The Impact of Stochastic Extraction Cost on the Value of an Exhaustible Resource: The Case of the Alberta Oil Sands

Abdullah Almansour [1] Margaret Insley[2]

May 18, 2011

Abstract

In a much cited paper, Brennan and Schwartz (1985) demonstrated the application of contingent claims analysis to the valuation of a nonrenewable natural resources project when the decision-maker has flexibility to choose from several modes of operations - open, closed and abandoned. The authors assumed fixed extraction costs and that the price of the resource follows Geometric Brownian Motion. The resulting stochastic optimal control problem must be solved numerically, such as with a finite difference approach. For natural resource extraction projects, uncertain costs are also important in optimal decisions and have been less studied in the literature. An example is the oils sands industry where natural gas is used as energy to extract the bitumen, and contributes more than 25 percent of the total per barrel cost. In this paper, we extend the Brennan and Schwartz (1985) model to account for stochastic extraction cost as well as stochastic convenience yield and resource price, and we study the impact on the value of an oil field and optimal decisions regarding extraction. We show that introducing stochastic extraction cost has a substantial impact on value and on the cut-off prices at which it is optimal for the field to switch from one operation mode to another. We use a relatively new method for the evaluation of American-type options - the Least Squares Monte Carol method - which can more easily deal with multiple stochastic factors than traditional numerical approaches.

[1]PhD candidate in Department of Economics, University of Waterloo, Waterloo ON, Canada
[2]Associate professor in Department of Economics, University of Waterloo, Waterloo ON, Canada

Introduction

Traditionally, valuing a natural resource project, or any project in general, is based on the simple net present value method. Using this method, expected future cash flows from operating the project are discounted to the current time using a constant risk adjusted discount rate and added up to give the value of the project. This procedure has been criticized for ignoring possible flexibilities in starting or operating a project. Examples of such ignored flexibilities are: the flexibility in starting the investment (the option to delay) and the flexibility to switch between different mode of operations (option to switch). In addition, the use of a constant risk adjusted discount rate is known to be inappropriate for valuing projects.

On the other hand, in real option valuation method, managerial flexibilities are taken into consideration when valuing a project. In addition, the real option approach provides a better treatment of risk. The real option method is based on the analogy between financial options and investment projects, and thus it uses the valuation tools developed for financial options. For more details on this method and its features, see Dixit and Pindyck (1994) and Schwartz and Trigeorgis (2004) .

In their seminal paper, Brennan and Schwartz (1985) set the ground for using contingent claims analysis for valuing a nonrenewable natural resources project when the decision-maker has flexibility to choose from multiple modes of operations. They assumed fixed extraction costs and that the price follows Geometric Bownian Motion (GBM). An analytical solution to such a problem is unavailable, so they solve the problem using a finite difference numerical method. Recent developments in valuing American options using simulation based methods enable researchers to explore more realistic extensions to the Brennan and Schwartz (1985) model that proved to be impractical to solve using the prevailing numerical methods such as finite difference or lattice methods.

The Least Square Monte Carlo (LSMC) method developed by Longstaff and Schwartz (2001) has proved to be an efficient tool for valuing complex real option problems. Gamba (2003) provide a comprehensive overview on how LSMC could be used to value various types of real options. For valuing exhaustible resources, Cortazar et al. (2008) explained the applicability of this method in valuing switching options. They extend Brennan and Schwartz (1985) to include the Cortazar and Schwartz (2003) three factor model. Their purpose was mainly to show how LSMC can be applied to value such complex problems. Tsekrekos et al. (2010) studied the Brennan and Schwartz (1985) valuation problem under different price model dynamics.

However, one aspect that seems to be ignored in this literature, valuing exhaustible resources using contingent clams analysis, is the possibility that extraction cost, along with other state variables like spot price and convenience yield, is stochastic and volatile as well. A perfect example where volatility of extraction cost appears to be salient is the oil sands industry.

The oil sands industry consumes substantial amounts of natural gas during production and upgrading activities. According to Canadian Energy Research Institute (CERI), natural gas, its price being highly volatile, contributes more than 25 percent of the total per barrel supply cost[3]. "In 2007, the oil sands industry accounted for approximately 1.0 bcf/d of natural gas demand, slightly more than 40 percent of Alberta total natural gas demand of 2.7 bcf/d"[4].

In this paper, we use the LSMC method and extend the Brennan and Schwartz (1985) model to account for stochastic stochastic extraction cost and study the impact of it on the value of an oil sands project. Moreover, in extending Brennan and Schwartz (1985) model, we also account for stochastic convenience yield. Casassus and Collin-Dufresne (2005) and Tsekrekos et al. (2010) have shown that failing to account for stochastic convenience yield have substantial impact on real options valuation.

The paper is organized as following: section one gives a background on oil sands production and how much natural gas is used. Next section specifies the model used to value an oil sands project. Third section shows the simulation results for a hypothetical oil sands projects. At the end, a conclusion is given summarizing the findings of the paper.

