Mergers and market valuation

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Abstract

This paper investigates the connection between market valuation and a type of the merger (stock, cash) using real options setup. I solve explicitly for the timing and terms of cash mergers in two different settings to demonstrate that cash mergers generally occur at low market valuations, whereas stock mergers that may be observed at both low and high valuations; the result holds with some differences for two dynamic setups. I also investigate the dynamics of the intra-industry mergers within the first setup. I solve for the optimal order of mergers inside an industry for different initial capital allocations to demonstrate that stock mergers in more concentrated industries occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries.

Keywords: real options, mergers, least squares Monte Carlo method

1 Introduction

Since the beginning of the ongoing financial crisis, the world has witnessed many once-strong firms being fire-sold to their former competitors, and quite often these deals were for cash. Cash takeovers normally follow the typical scheme: bidder makes an offer specifying price per target's share, takeover period etc. Target's shareholders either accept this offer agreeing to sell their shares at price offered, or reject it.

A recent all-cash takeover of *BG Group* over *Pure Energy Resources Limited* is a telling example: on 9 February 2009 *BG Group* announced all-cash offer for *Pure* of A$6.40 per share which was at that time superior to the offer made in December by a competing bidder *Arrow Energy Limited* (*Arrow*'s offer was A$2.70 in cash and 1.21 *Arrow* shares for each *Pure* share, being worth A$5.39 per Pure share on 6 February 2009).

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On February 18, *Pure* recommended *BG Group*'s offer of A$8 per share (this price increase by *BG Group* was a response to an earlier *Arrow*'s offer update of A$3.00 in cash and 1.57 *Arrow* shares for each *Pure* share).

On 6 April 2009 the takeover offer was closed; at that moment, *BG Group* owned 99.74% shares of *Pure* with the final price being A$8.25.[1]

As *BG Group* stated itself[2]:

> BG Groups Offer gives Pure shareholders the certainty of cash at a time of heightened uncertainty in world equity and financial markets.

Thus, both bidders and targets understand the superiority of cash deals over stock or stock-and-cash ones at the times of low market valuations.

The fact that periods of high market valuations often coincide with periods of intensive merger activity (especially stock merger activity) (so called 'merger waves') has been extensively documented in merger literature: see, for example, Andrade, Mitchell, and Stafford (2001) and Martynova and Renneboog (2008) for surveys on mergers.

The starting point of this paper can be formulated as follows: out of the last three completed merger waves examined in the literature (the 1960s, 1980s and 1990s), the waves of the 1960s and 1990s were characterized by high market valuations and dominance of stock as preferred medium of payment, whereas market valuations in the 1980s were lower with larger fraction of deals being paid by cash. The research questions is: Is it possible to build a dynamic model of mergers that would agree with existing empirical evidence on merger waves and market valuation? The answer is yes.

This paper investigates connection between market valuation and a type of merger (stock, cash) using real options setup. The study relates to the literature that uses real options approach to dynamically investigate merger decisions, in particular timing and terms of mergers. Lambrecht (2004), Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) model mergers as dynamic option exercise games between target and bidder(s) in which both timing and terms of mergers are determined endogenously.

In particular, Lambrecht (2004) studies mergers motivated by economies of scale under complete, symmetric information and explains the procyclicality of merger waves.

On the contrary, Morellec and Zhdanov (2005) relate to Shleifer and Vishny (2003) in assuming that outside investors have imperfect information about the parameters of the model (namely, about the synergy created by the merger); thus, both models generate short-run abnormal returns that conforms to empirical evidence.

Morellec and Zhdanov (2005) allow for competition between the bidders resulting in negative abnormal returns to the winning bidder; besides, they explain how outside investors update their

[1] See `http://www.bg-group.com/MediaCentre/Press/Pages/Releases.aspx` for more information on the deal.
[2] `http://www.bg-group.com/MediaCentre/Press/Pages/9Feb2009.aspx`

information about perceived synergy of merger observing actions (or, rather, inaction) of bidder(s); learning is also discussed in Grenadier (1999) and Lambrecht and Perraudin (2003).

In Lambrecht (2004), merger synergy comes from the production function that must display increasing returns to scale, whereas in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) merger surplus is linear in the pre-merger values of the firms and depends on the exogenous synergy parameter(s).

While Lambrecht (2004), Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) consider mergers for stock only (with Lambrecht (2004) examining both friendly and hostile stock mergers), this paper aims at analyzing both stock and cash mergers. Though neither Lambrecht (2004), nor Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) do not explicitly label mergers modeled in their papers as stock mergers, I believe that this is the case: in this type of merger each firm obtains shares in the new entity in exchange for the shares in the stand-alone firms (one risky asset is exchanged for another one), whereas in the cash merger the target is paid a lump-sum cash price (risky asset is exchanged for risk-free one). Literally, bidder in the cash merger is entitled to 100% of shares of the merged enrity; this situation can not be modeled within the original setup of Lambrecht (2004), Morellec and Zhdanov (2005) or Hackbarth and Morellec (2008) because in those models terms of merger are solved for endogenuously.

Thus, for the two setups considered (the first one by Lambrecht (2004) and the second one by Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008)), I extend the original model offering the opportunity of a cash merger to the players and then solve cash merger problem.

The model of Lambrecht (2004) depends on one stochastic process only and allows to obtain closed-form solutions. On the contrary, the setup of Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) depending on two correlated stochastic processes requires numerical solution; to this end, I use the Least Squares Monte Carlo approach (LSM) by Longstaff and Schwartz (2001).

In both setups, I solve for terms and timing of cash mergers. I compute option values to the players and introduce a measure of market valuation as weighed average of individual firm valuation in the second setup. I am able to demonstrate that in both setups, stock mergers should occur at high market valuation and at times of low market valuation cash mergers (or both types of mergers) should be observed. Thus, my conclusion agrees with existing empirical evidence on dominance of stock mergers at times of high market.

My results also partially accord with the prediction proposed in Shleifer and Vishny (2003) that one should observe more stock mergers at times of high markets and more cash takeovers at times of low markets.

I also investigate the dynamics of the intra-industry mergers within the first setup. I solve for the optimal order of mergers inside an industry for different initial capital allocations; I demonstrate that stock mergers in more concentrated industries occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries.

The paper is organized as follows: Section 2 examines cash mergers in the Lambrecht (2004) setup, Section 3 discusses cash merger in the Morellec and Zhdanov (2005) setup, Section 4 investigates the dynamics of the intra-industry mergers, Section 5 summarizes the results.

2 Stock vs. cash mergers under increasing returns to scale

This part of the paper is based on Lambrecht (2004) that examines the timing and terms of stock mergers (both friendly mergers and hostile takeovers) in partial equilibrium framework under complete information, increasing returns to scale (which are the only source of merger synergies) and risk-neutral firms. Lambrecht (2004) also assumes that mergers aim at maximizing shareholder value, thus avoiding the discussion of agency problem.

Lambrecht (2004) demonstrates that stock mergers are procyclical and provides closed-form solutions for the timing and terms of stock mergers. He also shows that stock mergers happen at globally efficient threshold.

In Section 2.1 I briefly re-state the setup and results of Lambrecht (2004); next, in Section 2.2, I augment the original model of Lambrecht (2004) with cash mergers. The aim is to demonstrate that cash mergers happen at lower market valuations than stock ones.

2.1 Stock mergers

In Lambrecht (2004), price-taking firm's instantaneous profits π_t are:

$$\pi_t = p_t L^a K^b - w_L L, \tag{1}$$

where p_t is the stochastic output price;
L and K are labor and capital inputs respectively;
w_L is the unit cost of labor;
a and b are positive constants such that $a < 1$ and $a + b > 1$, so that there are increasing returns to scale when both inputs are considered to be variable (as in the case of merger).

Thus, stochastic shock (output price) p_t is common for all the firms in the industry (as opposed to Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) where firms face correlated stochastic shocks) and is governed by the following geometric Brownian motion:

$$dp_t = \mu p_t dt + \sigma p_t dW_t, \tag{2}$$

where W_t is the standard Brownian motion;
μ and σ are constants such that $\mu < r$ and $\sigma > 0$ and r is the risk-free interest rate.

Firm's instantaneous profits maximized with respect to labor input are:

$$\pi_t^* = f(w_L, a) K^\theta p_t^\gamma \tag{3}$$

where $f(w_L, a)$ is a known function of w_L and a;
$\theta = \frac{b}{1-a} > 1$ and $\gamma = \frac{1}{1-a} > 1$.

