Investment Timing and Intensity under Progressive Taxation *

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Abstract

This paper examines a firm's investment intensity and timing decisions using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment with decreasing returns to scale. The project generates a stream of profit flows that is stochastic over time and increases with the intensity of investment. The firm's investment policy is characterized by an endogenously determined profit flow such that the investment option is exercised at the first instant when the firm's profit flow reaches this threshold level from below. Tax progression arises from an exogenously given tax exemption threshold that makes the average tax rate increase with the tax base. We show that corporate income taxes are not neutral when tax schedules are progressive.

JEL classification: D21; G31; H25

Keywords: Investment intensity; Investment timing; Progressive taxation; Real options

*This is a preliminary and incomplete version. Comments and suggestions are welcome.

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Abstract

This paper examines a firm's investment intensity and timing decisions using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment with decreasing returns to scale. The project generates a stream of profit flows that is stochastic over time and increases with the intensity of investment. The firm's investment policy is characterized by an endogenously determined profit flow such that the investment option is exercised at the first instant when the firm's profit flow reaches this threshold level from below. Tax progression arises from an exogenously given tax exemption threshold that makes the average tax rate increase with the tax base. We show that corporate income taxes are not neutral when tax schedules are progressive.

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1. Introduction

Corporate income tax schedules are by and large progressive and convex (see Graham and Smith, 1999). The real options literature on investment under uncertainty, however, usually either ignores tax effects or assumes a linear corporate income tax schedule (see Dixit and Pindyck, 1994). The purpose of this paper is to contribute to this literature by explicitly incorporating tax progression into a real options model of a firm that has to make irreversible investment decisions in continuous time with an infinite horizon.

Our real options model features a firm that possesses a perpetual option to invest in a project at any time by paying an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment, which is a choice variable of the firm. The project generates a stream of stochastic profit flows that follow a lognormal diffusion process and increase with the intensity of investment, as in Capozza and Li (1994, 2002) and Bar-Ilan and Strange (1999). We model tax progression by using an exogenously given tax exemption threshold, which may come from sources such as operating costs, depreciation allowances, R&D expenditures, and investment tax credits (see Agliardi and Agliardi, 2008, 2009; Alvarez and Koskela, 2008; Wong, 2009; Chu and Wong, 2010). The firm needs to pay corporate income taxes if its profit flow exceeds the tax exemption threshold; otherwise, the firm neither pays to nor receives tax rebates from the government. Even though the marginal corporate income tax rate is constant, the average tax rate as such increases with the tax base, thereby rendering tax progression.

The firm makes two decisions regarding the undertaking of the project: the timing and intensity of investment. The firm's investment timing decision is characterized by a threshold (the investment trigger) such that the project is undertaken at the first instant when the profit flow from the project reaches the investment trigger from below (see, e.g., McDonald and Siegel 1986; Dixit and Pindyck 1994). The firm's investment intensity decision affects the amount of the irreversible investment cost according to a known technology that exhibits decreasing returns to scale.

Within our real options model, we analytically show that the firm optimally chooses its threshold profit flow at which the investment option is exercised to be higher or lower than the exogenously given tax exemption threshold, depending on whether the tax exemption is below or above a unique critical level, respectively. We further show that the firm's optimal investment intensity and trigger are strictly increasing for all tax exemption thresholds above the critical level, and are U-shaped for all tax exemption thresholds below the critical level. Most interestingly, we numerically show that the value of the firm prior to the investment, i.e., the value of the investment option, is non-monotonic to the volatility of the firm's profit flows, which is in stark contrast to the extant literature that the value of an option increases with an increase in the volatility of the underlying uncertainty.

This paper is related to the burgeoning literature that incorporates different corporate income tax schemes to identify potential distortionary tax effects on corporate investment and liquidation decisions (see, e.g., MacKie-Mason, 1990; Alvarez et al., 1998; Hassett and Metcalf, 1999; Agliardi, 2001; Agliardi and Agliardi, 2008, 2009; Wong, 2009; Chu and Wong, 2010; and Wong and Wu, 2010). Alvarez and Koskela (2008), Niemann (1999), Niemann and Sureth (2004), Panteghini (2001, 2005), Sureth (2002) and others have established a tax neutrality result in that corporate investment decisions are invariant to changes in tax rates under a variety of corporate income tax schemes. Our results that corporate investment decisions are affected by the design of progressive corporate income tax schedules certainly contribute to this literature.

The rest of this paper is organized as follows. Section 2 delineates a continuous-time model of a firm that operates in an infinite horizon and possesses a perpetual option to invest in a project.

Section 3 characterizes the firm's optimal investment timing, keeping the firm's investment intensity fixed. Section 4 characterizes the firm's optimal investment intensity and timing and examines the effect of tax progression on the firm's investment policy. Section 5 offers numerical analysis. Section 6 concludes.

2. The model

Consider a risk-neutral firm that has monopoly access to a perpetual option to invest in a project.[1] Time is continuous and indexed by $t \in [0, \infty)$. The riskless rate of interest per unit time is constant at $r > 0$.

The firm's investment policy consists of two choices: the timing and intensity of the investment in the project. The firm's investment intensity, $q \geq 0$, affects the stochastic profit flow, qX, generated from the project over time, where $X \geq 0$ is a state variable specifying the project's profit flow per unit intensity of investment. We assume that X evolves over time according to the following geometric Brownian motion:

$$\mathrm{d}X = \mu X \ \mathrm{d}t + \sigma X \ \mathrm{d}Z, \tag{1}$$

where $\mu < r$ and $\sigma > 0$ are constant parameters, and $\mathrm{d}Z$ is the increment of a standard Wiener process under the risk-neutral probability space, $(\Omega, \mathcal{F}, \mathcal{Q})$.[2] Eq. (1) implies that the growth rate of X is normally distributed with a mean, $\mu\Delta t$, and a variance, $\sigma^2\Delta t$, over a time interval, Δt.

To undertake the project at endogenously chosen time, $t \geq 0$, and intensity, $q \geq 0$, the firm pays an irreversible investment cost, $I(q)$, at that instant, where $I(0) \geq 0$, $I'(0) = 0$, and $I'(q) > 0$ and $I''(q) > 0$ for all $q > 0$.[3] The strict convexity of $I(q)$ implies that the project exhibits decreasing returns to scale. Define the following two constants, α and β, that are derived from the fundamental quadratic equation, $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$:

$$\alpha = \frac{\mu}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 0, \tag{2}$$

[1] The assumption of risk neutrality is innocuous as long as there are arbitrage-free and complete financial markets in which assets can be traded to span the state variable that determines the value of the firm.

[2] The assumption that $\mu < r$ is needed to ensure that the value of the firm is finite.

[3] We allow for $I(0) > 0$ to account for some fixed set-up costs that are required to initiate the project.

and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{3}$$

To ensure that the optimal investment intensity is finite in the presence of the investment option, we assume that $[\beta/(\beta - 1)]I(q) - qI'(q)$ is a decreasing function of q, where β is given by Eq. (3). This assumption implies that the project has sufficient decreasing returns to scale in that $I''(q) > I'(q)/(\beta - 1)q$ for all $q > 0$.

We model progressive taxation by using an exogenously given tax exemption threshold, $D > 0$ (see Agliardi and Agliardi, 2008, 2009; Alvarez and Koskela, 2008; Wong, 2009; Chu and Wong, 2010). Sources of tax exemption include operating costs, depreciation allowances, R&D expenditures, investment tax credits, and many others. Succinctly, the firm needs to pay corporate income taxes at a constant rate, $\tau \in (0, 1)$, for its profit flow that exceeds the tax exemption threshold. When the firm's profit flow does not exceed the tax exemption threshold, the firm neither pays taxes to nor receives tax rebates from the government. The firm's after-tax profit flow, Π, is, therefore, given by

$$\Pi = qX - \tau \max(qX - D, 0), \tag{4}$$

where $\tau \max(qX - D, 0)$ is the corporate income tax flow paid to the government over time.[4] We summarize the progressive corporate income tax schedule by the pair, (τ, D).

To facilitate the exposition, we formulate the firm's decision problem as a two-stage optimization problem. In the first stage, we fix the firm's investment intensity, $q > 0$, and solve the optimal time, $t(q)$, at which the investment option is exercised. This is tantamount to solving the critical value of the state variable, $X(q)$, that triggers the exercise of the investment option at the first instant when X reaches $X(q)$ from below (see, e.g., McDonald and Siegel 1986; Dixit and Pindyck 1994). We refer to $X(q)$ as the investment trigger. In the second stage, we solve the firm's optimal investment intensity, q^*, taking the schedule of the investment triggers, $\{X(q) : q > 0\}$, characterized in the first stage as given. The complete solution to the model is, therefore, given by q^* and $X^* = X(q^*)$.

[4] While the marginal tax rate is constant at τ, the average tax rate increases with the tax base, as is evident from Eq. (4), thereby making the corporate income tax schedule progressive.

3. Investment triggers in the first stage

In this section, we solve the first-stage problem by fixing the firm's investment intensity at $q > 0$. This is done in two steps. First, we derive the value of the firm after the irreversible investment has been made. Second, we derive the value of the firm before the irreversible investment is made, taking the characterization of the investment trigger, $X(q)$, into account.

3.1 Firm value after investment

Let $V_0(q, X)$ be the value of the firm when currently no corporate income taxes are paid, i.e., for all $X \leq D/q$. Likewise, let $V_1(q, X)$ be the value of the firm when currently positive corporate income taxes are paid, i.e., for all $X \geq D/q$. Using the standard arbitrage arguments (see, e.g., Dixit and Pindyck, 1994), $V_0(q, X)$ and $V_1(q, X)$ must satisfy the following ordinary differential equations:

$$\frac{1}{2}\sigma^2 X^2 \, \frac{\partial^2 V_0(q, X)}{\partial X^2} + \mu X \, \frac{\partial V_0(q, X)}{\partial X} - rV_0(q, X) + qX = 0, \tag{5}$$

for all $X \leq D/q$, and

$$\frac{1}{2}\sigma^2 X^2 \, \frac{\partial^2 V_1(q, X)}{\partial X^2} + \mu X \, \frac{\partial V_1(q, X)}{\partial X} - rV_1(q, X) + qX - \tau(qX - D) = 0, \tag{6}$$

for all $X \geq D/q$, respectively, subject to the following boundary conditions:

$$V_0(q, 0) = 0, \tag{7}$$

$$\lim_{X \to \infty} \frac{V_1(q, X)}{X} < \infty, \tag{8}$$

$$V_0(q, D/q) = V_1(q, D/q), \tag{9}$$

and

$$\left.\frac{\partial V_0(q, X)}{\partial X}\right|_{X=D/q} = \left.\frac{\partial V_1(q, X)}{\partial X}\right|_{X=D/q}. \tag{10}$$

Eq. (7) reflects the fact that zero is an absorbing barrier for the diffusion process defined in Eq. (1). Eq. (8) rules out speculative bubbles. Eqs. (9) and (10) hold because the Brownian motion described in Eq. (1) can diffuse freely across $X = D/q$, rendering the value function to be continuously differentiable at $X = D/q$.[5]

Solving Eqs. (5) and (6) subject to Eqs. (7), (8), (9), and (10) yields our first proposition. All proofs of propositions are relegated to the appendix.

