Functional Dependencies in Vertical Industrial Structures and Readiness of Complementarity in Innovation*

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Abstract

We model investment under uncertainty in presence of complementarity with and without spillover effects. The associated functional dependence structure may create a low-level equilibrium trap. We derive efficient sharing (inter-firm transfer) arrangements that help resolving the trap.

Keywords: real options, industrial structure, vertical functional dependencies, innovation

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1 Introduction

The radical shift in recent years towards a new economy dominated by networks has challenged the *modus operandi* of many economic models both in academic and business world (see, e.g., Shapiro & Varian 1999, Shy 2001, amongst many books on the subject). One of the important traits of the networks is the complementarity of products that imposes functional dependency such that a product may not be worth much without complementors, or may not be worth anything without a platform. The functional dependence is not just unique to information technology and communication hardware and applications but is also prevalent in industrialisation (Murphy et al. 1989), in decisions to investment in human capital (Redding 1996), and is deeply rooted in economic development and growth (Aghion & Howitt 1996). Given the far-reaching consequences of network effects, it is surprising that complementarity has not yet been formally considered by the existing literature on investment under uncertainty. The objective of this paper is to provide a real options charaterisation of complementarity in investment and to analyse its implications for the pace of investment activity. Our formulation of complementarity in investment decisions provides a basis for understanding the low level equilibrium trap, which is found in many economic models, and develops tractable efficient sharing (inter-firm transfer) arrangements that help to resolve the impasse.

Underlying complementarity in products is complimentarity in investment decisions. Although complimentarity is often considered to be a desirable characteristic of products, it can be described as such only if the specific platform can be used by complementors to successfully develop their products. For producers of complementary products (unless, of course, they themselves constitute a platform for another layer of complementarity), complimentarity is essentially an additional impediment in investment uncertainty that must be overcome. This impediment is time-dependent; it may or may not affect the value of an investment opportunity.

A high degree of complementarity of products normally requires a new generation of technology to be introduced along the entire chain of linkages between complementary products otherwise the innovation may not be economically viable, it may even diminish the value of new technology. If complementary product owner invests independently of the state of platform technology, either the end users of the product will not be able to use it, or the producer will not be able to realise the full economic potential of the innovation. The fates of `Napster` and `iTunes` are obvious examples of impediment in complementarity in investment. Likewise, if platform owner invests independently of the state of complementary products it not only risks losing the network effect of its innovation but may even diminish its economic value.

The importance of complimentarity in investment is recognised by many high-tech companies that hold pre-release conferences, which "open up" the forthcoming products to the (trusted) potential producers of complementary products. This is best described in the words of Lars Rasmussen during the "Google Wave" demonstration at Google I/O well in advance of the actual product launch:

> "It is a little unusual for us to be showing [the product] this early. [...] We are doing this because [...] we are hoping we can persuade you [...] to start building cool things with [...] APIs while we are getting the product ready for launch [...] Because that way, when we do launch, our users and your users can enjoy both Google Wave and all the cool things that we hope you will build at the same time." (Google 2009)

Similar awareness of functional dependence in investment in platform and complementary was emphasized in the announcement Windows Phone 7 Series:

> "Of course, Microsoft's team still has plenty of time to either screw this up or have it screwed up for them. [...] Since Microsoft doesn't build phones,

> it's up to manufacturers such as HTC to develop phones which are also ahead of the curve. And then there's the application ecosystem [...] Microsoft needs to ensure the desktop software that partners with Windows Phone is as usable as Windows Phone itself (something Nokia and Sony Ericsson have never been able to do), and it needs to make sure developers want to build for its new platform. Even Google has struggled to do this for its Android phones." (Lanxon 2010)

The complimentarity in investment we consider in this paper draws on the literature on intertemporal optimisation and on real options. It commenced with a basic setup that treats an investment problem in isolation of future investment decisions or strategic interactions with other firms (Titman 1985, McDonald & Siegel 1986, Paddock et al. 1988, Ingersoll & Ross 1992). Subsequent literature considers investment as having sequential and/or compound nature (Myers 1977, Pindyck 1988, Dixit 1989, Smit 1996, Paxson 2007). Identification of various options implicit in investment projects (for example, options to abandon, to expand, to contract, to switch) entails the problem of aggregation in interactions of real options. A portfolio approach has been developed in an attempt to aggregate real options within and across available investment projects, and to single out strategies for sequential holding and extinguishment of options that would maximise shareholder value (see Brosch 2008, for a review).

Another of literature on real options focuses on strategic interactions of agents contemplating investment decisions in the same sector or geographical locale resulting in aggregate industrial organisation models (Smit & Ankum 1993, Dixit & Pindyck 1994, Grenadier 1996, Kulatilaka & Perotti 1998, Grenadier 2001, Weeds 2002). Further research has produced models of investment when the assumption of perfect information is relaxed (Grenadier 1999, Lambrecht & Perraudin 2003). The relationships between firms, however, do not necessarily have to bear only competitive flavour. Options embedded in joint ventures and partnerships have been considered by Kogut (1991) and Savva & Scholtes (2006), while Zavodov (2009*b*) has looked at the necessary conditions for the existence of the core in explicit and implicit cooperative arrangements that influence investment decisions.

