous ODE's. By using value matching and smooth-pasting conditions we can identify the unknowns A_1, B_1, B_2, C_1, C_2, D_1, D_2, F_2, X_L and X_H. Values $\widehat{X}$, $\underline{X}$ and $\overline{X}$ will be calculated by the profit maximization with respect to the given inverse demand function.

Using following value matching and smooth-pasting conditions we find corresponding constants and thresholds:

$$A_1\widehat{X}^{\beta_1} + Z[\widehat{X}] = B_1\widehat{X}^{\beta_1} + B_2\widehat{X}^{\beta_2} + Y[\widehat{X}] \tag{5}$$

$$\beta_1 A_1\widehat{X}^{\beta_1-1} + Z_{\widehat{X}} = \beta_1 B_1\widehat{X}^{\beta_1-1} + \beta_2 B_2\widehat{X}^{\beta_2-1} + Y_{\widehat{X}} \tag{6}$$

$$B_1 X_H^{\beta_1} + B_2 X_H^{\beta_2} + Y[X_H] = D_1 X_H^{\beta_1} + D_2 X_H^{\beta_2} + U[X_H] \tag{7}$$

$$\beta_1 B_1 X_H^{\beta_1-1} + \beta_2 B_2 X_H^{\beta_2-1} + Y_{X_H} = \beta_1 D_1 X_H^{\beta_1-1} + \beta_2 D_2 X_H^{\beta_2-1} + U_{X_H} \tag{8}$$

$$B_1 X_L^{\beta_1} + B_2 X_L^{\beta_2} + Z[X_L] - E_P = C_1 X_L^{\beta_1} + C_2 X_L^{\beta_2} + W[X_L] \tag{9}$$

$$\beta_1 B_1 X_L^{\beta_1-1} + \beta_2 B_2 X_L^{\beta_2-1} + Z_{X_L} = \beta_1 B_1 X_L^{\beta_1-1} + \beta_2 B_2 X_L^{\beta_2-1} + U_{X_L} \tag{10}$$

$$C_1\underline{X}^{\beta_1} + C_2\underline{X}^{\beta_2} + W[\underline{X}] = D_1\underline{X}^{\beta_1} + D_2\underline{X}^{\beta_2} + U[\underline{X}] \tag{11}$$

$$\beta_1 C_1\underline{X}^{\beta_1-1} + \beta_2 C_2\underline{X}^{\beta_2-1} + W_{\underline{X}} = \beta_1 D_1\underline{X}^{\beta_1-1} + \beta_2 D_2\underline{X}^{\beta_2-1} + U_{\underline{X}} \tag{12}$$

$$D_1\overline{X}^{\beta_1} + D_2\overline{X}^{\beta_2} + U[\overline{X}] = F_2\overline{X}^{\beta_2} + T[\overline{X}] \tag{13}$$

$$\beta_1 D_1\overline{X}^{\beta_1-1} + \beta_2 D_2\overline{X}^{\beta_2-1} + T_{\overline{X}} = \beta_2 D_2\overline{X}^{\beta_2-1} + Z_{\overline{X}} \tag{14}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are roots of the quadratic equation:

$$\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r = 0. \tag{15}$$

2.2 Optimal Production and Characterization of The Profit Functions

The monopolist's objective is to maximize the total discounted value of its' investment. Because of this objective, the monopolist will have to decide when to start&shut-down production and, afterwards, maximize its' profits whenever the generators are online. Therefore, we'll have two possible types of profit functions/flows. First, the profits when only the baseload generator is active, and second when both of the generators are active. According to this objective, the monopolist will face start-up and shut-down costs as well as the actual cost of production.

2.2.1 Baseload-Only Generation

For baseload-only generation, when $\omega = 0$, we have the profit function as:

$$\Pi^B[X(t), q_B^*(t)] = sup_{q_B}\{D[X(t), q_B(t)]q_B(t) - c_B q_B(t)\} \tag{16}$$

s.t. $0 \leq q_B \leq K_B$.