Proposition 1. *Fix the firm's investment intensity at $q > 0$ and the progressive corporate income tax schedule at (τ, D). The value of the firm after the irreversible investment has been made is given by*

$$V_0(q, X) = \frac{qX}{r-\mu} - \left[\frac{D}{r-\mu} - V_1(q, D/q)\right]\left(\frac{qX}{D}\right)^{\beta}, \tag{11}$$

for all $X \leq D/q$, and

$$V_1(q, X) = (1-\tau)\left(\frac{qX}{r-\mu}\right) + \frac{\tau D}{r} - \left[(1-\tau)\left(\frac{D}{r-\mu}\right) + \frac{\tau D}{r} - V_0(q, D/q)\right]\left(\frac{D}{qX}\right)^{\alpha}, \tag{12}$$

for all $X \geq D/q$, where

$$V_0(q, D/q) = V_1(q, D/q) = \frac{D}{r-\mu} - \left(\frac{1}{\alpha+\beta}\right)\left(\frac{\alpha}{\beta-1}\right)\frac{\tau D}{r}. \tag{13}$$

Eq. (11) has the following interpretation. The first term on the right-hand side of Eq. (11) is the value of the firm when there are no corporate income taxes. The expression, $(qX/D)^{\beta}$, can be interpreted as the stochastic discount factor that accounts for both the timing and the probability of one dollar received at the first instant when D/q is reached from below, starting off at the initial value, $X \leq D/q$. The second term on the right-hand side of Eq. (11) is the reduction in value arising from introducing corporate income taxes, where $D/(r-\mu) - V_1(q, D/q) = [\alpha/(\alpha+\beta)(\beta-1)]\tau D/r > 0$, is the loss in value at the instant when the firm starts to pay taxes, which is discounted by the stochastic discount factor, $(qX/D)^{\beta}$, to the present time.

Eq. (12) has the following interpretation. The first two terms on the right-hand side of Eq. (12) capture the value of the firm when the corporate income tax rate is constant at τ and the tax-shield benefits arising from the tax exemption threshold, D, can always be claimed by the firm. The expression, $(D/qX)^{\alpha}$, can be interpreted as the stochastic discount factor that accounts for both the timing and the probability of one dollar received at the first instant when the tax exemption

[5] See Dixit (1993) for a heuristic argument and Karatzas and Shreve (1988) for a rigorous proof.

threshold, D, is reached from above, starting off at the initial value, $X \geq D/q$. The last term on the right-hand side of Eq. (12) is the reduction in value due to the possibility of entering into the no-tax region in the future. Specifically, $(1-\tau)D/(r-\mu)+\tau D/r - V_0(q, D/q) = [\beta/(\alpha+\beta)(\alpha+1)]\tau D/r > 0$ is the loss in value at the instant when the firm starts to pay no corporate income taxes due to low profit flows, which is discounted by the stochastic discount factor, $(D/qX)^\alpha$, to the present time.

3.2 Firm value before investment

We now derive the value of the firm before the irreversible investment is made, taking the characterization of the investment trigger, $X(q)$, into account.

Let $F(q, X)$ be the value of the firm before the irreversible investment is made (i.e., the value of the investment option). Using the standard arbitrage arguments, $F(q, X)$ must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 \, \frac{\partial^2 F(q,X)}{\partial X} + \mu X \, \frac{\partial F(q,X)}{\partial X} - rF(q,X) = 0, \tag{14}$$

for all $X \in [0, X(q)]$, subject to the following boundary conditions:

$$F(q, 0) = 0, \tag{15}$$

$$F[q, X(q)] = \begin{cases} V_0[q, X(q)] - I(q) & \text{if } X(q) \leq D/q, \\ V_1[q, X(q)] - I(q) & \text{if } X(q) \geq D/q, \end{cases} \tag{16}$$

and

$$\left.\frac{\partial F(q,X)}{\partial X}\right|_{X=X(q)} = \begin{cases} \left.\frac{\partial V_0(q,X)}{\partial X}\right|_{X=X(q)} & \text{if } X(q) \leq D/q, \\ \left.\frac{\partial V_1(q,X)}{\partial X}\right|_{X=X(q)} & \text{if } X(q) \geq D/q. \end{cases} \tag{17}$$

Eq. (15) reflects the fact that zero is an absorbing barrier of the diffusion process defined in Eq. (1). Eq. (16) is the value-matching condition such that the value of the investment option equals the value of the firm net of the investment cost at the instant when the investment option is exercised. Eq. (17) is the smooth-pasting condition such that the investment trigger, $X(q)$, maximizes the value of the investment option.

To solve Eq. (14) subject to Eqs. (15), (16), and (17), we need to know which of the two pairs of the value-matching and smooth-pasting conditions is relevant, i.e., which of the two cases, (i) $X(q) \leq D/q$ and (ii) $X(q) \geq D/q$, prevails. Intuitively, we expect to see case (i) or case (ii), depending on whether D is sufficiently large or small, respectively. Define the following critical tax exemption threshold:

$$D(q) = (r - \mu)\left(\frac{\beta}{\beta - 1}\right)I(q). \tag{18}$$

Indeed, we show in the following proposition that $X(q)$ is less than, equal to, or greater than D/q if D is greater than, equal to, or less than $D(q)$, respectively.

Proposition 2. *Fix the firm's investment intensity at $q > 0$ and the progressive corporate income tax schedule at (τ, D). If the exogenously given tax exemption threshold, D, is no less than the critical level, $D(q)$, the firm's investment trigger, $X(q)$, is given by*

$$X(q) = (r - \mu)\left(\frac{\beta}{\beta - 1}\right)\frac{I(q)}{q}, \tag{19}$$

such that $X(q) \leq D/q$, and the value of the firm before the irreversible investment is made, $F(q, X)$, is given by

$$F(q, X) = \{V_0[q, X(q)] - I(q)\}\left[\frac{X}{X(q)}\right]^{\beta}, \tag{20}$$

for all $X \in [0, X(q)]$. If $D < D(q)$, then $X(q)$ is uniquely determined by

$$(1 - \tau)\left[\frac{qX(q)}{r - \mu}\right] + \frac{\tau D}{r - \mu} - \left(\frac{\beta}{\beta - 1}\right)\left\{I(q) - \left(\frac{1}{\alpha + 1}\right)\left\{1 - \left[\frac{D}{qX(q)}\right]^{\alpha}\right\}\frac{\tau D}{r}\right\} = 0, \tag{21}$$

such that $X(q) > D/q$, and

$$F(q, X) = \{V_1[q, X(q)] - I(q)\}\left[\frac{X}{X(q)}\right]^{\beta}, \tag{22}$$

for all $X \in [0, X(q)]$.

Proposition 2 says that if the exogenously given tax exemption threshold, D, is large enough, i.e., $D \geq D(q)$, the firm's investment trigger, $X(q)$, does not vary with the progressive corporate income tax schedule, (τ, D), and does not exceed D/q. Otherwise, for sufficiently small tax exemption thresholds, i.e., $D < D(q)$, $X(q)$ depends on (τ, D) and exceeds D/q. The expression, $[X/X(q)]^\beta$, in Eqs. (20) and (22) is the stochastic discount factor that accounts for both the timing and the probability of one dollar received at the first instant when the profit flow reaches $X(q)$ from below, starting off at the initial value, $X \in [0, X(q)]$. Depending on whether $X(q) \leq D/q$ or $X(q) > D/q$, the value of the investment option, $F(q, X)$, as in Eq. (20) or in Eq. (22), is equal to the net present value of the project at the instant when the investment option is exercised, $V_0[q, X(q)] - I(q)$ or $V_1[q, X(q)] - I(q)$, respectively, discounted by the stochastic discount factor, $[X/X(q)]^\beta$, to the present time.

If the tax exemption threshold, D, is no less than $D(q)$, we can substitute Eq. (11) into Eq. (20) to yield

$$F(X, q) = \left[\frac{qX(q)}{r-\mu} - I(q)\right]\left[\frac{X}{X(q)}\right]^\beta - \left[\frac{D}{r-\mu} - V_1(q, D/q)\right]\left(\frac{qX}{D}\right)^\beta, \tag{23}$$

for all $X \in [0, X(q)]$. Hence, the optimal investment trigger, $X(q)$, is the one that maximizes the first term on the right-hand side of Eq. (23), which does not depend on the progressive corporate income tax schedule, (τ, D). In the case that the investment intensity, q, is fixed and $D \geq D(q)$, Proposition 2 establishes the tax neutrality result that has been shown by Panteghini (2001, 2005) and Alvarez and Koskela (2008).

4. Optimal investment intensity and trigger

In this section, we solve the second stage problem, i.e., the choice of the optimal investment intensity, q^*, taking the schedule of the investment triggers, $\{X(q) : q > 0\}$, characterized in Proposition 2 as given:

$$\max_{q>0} F(q, X), \tag{24}$$

where $F(q, X)$ is given by Eq. (20) for all q such that $D(q) \leq D$, and by Eq. (22) for all q such that $D(q) \geq D$.

