Competition, Uncertainty(ies), and Corporate Cash Holdings

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Abstract

This work investigates the effects of competition on firms' cash holdings. We build a differentiated Cournot model where the intensity of competition depends on the degree of product substitutability. Firms are subject either to shocks (called common) that move their profitability in the same direction or shocks (called idisyncratic) that move profitabilities in opposite direction. Access to financial markets is imperfect and firms hold cash reserves to avoid inefficient liquidation. We find that the nature of uncertainty plays a key role to determine the cash policies. When shocks are common, the level of cash reserves either decreases or remains unaffected with respect to the degree of product substitutability. On the contrary, cash holdings increase with the intensity of competition when shocks are idiosyncratic. This happens because, if uncertainty is driven by idiosyncratic shocks, competition increases firm-level volatility and reinforces the precautionary motive for holding cash.

1 Introduction

When capital markets are perfect shareholders can raise external funds whenever it is optimal to do so. If liquidity is necessary to exploit new investment opportunities or to cushion losses in periods of distress, firms can always issue equity or raise debt. On the contrary, when access to capital markets is constrained, retained earnings and cash reserves may be the only means to finance new profitable projects and to cover operating losses. Ample empirical evidence (Opler, Pinkowitz, Stulz and Williamson (1999), Bates, Kahle and Stulz (2009)) documents that corporations hold substantial amounts of liquid assets. This work, under

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the assumption of imperfect capital markets, investigates how product market competition affects corporate cash holdings.

Although the precautionary motive is the primary reason for holding reserves of liquidity, interactions with other several factors can influence firms' cash policies. This consideration seems to be particularly relevant in light of the evidence documented in Bates, Kahle and Stulz (2009) that liquidity reserves of U.S. corporations have increased significantly in the period from 1986 to 2006. Other things being equal, the development of financial markets in the last thirty years and the consequent easier access to credit, should have implied a reduction in precautionary liquidity reserves. Given that the available empirical evidence suggests the opposite, it is relevant to investigate how other economic factors, interacting with imperfect capital markets, condition firms' willingness to hold cash. In this respect, product market competition is, potentially, one of the crucial determinants.

We analyze the effects of competition on cash holdings in a differentiated duopoly model where the intensity of competition depends on the degree of product substitutability. Firms compete in quantities *à la* Cournot and hold an option to irreversibly shut down their operations whenever market conditions become unfavorable. As in the standard real options models, irreversibility of the exit decision and uncertainty imply that firms face periods of negative profits but remain in the market in the perspective of future improvements. If profits fall below an endogenously determined threshold, firms are not economically viable and close down in the best interest of their shareholders.

In the model, the need of liquidity depends on the combination of uncertainty and financial markets imperfections. We assume that imperfect access to capital market implies the total inability of the firm to raise new external funds after its entry date. When it runs out of cash, a firm is forced out of the market without any consideration for its potential future profitability. In other words, with no reserves of liquidity a firm shuts down irreversibly even though it is still economically viable. Clearly, exit for liquidity reasons is inefficient because shareholders, if possible, would rather raise new funds to keep the operations running. To survive periods of financial distress and to avoid inefficient liquidation firms hoard cash.

Uncertainty is introduced via two types of shocks. One shock, that we shall call *common*, affects firms' profitability in the *same* direction, while the other, that we will define *idiosyncratic*, moves firms' profitability in the *opposite* direction. To clearly distinguish the effects of the two sources of uncertainty, we build two twin models in which shocks are either common or idiosyncratic. Our goal is to identify different aspects of competitive environments. On the one hand, when firms are operative in the same market, they share a common fate. As the demand for the product increases, all firms, up to some degree, experience a boost in profits. Both Toyota and GM, for instance, would benefit from a general surge in demand for automobiles or a rise in aggregate demand. But on the other hand, the success or survival of a firm depends, at least to some extent, on competitors' misfortunes. For example, at the end of 2009 and at the beginning of 2010 Toyota recalled millions of automobiles after that several vehicles experienced technical problems. This event, which directly affects only the Japanese automaker (a truly idiosyncratic shock), has an indirect influence on competitors. Indeed, due to the loss of credibility, Toyota may lose some of its traditional customers in favour of its competitors.[1]

[1] The Renault-Nissan CEO Carlos Ghosn after a Toyota's recall declared that "Obviously, competitors in the short term will benefit from these problems" (WSJ online, http://online.wsj.com/article/BT-CO-20100128-703451.html) Also, General Motors

In our work, we model what we define idiosyncratic shocks as the different ability of the firms to capture changes in consumers' preferences. Because of this asymmetry, and due to Cournot interaction, firms' profitability move in the opposite direction in response to shocks. Therefore, the shocks that we consider are not truly idiosyncratic (they potentially affect all firms in the market) but rather *idiosyncratic responses to common shocks*. To make a straight distinction with the common shock model, we will call them idiosyncratic.

From our analysis emerges that the nature of uncertainty is decisive to determine the effects of competition on cash holdings. If shocks are common, higher degree of product substitutability does not affect the level of cash reserves. However, when shocks are idiosyncratic, the optimal level of cash holdings increases with the intensity of competition. This happens because, with idisyncratic shocks, more intense competition increases profits volatility and reinforces the precautionary motive for holding cash. Intuitively, when product substitutability is easier and markets are more closely interwined, if a firm is hit by a negative shock, it experiences a rapid erosion in market share and profits. Symmetrically, shocks that negatively affect a competitor are greatly beneficial because a firm can easily attract competitor's costumers. Therefore, although currently adverse, market conditions can quickly become favorable and, for this reason, firms find it optimal to delay the exit time and increase the amount of precautionary reserves. This is the key finding of our work.

In the basic formulation of our model firms are financed only through equity. Later, we extend the analysis and allow firms to choose their optimal debt-equity mix. In order to do so, we amend the workhorse model of optimal capital structure by Leland (1994) in which

and Ford Motor offered $1,000 and low financing rates to attract Toyota customers worried about their recalled vehicles.

the firm issue debt to balance the tax benefits from deduction in interests payments and the costs of bankruptcy. We find that, when shocks are idiosyncratic, cash holdings are still increasing with the intensity of competition. The fundamental economic intuition for this result is the same of the model with all-equity firms. However, when shocks are common, cash holdings are now *decreasing* with competition. With easier product substitutability profits are lower so that also tax-shield benefits are smaller and, for this reason, firms issue lower debt. In its turn, lower leverage means smaller per-period interest payments and, therefore, smaller amount of precautionary reserves.

Our analysis is consistent with several empirical facts documented in the literature. Over the time horizon investigated by Bates, Kahle and Stulz (2009) the aggregate volatility was low (the so called "Great Moderation", see Stock and Watson (2002) among several others) while firm-level uncertainty displayed a substantial increase (Campbell, Lettau, Malkiel, and Xu (2001) , Chaney, Gabaix and Philippon (2005) , Comin and Philippon). At the same time, product market competition became more intense (Irvine and Pontiff (2009) , Comin and Philippon) and, as already mentioned, corporate cash holdings increased steadily. Bates, Kahle and Stulz (2009) attribute the rise of firms' liquidity reserves to the stronger preautionary motive induced by the larger firm-level volatility. Our model can bring together all these pieces of evidence.

We show that, when shocks are common (that is uncertainty is aggregate) more intense competition, if anything, should lead to a reduction of firms' cash holding. On the contrary, when shocks are idiosyncratic, stronger product market competition may indeed increase firm-level volatility inducing firms to hold more cash. On the basis of these theoretical predictions, we maintain that competition can be the keystone to explain the facts mentioned above. In a period of low aggregate volatility (when the main source of uncertainty are idiosyncratic movements) the rise in product market competition may have increased firm-level volatility (Irvine and Pontiff (2009), Comin and Philippon) and, through this channel, reinforced the precautionary motive for holding cash.

The model, being relatively rich (it includes exit strategies, competition, cash policies, capital structure), is related in various ways to the existing literature. The closest work to ours is a recent paper by Morellec and Nikolov (2000) which also investigates the effects of competition on cash holdings in a real option framework. Their focus, however, is mainly directed to the empirical analysis and the theoretical investigation is limited to a very stylized model which, inevitably, cannot capture the several effects identified here. Another related work is Murto and Terviö (2009) that introduces a liquidity constraint in the standard model of irreversible exit with stochastic cash flow and characterize the optimal default and dividend policy, both for a single firm and industry equilibrium. They find that the direct consequence of the liquidity constraint is to impose inefficient exit, while in industry equilibrium it creates a price distortion that causes inefficient survival. Boyle and Guthrie (2003) also introduce credit constraints in a real options model but investigate a firm entry choice, where uncertainty does not affect the ex-post investment cash-flows but the pre-entry availability of funds to cover investment costs.

