TENDER OFFER FOR WIND ENERGY IN BRAZIL: AN OPTION-GAMES EXPLANATION

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(This is a work in progress; we expect to present more detailed analysis and results by the time the final paper has to be submitted)

ABSTRACT:

The first tender-offer for wind energy in Brazil was held in December 2009 and exposed wind entrepreneurs to unprecedented rules and a fierce price competition. Near 450 new projects initially applied to the 2009 first tender offer for wind energy, structured as a reverse auction with ceiling price at R$ 189/MWh; supply was over 3X demand and 71 projects won the bid, selling at prices in the R$ 131-153/MWh range.

Among the winners, there are newcomers and companies that already operate wind farms in Brazil, including some which have wind equipment manufacturers as shareholders. Among the losers, some large companies such as Iberdrola, which has a significant experience in the wind industry. Apart from aggressive newcomers which were clearly outliers in the bid, the two groups - winners and losers - probably enjoy asymmetries not only in terms of investment costs, but also in terms of their beliefs on how the market for wind energy will evolve in Brazil.

Is there a risk that less viable wind farms won the bid? This paper attempts to analyze this problem in the light of option-games theory and, more specifically, based on works such as Huisman (2001), and Pawlina&Kort (2002) for asymmetric duopolies. We conclude that discrepancy of beliefs regarding future wind energy prices in Brazil may have let lower capex/more profitable projects out of the bid. The risk of preemption of less profitable projects would have been lower if the government had made the perspectives for wind energy clearer. In addition, when firms are less informed of competitors' actual views regarding the future, assuming that their own views prevail among players, the risk of preemption is lower, favoring the entry of more viable firms, but energy prices to consumers tend to be higher.

KEY WORDS: option-games; real options, wind energy, Brazil, asymmetric duopoly

1) Introduction:

Demand for power in Brazil will increase by 52% in the next 10 years and this challenge could be met by investing in wind energy, that today accounts for 0,3% of the country's power capacity. Brazil's power capacity is already 84% based on renewable sources, hydropower basically, but this number will drop to 80% in 2017 (MME-PDEE 2009) if wind energy and other renewable sources are not successful in surpassing carbon-based thermal plants in terms of costs and reliability.

The first significant support scheme (PROINFA) for alternative sources in Brazil was set in 2004 and offered fixed tariffs for 20-years, subsidized capital and other benefits, but results have been sluggish. Later attempts of the Brazilian government to contract wind energy failed because of the unfair competition, in the same tender offer, with other less expensive sources, such as hydrocarbon-based thermal plants, biomass & small hydro plants. The new rules established in 2009 for wind energy abandoned the fixed price system and introduced the tendering system as an attempt to force prices down, but still keeping wind farms insulated from energy spot price volatility, as well as other protective rules that actually reduce – but far from eliminate – the risk related to wind behavior.

This stop&go nature of the Brazilian support scheme for wind energy reflects the country's focus on low tariffs and reliability of supply, especially after the blackouts and forced energy rationing in years 2000/2001. Therefore, wind energy – regarded as an expensive and less reliable alternative - has lacked clear signs of long term support and the targets for the implantation of wind farms have been low. This, alone, might explain the rush to participate in the first tender offer for wind energy, held in December 2009, as reflected by the near 450 projects originally enrolled in the process, representing 14 GW of new energy, while the sector believed only 1-3 GW would be actually demanded.

In this paper, however, we assume that at least a significant part of the energy sector in Brazil, already used to the country's tendering system for energy from other sources, believes that the wind tendering system and its current contracting rules will persist for a long time[1]. Uncertainty, then, is translated here in the unknown future behavior of prices in the next tendering processes, rather than on the chances that the support scheme will die [2].

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[1] this is a feasible assumption, but we acknowledge that some players, especially the ones that have experienced the unsuccessful tendering systems for wind energy in China, UK and other countries , may have been influenced by this potential risk. In those specific examples, the tendering system was replaced by more protective systems which might, in fact, cause players to postpone investments in Brazil, in hope for more favorable rules. On the other hand, players that believe that new auctions will not happen soon would be willing to anticipate investments.

[2] when a project wins a tender offer, there is no longer price uncertainty; the project will sell at the price that was committed at the auction, corrected by inflation, throughout the 20-year life of the contract. Certain price penalties apply when energy output surpass a -10%/+30% tolerance range, but this

We then analyze the problem from the perspective of a duopoly, composed of two groups of projects, drawn from those 449 that initially registered for the tender-offer. The first group has a higher investment cost (Capex), for one or more of the following reasons: lower capacity to get capital, less ability to negotiate fiscal incentives at state/municipal layers or lower equipment prices, or the project is in a site with milder winds (and therefore the expected capacity factor[3] is lower) or is far from transmission lines. On the other hand, this group does not believe wind energy prices will vary significantly in the future. The second group has a lower Capex, but thinks that wind energy prices in Brazil will face a higher volatility or might experience slightly better prices in future auctions. These two groups, although purely fictional, will help us figure out if the unclear picture regarding wind prices might have favored the entry of less profitable wind farms. In this model, the tendering system will be regarded as just a device that forces competitors to reveal at what prices they would invest.

This paper is organized as follows: section 2 presents a brief literature review. Section 3 shows data that might support the assumption of asymmetric investment costs and beliefs regarding prices. Section 4 details the model of an asymmetric duopoly, including the derivation of value functions and the theoretic thresholds that trigger investments, while Section 5 discusses results and Section 6 compares them to the outcome of the first tender offer for wind energy in Brazil.

2) Literature Review

While Real Options theory deals with the problem of a firm making an irreversible investment in the presence of uncertainty, it in general ignores the fact that competitors also hold options which, if exercised, may affect the value of the firm's real option. Prior attempts to include the competition effect within Real Option models include Kester (1984), which considered that the entry of competitors might shorten the tenor of the real option, and Trigeorgis (1991), which considered that the entry of new competitors caused jump-downs in the firm's stochastic process. In such models, the effect of competition was random, instead of rational.

Smit & Trigeorgis (2004) remind us that value creation has two underlying sources: the general attractiveness of the industry in which the firm operates – fairly captured by the NPV and Real Options methods -, and on the competitive advantage over rivals and on how pervasive competitive forces may erode returns. The threat of new entrants in the market or price rivalry are examples of circumstances under which a project's value may be eroded, a problem that had been thoroughly analyzed in the literature on investment under competition, of which Reinganum (1981) and Fudenberg & Tirole (1985) are classic examples, but not in the literature of investment under uncertainty.

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uncertainty was considered here as a firm-specific risk. Therefore, in this paper price uncertainty refers to the risk of prices dropping or growing in future auctions.

[3] capacity factor= energy output, as compared to installed capacity. Wind farms in Brazil present capacity factors within the 35-48% range, in general.

Smets (1993)[4] was the first to combine Real Options Theory and Game Theory, modeling the competition effect endogenously in a problem where the two firms were considering entering a new market, but Smets assumed that competitors were symmetrical, that is, had the same production costs. Dixit & Pindyck (1994, ch.8) use Smets' model in a more heuristic way and analyze the equilibrium in pure strategies. Huisman & Kort (2001, ch.7)[5] followed Smets' steps but for companies that were considering the option to expand and extended Dixit & Pindyck(1994, ch.8)'s model, adding the analysis of equilibrium in mixed strategies[6], considering negative externalities and a first mover's advantage. Dias (2005) extended Joaquim & Buttler (2000) model and analyzed a duopoly in the oil industry where firms have asymmetric production costs and reach a Cournot equilibrium. Huisman & Nielsen (2001, ch.8) analyze a duopoly that is asymmetric in terms of the investment costs and in the presence of positive and negative externalities[7]. Pawlina & Kort (2002), still considering only investment cost asymmetry, adopt a more thorough analysis of the equilibrium strategies. All these papers solve their problems analytically, while other authors, such as Smit & Ankum (1993), use discrete-time models.[8]

Our work extends the models of Huisman & Nielsen (2001, ch.8) and Pawlina & Kort (2002), but considering asymmetry both in terms of investment costs and in terms of the stochastic price process, as to mimic the different views adopted by competitors regarding the market for wind energy in Brazil. Kong & Kwok (2006) also analyzed two asymmetries – in the sunk cost and in revenues, but not in the stochastic process. Miltersen & Schwartz (2004) analyze a duopoly with asymmetric R&D stochastic costs, but better suited to new technologies which are subject to technical shocks and a Poisson threat of termination (due to unacceptable side effects of the drugs).

Several of the above mentioned papers assume firms are risk-neutral, for the sake of notation simplicity. Some papers also assume that one or both firms are newcomers. This work relaxes these assumptions in order to obtain more general conclusions.

In the option-games models reviewed, the basic issue is to identify what is the net result of the trade-off between the advantages of an early preemptive investment and the gains from keeping the flexibility to wait and invest in a more advantageous

[4] Smets first discussed this issue in a working paper as of 1991.

[5] based on a 1999 Discussion Paper nr. 9992, Tilburg University, The Netherlands, which is usually referred to in other papers.

[6] Huisman&Kort (2001, ch.4&7) used a method that had been priorly used by Fudenberg & Tirole (1985) in order to derive the probability of simultaneous exercise of the option to invest. This method was drawn from the literature on optimal stochastic control.