1 Oil Sands Background

The oil sands are unevenly spread over 140,000 km2 (54,000 square miles) in Northern Alberta, Canada. The area contains an estimated 1.7 trillion barrels (initial volume-in-place) of an extremely heavy crude oil referred to as bitumen[5]. This reserve is believed to be promising given its size, the current and expected high prices of crude oil and the state of the global supply and demand of the oil

[3]The supply cost is the constant dollar price needed to recover all capital expenditures, operating costs, royalties, taxes, and earn a specified return on investment

[4]see McColl and Slagorsky, "Canadian Oil Sands Supply Costs and Development Projects (2008-2030)" Canadian Energy Research Institute, 2008

[5]Crude bitumen, or bitumen, is a term that reflects the heavy and highly viscous oil in the oil sands areas. The term "oil sands" includes the crude bitumen, minerals, and rocks that are found together with the bitumen (www.ERCB.com)

Table 1: Operation Cost for Bitumen In situ Production

Operating Cost (Excluding Energy)	
Fixed Operation Cost	C$ 47.6 Millions per year
Variable Operating Cost	6.6 per barrel
Natural Gas Cost	7.5 per barrel
Total (for capacity of 30,000 barrel per day)	18 per barrel
Total (WTI equivalent)	35 per barrel

Source: Canadian Energy Research Institute (CERI), 2008

market. According to Canadian Association of Petroleum Producers (CAPP), Capital expenditure in oil sands projects has risen from 4.2 billions in 2000 to 11.2 billions in 2009. [6]

Approximately 20 percent of of Alberta's oil sands can be found close enough under the surface (generally less than 75 meters) to permit mining production. On the other hand, around 80 percent of this reserve is found too deep below the surface for feasible mining operations. Bitumen in such deep deposits (typically 400 meters below the surface) needs to be recovered from the in situ (Latin: in place) position, similar to conventional oil, but by using a variety of special production techniques.

In in-situ extraction techniques, a high temperature steam is injected inside the bitumen deposit through horizontal or vetrical wells to reduce its viscosity and make it easier to be pumped up to the surface. The steam generators used within the process use natural gas as a fuel source. According to CERI, a rule-of-thumb commonly used in the industry is that 1.0 Mcf (thousand cubic feet) of natural gas is required to produce a barrel of bitumen. It is estimated that natural gas usage amounts to about 45 percent of total per-barrel operating cost. Table 1 shows the per-barrel of bitumen operating cost for a typical in situ project.

A typical in situ oil sands plant consists of multiple well pads containing a group of wells where bitumen is extracted and a central processing facility (CPF) where the extracted bitumen is processed to meet certain specifications. Steam from the CPF is transported by pipeline to the well pads and distributed to the various wells. Produced water and bitumen from the wells is then taken back for processing in the CPF. The majority of the bitumen is upgraded to produce Synthetic Crude Oil (SCO). Given this heavy dependency on natural gas in bitumen production in oil sands

[6] see 2011 Statistical Handbook in (http://www.capp.ca)

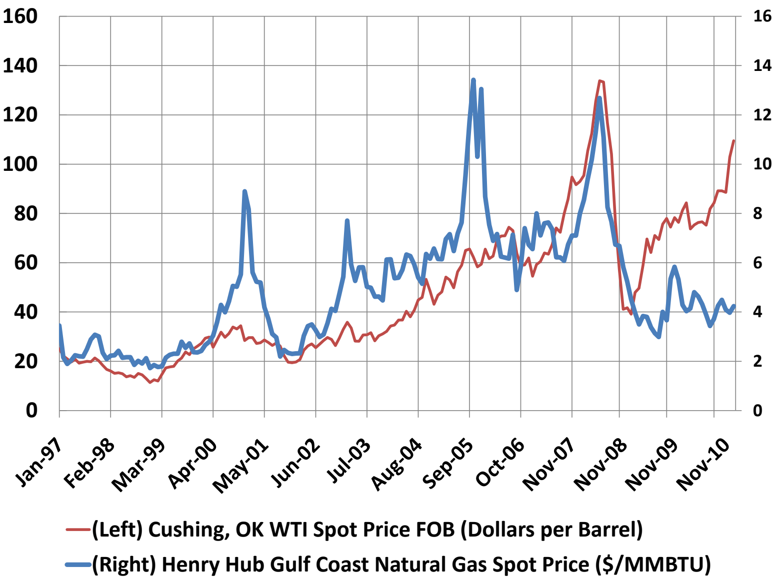


Figure 1: WTI Crude Oil and HH Natural Gas Prices

industry, uncertainty in natural gas price results in an important risk factor that need to account for. Natural gas prices are characterized by high volatility and high correlation with other energy markets especially oil market (see Pindyck (2004), Geman (2005) and Brown and Yucel (2007)). Figure 1 shows the price of natural gas at Henry Hub, a major trading point located in the south of the US on the Gulf of Mexico (the most active hub in the world), along with the price of WTI crude oil since 1997 until 2010. As can be seen from the graph, the natural gas price is highly volatile. Moreover, the two prices tend to move in the same direction. In fact, Villar and Joutz (2006) find a support to the presence of a cointegrating relationship between the crude oil and natural gas price time series.

In this paper, we study the impact of this risk factor on the value of an oil sands project and on the optimal operation under which it should be operated.

