Optimizing a Modular Expansion of a Wastewater Treatment Plant Using Option Theory and Moment Matching Approximation

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Abstract

We consider a municipality faced with the question of how big to make their new wastewater treatment facility to meet the demand of 10% expected growth in the number of new connections. Previously, we developed a real options framework for determining optimal plant size and showed that the model takes on the form of an Asian option. Furthermore, it was shown that if the connection rate growths are closely correlated with the market growth, then the penalty costs associated with having insufficient capacity to treat the wastewater can be effectively hedged, significantly reducing overall expected costs. In this study, we introduce an approximate analytical solution and optimize the plant size of a staged / modular expansion. Based on the given construction cost estimates, we show that a staged expansion has a minimal (expected) savings when connection growth rates are closely correlated to the market growth rates. However, as the correlation decreases to zero, or, alternatively, no attempt is made to hedge the penalty costs, a staged expansion has an expected savings of 20%.

Introduction

The municipal water and wastewater industrial sector is considered to be one of the most capital intensive industrial sectors and unfortunately the American Society of Civil Engineers (2005) rated the condition of the drinking water and wastewater infrastructure systems as poor, citing specifically a lack of investment in capital assets over a prolonged period of time. Clearly, methods based on sound financial principles that enhance capital asset allocation strategies can add significant value to municipal decision makers. It is recognized that projects face future uncertainties. The ability of project managers to react to these uncertainties at a future time adds intrinsic value to the project, and this value is not captured by standard discounted cash flow (DCF) / net present value (NPV) methods. To adequately account for the uncertainty and its impact on the project value, financial engineering methods applied in the financial markets can be utilized in "real" capital investment projects. Trigeorgis (1996) provides a thorough introduction and review of real option theory and how it can be utilized to enhance an entity's strategy in resource allocation.

While capital asset and project valuation using real options has seen a significant research focus over the last 15 years (see, for example Jacoby and Laughton (1992), Ingersoll and Ross (1992), Emhiellen and Alaouze (2003), and van Putten and MacMillan (2004)), real options theory has seen limited application in the municipal infrastructure sector. Of note, Schubert and Barenbaum (2007) discuss how public managers can employ real options technique to better value their capital budgeting opportunities and improve the efficacy of capital budgeting decisions.

Other studies, Ho and Liu (2002), Garvin and Cheah (2004), consider the application of real options to value public infrastructure projects under private management arrangements. Arboleda and Abraham (2006) propose a method using real option analysis to evaluate capital investments in public infrastructure projects managed by private operators. The proposed methodology develops a valuation based on deterioration curves of infrastructure and the associated value of flexibility to invest at optimal states within the model.

Recently, we presented a real option valuation method to determine the optimal size of a wastewater plant expansion required for a small municipality undergoing significant residential growth (Lawryshyn and Jaimungal (2009)). The community is located in a resort area and has experienced increases in the growth rates of approximately 10% over the last 15 years. Since a significant number of new dwellings are second "weekend" homes, the planners felt strongly that growth rates were tied to the strength of the market index[1]. We showed that the model takes the form of an Asian option, and numerical methods were used to solve the resulting partial differential equation. Hedging strategies were introduced to show the potential savings the municipality could theoretically realize.

In this study, we consider a staged investment approach, where the municipality can today, decide to build a plant of a smaller size, and then expand at a defined time in the future, if the residential growth warrants the expansion. Because of the significant computational effort required to solve the partial differential equation of the model, we introduce an approximate closed form solution and use this equation for optimization.

The following section provides a brief description of the cost dynamics of the problem. Next, in the Model Development section, the Asian option-like model developed previously is summarized, a closed form solution approximation is introduced and the

[1] Analysis of the data showed some correlation to the general stock index, but due to the limited amount of total connections, confidence intervals on the correlations were very broad. It is likely that a lag exists between the market index and the rate of connectivity and analysis of the data showed a lag of approximately 9 months. For the analysis presented here the lag was ignored.

optimization strategy is developed. A comparison of the numerical results of the exact model to the analytical approximate model and optimization results are presented in the Results section. Conclusions are presented in the last section.

Cost Description

As mentioned previously, this study focuses on developing an optimal wastewater plant expansion strategy for a small resort municipality. In the last 15 years, the municipal rate of wastewater connections has been growing by an average of 10% and the current infrastructure is inadequate to meet the expected demand in approximately 3 to 5 years. The present conditions are such that the municipality must build a new plant immediately. It is assumed that construction time to build a new plant is 3 years. The municipality is faced with the option of building a full-size (non-staged) plant of size K_f, or a modular plant of initial size K_1 with the option to expand up to a predetermined maximum size of $K_1 + K_{2max}$. The total timeframe for the analysis is assumed to be 23 years – i.e. the plant has a 20 year useful life. The staged expansion decision will be made 11 years from now and will require 2 years of construction time (see Figure 1).