[7] a negative externality means that the entry of the competitor reduces the company's profit due to, for example, loss of market-share. A positive externality occurs, for example, when the entry brings in synergies (eg.: more investment helps to create a local industry for equipment and investment costs drop, as a consequence)

[8] we also refer the reader to the site of Prof. Marco Antônio Dias/PUC-Rio (www.puc-rio.br/**marco**.ind*)*, which contains valuable hints on real options and option-games.

market condition. Smit & Trigeorgis (2004) highlight that early commitment can be an important isolating mechanism that protects profitability when first-mover advantages are present, while the wait-and-see flexibility can be an important late-mover advantage. Early works try to solve this dilemma by figuring-out the optimal strategy of each player, reflected in finding the condition that triggers his action (to invest), based on his expectation of what will the best rational response of the other players. This is a classic optimal-stopping problem, as discussed in Dixit&Pindyck (1994) .The problem is then solved backwards, trying to find the equilibrium of the game, based on rational expectations[9]. The effect of a player's action is then considered as endogenous to the model, instead of an exogenous variable.

This makes sense in our specific problem, as intuitive as it is that more firms entering a market tends to depress profits/market-share, reduce the chances of renewal of the contracts, hamper negotiations for lower taxes and maintenance costs, and curb players' future investment options.

3) uncertainties and asymmetric competitive conditions in the Brazilian wind industry

A wind firm is characterized by huge capital expenditures and very low operating costs. Uncertainty arises basically from the intermittent nature of wind behavior, which is a firm specific risk that we consider here as symmetric among the players we are analyzing. Other risks refer to a possible change in support schemes, but in this paper we consider that the recently instated policy will be kept for a fairly long time – and so think our game players.

Upon winning a tender-offer, a wind farm enjoys a fixed guaranteed price for 20 years in Brazil. However, tender-offers are expected to happen yearly from now on and the decision to invest is therefore also subject to the uncertainty if it is better to invest now, fixing prices at the current market scenario, or later.

There are contradictory signals regarding the future of wind prices, in Brazil and worldwide. In countries such as Germany, prices have dropped significantly and steadily, from 18,34 cent €/kWh in 1991, to the 7,14 cent €/kWh applicable to year 2013[10]. Although this country adopts the feed-in system that defines fixed prices based on certain investment cost and other technical criteria, the German example just highlights the obvious assumption that countries will exercise their power to pressure for lower tariffs. In contrast, in another country that adopted the tender offer system, China, prices first dropped by 35-45% in response to the tender-offers instated in

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[9] Myerson (1999, p.1069) highlights that the assumption of perfect rationality is certainly imperfect as a description of real human behavior, based on experimental studies. However, in the long run and when stakes are high, we should expect people's behavior to more closely approximate the ideal of perfect rationality, than in laboratory experiments.

[10] source: DEWI seminar, August 2009, Rio de Janeiro, Brazil.

2002/2003, but the contracted wind farms had persistent losses (Costa, Casotti & Azevedo, 2009). Lema & Ruby (2007) state that Chinese entrepreneurs, the winners of the tender offers at the expense of more experienced foreign wind firms, may have expected that the fixed price would be reinstated eventually, in order to protect the viability of the farms – and this actually happened recently [11].

In Brazil, 79% of energy capacity is hydro-power, so supply is strongly connected to rains and the hydrological system. Demand has also been erratic, especially after the downturns caused by the 2008/2009 financial crisis. Short term prices have been very volatile, in the range of R$ 16,31-502,45/MWh in 2008/2009. The spot price, although not affecting already contracted wind farms, signals the country's appetite for contracting or not more expensive wind energy: when reservoirs are full, as they are now, price pressure will be strong and tender-offers for alternative sources might even be cancelled[12] . On the other hand, Brazil has been steadily contracting other more expensive sources of energy, in order to counter-balance the volatility of rains and secure supply. Expensive thermo energy has also been contracted at tender offers, with upward prices, but when it comes to wind energy, there is no clear policy for the amount to be contracted in the future, even less regarding ceiling prices. Therefore, asymmetry in beliefs regarding future wind prices in Brazil is a reality.

Another important asymmetry is investment costs – from now on referred to as Capex. If wind behavior is milder, more turbines are necessary to produce a specified amount of energy, for example. Another reason for Capex asymmetry also carries game theory characteristics: among the winners of the 2009 tender offer, several had equipment suppliers as one of the shareholders, or even negotiated special conditions with turbine producers interested in securing demand in order to establish a foothold or expand their plants in Brazil. Although the size of Capex asymmetry is also a question mark, we assume here that investors' size and the location of competitors' projects give a good hint if their Capex is higher or lower, so we will consider this specific information as complete, in our analysis.

4) The problem, modeled as an asymmetric duopoly

a. basic assumptions

Each of two groups (or firms, to keep it simple), holds an American perpetual option to invest in wind energy in Brazil. First, let's assume that the output of any firm that operates in this sector is sold at a price that follows an inverse price-demand

[11] source: Suzlon, presentation to investor, Sep 2009, available at: http://www.suzlon.com/pdf/investor_p/Suzlon_Energy_Limited_Investor_Presentation.pdf.

[12] the first tender offer held in Dec 2009 did happen, though, in spite of spot prices that were at the R$ 16,31 floor.

function D, deterministic. However, a stochastic shock, $\tilde{Y}_i$, alters this function D, so that Firm *i*'s payoff π_i is defined as:

$\pi_i = \tilde{Y}_i . D_{i\,Ni,Nj}$ (1) , so the payoff is endogenously defined and varies not only with the total production of market players but also with some economic variable that changes with time ($\tilde{Y}_i$), so that:

- $\tilde{Y}_i$ is assumed to follow a Geometric Brownian Motion, that is,

  $dY_i = \alpha_i Y_i dt + \sigma_i Y_i dz_i$ (2);

  as D is deterministic, π_i follows a stochastic process with the same parameters as those of the stochastic process of *Y;*
- $D_{i\,Ni,Nj}$ is a deterministic inverse demand function that applies to Firm *i*, which depends only on the condition of the two parties in the duopoly, described by subscripts *Ni* and *Nj*. *Ni* is zero when Firm *i* has not invested yet, and takes the value of 1 when investment has already occurred. The same rule applies to *Nj*. Therefore:

  D_{i00} -> neither firms have invested yet

  D_{i10} -> Firm *i* as Leader, and Firm *j* has not invested yet

  D_{i01} -> Firm *i* as the potential Follower, that is, Firm *j* has already invested but Firm *i* has not invested yet

  D_{i11} -> both firms have already invested

We assume that: $D_{i10} > D_{i11} > D_{i00} > D_{i01}$, as to mimic the situation in which the best profit is obtained when Firm *i* is the first to invest (a first-mover advantage); when the competitor invests, as well, Firm *i*'s profit drops[13], but it is still above the profit it used to obtain before having invested. Finally, if the competitor is the first-mover, this deteriorates the profit of Firm *i*, compared to the situation where neither firms have invested yet. Now let Firm *j* ´s payoff function be similar to that of Firm *i*, but with different stochastic parameters:

$$\pi_j = \tilde{Y}_j . D_{jN_i,N_j} \qquad dY_j = \alpha_j Y_j dt + \sigma_j Y_j dz_j \qquad (3)$$

Likewise, we assume $D_{j01} > D_{j11} > D_{j00} > D_{j10}$. For the sake of simplicity, we consider that both firms' new output is the same, let's say, 25MW[14]. We also assume

[13] we are, therefore, considering that negative externalities surpass the positive externalities of both firms investing (see Note 5). Although in Brazil a wind farm that is already operating receives a fixed price, when newcomers enter the market they at least jeopardize the renewal of the firm's contract at maturity, or force prices down upon renewal, or pressures land and maintenance costs.

[14] there are certain fiscal incentives in Brazil that foster the decision for small plants. Although investors might still produce any quantity, they would rather split that in several small firms in order to take advantage of fiscal benefits. We will still assume that competitors will produce the same small amount

that $D_{i10} = D_{j01}; D_{i00} = D_{j00}; D_{i01} = D_{j10}; D_{i11} = D_{j11}$. This is not a strong assumption for wind farms, which incur in very low operating costs and are basically different just in terms of their capital expenditures[15]. Therefore, wind farms of the same size have similar profit structures which are impacted by competitors' moves in similar ways.

Different parameters for the stochastic behaviors of *Yi* and *Yj* mean that Firm *j*'s expectations regarding the future behavior of wind energy prices are different from Firm *i*'s expectations. This is the first asymmetry in our problem. None of the firms knows where the competitor stands regarding this issue, so this is a game of incomplete information. Instead of solving it for a Nash-Bayesian equilibrium (HARSANYI, 1968), which would involve new assumptions about the probability of each player being in one or in the other expectations condition[16] and a myriad of alternative scenarios, in this paper we will just evaluate how a firm would behave, considering that it believes the competitor either: - thinks the same about the future; - has a different view of future wind prices behavior.

The second asymmetry refers to investment costs (Capex): Firm *i*'s investment, I_i, is higher than I_j ,that is, $I_i = \rho . I_j$ and $\rho > 1$. Table 1 summarizes the asymmetries in our problem.

Table 1: summary of each Firm's conditions:

	Firm *i*	Firm *j*
Capex (known to competitor)	$I_i = \rho . I_j$ and $\rho > 1$	I_j
Beliefs on future payoff (unknown to competitor	$\pi_i = \tilde{Y}_i . D_{N_i, N_j}$ $dY_i = \alpha_i Y_i dt + \sigma_i Y_i dz_i$ α_i ; σ_i	$\pi_j = \tilde{Y}_j . D_{N_i, N_j}$ $dY_j = \alpha_j Y_j dt + \sigma_j Y_j dz_j$ $\alpha_j > \alpha_i$; $\sigma_j > \sigma_i$

A more general – and realistic – model might consider that competitor's Capex is also unknown, making the game even more incomplete. However, this would make analysis less parsimonious and hamper getting the intuition regarding the effects of an uncertain price scenario. Here, we believe that firms might have a good guess of competitor´s Capex based on the knowledge of the size of shareholders, experience in

of energy, as if one 25MW plant is competing with another one to sell the same amount of energy. By doing this, we are neglecting that investors might reach a Cournot or Stackelberg equilibrium, defining the energy outputs that maximize their profits. Here, the reason why payoffs change endogenously is actually related to the potential loss of fiscal benefits, higher maintenance/replacement costs or fewer chances of having the contract renewed, when other players enter the market.