This work also relates to the literature on exit in oligopolistic industries. Classical articles are Ghemawat and Nalebuff (1985 and 1990) and Fundenberg and Tirole (1985). While these early contributions investigate the problem in a deterministic scenario, Lambrecht (2001) and Murto (2004) extend the analysis in a stochastic continuous time framework. The common shock model presented in Section 3 is based on their results. Instead, the model with idiosyncratic shocks departs from their analysis and further contributes to the real option literature in a competitive setting (beside the already mentioned Lambrecht (2001) and Murto (2004), investment models include Aguerrevere (2003 and 2009), Grenadier (1996 and 2002) and Weeds (2002)).

Finally, the model also contributes to the literature of optimal capital structure in contingent claim models pioneered by Leland (1994) (other examples are Leland and Toft (1996), Fand and Sundaresan (2000), Hackbarth, Hennessy and Leland (2007), Sundaresan and Wang (2007a,b)), and show that the effect of competition on the optimal debt-equity mix also depends on the nature of uncertainty.

We organize the work as follows. Section 2 defines the general set up. Section 3 investigates the model with common shocks, while Section 4 analyzes the idiosyncratic shock model. Section 5 extends the analysis and investigates the optimal capital structure. Finally, Section 6 concludes.

2 Set up

2.1 Product market

Time is continuous and indexed by $t \in [0, \infty)$. We consider a monopolistic sector with two firms, labelled 1 and 2, that produce imperfect substitute goods. Market demands are derived from the preferences of a representative consumer that maximizes the instantaneous utility

$$u_t(q_1, q_2) = U_t(q_1, q_2) - P_{1,t}q_{1,t} - P_{2,t}q_{2,t},$$

where $q_{i,t}$, $i \in \{1, 2\}$, is the amount of good i and $P_{i,t}$ its price. The function $U_t(q_1, q_2)$ is quadratic and equal to

$$U_t(q_1, q_2) = z_{1,t}X_t q_{1,t} + z_{2,t}X_t q_{2,t} - \frac{1}{2}\left(q_{1,t}^2 + q_{2,t}^2 + 2\kappa q_{1,t}q_{2,t}\right).^2 \tag{1}$$

Utility maximization implies that the inverse demand functions are:

$$P_{1,t} = z_{1,t}X_t - q_{1,t} - \kappa q_{2,t}, \tag{2}$$

$$P_{2,t} = z_{2,t}X_t - q_{2,t} - \kappa q_{1,t}. \tag{3}$$

The parameter $\kappa \in [0, 1]$ is the degree of product substitutability and captures in the model the intensity of market competition. When $\kappa = 0$ there is no possibility of substitution between goods and firms are monopolists, while, when $\kappa = 1$, the two goods are perfect substitutes. Competition increases monotonically in k. The intercepts of the inverse demand functions depend on two components. A common component X which represents the strength of the sectoral (or aggregate) demand. An idiosyncratic component z_i that can be interpreted as the consumers' demand for characteristics specific of product i. Common and idiosyncratic components X and z_i are possibly stochastic. Furthermore, firms face fixed per-period costs of production F_1 and F_2, where $F_1 \neq F_2$.

The presence of fixed costs and the uncertain profitability imply that firms are subject to losses during their life. Managers have the option to shut the firm down in the best interest

[2] See, for example, Singh and Vives (1984).

of shareholders whenever market conditions deteriorate beyond an endogenously determined point. Exit is irreversible.

Firms do not have capacity limits and, in each period, compete in quantities by choosing simultaneously their optimal output without costs of adjusting production levels.[3] Asymmetry in the fixed cost introduces the possibility that the stronger firm may be tempted to follow limit or predatory pricing strategies to force the opponent out of the market. That is, the strongest firm may threat to produce in each period a quantity that would impose non-positive profits for the opponent. We assume that, if firms have deep pockets, given the simultaneous output choice and the absence of costs of adjusting production, such a commitment is not credible.[4] Hence, market interaction yields the standard Nash-Cournot outcome.

Nash-Cournot equilibrium implies that, when both firms are active in the market, optimal quantity and profits are

$$q_{1,t}\left(X, z_1, z_2\right) = \frac{X_t\left(2z_{1,t} - \kappa z_{2,t}\right)}{4-\kappa^2}, \tag{4}$$

$$q_{2,t}\left(X, z_1, z_2\right) = \frac{X_t\left(2z_{2,t} - \kappa z_{1,t}\right)}{4-\kappa^2}, \tag{5}$$

$$\pi^c_{1,t}\left(X, z_1, z_2\right) = q^2_{1,t} - F_1, \tag{6}$$

$$\pi^c_{2,t}\left(X, z_1, z_2\right) = q^2_{2,t} - F_2, \tag{7}$$

Firms discount profits at rate ρ.

[3] Maskin and Tirole (1987 and 1988) study dynamic Cournot models in which firms move alternatingly and are committed to their production choices for a finite period.

[4] Financial positions of the firms can play an important role. If firms have limited access to the capital market it is reasonable to think that the firm with strongest balance sheet may attempt to follow predatory strategies in the short run to force the opponent out of the market (see next section).

2.2 Capital market and exit strategies

When access to capital market is unconstrained, shareholders issue equity or raise debt to cover their financing needs, when they judge optimal to do so. However, if access to capital market is subject to restrictions, firms may be unable to raise sufficient liquidity to continue their operations and, although economically viable, may be forced out of the market for the impossibility to meet their payments. This suggests that financial market imperfection is a necessary ingredient in models where liquid assets play a role. In order to generate liquidity needs, we shall investigate a rather extreme scenario defined by the following assumption.

Assumption 1 *After the entry time firms do not have access to the capital market.*

Assumption 1 means that firms cannot rely on the capital market to finance their liquidity needs or, in other words, that cash reserves M are the only means to cushion negative shocks. Because of the constrained access to the capital market, firms shut down for two reasons. The first reason is that the value of equity falls to zero because firms are no longer profitable from shareholders' perspective. The second reason is lack of liquidity which happens when firms do not have enough cash to meet their payments but equity value is still positive. Exit for liquidity reasons is not efficient and, to prevent this possibility, firms accumulate cash. We assume that firms are forced to exit when liquidity reserves falls to zero, i.e. $M = 0$.

Due to the constrained access to the capital market and depending on the overall market conditions, if a firm has a substantial advantage in terms of larger amount of liquidity reserves, it may have an incentive to deviate from the equilibrium defined by (4)-(5). In particular, it may find it profitabile to pursue predatory pricing strategies to drain the opponent's reserves of cash and force it to exit. The following assumption rules out this possibility.

Assumption 2 *When a firm exits for liquidity reasons an unconstrained investor (e.g. a bank) takes control of the firm's asset at no costs.*

Roughly speaking, predation is profitable if, by charging a low price, a firm can push the opponent out of the market in a relatively short amount of time and charge the monopoly price afterwards. In our model, the stronger firm may depart from the standard Nash-Cournot and produce a larger quantity to impose losses to its competitor. The potential substitution of the current opponent with an unconstrained (deep-pocketed) investor means that there are no benefits of predation. For this reason, firms interact in the market according to the equilibrium (4)-(5).

As already said earlier, firms hoard cash to cushion negative shocks and to avoid inefficient liquidation. Within the firm cash reserves earn an interest at a free rate r. If the interest on liquidity reserves is below the discount rate, $r < \rho$ (for example, cash reserves may earn a lower rate of return for agency reasons (Jensen and Meckling (1976)), hoarding cash is costly and firms trade-off costs of holding liquid assets, due the liquidity premium, and benefits, deriving from the insurance provided against inefficient liquidation. Here we follow Mello and Parsons (2000) and Gryglewicz (2010) and assume away the liquidity premium. Therefore, the following assumption holds.

Assumption 3 *Firms' liquid reserves earn an interest equal to the discount rate, i.e.* $r = \rho$.

Assumption 3 means that hoarding cash is costless. Also, it implies that there exists a cash reserves level $\overline{M}_i$ such that, if $M_i < \overline{M}_i$, firm i finds it strictly optimal to retain cash, and, if $M_i \geq \overline{M}_i$, it is indifferent between retaining earnings or paying them out in form of dividends. We assume that, once $\overline{M}_i$ is reached, the residual cash is paid out and that, in case of exit, liquid assets are distributed to shareholders.