To solve program (24), we need to know which of the two cases, (i) $D(q^*) \leq D$ and (ii) $D(q^*) \geq D$, prevails at the optimal investment intensity, q^*. Define $q(D)$ as the solution to $D[q(D)] = D$, i.e.,

$$q(D) = I^{-1}\left[\left(\frac{\beta-1}{\beta}\right)\left(\frac{D}{r-\mu}\right)\right], \tag{25}$$

and q^0 as the critical investment intensity that solves the following equation:

$$\left(\frac{\beta}{\beta-1}\right)\left[1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\right]\frac{I(q^0)}{q^0}-I'(q^0)=0. \tag{26}$$

We show in the following proposition that $D(q^*)$ is less than, equal to, or greater than D if $D(q^0)$ is less than, equal to, or greater than D, respectively.

Proposition 3. *Fix the progressive corporate income tax schedule at (τ, D). If the exogenously given tax exemption threshold, D, exceeds the critical level, $D(q^0)$, the firm's optimal investment intensity and trigger, q^* and X^*, are uniquely determined by solving the following system of equations:*

$$\left(\frac{\beta}{\beta-1}\right)\left[1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\left(\frac{q^*X^*}{D}\right)^{\beta-1}\right]\frac{I(q^*)}{q^*}-I'(q^*)=0, \tag{27}$$

and

$$X^*=(r-\mu)\left(\frac{\beta}{\beta-1}\right)\frac{I(q^*)}{q^*}, \tag{28}$$

such that $q^0<q^*<q(D)$ and $X^*<D/q^*$, where $q(D)$ solves $D(q)=D$. If $D=D(q^0)$, then $q^*=q^0$ and $X^*=D(q^0)/q^0$. If $D<D(q^0)$, then q^* and X^* are uniquely determined by solving the following system of equations:

$$\left(\frac{\beta}{\beta-1}\right)\left\{\frac{I(q^*)}{q^*}-\frac{\tau D}{rq^*}\left[1-\left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{D}{q^*X^*}\right)^{\alpha}\right]\right\}-I'(q^*)=0, \tag{29}$$

and

$$(1-\tau)\left(\frac{q^*X^*}{r-\mu}\right)+\frac{\tau D}{r-\mu}-\left(\frac{\beta}{\beta-1}\right)\left\{I(q^*)-\left(\frac{1}{\alpha+1}\right)\left[1-\left(\frac{D}{q^*X^*}\right)^{\alpha}\right]\frac{\tau D}{r}\right\}=0, \tag{30}$$

such that $q^*>q(D)$ and $X^*>D/q^*$.

Proposition 3 shows that the firm's optimal intensity and trigger, q^* and X^*, are jointly determined by solving the system of equations, Eqs. (27) and (28), or Eqs. (29) and (30), depending on whether the exogenously given tax exemption threshold, D, is greater or less than the critical level, $D(q^0)$. We as such derive the complete solution to the firm's optimal investment policy under progressive taxation.

Let $Y^* = q^*X^*$ be the threshold profit flow at which the project is undertaken. The following proposition performs the comparative static exercise with respect to the progressive corporate income tax schedule.

Proposition 4. *Fix the progressive corporate income tax schedule at* (τ, D). *If the exogenously given tax exemption threshold,* D, *exceeds the critical level,* $D(q^0)$, *the firm's optimal investment intensity,* q^*, *and the threshold profit flow,* Y^*, *both increase with an increase in* D, *i.e.,* $\mathrm{d}q^*/\mathrm{d}D > 0$ *and* $\mathrm{d}Y^*/\mathrm{d}D > 0$, *and decrease with an increase in the tax rate,* τ, *i.e.,* $\mathrm{d}q^*/\mathrm{d}\tau < 0$ *and* $\mathrm{d}Y^*/\mathrm{d}\tau < 0$. *If* $D < D(q^0)$, *then* q^* *and* Y^* *are non-monotonic with respect to changes in* D *or* τ. *Specifically,* $\mathrm{d}q^*/\mathrm{d}D < 0$, $\mathrm{d}Y^*/\mathrm{d}D < 0$, $\mathrm{d}q^*/\mathrm{d}\tau > 0$, *and* $\mathrm{d}Y^*/\mathrm{d}\tau > 0$ *when* D *approaches zero, and* $\mathrm{d}q^*/\mathrm{d}D > 0$, $\mathrm{d}Y^*/\mathrm{d}D > 0$, $\mathrm{d}q^*/\mathrm{d}\tau < 0$, *and* $\mathrm{d}Y^*/\mathrm{d}\tau < 0$ *when* D *approaches* $D(q^0)$.

When the firm has to choose both the investment trigger and intensity, Proposition 4 shows that the progressive corporate income tax schedule cannot be neutral. Given the non-monotonic behavior of the firm's investment policy with respect to the progressive corporate income tax schedule, we can conclude that firms facing different tax rates or tax exemption thresholds may choose the same optimal investment policy.

5. Numerical analysis

Since there are no closed form solutions to the firm's optimal investment intensity and trigger, q^* and X^*, we conduct numerical analysis in this section. We set the investment cost function, $I(q) = 10 + q^4$, the annualized riskless rate of interest, $r = 7\%$, and the corporate income tax rate, $\tau = 30\%$. The state variable, X, takes on the initial value, $X_0 = 1$, with the annualized growth rate, $\mu = 3\%$, and the annualized standard deviation, $\sigma = 25\%$.

Figure 1 depicts the firm's optimal intensity and trigger, q^* and X^*), and the threshold profit flow, $Y^* = q^*X^*$, against the exogenously given tax exemption threshold, D. The 45° line shows us the point at which the threshold profit flow, Y^*, is exactly equal to the exogenously given tax exemption threshold, which defines $D(q^0)$. Figure 1 shows that $Y^* < (>)\ D$ for all $D < (>)\ D(q^0)$ and $Y^* = D(q^0)$ if $D = D(q^0)$. Figure 1 also shows that q^*, X^*, and Y^* are all increasing functions of D for all $D > D(q^0)$, and become non-monotonic for all $D < D(q^0)$. These results are consistent with those reported in Proposition 3.

(Insert Figure 1 here.)

Figure 2 depicts the firm's optimal intensity and trigger, q^* and X^*), against the standard derivation, σ, and against the tax rate, τ. Inspection of Figure 2 shows that q^* and X^* are both increasing functions of σ. On the other hand, q^* and X^* are both decreasing functions of τ, albeit with relatively small sensitivity.

(Insert Figure 2 here.)

Figure 3 depicts the value of the firm, $F(q^*, X_0)$, at $t = 0$ (i.e., the value of the investment option) against the standard derivation, σ, and against the tax rate, τ. Figure 3 shows that $F(q^*, X_0)$ is a decreasing function of τ, but is a U-shaped function of σ. The latter result is of shape contrast to the extant literature that the value of an option increases with the volatility of the underlying uncertainty.

(Insert Figure 3 here.)

6. Conclusion

In this paper, we have examined a firm's investment intensity and timing decisions using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment with decreasing returns to scale. The project generates a stream of profit flows that is stochastic over time and increases with the intensity of investment. The firm's investment policy is characterized by an endogenously determined profit flow (the investment trigger) such that the investment option is exercised at the first instant when the firm's profit flow reaches the investment trigger from below.

Within our real options model, we have shown that the investment trigger is higher or lower than the exogenously given tax exemption threshold, depending on whether the tax exemption is below or above a unique critical level, respectively. We have further shown that the firm's optimal investment intensity and trigger are strictly increasing for all tax exemption thresholds above the critical level, and are U-shaped for all tax exemption thresholds below the critical level. Most interestingly, we have numerically shown that the value of the firm prior to the investment, i.e., the value of the investment option, is non-monotonic to the volatility of the firm's profit flows, which is in stark contrast to the extant literature that the value of an option increases with an increase in the volatility of the underlying uncertainty. Corporate income taxes as such are not neutral in the presence of tax progression.

Appendix

A. Proof of Proposition 1

The solutions to Eqs. (5) and (6) are given by

$$V_0(q, X) = \frac{qX}{r - \mu} + A_0 X^{-\alpha} + B_0 X^{\beta}, \tag{A.1}$$

for all $X \leq D/q$, and

$$V_1(q, X) = (1 - \tau)\left(\frac{qX}{r - \mu}\right) + \frac{\tau D}{r} + A_1 X^{-\alpha} + B_1 X^{\beta}, \tag{A.2}$$

for all $X \geq D/q$, respectively, where A_0, B_0, A_1, and B_1 are four constants to be determined, and α and β are given by Eqs. (2) and (3), respectively. From Eqs (A.1) and (7), we have $A_0 = 0$. Likewise, from Eqs (A.2) and (8), we have $B_1 = 0$. Substituting Eq. (A.1) with $A_0 = 0$ and Eq. (A.2) with $B_1 = 0$ into Eqs. (9) and (10) yields

$$\frac{D}{r - \mu} + B_0\left(\frac{D}{q}\right)^{\beta} = (1 - \tau)\left(\frac{D}{r - \mu}\right) + \frac{\tau D}{r} + A_1\left(\frac{D}{q}\right)^{-\alpha}, \tag{A.3}$$

and

$$\frac{q}{r - \mu} + \beta B_0\left(\frac{D}{q}\right)^{\beta - 1} = (1 - \tau)\left(\frac{q}{r - \mu}\right) - \alpha A_1\left(\frac{D}{q}\right)^{-\alpha - 1}. \tag{A.4}$$

Solving Eqs. (A.3) and (A.4) for B_0 and A_1 yields

$$B_0 = -\left(\frac{1}{\alpha + \beta}\right)\left(\frac{r + \mu\alpha}{r - \mu}\right)\frac{\tau D}{r}\left(\frac{D}{q}\right)^{-\beta}, \tag{A.5}$$

and

$$A_1 = -\left(\frac{1}{\alpha+\beta}\right)\left(\frac{r-\mu\beta}{r-\mu}\right)\frac{\tau D}{r}\left(\frac{D}{q}\right)^{\alpha}. \tag{A.6}$$

Since $-\alpha$ and β are the negative and positive roots of the fundamental quadratic equation, $\frac{1}{2}\sigma^2 y(y-1)+\mu y - r = 0$, we have

$$r+\mu\alpha = \frac{1}{2}\sigma^2\alpha(\alpha+1), \tag{A.7}$$

and

$$r-\mu\beta = \frac{1}{2}\sigma^2\beta(\beta-1). \tag{A.8}$$

From Eqs. (2) and (3), we have

$$\alpha\beta = \left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2} - \left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2 = \frac{2r}{\sigma^2}, \tag{A.9}$$

and

$$(\alpha+1)(\beta-1) = \alpha\beta+\beta-\alpha-1 = \frac{2(r-\mu)}{\sigma^2}. \tag{A.10}$$

Using Eqs. (A.7) and (A.10), we have

$$\frac{r+\mu\alpha}{r-\mu} = \frac{\alpha}{\beta-1}. \tag{A.11}$$

Using Eqs. (A.8) and (A.10), we have

$$\frac{r-\mu\beta}{r-\mu} = \frac{\beta}{\alpha+1}. \tag{A.12}$$

Substituting Eq. (A.11) into Eq. (A.5) and Eq. (A.12) into Eq. (A.6) yields

$$B_0 = -\left(\frac{1}{\alpha+\beta}\right)\left(\frac{\alpha}{\beta-1}\right)\frac{\tau D}{r}\left(\frac{D}{q}\right)^{-\beta}, \tag{A.13}$$

and

$$A_1 = -\left(\frac{1}{\alpha+\beta}\right)\left(\frac{\beta}{\alpha+1}\right)\frac{\tau D}{r}\left(\frac{D}{q}\right)^{\alpha}. \tag{A.14}$$

Substituting Eq. (A.13) into Eq. (A.1) with $A_0 = 0$ and Eq. (A.14) into Eq. (A.2) with $B_1 = 0$ yields Eqs (11) and (12), respectively.

