Market segmentation under uncertainty*

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Abstract

This paper proposes a general model to value different strategies to enter a market, comparing alternative sequential segmentation paths to simultaneous investment in all segments. This general model also allows demand to evolve accordingly to an endogenous regime-switching process, under which it can behave differently before and after investment. It is shown how uncertainty, revenues and investment costs of each segment impact the choice between sequential or parallel investment, as well as the optimal path. The model also offers some insights for the valuation of growth options.

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Market segmentation under uncertainty

1 Introduction

When companies enter a market with a new product they face the decision to eventually segment demand, for example using pricing strategies. Many companies charge initially high prices to the top profitable segments of demand, those less sensitive to prices, and then reducing gradually the price,enter lower segments, with a strategy that is known as *market-skimming*. Other companies, instead, prefer to enter the market quickly, attracting a higher market share with a lower price and benefiting from economies of scale. This strategy is known as *market-penetration*. These companies can eventually expand to the top segments of demand. On the other hand, there are companies that prefer to stick themselves to a single segment of demand, either the more profitable or the less profitable but large-scaled.

Many companies in a wide variety of industries face this type of decision. One example is the airport industry. A significant and recent trend in the airport industry is the rise of the low-cost airlines. Neufville (2008) suggests that this trend, along with the long term forecasts uncertainty, has important implications for the airport design planning. Low cost carriers and passengers have different characteristics of the traditional full-service carriers. Usually they demand less quality of the infrastructure in exchange of a lower price. On the other hand these carriers have different strategies, preferring "point-to-point" flights over the traditional "hub" strategy.

The decision to serve one or both segments has important implications on the investments, largely irreversible, that are required when a new airport or an expansion are analyzed. This is even more important under uncertainty, which creates real options and determines optimal timing of investment. Real options embedded in airport projects have been studied by Smit (2003) combining real options and game theory to value airport expansion investments. Pereira, Rodrigues and Armada (2007) model an airport investment when the revenues and the number of passengers behave stochastically and negative or positive jumps occur randomly. Gil (2007) present a description of a wide range of real options embedded in airport investments.

This papers presents a general model that allow us to value alternative segmentation strategies, namely comparing different segmentation sequences and the simultaneous investment in all segments. Sequential versus parallel development has been studied, in a real options context, by Childs, Ott and Triantis (1998). They compare different strategies, including optimal ordering of sequential investment, exploring project interrelationships. Sequential investment is also analyzed in some detail in Dixit and Pindyck (1994, chapter 10).

Differently from Childs, Ott and Triantis (1998), it is considered that the underlying processes that drive demand of each segment are not correlated processes, but instead demand behaves differently before and after investment in each segment, under an endogenous regime-switching process. However, it can argued that there are some connections to the Childs, Ott and Triantis (1998) approach. A higher correlation between two projects (or segments is our case) implies a higher volatility after investing in both segments, while a lower correlation means a lower volatility, due to the diversification effect. We also include a cap for each segment demand. This, along with the fact that after investing in each segment the company benefits from a cash flow stream, allows, without the need to appeal to features such as "time to build", the model to produce a solution that implies that it is sometimes optimal to wait to invest in subsequent stages and not immediately after investing in the first stage as in Dixit and Pindyck (1994, chapter 10).

The paper unfolds as follows. Section 2 presents the model to value investment in a single segment, considering that demand growth ends after reaching a cap. Different strategies are then analyzed in section 3 comparing sequential to simultaneous investment. Section 4 presents a numerical example and section 5 concludes.

2 Investing in a single segment

Let Q, the demand of segment i, behave accordingly to the following geometric brownian motion:

$$dQ = \alpha_i Q dt + \sigma_i Q dW_i \tag{1}$$

where α_i is the expected growth rate, σ_i the respective standard deviation and dW_i increments of a Wiener process.

Each segment has a known size, C_i, after which growth ends. The revenue per unit of demand is R_i, and we assume it is constant.

The equivalent risk-adjusted process of equation 1 is:

$$dQ = (r - \delta_i)\, Q dt + \sigma_i Q dZ_i \tag{2}$$

where $\delta_i = \mu_i - \alpha_i$ and μ_i is the equilibrium rate of return.