[15] another great difference is the wind behavior at the site, but this also translates in a different Capex, in order to produce the same amount of energy

[16] in this case, "Nature" chooses randomly, based on a given probabilistic rule, what is the competitor's characteristic, and the game becomes a game of imperfect information.

the field and location of their projects. Annex 1 shows the representation of the incomplete game and clarifies the branches we are considering in our analysis.

We also assume that current Y (Y_0) is low enough, so that immediate investment is not optimal for either players (when Y_0 is high enough to reach regions when both firms are tempted to invest, then mixed strategies equilibria may occur, but this possibility is being neglected here).

b. duopoly alternatives we considered:

In this work we will derive the value functions and the equilibria solutions for each of these two alternatives (Table 2):

Table 2: scenarios to be modeled

	Model 1 (right guess)	Model 2 (wrong guess)
Firm *i* believes Firm *j*	has a lower Capex but has different views on future prices	has a lower Capex and it thinks the same about the future
Firm *j* believes Firm *i* is	has a higher Capex but has different views on future prices	has a higher Capex and thinks the same about the future

In Model 1, firms make a right guess about its competitor's beliefs on future prices and make their entry decisions accordingly. In Model 2, both guess wrongly. Again, other combinations are possible, but we opted to restrict the analysis to these extreme scenarios, in hope that they will help us get a clearer intuition of the problem.

In addition, we will compare results with the situation in which both parties actually share the same beliefs about future prices but still have and investment cost asymmetry. This might happen if the Brazilian government had sent reliable signals about this market's future[17], equalizing beliefs and, therefore, the stochastic process of π. We refer the reader to Pawlina & Kort (2002), who analyze this problem thoroughly.

In each scenario, we will derive the value functions for the following alternatives:

- Firm *i* is the first to invest (Leader), what makes Firm *j* the Follower;
- Firm *j* is the first to invest (Leader), what makes Firm *i* the Follower;

Therefore, we will find the prices at which the players would have an incentive to preempt the market (preemptive equilibrium). We will neglect the case when both firms invest simultaneously or even the sequential equilibrium focusing only on

[17] an example of such a policy is the German support scheme for wind energy, which signals falling prices based on certain known criteria.

identifying under which conditions a less profitable firm may preempt, given asymmetric views of future market conditions.

As usual in dynamic games, the problem will be solved backwards, first deriving the value function of the Follower, which has seen the Leader enter the market; then we obtain the value function of the Leader, who acts based on what he believes will be the sequential rational decision of the Follower.

c. Model 1 – both guess right

c.1 Firm *i* is the Leader, Firm *j* is the Follower

Considering that Firm *i* invested, Firm *j* will invest when Y_j is sufficiently large, that is, when it exceeds a certain threshold Y_{jF}^* (threshold when Firm *j* enters the market as a Follower), which occurs at time $t = \tau_{jF}^* = \inf(t \mid Y(t) \geq Y_{jF}^*)$.

c.1.i. follower (Firm *j*)

We may use the dynamic programming Bellman equation for a firm that holds an option to invest - an optimal stopping problem - and which gets a certain revenue while the option is alive (Annex 2 details the derivation of this differential function). Another alternative is to derive the same function using the Contingent Claims or the integral methods, which yield a similar result[18].Then:

$$\frac{1}{2}\sigma_j Y_j^2 F_{j_{Y_jY_j}} + \alpha_j Y_j F_{j_{Y_j}} - \rho F + Y_j D_{j10} = 0$$ or, using a simpler notation:

$$\frac{1}{2}\sigma_j Y_j^2 F_j^{''} + \alpha_j Y_j F_j^{'} - \rho F + Y_j D_{j10} = 0 \quad [19]\,[20] \qquad (4)$$

where $F_j^{''}$ and $F_j^{'}$ are, respectively, the second and first derivatives of *F* in regards to Y_j, and *F* is the value of the option holder.

Using Ito's Lemma and eq.(3), we get to eq. (4), a non-homogeneous partial differential equation, which solution is detailed below (Annex 2 describes the procedure, as well as the value of β_j):

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[18] the Contingent Claims method assumes that the firm's output can be traded in financial markets, which is not exactly the case here. Annex 2 highlights the slight differences between the value functions obtained in the two methods. Dias (2005) shows that the Differential method yields the same results as the Integral method.

[19] please recall that $r + \pi = \mu = \alpha + \delta$, that is, the risk free rate plus the risk premium equals the discount rate (that can be obtained through the CAPM model, for example), which is turn equals the drift α plus the dividends δ.

[20] when we assume the firm is neutral to risk, the value function looks alike, but *ρ* is replaced by *r*.A firm is neutral to risk when the asset's risk is totally diversifiable, that is, when it has zero correlation with the market risk.

$$\begin{cases} F_j(Y_j) = AY_j^{\beta_j} + \dfrac{Y_j D_{j10}}{\rho - \alpha_j} \quad (5) & \text{, in the continuation region}^{21}\text{, that is, for } Y_j \le Y_{jF}^* \\ \text{or} & \\ F_j(Y_j) = \dfrac{Y_j D_{j11}}{\rho - \alpha_j} - I_j \quad (6) & \text{, when } Y_j \ge Y_{jF}^* \end{cases}$$

In the right-hand side of expression (5), the second term reflects the value of Firm *j* if it never invests, while the first term reflects the option to invest; it is intuitive that the option value is positive, so the constant *A* should be positive.

At the threshold Y_{jF}^*, the Value Matching Condition (VMC) and the Smooth Pasting Condition (SPC) apply (for a heuristic discussion of these two conditions, please refer to Dixit & Pindyck, 1994, ch.4, p.130-132).

VMC: eq. (5)= eq. (6) => $AY_{jF}^{*\beta_j} + \dfrac{Y_{jF}^* D_{j10}}{\rho - \alpha_j} = \dfrac{Y_{jF}^* D_{j11}}{\rho - \alpha_j} - I_j$ (7)

SPC: the derivatives of eq.(5) and (6) are equal => $\beta_j A Y_{jF}^{*\,\beta_j - 1} + \dfrac{D_{j10}}{\rho - \alpha_j} = \dfrac{D_{j11}}{\rho - \alpha_j}$ (8)

Then we have: $A = \left(\dfrac{D_{11} - D_{10}}{\rho - \alpha_j} \right) \dfrac{Y_{jF}^{*\,1-\beta_j}}{\beta_j}$ (9)

which, applied to eq.(7), gives: $\left(\dfrac{D_{j11} - D_{j10}}{\rho - \alpha_j} \right) \dfrac{Y_{jF}^{*\,1-\beta_j}}{\beta_j} Y_{jF}^{*\beta_j} + \dfrac{Y_{jF}^* D_{j10}}{\rho - \alpha_j} = \dfrac{Y_{jF}^* D_{j11}}{\rho - \alpha_j} - I_j$.

As a result, $Y_{jF}^* = \dfrac{\beta_j}{(\beta_j - 1)} \dfrac{(\rho - \alpha_j) I_j}{(D_{j11} - D_{j10})}$ (10)

Recall from Annex 2 that: $\beta_j = \dfrac{\frac{1}{2}\sigma_j^{\,2} - \alpha_j + \sqrt{[\alpha_j - \frac{1}{2}\sigma_j^{\,2}]^2 + 2\rho\sigma_j^{\,2}}}{\sigma_j^{\,2}}$ (11)

Expressions (5), (6), (9), (10) and (11) allow us to draw the value function of Firm *j* as Follower. In expressions (9) and (10), $(D_{j11} - D_{j10}) > 0$, reflecting our assumption that the Follower's payoff is always higher when it invests, when compared to the situation

[21] here, the continuation region is the region where waiting to invest is optimal, so it is still better to keep the option alive.

in which only the Leader has invested. So, constant *A* and the threshold Y^*_{jF} are positive, as expected.

c.1.ii. leader (Firm *i*)

The Leader's value function is similar to eq. (4), but with state variable Y_i.

$$\frac{1}{2}\sigma_i Y_i^2 F_i^{''} + \alpha_i Y_i F_i^{'} - \rho F + Y_i D_{i10} = 0.$$

Following procedures similar to those adopted in section 2.1.i, we find the value functions that apply to the Leader:

$$F_i(Y_i) = BY_i^{\theta_i} + \left[\frac{Y_i D_{i10}}{\rho - \alpha_i} - I_i\right] \quad (12)$$, in the continuation region, that is, for $Y_j \le Y^*_{jF}$,

or

$$F_i(Y_i) = \frac{Y_i D_{i11}}{\rho - \alpha_i} - I_i \quad (13)$$, when $Y_j \ge Y^*_{jF}$

Please note that the Leader holds no option here (he has already exercised his option to invest!). Actually, the value functions here express what happens to the Leader's value if his competitor, the Follower, exercises his option to invest, too. Therefore, the second term in the right-hand side of eq.(12) reflects the Leader's value if the Follower never invests, while the first term reflects the erosion in the Leader's value, given the risk that the Follower will invest. So, it is intuitive that constant *B* is negative and that the value function in the continuation region is concave.