B. *Proof of Proposition 2*

The solution to Eq. (14) is given by

$$F(q, X) = A_2 X^{-\alpha} + B_2 X^{\beta}, \tag{A.15}$$

for all $X \in [0, X(q)]$, where A_2 and B_2 are two constants to be determined, and α and β are given by Eqs. (2) and (3), respectively. From Eqs. (15) and (A.15), we have $A_2 = 0$. Consider first case (i) that $X(q) \leq D/q$. Substituting Eq. (11) and Eq. (A.15) with $A_2 = 0$ into Eqs. (16) and (17) yields

$$B_2 X(q)^{\beta} = \frac{qX(q)}{r-\mu} - \left(\frac{\alpha}{\alpha+\beta}\right)\left(\frac{1}{\beta-1}\right)\frac{\tau D}{r}\left[\frac{qX(q)}{D}\right]^{\beta} - I(q), \tag{A.16}$$

and

$$\beta B_2 X(q)^{\beta-1} = \frac{q}{r-\mu} - \left(\frac{\alpha}{\alpha+\beta}\right)\left(\frac{\beta}{\beta-1}\right)\frac{\tau D}{rX(q)}\left[\frac{qX(q)}{D}\right]^{\beta}, \tag{A.17}$$

respectively. Multiplying β to Eq. (A.16) and $X(q)$ to Eq. (A.17), and subtracting the resulting equations yields Eq. (19). Substituting Eq. (A.16) into Eq. (A.15) with $A_2 = 0$ yields Eq. (20). It is evident from Eq. (19) that $X(q)$ does not depend on D. Hence, it follows from Eqs. (A.52) and (19) that $X(q) \leq D/q$ if, and only if, $D \geq D(q)$.

Now, we consider case (ii) that $X(q) \geq D/q$. Substituting Eq. (12) and Eq. (A.15) with $A_2 = 0$ into Eqs. (16) and (17) yields

$$B_2 X(q)^{\beta} = (1-\tau)\left[\frac{qX(q)}{r-\mu}\right] + \frac{\tau D}{r} - \left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{1}{\alpha+1}\right)\frac{\tau D}{r}\left[\frac{D}{qX(q)}\right]^{\alpha} - I(q), \tag{A.18}$$

and

$$\beta B_2 X(q)^{\beta-1} = (1-\tau)\left(\frac{q}{r-\mu}\right) + \left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau D}{rX(q)}\left[\frac{D}{qX(q)}\right]^{\alpha}, \tag{A.19}$$

respectively. Multiplying β to Eq. (A.18) and $X(q)$ to Eq. (A.19), and subtracting the resulting equations yields

$$(1-\tau)\left[\frac{qX(q)}{r-\mu}\right]-\left(\frac{\beta}{\beta-1}\right)\left\{I(q)-\frac{\tau D}{r}+\left(\frac{1}{\alpha+1}\right)\frac{\tau D}{r}\left[\frac{D}{qX(q)}\right]^{\alpha}\right\}=0. \tag{A.20}$$

Using Eqs. (A.9) and (A.10), we can write Eq. (A.20) as Eq. (21). Substituting Eq. (A.18) into Eq. (A.15) with $A_2 = 0$ yields Eq. (22). Define the following function:

$$G(X)=(1-\tau)\left(\frac{qX}{r-\mu}\right)+\frac{\tau D}{r-\mu}-\left(\frac{\beta}{\beta-1}\right)\left\{I(q)-\left(\frac{1}{\alpha+1}\right)\left[1-\left(\frac{D}{qX}\right)^{\alpha}\right]\frac{\tau D}{r}\right\}. \tag{A.21}$$

Inspection of Eqs. (21) and (A.21) reveals that the investment trigger, $X(q)$, for the case that $X(q) > D/q$ solves $G[X(q)] = 0$. Differentiating Eq. (A.21) with respect to X yields

$$G'(X)=(1-\tau)\left(\frac{q}{r-\mu}\right)+\frac{\tau D}{(r-\mu)X}\left(\frac{D}{qX}\right)^{\alpha}>0, \tag{A.22}$$

where we have used Eqs. (A.9) and (A.10). Since $G(0) = -\infty$ and $G(\infty) = \infty$, Eq. (A.22) implies that there is a unique point, $X(q)$, that solves $G[X(q)] = 0$. Evaluating Eq. (A.21) at $X = D/q$ yields

$$G(D/q)=\frac{D}{r-\mu}-\left(\frac{\beta}{\beta-1}\right)I(q)=\frac{D-D(q)}{r-\mu}, \tag{A.23}$$

where the second equality follows from Eq. (A.52). Eq. (A.23) implies that $X(q) > D/q$ if, and only if, $D < D(q)$. This completes our proof.

C. Proof of Proposition 3

Since $q(D)$ solves $D(q) = D$, $F(q, X)$ is given by Eq. (20) for all $q \in [0, q(D)]$. Differentiating Eq. (20) with respect to q yields

$$\begin{aligned}\frac{\partial F(q,X)}{\partial q} &= \left\{\left.\frac{\partial V_0(q,X)}{\partial q}\right|_{X=X(q)}+\left.\frac{\partial V_0(q,X)}{\partial X}\right|_{X=X(q)}X'(q)-I'(q)\right.\\ &\qquad\left.-\beta\{V_0[q,X(q)]-I(q)\}\frac{X'(q)}{X(q)}\right\}\left[\frac{X}{X(q)}\right]^{\beta}\\ &=\left\{\left.\frac{\partial V_0(q,X)}{\partial q}\right|_{X=X(q)}-I'(q)\right.\end{aligned}$$

$$+\left[\frac{\partial V_0(q,X)}{\partial X}\bigg|_{X=X(q)} - \frac{\partial F(q,X)}{\partial X}\bigg|_{X=X(q)}\right]X'(q)\bigg\}\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left[\frac{\partial V_0(q,X)}{\partial q}\bigg|_{X=X(q)} - I'(q)\right]\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left\{\frac{X(q)}{r-\mu} - \left(\frac{\alpha}{\alpha+\beta}\right)\left(\frac{\beta}{\beta-1}\right)\frac{\tau D}{rq}\left[\frac{D(q)}{D}\right]^{\beta} - I'(q)\right\}\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left\{\left(\frac{\beta}{\beta-1}\right)\left\{1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q)}{D}\right]^{\beta-1}\right\}\frac{I(q)}{q} - I'(q)\right\}\left[\frac{X}{X(q)}\right]^{\beta}, \tag{A.24}$$

where the second equality follows from Eq. (20), the third equality follows from Eq. (17), the fourth equality follows from Eqs. (11) and (A.52), and the last equality follows from Eqs. (A.52), (19), (A.9), and (A.10).

For all $q \geq q(D)$, $F(q,X)$ is given by Eq. (22). Differentiating Eq. (22) with respect to q yields

$$\frac{\partial F(q,X)}{\partial q} = \left\{\frac{\partial V_1(q,X)}{\partial q}\bigg|_{X=X(q)} + \frac{\partial V_1(q,X)}{\partial X}\bigg|_{X=X(q)} X'(q) - I'(q)\right.$$

$$\left. -\beta\{V_1[q,X(q)] - I(q)\}\frac{X'(q)}{X(q)}\right\}\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left\{\frac{\partial V_1(q,X)}{\partial q}\bigg|_{X=X(q)} - I'(q)\right.$$

$$\left. +\left[\frac{\partial V_1(q,X)}{\partial X}\bigg|_{X=X(q)} - \frac{\partial F(q,X)}{\partial X}\bigg|_{X=X(q)}\right]X'(q)\right\}\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left[\frac{\partial V_1(q,X)}{\partial q}\bigg|_{X=X(q)} - I'(q)\right]\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left\{(1-\tau)\left[\frac{X(q)}{r-\mu}\right] + \left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau D}{rq}\left[\frac{D}{qX(q)}\right]^{\alpha} - I'(q)\right\}\left[\frac{X}{X(q)}\right]^{\beta}$$

$$= \left\{\left(\frac{\beta}{\beta-1}\right)\left\{\frac{I(q)}{q} - \frac{\tau D}{rq}\left\{1-\left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{D}{qX(q)}\right]^{\alpha}\right\}\right\} - I'(q)\right\}\left[\frac{X}{X(q)}\right]^{\beta}, \tag{A.25}$$

where the second equality follows from Eq. (22), the third equality follows from Eq. (17), the fourth equality follows from Eq. (12), and the last equality follows from Eq. (21).