Gillot et al. (1999) and Alasino et al. (2007) provide detailed methodology to optimize the cost of constructing a wastewater plant by considering different unit processes. Most of the cost functions power functions of the form aQ^b, where Q is the flow rate (analogous to plant size) and a and b are constants for the given unit process. A simpler fixed / variable cost function is assumed in this study; however this has no loss of generality for the proposed method.

The present value cost, as of the initial construction date, to build a non-modular plant to size K_f is given by

$$C_f(K_f) = \alpha_f + \gamma_f K_f \,. \tag{1}$$

For a modular plant, the initial cost is given as

$$C_1(K_1, K_{2\max}) = \alpha_1 + \gamma_1 K_1 + \gamma_{12} K_{2\max} \tag{2}$$

and the cost to expand the modular plant (present value as of the date of initial construction expansion) in real dollars as of t_0 is

$$C_2(K_2 \le K_{2\max}) = \alpha_2 + \gamma_2 K_2 \,. \tag{3}$$

The parameters α_i and γ_i are the fixed cost and variable cost components. The present value of the salvage value is assumed to be captured by the parameters.

Model Development

This section consists for three sub-sections: 1) Summary of the Non-Staged Plant Expansion, which summarizes the model previously developed by Lawryshyn and Jaimungal (2009), 2) Analytical Approximation, which develops the analytical approximation to the Asian-like option model, and 3) Staged Optimization Model

Summary of the Non-Staged Plant Expansion

Geometric Brownian Motion (GBM) was assumed for both the stock index, S_t, and the wastewater connection rate, X_t,

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_t \tag{4}$$

$$dX_t = \mu_X X_t dt + \sigma_X X_t \left(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp} \right) \tag{5}$$

where W_t and $W^{\perp}_t$ are Weiner processes independent of each other. The growth, μ_i, volatility, σ_i, and correlation, ρ, parameters are assumed to be constant. Under the risk-neutral measure

$$d\tilde{W}_t \equiv \frac{\mu_S - r}{\sigma_S} dt + dW_t \tag{6}$$

and

$$dX_t = \bar{r} X_t dt + \sigma_X X_t \left(\rho d\tilde{W}_t + \sqrt{1-\rho^2} dW_t^{\perp} \right) \tag{7}$$

where $\bar{r} \equiv \mu_X - \frac{\rho \sigma_X}{\sigma_S}(\mu_S - r)$ and r is the risk-free rate.

Defining N_t as the total number of connections to the plant, N_0 as the current plant capacity, then

$$N_t = N_0 + \int_0^t X_u du, \tag{8}$$

and defining the penalty cost associated with over capacity, PC_t, as

$$PC_t = \max\left(0, (N_t - K) \cdot PC_0 e^{r_{cpi} t} \right), \tag{9}$$

led to the following expected present value of the penalty cost incurred from time t ot T,

$$\begin{aligned} \tilde{E}\left[PC_{t,T;K}^{PV} \right] &= \int_t^T PC_0 e^{-(r-r_{cpi})u} \cdot \tilde{E}\left[\left(N_u - K \right)_+ \right] du \\ &= \int_t^T PC_0 e^{-(r-r_{cpi})u} \cdot \tilde{E}\left[\left(N_0 + \int_0^u X_s \, ds - K \right)_+ \right] du \end{aligned} \tag{10}$$

where PC_0 is the current penalty cost rate associated with insufficient plant capacity per connection, K is the size of the plant expansion and r_{cpi} is the inflation rate. The term $\tilde{E}\left[\left(N_0 + \int_0^u X_s ds - K \right)_+ \right]$ takes on the form of an Asian option's payoff and defining

$$v(t, X_t, N_t) \equiv \tilde{E}\left[\left(N_t + \int_t^T X_s \, ds - K \right)_+ \middle| \mathscr{F}_t \right], \tag{11}$$

the solution to v(t,x,y), with X_t and N_t replaced by the dummy variables x and y, was given by the following partial differential equation (PDE)

$$\frac{\partial v}{\partial t} + \bar{r} x \frac{\partial v}{\partial x} + x \frac{\partial v}{\partial y} + \tfrac{1}{2} \sigma_X^2 x^2 \frac{\partial^2 v}{\partial x^2} = 0. \tag{12}$$

Analytical Approximation

Since the problem requires optimization of the parameter K of equation (10), which requires multiple solutions of the PDE of equation (12), an analytical approximation for $v(t,X_t,N_t)$ is developed.