Then the value of the firm equals to:

$$V(p_t) = \int_t^{\infty} \pi_t^* e^{-rt} dt = \frac{f(w_L, a) K^{\theta} p_t^{\gamma}}{r - \mu\gamma - \frac{\sigma^2 \gamma(\gamma-1)}{2}} = c p_t^{\gamma} K^{\theta}, \tag{4}$$

where $c = \frac{f(w_L,a)}{r-\mu\gamma-\frac{\sigma^2\gamma(\gamma-1)}{2}}$ s.t. $\gamma < \beta_2$ with β_2 being the positive root of the equation:

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \mu\beta - r = 0. \tag{5}$$

Thus, in the case of two firms with capital inputs equal K_1 and K_2 and lump-sum merger cost equal M_1 and M_2, the total merger surplus equals to:

$$\begin{aligned} S(p_t) &= \max\left(V_M(p_t) - V_1(p_t) - V_2(p_t) - M_1 - M_2,\, 0\right) \\ &= \max\left(c p_t^{\gamma}\left((K_1 + K_2)^{\theta} - K_1^{\theta} - K_2^{\theta}\right) - M_1 - M_2,\, 0\right), \end{aligned} \tag{6}$$

where V_M is the post-merger value of the new firm;
V_1 and V_2 are pre-merger values of firm 1 and firm 2.

In (6), total benefits of merger equal $c p_t^{\gamma}\left((K_1 + K_2)^{\theta} - K_1^{\theta} - K_2^{\theta}\right)$; they are positive since $\theta = \frac{b}{1-a} > 1$, i.e. for increasing returns to scale.

After the merger firm i ($i = 1, 2$) obtains fraction s_i of the new entity with $s_1 + s_2 = 1$. The surplus accruing to firm i equals:

$$\begin{aligned} S_i(p_t) &= \max\left(s_i V_M(p_t) - V_i(p_t) - M_i,\, 0\right) \\ &= \max\left(c p_t^{\gamma}\left(s_i (K_1 + K_2)^{\theta} - K_i^{\theta}\right) - M_i,\, 0\right). \end{aligned} \tag{7}$$

Since the merger surplus of each merging firm in a convex and increasing function of the stochastic output price p_t, the merger option is exercised by firm i the first time process p_t reaches the threshold p_i^* from below.

It is demonstrated in Lambrecht (2004) that the in the continuation region (for $p_t < p_i^*$) the option to merge of firm i OM_i satisfies:

$$rOM_i = \mu p_t OM_i' + \frac{\sigma^2}{2} p_t^2 OM_i'' \tag{8}$$

with the general solution being:

$$OM_i = B_1^i p_t^{\beta_1} + B_2^i p_t^{\beta_2} \tag{9}$$

where β_1 and β_2 are negative and positive root of (5).

Since $\lim_{p_t \to 0} OM_i = 0$, then $B_1^i = 0$; the value matching and smooth pasting conditions at p_i^* are:

$$OM_i(p_i^*) = c\,(p_i^*)^{\gamma}\left(s_i\,(K_1+K_2)^{\theta} - K_i^{\theta}\right) - M_i \tag{10}$$

$$OM_i'(p_i^*) = c\gamma\,(p_i^*)^{\gamma-1}\left(s_i\,(K_1+K_2)^{\theta} - K_i^{\theta}\right). \tag{11}$$

The solution is:

$$OM_i(p_t) = \left(cp_t^{\gamma}\left(s_i\,(K_1+K_2)^{\theta} - K_i^{\theta}\right) - M_i\right)\left(\frac{p_t}{p_i^*}\right)^{\beta_2} \tag{12}$$

with the merger threshold for firm i being:

$$p_i^* = \left(\frac{\beta_2}{\beta_2-\gamma}\frac{M_i}{c\left(s_i\,(K_1+K_2)^{\theta} - K_i^{\theta}\right)}\right)^{\frac{1}{\gamma}}. \tag{13}$$

Taking into account that merger threshold of firms should be equal $p_1^* = p_2^* = p^*$ and the fact that $s_1 + s_2 = 1$ allows to solve for the merger threshold p^* and for optimal shares s_1 and s_2:

$$p^* = \left(\frac{\beta_2}{\beta_2-\gamma}\frac{M_1+M_2}{c\left((K_1+K_2)^{\theta} - K_1^{\theta} - K_2^{\theta}\right)}\right)^{\frac{1}{\gamma}} \tag{14}$$

$$s_i = \frac{M_i\left((K_i+K_j)^{\theta} - K_j^{\theta}\right) + M_jK_i^{\theta}}{(M_i+M_j)\,(K_i+K_j)^{\theta}}. \tag{15}$$

It is demonstrated in Lambrecht (2004) that threshold p^* coincides with the socially optimal threshold derived from the point of view of social maximizer and based on total surplus $S(p_t)$ rather than on individual surplus of each firm $S_i(p_t)$; this means that the merger described by p^* and (s_1, s_2) is Pareto optimal and constitutes Nash equilibrium.

Merger threshold (14) will serve as benchmark for analysis of cash mergers in Section 2.2.

The choice of roles of bidder and target for this type of merger is completely immaterial not only for merger terms and timing, but also for welfare consequences (surplus distribution) of the merger. The solution does not directly involve 'the bidder' and 'the target'; it is enough to have two firms willing two merge.

2.2 Market valuation of stock vs. cash mergers

In the cash merger bidder buys the firm of target paying a lump-sum price P (and not a share of the merged entity as in the stock merger in Section 2.1).

The solution for the target is as follows: in the continuation region of the target (for $p_t > p_T$), differential equation (8) for the option of the target OT (instead of OM_i) holds with the general solution (9) and $B_2 = 0$ since $\lim_{p_t\to\infty} OT = 0$. Usual value-matching and smooth pasting conditions apply:

$$OT(p_T) = P - cp_T^\gamma K_T^\theta - M_T \tag{16}$$

$$OT'(p_T) = -c\gamma p_T^{\gamma-1} K_T^\theta. \tag{17}$$

The solution is:

$$OT(p_t) = \left(P - cp_T^\gamma K_T^\theta - M_T\right)\left(\frac{p_t}{p_T}\right)^{\beta_1}, \tag{18}$$

where β_1 is the negative root of (5);
p_T is the cash merger threshold of the target:

$$p_T = \left(\frac{\beta_1}{\beta_1 - \gamma}\frac{P - M_T}{cK_T^\theta}\right)^{\frac{1}{\gamma}}. \tag{19}$$

The solution for the bidder is as follows: in the continuation region of the bidder (for $p_t < p_B$), differential equation (8) for the option of the bidder OB (instead of OM_i) holds with the general solution (9) and $B_1 = 0$ since $\lim_{p_t \to 0} OB = 0$. Usual value-matching and smooth pasting conditions are:

$$OB(p_B) = cp_B^\gamma\left((K_B + K_T)^\theta - K_B^\theta\right) - M_B - P \tag{20}$$

$$OB'(p_B) = c\gamma p_B^{\gamma-1}\left((K_B + K_T)^\theta - K_B^\theta\right) \tag{21}$$

The solution is:

$$OB(p_B) = \left(cp_B^\gamma\left((K_B + K_T)^\theta - K_B^\theta\right) - M_B - P\right)\left(\frac{p_t}{p_B}\right)^{\beta_2}, \tag{22}$$

where p_B is the cash merger threshold of the bidder:

$$p_B = \left(\frac{\beta_2}{\beta_2 - \gamma}\frac{M_B + P}{c\left((K_B + K_T)^\theta - K_B^\theta\right)}\right)^{\frac{1}{\gamma}}. \tag{23}$$

The bidder is willing to exercise a cash merger option when the state variable p_t first hits the threshold p_B from below, whereas the target is willing to exercise when p_t first hits p_T from above; thus, for the merger to be exercised, the following condition should hold:

$$p_B \le p_t \le p_T, \tag{24}$$

or, after simplifications,

$$\frac{\beta_2}{\beta_2 - \gamma}\frac{M_B + P}{(K_B + K_T)^\theta - K_B^\theta} < \frac{\beta_1}{\beta_1 - \gamma}\frac{P - M_T}{K_T^\theta}. \tag{25}$$

Solving (25) for P yields:

$$P \geq \frac{\left((K_B + K_T)^{\theta} - K_B^{\theta}\right) M_T \frac{\beta_1}{\beta_1 - \gamma} + K_T^{\theta} M_B \frac{\beta_2}{\beta_2 - \gamma}}{\frac{\beta_1}{\beta_1 - \gamma} (K_B + K_T)^{\theta} - \frac{\beta_1}{\beta_1 - \gamma} K_B^{\theta} - \frac{\beta_2}{\beta_2 - \gamma} K_T^{\theta}} \tag{26}$$

provided the following inequality holds:

$$\frac{\beta_1}{\beta_1 - \gamma} (K_B + K_T)^{\theta} - \frac{\beta_1}{\beta_1 - \gamma} K_B^{\theta} - \frac{\beta_2}{\beta_2 - \gamma} K_T^{\theta} > 0. \tag{27}$$

Since $\frac{\beta_2}{\beta_2 - \gamma} > \frac{\beta_1}{\beta_1 - \gamma} > 0$ by the properties of the solution, inequality does not always hold implying that cash merger equilibrium does not always exists. Thus, depending on the model parameters, one can distinguish between two types of outcomes:

1. (27) holds; both stock and cash merger equilibria exist;

2. (27) does not hold; only stock merger equilibrium exists.