Recently, Patel & Zavodov (2010) have considered the co-evolutionary nature of innovation and suggested that functional dependencies may explain the pace of innovative activity. In this paper, we build on the latter insight and devise a formal model that helps to understand the impact of functional dependence in vertical industrial structure on investment decisions. This model can then be applied not only to capital budgeting but also to modelling of aggregate dynamics of the innovation process. The solution is scalable in the sense that the analytic formulation of the model allows for its integration into non-cooperative and cooperative games models, as well as for more complex problems in network theory, and analysis of credit default contagion.

The rest of the paper is organised as follows. In section 2, the basic model for the n-layer vertical industrial structure without spillover effects is developed. In section 3, the model is extended for the industrial structure with spillover effects. Section 4 provides a way of influencing investment in functional dependence structure with efficient (in the cooperative game theory sense) inter-firm transfers. Section 5 concludes.

2 Model of functional dependence curse

Consider a vertical industrial structure with functional dependencies. It can either be a supply chain or a hierarchy of complementary products similar to the one described by Patel & Zavodov (2010).[1] There is a platform product and $(n-1)$ layers of complementary products such that the platform product can be consumed without any complementary

[1] In endogenous growth theory, such relationship is often referred to as fundamental and secondary innovation (Aghion & Howitt 1996).

products, whereas the benefit from the use of complementary products is derived only if they are consumed together with the platform and the intermediate layers of complementary products (connectors). Our objective is to value an investment opportunity held by the owner of a complementary product pertaining to any layer in the above-described structure. This investment opportunity may come in a form of upgrade, or a completely new product development.[2]

The layers that connect (and include) platform and complementors are indexed by $i = 1, \ldots, n$, where 1 denotes the platform or upstream production. We assume that once project i is operational its value is given by $\{S_i(t) : t \geqslant 0\}$. This operating phase value process varies stochastically with time, and follows a geometric Brownian motion of the form:

$$dS_i(t) / S_i(t) = \alpha_i dt + \sigma_i dz_i(t), \quad S_i(0) \equiv S_i > 0, \quad \text{for all } i, \tag{1}$$

where α_i is the expected growth rate per unit time set at the level below the risk free rate, $r > 0$, $\sigma_i > 0$ denotes a measure of volatility per unit time such that $\alpha > 0.5\sigma_i^2$, and dz_i is an increment of a Gauss-Wiener process.

Application of Itô's lemma allows us to move from a geommetric Brownian motion to an arithmetic counterpart:

$$d \ln S_i(t) = \left(\alpha_i - 0.5\sigma_i^2\right) dt + \sigma_i dz_i(t), \quad \text{for all } i. \tag{2}$$

The individual value processes at the operating stage are correlated such that the correlation pairs are given by $\rho_{i,j} dt = \mathbb{E}\left[dz_i(t)\, dz_j(t)\right]$. The entire correlation structure

[2] Alternatively, the model can be viewed as a supply chain, whereby the n-th layer product pertains to the downstream of the production process, while the platform is viewed as an upstream activity. For example, a subcontracting chain in the production of an automobile or an aircraft, or a construction of a building can pertain to such an upstream-downstream characterisation.

is given by an $n \times n$ variance-covariance matrix of the form:

$$\mathbf{\Omega} = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{1,2} & \dots & \sigma_1\sigma_n\rho_{1,n} \\ \sigma_1\sigma_2\rho_{1,2} & \sigma_2^2 & \dots & \sigma_2\sigma_n\rho_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_1\sigma_n\rho_{1,n} & \sigma_2\sigma_n\rho_{2,n} & \dots & \sigma_n^2 \end{pmatrix} \tag{3}$$

To obtain the value of the project at the operating stage, investor i has to incur irreversible investment costs in the amount K_i. Under the assumption of complete arbitrage-free markets, the use of risk-neutral valuation framework (Cox & Ross 1976, Harrison & Kreps 1979, Harrison & Pliska 1981) results in the following optimal stopping problem:

$$V_i = \sup_{\tau \in \mathcal{T}} \mathbb{E}^{\mathbb{Q}} \left[e^{-r\tau} \left(S_i(\tau) - K_i \right)_+ \right], \tag{4}$$

where τ is an stopping time from the set of stopping times $\mathcal{T}$, and $\mathbb{E}^{\mathbb{Q}}[\cdot]$ denotes the expectation operator under the unique risk-neutral measure.