For the approximation, it is assumed that

$$\tilde{E}\left[\left(\int_t^T X_s\,ds - K'_t\right)_+ \middle| \mathscr{F}_t\right] \sim \tilde{E}\left[\left(X_t e^{\left(\tilde{\mu}-\frac{1}{2}\tilde{\sigma}^2\right)+\tilde{\sigma}Z} - K'_t\right)_+ \middle| \mathscr{F}_t\right] \tag{13}$$

where $K'_t = K - N_t$ and Z ~ N(0,1) so that the Black-Scholes analytical solution can be utilized. Moments are matched as follows. For a process given by $e^{\left(\tilde{\mu}-\frac{1}{2}\tilde{\sigma}^2\right)+\tilde{\sigma}Z}$,

$$E\left[e^{\left(\tilde{\mu}-\frac{1}{2}\tilde{\sigma}^2\right)+\tilde{\sigma}Z}\right] = e^{\tilde{\mu}} \tag{14}$$

and

$$E\left[\left(e^{\left(\tilde{\mu}-\frac{1}{2}\tilde{\sigma}^2\right)+\tilde{\sigma}Z}\right)^2\right] = e^{\left(2\tilde{\mu}+\tilde{\sigma}^2\right)}. \tag{15}$$

Also,

$$\tilde{E}\left[\int_t^T X_u\,du \middle| \mathscr{F}_t\right] = X_t\,\frac{e^{\overline{r}(T-t)}-1}{\overline{r}} \tag{16}$$

and

$$\begin{aligned}\tilde{E}\left[\left(\int_t^T X_u\,du\right)^2 \middle| \mathscr{F}_t\right] &= 2\int_t^T\int_t^u \tilde{E}\left[X_u X_v \middle| \mathscr{F}_t\right]dv\,dt \\ &= \frac{2X_t^2}{\overline{r}+\sigma_X^2}\left(\frac{e^{\left(2\overline{r}+\sigma_X^2\right)(T-t)}-1}{2\overline{r}+\sigma_X^2} - \frac{e^{\overline{r}(T-t)}-1}{\overline{r}}\right).\end{aligned} \tag{17}$$

Matching equations (14) and (16) gives

$$\tilde{\mu}_{t,T} \equiv \tilde{\mu} = \ln\left(e^{\overline{r}(T-t)}-1\right) - \ln\overline{r} \tag{18}$$

and equations (15), (17) and (18) gives

$$\tilde{\sigma}_{t,T} \equiv \tilde{\sigma} = \sqrt{\ln\left(\frac{2}{\overline{r}+\sigma_X^2}\right) + \ln\left(\frac{e^{\left(2\overline{r}+\sigma_X^2\right)(T-t)}-1}{2\overline{r}+\sigma_X^2} - \frac{e^{2\overline{r}(T-t)}-1}{\overline{r}}\right) + 2\ln\overline{r} - 2\ln\left(e^{\overline{r}(T-t)}-1\right)}. \tag{19}$$

From equation (13),

$$\begin{aligned}\tilde{E}\left[\left(X_t e^{\left(\tilde{\mu}_{t,T}-\frac{1}{2}\tilde{\sigma}_{t,T}^2\right)+\tilde{\sigma}_{t,T}Z} - K'_t\right)_+ \middle| \mathscr{F}_t\right] &= e^{\tilde{\mu}_{t,T}}\left(e^{-\tilde{\mu}_{t,T}}\tilde{E}\left[\left(X_t e^{\left(\tilde{\mu}_{t,T}-\frac{1}{2}\tilde{\sigma}_{t,T}^2\right)+\tilde{\sigma}_{t,T}^2 Z} - K'_t\right)_+ \middle| \mathscr{F}_t\right]\right) \\ &= e^{\tilde{\mu}_{t,T}}\left(X_t\Phi(d_{t,T;+}) - K'_t e^{-\tilde{\mu}_{t,T}}\Phi(d_{t,T;-})\right)\end{aligned} \tag{20}$$

where $d_{t,T;\pm} = \frac{\ln(X_t / K'_t) + \tilde{\mu}_{t,T} \pm \frac{1}{2}\tilde{\sigma}^2_{t,T}}{\tilde{\sigma}_{t,T}}$ and Φ is the normal cumulative distribution function. Thus, the penalty cost of equation (10) can be approximated as follows

$$\tilde{E}\left[PC^{PV}_{t,T;K}\right] \sim \int_t^T PC_0 e^{-(r-r_{cpi})u} \cdot e^{\tilde{\mu}_{0,u}}\left(X_0\Phi(d_{0,u;+}) - (K-N_0)e^{-\tilde{\mu}_{0,u}}\Phi(d_{0,u;-})\right)du\,, \quad (21)$$

or in the case where we are interested in a filtration, $\mathscr{F}_\tau$, where $\tau \le t$, equation (21) can be written as

$$\tilde{E}\left[PC^{PV}_{t,T;K}\middle|\mathscr{F}_\tau\right] \sim \int_t^T PC_\tau e^{-(r-r_{cpi})u} \cdot e^{\tilde{\mu}_{\tau,u}}\left(X_\tau\Phi(d_{\tau,u;+}) - (K-N_\tau)e^{-\tilde{\mu}_{\tau,u}}\Phi(d_{\tau,u;-})\right)du\,, \quad (22)$$

where $PC_\tau = PC_0 e^{r_{cpi}\tau}$. It should be emphasized that equation (22) represents the penalty cost as of time τ. The analytical approximation of equation (20) will be compared to the numerical solution of the PDE (equation (12)) in the Results section.