We have already derived the expression for Y^*_{jF} in section d.1.i and we still need to find constant *B*. We can do that by using the VMC, that is, by equaling equations (12) and (13), for $Y_i = Y_j = Y^*_{jF}$. It follows that:

$$BY_{jF}^{*\theta_i} + \left[\frac{Y^*_{jF} D_{i10}}{\rho - \alpha_i} - I_i\right] = \frac{Y^*_{jF} D_{i11}}{\rho - \alpha_i} - I_i$$

$$B = \frac{D_{i11} - D_{i10}}{\rho - \alpha_i} Y_{jF}^{*1-\theta_i} \quad (14)$$

$$\theta_i = \frac{\frac{1}{2}\sigma_i^2 - \alpha_i + \sqrt{[\alpha_i - \frac{1}{2}\sigma_i^2]^2 + 2\rho\sigma_i^2}}{\sigma_i^2} \quad (15)$$

It is worth noting that the root θ_i reflects Firm *i*'s own views of future market prospects; it expects the opponent to join the market at Y^*_{jF}, but the speed at which *Y* will reach this threshold is ruled by Firm *i*'s stochastic process. In summary, equations (10), (12), (13), (14) and (15) allow us to draw the value function of the Leader.

The time when firm *i* will invest is a bit more complicated to device, though. Firm *i* has an incentive to become the Leader as soon as its value as Leader becomes higher than its value as Follower (which is detailed in section c.2.i), that is, when $Y = Y_{iP}$, which happens when $t = \inf(t \mid F_{iLeader} \geq F_{iFollower})$. However, Firm *i* does not need to invest at that point if there is no risk that the opponent will preempt soon, forcing Firm *i* to be the lower value Follower. If the opponent is not likely to preempt the market, Firm *i* can wait for a clearer market condition and waiting to invest is still valuable, that is, it can wait until $t = \inf(t \mid F_{jLeader} \geq F_{jFollower})$, which happens at $Y = Y_{jP}$. As a result, Firm *i* is actually prone to invest at $Y_{jP} - \xi$.

There is another subtlety, though: is it really worth waiting? When Firm *i* still holds the option to invest as the Leader, its value is defined by the following function:

$$F_i(Y_i) = MY_i^{\theta_i} + \frac{Y_i D_{i00}}{\rho - \alpha_i} \qquad (16)$$

, and it is only worth waiting if the waiting value exceeds the value of Firm *i* as Leader, which is defined by expressions (12) and (13). The optimal stopping time, here, can again be obtained by using the VMC and the SPC in equations (12) and (16), at the point at which $Y = Y_i^*$, the point at which waiting is no longer worthy:

VMC: $MY_i^{*\theta_i} + \dfrac{Y_i^* D_{i00}}{\rho - \alpha_i} = BY_i^{*\theta_i} + \left[\dfrac{Y_i^* D_{i10}}{\rho - \alpha_i} - I_i\right]$

SPC: $\theta_i MY_i^{*\theta_i - 1} + \dfrac{D_{i00}}{\rho - \alpha_i} = \theta_i BY_i^{*\theta_i - 1} + \dfrac{D_{i10}}{\rho - \alpha_i}$

As a result: $Y_i^* = \dfrac{\theta_i}{\theta_i - 1} \dfrac{(\rho - \alpha_i) I_i}{D_{i10} - D_{i00}}$ (17)

$$M = B + \frac{Y_i^{*(1-\theta_i)}}{\theta_i} \frac{D_{i10} - D_{i00}}{\rho - \alpha_i} \qquad (18)$$

It is interesting to see that the trigger to stop waiting, Y_i^*, is the same as in the monopoly case, when there is no threat of competition eroding the firm's value (please refer to Annex 3 for a brief discussion of the impact on the firm's value, in each case). Depending on the parameters of the problem, Y_i^* may be higher or lower than Y_{jP}. Then, Firm *i* would actually enter as Leader at:

a) $Y_{iL}^* = Y_i^*$, if $Y_i^* \prec Y_{jP}$; in this case, Firm *i*'s decision to invest occurs at the same point as if it were in a monopoly.

b) $Y_{iL}^{*} = Y_{jP} - \xi$, if $Y_i^{*} \geq Y_{jP}$

c.2 Firm *j* is the Leader, Firm *i* is the Follower

Following procedures similar to those adopted in section c.1, we find the value functions that apply to the Leader and the Follower in this case (sections c.2.i and c.2.ii).

c.2.i. follower (Firm *i*)

$$\frac{1}{2}\sigma_i Y_i^2 F_{i_{Y_iY_i}} + \alpha_i Y_i F_{i_{Y_i}} - \rho F + Y_i D_{i01} = 0$$ or, using a simpler notation:

$$\frac{1}{2}\sigma_i Y_i^2 F_i^{"} + \alpha_i Y_i F_i^{'} - \rho F + Y_i D_{i01} = 0$$

$$F_i(Y_i) = CY_i^{\varphi_i} + \frac{Y_i D_{i01}}{\rho - \alpha_i} \quad (19)$$

, in the continuation region, that is, for $Y_i \leq Y_{iF}^{*}$

or

$$F_i(Y_i) = \frac{Y_i D_{i11}}{\rho - \alpha_i} - I_i \quad (20)$$

, when $Y_i \geq Y_{iF}^{*}$

$$C = \left(\frac{D_{i11} - D_{i01}}{\rho - \alpha_i}\right) \frac{Y_{iF}^{*\,1-\varphi_i}}{\varphi_i} \quad (21)$$

$$Y_{iF}^{*} = \frac{\varphi_i}{(\varphi_i - 1)} \frac{(\rho - \alpha_i) I_i}{(D_{i11} - D_{i01})} \quad (22)$$

$$\varphi_i = \theta_i = \frac{\frac{1}{2}\sigma_i^{\,2} - \alpha_i + \sqrt{[\alpha_i - \frac{1}{2}\sigma_i^{\,2}]^2 + 2\rho\sigma_i^{\,2}}}{\sigma_i^{\,2}} \quad (15)$$

c.2.ii. leader (Firm *j*)

$$\frac{1}{2}\sigma_j Y_j^2 F_j^{"} + \alpha_j Y_j F_j^{'} - \rho F + Y_j D_{j01} = 0 .$$

$$F_j(Y_j) = EY_j^{\phi_j} + \left[\frac{Y_j D_{j01}}{\rho - \alpha_j} - I_j\right] \quad (23)$$

, in the continuation region, that is, for $Y_i \leq Y_{iF}^{*}$

, or

$$F_j(Y_j)=\frac{Y_j D_{j11}}{\rho-\alpha_j}-I_j \qquad (24) \qquad \text{, when } Y_i \geq Y_{iF}^*$$

$$E=\frac{D_{j11}-D_{j01}}{\rho-\alpha_j}Y_{iF}^{*1-\phi_j} \quad (25)$$

$$\phi_j=\beta_j=\frac{\frac{1}{2}\sigma_j^{\,2}-\alpha_j+\sqrt{[\alpha_j-\frac{1}{2}\sigma_j^{\,2}]^2+2\rho\sigma_j^{\,2}}}{\sigma_j^{\,2}} \qquad (11)$$

Again, we need to obtain the monopoly threshold for the Leader, Firm *j*. Following the same steps as in section c.1.ii, we get to:

$$Y_j^*=\frac{\beta_i}{\beta_i-1}\frac{\left(\rho-\alpha_j\right)I_j}{D_{j01}-D_{j00}} \qquad (26)$$

, while the value of Firm j , while it still holds the option to become the leader is:

$$F_j(Y_j)=NY_j^{\beta_j}+\frac{Y_j D_{j00}}{\rho-\alpha_j} \qquad (27)$$

, and: $N=E+\frac{Y_j^{*(1-\beta_j)}}{\beta_j}\frac{D_{j01}-D_{j00}}{\rho-\alpha_j}$ (28)

d. Model 2- both guess wrongly – the impact of misinformation

Procedures are similar to those adopted in section c, except that each player, not knowing what are the competitor's beliefs regarding future behavior of prices, assumes the competitor adopts the same stochastic process for prices as he does. This basically changes the expected preemption and follower thresholds of the opponent, thus affecting how much time the Leader believes he will able to reap the benefits of being the first-mover.

The Leader expects the opponent will enter at one of those expected thresholds, but the opponent will obviously make his own decisions based on his true stochastic process. The problem is thus solved separately for each firm, as if there was only one asymmetry, Capex, and each player is therefore surprised by an unexpected behavior of its opponent.

The leader entry thresholds can be obtained in ways similar to those presented in section c, Model 1, and sections d.1 and d.2 exemplify the changes in the Leader's value functions.

d.1. Firm *i* is the Leader, Firm *j* is the Follower

Firm i will assume the competitor enters the market as Follower at:

$$Y_{jF}^{*} = \frac{\theta_i}{(\theta_i - 1)} \frac{(\rho - \alpha_i) I_j}{(D_{j11} - D_{j10})} \quad (29)\ ; \qquad \theta_i = \frac{\frac{1}{2}\sigma_i^{\,2} - \alpha_i + \sqrt{[\alpha_i - \frac{1}{2}\sigma_i^{\,2}]^2 + 2\rho\sigma_i^{\,2}}}{\sigma_i^{\,2}}$$

Expressions (12), (13), (14), (15) and (29) are used to calculate the Leader's value function.

d.2. Firm *j* is the Leader, Firm *i* is the Follower

Likewise, the Follower's investment is expected to be triggered at:

$$Y_{iF}^{*} = \frac{\phi_j}{(\phi_j - 1)} \frac{(\rho - \alpha_j) I_i}{(D_{i11} - D_{i01})} \quad (30)\ ; \qquad \phi_j = \frac{\frac{1}{2}\sigma_j^{\,2} - \alpha_j + \sqrt{[\alpha_j - \frac{1}{2}\sigma_j^{\,2}]^2 + 2\rho\sigma_j^{\,2}}}{\sigma_j^{\,2}}$$

Expressions (11), (23), (24) , (25) and (30) define the value of the Leader.