Assume that (27) holds i.e. cash merger equilibrium exists; to determine the relationship between the stock merger trigger p^* in (14) and the cash merger corridor $[p_B, p_T]$ solve $p^* < p_B$ to obtain:

$$P \geq \frac{\left((K_B + K_T)^{\theta} - K_B^{\theta}\right) M_T + K_T^{\theta} M_B}{(K_B + K_T)^{\theta} - K_B^{\theta} - K_T^{\theta}}. \tag{28}$$

The fact that $\frac{\beta_2}{\beta_2 - \gamma} > \frac{\beta_1}{\beta_1 - \gamma} > 0$ means that (26) implies (28) and, consequently:

$$p^* < p_B < p_T. \tag{29}$$

Consider an example with $a = 0.4$, $b = 1.9$, $\mu = 0.01$, $r = 0.08$, $\sigma = 0.2$, $K_B = K_T = 100$, $M_B = M_T = 3$, $c = 1$; (27) holds and price P should satisfy $P \geq 73.5$.

Setting $P = 200$, one obtains the following values for bidder's and target's threshold: $p_B = 0.00245471$, $p_T = 0.002532$. The stock merger threshold p^* (see 14) equals $p^* = 0.000321576$ and (29) holds.

Inequality (29) suggests that when market valuation (as measured by the state variable p_t) is in the interval $[p_B, p_T]$, both cash and stock mergers are observed; as p_t increases, only stock mergers should be observed. This conclusion agrees quite well with empirical evidence on procyclicality of merger waves and dominance of stock mergers at high market valuations.

Now I proceed to a more complicated setup of Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) that employs two correlated stochastic processes (instead of one in this setup) and is based on linear synergy instead of synergy stemming from economies of scale.

3 Stock vs. cash mergers under linear merger synergy

I follow Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) in the setup of my model. Consider an industry consisting of two firms (bidder and target) with capital stock K and Q; present value of the cash flows of the firms are X and Y that are governed by the stochastic differential equations:

$$dX_t = \mu_X X_t dt + \sigma_X X_t dW_t^X \tag{30}$$

$$dY_t = \mu_Y Y_t dt + \sigma_Y Y_t dW_t^Y \tag{31}$$

where W_t^X and W_t^Y are standard correlated Brownian motions with correlation coefficient ρ; μ_X, μ_Y, σ_X and σ_Y are constants such that $\mu_X < r$, $\mu_Y < r$, $\sigma_X > 0$ and $\sigma_Y > 0$ and r is the risk-free interest rate.

Assume also that investors are risk-neutral.

In case of a merger, combined value of the merged firms equals:

$$V(X,Y) = KX + QY + \alpha (K+Q)(X-Y), \tag{32}$$

where KX is the pre-merger value of the bidder;
KY is the pre-merger value of the target;
α is positive and reflects merger synergy;
$\alpha (K+Q)(X-Y)$ is the merger surplus.

It follows from (32) that for the merger to be profitable, the bidder should have higher valuations per unit capital than the target; it means that the roles of bidder and target are pre-determined as opposed to Lambrecht (2004) where any of the firms can act as a bidder. 'Valuation per unit capital' may be thought of as Tobin's q or M/B ratio.

I choose the complete information setup of Hackbarth and Morellec (2008) as opposed to incomplete information with learning as in Morellec and Zhdanov (2005) for better comparison with the results from the previous section; both Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) assume that the option to merger has infinite horizon.

First I briefly repeat the results of the stock merger as in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008), and then I solve this model for the cash takeover game (I follow the same order as in Section 2).

3.1 Stock mergers

Stock merger is modeled as a simultaneous game with bidder and target giving up their pre-merger values of the firms to get a share in the new merged entity.

Payoffs to the bidder P_B^s and to the target P_T^s at the stock merger are as follows:

$$\begin{aligned} P_B^s(X,Y) &= \max(s_B V(X,Y) - KX, 0) \\ P_T^s(X,Y) &= \max((1-s_B)V(X,Y) - QY, 0), \end{aligned} \tag{33}$$

where s_B is the share of the merged entity accruing to the bidder.

In the continuation region option to the bidder O^{Bs} and to the target O^{Ts} satisfy the following differential equations:

$$rO^{Bs} = \frac{1}{2}\sigma_X^2 X^2 O_{XX}^{Bs} + \frac{1}{2}\sigma_Y^2 Y^2 O_{YY}^{Bs} + \rho\sigma_X\sigma_Y XY O_{XY}^{Bs} + \mu_X O_X^{Bs} + \mu_Y O_Y^{Bs} \tag{34}$$

$$rO^{Ts} = \frac{1}{2}\sigma_X^2 X^2 O_{XX}^{Ts} + \frac{1}{2}\sigma_Y^2 Y^2 O_{YY}^{Ts} + \rho\sigma_X\sigma_Y XY O_{XY}^{Ts} + \mu_X O_X^{Ts} + \mu_Y O_Y^{Ts} \tag{35}$$

subject to the following value-matching conditions:

$$O^{Bs}(X^s, Y^s) = s_B V(X^s, Y^s) - KX^s \tag{36}$$

$$O^{Ts}(X^s, Y^s) = (1-s_B)V(X^s, Y^s) - QY^s, \tag{37}$$

where X^S and Y^S is the stock exercise bound.

Though options to both bidder and target depend on two stochastic processes X (30) and Y (31), but since the payoffs P_B^s and P_T^s are both linear in X and Y, it is demonstrated in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) that terms and timing of the mergers can be solved in terms of the ratio $R = \frac{X}{Y}$.

In particular, option values to bidder and target satisfy:

$$\begin{aligned} O^{Bs}(X,Y) &= Y(s_B V(R^*, 1) - KR^*)\left(\tfrac{R}{R^*}\right)^{\lambda_2} \\ O^{Ts}(X,Y) &= Y((1-s_B)V(R^*, 1) - Q)\left(\tfrac{R}{R^*}\right)^{\lambda_2}, \end{aligned} \tag{38}$$

where R^s is the stock merger threshold:

$$R^s = \frac{\lambda_2}{\lambda_2 - 1}, \tag{39}$$

s_B is the share of the merged firm accruing to the bidder:

$$s_B = \frac{K}{K+Q}, \tag{40}$$

and λ_2 is the positive root of the equation:

$$\frac{1}{2}\left(\sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2\right)\lambda(\lambda-1) + (\mu_X - \mu_Y)\lambda = r - \mu_Y. \tag{41}$$

Merger occurs as soon as process $R = \frac{X}{Y}$ first hits the threshold $R^s = \frac{\lambda_2}{\lambda_2 - 1}$ from below. This result is similar in spirit to the one in Lambrecht (2004) where the state variable p_t also needs to hit the threshold p^* from below.

Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) also demonstrate that the stock merger equilibrium coincides with the central-planner equilibrium where the central planner is maximizing merger surplus. It means that the payoff of the central planner equals:

$$\begin{aligned} P^s_{CP}(X,Y) &= \max\left(V(X,Y) - KX - QY, 0\right) = \\ &= \max\left(\alpha (K+Q)(X-Y), 0\right), \end{aligned} \tag{42}$$

and the option to the central planner O^{CPs} equals the sum of bidder's O^{Bs} and target's O^{Ts} options:

$$\begin{aligned} O^{CPs}(X,Y) &= Y\left(V(R^*,1) - KR^* - Q\right)\left(\tfrac{R}{R^*}\right)^{\lambda_2} = \\ &= Y\left(\alpha (K+Q)(R^*-1)\right)\left(\tfrac{R}{R^*}\right)^{\lambda_2}. \end{aligned} \tag{43}$$

Solution to the stock merger problem summarized in this section will provide the benchmark for the cash merger problem presented in the next section.