Define firm i's profits at exit time $\underline{\pi}_i$ such that, given the Cournot-Nash equilibrium (4)-(5), if $\pi_{i,t} \geq \underline{\pi}_i$ it is optimal for shareholders of firm i to continue (equity value is strictly positive), while if $\pi_{i,t} < \underline{\pi}_i$ it is optimal to exit (equity value is zero). Typically, profits at the exit time are negative, i.e. $\underline{\pi}_i < 0$ because firms should suffer substantial losses before deciding to abandon their operations. Therefore, during their lifes, firms face periods in which profits are negative but it is nevertheless optimal to remain in the market, i.e. $\underline{\pi}_i < \pi_{i,t} < 0$. In these periods shareholders are willing to inject liquidity in the firm to cover losses and to keep it alive.

Given the assumption of no costs of hoarding liquid assets, $\overline{M}_i$ is the minimum level of cash that allow firm i to avoid exit for liquidity reasons. This quantity is given by the discounted worst-case losses for a firm that follows a first best (unconstrained) policy and equals

$$\overline{M}_i = \max\left\{0, -\frac{\underline{\pi}_i}{r}\right\} \tag{8}$$

(see also Murto and Terviö (2009)). Intuitively, a firm could stay arbitrarily close to the exit boundary $\underline{\pi}_i$ for an infinite amount of time without crossing it. If the exit boundary is such that profits are positive at the time when it closes down (i.e. $\underline{\pi}_i > 0$, an example when this happens will be provided in the next section), then, although constrained, the firm has no need to hold cash reserves, i.e. $\overline{M}_i = 0$.

Assumption 4 *At the initial date $t = 0$ firms raise liquidity to cover cash reserves $\overline{M}_i$.*

Assumption 4 implies that closure for liquidity reasons never occurs. The fact that firms are not at liquidity risk has important implications for the solution of the model. *Ceteris paribus*, firm i's value (net of liquidity reserves) and exit strategy coincide to those of a firm with perfect access to capital markets. Once firm i holds sufficient cash to avoid inefficient liquidation, its value depend only on fundamentals and not on its financial stance. From a technical point of view, this implies that firm i's net value can be found as a solution of an ordinary differential equation. When firm i is at liquidity risk, i.e. $M_i < \overline{M}_i$, its value depends on the level of liquidity reserves and it must be found as a (numerical) solution of a *partial* differential equation (see Murto and Terviö (2009)).

3 Common shocks

Consider the market for automobiles mentioned in the Introduction. A growth of aggregate demand, a sectoral increase of the market for autovehicles, or a fall in production costs (for instance a decline in the price of raw materials) benefit all the producers active in the market. In this sense, firms that operate in the same sector share a common destiny. This section investigates firms' strategies when they are subject to common shocks, that is shocks that, *given the duopoly market structure,* move their profitability in the same direction.[5]

[5] This is a crucial remark. A common negative shock, for example a general fall in sectoral demand, unambiguously lowers profitability of firms if both of them remain in the market. But if the shock is such that one of the is pushed out, for the surviving firm the negative effect of a lower aggregate demand may be more than offset by the change in market structure. In other words, the surviving firm may benefit from a "negative" common shock.

In order to analyze this scenario, we assume that the common component X of the demand functions is stochastic and fluctuates according to

$$\frac{dX_t}{X_t} = \mu dt + \sigma dZ_t, \tag{9}$$

where dZ_t is a standard Wiener increment. On the contrary, the idiosyncratic components $z_{1,t}$ and $z_{2,t}$ are constant over time and, for simplicity, we set them both equal to one, i.e. $z_1 = z_2 = 1$. This not alter the conclusions about firms' cash policies. Given that z_1 and z_2 are equal, asymmetry in the production side is eliminated and firms produce the same per-period quantity equal to

$$q_t(X) = \frac{X_t(2-\kappa)}{4-k^2}$$

(see also (4) and (5)). Although the quantity produced is the same, firms are asymmetric because they face different fixed costs. In this respect, we assume that Firm 2 is less efficient, i.e. $0 \leq F_1 < F_2$. Due the presence of positive fixed costs firms irreversibly exit when X falls to sufficiently low levels.

This scenario closely resembles the exit problem in an asymmetric duopoly investigated by Murto (2004). He shows that when firms' strategies space is a connected set a unique "natural" equilibrium exists in which the less efficient firm exits first. A similar model is investigated, for example, in Lambrecht (2001). But if the analysis is extended to a more general setting where we allow firms' strategies space to be a disconnected set, and if uncertainty and firms' asymmetry are sufficiently large, another equilibrium appears in which the *more* efficient firm exits first. Murto emphasizes, however, that this equilibrium can only be reached by a "mistake", that is suboptimal behavior, or when firms' profitability is low at the initial time. On the basis of these considerations, in the present work, we restrict our attention to the case where the the strategy space is a connected set so that the more efficient firm outlives the less efficient one.

The scenario that we investigate is, therefore, the following. Firms choose their exit optimal boundaries $\underline{X}_1$ and $\underline{X}_2$. Given that Firm 2 is less efficient it will exit first, i.e. it holds that $\underline{X}_2 > \underline{X}_1$. The common demand component X fluctuates stochastically in the region above $\underline{X}_2$ and, as soon as X falls below $\underline{X}_2$, Firm 2 exits leaving Firm 1 alone in the market.

Before proceding with the investigation of the model, a further consideration is necessary. After Firm 2's exit, Firm 1 becomes a monopolist. Therefore, its profit at exit time, and its liquidity reserves $\overline{M}_1$ (if positive), do not depend on the intensity of competition. Given that our goal is to investigate the effects of competition on firms' cash holdings, we leave Firm 1 in the background and focus our attention on exit and financing strategies of Firm 2.

3.1 Firm 2's valuation

As already explained, by holding $\overline{M}_2$, Firm 2 surely avoids inefficient closure. This implies that its total value equals the level of cash reserves plus the value of an unconstrained and cashless firm $V_2(X)$. Also, given that it exits first, Firm 2 earns duopoly profits during its entire existence. It follows that $V_2(X)$ satisfies

$$\frac{1}{2}\sigma^2 X^2 V_2''(X) + \mu X V_2'(X) - rV_2(X) + \frac{X^2 (2-\kappa)^2}{(4-\kappa^2)^2} - F_2 \ = 0 \tag{10}$$

subject to

$$\lim_{X\to\infty} V_2(X) = \left(\frac{X^2}{r-2\mu-\sigma^2} \frac{(2-\kappa)^2}{(4-\kappa^2)^2} - \frac{F_2}{r} \right) \tag{11}$$

$$V_2(\underline{X}_2) = 0 \tag{12}$$

$$V_2'(\underline{X}_2) = 0 \tag{13}$$

Equation (11) means that, when X grows larger, the probability of exit becomes negligible and Firm 2's value is given by the discounted stream of duopoly profits. Equation (12) implies that at exit time the value of the firm is zero, while equation (13) is the optimality (smooth pasting) condition. Solution of (10)-(13) yields

$$\begin{aligned} V_2(X) \quad &= \quad \left(\frac{X^2}{r - 2\mu - \sigma^2} \frac{(2-\kappa)^2}{(4-\kappa^2)^2} - \frac{F_2}{r} \right) - \\ &\quad - \left(\frac{\underline{X}^2}{r - 2\mu - \sigma^2} \frac{(2-\kappa)^2}{(4-\kappa^2)^2} - \frac{F_2}{r} \right) \left(\frac{X}{\underline{X}_2} \right)^{\beta_2}, \end{aligned} \tag{14}$$

and

$$\underline{X}_2 = \frac{4-\kappa^2}{(2-\kappa)} \sqrt{\frac{\beta}{\beta - 2} \frac{r - 2\mu - \sigma^2}{r} F_2}, \tag{15}$$

where $\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left[\frac{\mu}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0$ and we assume that $r - 2\mu - \sigma^2 > 0$ to ensure that Firm 2's value has a finite solution. From (15) note that $\underline{X}_2$ is increasing in κ. Indeed, more intense competition lowers profits and induce Firm 2 to exit at a higher level of X.

Using (15), the expression for the optimal amount of cash reserves becomes

$$\overline{M}_2 = \max \left\{ 0, \frac{F_2}{r} \left[1 - \frac{\beta}{\beta - 2} \left(r - 2\mu - \sigma^2 \right) \right] \right\} \tag{16}$$

It is immediate to see that $\overline{M}_2$ is not affected by the intensity of competition. Optimality implies that Firm 2 chooses the exit threshold in a way that the per-period profit $\underline{\pi}_2$ at the exit time, and therefore also the optimal liquidity reserves, does not depend on κ. Other things being equal, higher κ means lower profits and this tends to decrease $\underline{\pi}_2$. However, higher κ also implies that Firm 2 will exit for a larger X and this effect tends to increase $\underline{\pi}_2$. The two effects exactly offset eachother so that both $\underline{\pi}_2$ and $\overline{M}_2$ are independent of κ.