Using the standard procedures the value of the project after investing is[1]:

$$V_i(Q) = \begin{cases} A_i Q^{\beta_i} + \dfrac{R_i Q_i}{\delta_i} & \text{for } Q < C_i \\ \\ B_i Q^{\gamma_i} + \dfrac{R_i C_i}{r} & \text{for } Q \geqslant C_i \end{cases} \tag{3}$$

[1] Please refer to Armada, Pereira and Rodrigues (2008) for a similar model.

where:

$$A_i = \frac{C_i^{1-\beta_i}}{\gamma_i - \beta_i} R_i \left(\frac{\gamma_i}{r} - \frac{(\gamma_i - 1)}{\delta_i} \right) \tag{4}$$

$$B_i = \frac{C_i^{1-\gamma_i}}{\gamma_i - \beta_i} R_i \left(\frac{\beta_i}{r} - \frac{(\beta_i - 1)}{\delta_i} \right) \tag{5}$$

$$\beta_i = \frac{1}{2} - \frac{r - \delta_i}{\sigma_i^2} + \sqrt{\left(-\frac{1}{2} + \frac{r - \delta_i}{\sigma_i^2} \right)^2 + \frac{2r}{\sigma_i^2}} > 1 \tag{6}$$

$$\gamma_i = \frac{1}{2} - \frac{r - \delta_i}{\sigma_i^2} - \sqrt{\left(-\frac{1}{2} + \frac{r - \delta_i}{\sigma_i^2} \right)^2 + \frac{2r}{\sigma_i^2}} < 0 \tag{7}$$

Demand can behave differently before and after investment. Before investment, demand evolves accordingly to the following stochastic process:

$$dQ = (r - \delta_0)\, Q dt + \sigma_0 Q dW_0 \tag{8}$$

After investment Q is driven by equation 1.

Let $F_i(Q)$ be the value the option to invest in segment i:

$$F_i(Q) = D_i Q^{\beta_0} \tag{9}$$

The following boundary conditions are used to find D_i and the optimal trigger value, Q_i^*:

$$F_i(Q_i^*) = V_i(Q_i^*) - K_i \tag{10}$$

$$F_i'(P^*) = V_i'(Q_i^*) \tag{11}$$

where K_i is the invesment cost to enter segment i.

When δ_0 is positive, as is commonly assumed, there are two possible solutions, depending whether the trigger value, Q_i^*, is lower or greater than segment size, C_i.

Case 1: $Q_i^* < C_i$ and $\delta_0 > 0$

For $Q_i^* < C_i$, the trigger value of Q is the solution to the following nonlinear equation:

$$(\beta_0 - \beta_i)\, A_i Q_i^{*\beta_i} + (\beta_0 - 1)\, \frac{R_i}{\delta_i} Q_i^* - \beta_0 K_i = 0, \tag{12}$$

and D_i is:

$$D_i = A_i Q_i^{*(\beta_i - \beta_0)} + \frac{R_i}{\delta_i} Q_i^{*1-\beta_0} - K_i Q_i^{*-\beta_0} = 0. \tag{13}$$

The value-function for the option to invest is:

$$F_i(Q) = \begin{cases} \left(A_i Q^{*\beta_i} + \dfrac{R_i}{\delta_i} Q_i^* - K_i\right)\left(\dfrac{Q}{Q_i^*}\right)^{\beta_0} & \text{for } Q < Q_i^* \\ A_i Q^{\beta_i} + \dfrac{R_i Q}{\delta_i} - K_i & \text{for } Q \geqslant Q_i^* \wedge Q \leqslant C_i \\ B_i Q^{\gamma_i} + \dfrac{R_i C_i}{r} - K_i & \text{for } Q \geqslant Q_i^* \wedge Q \geqslant C_i \end{cases} \quad (14)$$

For $\beta_i = \beta_0$, for example when $\delta_i = \delta_0$ and $\sigma_i = \sigma_0$, Q_i^* and D_i have the following closed form solution:

$$Q_i^* = \frac{\beta_0}{\beta_0 - 1} \frac{\delta_0}{R_i} K_i \quad (15)$$

$$D_i = A_i + \frac{K_i}{\beta_0 - 1} Q_i^{*-\beta_0} \quad (16)$$

Case 2: $Q_i^* \geqslant C_i$ and $\delta_0 > 0$

The trigger value of Q, when it is greater than C_i, is:

$$Q_i^* = \left[\frac{\beta_0}{B_i(\gamma_i - \beta_0)}\left(\frac{R_i C_i}{r} - K_i\right)\right]^{\frac{1}{\gamma_i}} \quad (17)$$