If the industry is at a state where $\omega = 0$ then, by setup, $q_B^*(t) = K_B$ as long as we have $X \geq \widehat{X}$. And if we have $X \leq \widehat{X}$ then $q_B^*(t) \leq K_B$.

Note: Observe that $\underline{X} \geq \widehat{X}$. This result is an application of *Proposition 1.* If it is profitable to operate the peaker at or above minimum level, the baseload must be already operating at full capacity.

2.2.2 Baseload and Peaker Generation

According to the monopolist's objective, the peaker generator will be started whenever it yields *positive additional* profits compared to the baseload-only production case. In this case, the total profit function consists of two parts: baseload profits and peaker profits.

$$\Pi^{B+P}[X(t), q_B^*(t), q_P^*(t)] = sup_{q_B(t),q_P(t)}\{\Pi^B[X(t), q_B(t), q_P(t)] + \Pi^P[X(t), q_B(t), q_P(t)]\} \tag{17}$$

Note: When peaker is operational(i.e, $q_P(t) > 0$) this will negatively affect the profits coming from baseload generation since the price will fall when output increases($\partial D/\partial Q < 0$).

Knowing that baseload generator will be producing at full capacity whenever the peaker is acive, a detailed formula for $\Pi^P[X(t), q_P(t)]$ is:

$$\Pi^{B+P}[X(t), q_B(t), q_P(t)] = D[X(t), q_P(t) + K_B]q_P(t) - c_P q_P(t) + D[X(t), q_P(t) + K_B]K_B - c_B K_B \tag{18}$$

Therefore, when both of the generators are online, the optimal generation for the peaker will be:

$$q_P^*(t) = \begin{cases} 0 & \text{if } X < X_L \\ \overline{q}_P & \text{if } \underline{X} > X \geq X_L \\ \overline{q}_P \leq q_P(t) < K_P & \text{if } \overline{X} > X \geq \underline{X} \\ K_P & \text{if } X \geq \overline{X} \end{cases} \tag{19}$$

3 Social Planning and Competitive Equilibrium

In this section, we'll investigate the social planner's problem. Our aim is to use the solution to the social planner's problem as a benchmark. This way, we'll be able to identify the effects of market power for the corresponding monopolist case compared to the benchmark.

In the social planner's problem, we'll need to maximize the discounted total expected consumer surplus. In that regard, we'll use a similar approach to Dixit&Pindyck(Chapter 9). So, we'll need to find the total social surplus, total consumer surplus and total cost for a given production level.

Total social surplus for a given production level, $Q(t)$:

$$U[X(t), Q(t)] = \int_0^{Q(t)} D[X(t), Q(t)]dq. \tag{20}$$

Total consumer surplus for a given production level, $Q(t)$:

$$S^\omega[X(t), Q(t)] = max_Q \{U[X(t), Q(t)] - C^\omega[Q(t)]\} \tag{21}$$

as before, superscript $\omega = 0, 1$ shows whether both of the generators are online. In this setup instantaneous consumer surplus at time t, $S^\omega[X(t), Q(t)]$, will be analogous to the profit flow of a firm. Therefore, using standard real options analysis as before, we can derive $W^0[X(t), q_B^*(t)]$ as the expected net present value of the total investment when the peaker generator is in idle state with future optimal strategies. Similarly, $W^1[X(t), q_B^*(t), q_P^*(t)]$ is the expected net present value of the total investment when the peaker generator is in active state with future optimal strategies. Using standard real options techniques, $W^\omega[X(t), q_B^*(t)]$ will be the solution to Ordinary Differential Equation(ODE):

$$\frac{1}{2}\sigma^2 X(t)^2 W_{XX}^\omega + \alpha X(t) W_X^\omega - rW^\omega + S^\omega[X(t), Q(t)] = 0 \text{ for } \omega = 0, 1 \tag{22}$$

