Partial Divestment and Firm Sale under Uncertainty

Sebastian Gryglewicz*
Erasmus University Rotterdam

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Abstract

This paper studies optimal divestment policy of an investor in a firm that may partially and gradually divest its capital or sell the whole firm at once. Partial divestment offers greater flexibility while a whole-firm transaction provides a price premium. We show that, if the price premium includes both a fixed and a proportional component, a large firm optimally starts to divest partial capital before choosing to sell the whole-firm. Full-firm divestment is preferable over partial divestment with higher profit volatility, in more declining markets and if capital is less industry-specific.

1 Introduction

Firms can downgrade their operations and release the capital to investors in response to unfavorable market conditions or a deterioration of efficiency relative to competitors. In essence, corporate assets can be either divested and sold gradually over time or the whole firm can be sold at once. These two alternative phase-out modes differ in two key aspects. On the one hand, gradual divestment allows firms to maintain flexibility and to benefit from possible future positive market developments. In this respect gradual divestment is advantageous compared to firm sale. On the other hand, partial displaced assets are sold with a discount on secondary markets whereas firms are sold with a substantial takeover premium. In this paper we study optimal divestment directly addressing the trade-off between the flexibility of gradual divestment and the premium of whole firm sale.

The flexibility advantage of gradual divestment is related to the optionality of the irreversible (dis-)investment decisions. The real options approach to investment stresses the value created by uncertainty when investment timing is flexible. In the case of

*Finance Group, Erasmus School of Economics, Erasmus University Rotterdam, PO Box 1738, 3000 DR Rotterdam, the Netherlands; Tel.: +31 (0)10 4081435; E-mail: gryglewicz@few.eur.nl.

gradual divestment, the firm holds a bundle of options to sell its partial assets. A marginal sale of assets leaves the options to sell the remaining assets and allows the firm to benefit from their optimal execution in the future. In the case of firm sale, the decision is also an option at owners discretion. The available evidence on takeover transactions supports the stance we adopt in this paper. Andrade, Mitchell and Stafford (2001) show that 94 percent of takeover transactions are initiated by the selling party.[1] While the timing of firm sales is flexible, all flexibility is lost after the firm sale and exit.

If the whole firm is sold at the same price as the sum of partial asset sales, gradual divestment is always a preferable choice. This is no longer the case if partial asset sale is associated with a discount in comparison to whole firm sale. The literature on asset sale provides strong empirical evidence for the partial asset sale discount and the firm sale premium. The discount for partial displaced capital stems from firm and sectorial capital specificity, the thinness of the used capital market and costs of redeploying the capital. For example, Ramey and Shapiro (2001) cite such reasons for substantially discounted prices of used capital relative to replacement value found in the aerospace industry. Pulvino (1998) shows that financial constraints add to depress selling prices for used aircraft in transactions between airlines. Firm sales, on the other hand, are attributed with premiums relative to some benchmark values. The two main sources of the premium are the following. First, a firm is sold with a premium over the selling price of partial physical capital because many types of intangible assets are purchased only with the full corporate entity. These assets include mainly competitive intangibles such as customer and suppliers relations, know-how and organization, and may account to a significant portion of firm value (see, e.g., Hand and Lev (2003)). Second, persistent empirical evidence documents substantial takeover premiums defined as the difference between the selling price and the value of the target firm before the transaction. A recent study of Boone and Mulherin (2007) reports a mean premium of 40 percent in the announced transaction price relative to the price of the target firm 4 weeks before the first announcement of the takeover. This means that even after controlling for intangible assets (included in the pre-announcement firm value), whole firms are sold with premiums. These takeover premiums are typically explained as originating from strategic synergies or higher productivity of the buying firm coupled with bargaining power of the selling party. Part of the surplus created by a merger is paid out to the target firm owners.

[1] Using a smaller sample, but with more detailed information, Boone and Mulherin (2007) document that 15 percent of takeover bids are unsolicited. However small is the fraction of unsolicited takeover bids, even these sale transactions leave some flexibility and discretion in the hands of the selling party. Boone and Mulherin (2007) report that most of the unsolicited bids are executed by competitive auctions to solicit bids from other potential buyers. Furthermore, Schwert (2000) shows that the so-called hostile takeover deals are economically equivalent to friendly takeovers and hostility is mostly related to strategic negotiations.