Consider first the case that $D(q^0) \leq D$ and thus $q^0 \leq q(D)$, where the equality holds only when $D = D(q^0)$. Suppose that $D(q^*) \leq D$ so that q^* solves the first-order condition that $\partial F(q, X)/\partial q = 0$, where $\partial F(q, X)/\partial q$ is given by Eq. (A.24). Then, q^* solves

$$\left(\frac{\beta}{\beta-1}\right)\left\{1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q^*)}{D}\right]^{\beta-1}\right\}\frac{I(q^*)}{q^*} - I'(q^*) = 0. \tag{A.26}$$

Define the following function:

$$H(q) = \left(\frac{\beta}{\beta-1}\right)\left\{1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q)}{D}\right]^{\beta-1}\right\}I(q) - qI'(q), \tag{A.27}$$

for all $q \in [0, q(D)]$. Inspection of Eqs. (A.26) and (A.27) reveals that the optimal investment intensity, q^*, for the case that $D(q^*) \leq D$ solves $H(q^*) = 0$. Differentiating Eq. (A.27) with respect to q yields

$$\begin{aligned} H'(q) &= \left\{\left(\frac{\beta}{\beta-1}\right)\left\{1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q)}{D}\right]^{\beta-1}\right\} - 1 - \tau\beta\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q)}{D}\right]^{\beta-1}\right\}I'(q) - qI''(q) \\ &\leq \left\{\frac{1}{\beta-1} - \tau\beta\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q)}{D}\right]^{\beta-1}\right\}I'(q) - qI''(q) < 0, \end{aligned} \tag{A.28}$$

where the first inequality follows from the fact that $D(q) \leq D$ for all $q \in [0, q(D)]$, and the second inequality follows from the fact that $[\beta/(\beta-1)]I(q) - qI'(q)$ is strictly decreasing in q. Evaluating Eq. (A.27) at $q = q^0$ yields

$$H(q^0) = \tau\left(\frac{\beta}{\beta-1}\right)\left(\frac{\alpha+1}{\alpha+\beta}\right)\left\{1-\left[\frac{D(q^0)}{D}\right]^{\beta-1}\right\}I(q^0) \geq 0, \tag{A.29}$$

where we have used Eq. (26) and the inequality follows from the fact that $D(q^0) \leq D$. Eq. (A.29) implies that $H(q^0) = 0$ only when $D = D(q^0)$. Evaluating Eq. (A.27) at $q = q(D)$ yields

$$H[q(D)] = \left(\frac{\beta}{\beta-1}\right)\left[1-\tau\left(\frac{\alpha+1}{\alpha+\beta}\right)\right]I[q(D)] - q(D)I'[q(D)] \leq 0, \tag{A.30}$$

where the inequality follows from Eq. (26), and the fact that $q^0 \leq q(D)$ and $[\beta/(\beta-1)]I(q) - qI'(q)$ is strictly decreasing in q. Eqs. (26) and (A.30) imply that $H[q(D)] = 0$ only when $D = D(q^0)$. From Eqs. (A.28), (A.29), and (A.30), there must exist a unique point, $q^* \in [q^0, q(D)]$, that solves $H(q^*) = 0$. Hence, it is indeed that $D(q^*) < D$ if $D(q^0) < D$ and $D(q^*) = D$ if $D(q^0) = D$. We as such conclude that q^* and X^* are uniquely determined by solving Eqs. (27) and (28) if $D > D(q^0)$ such that $q^0 < q^* < q(D)$ and $X^* < D/q^*$. If $D = D(q^0)$, we have $q^* = q^0$ and $X^* = D(q^0)/q^0$.

Consider now the case that $D(q^0) \geq D$ and thus $q^0 \geq q(D)$, where the equality holds only when $D = D(q^0)$. Suppose that $D(q^*) \geq D$ so that q^* solves the first-order condition that $\partial F(q, X)/\partial q = 0$, where $\partial F(q, X)/\partial q$ is given by Eq. (A.25). Then, q^* solves

$$\left(\frac{\beta}{\beta - 1}\right)\left\{\frac{I(q^*)}{q^*} - \frac{\tau D}{rq^*}\left\{1 - \left(\frac{\beta}{\alpha + \beta}\right)\left[\frac{D}{q^* X(q^*)}\right]^{\alpha}\right\}\right\} - I'(q^*) = 0, \tag{A.31}$$

where $X(q^*)$ is given by Eq. (21) with $q = q^*$. Define the following two functions:

$$K_1(q, Y) = \left(\frac{\beta}{\beta - 1}\right)\left\{I(q) - \frac{\tau D}{r}\left[1 - \left(\frac{\beta}{\alpha + \beta}\right)\left(\frac{D}{Y}\right)^{\alpha}\right]\right\} - qI'(q), \tag{A.32}$$

and

$$K_2(q, Y) = (1 - \tau)\left(\frac{Y}{r - \mu}\right) + \frac{\tau D}{r - \mu} - \left(\frac{\beta}{\beta - 1}\right)\left\{I(q) - \left(\frac{1}{\alpha + 1}\right)\left[1 - \left(\frac{D}{Y}\right)^{\alpha}\right]\frac{\tau D}{r}\right\}. \tag{A.33}$$

Inspection of Eqs. (21), (A.31), (A.32), and (A.33) reveals that the optimal investment intensity and trigger, q^* and X^*, for the case that $D(q^*) \geq D$ solve $K_1(q^*, q^*X^*) = 0$ and $K_2(q^*, q^*X^*) = 0$. Totally differentiating $K_1(q, Y) = 0$ with respect to q yields

$$\frac{\mathrm{d}Y}{\mathrm{d}q} = \left[\frac{I'(q)}{\beta - 1} - qI''(q)\right]\Big/\left(\frac{\beta}{\beta - 1}\right)\left(\frac{\alpha\beta}{\alpha + \beta}\right)\left(\frac{D}{Y}\right)^{\alpha}\frac{\tau D}{rY} < 0, \tag{A.34}$$

since $[\beta/(\beta - 1)]I(q) - qI'(q)$ is strictly decreasing in q. Totally differentiating $K_2(q, Y) = 0$ with respect to q yields

$$\frac{\mathrm{d}Y}{\mathrm{d}q} = \left(\frac{\beta}{\beta - 1}\right)I'(q)\Big/\left[\frac{1 - \tau}{r - \mu} + \left(\frac{\alpha}{\alpha + 1}\right)\left(\frac{\beta}{\beta - 1}\right)\left(\frac{D}{Y}\right)^{\alpha}\frac{\tau D}{rY}\right] > 0, \tag{A.35}$$

As Y approaches infinity, $K_1(q, Y) = 0$ implies that q is finite, whereas $K_2(q, Y) = 0$ implies that q is infinite. When $Y = D$, solving $K_1(q_1, D) = 0$ yields

$$\left(\frac{\beta}{\beta - 1}\right)I(q_1) - q_1 I'(q_1) = \left(\frac{\alpha + 1}{\alpha + \beta}\right)\left(\frac{\tau D}{r - \mu}\right), \tag{A.36}$$

and solving $K_2(q_2, D) = 0$ yields

$$\left(\frac{\beta}{\beta - 1}\right)I(q_2) = \frac{D}{r - \mu}. \tag{A.37}$$

Using Eq. (A.52), we can write Eq. (26) as

$$\left(\frac{\beta}{\beta - 1}\right)I(q^0) - q^0 I'(q^0) = \left(\frac{\alpha + 1}{\alpha + \beta}\right)\left[\frac{\tau D(q^0)}{r - \mu}\right]. \tag{A.38}$$

Since $[\beta/(\beta-1)]I(q) - qI'(q)$ is strictly decreasing in q, Eqs. (A.36) and (A.38) imply that $q_1 \geq q^0$ for all $D \leq D(q^0)$, where the equality holds only when $D = D(q^0)$. Eqs. (A.52) and (A.37) imply that $q_2 \leq q^0$ for all $D \leq D(q^0)$, where the equality holds only when $D = D(q^0)$. Since $q_1 \geq q_2$, there must exist a unique point, (q^*, X^*), that solves the system of equations, $K_1(q^*, q^*X^*) = 0$ and $K_2(q^*, q^*X^*) = 0$, with $q^*X^* \geq D$, for the case that $D \leq D(q^0)$. Hence, it is indeed that $D(q^*) > D$ if $D(q^0) > D$ and $D(q^*) = D$ if $D(q^0) = D$. We as such conclude that q^* and X^* are uniquely determined by solving Eqs. (29) and (30) if $D < D(q^0)$ such that $q^* > q(D)$ and $X^* > D/q^*$. If $D = D(q^0)$, then $q^* = q^0$ and $X^* = D(q^0)/q^0$.

D. *Proof of Proposition 4*

Consider first the case that $D > D(q^0)$. Differentiating Eq. (A.26) with respect to D and rearranging terms yields

$$\frac{\mathrm{d}q^*}{\mathrm{d}D} = -\tau\beta\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q^*)}{D}\right]^{\beta-1}\frac{I(q^*)}{D}\Big/ H'(q^*) > 0, \tag{A.39}$$

where $H'(q^*) < 0$ from Eq. (A.28). Differentiating Eq. (28) with respect to D yields

$$\frac{\mathrm{d}Y^*}{\mathrm{d}D} = (r-\mu)\left(\frac{\beta}{\beta-1}\right)I'(q^*)\,\frac{\mathrm{d}q^*}{\mathrm{d}D} > 0, \tag{A.40}$$

where the inequality follows from Eq. (A.39) and $I'(q) > 0$.

Differentiating Eq. (A.26) with respect to τ and rearranging terms yields

$$\frac{\mathrm{d}q^*}{\mathrm{d}\tau} = \left(\frac{\beta}{\beta-1}\right)\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\frac{D(q^*)}{D}\right]^{\beta-1} I(q^*)\Big/ H'(q^*) < 0, \tag{A.41}$$

where $H'(q^*) < 0$ from Eq. (A.28). Differentiating Eq. (28) with respect to τ yields

$$\frac{\mathrm{d}Y^*}{\mathrm{d}\tau} = (r-\mu)\left(\frac{\beta}{\beta-1}\right)I'(q^*)\,\frac{\mathrm{d}q^*}{\mathrm{d}\tau} < 0, \tag{A.42}$$

where the inequality follows from Eq. (A.41) and $I'(q) > 0$.