For every firm i there is an optimal stopping time τ_i^* such that the problem in equation (4) is given by

$$\begin{aligned} V_i &= \left(S_{i,\mathrm{ND}}^* - K_i \right) \mathbb{E}^{\mathbb{Q}} \left[e^{-r\tau_i^*} \right] \\ &= \left(S_{i,\mathrm{ND}}^* - K_i \right) \int_0^\infty e^{-r\tau_i^*} \phi\left(\tau_i^*\right) d\tau_i^*, \end{aligned} \tag{5}$$

where $S_{i,\mathrm{ND}}^*$ is the optimal exercise trigger of firm i given the vertical structure under consideration, and $\phi(\tau_i^*)$ denotes the probability density function of the first passage time τ_i^* out of the continuation region.

Since for complementary product investment i to start generating revenue all platforms $0 \leqslant k < i$ have to complete their respective investment programmes, optimal exercise time can be formally defined as $\tau_i^* \equiv \inf\{t \geqq 0 : S_k(t) \geqslant S_k^* \text{ for all } 1 \leqslant k \leqslant i\}$.

In words, the optimal stopping time for the i-th layer investment occurs when the operating stage value process hits a certain exercise trigger for all layers $1 \leqslant k < i$, as well as the layer i itself.

Solution of the optimal stopping problem in equation (5) can be approached in two different ways. One approach is to transform the correlated one-dimensional processes in equation (2) into a single multi-dimensional process. The first passage time probability density function of mutlti-dimensional process out of the continuation region defined by the exercise thresholds of all layers $1 \leqslant k \leqslant i$ is then found using the standard techniques.[3] Maximisation over the exercise threshold yields the value of the option to wait to invest in the i-th layer. Unfortunately, this approach is computationally challenging since even for two-dimensional problems numerical procedure is required for calculating the option value once the probability density function is approximated.[4]

We propose an alternative approach that results in a closed-form solution (or simple numerical solution in the case with spillover effects). Our solution proceeds in two stages: first, we transform the stochastic differential equation (2) with correlated Gauss-Wiener increments into a stochastic equation with uncorrelated Gauss-Wiener increments, and, second, we solve the optimal stopping problem in equation (5).

The transformation requires the assumption that assets under consideration are distinct such that return on any asset i cannot be replicated by combining any assets that are not i (Merton 1973). Under this assumption $\mathbf{\Omega}$ can be Cholesky factorised as: $\mathbf{\Omega} = \mathbf{A}\mathbf{A}^{\mathrm{T}}$, where $\mathbf{A}$ is a lower triangular matrix. Following Ekvall (1996) we can multiply the n-dimensional process in equation (2) by the inverse of $\mathbf{A}$ (i.e., $\mathbf{A^{-1}}$) to obtain a system of

[3] For a review of the derivation procedure of the first-passage time probability density function for a simpler case of one-dimensional problems see, for example, Appendix B in Wilmott et al. (1993).

[4] The attempts at deriving a similar class of two-dimensional first-passage time probability density functions can be traced through the works of Buckholtz & Wasan (1979), Iyengar (1985), He et al. (1998), Zhou (2001).

uncorrelated processes such that

$$d \ln S_i(t) = \left(\alpha_i - 0.5\sigma_i^2\right) dt + v_i dw_i(t), \tag{6}$$

where $v_i \equiv 1/\left(\sum_{j=1}^{n} A_{i,j}\right)$, $A_{i,j}$ denotes the i-th row and j-th column of matrix $\mathbf{A^{-1}}$, and $\mathbb{E}\left[dw_i(t)\, dw_j(t)\right] = 0$ for all $i \neq j$.

The key ingredient to the solution of valuation problem in equation (5) is finding, which of the firms $k \leqslant i$ will be the last to invest as if there were no functional dependencies, since its investment time will determine the optimal investment time for firm i:

$$\begin{aligned} h &= \underset{0\leqslant k\leqslant i}{\arg\min} \int_0^\infty e^{-r\tau_k^*} \frac{\ln\left(S_k^*/S_k\right)}{v_k\sqrt{2\pi\left(\tau_k^*\right)^3}} e^{-\frac{\left[\ln\left(S_k^*/S_k\right)-\left(\alpha_k-0.5\sigma_k^2\right)\tau_k^*\right]^2}{2v_k^2\tau_k^*}} d\tau_k^* \\ &= \underset{0\leqslant k\leqslant i}{\arg\min} \left(\frac{S_k}{S_k^*}\right)^{\beta_k}, \end{aligned} \tag{7}$$

where:

$$\begin{aligned} \beta_k &= \frac{-\left(\alpha_k - 0.5\sigma_k^2\right) + \sqrt{\left(\alpha_k - 0.5\sigma_k^2\right)^2 + 2rv_k^2}}{v_k^2} > 1, \\ S_k^* &= \frac{\beta_k K_k}{\beta_k - 1}. \end{aligned}$$

The solution of the optimal stopping problem in equation (4) in the continuation region (i.e., where $\tau_i^* > 0$) can thus be obtained.