2 Model Specification

Consider a competitive firm that operates an oil sand project to extract bitumen from known inventory of Q units. The is projects under operation which means that initial cost to build the facility is sunk. The spot price of crude oil is governed by the following stochastic differential equations proposed by Gibson and Schwartz (1990):

$$dS_t = (\mu - \delta_t)\, S_t dt + \sigma_s S_t dz_{st} \tag{1}$$

$$d\delta_t = \kappa(\theta - \delta_t)dt + \sigma_\delta dz_{\delta t} \tag{2}$$

$$dz_{st} dz_{\delta t} = \rho_{s\delta} dt \tag{3}$$

where S_t is the spot price with μ rate of return and volatility of σ_s. δ_t is the convenience yield for having one unit of the commodity at hand. It reverts to the long run value of θ with a speed of κ and it has volatility of σ_δ. dz_{st} and $dz_{\delta t}$ are increments of correlated Brownian motions[7] with correlation factor of $\rho_{s\delta}$

When the project is in operation, the profit flow rate generated by selling the produced amount from t to $t + dt$ is given by:

$$\Pi_t = (1 - \tau)q_t\big(S_t - C_t\big) \tag{4}$$

where q_t is the optimal rate of production in barrels per unit of time which is assumed to be known to the management. C_t is the cost of producing one barrel in dollars and τ is the total taxes of income tax plus royalties, $\tau = \tau_{inc} + \tau_r$.

Cost of production, C_t, is assumed to have a deterministic component, C_F, and, unlike existing models in the literature, a stochastic component, v_t. That is:

$$C_t = C_F + v_t \tag{5}$$

where

$$dv_t = (\mu_v - \delta_v)v_t dt + \sigma_v v_t dz_{vt} \tag{6}$$

[7] Brownian motion is a continuous-time stochastic process that has independent increments of normal distribution with mean of zero and variance of time difference, i.e. if $z(t)$ is a Brownian motion then $dz(t) \sim N(0, dt)$. For more details see Klebaner (2005)

and

$$dz_{st}dz_{vt} = \rho_{sv}dt \tag{7}$$

where v_t is the variable per barrel cost of production. It has μ_v rate of return, δ_v convenience yield and a volatility of σ_v. dz_{vt} is an increment of a Brownian motion that has a correlation factor of ρ_{sv} with the spot price process increment and is independent of the convenience yield process. In the case of the oil sand industry, v_t corresponds mainly to the price of natural gas used in extraction. Our main objective is to study the impact of extraction cost being stochastic and thus we abstract from the complexities in modeling natural gas prices (such as stochastic volatility and stochastic convenience yield, see for example Geman (2005)) which can be studied in future research.

Depending on the profitability of the crude oil price, the decision-maker has the option to switch between different modes of operation. When the price goes low enough, the decision-maker can incur a fixed cost, K_{oc}, and suspend the operation until the price level goes back up to profitable levels. During suspension, the decision-maker should also incur a flow of maintenance cost, M. If the price drops dramatically to very low levels, the decision-maker has the option to abandon the project permanently. On the other hand, if the resource currently is closed and the price recovers to a profitable level, the decision-maker has the option to reopen the field again by paying another fixed cost of K_{co}.

One fundamental result in option pricing theory is that, under the assumption of no arbitrage, there exists a probability measure such that the option value is the sum of all expected future cash flows discounted at the risk free rate[8]. This measure is called the risk-neutral measure. If the option is of the American type, its value is the the sum of all expected future cash flows from pursuing the optimal exercise policy discounted at the risk-free rate. The above project can be seen as an American option where the underlying assets are the three stochastic variables: the spot price, S_t, the convenience yield, δ_t, and the cost process, C_t. Thus, the value of the project is the sum of all expected future cash flows discounted at the risk free rate, provided that the optimal policy of switching between operation modes is pursued.

Let X_t denote the vector of the values of state variables at time t, i.e $X_t = [S_t, \delta_t, C_t]$. The value of the project would then be governed by the following two Bellman equations for currently open and closed projects respectively:

[8]For more details, see Björk (2003)

$\mathbf{V}_{open}\big(X_t, Q, t\big) =$

$$max \begin{cases} \Pi(t)dt + e^{-(r+\tau_o)dt} E_t\big[\mathbf{V}_{open}(X_{t+dt}, Q - qdt, t + dt)\big] & \text{open} \\ -Mdt - K_{oc} + e^{-(r+\tau_c)dt} E_t\big[\mathbf{V}_{closed}(X_{t+dt}, Q, t + dt)\big] & \text{close} \\ 0 & \text{abandon} \end{cases} \tag{8}$$