3.2 Cash mergers and market valuation

The bidder offers a lump-sum price P for the whole firm of the target. Payoffs to the bidder P^c_B and to the target P^c_T at the cash merger are as follows:

$$\begin{aligned} P^c_B(X,Y) &= \max\left(V(X,Y) - KX - P, 0\right) = \\ &= \max\left(QY + \alpha (K+Q)(X-Y) - P, 0\right) \\ P^c_T(X,Y) &= \max\left(P - QY, 0\right) \end{aligned} \tag{44}$$

In the continuation region options to the bidder O^{Bc} and to the target O^{Tc} satisfy the following differential equations:

$$rO^{Bc} = \frac{1}{2}\sigma_X^2 X^2 O^{Bc}_{XX} + \frac{1}{2}\sigma_Y^2 Y^2 O^{Bc}_{YY} + \rho\sigma_X\sigma_Y XY O^{Bc}_{XY} + \mu_X O^{Bc}_X + \mu_Y O^{Bc}_Y \tag{45}$$

$$rO^{Tc} = \frac{1}{2}\sigma_Y^2 Y^2 O^{Tc}_{YY} + \mu_Y O^{Tc}_Y \tag{46}$$

subject to the following value-matching conditions:

$$O^{Bc}(X^c, Y^c) = QY^c + \alpha (K+Q)(X^c - Y^c) - P \tag{47}$$

$$O^{Tc}(X^c, Y^c) = P - QY^c, \tag{48}$$

where X^c and Y^c is the exercise boundary.

Since the value function of the bidder $O^{Bc}(X^c, Y^c)$ (47) is not homogeneous neither in X, nor in Y, it is not possible to reduce the solution to the ratio $\frac{X}{Y}$ as it was done for the stock mergers in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) (see Section 3.1) and I have to rely on numerical methods to solve this problem.

I use Longstaff and Schwartz (2001) least squares Monte Carlo (LSM) approach that is relatively simple and convenient for multi-factor models[3].

Parameter calibration for LSM Least squares approach requires setting a finite time horizon (as opposed to infinite horizon in the original papers); for the major part of the solution I choose a horizon of 5 years (T=5). Remaining parameters are set as follows:

- risk-free interest rate $r = 0.06$, dividend payout rates for the bidder $\delta_X = 0.005$ and for the target $\delta_Y = 0.035$ implying drifts of $\mu_X = 0.055$ and $\mu_Y = 0.025$, volatilities $\sigma_X = 0.2$ and $\sigma_Y = 0.2$, correlation between stochastic processes of the firms $\rho = 0.75$ are set as in Hackbarth and Morellec (2008) (see Table I on page 1227);

- estimation is based on $N = 100,000$ paths as in Longstaff and Schwartz (2001) with $y = 10$ exercise points per year for the sample simulation in Table 5 and interchangeably $y = 10$ and $y = 50$ otherwise[4];

- though Hackbarth and Morellec (2008) consider mergers of equals with $K = Q$, Andrade, Mitchell, and Stafford (2001) report that the median relative size of the target was 11.7% in 1973-1998; that is why I set the capital stock of the bidder $K = 100$ and of the target $Q = 12$;

- initial values of X and Y are set to $X_0 = Y_0 = 1$ implying that 1) both bidder and target are neither undervalued, nor overvalued and 2) initial merger synergy computed as $\alpha\left(X_0 - Y_0\right)(K + Q)$ is zero;

- lump-sum price P offered for the whole firm of the target is set to $P = 12$ implying zero merger premium for the target;

- synergy parameter α is set to $\alpha = 0.4$ resulting in reasonable merger premium of 22% for cash merger and 52% for stock merger over a 5-year horizon (see Table 2).

Since the solution for the cash merger based on LSM hinges on the assumption about chosen finite horizon, it is not directly comparable to the infinite-horizon solution derived in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) and presented in Section 3.1. Thus, I need to solve stock merger problem using LSM with finite horizon too.

[3]MATLAB codes for LSM estimation are available from the author on demand.
[4]y=50 was used in the original paper by Longstaff and Schwartz (2001).

LSM: short algorithm description for both cash and stock merger problems

1. simulate X and Y obtaining N simulation paths for y exercise points per year;

2. compute state variables and payoffs:
   cash: state variables for the bidder $S_B = V(X,Y) - KX = QY + \alpha(K+Q)(X-Y)$, for the target $S_T = QY$;
   payoffs to the bidder $P_B^c = \max(QY + \alpha(K+Q)(X-Y) - P, 0)$ and to the target $P_T^c = \max(P - QY, 0)$;
   stock: state variable for the central planner $S_{CP} = \alpha(X-Y)(K+Q)$;
   payoff to the central planner $P_{CP} = \max(\alpha(K+Q)(X-Y), 0)$.

   Appendix A explains in detail why it is possible to solve the stock merger problem from the point of view of central planner setting $s_B = \frac{K}{K+Q}$ and provided $\alpha(K+Q) > Q$ (that can be rewritten as $\frac{K}{K+Q} > 1-\alpha$) is satisfied; in this paper $\frac{K}{K+Q} = 0.8929 > 1-\alpha = 0.6$.

3. apply LSM as follows[5]:
   cash: for the merger option to be exercised at some point both bidder and target should independently prefer immediate exercise to option continuation at this point;
   stock: central planner should prefer immediate exercise to continuation;

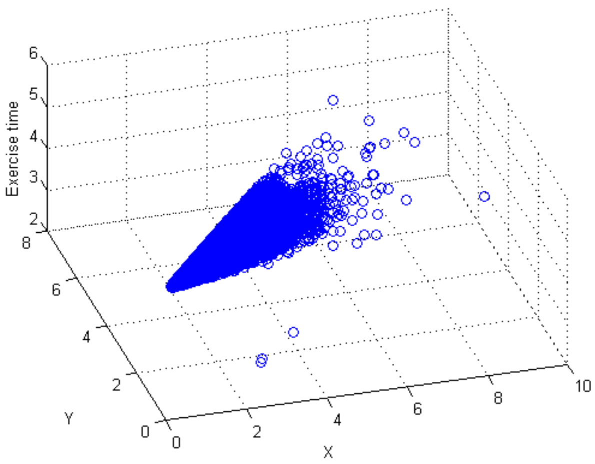
4. compute option values:
   cash: to the bidder O_{LSM}^{Bc} and to the target O_{LSM}^{Tc} as sample mean;
   stock: to the central planner O_{LSM}^{CPs} as sample mean; separately to the bidder O_{LSM}^{Bs} and to the target O_{LSM}^{Ts} as sample mean of discounted payoffs at exercise to the bidder $(s_B V(X^s, Y^s) - KX^s)\, e^{-rt}$ and to the target $((1-s_B) V(X^s, Y^s) - QY^s)\, e^{-rt_{ex}}$ (t_{ex} is the time of option exercise);

5. compute average market valuation[6] $MARKET_{LSM}^c$ ($MARKET_{LSM}^s$), average merger premium $PREM_{LSM}^c$ ($PREM_{LSM}^s$), and average ratio R_{LSM}^c (R_{LSM}^s) at the time of exercise using the sub-sample of paths where the merger is exercised at some point as a mean of the following quantities: $\frac{KX_{LSM}^c + QY_{LSM}^c}{K+Q}$ $\left(\frac{KX_{LSM}^s + QY_{LSM}^s}{K+Q}\right)$, $\frac{P}{QY^c} - 1$ $\left(\frac{(1-s_B)V(X^s,Y^s)}{QY^s} - 1\right)$ and $\frac{X^c}{Y^c}$ $\left(\frac{X^s}{Y^s}\right)$.