Proposition 1. *Under the above assumptions, the value of the option to wait to invest for firm i in the continuation region is given by*

$$V_i = \left(S_i e^{\left(\alpha_i - 0.5\sigma_i^2\right)\ln\left[S_h^*/S_h\right]/\left(\alpha_h - 0.5\sigma_h^2\right)} - K_i\right)\left(\frac{S_h}{S_h^*}\right)^{\beta_h}, \tag{8}$$

where h is given in equation (7),

$$\beta_h = \frac{-\left(\alpha_h - 0.5\sigma_h^2\right) + \sqrt{\left(\alpha_h - 0.5\sigma_h^2\right)^2 + 2rv_h^2}}{v_h^2} > 1, \tag{9}$$

$$S_h^* = \frac{\beta_h K_h}{(\beta_h - 1)}. \tag{10}$$

Proof. From the above calculations we have:

$$\begin{aligned} V_i &= \left(S_{i,\mathrm{ND}}^* - K_i\right)\left(\frac{S_h}{S_h^*}\right)^{\beta_h} \\ &= \left(S_i e^{\left(\alpha_i - 0.5\sigma_i^2\right)\mathbb{E}\left[\tau_i^*\right]} - K_i\right)\left(\frac{S_h}{S_h^*}\right)^{\beta_h}. \end{aligned} \tag{11}$$

Since τ_i^* is the time when all layers $1 \leqslant k \leqslant i$ have embarked upon investment, then it is simply:

$$\begin{aligned} \mathbb{E}\left[\tau_i^*\right] &= \max_{0\leqslant k\leqslant i} \int_0^\infty \tau_k^* \frac{\ln\left(S_k^*/S_k\right)}{v_k\sqrt{2\pi\left(\tau_k^*\right)^3}} e^{-\frac{\left[\ln\left(S_k^*/S_k\right) - \left(\alpha_k - 0.5\sigma_k^2\right)\tau_k^*\right]^2}{2v_k^2\tau_k^*}} d\tau_k^* \\ &= \max_{0\leqslant k\leqslant i} \ln\left[S_k^*/S_k\right] / \left(\alpha_k - 0.5\sigma_k^2\right) \\ &= \ln\left[S_h^*/S_h\right] / \left(\alpha_h - 0.5\sigma_h^2\right). \end{aligned} \tag{12}$$

Substituting equation (12) into equation (11) we obtain in Proposition 1. ***Q.E.D.***

It is worth noting the requirement that $\beta_h > 1$, since otherwise the solution will not have economic sense. One of the limits of Cholesky factorisation employed is that this condition may not necessarily hold, which somewhat limits the applicability of the solution methodology.

Example 1. *Consider a vertical industrial structure comprised of two firms (or, depending on the context, agents). Suppose platform owner holds an investment opportunity that costs 100 to set up, the current value of the operating project is 100, expected annualised* *growth rate of current value is 4%, and volatility of 20%. The second firm holds an opportunity to produce a complementary product that costs 50 to set up, the expected growth rate of current value is 4%, and volatility of 20%. Risk-free rate of interest is given at 5%, and correlation between changes in the two revenue processes is* 0.

Figure 1 demonstrates the sensitivity of the complementary project value to changes in the current value of the operating project. The dashed line pertains to the value of complementary product owner's option to wait to invest without functional dependancies, whereas the solid line indicates the value of the option to wait to invest given the requirement that platform owner has to invest first. The difference between the two lines shows the loss to the complementary product owner as a result of functional dependence on the platform. The value of this loss represents the accumulated loss of revenue from the point of optimal exercise by the complementary product to the time, when the market is ready to acquire it, i.e., when platform owner has invested. Given the cost structure of the complementary product owner, at certain values of current value of the operating project its expected time to investment will be less than that of the platform owner, and the above-mentioned loss will result.

Proposition 2. *Vertical functional dependence without spillover effects does not increase the value of investment opportunities. Furthermore, the probability of investment by i-th complementary product owner is non-increasing in the number of layers that separate it from platform 1.*

Proof. The first part of proposition follows from the observation that $V_i^C = V_i^0$, where V_i^0 denotes the value of the project as if there were no functional dependencies, only if $h = i$. The second part of proposition follows from Patel & Zavodov (2010). ***Q.E.D.***

The main implication of Proposition 2 is that as vertical industrial structure becomes more complex, at some point investment may vanish and, as a result, the number of