Consider now the case that $D > D(q^0)$. Differentiating the system of equations, $K_1(q^*, Y^*) = 0$ and $K_2(q^*, Y^*) = 0$, with respect to D yields

$$\frac{\partial K_1(q^*, Y^*)}{\partial q}\,\frac{\mathrm{d}q^*}{\mathrm{d}D} + \frac{\partial K_1(q^*, Y^*)}{\partial Y}\,\frac{\mathrm{d}Y^*}{\mathrm{d}D} + \frac{\partial K_1(q^*, Y^*)}{\partial D} = 0, \tag{A.43}$$

and

$$\frac{\partial K_2(q^*,Y^*)}{\partial q}\frac{\mathrm{d}q^*}{\mathrm{d}D}+\frac{\partial K_2(q^*,Y^*)}{\partial Y}\frac{\mathrm{d}Y^*}{\mathrm{d}D}+\frac{\partial K_2(q^*,Y^*)}{\partial D}=0, \tag{A.44}$$

where $K_1(q,Y)$ and $K_2(q,Y)$ are given by Eqs. (A.32) and (A.33), respectively. Using Eqs. (A.32) and (A.33), we have

$$\frac{\partial K_1(q^*,Y^*)}{\partial q}=\frac{I'(q^*)}{\beta-1}-q^*I''(q^*)<0, \tag{A.45}$$

$$\frac{\partial K_1(q^*,Y^*)}{\partial Y}=-\left(\frac{\beta}{\beta-1}\right)\left(\frac{\alpha\beta}{\alpha+\beta}\right)\left(\frac{D}{Y^*}\right)^{\alpha}\frac{\tau D}{rY^*}<0, \tag{A.46}$$

$$\frac{\partial K_1(q^*,Y^*)}{\partial D}=-\left(\frac{\beta}{\beta-1}\right)\left[1-(\alpha+1)\left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{D}{Y^*}\right)^{\alpha}\right]\frac{\tau}{r}, \tag{A.47}$$

$$\frac{\partial K_2(q^*,Y^*)}{\partial q}=-\left(\frac{\beta}{\beta-1}\right)I'(q^*)<0, \tag{A.48}$$

$$\frac{\partial K_2(q^*,Y^*)}{\partial Y}=\frac{1-\tau}{r-\mu}+\left(\frac{\beta}{\beta-1}\right)\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{D}{Y^*}\right)^{\alpha}\frac{\tau D}{rY^*}>0, \tag{A.49}$$

and

$$\frac{\partial K_2(q^*,Y^*)}{\partial D}=\frac{\tau}{r-\mu}+\left(\frac{\beta}{\beta-1}\right)\left[\frac{1}{\alpha+1}-\left(\frac{D}{Y^*}\right)^{\alpha}\right]\frac{\tau}{r}. \tag{A.50}$$

Note that

$$H=\frac{\partial K_1(q^*,Y^*)}{\partial q}\frac{\partial K_2(q^*,Y^*)}{\partial Y}-\frac{\partial K_1(q^*,Y^*)}{\partial Y}\frac{\partial K_2(q^*,Y^*)}{\partial q}<0, \tag{A.51}$$

where the equality follows from Eqs. (A.45), (A.46), (A.48), and (A.49). Applying Cramer's rule to Eqs. (A.43) and (A.44), we have

$$\frac{\mathrm{d}q^*}{\mathrm{d}D}=\left[\frac{\partial K_2(q^*,Y^*)}{\partial D}\frac{\partial K_1(q^*,Y^*)}{\partial Y}-\frac{\partial K_1(q^*,Y^*)}{\partial D}\frac{\partial K_2(q^*,Y^*)}{\partial Y}\right]\Big/H, \tag{A.52}$$

and

$$\frac{\mathrm{d}Y^*}{\mathrm{d}D}=\left[\frac{\partial K_1(q^*,Y^*)}{\partial D}\frac{\partial K_2(q^*,Y^*)}{\partial q}-\frac{\partial K_2(q^*,Y^*)}{\partial D}\frac{\partial K_1(q^*,Y^*)}{\partial q}\right]\Big/H, \tag{A.53}$$

where $H<0$ is given by Eq. (A.51). When D approaches zero, Eqs. (A.46), (A.47), and (A.50) imply that $\partial K_1(q^*,Y^*)/\partial Y=0$, $\partial K_1(q^*,Y^*)/\partial D<0$, and $\partial K_2(q^*,Y^*)/\partial D>0$. Hence, it follows from Eqs. (A.52) and (A.53) that $\mathrm{d}q^*/\mathrm{d}D<0$ and $\mathrm{d}Y^*/\mathrm{d}D<0$. When D approaches

$D(q^0)$ so that Y^* also approaches $D(q^0)$, Eqs. (A.47) and (A.50) imply that $\partial K_1(q^*, Y^*)/\partial D > 0$ and $\partial K_2(q^*, Y^*)/\partial D = 0$. Hence, it follows from Eqs. (A.52) and (A.53) that $\mathrm{d}q^*/\mathrm{d}D > 0$ and $\mathrm{d}Y^*/\mathrm{d}D > 0$.

Differentiating the system of equations, $K_1(q^*, Y^*) = 0$ and $K_2(q^*, Y^*) = 0$, with respect to τ yields

$$\frac{\partial K_1(q^*, Y^*)}{\partial q} \frac{\mathrm{d}q^*}{\mathrm{d}\tau} + \frac{\partial K_1(q^*, Y^*)}{\partial Y} \frac{\mathrm{d}Y^*}{\mathrm{d}\tau} + \frac{\partial K_1(q^*, Y^*)}{\partial \tau} = 0, \tag{A.54}$$

and

$$\frac{\partial K_2(q^*, Y^*)}{\partial q} \frac{\mathrm{d}q^*}{\mathrm{d}\tau} + \frac{\partial K_2(q^*, Y^*)}{\partial Y} \frac{\mathrm{d}Y^*}{\mathrm{d}\tau} + \frac{\partial K_2(q^*, Y^*)}{\partial \tau} = 0. \tag{A.55}$$

Using Eqs. (A.32) and (A.33), we have

$$\frac{\partial K_1(q^*, Y^*)}{\partial \tau} = -\left(\frac{\beta}{\beta - 1}\right)\left[1 - \left(\frac{\beta}{\alpha + \beta}\right)\left(\frac{D}{Y^*}\right)^{\alpha}\right]\frac{D}{r} < 0, \tag{A.56}$$

and

$$\frac{\partial K_2(q^*, Y^*)}{\partial \tau} = -\left(\frac{Y^* - D}{r - \mu}\right) + \left(\frac{\beta}{\beta - 1}\right)\left(\frac{1}{\alpha + 1}\right)\left[1 - \left(\frac{D}{Y^*}\right)^{\alpha}\right]\frac{D}{r}. \tag{A.57}$$

Applying Cramer's rule to Eqs. (A.54) and (A.55), we have

$$\frac{\mathrm{d}q^*}{\mathrm{d}\tau} = \left[\frac{\partial K_2(q^*, Y^*)}{\partial \tau} \frac{\partial K_1(q^*, Y^*)}{\partial Y} - \frac{\partial K_1(q^*, Y^*)}{\partial \tau} \frac{\partial K_2(q^*, Y^*)}{\partial Y}\right] \Big/ H, \tag{A.58}$$

and

$$\frac{\mathrm{d}Y^*}{\mathrm{d}\tau} = \left[\frac{\partial K_1(q^*, Y^*)}{\partial \tau} \frac{\partial K_2(q^*, Y^*)}{\partial q} - \frac{\partial K_2(q^*, Y^*)}{\partial \tau} \frac{\partial K_1(q^*, Y^*)}{\partial q}\right] \Big/ H, \tag{A.59}$$

where $H < 0$ is given by Eq. (A.51). When D approaches zero, Eqs. (A.46), (A.56), and (A.57) imply that $\partial K_1(q^*, Y^*)/\partial Y = 0$, $\partial K_1(q^*, Y^*)/\partial \tau = 0$, and $\partial K_2(q^*, Y^*)/\partial \tau < 0$. Hence, it follows from Eqs. (A.58) and (A.59) that $\mathrm{d}q^*/\mathrm{d}\tau > 0$ and $\mathrm{d}Y^*/\mathrm{d}\tau > 0$. When D approaches $D(q^0)$ so that Y^* also approaches $D(q^0)$, Eqs. (A.56) and (A.57) imply that $\partial K_1(q^*, Y^*)/\partial \tau < 0$ and $\partial K_2(q^*, Y^*)/\partial \tau = 0$. Hence, it follows from Eqs. (A.58) and (A.59) that $\mathrm{d}q^*/\mathrm{d}\tau < 0$ and $\mathrm{d}Y^*/\mathrm{d}\tau < 0$.

References

Agliardi, E., 2001. Taxation and investment decisions: a real options approach. Australian Economic Papers 40, 44–55.

Agliardi, E., Agliardi, R., 2008. Progressive taxation and corporate liquidation policy. Economic Modelling 25, 532–541.

Agliardi, E., Agliardi, R., 2009. Progressive taxation and corporate liquidation: Analysis and policy implications. Journal of Policy Modeling 31, 144–154.

Alvarez, L.H.R., Kanniainen, V., Södersten, J., 1998. Tax policy uncertainty and corporate investment: A theory of tax-induced investment spurts. Journal of Public Economics 69, 17–48.

Alvarez, L.H.R., Koskela, E., 2008. Progressive taxation, tax exemption, and irreversible investment under uncertainty. Journal of Public Economic Theory 10, 149–169.

Bar-Ilan, A., Strange, W.C., 1996. Investment lags. American Economic Review 86, 611–622.

Bar-Ilan, A., Strange, W.C., 1999. The timing and intensity of investment. Journal of Macroeconomics 21, 57–77.

Capozza, D. R., Li, Y., 1994. The intensity and timing of investment: The case of land. American Economic Review 84, 889–904.

Chu, K. C., Wong, K. P., 2010. Progressive taxation and corporate liquidation policies with mean-reverting earnings. Economic Modelling 27, in press.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainty. Princeton University Press, Princeton, NJ.

Hassett, K. A., Metcalf, G. E., 1999. Investment with uncertain tax policy: Does random tax policy discourage investment? Economic Journal 109, 373–393.

Mackie-Mason, J. K., 1990. Some nonlinear tax effects on asset values and investment decisions under uncertainty. Journal of Public Economics 42, 301–328.

McDonald, R., Siegel, D., 1986. The value of waiting to invest. Quarterly Journal of Economics 101, 707–727.