The value of the option to invest is, for $Q_i^* \geqslant C_i$:

$$F_i(Q) = \begin{cases} \dfrac{\gamma_i}{\gamma_i - \beta_0}\left(\dfrac{R_i C_i}{r} - K_i\right)\left(\dfrac{Q}{Q_i^*}\right)^{\beta_0} & \text{for } Q < Q_i^* \\ B_i Q^{\gamma_i} + \dfrac{R_i C_i}{r} - K_i & \text{for } Q \geqslant Q_i^* \geqslant C_i \end{cases} \quad (18)$$

3 Alternative strategies

When a company enters the market, it can choose to invest sequentially in segment i and j or simultaneously in both segments. This section compares these alternatives strategies.

3.1 Sequential investment

Valuation of sequential investment needs to be done backwards, starting with the option to invest in segment j, $F_j(Q)$. Q is the cumulative demand of segments i and j. Before investing in j, demand is governed by the following stochastic process, under the risk-neutral measure:

$$dQ = (r - \delta_i)\, Q dt + \sigma_i Q dZ_i \quad (19)$$

After investing in j, demand, starting at the level of Q at that moment, follows a similar process under different parameters:

$$dQ = (r - \delta_j)\, Q dt + \sigma_j Q dZ_j \tag{20}$$

The value of the option to expand to segment j, takes the form:

$$F_j(Q) = D_j Q^{\beta_i} \tag{21}$$

The following value-matching and smooth-pasting conditions are used to find the optimal trigger value, Q_j^*, which corresponds to the value Q at which it is optimal to expand to segment j, and D_j:

$$F_j(Q_j^*) = V_j(Q_j^*) - V_i(Q_j^*) - K_j \tag{22}$$

$$F_j'(Q_j^*) = V_j'(Q_j^*) - V_i'(Q_j^*) \tag{23}$$

Expanding to segment j, paying the irreversible cost K_j, the firm exchanges $V_i(Q_j^*)$ for $V_j(Q_j^*)$. It is reasonable to assume that, expanding, it will benefit from a larger demand size, i.e. $C_j > C_i$. Therefore, depending on the parameters, the trigger value can be in the $[0, C_i[$, $[C_i, C_j[$ or $[C_j, \infty[$ intervals.

Case 1: $Q_j^* < C_i < C_j$

For the first case, the solution is found with the following value-matching and smooth-pasting conditions:

$$D_j Q_j^{*\beta_i} = A_j Q_j^{*\beta_j} + \frac{R_j Q_j^*}{\delta_j} - A_i Q_j^{*\beta_i} - \frac{R_i Q_j^*}{\delta_i} - K_j \tag{24}$$

$$\beta_i D_j Q_j^{*\beta_i - 1} = \beta_j A_j Q_j^{*\beta_j - 1} + \frac{R_j}{\delta_j} - \beta_i A_i Q_j^{*\beta_i - 1} - \frac{R_i}{\delta_i} \tag{25}$$

Q_j^* is the solution to the following nonlinear equation:

$$(\beta_i - \beta_j)\, A_j Q_j^{*\beta_j} + (\beta_i - 1)\left(\frac{R_j}{\delta_j} - \frac{R_i}{\delta_i}\right) Q_j^* - \beta_i K_j = 0, \tag{26}$$

and D_j is:

$$D_j = A_j Q_j^{*(\beta_j - \beta_i)} - A_i + \left(\frac{R_j}{\delta_j} - \frac{R_i}{\delta_i}\right) Q_j^{*(1-\beta_i)} - K_j Q_j^{*-\beta_i}. \tag{27}$$

Note that when $\beta_j = \beta_i$, for example when $\delta_j = \delta_i$ and $\sigma_j = \sigma_i$, there is a closed form solution:

$$Q_j^* = \frac{\beta_i}{\beta_i - 1}\frac{\delta_i}{R_j - R_i}K_j \tag{28}$$

$$D_j = A_j - A_i + \frac{K_i}{\beta_i - 1}Q_j^{*-\beta_i} \tag{29}$$

Case 2: $C_i \leqslant Q_j^* < C_j$

For the second case, the following value-matching and smooth-pasting conditions allow gives the solution:

$$D_j Q_j^{*\beta_i} = A_j Q_j^{*\beta_j} + \frac{R_j Q_j^*}{\delta_j} - B_i Q_j^{*\gamma_i} - \frac{R_i C_i}{r} - K_j \tag{30}$$

$$\beta_i D_j Q_j^{*\beta_i - 1} = \beta_j A_j Q_j^{*\beta_j - 1} + \frac{R_j}{\delta_j} - \gamma_i B_i Q_j^{*\gamma_i - 1} \tag{31}$$