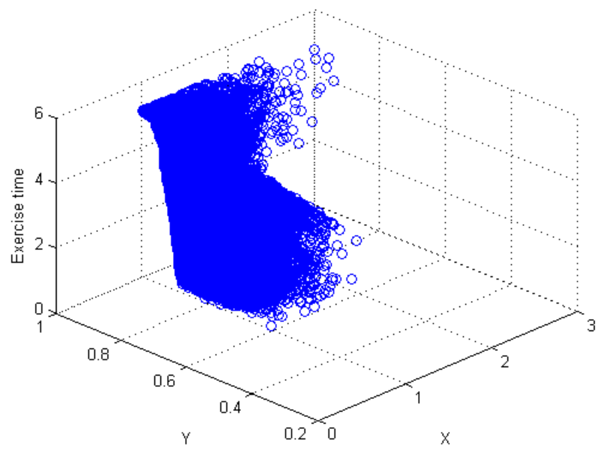
Table 1 presents the results of LSM simulations over different time horizons: 1, 5, 25 and 50 years together with the result for infinite horizon based on Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) (see Section 3.1 for detailed derivations).

Option values to the players increase as time horizon becomes longer; for the stock merger case, option values converge monotonically to the infinite horizon option which is perfectly intuitive. 'Total' for cash merger options is always lower than for respective stock merger options reflecting the fact that stock merger is the 'first-best' choice as shown in Morellec and Zhdanov (2005), Hackbarth and Morellec (2008) and, though for a different setup, in Lambrecht (2004).

[5]As regressors, I use a constant and the first three powers of the state variable.
[6]There are only two firms in the model and, consequently, market valuation depends on X and Y only.



(a) Stock merger



(b) Cash merger

Figure 1: Exercise boundaries for the stock and cash mergers separately at $T = 5$, $y = 10$ and remaining parameters as above

Market valuation as measured by $MARKET^c_{LSM}$ for cash merger and $MARKET^s_{LSM}$ for stock mergers demonstrates desired behavior: for all of the estimated time horizons, market valuation for cash merger is always lower than the market valuation for the stock merger; to prove that this relationship holds generally, I will simulate a cross-section of mergers and conduct regression analysis later in the paper.

The behavior of merger premium of the stock merger $PREM^s_{LSM}$ has one quite striking property: while the size of the premium remains quite reasonable over 1-year and 5-year horizon (18% and 52% respectively), it becomes very high over 50-year horizon (689%) and reaches even higher level of 1066% over an infinite horizon.

Average ratio $\frac{X^s}{Y^s}$ for the stock merger (R^s_{LSM}) also climbs very high over an infinite horizon reaching the level of 9.24, whereas the same ratio for the cash merger remains reasonable.

These large (and unrealistic) magnitudes may suggest that firms do not really consider horizons of such length; that is why the choice of 5-year horizon seems quite appropriate.

Table 2 compares the results of LSM estimation over 1-year and 5-year horizon for different number of exercise point per year: 10 and 50. Results suggest that loss in computational accuracy when switching from 50 to 10 exercise points per year is acceptable, whereas gains in computational speed are significant; henceforth, I conduct LSM estimation based on 10 exercise points per year ($y = 10$).

Finally, Figures 1 illustrates exercise boundaries for the stock and cash mergers separately for the benchmark example with $T = 5$ and $y = 10$ (for estimation results see row 4 of Table 2), whereas Figure 2 puts them together for better comparison.

Figure 2 demonstrates that when capital valuations of both bidder and target are relatively high, only stock mergers should be observed; on the contrary, when valuations are relatively low, both

Table 1: LSM simulation results over different time horizons

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O^{Bs}_{LSM}	O^{Ts}_{LSM}	total	O^{Bc}_{LSM}	O^{Tc}_{LSM}	total	cash	stock	cash	stock	cash	stock
1	10	2.09	1.10	3.19	1.43	0.49	1.92	0.99	1.09	0.11	0.18	1.11	1.14
5	10	5.71	3.03	8.74	3.47	0.94	4.40	1.02	1.38	0.22	0.53	1.26	1.41
25	10	14.43	7.64	22.07	7.64	1.08	8.72	1.25	3.82	0.44	2.56	1.77	2.98
50	10	18.02	9.55	27.57	9.00	0.97	9.97	1.54	13.21	0.63	8.65	2.40	7.69
infinite horizon		19.94	10.56	30.51							10.66		9.24

Table 2: Comparison of LSM simulation results over 1-year and 5-year time horizon for 10 and 50 exercise points per year

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O^{Bs}_{LSM}	O^{Ts}_{LSM}	total	O^{Bc}_{LSM}	O^{Tc}_{LSM}	total	cash	stock	cash	stock	cash	stock
1	50	2.09	1.11	3.20	1.42	0.50	1.92	0.99	1.09	0.10	0.18	1.09	1.14
1	10	2.09	1.10	3.19	1.43	0.49	1.92	0.99	1.09	0.11	0.18	1.11	1.14
5	50	5.67	3.00	8.67	3.46	0.94	4.40	1.02	1.38	0.20	0.52	1.23	1.40
5	10	5.71	3.03	8.74	3.47	0.94	4.40	1.02	1.38	0.22	0.53	1.26	1.41

Table 3: LSM simulation results over different time horizons: P=-0.6 + 1.35QY

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O_{LSM}^{Bs}	O_{LSM}^{Ts}	total	O_{LSM}^{Bc}	O_{LSM}^{Tc}	total	cash	stock	cash	stock	cash	stock
1	10	2.09	1.10	3.19	1.53	1.24	2.77	1.09	1.09	0.30	0.18	1.18	1.14
5	10	5.71	3.03	8.74	6.27	1.69	7.96	1.31	1.38	0.29	0.53	1.37	1.41
25	10	14.43	7.64	22.07	20.32	1.25	21.57	3.37	3.82	0.28	2.56	2.50	2.98
50	10	18.02	9.55	27.57	26.91	0.66	27.57	11.92	13.21	0.29	8.65	5.48	7.69
infinite horizon		19.94	10.56	30.51							10.66		9.24

Table 4: Comparison of LSM simulation results over 1-year and 5-year time horizon for 10 and 50 exercise points per year: P=-0.6 + 1.35QY

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O_{LSM}^{Bs}	O_{LSM}^{Ts}	total	O_{LSM}^{Bc}	O_{LSM}^{Tc}	total	cash	stock	cash	stock	cash	stock
1	50	2.09	1.11	3.20	1.46	1.29	2.75	1.08	1.09	0.30	0.18	1.17	1.14
1	10	2.09	1.10	3.19	1.53	1.24	2.77	1.09	1.09	0.30	0.18	1.18	1.14
5	50	5.67	3.00	8.67	6.09	1.71	7.81	1.29	1.38	0.29	0.52	1.35	1.40
5	10	5.71	3.03	8.74	6.27	1.69	7.96	1.31	1.38	0.29	0.53	1.37	1.41

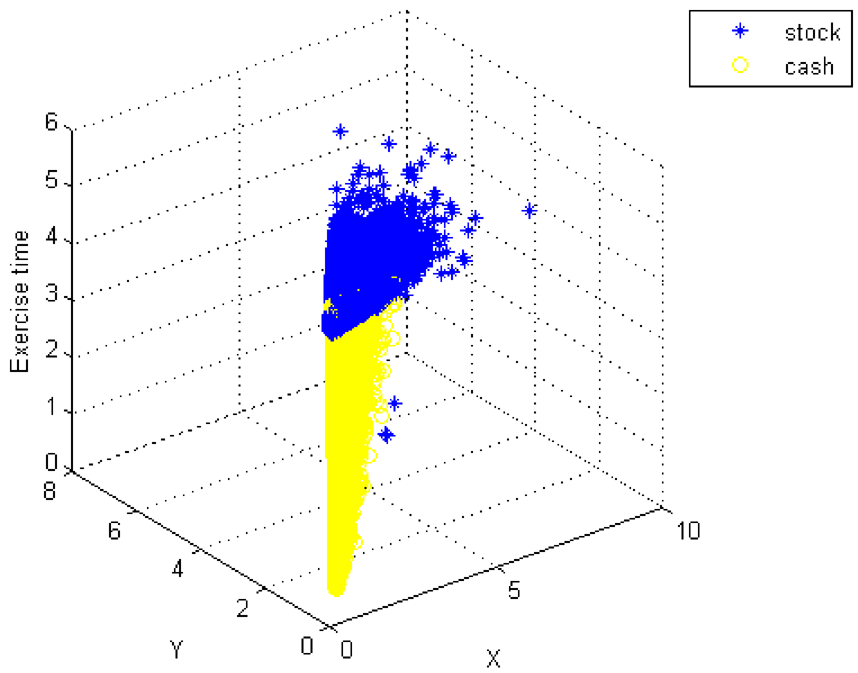


Figure 2: Exercise boundaries for the stock and cash mergers at $T = 5$, $y = 10$ and remaining parameters as above

cash and stock merger may be observed. Taking into account the fact that correlation between X and Y is positive in this example ($\rho = 0.75$), one concludes that when market valuation (as measured by weighted average of firm's valuations) is high, stock mergers should be observed, whereas at low market valuations both types of merger may be observed.