A special scenario is represented by the case when the drift μ of the process (9) satisfies $\mu < -\frac{1}{2}\left(\frac{\beta-2}{\beta} + r - \sigma^2\right)$. When this happens the fall in profitability is so rapid that Firm 2 exits when it is still making positive profits, i.e. $\underline{\pi}_2 > 0$. Given that Firm 2 never experiences losses during its (presumably short) existence, exit for liquidity motives cannot occur. Therefore, Firm 2 has no need to hold cash reserves, i.e. $\overline{M}_2 = 0$.

4 Idiosyncratic shocks

An important aspect of competitive environments is that competitors' misfortunes often represent good news. When firms compete in the same sector and one of them is affected by negative events, sales and profits of the competitors may experience a boost. In the common shock model this important aspect remains neglected. There, firms only experience demand fluctuations that affect their profitability in the same direction. Contrary to the previous section, here we investigate the case when firms are hit by shocks which affect firms' profitability in the opposite direction.

In order to do so, we adapt the general framework outlined in Section 2 in the following way. First, we assume that the common component X is a strictly positive constant that we set, for simplicity, equal to one. Second, Firm 2's idiosyncratic component is constant over time, $z_{2,t} = z_2$. Third, Firm 1's idiosyncratic component $z_{1,t}$ is stochastic and evolves according to

$$\frac{dz_{1,t}}{z_{1,t}} = \alpha dt + \varsigma dW_t. \tag{17}$$

Fourth, Firm 1 has no fixed costs of production, $F_1 = 0$, and therefore never exits. However, despite that it remains in the market forever, Firm 1 will not be always active. To see this, define

$$z^* = \frac{\kappa z_2}{2}. \tag{18}$$

From equation (4) it is immediate to see that, if $z_1 \leq z^*$, Cournot interaction implies that Firm 1 remains idle, i.e. $q_1 = 0$. Fifth, Firm 2 has strictly positive fixed costs that satisfy

$$F_2 \in \left(0, \frac{z_2^2}{4}\right). \tag{19}$$

Condition (19) implies that Firm 2 irreversibly exits the market as soon as $z_{1,t}$ grows above an endogenously determined threshold $\overline{z}$. Also, when $z_{1,t}$ drops to zero, Firm 2 makes positive monopoly profits forever.

The framework outlined above implies that Firm 2's strategy space is defined by three regions (see also figure 3). As long as $z_{1,t} \in (z^*, \overline{z})$, both firms are active and compete in quantities as described in Section 2. When z hits $\overline{z}$, Firm 2 irreversibly exits. If $z_{1,t} \leq z^*$, Firm 2 is (temporarily) a monopolist and obtains per-period profits equal to

$$\pi_2^m = \frac{z_2^2}{4} - F_2. \tag{20}$$

Note how the stronger the competition (higher κ) the sooner Firm 1 suspends its production, i.e z^* is increasing in κ. The economic intuition is straightforward. With low product substitutability, Firm 2's output has a weak influence on Firm 1's demand, and Firm 2 pushes Firm 1 to inaction only when its demand is substantially stronger (z_1 is low). On the contrary, when product substitutability is high, fortunes of the two firms are more closely interwined, so that it is easier for Firm 2 to force Firm 1 (temporarily) out of the market when conditions become favorable.

Given that Firm 1 has no fixed costs and never exits, as in the previous section our attention will be solely focused on exit and financing strategies of firms 2.

4.1 Firm 2's valuation

For the already known reason, Firm 2's value, net of cash reserves, equals the value of an unconstrained and cashless firm. Standard arguments imply that this value must satisfy

$$\frac{1}{2}\varsigma^2 z_1^2 V_2''(z_1) + \alpha z_1 V_2'(z_1) - rV_2(z_1) + \pi_2^j = 0, \tag{21}$$

where $j = c$ if $z_1 > z^*$, and $j = m$ otherwise. The general solution is

$$V_2(z_1) = \begin{cases} Az_1^{\theta_1} + Bz_1^{\theta_2} + \Pi_2^c(z_1) \text{ if } z_1 \geq z^* \\ Cz_1^{\theta_1} + Dz_1^{\theta_2} + \Pi_2^m(z_1) \text{ if } z_1 \leq z^* \end{cases}$$

where

$$\Pi_2^c(z_1) = \frac{1}{(4-\kappa^2)^2}\left[\frac{z_1^2}{r - 2\alpha - \zeta^2} - 2\frac{z_1 z_2}{r-\alpha} + \frac{z_2^2}{r}\right] - \frac{F_2}{r}$$

$$\Pi_2^m(z_1) = \frac{z_z^2}{4r} - \frac{F_2}{r}, \tag{22}$$

$$\theta_1 = \frac{1}{2} - \frac{\alpha}{\varsigma^2} + \sqrt{\left[\frac{\alpha}{\varsigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\varsigma^2}} > 1,$$

$$\theta_2 = \frac{1}{2} - \frac{\alpha}{\varsigma^2} - \sqrt{\left[\frac{\alpha}{\varsigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\varsigma^2}} < 0.$$

Equation (21) must be solved subject to the following boundary conditions:

$$V_2(\overline{z}) = 0 \tag{23}$$

$$V_2'(\overline{z}) = 0 \tag{24}$$

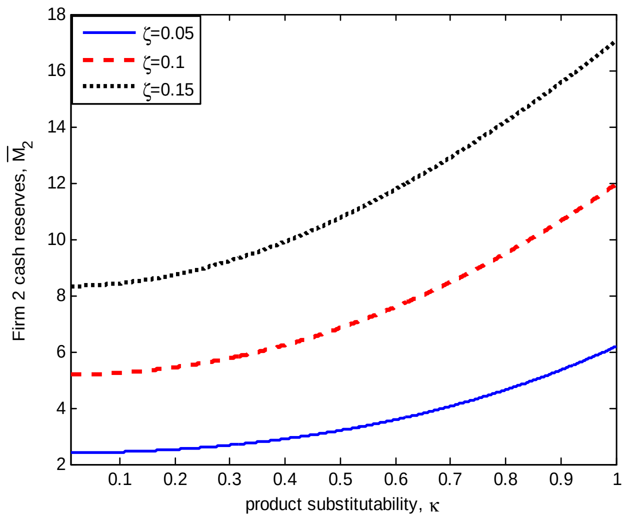

Figure 1: Firm 2's cash reserves. Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0$.

$$\lim_{z \downarrow z^*} V_2(z_1) = \lim_{z \uparrow z^*} V_2(z_1) \tag{25}$$

$$\lim_{z \downarrow z^*} V_2'(z_1) = \lim_{z \uparrow z^*} V_2'(z_1) \tag{26}$$

$$V_2(0) = \frac{z_2^2}{4r} - \frac{F_2}{r}. \tag{27}$$

Condition (27) means that if z_1 falls to zero Firm 2 will enjoy monopoly profits forever and implies $D = 0$. Conditions (23) and (24) are value matching and smooth pasting conditions at the exit threshold $\overline{z}$. Conditions (25) and (26) guarantee continuity and smoothness of the value function at z^*. Firm 2's value and exit threshold $\overline{z}$ are easily found numerically.

4.2 Model analysis

To investigate the properties of the model we choose the following parametrization.[6] As already mentioned, $F_1 = 0$ and $X = 1$. Firm 2's fixed cost is set equal to $F_2 = 1$ and

[6]Results are robust for different parametric specifications.

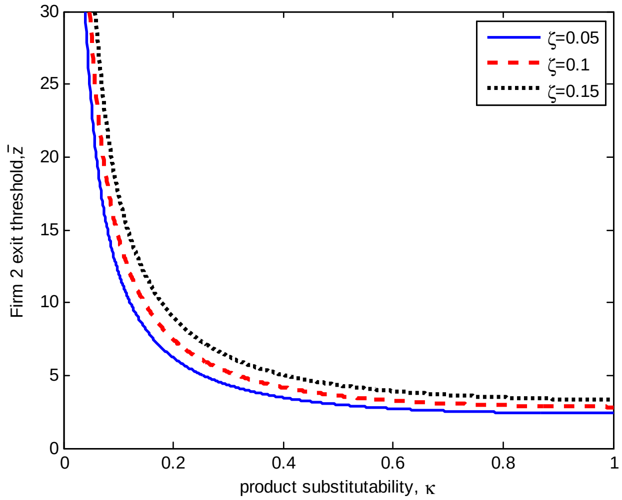

Figure 2: Firm 2's strategy space. Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0$.