Q_j^* is the solution to the following nonlinear equation:

$$(\beta_i - \beta_j) A_j Q_j^{*\beta_j} - (\beta_i - \gamma_i) B_i Q_j^{*\gamma_i} + (\beta_i - 1)\frac{R_j}{\delta_j}Q_j^* - \beta_i\left(\frac{R_i C_i}{r} + K_j\right) = 0, \tag{32}$$

and D_j is:

$$D_j = A_j Q_j^{*(\beta_j - \beta_i)} + \frac{R_j}{\delta_j}Q_j^{*(1-\beta_i)} - B_i Q_j^{*(\gamma_i - \beta_i)} - \left(\frac{R_i C_i}{r} + K_j\right)Q_j^{*-\beta_i}. \tag{33}$$

Note that even for $\delta_j = \delta_i$ and $\sigma_j = \sigma_i$, Q_j^* is still the solution to a nonlinear equation:

$$(\beta_i - \gamma_i) B_i Q_j^{*\gamma_j} + (\beta_i - 1)\frac{R_j}{\delta_j}Q_j^* - \beta_i\left(\frac{R_i C_i}{r} + K_j\right) = 0 \tag{34}$$

Case 3: $C_i < C_j \leqslant Q_j^*$

Finally, for $C_i < C_j \leqslant Q_j^*$, the solution is found with the following value-matching and smooth-pasting conditions:

$$D_j Q_j^{*\beta_i} = B_j Q_j^{*\gamma_j} + \frac{R_j C_j}{r} - B_i Q_j^{*\gamma_i} - \frac{R_i C_i}{r} - K_j \tag{35}$$

$$\beta_i D_j Q_j^{*\beta_i - 1} = \gamma_j B_j Q_j^{*\gamma_j - 1} - \gamma_i B_i Q_j^{*\gamma_i - 1} \tag{36}$$

Q_j^* is the solution to the following nonlinear equation:

$$(\beta_i - \gamma_j) B_j Q_j^{*\gamma_j} - (\beta_i - \gamma_i) B_i Q_j^{*\gamma_i} + \beta_i\left(\frac{R_j C_j - R_i C_i}{r} - K_j\right) = 0, \tag{37}$$

and D_j is:

$$D_j = B_j Q_j^{*(\gamma_j - \beta_i)} - B_i Q_j^{*(\gamma_i - \beta_i)} + \left(\frac{R_j C_j - R_i C_i}{r} - K_j \right) Q_j^{*-\beta_i}. \tag{38}$$

When $\delta_j = \delta_i$ and $\sigma_j = \sigma_i$, $\gamma_j = \gamma_i$ and Q_j^* is:

$$Q_j^* = \left[\frac{\beta_i}{(B_j - B_i)(\gamma_i - \beta_i)} \left(\frac{R_j C_j - R_i C_i}{r} - K_j \right) \right]^{\frac{1}{\gamma_i}} \tag{39}$$

In either case, option value to invest in segment j is:

$$F_j(Q) = \begin{cases} D_j Q^{\beta_i} & \text{for } Q < Q_j^* \\ \\ V_j(Q) - V_i(Q) - K_j & \text{for } Q \geqslant Q_j^* \end{cases} \tag{40}$$

Let $F_i(Q)$ be the value option to invest in segment i. Before investment, demand evolves accordingly to the following stochastic process:

$$dQ = (r - \delta_0)\, Q dt + \sigma_0 Q dW_0 \tag{41}$$

The solution for the value of option to invest in segment i takes the form:

$$F_i(Q) = D_i Q^{\beta_0} \tag{42}$$

The following boundary conditions are used to find the optimal trigger value (Q_i^*), i.e. the value Q at which it is optimal to invest in segment i:

$$F_i(Q_i^*) = V_i(Q_i^*) + F_j(Q_i^*) - K_i \tag{43}$$
$$F_i'(Q_i^*) = V_i'(Q_i^*) + F_j'(Q_i^*) \tag{44}$$