$\mathbf{V}_{closed}\big(X_t, Q, t\big) =$

$$max \begin{cases} \Pi(t)dt - K_{co} + e^{-(r+\tau_o)dt} E_t\big[\mathbf{V}_{open}(X_{t+dt}, Q - qdt, t + dt)\big] & \text{re-open} \\ -Mdt + e^{-(r+\tau_c)dt} E_t\big[\mathbf{V}_{closed}(X_{t+dt}, Q, t + dt)\big] & \text{close} \\ 0 & \text{abandon} \end{cases} \tag{9}$$

where τ_i , $i = o$ or c, is the property tax rates proportional to project value when it is open and when it is closed respectively. As mentioned above, the expectations are taken under the risk-neutral measure. Under such measure, the states variables behave as follows [9]:

$$dS_t = (r - \delta_t)\, S_t dt + \sigma_s S_t d\hat{z}_{st} \tag{10}$$

$$d\delta_t = (\kappa(\theta - \delta_t) - \lambda_\delta)dt + \sigma_\delta d\hat{z}_{\delta t} \tag{11}$$

$$dv_t = (r - \delta_v)v_t dt + \sigma_v v_t d\hat{z}_{vt} \tag{12}$$

$$d\hat{z}_{st} d\hat{z}_{\delta t} = \rho_{s\delta} dt \tag{13}$$

$$d\hat{z}_{st} d\hat{z}_{vt} = \rho_{sv} dt \tag{14}$$

where r is the risk free rate and λ_δ is the market price of risk associated with the convenience yield process and assumed to be constant. $d\hat{z}_{st}$, $d\hat{z}_{\delta t}$ and $d\hat{z}_{vt}$ are increments of Brownian motion under the risk neutral measure.

Analytical solutions to equations (8) and (9) are unavailable, thus numerical methods should be used. For such complex problems, finite difference or lattice methods have proved to be impractical. On the other hand, the Least Square Monte Carlo (LSMC) method developed by Longstaff and Schwartz (2001) has proved to be an efficient tool for such problems (see Cortazar et al. (2008)).

[9] Details on deriving the risk neutral process for the purpose of derivative pricing can be found in Björk (2003).

The procedure starts by simulating a large number of paths of X_t from the current time to time T when the project is over. Then, backward induction is carried out starting from time T up to the current time using the two Bellman equations stated above. The essence of the LSMC method is in the way it calculates the expectation of the project values in each simulated path at each time step. It achieves this task by path-wise regression of the project value at each node, on a linear combination of basis functions of the state variables at each node. That is:

$$E_t[\mathbf{V}(\omega, X_{t+dt})] = \sum_{j=1}^{N} a_j \Psi_j(\omega, X_t) \tag{15}$$

where ω is a simulated path, a_j are constants and N is the number of the basis functions. At each time step, the values of a_j are estimated by regressing the discounted value of the project at the next time step for each path, which is the sum of all discounted future cash flows along the path, on $\Psi_j(\omega, X_t)$. Although the choice of the basis functions is arbitrary, Tsekrekos et al. (2010) shows that the procedure is robust to different choices and that simple power functions are enough for reasonable results.

3 Results

To accomplish the objective of this study, we consider valuing a hypothetical oil sands project that has a capacity of 4 millions barrel per year. Per unit initial deterministic cost, C_F, is assumed to be \$21 and it rises at the inflation rate of 2.5%. Per unit initial variable cost, v_0, is assumed to be \$14. When the operation is suspended and the project is temporally closed, the maintenance cost is assumed to be at a rate of \$4 million per year. These assumptions are adopted from a CERI report on Canadian oil sands supply costs[10]. The costs of switching from open to suspended and visa versa is assumed to be \$1 million. Moreover, for simplicity, we assume that the project could be abandoned at no cost. Switching and maintenance costs all rise at the rate of inflation. Since the profit flow, Π_t, is linear in the extraction rate, q_t, the optimal rate of extraction is either to extract at capacity or not to extract depending on whether the price is higher or lower than marginal value of the reserve (see Pindyck (1980)). Thus, whenever it is in operation, the project would extract at

[10]See McColl and Slagorsky "Canadian Oil Sands Supply Costs and Development Projects (2008-2030)" Canadian Energy Research Institute, November 2008

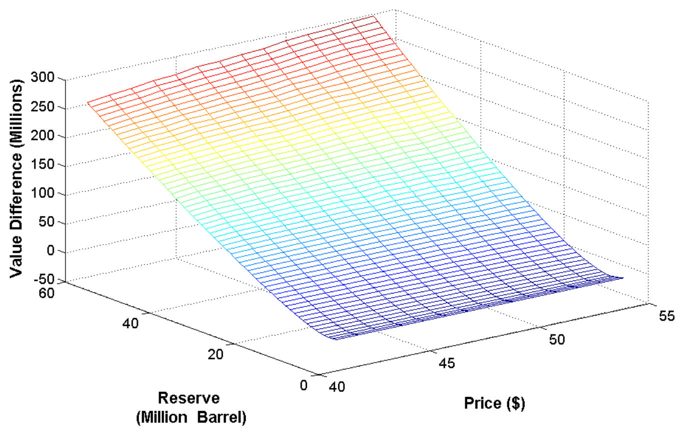
Table 2: Parameter Values for Valuating a Hypothetical Oil Sands Project