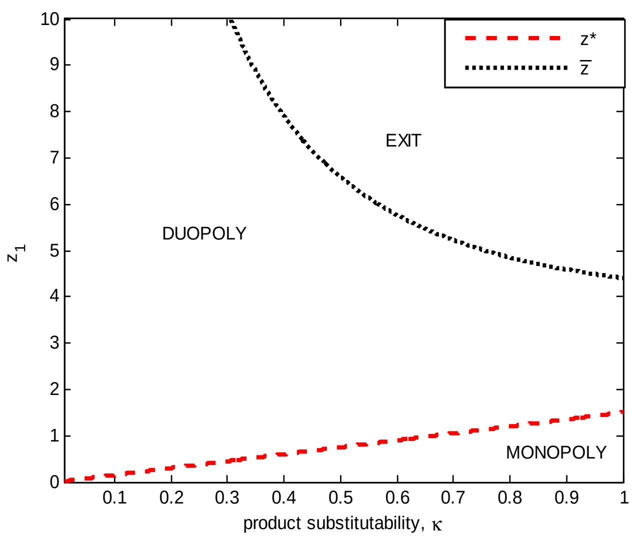

Figure 3: Firm 2's strategy space. Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0$.

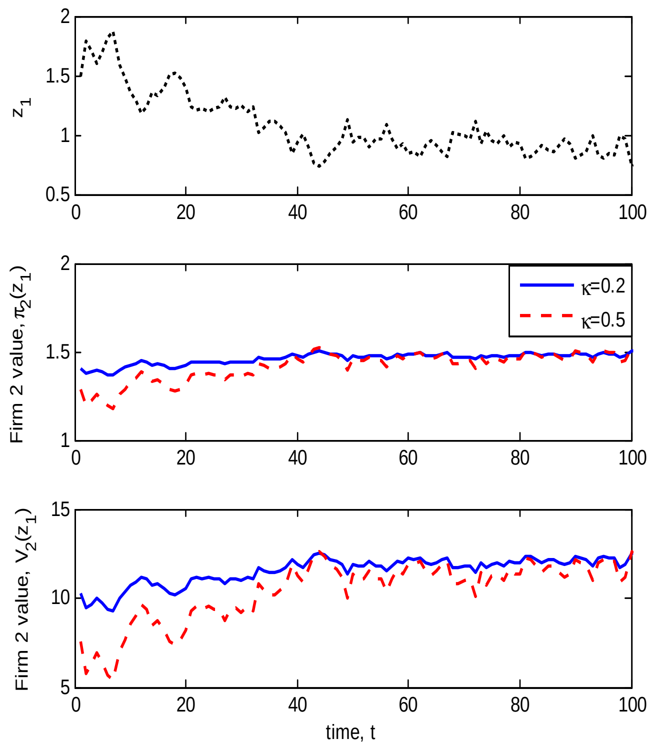

Figure 4: Firm 2's per-period profits $\pi_2(z_1)$ and value $V_2(z_1)$ for a random realization of z_1 over a horizon of 100 time periods. Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0$. Exit threshold and the boundary at which firm 2 becomes a monopolist are $\overline{z} = 7.49$, $z^* = .25$ if $\kappa = 0.2$, and $\overline{z} = 3.61$, $z^* = .625$ if $\kappa = 0.5$. For this random path idiosyncratic shock z_1 reaches a minimum of $z_1 = 0.733$ and a maximum of $z_1 = 1.866$, therefore firm 2 neither exits nor becomes a monopolist. When $\kappa = 0.2$ the standardized variance of firm 2's profits and value is 0.00005 and 0.0045. When $\kappa = 0.5$ the standardized variances are 0.0034 and 0.0276.

$z_2 = 2.5$. The risk-free interest rate equals $r = 0.04$ and $\alpha = 0$.

We refer to figure 1 and immediately state the main result of this section. Liquidity reserves increase monotonically with the intensity of competition. Contrary to the common shocks model, where increased competition had no effects on the level of cash, here larger product substitutability implies that Firm 2 is willing to hold a larger amount of liquidity reserves.[7]

Recall that, when positive, cash reserves are equal to $\overline{M}_2 = -\underline{\pi}_2$ (see equation (8)). The fact that $\overline{M}_2$ is increasing in κ means that Firm 2 is willing to absorb a larger amount of losses when competition becomes more intense or, in other words, that it wants to remain relatively longer in the market. Therefore, the crucial point of our analysis is to understand where this incentive to stay longer alive comes from.

We claim that volatility is the channel through which competition affects cash holdings. Specifically, when shocks are idiosyncratic, competition increases profits volatility and, by rising the option value to remain in the market, increases the optimal level of cash.

The link between competition, volatility and cash holdings is better understood in connection with the results of the common shock model. Consider the instantaneous change of Firm 2's per-period profits $d\pi_{2,t}(\cdot) = a_t(\cdot)\,dt + b_t(\cdot)\,dB_t$, where $dB_t \in \{dZ_t, dW_t\}$, and define $\Sigma(\kappa) = \frac{b_t(\cdot)}{\pi_2(\cdot)}$ as the standardized volatility of the profit process. The following Proposition (proven in the Appendix) holds.

Proposition 1 *When shocks are common the standardized volatility is constant and equal*

[7] The case $\kappa = 0$, that is a when firms are monopolist, is of no interest because firm 2 makes always positive profits and does not need precautionary cash. Therefore, all the figures are plotted for a degree of product substitutability within the range $\kappa \in [0.001, 1]$

to, $\Sigma(\kappa) = 2\sigma$. When shocks are idiosyncratic $\Sigma(\kappa)$ increases in κ.

Proposition 1 establishes the link between competition and volatility and suggests that volatility is indeed the channel through which competition increases cash holdings. In the common shock model, $\Sigma(\kappa)$ is equal to 2σ and therefore is independent of κ. Consistently, also the level of cash holdings is independent of the intensity of competition. On the contrary, when shocks are idiosyncratic, competition magnifies the intensity of shocks driven by the Brownian component dB_t, i.e. $\Sigma(\kappa)$ increases. Given that profits are more volatile, a firm waits longer before taking the irreversible decision to exits, that is it is willing to absorb a larger amount of losses.[8] With higher volatility, losses are substantial in case of negative shocks but, as soon as conditions get favorable, market shares and profits rise faster. Given that market conditions can rapidly improve a too hasty exit is not optimal.

Figure 4 plots Firm 2's per-period profits $\pi_2(z_1)$ and value $V_2(z_1)$ for a random path for z_1, when $\kappa = 0.2$ and $\kappa = 0.5$. The reader can immediately notice that, when the environment is more competitive (i.e. for $\kappa = 0.5$) profits and value are subject to wider fluctuations.

The relation between competition, volatility and exit strategies also suggests an analogy with the standard effect of uncertanty in real option models. In investment models larger uncertainty typically implies that a higher expected profitability is needed to induce firms to undertake the project. Analogously, in exit models a firm waits to suffer a greater amount of losses before abandoning its operations. Competition in combination with idiosyncratic shocks gives rise to a similar effect. Easier product substitutability makes profitability more

[8] A similar relation between volatility and competitiveness is found in Raith (2003) and Irvine and Pontiff (2009).

volatile and, for this reason, it delays exit.

Our model also accounts for the often neglected fact that competition can *decrease* the number of firms in the market through a selection effect (see Boone (2000)for a more detailed discussion). When competition is tighter, it is easier for firms to gain and lose market shares, and the less efficient firms are forced out of the market more rapidly. A similar mechanism is at work here. With higher product substitutability, that is more intense competition, output reallocation occurs faster so that Firm 1 is forced to suspend the production sooner and Firm 2 exits for a lower $\overline{z}$ (see figure 3). In other words, the region in which the two incumbent firms can be both active becomes smaller. Figure 3 also helps to reinforce the main intuition of the mechanism at work. The region in which both firms are active in the market shrinks as κ rises. This implies that when the degree of product substitutability is high, positive shocks quickly move Firm 2 close to the monopoly region (the monopoly threshold z^* is indeed higher), while negative shocks rapidly bring it close to the exit threshold. For this reason, when κ is larger, Firm 2's value is very responsive to variations in z_1, that is it is highly volatile.

To summarize, we found that, when firms are subject to idiosyncratic shocks, precautionary cash $\overline{M}_2$ increases with the intensity of competition. This means that Firm 2 exits relatively later and is willing to absorb greater losses before to irreversibly abandon its operations. We interpret the result with the fact that, when markets are strongly interconnected, firms profitability is more volatile so that the option to stay alive is more valuable. This means that it is optimal to absorb more severe of losses and to keep a larger amount of cash to prevent inefficient liquidation.

5 Optimal capital structure

In the previous section, the exit and financing strategies are investigated under the assumption that firms are financed only through equity. Here we extend the analysis and allow for the possibility to choose the optimal debt-equity mix. In order have an economically meaningful problem, we assume that profits are taxed at a rate $\tau \in (0, 1)$. With positive tax rate firms issue an optimal amount of debt to trade-off bankruptcy costs and benefit from a tax shield on debt interest payments.