Note again that, depending on the parameters, the trigger value can be in the $[0, C_i[$, $[C_i, C_j[$ or $[C_j, \infty[$ intervals, and can also be lower or higher than Q_j^*, which means that it may be optimal either to invest in the second segment immediately after entering the first, or wait a bit more. In that case, given that $F_j(Q_i^*) = V_j(Q_i^*) - V_i(Q_i^*) - K_j$, the value-matching and smooth-pasting equations are:

$$F_i(Q_i^*) = V_j(Q_i^*) - K_j - K_i \tag{45}$$
$$F_i'(Q_i^*) = V_j'(Q_i^*) \tag{46}$$

Case 1: $Q_i^* < Q_j^*$ **and** $Q_i^* < C_i < C_j$

Value-matching and smooth-pasting conditions produce the following nonlinear equation, whose solution is Q_i^*:

$$(\beta_0 - \beta_i)(A_i + D_j) Q_i^{*\beta_i} + (\beta_0 - 1) \frac{R_i}{\delta_i} Q_i^* - \beta_0 K_i = 0 \tag{47}$$

D_i is:

$$D_i = (A_i + D_j) Q_i^{*(\beta_i - \beta_0)} + \frac{R_i}{\delta_i} Q_i^{*(1-\beta_0)} - K_i Q_i^{*-\beta_0} \tag{48}$$

where D_j is given by equations 27, 33, and 38 respectively for $Q_j^* < C_i < C_j$, $C_i \leqslant Q_j^* < C_j$, and $C_i < C_j \leqslant Q_j^*$.

For $\beta_i = \beta_0$, the following closed form solution is found:

$$Q_i^* = \frac{\beta_0}{\beta_0 - 1} \frac{\delta_0}{R_i} K_i \tag{49}$$

$$D_i = A_i + D_j + \frac{K_i}{\beta_0 - 1} Q_i^{*-\beta_0} \tag{50}$$

This trigger value is the same as that of investing in a single segment (see equation 15). The option to expand to the following segments does not change the level of Q for which it is optimal to invest, although it changes the value of the investment opportunity: D_i depends now on D_j. For the particular case when the demand risk-neutral process is unaffected by the act of investing, future growth options have no impact on the investment optimal timing, although they make investment more valuable.

Case 2: $Q_i^* < Q_j^*$ **and** $Q_i^* \geqslant C_i$

Value-matching and smooth-pasting conditions produce the following nonlinear equation, whose solution is Q_i^*:

$$(\beta_0 - \gamma_i) B_i Q_i^{*\gamma_i} + (\beta_0 - \beta_i) D_j Q_i^{*\beta_i} + \beta_0 \left(\frac{R_i C_i}{r} - K_i \right) = 0 \tag{51}$$

D_i is:

$$D_i = B_i Q_i^{*(\gamma_i - \beta_0)} + D_j Q_i^{*(\beta_i - \beta_0)} + \left(\frac{R_i C_i}{r} - K_i \right) Q_i^{*-\beta_0} \tag{52}$$

where D_j is given by equations 33, and 38 respectively for $C_i \leqslant Q_j^* < C_j$, and $C_i < C_j \leqslant Q_j^*$.

For $\beta_i = \beta_0$, this produces a similar result as in the previous case: the trigger value of Q is the same as that of investing in a single segment, although the value of the project is greater, benefiting from the value of the options to expand to subsequent segments.

Case 3: $Q_i^* < C_j$ **and** $Q_i^* \geqslant Q_j^*$

Q_i^* is the solution of the following nonlinear equation:

$$(\beta_0 - \beta_j) A_j Q_i^{*\beta_j} + (\beta_0 - 1) \frac{R_j}{\delta_j} Q_i^* - \beta_0 (K_j + K_i) = 0 \tag{53}$$

D_i is:

$$D_i = A_j Q_i^{*(\beta_j - \beta_0)} + \frac{R_j}{\delta_j} Q_i^{*(1-\beta_0)} - (K_i + K_j) Q_i^{*-\beta_0} \tag{54}$$

For $\beta_j = \beta_0$, for example when $\delta_j = \delta_0$ and $\sigma_j = \sigma_0$, there is a closed form solution:

$$Q_i^* = \frac{\beta_0}{\beta_0 - 1} \frac{\delta_0}{R_j} (K_j + K_i) \tag{55}$$

$$D_i = A_j + \frac{K_i + K_j}{\beta_0 - 1} Q_i^{*-\beta_0} \tag{56}$$

The solution for Q_i^* and D_i is equivalent to equation 15 and equation16, replacing K_i with $K_j + K_i$ and R_i with R_j.