State Processes Parameters		
Risk free rate	r	4%
Volatility of $S(t)$	σ_s	40%
Volatility of $\delta(t)$	σ_δ	66%
Speed of the mean reversion of $\delta(t)$	κ	2.4
Long run convenience yield	θ	1%
Market price of convenience yield risk	λ_δ	0
Convenience yield of $v(t)$	δ_v	2%
Taxes		
Income tax	τ_{inc}	19%
Royalty tax	τ_r	30%
Property tax when the field is open	τ_o	1% per year
Property tax when the field is closed	τ_c	1% per year
Inflation Rate	π	2.5%

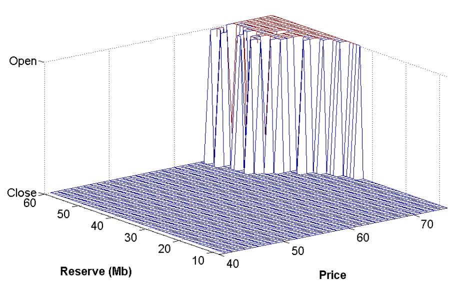
the capacity rate.

For the oil price process and convenience yield process parameters, we run a seemingly unrelated regression (SUR) on daily data of the two processes from the beginning of 2000 until the end of 2009. For the parameters of the stochastic cost process, we estimated the unconditional volatility of natural gas and the correlation factor between crude oil and natural gas return series to set the values of σ_v and ρ_{sv} respectively. In the simulation process, we generate 100,000 paths of X_t (50,000 plus 50,000 antithetic). Moreover, we assume that the decision-maker, for simplicity, has four opportunities per year to switch between operating modes. Table 2 lists these values along with other assumptions for other parameters values needed for for the valuating process. We obtained the value of the project as a function of the remaining reserve (Q) and the oil price.

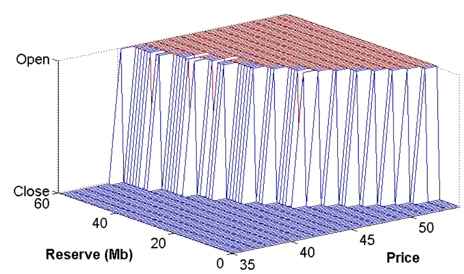
Figure 2(a) confirms the findings of Casassus and Collin-Dufresne (2005) and Tsekrekos et al. (2010) that the modeling choice of convenience yield has a substantial impact on real options valuation. The figure shows the impact of the convenience yield of the crude oil being stochastic. It plots the value of the project under the Brennan and Schwartz (1985) one factor model when convenience yield is constant, $\delta_t = \theta$, and its value under the Gibson and Schwartz (1990) two factors model presented in equations (1) and (2). Although the constant convenience yield in the Brennan and Schwartz (1985) one factor model and the long run convenience yield in the Gibson and Schwartz (1990) two factors model are equal, there is a substantial difference in the value of the project under both models. In relation to the the optimal operating policy, Figure 2(b) and 2(c) shows that, under the one factor model, the field should be closed until much higher prices than it should be under the two factors model when stochastic convenience yield is introduced. When $Q = 6$ million, the project would extract at around \$40 under the two factor model while it is optimal to wait until the price reaches \$66 under the one factor model. The above result is due to the fact that high correlation between spot and convenience yield induces mean reversion in the price while it is expected to rise at a constant rate under the one factor model. Mean reversion is observed because if the current price is high, the correlated convenience yield goes high and higher convenience yield, in turn, reduces the drift rate of, S_t, and hence future prices.



(a) The value of an operating oil sands project under the one factor model minus its value under the two factor model when stochastic convenience yield is included ($\kappa = 2.4$, $\theta = 1\%$ and $\sigma_\delta = 66\%$), stochastic cost is included in both.



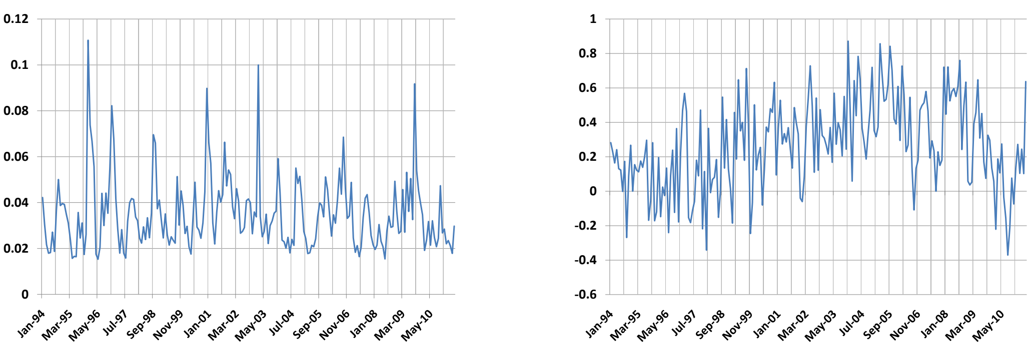
(b) Constant Convenience yield, $\delta = 1\%$



(c) Stochastic Convenience yield $\kappa = 2.4$, $\theta = 1\%$ and $\sigma_\delta = 66\%$

Figure 2: The Impact Of Stochastic Convenience Yield

The impact of stochastic cost depends on the volatility of the natural gas market and its co-movement with oil market. The volatility of natural gas and the correlation between crude oil and natural gas is not constant and depends on supply and demand factors in both markets (see Villar and Joutz (2006)). Figure 3 shows the monthly realized volatility of daily natural gas returns and the monthly correlation between daily crude oil and natural gas returns from 1994 until 2011. It is clear that both parameters have high variation. Given this fact, we calculate the value of the project for different values of σ_v and ρ_{sv}.