Therefore, as before, we'll have,

$$W^0 = \begin{cases} A_1 X^{\beta_1} + Z[X] & \text{if } X < \widehat{X} \\ B_1 X^{\beta_1} + B_2 X^{\beta_2} + Y[X] & \text{if } \widehat{X} \leq X < X_H \\ W^1 - I_P & \text{if } X \geq X_H \end{cases}$$

Similarly,

$$W^1 = \begin{cases} W^0 - E_P & \text{if } X < X_L \\ C_1 X^{\beta_1} + C_2 X^{\beta_2} + W[X] & \text{if } \underline{X} > X \geq X_L \\ D_1 X^{\beta_1} + D_2 X^{\beta_2} + U[X] & \text{if } \overline{X} > X \geq \underline{X} \\ F_2 X^{\beta_2} + T[X] & \text{if } X \geq \overline{X} \end{cases}$$

As it can be shown, main difference of social planner's problem will entail different solutions for corresponding non-homogeneous ODE's. We'll show the exact difference, given a specific example, in the following section.

4 A Specific Example

In this specific example, we use a linear inverse demand function satisfying assumptions in Section 2.

$$P(t) = X(t) - \gamma Q(t) \quad \text{with } \gamma > 0 \tag{23}$$

For simplification, let's assume that $\overline{q}_B = 0$ and $c_B = 0$, $c_P \geq 0$.

4.1 Monopolist production

One can verify that for $\omega = 0$, when the peaker is idle, we have (given that the optimal production level for the monopolist must satisfy $0 \leq q_B^*(t) \leq K_B$):

$$q_B^*(t) = \frac{X(t)}{2\gamma} \quad , \quad P(t) = \frac{X(t)}{2} \quad \text{and} \quad \Pi^B(t) = \frac{[X(t)]^2}{4\gamma} \tag{24}$$

So,

$$q_B^*(t) = \begin{cases} \dfrac{X(t)}{2\gamma} & \text{if } X < \widehat{X} \\ K_B & \text{if } X \geq \widehat{X} \end{cases} \tag{25}$$

Therefore, $\widehat{X} = 2\gamma K_B$.

Hence,

$$\Pi^B = \begin{cases} \dfrac{[X(t)]^2}{4\gamma} & \text{if } X < 2\gamma K_B \\ (X(t) - \gamma K_B)K_B - c_B K_B & \text{if } X \geq 2\gamma K_B \end{cases} \tag{26}$$

Similarly, for the optimal production levels of the peaker when both of the generators are active:

$$q_P^*(t) = \begin{cases} 0 & \text{if } X < X_L \\ \overline{q}_P & \text{if } \underline{X} > X \geq X_L \\ \dfrac{X(t) - c_P}{2\gamma} - K_B & \text{if } \overline{X} > X \geq \underline{X} \\ K_P & \text{if } X \geq \overline{X} \end{cases} \tag{27}$$

One can verify that $\overline{X} = [2\gamma(K_B + K_P) + c_P]$ and $\underline{X} = [2\gamma(K_B + \overline{q}_P) + c_P]$. And the total profits at time t becomes:

$$\Pi^{B+P} = \begin{cases} 0 & \text{if } X < X_L \\ (X(t) - \gamma K_B - \gamma \overline{q}_P)(K_B + \overline{q}_P) - c_P \overline{q}_P & \text{if } 2\gamma(K_B + \overline{q}_P) + c_P > X \geq X_L \\ \dfrac{(X(t) - c_P)^2}{4\gamma} + c_P K_B & \text{if } 2\gamma(K_B + K_P) + c_P > X \geq 2\gamma(K_B + \overline{q}_P) + c_P \\ (X(t) - \gamma K_B - \gamma K_P)(K_B + K_P) - c_P K_P & \text{if } X \geq 2\gamma(K_B + K_P) + c_P \end{cases} \tag{28}$$