For the non-modular plant, determining the optimal plant size, K_f can be done directly by minimizing the present value of both the plant cost and the expected penalty cost (equations (1) and (21)),

$$K_f^{Opt} = \min_{K_f}\left(C_f(K_f) + \tilde{E}\left[PC^{PV}_{t_1,T;K_f}\right]\right). \quad (23)$$

Equation (23) is easily solved using numerical non-linear minimization.

Staged Optimization Model

The timeline for the optimization problem is depicted in Figure 1. A sample path for X_t is presented in Figure 2 highlighting the decision points. The initial plant size and the maximum plant size decision must be made at t_0, i.e. K_1 and K_{2max} must be determined at this time. Stage 1 construction will take place from t_0 to t_1. Note that during this timeframe, incurred penalty costs for lack of plant capacity have no impact on decision making, since the decision making process has no impact on the current plant capacity. From t_1 to t_3 the new modular plant will operate with a capacity of K_1. Note that N_0 denotes the number of current customers (connections), as of t_0, that the municipality will redirect towards the new plant. At t_2 a decision will need to be made with respect to the added plant capacity, K_2, with $K_2 \le K_{2max}$. Stage 2 construction will take place from t_2 to t_3 and Stage 2 operation, with total plant capacity of $K_1 + K_2$, will occur from t_3 to T.

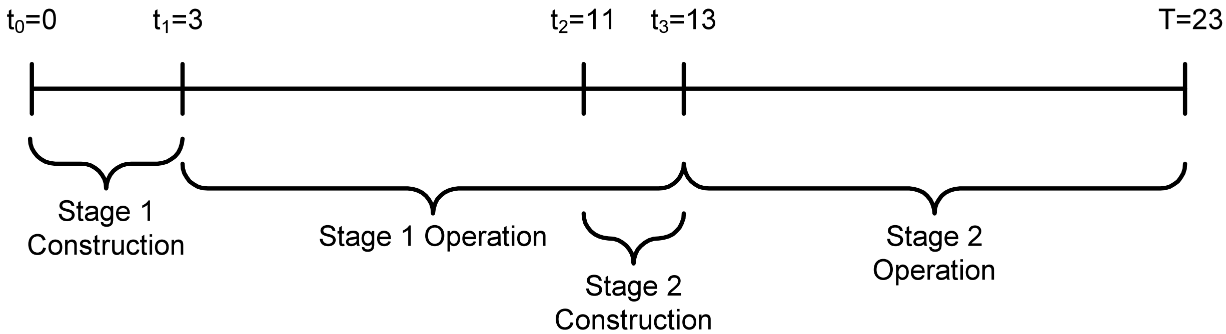

Figure 1. Timeline for plant construction and operation.