Tables 3 and 4 provide estimation results for the price P computed as $P = -0.6 + 1.35QY$ with parameters estimated from a sample of cash mergers; one can see that for this functional form of P cash mergers demonstrate the same long-run behavior as stock mergers: $MARKET^c_{LSM}$ and R^c_{LSM} increase significantly over 50-year horizon; on the contrary, cash merger premium $PREM^c_{LSM}$ stays under 35% due to the scific functional form of P. However, on 5-year horizon there is no huge qualitative difference between constant price P as in Tables 1 and 2 and linear price P as in Tables 3 and 4; for the rest of the Section, I use the first setup with constant price P.

In order to conduct more general test of a hypothesis that stock mergers should be observed at high market valuations, and cash mergers should occur at low market valuations, I simulate a sample of 1000 merger situations (without initially specifying the type of a merger) with majority of input parameters drawn from independent uniform distributions (see Table 5 for details).

A fragment of 15 simulated merger situations is presented in Appendix B, in Table 9 (estimation results) and Table 10 (input parameters).

One can easily see that though the sum of bidder's and target's stock merger options is always greater than the sum of cash merger options (reflecting the fact that stock merger is the 'first-best'),

Table 5: Parameters calibration

Parameter	Benchmark example	Sample simulation rule		
		$U[a,b]$		Other
		a	b	
T	5			$=5$
y	10			$=10$
r	0.06	0.04	0.08	
δ_X	0.005	0	0.01	
δ_Y	0.035	0.03	0.04	
σ_X	0.2	0.15	0.25	
σ_Y	0.2	0.15	0.25	
ρ	0.75	-1	1	
K	100	75	125	
Q	12	9	15	
α	0.4	0.35	0.45	
X_0	1	0.5	2.5	
Y_0	1			$=X_0$
P	12			$=QY_0$

$U[a,b]$ stands for uniform distribution with parameters a and b.
Parameters Y_0 and P are set so as to ensure that initial synergy and initial merger premium are both equal to zero (as in estimations in Tables 1 and 2).
Drifts are $\mu_X = r - \delta_X$, $\mu_Y = r - \delta_Y$.

Table 6: Payoff matrix

		Target: CASH	Target: STOCK
Bidder	CASH	O^{Bc}_{LSM} , O^{Tc}_{LSM}	0 , 0
	STOCK	0 , 0	O^{Bs}_{LSM} , O^{Ts}_{LSM}

but in some cases both bidder's and target's stock merger options are greater than respective cash merger options, and in the remainder of cases target's stock option is greater than target's cash options, whereas bidder's stock option is smaller than bidder's cash option.

Generally, when both stock and cash merger options are available to the players, the payoff matrix of the game looks like the one presented in Table 6. Each player has two pure strategies: play $CASH$ or play $STOCK$; if players' strategies do not match, then payoffs to both players are zero.

It is easy to see that there are two Nash equilibria in this game: play $CASH, CASH$ and play $STOCK, STOCK$. Depending on the relative size of payoffs, one needs to distinguish between two following situations in order to formulate rules of equilibrium selection:

1. $O^{Bs}_{LSM} \geq O^{Bc}_{LSM}$ and $O^{Ts}_{LSM} > O^{Tc}_{LSM}$; see, for example, row 2 of Table 9. It means that Nash equilibrium $STOCK, STOCK$ is both payoff and risk dominant over Nash equilibrium

Table 7: Regression analysis

	Probit	Logit
constant	-10.24256**	-16.82325**
	(-7.97)	(-7.92)
$MARKET_{LSM}$	-11.72748**	-21.26603**
	(-13.18)	(-11.99)
r	76.8797**	138.9899**
	(8.23)	(7.91)
δ_X	-111.3395**	-198.6983**
	(-4.00)	(-3.97)
σ_X	24.03141**	42.40887**
	(7.54)	(7.29)
σ_Y	-6.76513*	-11.16768*
	(-2.38)	(-2.23)
ρ	-3.874487**	-7.021367**
	(-13.32)	(-12.12)
Q	.0093608	-
	(1.70)	
X_0	16.77087**	30.33026**
	(12.85)	(11.78)
Pseudo R^2	0.6747	0.6726
LR χ^2	755.82	753.5
p-value of LR	0.0000	0.0000
Number of observations	1000	1000
Goodness-of-fit Pearson test	OK	OK

Only last specification shown; z-score values in parentheses; 5% and 1% significance levels denoted by * and ** respectively.

$CASH, CASH$; thus, rational players should both agree on playing $STOCK, STOCK$.

2. $O^{Bs}_{LSM} < O^{Bc}_{LSM}$ and $O^{Ts}_{LSM} > O^{Tc}_{LSM}$; see, for example, row 1 of Table 9. It means that neither of equilibria is payoff dominant; thus, Nash equilibria $CASH, CASH$ and $STOCK, STOCK$ are played with the same probability.

Having established equilibrium selection rules, I proceed to regression analysis using the simulated sample.

Table 7 demonstrates that $MARKET_{LSM}$ has negative effect on probability of a cash merger in both probit and logit models: coefficient on $MARKET_{LSM}$ is negative and statistically significant at the 1% significance level.

Thus, regression analysis in Table 7 shows that in this setup, cash mergers should be observed at low market valuations, and stock mergers should be observed at high market valuations agreeing with empirical evidence on dominance of stock mergers at high market valuations.

4 Dynamics of the intra-industry mergers

This section is based on the results for stock mergers obtained in Lambrecht (2004) and uses the same definitions as Section 2, in particular:
option value to firm 1 equals:

$$OM_1(p_t) = \left(c\,(p^*)^\gamma \left(s_1\,(K_1+K_2)^\theta - K_1^\theta\right) - M_1\right)\left(\frac{p_t}{p^*}\right)^{\beta_2} \tag{49}$$

option value to firm 2 equals:

$$OM_2(p_t) = \left(c\,(p^*)^\gamma \left((1-s_1)\,(K_1+K_2)^\theta - K_2^\theta\right) - M_2\right)\left(\frac{p_t}{p^*}\right)^{\beta_2} \tag{50}$$

globally optimal threshold p^* is:

$$p^* = \left(\frac{\beta_2}{\beta_2-\gamma}\frac{M_1+M_2}{c\left((K_1+K_2)^\theta - K_1^\theta - K_2^\theta\right)}\right)^{\frac{1}{\gamma}} \tag{51}$$

share of firm 1 equals:

$$s_1 = \frac{M_1\left((K_1+K_2)^\theta - K_2^\theta\right) + M_2K_1^\theta}{(M_1+M_2)\,(K_1+K_2)^\theta}. \tag{52}$$

Consider an industry consisting of three firms that differ in capital stock (K_1, K_2 and K_3) and merger costs (M_1, M_2 and M_3). Assume that only two firms can merge at a time, but later this combined entity may merge with the third firm. The questions are: What is the optimal order in which firms should merge? How does it change with changes in initial capital allocation? How does market valuation influence merger process in the industry?

Without loss of generality, assume that in the first step firm 1 merges with firm 2 creating firm 12; in the second step, the merged firm 12 merges with firm 3.

Solving backwards, one needs first to determine the terms and timing of the merger between firm 12 and firm 3; using the formulas above yields:

$$p_2 = \left(\frac{\beta_2}{\beta_2-\gamma}\frac{M_1+M_2+M_3}{c\left((K_1+K_2+K_3)^\theta - (K_1+K_2)^\theta - K_3^\theta\right)}\right)^{\frac{1}{\gamma}} \tag{53}$$

for the optimal timing of merger p_2;

$$\begin{aligned} OM_{12}(p_t) \; &= \left(cp_2^\gamma\left(s_{12}\,(K_1+K_2+K_3)^\theta - (K_1+K_2)^\theta\right) - \right. \\ &\left. -M_1 - M_2\right)\left(\tfrac{p_t}{p_2}\right)^{\beta_2} = Bp_t^{\beta_2} \end{aligned} \tag{54}$$

for the option value to firm[7] 12 OM_{12};

$$OM_3(p_t) = \left(cp_2^\gamma\left(\left(1-s_{12}\right)\left(K_1+K_2+K_3\right)^\theta - K_3^\theta\right) - M_3\right)\left(\frac{p_t}{p_2}\right)^{\beta_2} \tag{55}$$

for the option value to firm 3 OM_3;

$$s_{12} = \frac{\left(M_1+M_2\right)\left(\left(K_1+K_2+K_3\right)^\theta - K_3^\theta\right) + M_3\left(K_1+K_2\right)^\theta}{\left(M_1+M_2+M_3\right)\left(K_1+K_2+K_3\right)^\theta} \tag{56}$$

for the share of firm 12 in the new entity s_{12}.