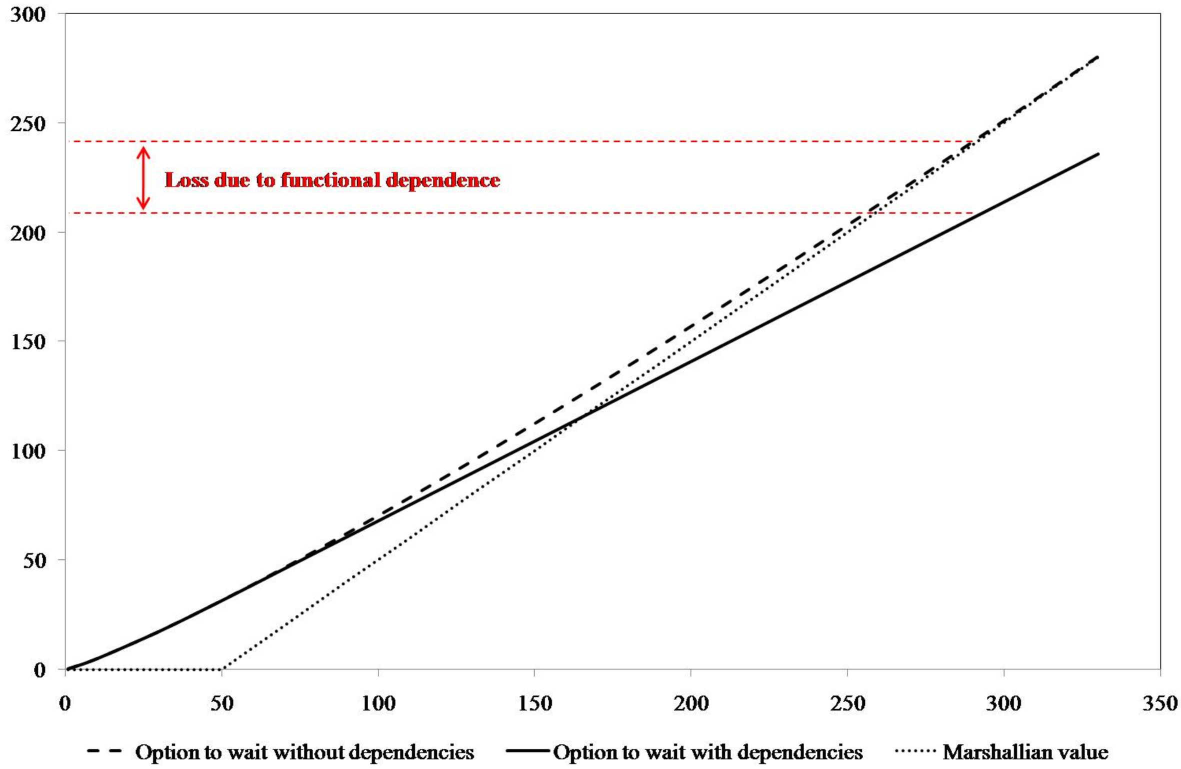

Figure 1: Effects of functional dependence on complementor's value

complementary products may be bounded from above by the costs of technological advancement of platform products. In fact, in such an industrial structure it may be better to have a single firm that internalises functional dependencies rather than an array of specialised producers of complementary products. Innovation process in complementary products may still be led by independent firms but as soon as other complementary products develop on top of them, it may in the interest of the society for the platform to acquire the next-in-the-order complementary product. Such a situation may be referred to as functional dependence curse.

3 Model of a functional dependence opportunity

Suppose now that there are spillover effects from functional dependence such that whenever a complementary product becomes available platform receive an additional revenue from sales that increases its value. This increase in value, λ_i, is proportional to the no-dependence value: $\lambda_i S_i(t)$. As a result, depending where in the industrial structure the product is, its value at the exercise will be:

$$S^*_{i,\mathrm{D}} + \sum_{i<j\leqslant n} \lambda_j e^{-(r-\alpha_i)\left(\tau^*_{j,\mathrm{D}}-\tau^*_{i,\mathrm{D}}\right)} S^*_{i,\mathrm{D}} = S^*_{i,\mathrm{D}}\left(1+\sum_{i<j\leqslant n} \lambda_j e^{-(r-\alpha_i)\left(\tau^*_{j,\mathrm{D}}-\tau^*_{i,\mathrm{D}}\right)}\right), \quad \text{for all } i, \tag{13}$$

where the discount factor is applied to the time until the next complementary product becomes available.

We can thus adjust the exercise trigger obtained in the previous section for the spillover effect:

$$S^*_{i,\mathrm{D}} = \frac{\beta_i K_i}{(\beta_i-1)\left(1+\sum_{i<j\leqslant n} \lambda_j e^{-(r-\alpha_i)\left(\tau^*_{j,\mathrm{D}}-\tau^*_{i,\mathrm{D}}\right)}\right)}, \quad \text{for all } i, \tag{14}$$

where $\tau^*_{i,\mathrm{D}}$ and $\tau^*_{j,\mathrm{D}}$ are optimal investment dates of platform product and its complementors, respectively, given backward dependencies (spillover effects).

Following the approach developed in the previous section, these are defined as $\tau^*_{i,\mathrm{D}} \equiv \inf\left\{t \geqq 0 : S_k(t) \geqslant S^*_{k,\mathrm{D}} \text{ for all } 0 \leqslant k \leqslant i\right\}$. Following a similar line of logic used to establish Proposition 1, we can find the value of the option to wait to invest given vertical functional dependencies and spillover effects from complementors.