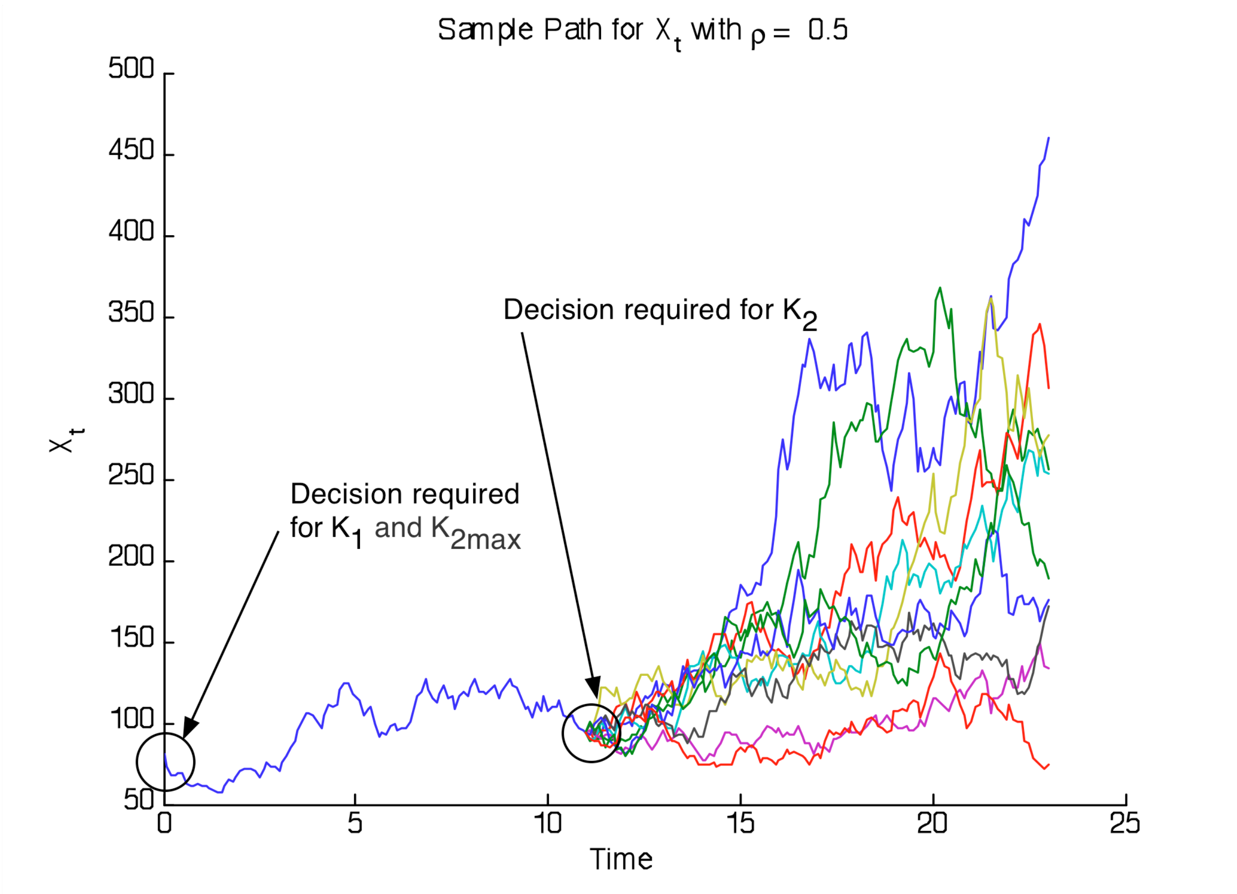

Figure 2. Simulation of X_t.

The expected cost for the modular plant can be written as

$$
\begin{aligned}
C_{mod} &= C_1\left(K_1, K_{2\max}\right) + \tilde{E}\left[PC^{PV}_{t_1,t_3;K_1}\right] \\
&+ \int_{-\infty}^{\infty}\int_0^{\infty}\left(\tilde{E}\left[PC^{PV}_{t_3,T;K_1+\min\left(K_2^{Opt},K_{2\max}\right)}\middle|\mathscr{F}_{t_2}\right]e^{-rt_2} + C_2\left(\min\left(K_2^{Opt},K_{2\max}\right)\right)\right)\cdot f_{X,N}(X_{t_2},N_{t_2})\,dX_{t_2}\,dN_{t_2}
\end{aligned}
\tag{24}
$$

where $f_{X,N}(X_{t_2},N_{t_2})$ is the joint probability density function (PDF) of X_t and N_t at $t = t_2$ and K_2^{Opt} is the optimized size of the plant at Stage 2 for a given X_{t2} and N_{t2}, determined by

$$K_2^{Opt} = \min_{K_2}\left(C_2(K_2) + \tilde{E}\left[PC^{PV}_{t_3,T;K_1+\min(K_2,K_{2max})}\middle|\mathscr{F}_{t_2}\right]e^{-rt_2}\right). \tag{25}$$

For notational efficiency, we define

$$g\left(X_{t_2},N_{t_2};K_1,K_{2max}\right) \equiv \tilde{E}\left[PC^{PV}_{t_3,T;K_1+\min\left(K_2^{Opt},K_{2max}\right)}\middle|\mathscr{F}_{t_2}\right]e^{-rt_2} + C_2\left(\min\left(K_2^{Opt},K_{2max}\right)\right) \tag{26}$$

Optimization now proceeds by minimizing C_{mod} as a function of K_1 and K_{2max}.

One unresolved issue is the estimation of the joint PDF, $f_{X,N}(X_{t_2},N_{t_2})$. A simple approach would be to estimate the function through simulation. We propose a slightly more elegant approach by assuming that N_t is approximately log-normally distributed and applying moment matching. Specifically, we assume that $X_t \sim e^{z_1}$ and $N_t \sim e^{z_2}$, where $z_1 = m_1 + \sigma_1 W_1$ and $z_2 = m_2 + \sigma_2\left(\bar{\rho}W_1 + \sqrt{1-\bar{\rho}^2}W_2\right)$, W_1 and W_2 are uncorrelated N(0,1) and, m_1, m_2, σ_1, σ_2 and $\bar{\rho}$ need to be determined. Letting $E\left[X_t\right] = E\left[e^{z_1}\right]$, $E\left[N_t\right] = E\left[e^{z_2}\right]$, $E\left[X_t^2\right] = E\left[e^{2z_1}\right]$, $E\left[N_t^2\right] = E\left[e^{2z_2}\right]$ and $E\left[X_tN_t\right] = E\left[e^{z_1}e^{z_2}\right]$ we can solve for the parameters as follows (see Appendix for details)