5) Results

Using a certain specified turbine, a good site in Northeastern Brazil might yield a net capacity factor of 45%. If this project sold 25 MW at the auction, investments in a 56 MW plant will be necessary; other projects in regions with milder winds would have to invest in more turbines to produce the same amount of energy, which might explain part of the Capex asymmetry. We estimate the investment cost at USD 2360 per installed kW[22]. We got a rough estimate of the D_{11} parameters, in our models, by projecting the cash-flow of this hypothetical wind farm in Brazil and transforming the (NPV+Capex) of this 20-year project into a perpetuity with no growth[23]. For that steady yearly cash-flow, we assumed that the stochastic parameter Y would be 1, which would be equivalent to the price we used in our projections, R$ 153/MWh (the highest price contracted in the 2009 tender offer).

Finally, we just checked if followers' and leaders' thresholds, obtained in the model with just the Capex asymmetry (equivalent to Pawlina & Kort (2002) model), would fit in the range of prices of the 217 firms that actually bid at the 2009 auction – R$ 131-189/MWh (~USD 74-108/MWh), which are equivalent to Y in the range of 0,85-1,24. Other parameters, especially those related to the stochastic processes, are

[22] the first wind farms built in Brazil reached higher levels of Capex, near USD 2840, but prices are going down, nearer international levels, due to the slow but steady entry of new equipment suppliers and to the international financial crisis, which caused prices to drop by 18%.

[23] upon signature of the contract, price is fixed for 20 years. In order to consider this feature in the model, we considered an artificial PMT in the value functions of the Follower and the Leader: instead of using the real *D,* we used $\frac{D(\rho-\alpha)}{\rho}$. As a result, the impact of drift α is still accounted for when the firm has not invested yet, but it is neglected after the firm has fixed its selling price.

just arbitrarily chosen at initially low values and varied in order to get a sensitivity perspective of their effects on the entry decision.

Examples will illustrate the impact of certain slight changes on the investment decision of each firm. First, Figure 1 shows the value functions for Firm *i* (stated as HCpx, to remind us that it is the high capex firm) and Fim *j* (stated as LCpx), considering only the Capex asymmetry.

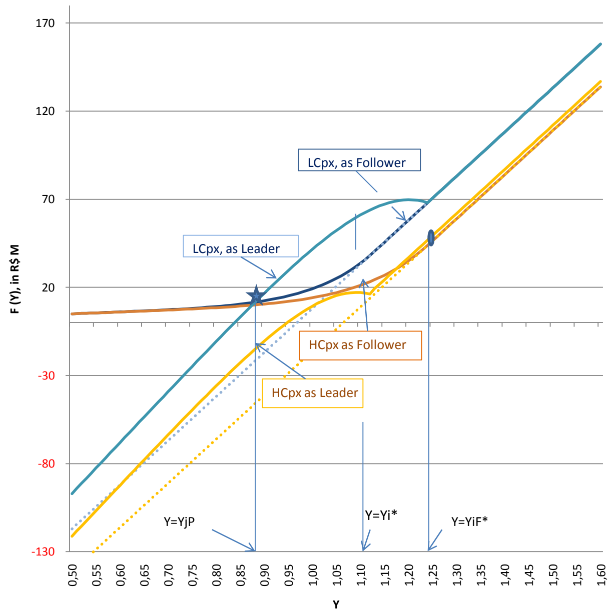

Figure 1: Capex asymmetry, only, for the set of parameters: $\alpha_i = \alpha_j = 0; \sigma_i = \sigma_j = 0.05; \rho = 0.10; D_{i11} = D_{j11} = 25; D_{j10} = 29; D_{i00} = D_{j00} = 2; D_{i01} = D_{j10} = 1.$ $I_i = 1.1 I_j = R\$266M$

The Leader is Firm j , the low Capex project, which invests at its trigger in the monopoly case $Y_j^* = 1.10$ (therefore, later than its preemption trigger, $Y_{jP} = 0{,}89$). As a result, Firm *j* invests at R$ 168/MWh [24]. The Follower, the high Capex Firm i , invests at its follower trigger, R$ 184/MWh. Varying the assumed set of parameters within a certain range, the high Capex Firm *i* investment trigger is always later than Firm *j* entry threshold, so the low Capex Firm *j* always enters first, in the example above.

5.1. Model 1

Figure 2 shows what happens when we introduce asymmetry in the firms' beliefs regarding the future of the sector, translated here as price uncertainty. Here, we assume that firms have a good feeling on how the competitor really stands in terms of its views about the future.

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[24] Although prices obtained from our models are purely fictional, we opted to present them just as an illustration.

If the high Capex Firm *i* thinks the market will be less volatile, it may also invest before the low Capex Firm *j* as depicted in Figure 2. It is noteworthy that, given the contradictory signals regarding future wind prices, such an asymmetry is feasible, not to say very probable.

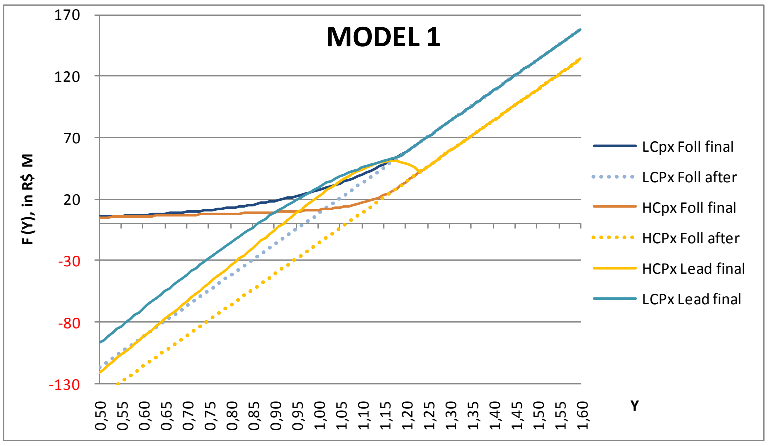


Figure 2: same set of parameters as in Figure 1, except for volatilities, which are now set at: $\sigma_i = 0,03; \sigma_j = 0,09.$ In this case, the high Capex Firm *i* preempts the market at Y=0,97 (R$ 148/MWh, or USD 85/MWh) and the low Capex Firm *j* follows at Y=1,23 (R$ 189/MWh, or USD 108/MWh).

Figure 3 shows another feasible situation: if the Capex asymmetry is lower (ex.: the high Capex firm has to invest 5% more than the low Capex firm) and the subjective payoff asymmetry and other parameters are kept fixed, preemption of the high Capex Firm *i* also happens, but it surprisingly occurs later, at Y=1,01 , or R$ 154/MWh, while the more profitable Firm *j* enters again only at *Y*=1,23, or R$ 189/MWh, its threshold as Follower. The expectation that Firm *i* will enter in the market soon, impairing Firm *j*'s benefits as first mover, makes the low Capex Firm *j* opt to invest as Follower; aware of that, the opponent Firm *i* , can wait a bit longer, investing only at its monopoly threshold. Firm *i*'s decision is detailed in Figure 4.

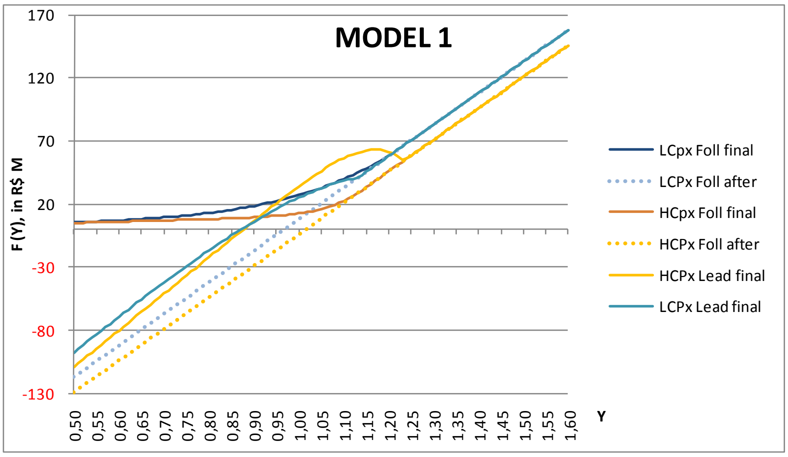


Figure 3: same set of parameters as in Figure 1, except for volatilities, which are now set at $\sigma_i = 0,03; \sigma_j = 0,09$. Capex asymmetry is also different: $I_i = 1.05 I_j$. In this case, the high Capex Firm *i* preempts the market at Y= 1,01, or R$ 154/MWh, while the low Capex Firm *j* follows at Y=1, 23, or R$ 189/MWh.

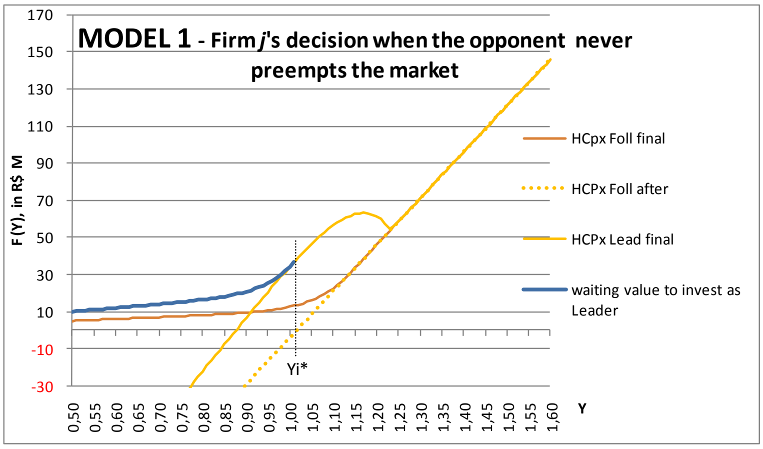


Figure 4: same set of parameters as in Figure 3, but detailing only the value curves of Firm *j*. When there is no threat of preemption, Firm *j* waits until the curve that represents the value of waiting touches the curve as leader (equivalent to the monopoly threshold).