Niemann, R., 1999. Neutral taxation under uncertainty — a real options approach. FinanzArchiv

56, 51–66.

Niemann, R., Sureth, C., 2004. Tax neutrality under irreversibility and risk aversion. Economics Letters 84, 43–47.

Panteghini, P. M., 2001. On corporate tax asymmetries and neutrality. German Economic Review 2, 269–286.

Panteghini, P. M., 2005. Asymmetric taxation under incremental and sequential investment. Journal of Public Economic Theory 7, 761–779.

Pennings, E., 2000. Taxes and stimuli of investment under uncertainty. European Economic Review 44, 383–391.

Sarkar, S., 2008. Can tax convexity be ignored in corporate financing decisions? Journal of Banking and Finance 32, 1310–1321.

Sarkar, S., Goukasian, L., 2006. The effect of tax convexity on corporate investment decisions and tax burdens. Journal of Public Economic Theory 8, 293–320.

Sureth, C., 2002. Partially irreversible investment decisions and taxation under uncertainty: A real option approach. German Economic Review 3, 185–221.

Wong, K. P., 2007. The effect of uncertainty on investment timing in a real options model. Journal of Economic Dynamics and Control 31, 2152–2167.

Wong, K. P., 2009. Progressive taxation, tax exemption, and corporate liquidation policy. Economic Modelling 26, 295–299.

Wong, K. P., Wu, Y., 2010. Tax convexity, investment, and capital structure. Working Paper, University of Hong Kong.

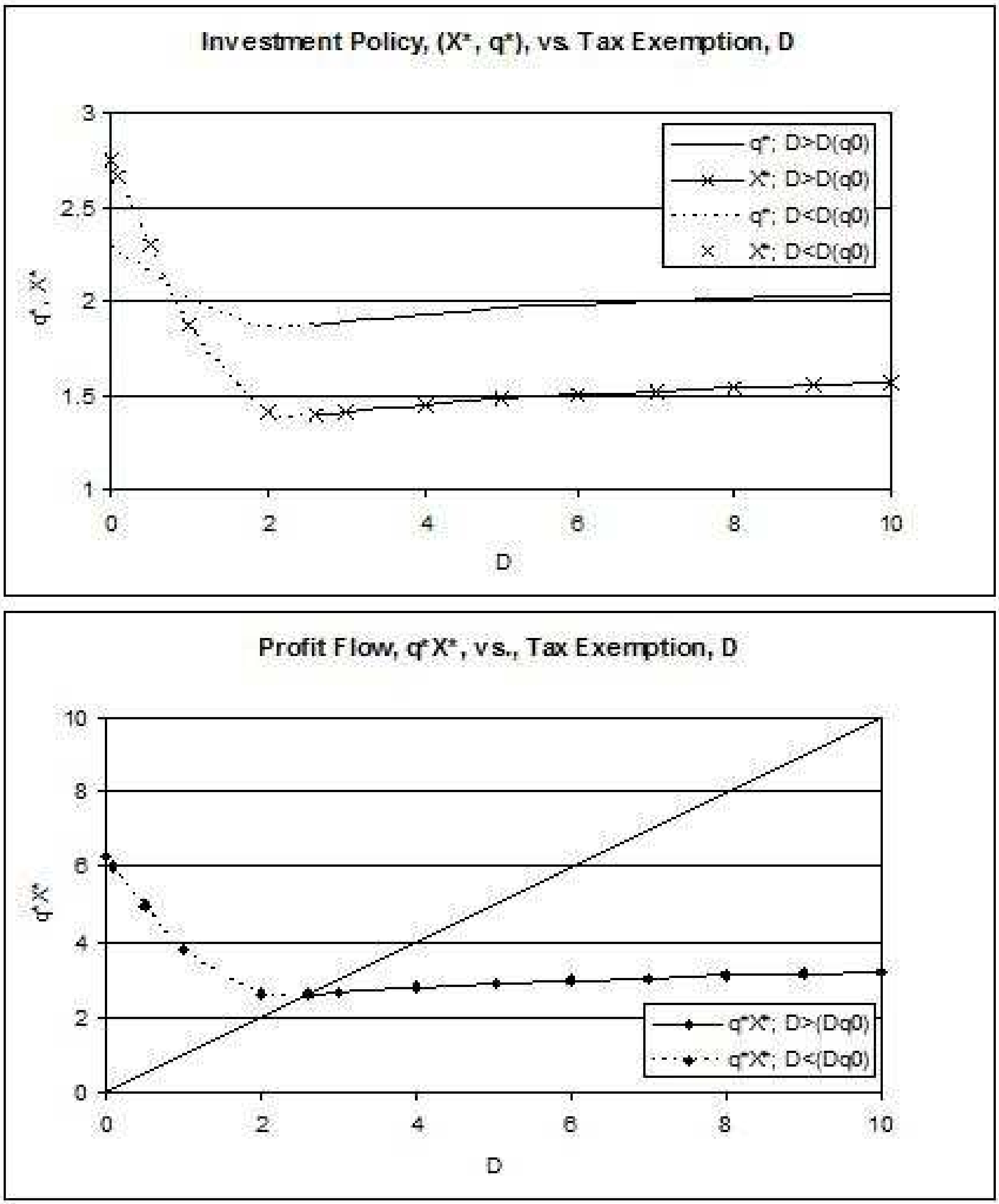
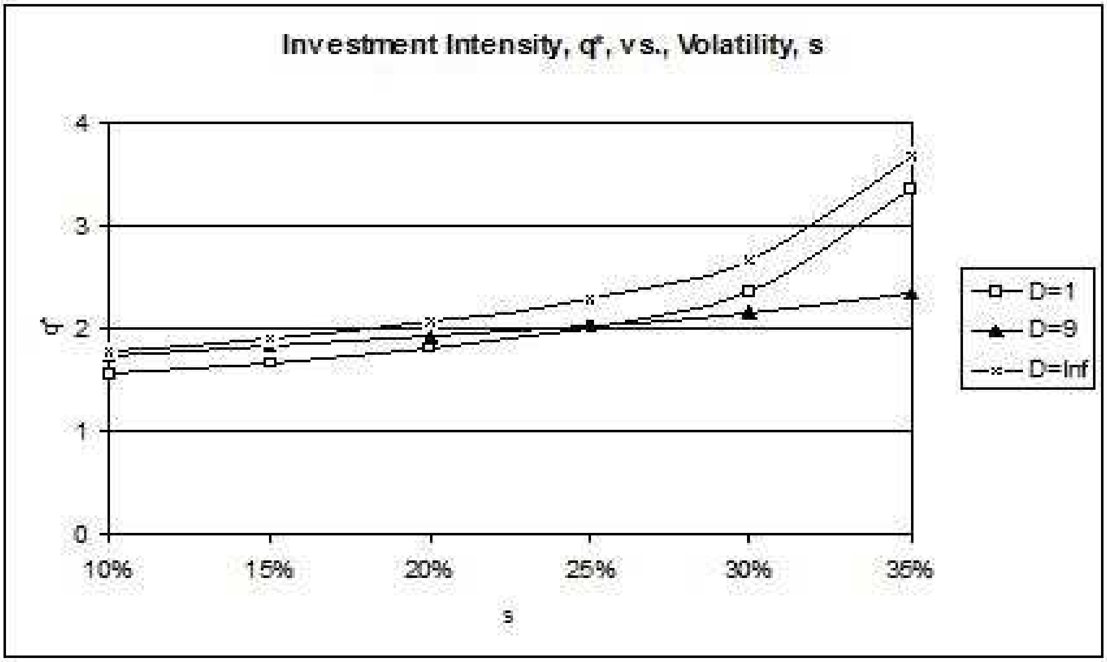
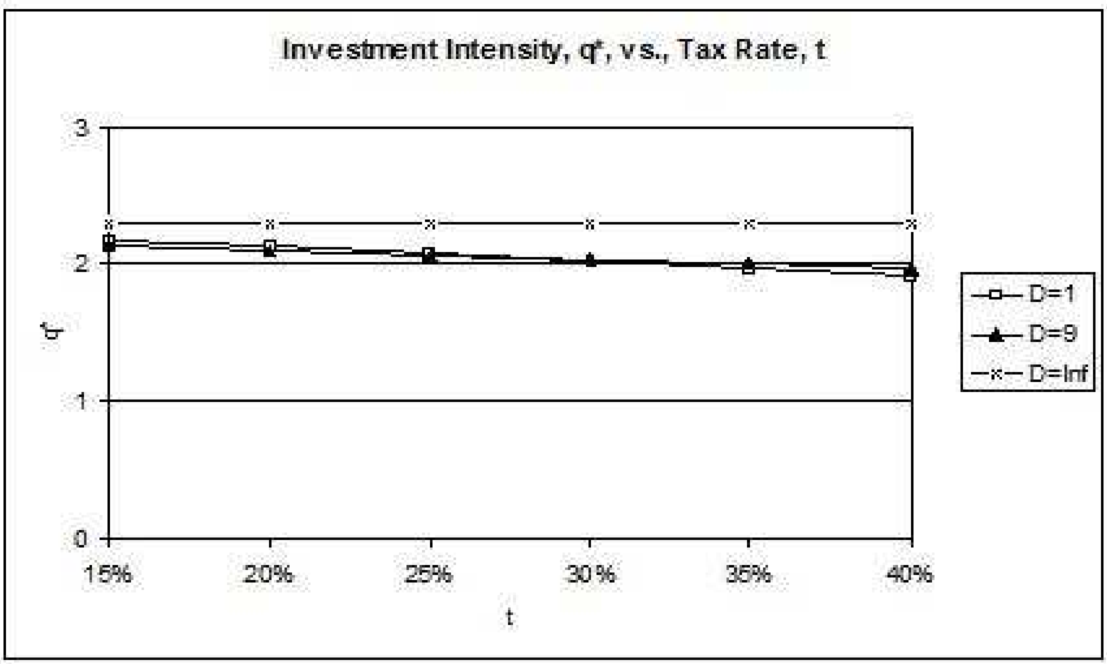
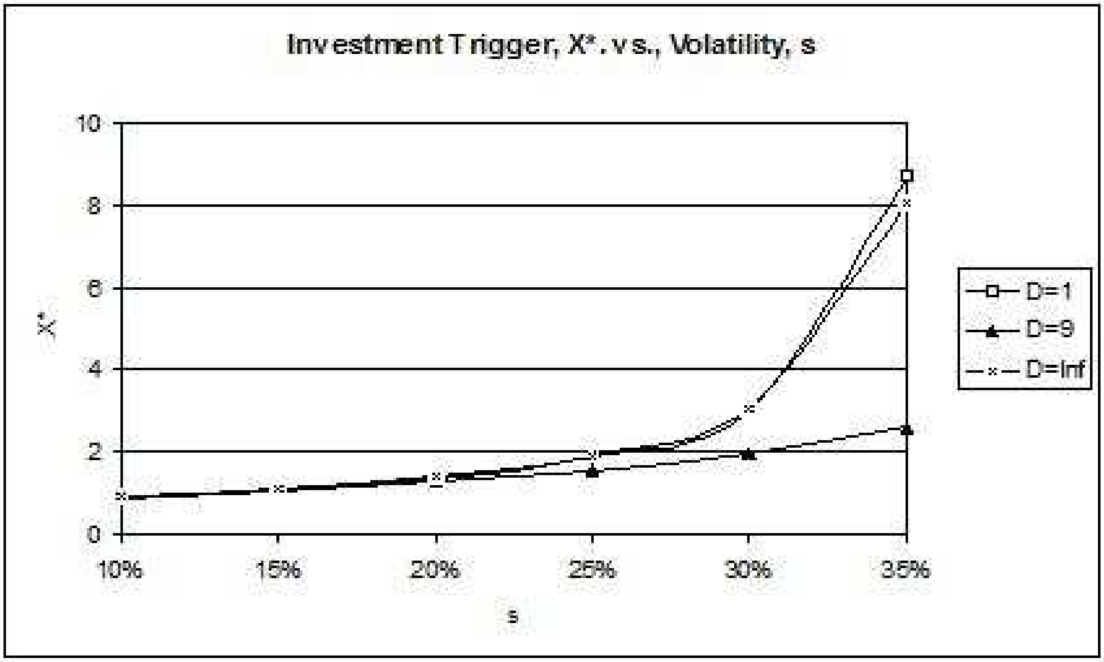