$$\begin{aligned}
m_1 &= 2\ln\left(E\left[X_t\right]\right) - \tfrac{1}{2}\ln\left(E\left[X_t^2\right]\right)\\
m_2 &= 2\ln\left(E\left[N_t\right]\right) - \tfrac{1}{2}\ln\left(E\left[N_t^2\right]\right)\\
\sigma_1^2 &= -2\ln\left(E\left[X_t\right]\right) + \ln\left(E\left[X_t^2\right]\right)\\
\sigma_2^2 &= -2\ln\left(E\left[N_t\right]\right) + \ln\left(E\left[N_t^2\right]\right)\\
\bar{\rho} &= \frac{\ln\left(E\left[X_tN_t\right]\right) - \left(m_1 + m_2 + \tfrac{1}{2}\left(\sigma_1^2 + \sigma_2^2\right)\right)}{\sigma_1\sigma_2}
\end{aligned} \tag{27}$$

Therefore, equation (24) can be modified as follows,

$$\begin{aligned}
C_{mod} = C_1\left(K_1,K_{2max}\right) &+ \tilde{E}\left[PC^{PV}_{t_1,t_3;K_1}\right]\\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g\left(e^{z_1},e^{z_2};K_1,K_{2max}\right)\cdot\bar{\phi}(z_1,z_2)\,dz_1dz_2
\end{aligned} \tag{28}$$

where $\bar{\phi}(x,y)$ is the joint normal PDF with respective means of m_1 and m_2, respective standard deviations of σ_1 and σ_2, and a correlation of $\bar{\rho}$. For faster numerical integration, we rewrite equation (28) as

$$
\begin{aligned}
C_{mod} &= C_1\left(K_1,K_{2\max}\right)+\tilde{E}\left[PC^{PV}_{t_1,t_3;K_1}\right] \\
&\quad+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g\left(e^{m_1+\sigma_1 W_1}, e^{m_2+\sigma_2\left(\bar{\rho}W_1+\sqrt{1-\bar{\rho}^2}W_2\right)}; K_1, K_{2\max}\right)\cdot\phi(W_1)\phi(W_2)\,dW_1 dW_2
\end{aligned} \tag{29}
$$

where $\phi(x)$ is the standard normal PDF.

Results

The time to construct the initial plant was estimated to be 3 years, thus t_1 = 3. For the modular case, it was determined that a decision regarding staged expansion would occur 8 years after the commissioning date of the first stage, thus, t_2 = 11. Two years will be required for the stage expansion construction, and thus, t_3 = 13. The horizon time, T, was taken to be 23 years. Since the existing plant still had capacity for 200 connections, the value for N_0 was -200 connections. Based on historical data, X_0 was estimated to be 81 connections / year, the growth rate, μ_X, was estimated to be 0.1 and the volatility, σ_X, was estimated to be 0.16. The penalty cost rate, PC_0, was estimated to be \$5000 per year per connection The following market parameters were used: r = 5%, r_{cpi} = 3%, μ_S = 8% and σ_S = 0.05.

Based on data provided by engineers, the following parameters were estimated for the construction cost equations (see equations (1), (2) and (3)).

Table 1. Construction Cost Parameters.

Parameter	Value
α_f	\$3,500,000
α_1	\$3,500,000
α_2	\$525,000
γ_f	\$860/connection
γ_1	\$860/connection
γ_{12}	\$258/connection
γ_2	\$900/connection

The approximate solutions to $v(t,X_t,N_t)$ using equation (21) versus the PDE solutions obtained by solving equation (11) are plotted in Figure 3 for varying K. Clearly, the solutions match very closely. The present value of the expected total cost for the modular case, C_{mod}, for ρ = 0.5 is plotted in Figure 4, as a function of K_1 and K_{2max}.