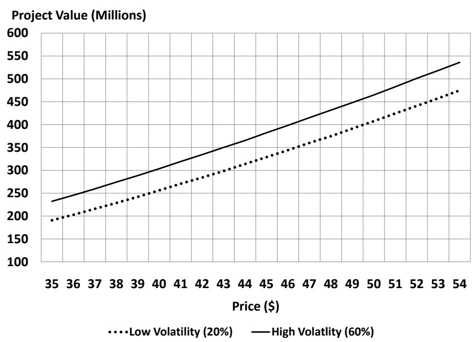


(a) Monthly Volatility of HH Natural Gas return

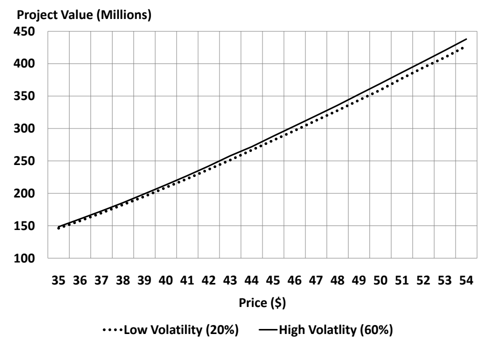
(b) Monthly Correlation between WTI Crude oil and HH Natural Gas Returns

Figure 3: Time Series of Natural Gas Volatility and Correlation with Crude Oil

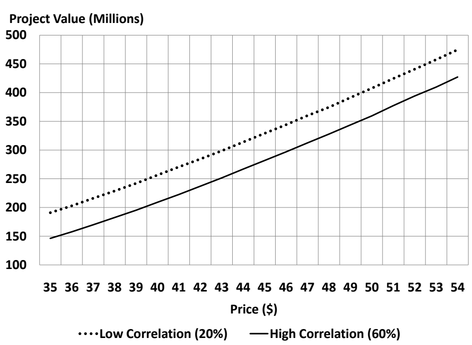
In general, when extraction cost is stochastic, there will be a trade off between the impact of volatility, σ_v, and the impact of correlation, ρ_{sv}, this is shown in Figure 4. Holding correlation constant, the higher the volatility the higher the value of the project. This is because the value of the operating options increases as a function of volatility. Holding correlation at 0.2, the value of the project when volatility is high is around 50 million higher than its value when the volatility is relatively low, about 11% at the price of \$53. That is to say, when the prices of crude oil and natural gas tend to move independently, the operating options become more valuable. On the other hand, when correlation is high ($\rho_{sv} = 0.6$), the value of the project is almost the same whether there is high volatility in the extraction cost or not. The impact of the correlation is more critical, under relatively low volatility, $\sigma_v = 0.2$, higher correlation reduces the value of the project about 50 millions dollar, which is about 12.5 % at the price of \$53, while the reduction reaches the double



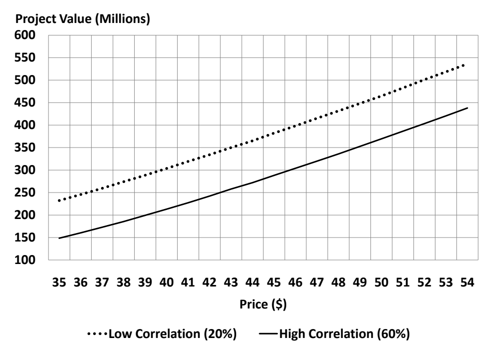
(a) The impact of Volatility when $\rho_{sv} = 20\%$



(b) The impact of Volatility when $\rho_{sv} = 60\%$



(c) The impact of Correlation when $\sigma_v = 20\%$



(d) The impact of Correlation when $\sigma_v = 60\%$

Figure 4: The Impact of the Stochastic Cost on the Value of an Operating Project

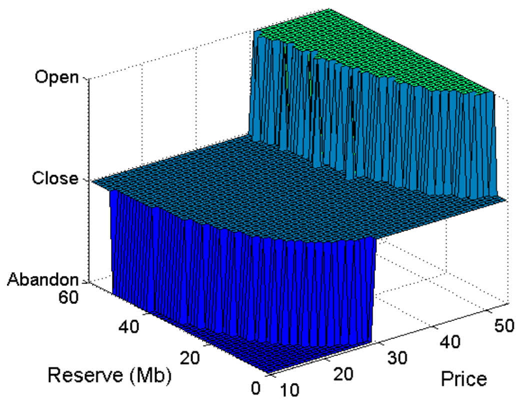
All graphs is for $Q = 6$ million barrels, the specification of oil price process and oil convenience yield is the same as Table 2 when volatility is 0.6, about 25 % at the price of \$53. This is because at higher correlation, future cash flows are reduced and the reduction becomes higher at higher volatility.