We might also consider that the asymmetry regarding the views on future market prospects can be reflected in the drift of the stochastic process of Y. Let's assume that the example depicted in Figure 1 is now changed as to reflect that Firm *j* now expects that the payoff tends to increase by 1,5%, on average, in future auctions ($\alpha_j = 0{,}15$). Other parameters remaining equal, this slight change in perception is already enough to make the high Capex Firm i enter first in the market. This example is detailed in Figure 5.

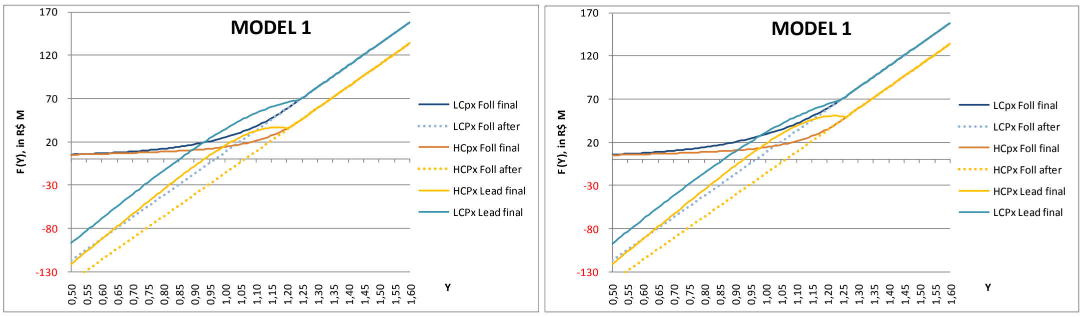


Figure 5: same set of parameters as in Figure 1, except for the introduction of asymmetry in the drift of the stochastic process of Y. In the left-hand side, Firm *j* assumes a $\alpha_j = 0{,}010$ and Firm *j* is still the first to invest. If the drift is a bit higher, $\alpha_j = 0{,}015$, then the first to invest is the high Capex Firm i, as shown in the right-hand side graph: Firm *i* would invest at Y=0,98, or R$ 150/MWh and Firm *j* would only invest at Y=1,26, or R$ 193/MWh. Other parameters were kept equal to those of Figure 1.

5.2) Model 2

The impact of misinformation will be illustrated, again, through an example. Figure 6 depicts the value functions when parameters are the same as in Figure 3, but now, although each firm will still make its entry decision based on a rational expectation about the competitor's actions, their assumptions about the competitor are unfortunately wrong.

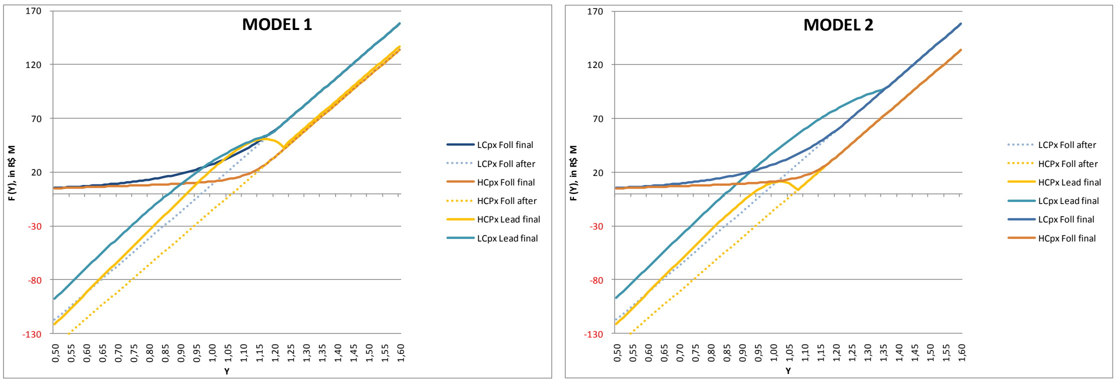


Figure 6: same parameters as in Figure 2, reproduced here in the left-hand side for ease of comparison. The right-hand side reflects the value curves each firm will expect to have, either as Follower or as Leader, by making a wrong guess about what the competitor expects for the market. $Ii = 1.1 I_j$.

The right-hand side of Figure 6 shows value curves that will not materialize, after all, because the opponents will not invest as expected. By making wrong assumptions on when the opponent will enter the market, Firm *j* imagines that its opponent also thinks the market will be more volatile in the future and will delay entry, enabling Firm *j* to reap the first-mover advantage for a longer time. As a result, Firm *j* will be tempted to enter as Leader, at Y=1,10, or R$ 168/MWh, while if it knew the correct stochastic process used by its opponent, it would have entered as Follower at Y=1,23, or R$ 189/MWh. At Y=1,10, Firm *j* expects to reap a value of R$ 60M upon the new investment.

Firm *i* , on the other hand, assuming that the opponent also thinks the market will not be significantly volatile in the future and knowing that is has a lower Capex, imagines the opponent will be tempted to invest sooner at Y=0,96, or R$ 147/MWh, and verifies that in this case it is never optimum to enter the market as Leader. Therefore, it plans to enter as Follower at Y=1,19, or R$ 181/MWh, sooner than originally expected by Firm *j*. As a result, instead of reaping the value of R$ 60M from the investment at Y=1,10, Firm *j* will actually value only R$ 42M (calculated as in c.2.ii).

Table 3 summarizes the actual as well as the wrongly guessed threshold values of *Y* for each firm as Follower, as well as the *Y* value that triggers each firm's entry decision.

Table 3: comparing the two models, for $\sigma_i = 0.03; \sigma_j = 0.09; \alpha_i = \alpha_j = 0; I_i = 1.1 I_j$. $\rho = 0.10; D_{i11} = D_{j11} = 25; D_{j10} = 29; D_{i00} = D_{j00} = 2; D_{i01} = D_{j10} = 1$.

	Modelo 1 – both guess right		Modelo 2 – both guess wrong	
	Firm *i* – high capex	Firm *j* –low capex	Firm *i* – high capex	Firm *j* –low capex
Y that triggers entry as Follower	$Y^*_{iF} = 1{,}186$ $->R\$181/MWh$	$Y^*_{jF} = 1{,}233$ $->R\$189/MWh$	$Y^*_{iF} = 1{,}186$ $->R\$181/MWh$ (but the opponent assumes it is at Y=1,356)	$Y^*_{jF} = 1{,}233$ $->R\$189/MWh$ (but the opponent assumes it is at Y=1,078)
Y that allows for preemption	$Y_{iP} = 0{,}960->$ $R\$147/MWh$	$Y_{jP} = 0{,}980->$ $R\$150/MWh$	It is never optimum to preempt (and the opponent assumes the same)	$Y_{jP} = 0{,}930->$ $R\$150/MWh$ (but the opponent assumes it is at Y=0,87)
monopolist Y	$Y^*_i = 1{,}054->$ $R\$161/MWh$	$Y^*_j = 1{,}096->$ $R\$168/MWh$	$Y^*_i = 1{,}054->$ $R\$161/MWh$	$Y^*_j = 1{,}096->$ $R\$168/MWh$
Y that triggers entry	**as Leader:** $Y^*_{iL} = 0{,}970->$ $R\$148/MWh$	as Follower: $Y^*_{jF} = 1{,}233$ $->R\$189/MWh$	as Follower: $Y^*_{iF} = 1{,}186$ $->R\$181/MWh$	**as Leader:** $Y^*_{jL} = 1{,}096->$ $R\$168/MWh$

In Model 1, the high Capex Firm *i* enters first, a bit before the point at which the opponent would consider entering as Leader. In Model 2, the low Capex Firm *j* is the one that becomes the Leader. In summary, a group of investors less informed of competitors' views tends to privilege the entry of more profitable firms in the market. In this case, however, the Leader will reap its first-mover advantage for a shorter period of time and, therefore, its value is lower than originally forecasted.

From the strict point of view of prices for energy consumers, the situation depicted in Model 1 is better, though, because it allows not only for a lower price when the first company expands – R$ 148/MWh – but also for a lower average price of R$ 169/MWh (when the second firm also expands), while in Model 2, the average price is R$ 175/MWh. However, this price advantage is at risk if we consider that the Leader, the high Capex Firm *i*, is less robust to face the firm-specific risks[25].

Finally, it is worth mentioning that if there were no asymmetry regarding the stochastic process and both firms did share the expectation of a volatility of up to 4%, not only the most viable firm would preempt the market, but also the average price to consumers would be the lowest (inferior to R$ 168/MWh). Table 4 presents the outcomes predicted by both models under the conditions depicted in Figures 1-5 and in other new scenarios.