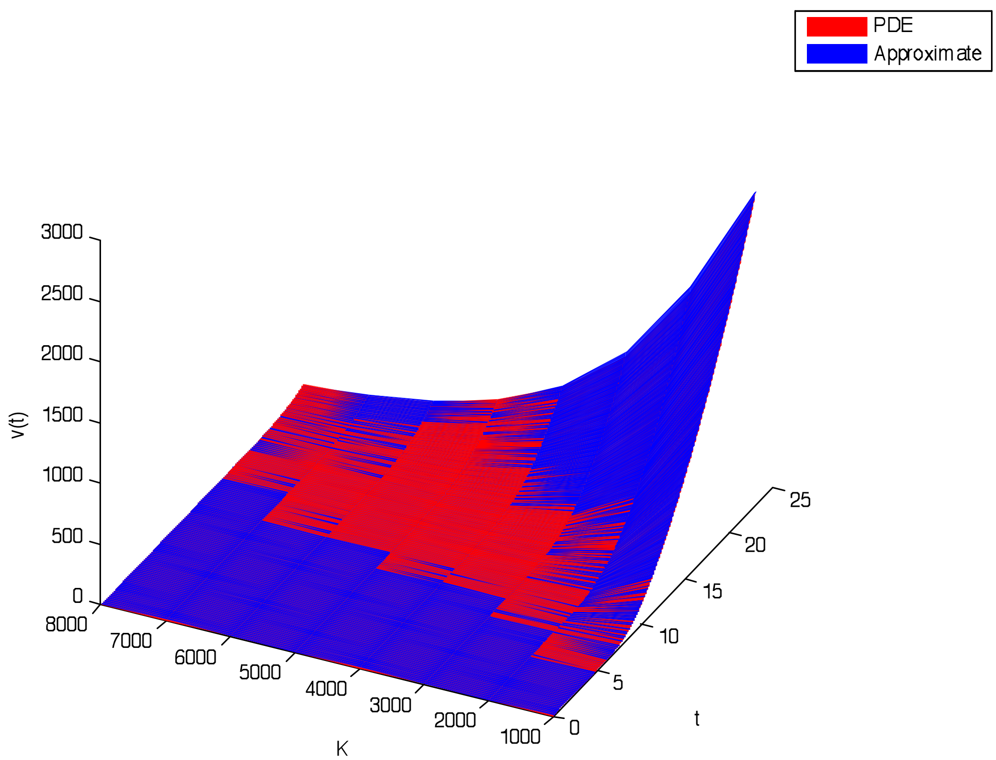

Figure 3. Comparison of approximate solution to the solution determined via solution of the PDE.

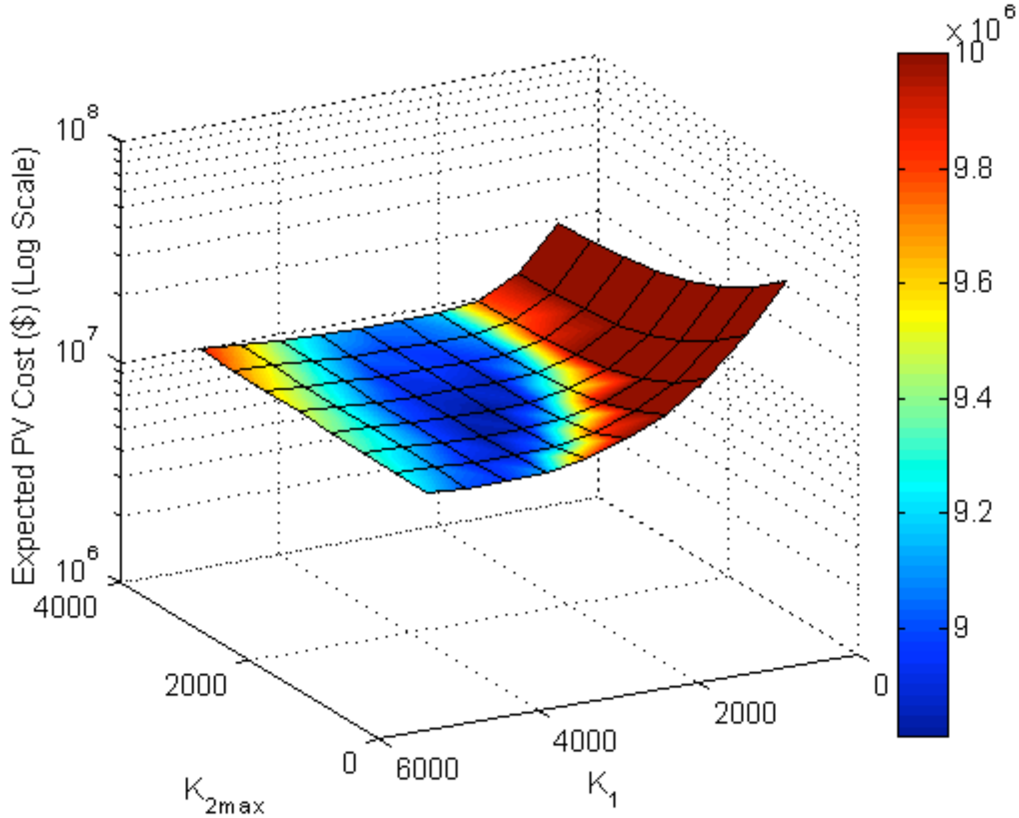

Figure 4. Present value of the expected total cost for the modular plant, C_{mod}, for ρ = 0.5 versus K_1 and K_{2max}.

The optimal plant sizes for ρ = 1.0, 0.5 and 0.0 for the non-modular (fixed) case are presented in Table 2 and for the modular case are presented in Table 3. For the case of ρ = 1.0, where the connection rate is perfectly correlated to the market, the expected cost savings associated with building a modular plant is less than 4%. The expected savings increases to approximately 10% of total costs for the case of ρ = 0.5 and to over 20% for the case of ρ = 0.0.