Case 4: $Q_i^* \geqslant Q_j^* \geqslant C_j$

For this case, a closed form solution is found:

$$Q_i^* = \left[\frac{\beta_0}{B_i (\gamma_j - \beta_0)} \left(\frac{R_j C_j}{r} - K_i - K_j \right) \right]^{\frac{1}{\gamma_j}} \tag{57}$$

$$D_i = B_j Q_i^{*(\beta_j - \beta_0)} + \left(\frac{R_j C_j}{r} - K_i - K_j \right) Q_i^{*-\beta_0} \tag{58}$$

3.2 Simultaneous investment

In the previous section it is shown that it can be optimal to invest immediately in the second segment if all parameters remain the same. However it can be argued that growth rates, volatility, investment costs and segment size, for the case of simultaneous investment in both segments, are not the same as those of the second segment.

Let, as above, demand before investment be given by equation 41 and after investment evolve accordingly to the following stochastic process:

$$dQ = (r - \delta_s) Q dt + \sigma_s Q dW_s \tag{59}$$

whrere δ_s and σ_s are not necessarily the same as δ_j and σ_j .

Market size, C_S can be a multiple of segment i size, $C_s = \psi_1 C_j$, and investment costa can also be a multiple of the sum investment in both segments, $K_s = \psi_2 (K_i + K_j)$. ψ_1 and ψ_2 can be either greater or smaller than 1.

	Before Investment	Segment 1	Segment 2	Simultaneous
δ	0.02	0.02	0.02	0.02
σ	0.12	0.1	0.08	0.09
R		1	0.9	0.9
C		20	30	ψ_1 30
K		200	90	ψ_2 90
ψ_1				1
ψ_2				0.9
r		0.03		
$Q(0)$		5		

Table 1: Base-case parameters

Simultaneous investment is valued as in section 2 with δ_i, σ_i, C_i, and K_i replaced by δ_s, σ_s, C_s, and K_s, respectively.

4 Numerical example

As in almost every model, results are sensitive to the parameters, and it is sometimes difficult to draw general conclusions based on comparative statics. Table 1 presents the base-case parameters. δ is taken as a market parameter independent of σ. The analysis would have been different had it been allowed to change with σ. Segment 1 is bigger than segment 2. After investing in segment 1, expanding to segment 2 reduces risk and revenues. Whereas the latter means that this segment is less profitable, the first does not means necessarily that it is less riskier. It has to be considered that if the two segments are not perfectly correlated, risk can be lower after expanding to segment 2, even if segment 2 is riskier. When investing simultaneously in both segments, the company benefits from the same segments size but with more risk and it is assumed that there are some economies of scale at the investment level ($\psi_2 = 0.9$).

The trigger values of Q for which it is optimal to invest are 8.46 for segment 1 and 29.37 for segment 2. Figure 1 shows how the value of each segment and options to invest in them behave with "moneyness".

The effect of volatility on the trigger values is shown in Figure 2. Obviously, demand volatility before any investment has no effect on the trigger to invest in segment 2, while it increases the trigger value to invest in segment 1. As the volatility of the first segment increases, and becomes greater than demand volatility after expansion to the second segment, Q_1^* increases slightly, whereas Q_2^* decreases more significantly approaching segment 1 size. For lower values of σ_1, expanding to segment 2 is only optimal after demand reaches segment 1 size, and after a certain volatility level it becomes optimal to expand before demand hits the previous segment size. Volatility of the last segment does not change