[25] From a economic/welfare point of view, the value of the firms should also be considered in this comparative analysis

Table 4: outcomes predicted by Model 1 and Model 2:

parameters	Model 1				Model 2			
	high Capex Firm *i* invests at Y=	low Capex Firm *j* invests at Y=	first price	average energy price for consumers (R$/MWh)	high Capex Firm *i* invests at Y=	low Capex Firm *j* invests at Y=	first price	average energy price for consumers (R$/MWh)
(1) αi=αj=0; σi=σj=5% Ij=242; Ii=1.1Ij; D00=2; D01=1; D11=25;D10=29	1.24	**1.00**	153	172	idem	-		-
(2) Same as in (1), except for σi=3%;σj=9%	**0.97**	1.23	148	169	1.19	**1.10**	168	175
Same as in (2), except for Ii=1.05Ij	**1.01**	1.23	155	171	1.13	**1.03**	158	165
Same as in (1), except for αj=1,5%	**1.01**	1.26	155	174	1.24	**1.12**	171	181
Same as in (1), except for σi=4%;σj=4%	1.21	**0.98**	150	168	**-**	**-**		-
αi=0; αj=1,5%; σi=3; σj=9% Ij=242; Ii=1.1Ij; D00=2; D01=1; D11=25;D10=29	**1.05**	1.37	161	185	**1.19**	1.22	182	184

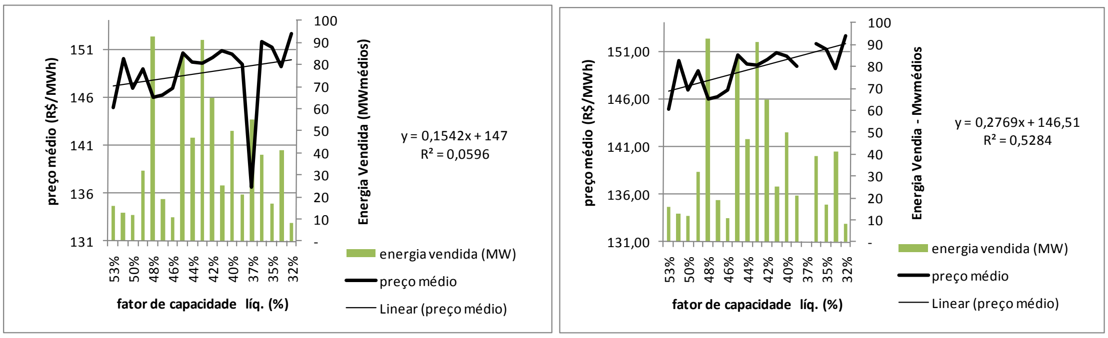
6) the 2009 tender offer results

At the ceiling price of R$ 189/MWh (*Y*=1,24), players offered 3,32 x actual demand, if we consider only those that went through all the qualification process. If we take into account the players that gave up even before the auction, this number would have been even more impressive: 7,78 x demand. Figure 6 shows how the "selection" process evolved:

449 projects applied / 14 GW → 339 were qualified / 10 GW → 217 actually participated / 6 GW → 71 winners / 1,8 GW

Figura 6: wind projects which intended to sell energy in the 2009 tender offer, in number of players and proposed installed capacity.

Competition led to contracts in the range of R$ 131 a R$ 153/MWh after an 8-hour auction, with a weighted average price of R$ 148/MWh, R$ 10/MWh below the price level obtained in the tender offer for biomass energy, held in 2008. Contracted farms will provide 738 MW, and both experienced players and newcomers won the bid, although the absence of some worldwide large players was outstanding. The average net capacity factor of the contracted farms is as high as 42%, but some winners have capacity levels much lower than other players that stayed out of the bid. Figure 7 details the average prices as per capacity factor of winners.



(a) (b)

Figura 7: weighted average prices contracted at the 2009 tender offer, grouped by net capacity factor; (a) including all winners ; (b) excluding outliers.

7) Conclusions and recommendations

Smit & Trigeorgis (2004, p.52) highlight that the important contribution of the option-games framework in strategic management is that it enables a quantification of qualitative strategic thinking, which is exactly what we attempted to do in this paper, when analzysing the recent developments in the Brazilian wind industry.

The results of the 2009 tender-offer surprised the sector and the government. Winners are still in the financial habilitation process, so it is still early to tell if all projects will be financially viable. The threat that Brazil might repeat the experiences of China and UK, which upon implementation of the tender offer system had to deal with several financially unviable projects, was the teaser that led us to reflect about this problem in the light of game options theory.

Interpreting the problem as a duopoly and performing a sensitivity analysis to figure out the conditions that would change the outcome, we concluded that when players are asymmetric both in terms of its investment costs and on the subjective perception of future market prospects, the risk of less profitable wind farms preempting the market grows significantly. Even minor asymmetries may cause preemption.

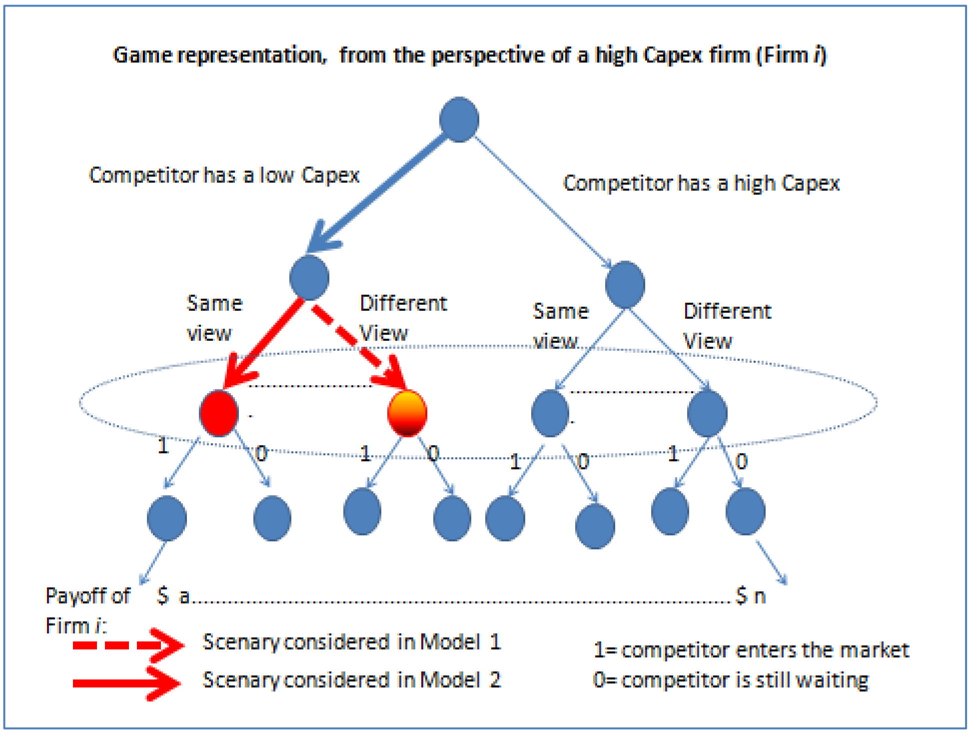
In addition, when players are less informed of competitors' views, this privileges the entry of more profitable/viable firms in the market, although this might occur at the expense of offering a higher energy price to consumers. Regarding the Dec 2009 tender-offer, dozens of seminars were held along 2009, congregating players at forums that discussed the contracting rules and market perspectives. The authors even witnessed a meeting where Iberdrola executives publicly stated their concerns that technical and financial details were not being thoroughly examined by market players. In summary, some less profitable players may have actually had a good hint of competitors' subjective views and decided to enter earlier and at lower prices to consumers.

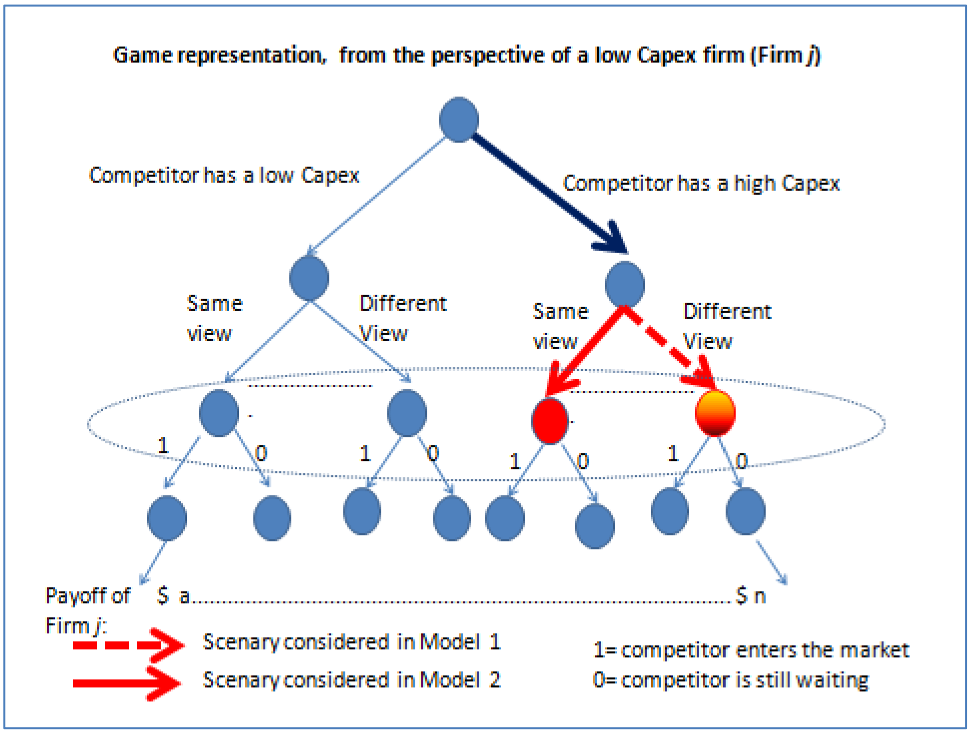
Finally, it is worth mentioning that under the very specific parameters used our examples, the better option to reduce the average price to consumer would be to eliminate the asymmetry in the stochastic process, provided that expectations about the market's uncertainty are kept at low levels.

As a way to avoid the perverse effect of contracting less viable firms, the Brazilian government might send a clear – and credible – signal of future market prospects for wind energy. This would make the problem fit in the example shown in Figure 1, which considers a market with complete information and only Capex asymmetry.

This work has several limitations: first, we have not considered firm-specific uncertainties, which may vary significantly among players, especially regarding the volatility of wind behavior at their projects' sites. In addition, waiting may be the optimal decision when technical uncertainty is high: some players may have opted to wait and get a longer history of wind behavior at their sites before actually investing, which is a good theme for future research. Finally, in this work we looked at two very specific scenarios that do not represent the myriad of alternatives that actually yield the optimization problem equilibria.