Table 2. Fixed Plant Optimal Size and Cost Estimates

ρ	Optimal Plant Size (K_f)	Construction Cost	Expected (Hedged) Total PV Cost
ρ = 1.0	3088	$6,155,508	$6,758,394
ρ = 0.5	5792	$8,481,202	$9,807,298
ρ = 0.0	11,610	$13,484,821	$16,555,133

Table 3. Modular Plant Optimal Size and Cost Estimates

ρ	Optimal K_1	Optimal K_{2max}	Initial Construction Cost	Expected (Hedged) Total PV Cost
ρ = 1.0	1655	1665	$5,352,929	$ 6,489,406
ρ = 0.5	3096	2526	$6,814,393	$8,843,789
ρ = 0.0	6905	3351	$10,303,244	$13,058,365

Discussion and Conclusion

A key contribution of this paper is the introduction of approximate methods to allow for the solution of a somewhat complicated real option problem that has significant similarities to an Asian option. Without the approximation, a PDE would be required to be solved at every optimization step, making the analysis intractable. As discussed in a previous paper (Lawryshyn and Jaimungal (2009)), it is not likely that many municipalities will opt to build smaller wastewater treatment plants and try to hedge away their potential penalty costs in the stock market. However, the approach developed here highlights how private investment would value the cost of a plant expansion. Capital investment theory is based on the assumption that investors will only invest in a project if its expected payoffs, for a give risk level, are better than what could be achieved in the market. In situations where private investors are burdened with the financial costs of a design-build-operate system, it may be important to ensure penalty costs associated with not meeting treatment are adequately adjusted at the outset.

In the case where no attempt will be made to hedge the penalty cost, we have showed that an expected 20% total savings may be achieved by building a modular plant. This result is consistent with real option theory – while the total construction cost per unit treated is greater in the case of the modular plant, the value of waiting to observe the total number of connections over the 11 years and then adding to the plant, accordingly, is greater and the modular options should be pursued. The final recommendation resulted in building a plant with K_1 = 7300 and K_{2max} = 2700.

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Appendix

Here we provide details for the moment matching.

Clearly,

$$\tilde{E}\left[X_t\right] = X_0 e^{\bar{r}t} \tag{30}$$

and

$$\tilde{E}\left[X_t^2\right] = X_0^2 e^{\left(2\bar{r}+\sigma_X^2\right)t}. \tag{31}$$

Since $dN_t = X_t dt$ or $N_t = N_0 + \int_0^t \tilde{E}\left[X_s\right] ds$, we have

$$\tilde{E}\left[N_t\right] = N_0 + X_0 \frac{e^{\bar{r}t} - 1}{\bar{r}}. \tag{32}$$

To calculate $\tilde{E}\left[X_t N_t\right]$, we apply Ito's lemma so that

$$d(X_t N_t) = \left(\bar{r} X_t N_t + X_t^2\right)dt + \sigma_X X_t N_t \left(\rho d\tilde{W}_t + \sqrt{1-\rho^2}\, dW_t^{\perp}\right) \tag{33}$$

or

$$\tilde{E}\left[X_t N_t\right] = X_0 N_0 + \bar{r}\int_0^t \tilde{E}\left[X_s N_s\right] ds + \int_0^t \tilde{E}\left[X_s^2\right] ds\,. \tag{34}$$

Differentiating equation (34) with respect to t and substituting equation (31) for $\tilde{E}\left[X_t^2\right]$, gives the following linear ODE

$$\frac{d\tilde{E}\left[X_t N_t\right]}{dt} = \bar{r}\tilde{E}\left[X_s N_s\right] + X_0 e^{\left(2\bar{r}+\sigma_X^2\right)t}\,, \tag{35}$$

the solution of which gives

$$\tilde{E}\left[X_t N_t\right] = X_0 N_0 e^{\bar{r}t} + \frac{X_0^2}{\bar{r}+\sigma_X^2}\left(e^{\left(2\bar{r}+\sigma_X^2\right)t} - e^{\bar{r}t}\right). \tag{36}$$

Finally, for $\tilde{E}\left[N_t^2\right]$, Ito's lemma gives

$$d(N_t^2) = 2X_t N_t dt \tag{37}$$

so that

$$\begin{aligned}\tilde{E}\left[N_t^2\right] &= N_0^2 + 2\int_0^t \tilde{E}\left[X_s N_s\right] ds \\ &= N_0^2 + 2X_0 N_0 \frac{\left(e^{\bar{r}t}-1\right)}{\bar{r}} + \frac{2X_0^2}{\bar{r}+\sigma_X^2}\left(\frac{e^{\left(2\bar{r}+\sigma_X^2\right)t}-1}{2\bar{r}+\sigma_X^2} - \frac{e^{\bar{r}t}-1}{\bar{r}}\right).\end{aligned} \tag{38}$$