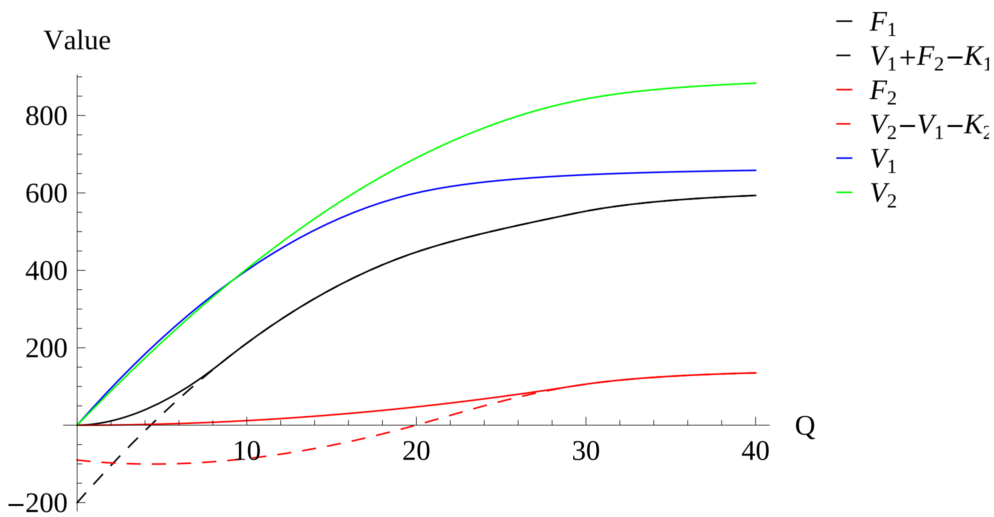
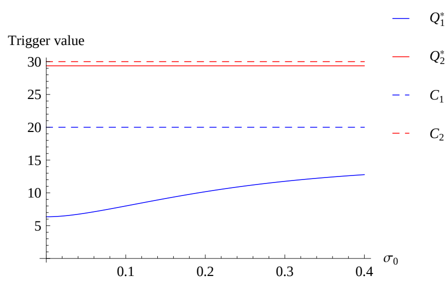
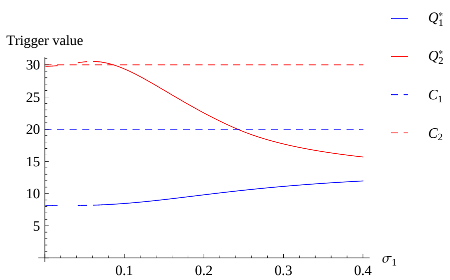


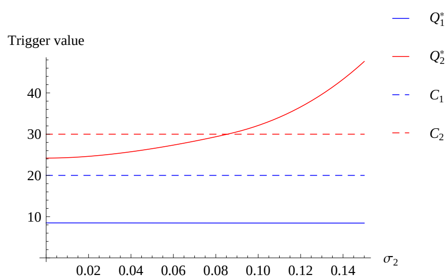
Figure 1: Sensitivity of the value of each segment and options to invest to demand.



(a) Volatility before investment



(b) Volatility of segment 1



(c) Volatility of segment 2

Figure 2: Sensitivity of trigger values to volatility before investment and segments volatility.

	Before Investment	Segment 2	Segment 1
δ	0.02	0.02	0.02
σ	0.12	0.08	0.1
R		0.7	0.9
C		10	30
K		90	200

Table 2: Parameters for sequential investment in segment 2 and segment 1.

the optimal level to enter the market (Q_1^* remains unchanged) and increases the optimal level to expand to it. This effect seems to be very pronounced: for relatively low levels of volatility it only becomes optimal to expand after demand reaches the segment size. This arises as a result of two effects: the first relates to the well known effect of the value of waiting and the second is a consequence of imposing an upper limit to demand growth, which makes the value of segment a decreasing function of volatility.

Another interesting issue is the comparison of different sequences of segmentation and simultaneous investment. For sequential investment there are two alternatives: a) invest in segment 1 and expand to segment 2, b) investment in segment 2 and expand to segment 1. Parameters for a) are those presented in Table 1. Note that column under segment 2 shows the parameters for demand behavior after expansion to segment 2. If we change the sequence, we have to know which parameters characterize segment 2 when we invest in it before expanding to segment 2. We assume that with this sequence we start with a less riskier and less profitable segment (Table 2).

Figure 3 shows the value of each strategyand Figure 4 the trigger values for different volatility levels of demand before investment. For lower values of demand, investing first in the smaller, less riskier and less profitable segment is preferable. For higher levels of demand simultaneous investment produces a higher value, due to investment cost economies of scale. For intermediate levels, the bigger, riskier and more profitable segment is the best choice to enter the market.