Therefore we'll have,

$$V^0 = \begin{cases} A_1 X^{\beta_1} + \frac{X^2}{4\gamma(r - 2\alpha - \sigma^2)} & \text{if } X < 2\gamma K_B \\ B_1 X^{\beta_1} + B_2 X^{\beta_2} + \frac{K_B}{r - \alpha} X - \frac{(c_B + \gamma K_B)K_B}{r} & \text{if } 2\gamma K_B \leq X < X_H \\ V^1 - I_P & \text{if } X \geq X_H \end{cases}$$

Similarly,

$$V^1 = \begin{cases} B_1 X^{\beta_1} + B_2 X^{\beta_2} + \frac{K_B}{r - \alpha} X - \frac{(c_B + \gamma K_B)K_B}{r} - E_P & \text{if } X < X_L \\ C_1 X^{\beta_1} + C_2 X^{\beta_2} + \frac{K_B + \overline{q}_P}{r - \alpha} X - \frac{\gamma(K_B + \overline{q}_P)^2 + c_P \overline{q}_P}{r} & \text{if } 2\gamma(K_B + \overline{q}_P) + c_P > X \geq X_L \\ D_1 X^{\beta_1} + D_2 X^{\beta_2} + \frac{1}{4\gamma}\left[\frac{X^2}{r - 2\alpha - \sigma^2} - \frac{2c_P}{r - \alpha} X + \frac{c_P^2}{r}\right] + \frac{c_P K_B}{r} & \text{if } 2\gamma(K_B + K_P) + c_P > X \geq 2\gamma(K_B + \overline{q}_P) + c_P \\ F_2 X^{\beta_2} + \frac{K_B + K_P}{r - \alpha} X - \frac{\gamma(K_B + K_P)^2 + c_P K_P}{r} & \text{if } X \geq 2\gamma(K_B + K_P) + c_P \end{cases}$$

Using above value functions and deriving value matching and smooth pasting conditions, we'll have a system of 10 equations.

4.2 Social planner's production

Similarly, when the peaker is idle, social planner's problem will entail:

$$q_B^*(t) = \begin{cases} \dfrac{X(t)}{\gamma} & \text{if } X < \widehat{X} \\ K_B & \text{if } X \geq \widehat{X} \end{cases} \tag{29}$$

where, $\widehat{X} = \gamma K_B$.

Note: Note that in the social welfare maximization, production is higher for a given demand shock. Also, the generator reaches to full capacity at a lower demand shock level.

Additionally, when the peaker is active, social planner's problem will entail:

$$q_P^*(t) = \begin{cases} 0 & \text{if } X < X_L \\ \overline{q}_P & \text{if } \underline{X} > X \geq X_L \\ \dfrac{X(t) - c_P}{\gamma} - K_B & \text{if } \overline{X} > X \geq \underline{X} \\ K_P & \text{if } X \geq \overline{X} \end{cases} \tag{30}$$

where $\overline{X} = [\gamma(K_B + K_P) + c_P]$ and $\underline{X} = [\gamma(K_B + \overline{q}_P) + c_P]$.

Therefore, value functions will be given by:

$$W^0 = \begin{cases} A_1 X^{\beta_1} & \text{if } X < \gamma K_B \\ B_1 X^{\beta_1} + B_2 X^{\beta_2} + \frac{K_B}{r-\alpha} X - \frac{(c_B + \gamma K_B) K_B}{r} & \text{if } \gamma K_B \leq X < X_H \\ W^1 - I_P & \text{if } X \geq X_H \end{cases}$$

Similarly,

$$W^1 = \begin{cases} B_1 X^{\beta_1} + B_2 X^{\beta_2} + \frac{K_B}{r-\alpha} X - \frac{(c_B + \gamma K_B) K_B}{r} - E_P & \text{if } X < X_L \\ C_1 X^{\beta_1} + C_2 X^{\beta_2} + \frac{K_B + \overline{q}_P}{r-\alpha} X - \frac{\gamma(K_B + \overline{q}_P)^2 + c_P \overline{q}_P}{r} & \text{if } \gamma(K_B + \overline{q}_P) + c_P > X \geq X_L \\ D_1 X^{\beta_1} + D_2 X^{\beta_2} + \frac{c_P K_B}{r} & \text{if } \gamma(K_B + K_P) + c_P > X \geq \gamma(K_B + \overline{q}_P) + c_P \\ F_2 X^{\beta_2} + \frac{K_B + K_P}{r-\alpha} X - \frac{\gamma(K_B + K_P)^2 + c_P K_P}{r} & \text{if } X \geq \gamma(K_B + K_P) + c_P \end{cases}$$