We make two simplifying assumptions. First, as customary in contingent claim models (for example, Leland (1994), Leland and Toft (1996), Sundaresan and Wang (2007a,b) among others), the optimal debt-equity mix is choosen at the initial date $t = 0$ to maximize the value of the initial equity holders. Debt has infinite maturity and pays a constant coupon b_i. Afterwards, for any $t > 0$, firms can neither issue equity nor raise debt. Also, firms can be only net borrowers, i.e. $b_i > 0$. In case of default, firms are liquidated, and the liquidation value is a fraction $(1 - \phi)$ of their value at exit time. The parameter $\phi \in (0, 1)$ is the proportional liquidation cost. Upon liquidation, debtholders have absolute priority and shareholders obtain either the residual value after debtholders claims, if positive, or nothing otherwise.

Second, we assume that firms enter sequentially and correctly believe that the first entrant can credibly commit to remain in the market longer than the second. In other words, as in Abbring and Campbell (2007 and 2010) and Lambrecht (2001), we investigate a "Last-in-First-Out" (LIFO) equilibrium in which firms exit in the reverse order they entered in the market. Without loss of generality, we consider the case in which Firm 1 enters first and, thus, outlives Firm 2.

We introduce the LIFO assumption to select one of the several equilibria that can emerge in a competitive model of optimal capital structure. In a competitive setting, the choice of the optimal debt-equity mix does not only depend on the demand strength at the initial date (as we will show shortly), but may also depend on the order in which firms enter in the market and on strategic considerations. In other words, firms may have an incentive to link their optimal capital structure to the leverage of their competitors (for example, they can try to reduce their level of debt to lower fixed costs and outlive other firms[9]). Depending on the model assumptions a large number of equilibria are possible. The investigation of such a model is far beyond the scope of this work and, therefore, we select one of the possible equilibria by introducing the LIFO exit rule.

A LIFO equilibrium can also be motivated on the ground of the consistent empirical evidence that exit rates of older firms are substantially lower than their younger counterparts (Dunne, Roberst and Samuelson (1988), Jarmin, Klimek and Miranda (2003)). Well established firms may, indeed, benefit from several advantages as, for instance, consumers' brand fidelity or easier access to the credit market. In our example, we may assume that early entry gives Firm 1 the ability to renegotiate its debt in case of distress. Given that debt-renogatiation is always optimal *ex-post* (in absence of fixed costs of production), Firm 1 never defaults, while Firm 2 exits whenever its profitability falls below and endogenously determined threshold. Beyond the underlying reasons that can motivate a LIFO equilib-

[9] In a different context, Brander and Lewis (1986) argue that firms have incentives to use their financial structure to influence the output market. In particular, they can use debt as a variable to commit to a higher level of production and influence, in this way, the output of its competitors.

rium, what matters for our investigation is that in such scenario firms choose their financial structure knowing that Firm 1 will remain in the market longer. As in the previous section, our analysis will focus entirely on Firm 2's strategies.

To complete the model, define λ to be the proportional issuance costs, L the fixed issuance cost, and η the fraction of equity obtained by the new equity holders. Also, indicate $E_2(\cdot,\cdot)$ and $DBT_2(\cdot,\cdot)$ Firm 2's equity and debt value, and allow all the relevant variables to be explicitly dependent on b_2. In what follows, if not strictly necessary, the dependence on b_2 will be omitted for notational convenience. Finally, call $W \in \{X, z_1\}$ the stochastic profitability index (X in the common shock model, and z_1 in the idiosyncratic shock model),

If corporate earnings are taxed at rate τ, Firm 2's per-period profit becomes

$$\pi_2^c\left(W, b_2\right) = \left(1-\tau\right)\left(q_{2,t}^2 - b_2\right)$$

The following funding condition holds:

$$I + \overline{M}_2\left(b_2\right) = \left(1-\lambda\right)\left(\eta E_2(W_0, b_2) + DBT_2(W_0, b_2)\right) - L_2,$$

where W_0 is the profitability index at Firm 2's entry time. The initial value for equity holders can be written as

$$\left(1-\eta\right) E_2(W_0, b_2) = E_2(W_0, b_2) + DBT_2(W_0, b_2) - \frac{L+I}{(1-\lambda)} - \frac{\overline{M}_2\left(b_2\right)}{(1-\lambda)}. \tag{28}$$

The optimal coupon b_2^* maximizes (28) with respect to b_2.

5.1 Common shocks

Having in mind the general framework outlined in the introductory part, let us move to the analysis of the model with common shocks. In order to obtain closed form solutions, we further assume that Firm 2's fixed costs F_2 are equal to zero. With strictly positive fixed costs the solution would be numerical but its qualitative conclusions would not be change.

For a thorough analysis of the model, a more detailed discussion and interpretation we refer to the Appendix. There we show that, depending on the relation between parameters, three different scenarios may occur. Here, we focus on the only one immediately relevant for our analysis. In this scenario, the following expressions define the value equity net of liquid assets, debt, optimal coupon bond, exit threshold and cash reserves:

$$\begin{aligned} E_2(X) &= (1-\tau)\left(\frac{X^2}{r-2\mu-\sigma^2}\frac{(2-\kappa)^2}{(4-\kappa^2)^2}-\frac{b_2^*}{r}\right)- \\ &\quad -(1-\tau)\left(\frac{\underline{X}^2}{r-2\mu-\sigma^2}\frac{(2-\kappa)^2}{(4-\kappa^2)^2}-\frac{b_2^*}{r}\right)\left(\frac{X}{\underline{X}_2}\right)^{\beta}, \end{aligned} \tag{29}$$

$$DBT_2(X)=\frac{b_2^*}{r}+\left[(1-\phi)(1-\tau)\frac{\underline{X}^2}{r-2\mu-\sigma^2}\frac{(2-\kappa)^2}{(4-\kappa^2)^2}-\frac{b_2^*}{r}\right]\left(\frac{X}{\underline{X}_2}\right)^{\beta}, \tag{30}$$

$$\begin{aligned} b_2^* &= \frac{\beta-2}{\beta}\frac{r}{r-2\mu-\sigma^2}\frac{X_0^2(2-\kappa)^2}{(4-k^2)^2}\left[2-\beta-\phi\beta\frac{(1-\tau)}{\tau}\right]^{\frac{2}{\beta}}\times \\ &\quad \times\left(1-\frac{1}{1-\lambda}\frac{(1-\tau)}{\tau}\left(1-\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}\right)\right)^{-\frac{2}{\beta}}, \end{aligned} \tag{31}$$

$$\underline{X}_2=\frac{4-k^2}{(2-\kappa)}\sqrt{\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}b_2^*}, \tag{32}$$

$$\overline{M}_2=\frac{(1-\tau)}{r}b_2^*\left[1-\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}\right]. \tag{33}$$

Expression (33) for cash is analogous to (16) except for the fact that the optimally chosen coupon b_2^* substitutes the fixed cost F_2. Given that b_2^* declines in κ, the effect of competition on cash is unambiguous. In enviroments where competition is more intense the level of liquidity reserves is lower. The intuition is straightforward. As product substitutability

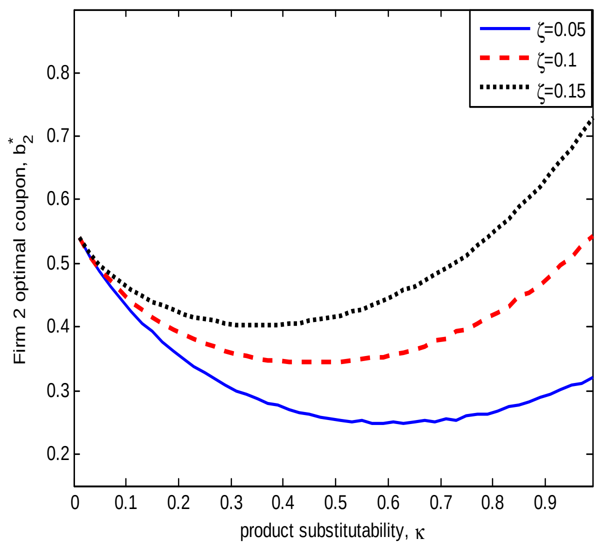

Figure 5: Firm 2's optimal coupon bond, b_2^* . Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0.$, $\varphi = 0.5$, $\lambda = 0.1$, $\tau = 0$.

increases, Firm 2's profits are lower and, therefore, the tax-shield benefits decrease. This implies that it will choose a less risky financial strategy by issuing lower debt. With a lower leverage Firm 2 has smaller per-period payments, i.e. smaller coupon, and needs a lower amount of liquid assets to avoid an inefficient shutdown.