5 Conclusion

This paper proposes a real option model for the valuation of alternative strategies for the market introduction of a new product. It compares sequential segmentation paths to simultaneous investment in all segments. Unlike most real options models the model assumes that demand can only grow up to a limit, determined by segment size. Additionally demand can behave differently before and after investment in each segment.

Optimal timing and value for each strategy are computed. It is shown that, depending on the parameters, it can be optimal to expand to subsequent segments immediately after

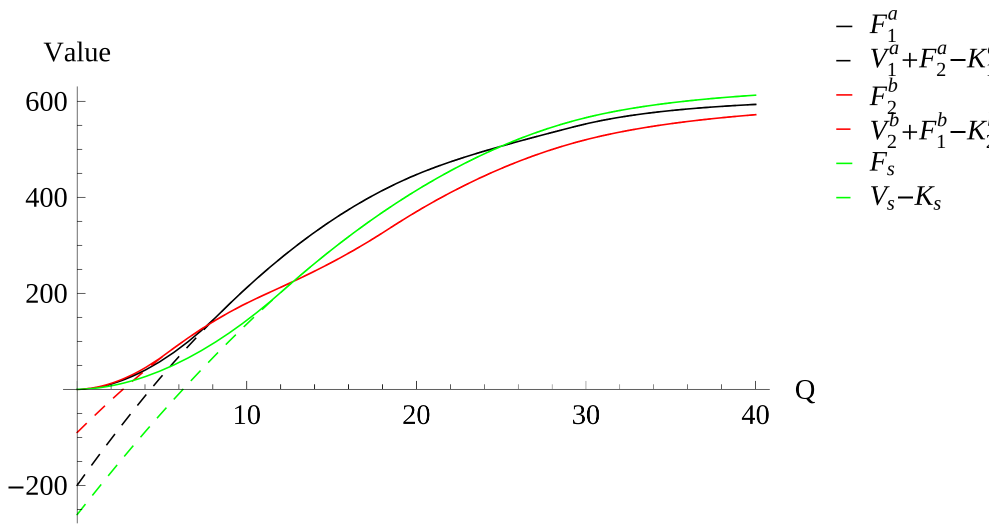


Figure 3: Value of different strategies.

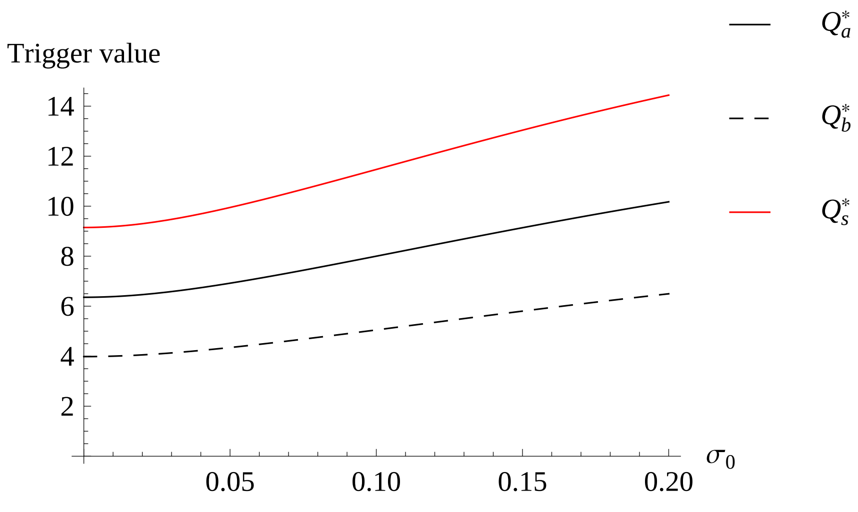


Figure 4: Trigger values for different strategies.

enter the previous or wait until demand reaches some specified trigger value. Under some particular assumptions future growth options have no impact on optimal timing to enter initial segments, although value is greater with more valuable future growth opportunities.

Most results are sensitive to the parameters, and it is sometimes difficult to draw general conclusions based on comparative statics. Using an example, it is shown that for lower values of demand, investing first in a smaller, less riskier and less profitable segment is more profitable, while for higher levels of demand simultaneous investment produces a higher value, if there are investment costs economies of scale. For intermediate levels, the bigger, riskier and more profitable segment is the best choice to enter the market.

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