5 Numerical Results

For the purpose of getting numerical results, we use the derivations in the previous section by taking: $\alpha = 0.01$, $\sigma = 0.05$, $r = 0.05$, $\gamma = 0.1$, $K_B = 400$, $K_P = 100$, $\overline{q}_B = 0$, $\overline{q}_P = 20$, $c_B = 0$, $c_P = 38$, $I_P = 1000$ and $E_P = 100$.

5.1 Monopolist production

Given above values for the parameters we have, $\widehat{X} = 80$, $\underline{X} = 122$ and $\overline{X} = 138$. Therefore, trigger demand shock level for switching on the peaker is, $X_H \in [122, 138]$. Hence, trigger price for switching on the peaker is, $P_H \in [82, 98]$. Similarly, trigger demand shock level for switching off the peaker is, $X_L \in [80, 122]$. Hence, trigger price for switching off the peaker is, $P_L \in [38, 80]$.[6]

5.2 Social planner's production

Given above values for the parameters we have, $\widehat{X} = 40$, $\underline{X} = 80$ and $\overline{X} = 88$. Therefore, trigger demand shock level for switching on the peaker is, $X_H \in [80, 88]$. Hence, trigger price for switching on the peaker is, $P_H \in [40, 48]$. Similarly, trigger demand shock level for switching off the peaker is, $X_L \in [40, 80]$. Hence, trigger price for switching on the peaker is, $P_L \in [0, 38]$.

6 Discussion & Conclusion

In our method above we have showed that, by using real options analysis, the firm(s) will wait until prices are well above marginal costs to start generation at peaker level. Therefore our theoretical model supports what Harvey&Hogan (2001) argue on economic withholding.

Looking at the numerical results we can see some interesting and intuitive findings. First, unsurprisingly, price(or underlying demand shock) trigger for switching on the peaker is higher in the monopoly case. We can argue that the difference between monopoly and social planner cases, stems from the existence and exercise of market power in the monopoly case. Second, even in the benchmark social planner problem, trigger price for starting the generator exceeds the marginal cost. Therefore, even in social planner's case, uncertainty and operational constraints have to be

[6] Here, we use the properties that $\underline{X} \leq X_H \leq \overline{X}$ and $\widehat{X} \leq X_L \leq \underline{X}$.

taken into account and the peaker have to be switched on when the market price is above their marginal cost.

A Appendix Additional Model Details and Results

A.1 Monopolist production

$$A_1\widehat{X}^{\beta_1} + \frac{\widehat{X}^2}{4\gamma(r-2\alpha-\sigma^2)} = B_1\widehat{X}^{\beta_1} + B_2\widehat{X}^{\beta_2} + \frac{K_B}{r-\alpha}\widehat{X} - \frac{(c_B+\gamma K_B)K_B}{r} \tag{31}$$

$$\beta_1 A_1\widehat{X}^{\beta_1-1} + \frac{2\widehat{X}}{4\gamma(r-2\alpha-\sigma^2)} = \beta_1 B_1\widehat{X}^{\beta_1-1} + \beta_2 B_2\widehat{X}^{\beta_2-1} + \frac{K_B}{r-\alpha} \tag{32}$$