The above result suggests that there is a trade off between the impact of volatility and the impact of correlation. The increase in the operating options due to higher volatility will be offset by the reduction in the cash flows caused by higher correlation. In such an environment, the operator of an oil sands project should keep a close watch on the factors that affect correlation between the two markets. Brown and Yucel (2007) shows that oil and natural gas prices have a powerful relationship, but the relationship is conditioned by weather, seasonality and natural gas storage. In addition, the co-movement between the two markets may depend on the oil sands production. If oil sands production is high enough to affect the supply of oil and the demand in natural gas, oil prices would fall while natural gas prices would rise which may reduces the correlation between the two markets.

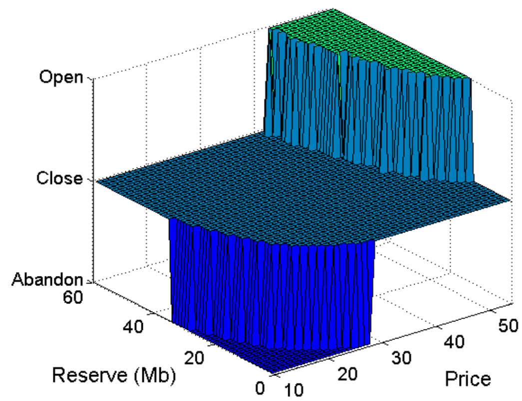
The impact of the stochastic cost on the optimal operation follows the same analysis. When the market of natural gas is less volatile and the correlation between oil and gas is high, the project should start extraction at lower prices (around \$38) and it should be abandoned at higher prices (around \$12), Figure 5(c). In contrast, if σ_v is high and ρ_{sv} is low, the project should delay extraction until at higher prices (around \$42) and it should be abandoned at lower prices (around \$9), Figure 5(b). Situations in between have mixed impacts, Figure 5(a) and Figure 5(d).

It is interesting to see the effect of reducing the stochastic component in extraction cost, i.e reducing the share of v_t in C_t, by, for example, employing extraction techniques that have less dependency on natural gas[11]. Figure 6 shows this impact. When volatility is low, the reduction in the stochastic component of the extraction cost will not have a much impact when correlation low, Figure 6(a), but it will increase the value of the project when correlation is high, Figure 6(c). However, when volatility is high, reduction in stochastic component of the extraction cost will reduce the value of the project even under high correlation, Figure 6(b) and 6(d). This seems counterintuitive at first glance. The reason for this observation is that lower uncertainty would reduce the value of operating options. In other word, if the company operating the project has the flexibility to shut down the project when prices are low, then it is favorable to the company to work under high uncertain environment.

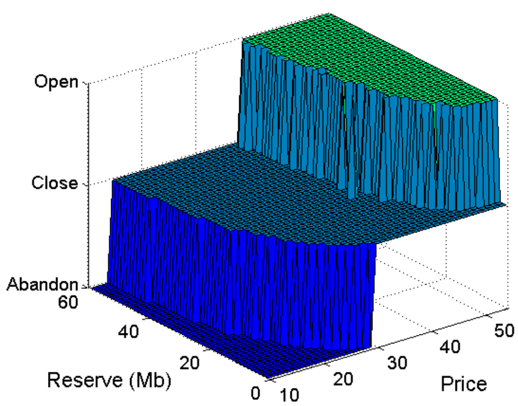
[11] According to CERI, the majority of in situ systems currently use natural gas as their fuel source. Other fuel sources are being considered but they are far from being feasible



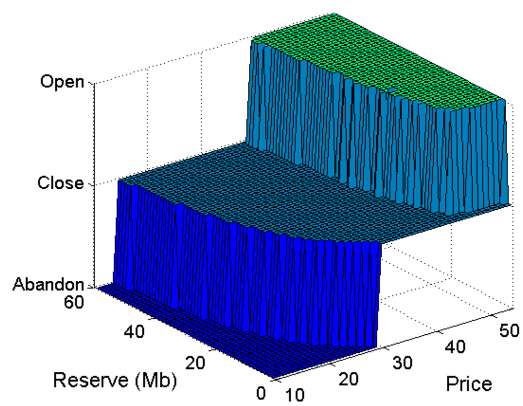
(a) Optimal Operation when $\sigma_v = 20\% \rho_{sv} = 20\%$



(b) Optimal Operation when $\sigma_v = 60\% \rho_{sv} = 20\%$

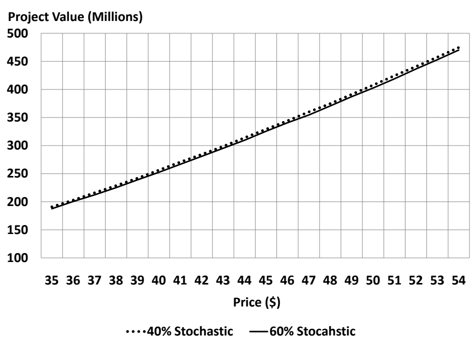


(c) Optimal Operation when $\sigma_v = 20\% \rho_{sv} = 60\%$



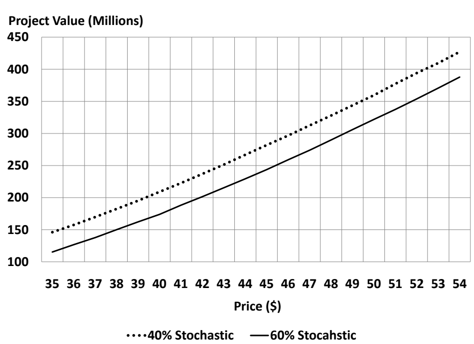
(d) Optimal Operation when $\sigma_v = 60\% \rho_{sv} = 60\%$