Proposition 3. *Under the above assumptions, the value of the option to wait to invest for firm i with spillover effects in the continuation region is given by*

$$V^C_{i,D} = \left[S_i e^{\left(\alpha_i-0.5v_i^2\right)\mathbb{E}\left[\tau^*_{i,D}\right]}\left(1+\sum_{i<j\leqslant n} \lambda_j e^{-(r-\alpha_i)\left(\mathbb{E}\left[\tau^*_{j,D}\right]-\mathbb{E}\left[\tau^*_{i,D}\right]\right)}\right) - K_i\right]\left(\frac{S_{h_D,D}}{S^*_{h_D,D}}\right)^{\beta_h}, \quad \textit{for all } i, \tag{15}$$

where:

$$h_D = \underset{0 \leqslant k \leqslant i}{\arg\min} \left(\frac{S_k}{S^*_{k,D}} \right)^{\beta_k}. \tag{16}$$

Proof. The result follow from application of equation (14) to Proposition 1. ***Q.E.D.***

It is worth pointing out that Proposition 3 does not provide an explicit solution but can be used for rapid numerical evaluation. An example below is indicative of the implications that spillover effects have on the value of investment opportunity.

Example 2. *Maintaining the assumptions of Example 1, consider a vertical industrial structure with (demand) spillovers from complementary product investment to platform. The magnitude of this spillover effect is* $\lambda = 0.5$.

As with the previous example, checking the sensitivity of project value to changes in the current value of operating project is instructive. We observe that spillover from complementor's investment to platform increases the value of the latter (Figure 2). The intuition for this observation is straightforward: probability of gaining additional (non-zero) value increase the expected present value of a project. A more interesting relation is observed in Figure 3, whereby the spillover above-described spillover effect also increases the value of complementor relative to the no-spillover-effect scenario. This happens because spillover effect increases the probability of investment by platform, which may reduce the time-to-investment for complementor. If this happens, discount due to functional dependence that we observed in Figure 1 is reduced. Thus, although spillover effect (in our two-firm model) will not increase the value of complementor beyond the no-functional-dependence value, it may bring it relatively closer to this benchmark. Formally, this results in the following proposition.

Proposition 4. *Probability of investment of i-th firm increases in the magnitude of spillover effects such that it may exceed the probability of investment in no functional*

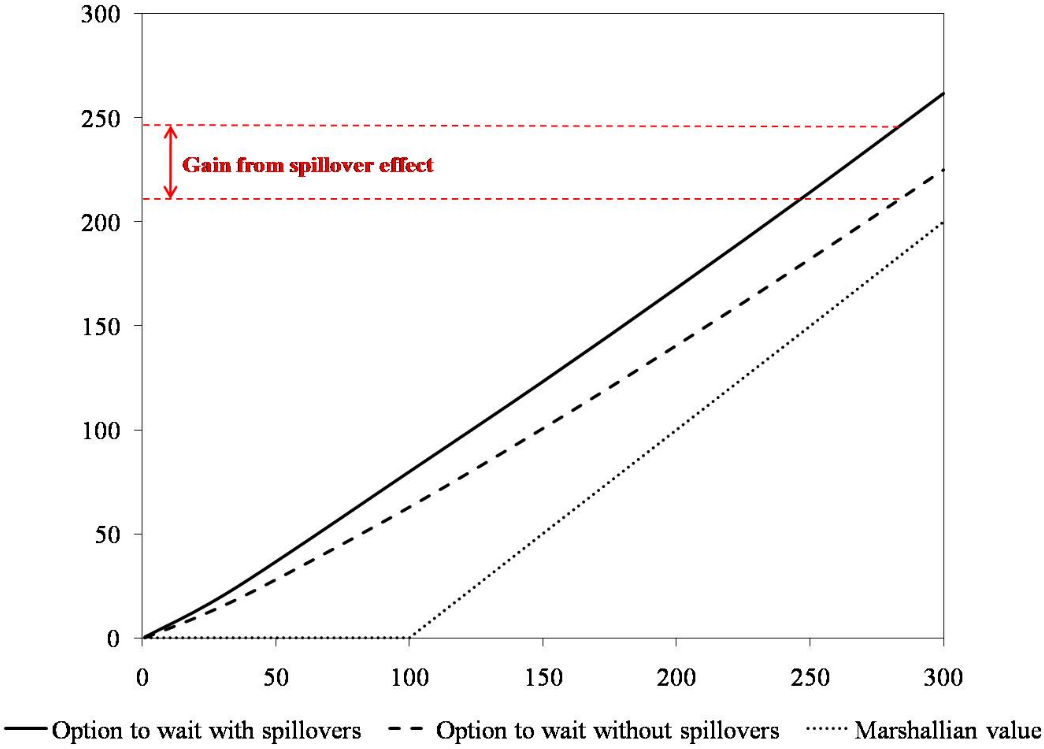

Figure 2: Effect of demand spillovers on platform's value

dependence scenario for all $1 \leqslant i < n$*, whenever optimal exercise time is finite for all* $1 \leqslant j \leqslant n$*. While probability of investment of* n*-th firm increases in the magnitude of spillover effects to the point where it equals the probability of investment in no functional dependence scenario, whenever optimal exercise time is finite for all* $1 \leqslant j \leqslant n$*.*