Given the above characteristics of corporate divestments, some interesting questions remain unanswered. What does the optimal downsizing path look like? How does the structure of the price discount-premium affect the choice between partial divestment and firm sale? Should firms with more volatile profits divest partially or sell at once? Do firms in more declining markets prefer gradual divestment or firm sale? Do firms with more industry-specific capital opt for gradual exit or takeover sale?

To answer these questions we construct a stylized real options model in which a firm faces a stochastic profit flow and optimally chooses its divestment path. Marginal units of capital may be released and sold at a discounted unit price. Alternatively, the whole firm can be sold at a premium price that depends on the capital level at the transaction time. To focus on the main trade-off problem between partial divestment and firm sale we assume that both decisions are irreversible. From a technical point of view, the problem is not trivial as it involves two different stochastic control technics. Partial divestment is modeled as a barrier control, and the firm adjusts capital level at each time the underlying profitability state variable reaches a new minimum on a barrier. On the other hand, whole-firm sale is a discrete decision, and the firm's problem takes the form of an optimal stopping problem.[2]

Our analysis indicates that the optimal divestment policy depends critically on the structure of the discount-premium of the capital price. We first study the simplest case, in which the firm-sale premium is linear (proportional in the level of capital). In this case, the optimal policy is either to divest only gradually if the proportional premium is below a certain threshold or to divest the whole firm if the proportional premium is sufficiently large (it is assumed here that the firm has followed the optimal divestment path before and does not start off the optimal policy path).

The optimal divestment policy takes a notably different form if the firm-sale premium is affine, i.e. if the premium consists of both proportional and fixed components. The fixed part of the premium arises because of, e.g., non-tangible assets sold only with the whole firm. In this case, if the proportional premium is sufficiently large, the firm optimally decides to use only the firm-sale option, as the premium offsets the gains from the flexibility of gradual divestment. But if the proportional premium is not too high, the firm optimally divests marginal units of capital in a declining market until its size reaches a certain threshold. Subsequently, the remaining capital is sold with the whole firm, but this only happens after an anticipation phase in which the market falls to a sufficiently low level. Intuitively, while at high levels of capital the firm prefers to maintain the flexibility of partial divestment against a moderate firm-sale premium, at

[2] Two other recent papers study corporate investment as mixed stochastic control problems. Guo and Pham (2005) analyze optimal entry and subsequent investment, and Décamps and Villeneuve (2007) deal with dividend choice and optimal exercise of a growth option of a financially constrained firm.

lower levels of capital the benefit of a positive fixed premium will offset the flexibility advantage of gradual adjustments.

The model generates some new predictions on the optimal choice of divestment policy and, specifically, on the choice between partial divestment and firm sale. We find that in more uncertain markets the value-maximizing firm is more inclined to divest its capital fully at once. This means that, somewhat surprisingly, the value of flexibility of partial divestment does not become more valuable in more volatile markets compared to one-time firm sale. This effect arises because firm sale, being less flexible, has a higher value of waiting, which is directly and positively affected by uncertainty. We also show that firm sale is more preferable over partial divestment in more declining markets. This is because in a declining market there is less room to benefit from the flexibility of gradual divestment.

We extend the model to allow the selling price of capital to be correlated with the market state variable. The correlation coefficient between the market state and the price level is interpreted as a measure of industry-specificity of capital. We model in a reduced form the effect that, in a declining market, the demand for used capital decreases, and consequently prices also fall. We are interested how the industry-specificity of capital affects optimal divestment policies. We obtain that the more industry-specific is capital, the more preferable is partial divestment over firm sale. The explanation for this result is again related to the large value of waiting in the option to sell the firm at once. Because the specificity of capital affect the values of alternative strategies mostly via the values of waiting, and increasing specificity decreases these values, firm sale becomes less desirable.

The distinction between incremental capital adjustment and full-firm sale has been noted by several previous authors. In a series of two papers Ghemawat and Nalebuff (1985, 1990) study divestment and exit in declining industries. Ghemawat and Nalebuff (1985) consider the equilibrium order of full-firm exit in an oligopolistic market, while Ghemawat and Nalebuff (1990) allows firms to adjust their capital incrementally. In contrast, our paper incorporates both modes of capital adjustment in one model with uncertain demand, but we choose not to focus on the competitive effects. Lieberman (1990) and Maksimovic and Phillips (2001) empirically study the choice between partial and whole-firm divestment. While these studies do not test the whole set of predictions implied by our model, they nevertheless provide some supporting evidence. In particular, Lieberman (1990) and Maksimovic and Phillips (2001) show that large firms adjust capital partially and small firms tend to sell their all assets at once.