5.2 Idiosyncratic shocks

Let us consider, now, the model with idiosyncratic shocks. The equity value net of cash reserves satisfy

$$\frac{1}{2}\varsigma^2 z_1^2 E_2''(z_1) + \alpha z_1 E_2'(z_1) - rE_2(z_1) + \pi_2^j = 0 \tag{34}$$

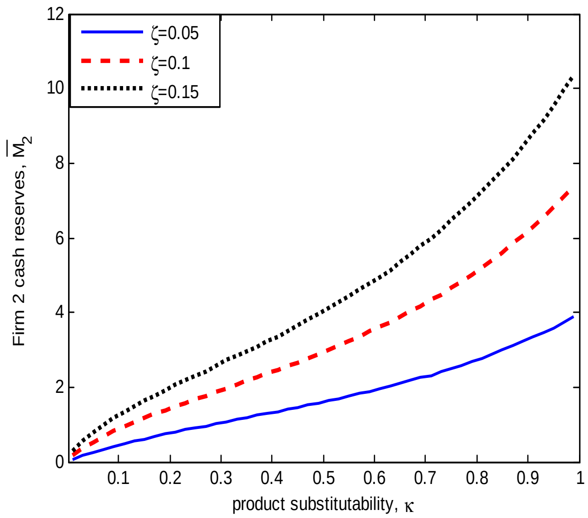

Figure 6: Firm 2's liquidity reserves. Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0.$, $\varphi = 0.5$, $\lambda = 0.1$, $\tau = 0$.

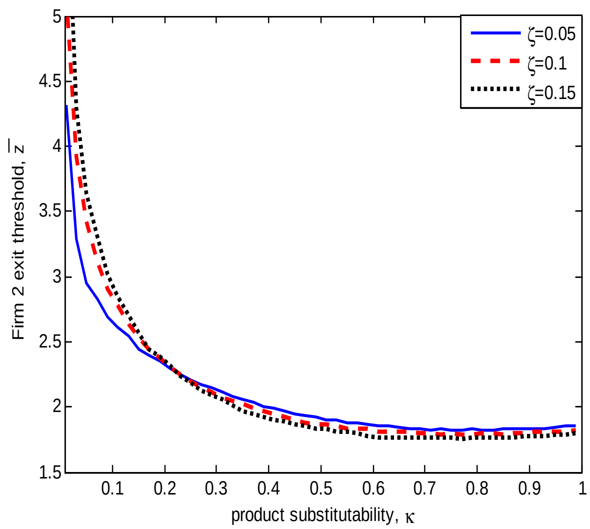

Figure 7: Firm 2's exit threshold. Parameters values are $F_1 = 0$, $X = 1$, $F_2 = 1$, $z_2 = 2.5$, $r = 0.04$, $\alpha = 0.$, $\varphi = 0.5$, $\lambda = 0.1$, $\tau = 0$.

where $j = c$ if $z_1 > z^*$, and $j = m$ if $z_1 \leq z^*$. The general solution is

$$E_2(z_1) = \begin{cases} \widetilde{A} z_1^{\theta_1} + \widetilde{B} z_1^{\theta_2} + \Pi_2^c(z_1) \text{ if } z_1 \geq z_1^* \\ \widetilde{C} z_1^{\theta_1} + \widetilde{D} z_1^{\theta_2} + \Pi_2^m(z_1) \text{ if } z_1 \leq z_1^* \end{cases}$$

where

$$\Pi_2^c(z_1) = \frac{1}{(4-\kappa^2)^2}\left[\frac{z_1^2}{r - 2\alpha - \zeta^2} - 2\frac{z_1 z_2}{r-\alpha} + \frac{z_2^2}{r}\right] - \frac{F_2 + b_2}{r}$$

$$\Pi_2^m(z_1) = \frac{z_2^2}{4r} - \frac{F_2 + b_2}{r}. \tag{35}$$

Conditions analogous to (23)-(27) determine the constants $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, and $\widetilde{D}$ and the exit threshold $\overline{z}$. The optimal coupon b_2^* maximizes (28) with respect to b_2.

The expressions for debt and liquidity reserves are as follows:

$$DBT_2(z_1, b_2^*) = \frac{b_2^*}{r} + \left[(1-\phi)(1-\tau)\max\left\{0, \Pi_2^c(\overline{z}) - \phi\frac{b_2^*}{r}\right\}\right]\left(\frac{z_{.1}}{\overline{z}}\right)^{\theta_1}, \tag{36}$$

$$\overline{M}_2 = \max\left\{0, -\frac{\pi_2^c}{r}\right\}, \tag{37}$$

where

$$\pi_{2,t}^c(z_1) = q_{2,t}^2 - (F_2 + b_2^*). \tag{38}$$

In the common shock case, we have seen that higher product substitutability, by reducing profits, leads Firm 2 to choose a lower coupon. This implies that its per-period payments and precautionary cash holdings are lower. We anticipate that the introduction of debt does not change the qualitative effects of competition on liquidity reserves with respect to the model with all-equity firm. Hence, $\overline{M}_2$ is increasing in κ (see Figure 6).

However, let us procede step by step and investigate, first, the choice of capital structure. Where possible, we use the same parametrization of Section 4.2. Furthermore, we set the floatation cost equal to $\lambda = 0.1$, the fraction of firm's value lost at default $\varphi = 0.5$, the corporate tax rate $\tau = 0.3$ and the idiosyncratic shock at Firm 2's entry time $z_{1,0} = 2$.

Figure 5 plots the optimal coupon b_2^* as a function of product substitutability κ. A striking difference with respect to common model arises. While in that case the optimal coupon is monotonically decreasing in κ, here b_2^* is a U-shaped function of product substitutability. On the one hand, stronger competition generally implies lower profits, lower tax-shield benefits and therefore, as in the common shock model, a smaller optimal coupon. But on the other hand, when shocks are idiosyncratic fiercer competition also means higher volatility. This implies that, even if it is currently earning lower profits, Firm 2's may be willing to pay a sizeable coupon anticipating to obtain larger profits in the future. In other words, Firm 2 has an incentive to "bet" on improvements of market conditions (i.e. on decreases in z_1) and to commit to pay a larger coupon. Depending on which of the two effects prevalis, b_2^* may increase or decrease with product substitutability.

It is also useful to notice the striking analogy with the relation between risk and leverage identified by Leland (1994). In his model of optimal capital structure, he finds that the optimal coupon is a U-shaped function of firm riskiness (figure 8 in his article). Firms with little or very high risk pay a large coupon, while the opposite holds for firms with an intermediate level of risk. Higher competition, by increasing profit volatility, gives rise to a similar effect.

Figure 6 shows the effects of the intensity of competition on precautionary cash holdings. As anticipated, the $\overline{M}_2$ curves are increasing with product substitutability and the interpretation of the result is the same as the model with all-equity firms. Note that in the region where the optimal coupon is decreasing in κ, the fact that Firm 2 faces lower per-period payments could potentially reverse the relation between competition and cash holdings. In other words, smaller fixed cost could imply that a lower amount of cash is necessary to avoid liquidity default. This is not the case and $\overline{M}_2$ monotonically increase with product substitutability.

Finally, figure 7 shows that the shape of the exit threshold is substantially analogous to the model without debt (compare with figure 2), and $\overline{z}$ decreases with the degree of product substitutability.

6 Conclusions

This work investigates how competition affects firms' willingness to hold liquid assets. We build a differentiated duopoly model where firms compete *à la* Cournot and the intensity of competition depends on the degree of product substitutability. Firms can voluntarily and irreversibly abandon the market when profitability falls to sufficiently low levels, but are forced to exit when they lack liquidity to continue their operations. In order to avoid inefficient liquidation firms hold cash.

We introduce uncertainty through two different types of shocks. A common shock, which moves firms profitability in the same direction, and an idiosyncratic shock that moves profitabilities in opposite directions. The idea is to capture the fact that, in competitive markets, while firms share a common fate (they share benefits and losses related the movements of the aggregate economy and/or the strength of sectoral demand), they also experience opposite destinies. Indeed, the success of a firm may depend, to some extent, on the failure of its competitors, and the other way around. To clearly distinguish the effects of different sources of uncertainty, we analyze two twin models where shocks are either common or idiosyncratic.

We find that the nature of the shock crucially affects the direction in which competition affects cash holdings. In the basic model, where firms are financed only through equity, if shocks are common liquidity reserves are not affected by the intensity of competition. But if shocks are idiosyncratic, cash increases with competition. The reason is that, in the model with idiosyncratic shocks, competition makes firm value more volatile. When markets are closely interconnected, that is when competition is intense, firms values are very responsive to stochastic fluctuations. In case of positive (negative) shocks a firm gains (losses) more rapidly market shares and profits. This larger volatility reinforces the precautionary motive for holding cash.