$$B_1X_H^{\beta_1} + B_2X_H^{\beta_2} + \frac{K_B}{r-\alpha}X_H - \frac{(c_B+\gamma K_B)K_B}{r} = D_1X_H^{\beta_1} + D_2X_H^{\beta_2} + \frac{1}{4\gamma}\left[\frac{X_H^2}{r-2\alpha-\sigma^2} - \frac{2c_P}{r-\alpha}X_H + \frac{c_P^2}{r}\right] + \frac{c_PK_B}{r} \tag{33}$$

$$\beta_1B_1X_H^{\beta_1-1} + \beta_2B_2X_H^{\beta_2-1} + \frac{K_B}{r-\alpha} = \beta_1D_1X_H^{\beta_1-1} + \beta_2D_2X_H^{\beta_2-1} + \frac{1}{4\gamma}\left[\frac{2X_H}{r-2\alpha-\sigma^2} - \frac{2c_P}{r-\alpha}\right] \tag{34}$$

$$B_1X_L^{\beta_1} + B_2X_L^{\beta_2} + \frac{K_B}{r-\alpha}X_L - \frac{(c_B+\gamma K_B)K_B}{r} - E_P = C_1X_L^{\beta_1} + C_2X_L^{\beta_2} + \frac{K_B+\overline{q}_P}{r-\alpha}X_L - \frac{\gamma(K_B+\overline{q}_P)^2 + c_P\overline{q}_P}{r} \tag{35}$$

$$\beta_1B_1X_L^{\beta_1-1} + \beta_2B_2X_L^{\beta_2-1} + \frac{2X_L}{4\gamma(r-2\alpha-\sigma^2)} = \beta_1C_1X_L^{\beta_1-1} + \beta_2C_2X_L^{\beta_2-1} + \frac{K_B+\overline{q}_P}{r-\alpha} \tag{36}$$

$$C_1\underline{X}^{\beta_1} + C_2\underline{X}^{\beta_2} + \frac{K_B+\overline{q}_P}{r-\alpha}\underline{X} - \frac{\gamma(K_B+\overline{q}_P)^2 + c_P\overline{q}_P}{r} = D_1\underline{X}^{\beta_1} + D_2\underline{X}^{\beta_2} + \frac{1}{4\gamma}\left[\frac{\underline{X}^2}{r-2\alpha-\sigma^2} - \frac{2c_P}{r-\alpha}\underline{X} + \frac{c_P^2}{r}\right] + \frac{c_PK_B}{r} \tag{37}$$

$$\beta_1C_1\underline{X}^{\beta_1-1} + \beta_2C_2\underline{X}^{\beta_2-1} + \frac{K_B+\overline{q}_P}{r-\alpha} = \beta_1D_1\underline{X}^{\beta_1-1} + \beta_2D_2\underline{X}^{\beta_2-1} + \frac{1}{4\gamma}\left[\frac{2\underline{X}}{r-2\alpha-\sigma^2} - \frac{2c_P}{r-\alpha}\right] \tag{38}$$

$$D_1\overline{X}^{\beta_1} + D_2\overline{X}^{\beta_2} + \frac{1}{4\gamma}\left[\frac{\overline{X}^2}{r-2\alpha-\sigma^2} - \frac{2c_P}{r-\alpha}\overline{X} + \frac{c_P^2}{r}\right] + \frac{c_PK_B}{r} = F_2\overline{X}^{\beta_2} + \frac{K_B+K_P}{r-\alpha}\overline{X} - \frac{\gamma(K_B+K_P)^2 + c_PK_P}{r} \tag{39}$$

$$\beta_1D_1\overline{X}^{\beta_1-1} + \beta_2D_2\overline{X}^{\beta_2-1} + \frac{1}{4\gamma}\left[\frac{2\overline{X}}{r-2\alpha-\sigma^2} - \frac{2c_P}{r-\alpha}\right] = \beta_2F_2\overline{X}^{\beta_2-1} + \frac{K_B+K_P}{r-\alpha} \tag{40}$$

where $\beta_1 > 1,\ \beta_2 < 0,\ \widehat{X} = 2\gamma K_B,\ \overline{X} = [2\gamma(K_B+K_P)+c_P]$ and $\underline{X} = [2\gamma(K_B+\overline{q}_P)+c_P]$.