Fig. 1. The investment intensity, q^*, the investment trigger, X^*, and the threshold profit flow, $Y^* = q^*X^*$, as functions of the exogenously given tax exemption threshold, D. The investment cost function is $I(q) = 10 + q^4$. The parameter values are: the riskless rate of interest, r, is 7%; the corporate income tax rate, τ, is 30%; and the state variable, X, takes on the initial value, $X_0 = 1$, with the growth rate, $\mu = 3\%$, and the standard deviation, $\sigma = 25\%$. For this set of parameters, $q^0 = 1.648$ and $D(q^0) = 1.571$.


Investment Intensity, q*, vs., Volatility, s
4
3
2
1
0
q*
10%
15%
20%
25%
30%
35%
s
D=1
D=9
D=Inf


Investment Intensity, q*, vs., Tax Rate, t
3
2
1
0
q*
15%
20%
25%
30%
35%
40%
t
D=1
D=9
D=Inf



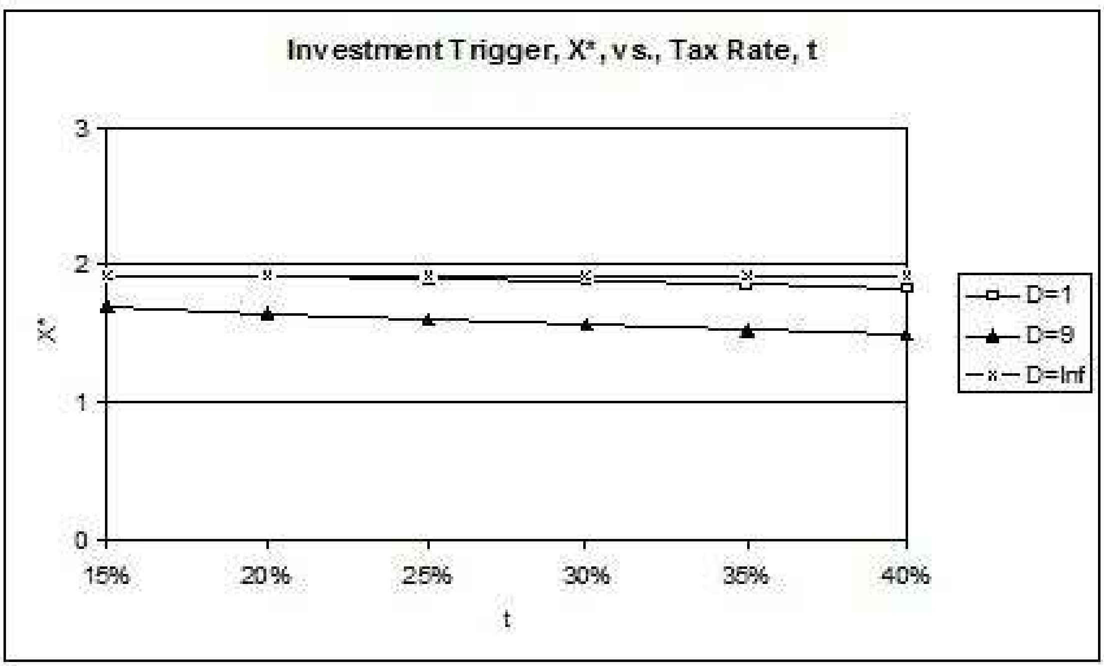


Fig. 2. The investment intensity, q^*, and the investment trigger, X^*, as functions of the standard derivation, σ, and as functions of the tax rate, τ, for three different values of the exogenously given tax exemption threshold, D. The investment cost function is $I(q) = 10 + q^4$. The parameter values are: the riskless rate of interest, r, is 7%; the corporate income tax rate, τ, is 30%; and the state variable, X, takes on the initial value, $X_0 = 1$, with the growth rate, $\mu = 3\%$, and the standard deviation, $\sigma = 25\%$.

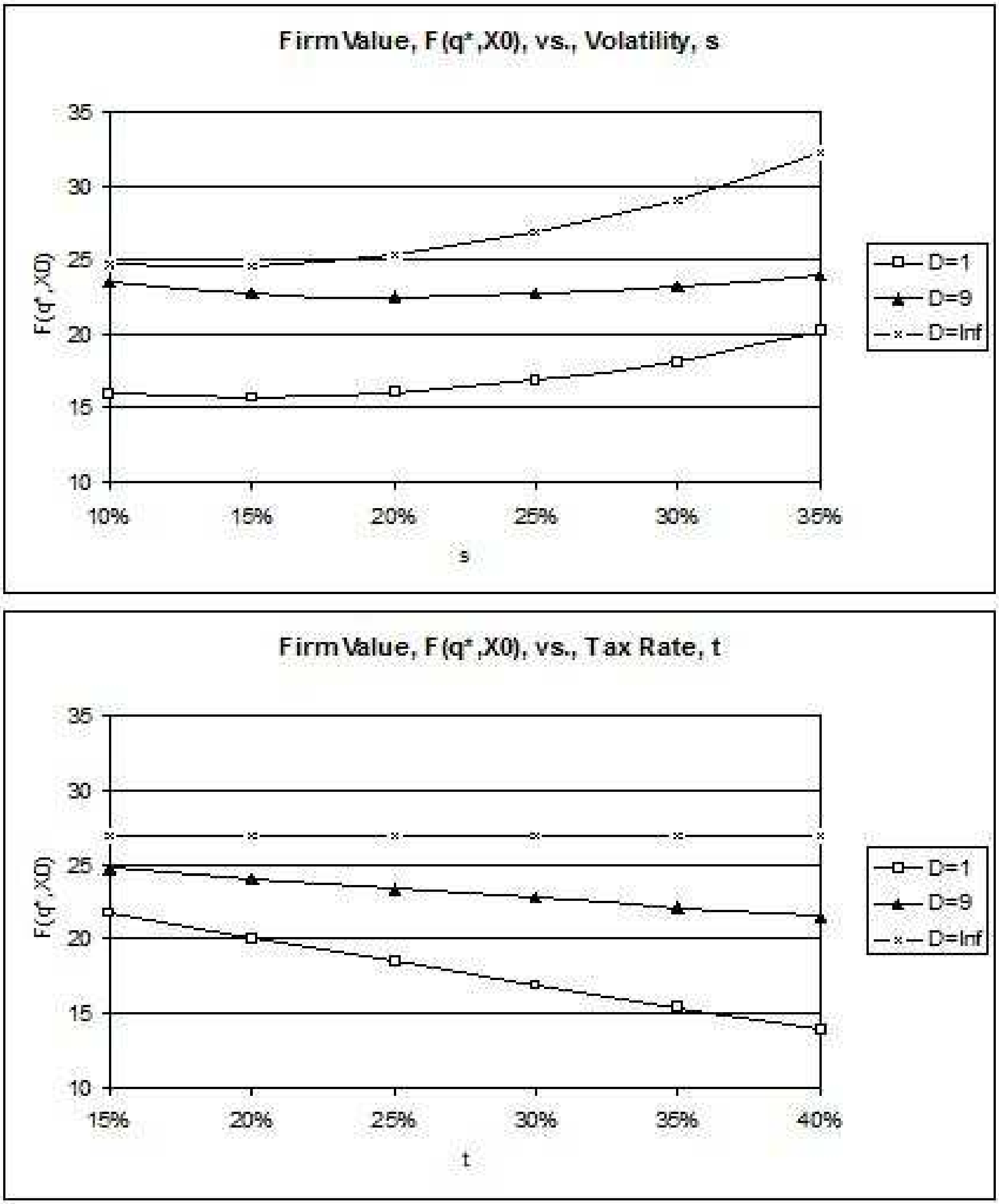


Fig. 3. The value of the firm, $F(q^*, X_0)$, at $t = 0$ as functions of the standard derivation, σ, and as functions of the tax rate, τ, for three different values of the exogenously given tax exemption threshold, D. The investment cost function is $I(q) = 10 + q^4$. The parameter values are: the riskless rate of interest, r, is 7%; the corporate income tax rate, τ, is 30%; and the state variable, X, takes on the initial value, $X_0 = 1$, with the growth rate, $\mu = 3\%$, and the standard deviation, $\sigma = 25\%$.