Under a Last-In-First-Out equilibrium, we allow firms to also issue debt and extend the analysis within the context of a capital structure model in the same spirit of Leland (1994). The results of these extensions confirm the findings of the model with all-equity firms.

A Appendix

A.1 Proof of proposition 1

When shocks are common, the instantaneous change in profits is

$$d\pi_{2,t}(X) = \pi_2'(X)\, dz_1 + \frac{1}{2}\pi_2'(X)(dz_1)^2 = \pi_{2,t}(X)[2\mu + \sigma]\, dt + \pi_{2,t}(X)\, 2\sigma dZ_t.$$

The profit process follows a geomeric Brownian motion with drift $2\mu + \sigma$ and constant volatility 2σ. That is, the standardized volatility $\Sigma(\kappa) = \frac{b_t(\kappa)}{\pi_2(X)} = 2\sigma$ is independent of κ.

When shocks are idiosyncratic, the instantaneous change in profits is given by

$$d\pi_{2,t}\left(z_1\right) = \pi_2'\left(z_1\right) dz_1 + \frac{1}{2}\pi_2'\left(z_1\right)\left(dz_1\right)^2 = a_t(z_1,\kappa)dt + b_t(z_1,\kappa)dW_t$$

where $a_t(z_1,\kappa) = -\frac{2\kappa}{4-\kappa^2} z_{1,t}\left[\frac{\left(2z_2-\kappa z_{1,t}\right)}{4-\kappa^2}\alpha + \frac{1}{2}\frac{\kappa}{4-\kappa^2} z_{1,t}\varsigma\right]$ and $b_t(z_1,\kappa) = -\frac{2\kappa}{4-\kappa^2}\varsigma z_{1,t}\frac{\left(2z_2-\kappa z_{1,t}\right)}{4-\kappa^2}$. The volatility of the profit process is given by the coefficient $b_t(\kappa)$. Consider the standardized volatility $\Sigma\left(\kappa\right) = \frac{b_t(\kappa)}{\pi_2(z_1)}$. Differentiating $\Sigma\left(\kappa\right)$ with respect to κ yields

$$\frac{\partial\Sigma\left(\kappa\right)}{\partial\kappa} = \frac{2\varsigma}{\left(2z_{2,t}-\kappa z_{1,t}\right)} + \frac{2\kappa z_{1,t}\varsigma}{\left(2z_{2,t}-\kappa z_{1,t}\right)^2}$$

which is surely positive if Firm 1 is active in the market, i.e. $z_1 > z^*$. ■

A.2 Optimal capital structure: common shocks

Firm 2 holds liquidty reserves sufficient to avoid exit for liquidity reason. Hence value of equity E_2 and debt D_2 depend only on the state of the market conditions, represented by the common shock X. Equity value satisfy

$$\frac{1}{2}\sigma^2 X^2 E_2''(X) + \mu X E_2'(X) - rE_2(X) + \frac{X^2\left(2-\kappa\right)^2}{\left(4-\kappa^2\right)^2} - b = 0, \tag{39}$$

subject to

$$\lim_{X\to\infty} E_2(X) = \frac{X^2}{r-2\mu-\sigma^2}\frac{\left(2-\kappa\right)^2}{\left(4-\kappa^2\right)^2} - \frac{b_2}{r}$$

$$E_2(\underline{X}) = 0$$

$$E_2'(\underline{X}) = 0$$

Debt satisfies

$$\frac{1}{2}\sigma^2 X^2 D_2''(X) + \mu X D_2'(X) - rD_2(X) + b_2 = 0. \tag{40}$$

$$\lim_{X\to\infty} D_2(X) = \frac{b_2}{r}$$

$D_2(\underline{X}) = (1-\phi)(1-\tau)\frac{\underline{X}^2}{r-2\mu-\sigma^2}\frac{(2-\kappa)^2}{(4-\kappa^2)^2}$.

For further reference define $\underline{\tau} = \frac{1-\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}}{2-\lambda-\left(1-\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}\right)}$ and X^* the level of demand strength such that Firm 2 just breaks even, i.e. $\pi_2^c(X^*) = 0$. Three different scenario may occur.

1. If $\mu \leq -\frac{1}{2}\left(\frac{\beta-2}{\beta} + r - \sigma^2\right)$

$$b_2^* = \frac{\beta-2}{\beta}\frac{r}{r-2\mu-\sigma^2}\frac{X_0^2(2-\kappa)^2}{(4-k^2)^2}\left[2-\beta-\phi\beta\frac{(1-\tau)}{\tau}\right],^{\frac{2}{\beta}} \tag{41}$$

$$\underline{X}_2 = \frac{4-k^2}{(2-\kappa)}\sqrt{\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}b_2^*},$$

$$\overline{M}_2 = 0.$$

2. If $\mu > -\frac{1}{2}\left(\frac{\beta-2}{\beta} + r - \sigma^2\right)$ and $\tau < \underline{\tau}$

$$b_2^* = 0,$$

$$\underline{X}_2 = 0,$$

$$\overline{M}_2 = 0.$$

3. If $\mu > -\frac{1}{2}\left(\frac{\beta-2}{\beta} + r - \sigma^2\right)$ and $\tau \geq \underline{\tau}$

$$\begin{aligned} b_2^* &= \frac{\beta-2}{\beta}\frac{r}{r-2\mu-\sigma^2}\frac{X_0^2(2-\kappa)^2}{(4-k^2)^2}\left[2-\beta-\phi\beta\frac{(1-\tau)}{\tau}\right]^{\frac{2}{\beta}} \times \\ &\times\left(1-\frac{1}{1-\lambda}\frac{(1-\tau)}{\tau}\left(1-\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}\right)\right)^{-\frac{2}{\beta}}, \end{aligned} \tag{42}$$

$$\underline{X}_2 = \frac{4-k^2}{(2-\kappa)}\sqrt{\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}b_2^*},$$

$$\overline{M}_2 = \frac{(1-\tau)}{r}b_2^*\left[1-\frac{\beta}{\beta-2}\frac{r-2\mu-\sigma^2}{r}\right]. \tag{43}$$

In scenario 1 the exit boundary is non-lower than the break even point ($\underline{X}_2 \geq X^*$). This implies that Firm 2 never makes losses and therefore does not need to keep cash. The optimal coupon b_2^* maximizes (28), where $\overline{M}_2$ is set equal to zero. Note that this solution correspond to the optimal financing strategy of an unconstrained firm (see Leland (1994)) because, given that $\underline{X}_2 \geq X^*$, the ex-post financing constraint does not play any role. Firm 2 does not need to raise cash for preacutionary reasons and chooses the optimal coupon b_2^* as it was unconstrained. The financing strategy correspond to the optimal compromise between tax-benefits and bankruptcy costs (standard trade-off theory).

In scenario 2 the tax rate is so low ($\tau < \underline{\tau}$) that pure maximization (see equation (42)) would imply a negative coupon, ruled out by assumption and, therefore, $b_2^* = 0$. Note that in standard trade-off models, for instance Leland (1994), optimality implies that firms are always willing to become net borrower (i.e. the optimal coupon is strictly positive). In absence of other costs production always generates positive profits. This implies two things. First, exit occurs when demand falls zero, $\underline{X}_2 = 0$. Second, firm is not at liquidity risk and does not keep cash reserves, $\overline{M}_2 = 0$.

Finally, in scenario 3 the exit boundary is above the break even point ($\underline{X}_2 \geq X^*$) and, therefore, Firm 2 needs to keep cash reserves to be insured against liquidity default. For this reason, this scenario is the relevant one for our analysis and was already presented in the main text. Again, the optimal coupon $b_2^* > 0$ maximizes (28) with respect to b_2. From (28), it is immediate to see that, when the proportional issuance cost λ is strictly positive, the need of raising $\overline{M}_2$ lowers the initial shareholders' value. This implies that, contrary to scenario 1, constrained shareholders distort the choice of optimal coupon with respect the unconstrained case. Now, b_2^* represents the optimal compromise between three factors: tax-shield benefits, bankruptcy costs and costs of raising $\overline{M}_2$. Given that a larger coupon increases the financing needs issuing debt is more costly respect to the unconstrained case. For this reason, while for an unconstrained firm, corresponding to scenario 1, it holds $\lim_{\tau \to 0} b_2^* = 0$, now the optimal coupon drops to zero for a strictly positive tax rate ($\tau = \underline{\tau}$). The effect of competition on cash-holding is clear. Given that b_2^* declines in κ, a raise in competition lowers Firm 2's cash reserves.

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