Now we are back to the first stage: firm 1 is merging with firm 2 to create a new firm 12. The benefit from merging is twofold: first, participating firms share synergy stemming directly from the merger; second, they acquire the opportunity to merge with the firm 3 later to get even more benefits from this new merger.

The option to merge of firm 1 OM_1 reflects this twofold benefit and satisfies the following value matching and smooth pasting conditions:

$$\begin{aligned} OM_1(p_1) &= cp_1^\gamma\left(s_1\left(K_1+K_2\right)^\theta - K_1^\theta\right) - M_1 + s_1 OM_{12} = \\ &= cp_1^\gamma\left(s_1\left(K_1+K_2\right)^\theta - K_1^\theta\right) - M_1 + s_1 B p_1^{\beta_2} \end{aligned} \tag{57}$$

$$OM_1'(p_1) = c\gamma p_1^{\gamma-1}\left(s_1\left(K_1+K_2\right)^\theta - K_1^\theta\right) + s_1 B\beta_2 p_1^{\beta_2-1} \tag{58}$$

where p_1 is the merger threshold for the merger between firm 1 and firm 2;
s_1 is the share of the new merged entity accruing to firm 1.

Applying the same logic as in Section 2.1 yields that the option to firm 1 is of the form $OM_1 = Ap_t^{\beta_2}$. Writing down corresponding conditions for firm 2 and solving for OM_1 and for OM_2 yields:

$$OM_1(p_t) = \left(cp_1^\gamma\left(s_1\left(K_1+K_2\right)^\theta - K_1^\theta\right) - M_1 + s_1 B p_1^{\beta_2}\right)\left(\frac{p_t}{p_1}\right)^{\beta_2} \tag{59}$$

$$\begin{aligned} OM_2(p_t) &= \left(cp_1^\gamma\left(\left(1-s_1\right)\left(K_1+K_2\right)^\theta - K_2^\theta\right) - M_2 + \right. \\ &\left. + \left(1-s_1\right) B p_1^{\beta_2}\right)\left(\tfrac{p_t}{p_1}\right)^{\beta_2} \end{aligned} \tag{60}$$

where the merger threshold p_1 equals:

$$p_1 = \left(\frac{\beta_2}{\beta_2-\gamma}\frac{M_1+M_2}{c\left(\left(K_1+K_2\right)^\theta - K_1^\theta - K_2^\theta\right)}\right)^{\frac{1}{\gamma}} \tag{61}$$

[7] $B = \frac{cp_2^\gamma\left(s_{12}\left(K_1+K_2+K_3\right)^\theta - \left(K_1+K_2\right)^\theta\right) - M_1 - M_2}{p_2^{\beta_2}}$

and the share of the new firm 12 accruing to firm 1 is:

$$s_1 = \frac{M_1 \left((K_1 + K_2)^\theta - K_2^\theta \right) + M_2 K_1^\theta}{(M_1 + M_2)(K_1 + K_2)^\theta}. \tag{62}$$

It is important to notice that neither merger threshold p_1, nor share of firm 1 s_1 change as compared to the baseline model without an option to merge with firm 3 later (compare p_1 to p^* in (51) and s_1 to s_1 in (52)). This means that the extension of the original model with an extra option does not drive away the equilibrium from being Pareto-optimal.

The strategy of the players now can be summarized as follows:

1. $p_1 < p_2$ means that both mergers occur at optimal thresholds:
   - Firm 1 merges with firm 2 at p_1 to establish a new firm 12;
   - Firm 12 merges with firm 3 at p_2.
2. $p_1 \geq p_2$ means that one of the mergers happens at sub-optimal threshold[8]:
   - Firm 1 merges with firm 2 at p_1 to establish a new firm 12;
   - Firm 12 merges with firm 3 at the same (sub-optimal in this stage) threshold p_1. Option value of this merger is computed based on (54) and (55) using p_1 rather than p_2 as it would be at the optimal threshold.

Option values to the firms are OM_1, OM_2 and OM_3 as in (59), (60) and (55) respectively.

Table 8 presents numerical examples for different initial capital allocations in the industry: equal firms with $K_1 = K_2 = K_3 = 33$ (Panel A, least concentrated industry), firms of comparable size with $K_1 = 20$, $K_2 = 30$, $K_3 = 50$ (Panel B), one large firm with $K_3 = 80$ and two small firms with $K_1 = K_2 = 10$ (Panel C, most concentrated industry).

The main conclusion drawn from Table 8 is that stock mergers in more concentrated industries (Panel C) occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries (Panel A). Table 8 also demonstrates that total value of options to merge OM is highest in Panel A and decreasing in industry concentration. Analysis in this Section should be extended to the industries with larger number of firms to obtain clearer picture.

[8] I assume here that this is the merger in the second stage that occurs at sub-optimal threshold, but it can also be vice versa.

Table 8: Intra-industry mergers

p_1	p_2	Firm 1	Firm 2	Firm 3	Total	Stage 1	Stage 2
Panel A (HHI=3333)							
		K_1	K_2	K_3	K		
		33	33	33	99		
		OM_1	OM_2	OM_3	OM		
1.928	1.947	17.131	17.131	8.500	42.763	1+2	12+3
1.928	1.947	17.131	8.500	17.131	42.763	1+3	13+2
1.928	1.947	8.500	17.131	17.131	42.763	2+3	23+1
Panel B (HHI=3800)							
		K_1	K_2	K_3	K		
		20	30	50	100		
		OM_1	OM_2	OM_3	OM		
2.053	1.817	10.229	15.945	14.280	40.454	1+2	12+3
2.150	2.009	8.805	7.370	23.935	40.109	1+3	13+2
1.947	2.356	3.872	13.219	22.887	39.978	2+3	23+1
Panel C (HHI=6600)							
		K_1	K_2	K_3	K		
		10	10	80	100		
		OM_1	OM_2	OM_3	OM		
2.286	2.356	3.961	3.961	15.487	23.409	1+2	12+3
3.186	3.319	2.046	1.158	19.450	22.654	1+3	13+2
3.186	3.319	1.158	2.046	19.450	22.654	2+3	23+1

HHI denotes Herfindahl-Hirschman index: $HHI = 10000\frac{K_1^2+K_2^2+K_3^2}{K^2}$.
K is total capital in the industry.
OM is the total value of options to merge.
Parameters are set as follows: merger costs $M_i = 0.05K_i$, $c = 1$, $\gamma = 1.4$, $\theta = 1.2$, $\mu = 0.03$, $\sigma = 0.2$, $r = 0.06$ implying $\beta_2 = 1.5$ so that the condition $\gamma < \beta_2$ holds, $p_0 = 1$.
Value $1 + 2$ in the column Stage 1 means that in the first stage firm 1 merges with firm 2 to create a new firm 12; analogously, value $12 + 3$ in the column Stage 2 means that in the second stage firm 12 merges with firm 3.

5 Conclusions

In this paper I have compared the terms and timing of cash vs. stock mergers for two different settings: in the first one by Lambrecht (2004), the synergy comes from increasing returns to scale and stochastic shock is the same for both bidder and target; the second one, with the synergy linear in pre-merger valuations of the firms, encompasses correlated stochastic processes for the firms and is based on Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008).

I have demonstrated that cash mergers should generally happen at low market valuation, and stock mergers may happen at both low and high market valuations; this conclusion conforms to existing empirical evidence. It partially supports prediction made by Shleifer and Vishny (2003) for the static model.

I have investigated the dynamics of the intra-industry mergers within the first setup. I solved for the optimal order of mergers inside an industry for different initial capital allocations to demonstrate that stock mergers in more concentrated industries occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries.