Proof. Note that probability of investment increases as exercise trigger falls. Since $\partial S^*_{i,D}/\partial \lambda_j < 0$, for all $1 \leqslant i < j \leqslant n$, the first part of Proposition 4 follows. The second part of the proposition is easy to arrive at by noticing that there are no complementors to layer n. ***Q.E.D.***

The above discourse suggests that rather than being a curse, functional dependence may become a virtue (at least for firms $1 \leqslant i < n$). But what if spillover effects are not sufficient to induce a platform or connector to extinguishing its option to wait to invest? This would require efficient sharing arrangements that enable a conversion of functional

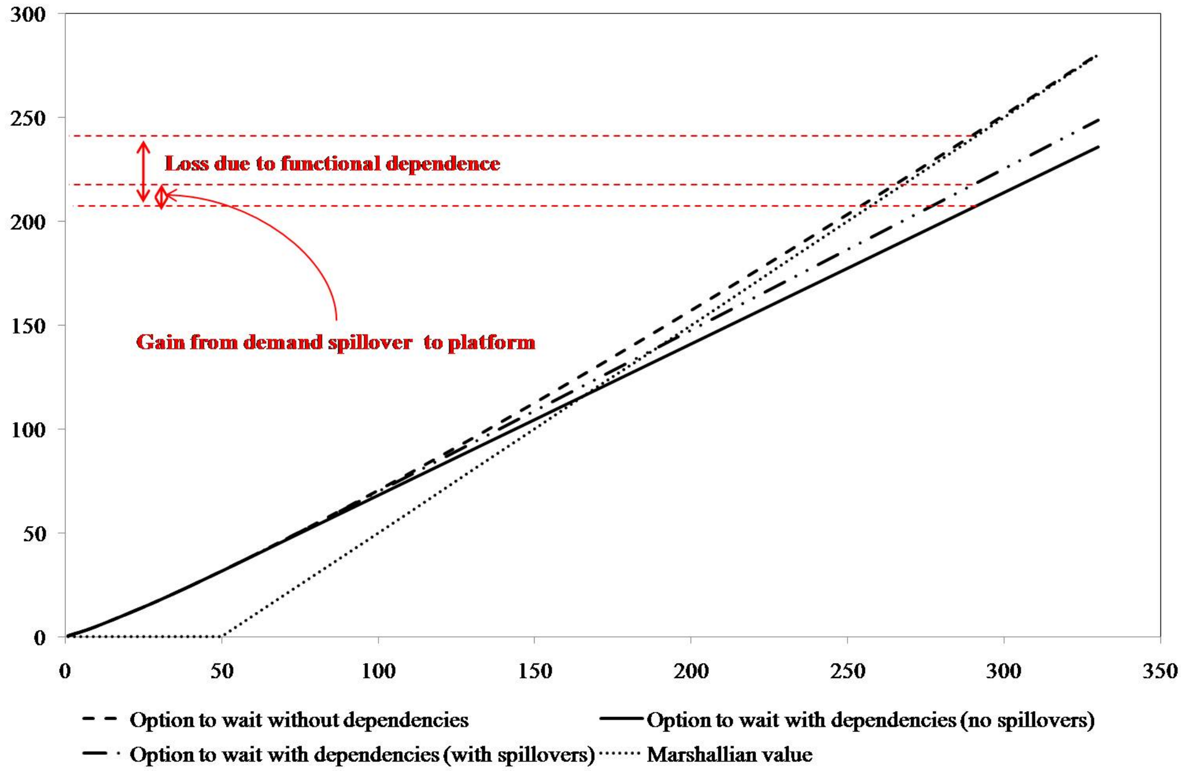

Figure 3: Effect of demand spillovers on complementor's value

dependence curse into an opportunity, and is the subject of the next section.

4 Using transfers to turn a curse into an opportunity

It is often the case in functional dependence structures that development of platform product is very costly, while complementary products can be introduced relatively quickly and inexpensively. Would this stifle innovative activity, or, in the parlance of strategic complementarity literature, would the industry be locked in the low-level equilibrium trap? Not necessarily, because an efficient licensing fee, or another form of inter-firm transfer from upper layers of complementarity to the platform level can be developed.

Suppose the platform holds an out of exercise region investment project, while complementors hold projects that would all be in the exercise region, if they were not bounded by functional dependence structure. In fact, platform's revenue may be $S_1(t) = 0$ for all t, if it is a fundamental research institute. Then complementors (or following our example commercialisation firms) can induce the platform to investment by means of a monetary transfer. Although this transfer can take various forms following the literature commenced by Aghion & Tirole (1994), we will consider two types of claims: licensing fee with present value F, and a royalty, ϕ with present value $\phi S_j(t)$ for all $1 < j \leqslant n$. Clearly, the former is time-invariant, whereas the latter varies with the underlying complementor's process. The conditions that determine an efficient transfer that would induce the platform to an immediate investment pertain to the core of a cooperative option game as outlined by (Zavodov 2009*a*).