Figure 5: The Impact of Stochastic Cost on the Optimal Operation Policy

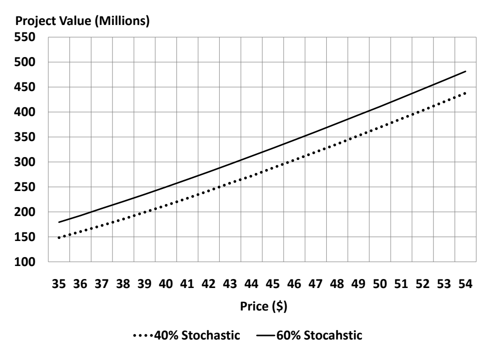


(a) $\sigma_v = 20\%$ and $\rho_{sv} = 20\%$

(b) $\sigma_v = 60\%$ and $\rho_{sv} = 20\%$



(c) $\sigma_v = 20\%$ and $\rho_{sv} = 60\%$



(d) $\sigma_v = 60\%$ and $\rho_{sv} = 60\%$

Figure 6: The Impact of Reduction of the Stochastic Component of the Extraction Cost

All graphs is for Q = 6millions barrel, the specification of oil price process and oil convenience yield is the same as table 2

4 Conclusion

In this paper, we extend Brennan and Schwartz (1985) valuation model to account for stochastic extraction cost, the situation prevails in oil sands industry where natural gas is heavily used in production and upgrading process. We study the value of a hypothetical oil sands project under different dynamics of the volatility of natural gas price and its movement in relation to crude oil markets. We found that accounting for stochastic cost has critical impact on the value of a exhaustible resource such as oil sands and in the optimal way under which it should be operated. In particular, we found that there is a trade off between the impact of natural gas volatility and the correlation between natural gas and oil prices.

Natural gas market is complex. This is reflected in natural gas prices which exhibits more complex behavior than can be modeled by GBM. Studying the impact of different price dynamics on the value of the project is a suggested avenue for research. Another avenue is to account for the impact of the oil sands production on both oil and natural gas markets. That is the price of both might be endogenous due to the large expected production of oil sands given its promising reserves and the current situation of oil tight supply and high demand. Moreover, with large supplies of shale gas one may expect the correlation between the two markets to fall.

References

Björk, T. (2003). *Arbitrage Theory in Continuous Time*. Oxford University Press, second edition.

Brennan, M. J. and Schwartz, E. S. (1985). Evaluating natural resource investments. *The Journal of Business*, 58(2):135–157.

Brown, S. P. A. and Yucel, M. K. (2007). What drives natural gas prices? Working paper, Federal Reserve Bank Of Dallas.

Casassus, J. and Collin-Dufresne, P. (2005). Stochastic convenience yield implied from commodity futures and interest rates. *The Journal of Finance*, 60(5):2283–2331.

Cortazar, G., Gravet, M., and Urzua, J. (2008). The valuation of multidimensional american real options using the lsm simulation method. *Computers and Operations Research*, 35(1):113–129.

Cortazar, G. and Schwartz, E. S. (2003). Implementing a stochastic model for oil futures prices. *Energy Economics*, 25(3):215 – 238.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton University Press.

Gamba, A. (2003). Real options valuation: a monte carlo approach. working paper, University of Calgary.

Geman, H. (2005). *Commodities and Commodity Derivatives Modeling and Pricing for Agriculturals, Metals and Energy*. John Wiley & Sons, England.

Gibson, R. and Schwartz, E. S. (1990). Stochastic convenience yield and the pricing of oil contingent claims. *The Journal of Finance*, 45(3):959–976.

Klebaner, F. C. (2005). *Introduction to Stochastic Calculus with Applications*. Imperial College Press, second edition.

Longstaff, F. A. and Schwartz, E. S. (2001). Valuing american options by simulation: a simple least-squares approach. *Review of Financial Studies*, 14(1):113–147.

Pindyck, R. S. (1980). Uncertainty and exhaustible resource markets. *The Journal of Political Economy*.

Pindyck, R. S. (2004). Volatility and commodity price dynamics. *The Journal of Futures Markets*, 24(11):1029–1047.

Schwartz, E. S. and Trigeorgis, L. (2004). *Real Options and Investment Under Uncertainty: Classical Readings and Recent Contributions*. MIT Press.

Tsekrekos, A. E., Shackleton, M. B., and Wojakowski, R. (2010). Evaluating natural resource investments under different model dynamics: Managerial insights. *European Financial Management*.

Villar, J. A. and Joutz, F. L. (2006). The relationship between crude oil and natural gas prices. Working paper, Energy Information Administration, October.