Annex 1: Extensive form representation of the Game, showing alternatives considered in Model 1 and Model 2





Annex 2: partial differential equation that describes the value of a project subject to a variable *Y*

A firm holds a perpetual American option to invest in a project that is subject to a variable Y that follows a Geometric Brownian Motion stochastic process, described by:

$dY = \alpha Y dt + \sigma Y dz$ (1) , where $dz = \xi\sqrt{dt}$, for $\xi \sim N(0,1)$

The value of the rights to this project's cash-flow is *V:*

$$V(Y(t)) = \max\{\Omega(Y); \pi(Y) + \frac{1}{1+\rho} E[V(Y(t+1)) \mid Y(t))\}$$

The first term in the maximization function refers to the payoff if the company invests, while the second term describes the continuation region, that is, the region when it is optimal to continue waiting, instead of investing. In this continuation region,

$$V(Y(t)) = \pi(t)dt + \frac{1}{1+\rho dt} E[V(Y(t+dt))$$

$$V(Y(t)).(1+\rho dt) = \pi(t)dt.(1+\rho dt) + E[V(Y(t)) + dV]$$

$V(Y(t)) + \rho V(Y(t))dt = \pi(t)dt(1+\rho dt) + V(Y(t)) + E[dV]$. It follows that:

$$\rho V(Y(t))dt = \pi(t)dt + \pi(t)\rho dt^2 + E[dV]$$

Eliminating terms in dt^2 , which tend to zero, and simplifying notation, we get to:

$\rho V dt = \pi dt + E[dV]$, or: $\boxed{\rho V = \pi + \frac{1}{dt} E[dV]}$ (2)

Using Ito's Lemma:

$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial Y} dY + \frac{1}{2}\frac{\partial^2 V}{\partial Y^2}(dY)^2$; or, in a simpler notation:

$\boxed{dV = V_t dt + V_Y dY + \frac{1}{2} V_{YY} (dY)^2}$ (3)

From equation (1),

$dY^2 = \alpha^2 Y^2 dt^2 + 2\alpha Y^2 \sigma dt dz + \sigma^2 Y^2 dz^2$; the first two terms can be neglected because dt is factored to orders higher than 1 and, therefore, they tend to zero, reducing the expression to: $dY^2 = \sigma^2 Y^2 dz^2$ (4)

Let's recall that:

$dz^2 = \xi^2 dt$, $\xi \sim N(0,1)$. Therefore, $E[\xi] = 0$ and

$$VAR(\xi) = 1 => E\left[\xi^2\right] - E^2\left[\xi\right] = 1 => E\left[\xi^2\right] = 1 + 0 = 1$$

It follows that: $E[dz^2] = E[\xi^2 dt] = dt.E[\xi^2] = dt$

and $VAR(dz^2) = VAR(\xi^2 dt) = dt^2.VAR(\xi^2) \sim zero$, so we can say that $dz^2 = dt$.

Therefore, equation (4) can be re-written as: $\boxed{dA^2 = \sigma^2 A^2 dt}$ (5)

Combining eq. (1), (3) and (5), we get:

$$dV = V_t dt + V_Y dY + \frac{1}{2} V_{YY} (dY)^2$$

$$dV = V_t dt + V_Y \left(\alpha Y dt + \sigma Y dz\right) + \frac{1}{2} V_{YY} \left(\sigma^2 Y^2 dt\right)$$

$$E[dV] = E[V_t dt + V_Y \left(\alpha Y dt + \sigma Y dz\right) + \frac{1}{2} V_{YY} \left(\sigma^2 Y^2 dt\right)]$$

$$E[dV] = V_t dt + \alpha Y V_Y dt + \frac{1}{2} V_{YY} \left(\sigma^2 Y^2 dt\right) + \underbrace{\sigma Y V_Y \sqrt{dt}.E[\xi]}_{\text{Zero}}$$

$$E[dV] = V_t dt + \alpha Y V_Y dt + \frac{1}{2} \sigma^2 Y^2 V_{YY} dt \qquad (6)$$

Combining equations (2) and (6):

$\rho V = \pi + \frac{1}{dt}\left\{V_t dt + \alpha Y V_Y dt + \frac{1}{2}\sigma^2 Y^2 V_{YY} dt\right\}$, which leads us to:

$$\frac{1}{2}\sigma^2 Y^2 V_{YY} + \alpha Y V_Y + V_t - \rho V + \pi = 0$$

In a perpetual option, postponing the decision just leads to a new perpetual option that is exactly the same; therefore, the value of the option does not vary with time and $V_t = 0$. It follows that:

$$\underbrace{\frac{1}{2}\sigma^2 Y^2 V_{YY} + \alpha Y V_Y - \rho V}_{\text{homogeneous PDE}} + \underbrace{\pi}_{\text{a non-homogeneous term}} = 0 \qquad (7)$$

homogeneous PDE + a non-homogeneous term

A solution for the homogeneus PDE is: $V_{\text{hom}} = A.Y^{\beta}$ (8)

Deriving expression (8) one and two times and substituting the results in the homogeneous PDE leads to:

$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - \rho = 0$ (9) , a quadratic expression which roots are:

$$\beta_1 = \frac{\frac{1}{2}\sigma^2 - \alpha + \sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\rho}}{\sigma^2} \quad \text{and} \quad \beta_2 = \frac{\frac{1}{2}\sigma^2 - \alpha - \sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\sigma^2\rho}}{\sigma^2} \qquad (10)$$

It can be proved that $\beta_1 > 1$ and $\beta_2 < 0$ in the general solution of the homogeneous part of the PDE,

$V_{homogenea} = A_1 Y^{\beta_1} + A_2 Y^{\beta_2}$, but we know that if $Y \to 0$, $V \to 0$, and this is only feasible if A_2 is zero (because the negative root β_2 will make the second term tend to ∞ when Y tends to zero).

The non-homogeneous part of the expression may be accounted for by including any term that makes expression (7) work. A natural alternative is to consider that solution that reflects the value of the project in case it is never optimal to exercise the option, that is: $\frac{\pi}{\rho - \alpha}$, a perpetuity.

Therefore, the solution of expression (7) is:

$$\boxed{V = AY^{\beta_1} + \frac{\pi}{\rho - \alpha}} \qquad (11)$$

The same expressions above can be derived using Contingent Claims, which assumes that the problem can be mimicked by a replicating portfolio or spanning assets (please refer to Dixit&Pindyck, 1999, p.114-119). Results are similar: just substitute the drift α for $(r - \delta)$ and ρ for r in expressions (7), (10) and (11).

If firms are assumed to be risk-neutral, the same expressions apply, but ρ must be replaced for r in expressions (7), (10) and (11).

Annex 3: value functions of a Firm *i* with an option to invest as a Monopolist

Following the dynamic programming solution stated in Annex 2, the value of a Firm with an option to invest as a monopolist is stated by:

$$\begin{cases} F_i(Y_i) = mY_i^{\theta_i} + \dfrac{Y_i D_{i00}}{\rho - \alpha_i} \quad , \text{for } Y \prec Y_i^* & (16) \text{ , or} \\ F_i(Y_i) = \dfrac{Y_i D_{i10}}{\rho - \alpha_i} - I_i \ , \quad \text{for } \geq Y_i^* \end{cases}$$

The optimal stopping time can be obtained by using the VMC and the SPC in equations (12) and (16), at the point at which $Y = Y_i^*$, which is the point at which waiting is no longer worthy:

VMC: $mY_i^{*\theta_i} + \dfrac{Y_i^* D_{i00}}{\rho - \alpha_i} = \dfrac{Y_i^* D_{i10}}{\rho - \alpha_i} - I_i$

SPC: $\theta_i m Y_i^{*\theta_i - 1} + \dfrac{D_{i00}}{\rho - \alpha_i} = \dfrac{D_{i10}}{\rho - \alpha_i}$

As a result: $Y_i^* = \dfrac{\theta_i}{\theta_i - 1} \dfrac{(\rho - \alpha_i) I_i}{D_{i10} - D_{i00}}$, equal to expression (17), in section 2.1.ii

$$m = \frac{Y_i^{*(1-\theta_i)}}{\theta_i} \frac{D_{i10} - D_{i00}}{\rho - \alpha_i}$$

From expression (18) in section 2.1.ii, we can see that $M = m + B$. As B, given by expression (14), is a strictly negative constant, $M \prec m$. So, as expected, the value of a firm with an option to invest as Leader, stated by expression (12) is lower than the value of a firm with an option to invest as a monopolist, but the threshold at which the firm invests, Y_i^*, is the same in the two cases.

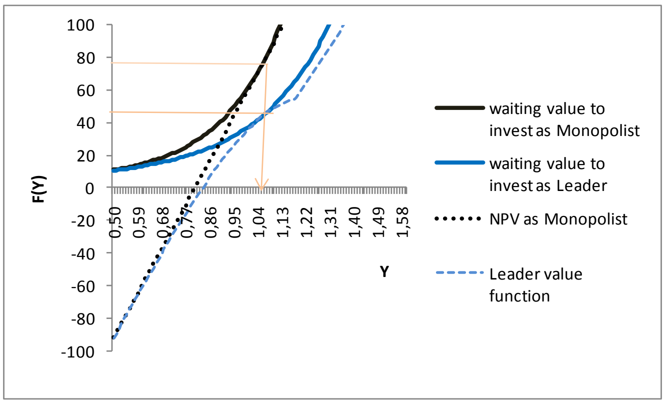


Figure 8: example of the threshold to invest, either as a monopolist or to become a leader, and respective values of the firm in the two cases.

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