We start with a set of conditions that come from the deterministic game theory and include: *collective rationality*, whereby the value of the integrated functional dependence structure has to be maximised; *individual rationality*, whereby each players' payoff under cooperative scenario (i.e., with a transfer) has to be at least as large as under a non-cooperative scenario (i.e., without a transfer); and *Pareto efficiency*, whereby there should be no industrial structure payoff that is left undistributed. Since our game is set up in the stochastic environment, the *subgame consistency* condition in the sense of Yeung & Petrosyan (2004), which is a stochastic equivalent of the time consistency condition in dynamic games, has to hold. Finally, the real options setting imposes the last condition of *immediate exercise* that requires the payoff to be located outside continuation region.

Fulfilling the above conditions results in the following two proposition describing efficient transfers.

Proposition 5. *Under the above assumptions, efficient licencing, F, is given by the* *following set of inequalities:*

$$F \geqslant \left[K_1 - \frac{S_1(\beta_1 - 1)\left(1 + \sum_{1<j\leqslant n}\lambda_j\right)}{\beta_1}\right] / (n-1) > 0, \tag{17}$$

$$0 < F \leqslant \frac{S_i(\beta_i - 1)\left(1 + \sum_{i<j\leqslant n}\lambda_j\right)}{\beta_i} - K_i, \quad \textit{for all } 1 < i \leqslant n. \tag{18}$$

Proof. Since all of complementors' options are in the exercise region there exists a licensing fee, $F > 0$, that, if charged, would keep these options outside continuation region at current revenue process levels (i.e., $S_i = S^*_{i,D}$ for all $1 < i \leqslant n$). Equation (18) follows.

Since platform's option to wait to invest is in the continuation region, there exists a licensing fee, $F > 0$, that, if charged, would move this option into the exercise region at current revenue process level (i.e., $S_1 = S^*_{1,D}$). Equation (17) follows. ***Q.E.D.***

Efficient royalty can be found in a similar fashion.

Proposition 6. *Under the above assumptions, efficient royalty, ϕ, is given by the following set of inequalities:*

$$1 > \phi \geqslant \frac{\left[K_1 - \frac{S_1\left(1+\sum_{1<j\leqslant n}\lambda_j\right)(\beta_1-1)}{\beta_1}\right]}{\left[\sum_{1<i\leqslant n}\frac{S_i\left(1+\sum_{i<j\leqslant n}\lambda_j\right)(\beta_i-1)}{\beta_i}\right]} > 0, \tag{19}$$

$$0 < \phi \leqslant 1 - \frac{\beta_i K_i}{S_i\left(1 + \sum_{i<j\leqslant n}\lambda_j\right)(\beta_i - 1)} < 1, \quad \textit{for all } 1 < i \leqslant n. \tag{20}$$

Proof. Given the nature of royalty, the optimal stopping problem will involve additive revenue components with distinct log-normal diffusions. Applying separation argument of Olsen & Stensland (1992), this problem can be solved for the exercise region. Then, setting $S_i = S^*_{i,D}$ for all $1 \leqslant i \leqslant n$, equation (19) results.

Royalty reduces the revenue for complementor by $\phi S_i(t)$ for all $1 < i \leqslant n$. Solving for the maximum ϕ that maintains complementor's option in the exercise region yields equation (20). ***Q.E.D.***

The above described situation, where a non-zero monetary transfer is required to enable investment along the functional dependence structure, can be described as a low-level equilibrium trap. In this situation, although each party holds a valuable opportunity to invest neither of them does. The problem is resolved by a simple revenue sharing arrangement across the functional dependence structure that is orchestrated privately. However, it need not be so. If the structure is complex, it may be difficult to bring all parties together and reach a mutually acceptable agreement (especially, if they are operating in a competitive environment). Instead, the pivotal role may be played by the government in designing taxes and subsidies (that are effectively royalties) in a localised way proposed by Pennings (2000). Alternatively, a dominant player, who is not the platform, may lead the way. Given the excerpt from Lanxon (2010) quoted in the introductory section, Microsoft could solve its problem by designing monetary transfers for platforms and developers of applications.

5 Conclusion

In this paper, the valuation formula for an option to wait to invest given functional dependencies in n-layer industrial structure has been derived. We show that functional dependencies may constitute a curse as well as a virtue depending on presence and magnitude of spillover effects between complementors and platforms. When spillover effects are insufficient firms (or government) may design efficient transfers between layers of complementarity to move a given industry from low-level equilibrium trap, and stimulate economic growth.

The analytic characterisation of the problems allows for its extensions to a strategic (non-cooperative) interaction without any major difficulty. This may yield a more detailed analysis of results but given the simplicity of such an extension we intentionally leave it out from the present discourse. Further application of our results can be envisaged in the area of modelling credit default contagion, and joint probability of default in the first passage time framework. Again, it can be done in a straightforward way, and we do not address this issue specifically. Finally, our cooperative option game with functional dependence provides a simpler intuition and a richer analytical environment for strategic complementarity models with multiple (low- and high-level) equilibria.

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