A.2 Social planner's production

$$A_1\widehat{X}^{\beta_1} = B_1\widehat{X}^{\beta_1} + B_2\widehat{X}^{\beta_2} + \frac{K_B}{r-\alpha}\widehat{X} - \frac{(c_B+\gamma K_B)K_B}{r} \tag{41}$$

$$\beta_1A_1\widehat{X}^{\beta_1-1} = \beta_1B_1\widehat{X}^{\beta_1-1} + \beta_2B_2\widehat{X}^{\beta_2-1} + \frac{K_B}{r-\alpha} \tag{42}$$

$$B_1X_H^{\beta_1} + B_2X_H^{\beta_2} + \frac{K_B}{r-\alpha}X_H - \frac{(c_B+\gamma K_B)K_B}{r} = D_1X_H^{\beta_1} + D_2X_H^{\beta_2} + \frac{c_PK_B}{r} \tag{43}$$

$$\beta_1 B_1 X_H^{\beta_1 - 1} + \beta_2 B_2 X_H^{\beta_2 - 1} + \frac{K_B}{r-\alpha} = \beta_1 D_1 X_H^{\beta_1 - 1} + \beta_2 D_2 X_H^{\beta_2 - 1} \tag{44}$$

$$B_1 X_L^{\beta_1} + B_2 X_L^{\beta_2} + \frac{K_B}{r-\alpha} X_L - \frac{(c_B + \gamma K_B) K_B}{r} - E_P = C_1 X_L^{\beta_1} + C_2 X_L^{\beta_2} + \frac{K_B + \overline{q}_P}{r-\alpha} X_L - \frac{\gamma (K_B + \overline{q}_P)^2 + c_P \overline{q}_P}{r} \tag{45}$$

$$\beta_1 B_1 X_L^{\beta_1 - 1} + \beta_2 B_2 X_L^{\beta_2 - 1} + \frac{K_B}{r-\alpha} = \beta_1 C_1 X_L^{\beta_1 - 1} + \beta_2 C_2 X_L^{\beta_2 - 1} + \frac{K_B + \overline{q}_P}{r-\alpha} \tag{46}$$

$$C_1 \underline{X}^{\beta_1} + C_2 \underline{X}^{\beta_2} + \frac{K_B + \overline{q}_P}{r-\alpha} \underline{X} - \frac{\gamma (K_B + \overline{q}_P)^2 + c_P \overline{q}_P}{r} = D_1 \underline{X}^{\beta_1} + D_2 \underline{X}^{\beta_2} + \frac{c_P K_B}{r} \tag{47}$$

$$\beta_1 C_1 \underline{X}^{\beta_1 - 1} + \beta_2 C_2 \underline{X}^{\beta_2 - 1} + \frac{K_B + \overline{q}_P}{r-\alpha} = \beta_1 D_1 \underline{X}^{\beta_1 - 1} + \beta_2 D_2 \underline{X}^{\beta_2 - 1} \tag{48}$$

$$D_1 \overline{X}^{\beta_1} + D_2 \overline{X}^{\beta_2} + \frac{c_P K_B}{r} = F_2 \overline{X}^{\beta_2} + \frac{K_B + K_P}{r-\alpha} \overline{X} - \frac{\gamma (K_B + K_P)^2 + c_P K_P}{r} \tag{49}$$

$$\beta_1 D_1 \overline{X}^{\beta_1 - 1} + \beta_2 D_2 \overline{X}^{\beta_2 - 1} = \beta_2 F_2 \overline{X}^{\beta_2 - 1} + \frac{K_B + K_P}{r-\alpha} \tag{50}$$

where $\beta_1 > 1,\ \beta_2 < 0,\ \widehat{X} = \gamma K_B,\ \overline{X} = [\gamma(K_B + K_P) + c_P]$ and $\underline{X} = [\gamma(K_B + \overline{q}_P) + c_P]$.

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