A Derivations for Section 3.2

Recall from (33) that payoffs to the bidder P_B^s and to the target P_T^s at the stock merger are:

$$P_B^s(X,Y) = \max\left(s_B V(X,Y) - KX, 0\right) \tag{63}$$

$$P_T^s(X,Y) = \max\left((1-s_B)V(X,Y) - QY, 0\right). \tag{64}$$

Then, the state variables for the bidder and the target for LSM would be:

$$\begin{aligned}
&S_B^s(X,Y) = s_B V(X,Y) - KX = \\
&= s_B\left(KX + QY + \alpha(K+Q)(X-Y)\right) - KX = \\
&= X\left(s_B\left(K + \alpha(K+Q)\right) - K\right) + Y s_B\left(Q - \alpha(K+Q)\right) = \\
&= Xk_1 + Yk_2
\end{aligned} \tag{65}$$

$$\begin{aligned}
&S_B^s(X,Y) = (1-s_B)V(X,Y) - QY = \\
&= (1-s_B)\left(KX + QY + \alpha(K+Q)(X-Y)\right) - QY = \\
&= X(1-s_B)\left(K + \alpha(K+Q)\right) + \\
&+ Y\left((1-s_B)\left(Q - \alpha(K+Q)\right) - Q\right) = \\
&= Xq_1 + Yq_2.
\end{aligned} \tag{66}$$

Solving for the share s_B such that $\frac{k_1}{k_2} = \frac{q_1}{q_2}$ yields:

$$s_B = \frac{K}{K+Q}. \tag{67}$$

Substituting $s_B = \frac{K}{K+Q}$ into the expressions for the state variables (65) and (66) yields:

$$S_B^s(X,Y) = \frac{(X-Y)(\alpha K(K+Q) - KQ)}{K+Q} = (X-Y)\, a_B \tag{68}$$

$$S_T^s(X,Y) = \frac{(X-Y)(\alpha Q(K+Q) + KQ)}{K+Q} = (X-Y)\, a_T \tag{69}$$

summing up to $S_B^s + S_T^s = (X-Y)\,\alpha(K+Q) = (X-Y)(a_T + a_B) = S_{CP}$.

Thus, at $s_B = \frac{K}{K+Q}$ all state variables (S_B^s, S_T^s and S_{CP}) depend on exactly the same stochastic process $X - Y$; it means that the same matrix $P_R = R(R'R)^{-1} R'$ will be used to compute fitted values in regressions based on either of these state variables[9].

It is clear that for $s_B = \frac{K}{K+Q}$ the payoffs to the players are as follows:

$$P_B^s(X,Y) = \max\left(\frac{(X-Y)(\alpha K(K+Q) - KQ)}{K+Q}, 0\right) \tag{70}$$

$$P_T^s(X,Y) = \max\left(\frac{(X-Y)(\alpha Q(K+Q) + KQ)}{K+Q}, 0\right) \tag{71}$$

$$P_{CP} = \max\left(\alpha(K+Q)(X-Y), 0\right). \tag{72}$$

Payoffs (70)-(72) have the same sign if the condition $\alpha(K+Q) > Q$ holds; they are all positive for $X > Y$ (which is the necessary condition for the synergy to be positive ans for the merger to be economically meaningful) and they are all negative for $X < Y$. Thus, positiveness of the payoff to the central planner P_{CP} implies positiveness of P_B^s and P_T^s given $\alpha(K+Q) > Q$. Besides, the ratios $\frac{P_B^s}{P_{CP}}$ and $\frac{P_T^s}{P_{CP}}$ are constant over time for $X > Y$ (and not defined otherwise).

This means that one can solve then stock merger problem from the point of view of central planner (instead of solving it for the bidder and the target) setting $s_B = \frac{K}{K+Q}$ and provided that condition $\alpha(K+Q) > Q$ holds.

It is not surprising that the same conclusion along with the same share s_B and the same necessary condition $\alpha(K+Q) > Q$ appear in the original infinite horizon model by Morellec and Zhdanov (2005).

Thus, one can choose $s_B = \frac{K}{K+Q}$ provided $\alpha(K+Q) > Q$ (that can be rewritten as $\frac{K}{K+Q} > 1-\alpha$) is satisfied; in this paper $\frac{K}{K+Q} = 0.8929 > 1 - \alpha = 0.6$.

B LSM sample simulation

[9] R denotes the matrix of regressors.

Table 9: LSM sample simulation: results

n	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
	O_{LSM}^{Bs}	O_{LSM}^{Ts}	total	O_{LSM}^{Bc}	O_{LSM}^{Tc}	total	cash	stock	cash	stock	cash	stock
1	11.43	5.24	16.67	12.55	1.04	13.59	1.31	1.49	0.27	1.26	1.79	1.94
2	20.19	9.47	29.66	16.30	2.77	19.07	2.72	3.52	0.30	0.86	1.44	1.66
3	19.40	6.29	25.69	8.40	1.43	9.83	2.46	3.83	0.19	0.55	1.23	1.42
4	20.39	7.27	27.66	12.84	2.59	15.42	2.50	3.26	0.31	0.66	1.32	1.49
5	6.13	2.41	8.53	5.44	0.75	6.19	0.75	0.96	0.41	1.30	1.67	2.01
6	4.34	3.64	7.98	3.86	0.80	4.66	1.02	1.39	0.21	0.85	1.41	1.69
7	6.23	3.22	9.45	3.13	1.04	4.17	1.33	1.75	0.19	0.38	1.19	1.29
8	10.46	6.46	16.93	12.31	2.08	14.39	1.94	2.17	0.45	1.36	1.83	2.08
9	12.95	8.00	20.95	15.93	1.96	17.89	1.93	2.18	0.36	1.60	1.91	2.27
10	9.32	7.69	17.00	9.74	2.81	12.55	1.92	2.32	0.33	0.87	1.50	1.70
11	12.36	8.64	21.00	15.60	1.97	17.56	2.02	2.29	0.31	1.45	1.84	2.16
12	12.39	8.59	20.99	10.18	1.58	11.76	2.42	3.33	0.17	0.80	1.39	1.65
13	16.54	9.84	26.38	23.17	1.74	24.91	3.13	3.17	0.32	1.72	2.36	2.36
14	8.91	2.96	11.87	4.35	1.98	6.33	1.45	2.09	0.39	0.34	1.22	1.25
15	12.88	3.55	16.44	13.28	0.63	13.91	1.20	1.35	0.28	1.54	1.95	2.11

Table 10: LSM sample simulation: input parameters

n	r	δ_X	δ_Y	σ_X	σ_X	ρ	K	Q	α	P	X_0	Y_0
1	0.065	0.0074	0.0338	0.193	0.167	-0.535	113.538	13.075	0.438	12.648	0.967	0.967
2	0.054	0.0089	0.0332	0.246	0.225	0.480	109.290	11.913	0.401	29.277	2.458	2.458
3	0.080	0.0024	0.0399	0.222	0.198	0.778	118.383	9.208	0.388	21.971	2.386	2.386
4	0.049	0.0013	0.0372	0.208	0.215	0.720	104.025	9.687	0.438	23.850	2.462	2.462
5	0.066	0.0023	0.0341	0.204	0.247	0.194	106.305	9.460	0.375	5.997	0.634	0.634
6	0.064	0.0035	0.0310	0.221	0.181	0.310	80.557	12.647	0.366	11.262	0.891	0.891
7	0.055	0.0029	0.0373	0.151	0.158	0.830	85.685	10.401	0.416	14.008	1.347	1.347
8	0.046	0.0093	0.0364	0.228	0.229	-0.134	76.818	9.852	0.376	15.509	1.574	1.574
9	0.041	0.0005	0.0307	0.243	0.187	-0.420	97.227	12.181	0.366	17.377	1.427	1.427
10	0.057	0.0059	0.0312	0.151	0.209	0.264	91.329	14.582	0.378	25.545	1.752	1.752
11	0.047	0.0016	0.0398	0.232	0.168	-0.409	89.365	12.701	0.380	19.117	1.505	1.505
12	0.069	0.0084	0.0350	0.227	0.158	0.244	99.823	13.139	0.351	27.377	2.084	2.084
13	0.055	0.0017	0.0302	0.250	0.165	-0.905	84.085	10.407	0.373	19.988	1.921	1.921
14	0.074	0.0050	0.0305	0.173	0.233	0.989	121.690	10.241	0.417	16.783	1.639	1.639
15	0.069	0.0100	0.0314	0.242	0.180	-0.586	122.009	9.224	0.448	7.510	0.814	0.814

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