The remainder of the paper is organized as follows. In Section 2 we set up a model of a firm with both partial and full-firm divestments. Section 3 derives the optimal divestment policies and the corresponding firm values. Section 4 discusses the implications of the model for divestment policies. Section 5 studies the effects of industry-specificity of capital. Section 6 concludes and the Appendix provides the proofs omitted in the main text.

2 Model

Consider a firm that produces a uniform non-storable good and faces stochastic demand. To produce the good the firm uses capital and possibly other variable inputs. The firm's operating profit at time t depends on the installed capital stock K_t and the market conditions variable X_t and is given by

$$\pi_t = \pi(X_t, K_t) = X_t K_t^{\gamma}, \quad \gamma \in (0,1). \tag{1}$$

The formulation has been frequently employed in previous studies (Bertola and Caballero (1994), Abel and Eberly (1996), Abel and Eberly (1999), Guo, Miao and Morellec (2005)) and is consistent with either a monopolist facing an isoelastic demand function and production technology with non-increasing returns to scale, or a price taking firm with decreasing returns to scale technology.[3] The investors are risk neutral and discount cash flows at a constant rate r.

The market conditions variable X_t captures the exogenous time varying business environment; more specifically X_t reflects demand shocks, but can also include productivity shocks and the prices of variable inputs (see footnote 3). We assume that the process $(X_t)_{t\geq 0}$ evolves according to the geometric Brownian motion

$$dX_t = \mu X_t dt + \sigma X_t dZ_t,$$

where Z_t is the standard Brownian motion, μ is the drift parameter and $\sigma > 0$ is the volatility parameter. We denote the filtration (the σ-algebra) generated by $(X_t)_{t\geq 0}$ with $(\mathcal{F}_t)_{t\geq 0}$. To ensure convergence of the problem, it is assumed that $\mu < r$.

Marginal units of capital can be sold at a price normalized to 1. Selling the whole firm at once results in a premium with a fixed component $A \geq 0$ and a unit price

[3] Suppose that the production function is $Q_t = K_t^{\phi}$, where Q_t is output produced at time t and $\phi \in (0,1]$ measures the degree of returns to scale. The inverse demand function is given by $P_t = X_t Q_t^{-\frac{1}{\varepsilon}}$, so that for a given quantity the price evolves in time together with the demand shock X_t. $\varepsilon > 1$ is the constant price elasticity of demand. Then instantaneous operating profit at time t is

$$\pi_t = P_t Q_t = X_t Q_t^{\frac{\varepsilon-1}{\varepsilon}} = X_t K_t^{\phi\frac{\varepsilon-1}{\varepsilon}}.$$

Defining $\gamma = \phi - \phi/\varepsilon$ we obtain (1) with $\gamma \in (0,1)$ if either the firm has a monopoly power (that is if $\varepsilon < \infty$) or the technology exhibits decreasing returns to scale ($\phi \in (0,1)$). As shown by Abel and Eberly (2004) the argument can be extended to the case with variable outputs in the production function (e.g. labor) and time varying productivity.

of capital equal to $a \geq 1$.[4] This means that the owners of the firm with a level of capital k divesting at once receive $ak + A$. The fixed premium may stem from the non-tangible assets or from a part of the takeover premium. It must be understood that our formulation incorporates the discount for partial displaced capital in the difference between a and 1, so the normalization of the selling price of partial capital is without loss of generality. Capital divestment, either marginal or complete exit, is irreversible.

The objective of the firm is to maximize the value of the original owners. The control policy comprises the choice of capital and the exit time. The admissible capital contraction is a non-increasing process $K = (K_t)_{t \geq 0}$ that is progressively measurable with respect to filtration $(\mathcal{F}_t)_{t \geq 0}$. The exit time τ is a stopping time with respect to $(\mathcal{F}_t)_{t \geq 0}$. The value of the firm following the optimal investment policy is the solution to the following optimization problem:

$$W(X_t, K_t) = \sup_{\tau} \sup_{\{K_{t+s}\}} \mathbb{E}_t \left[\int_0^{\tau - t} e^{-rs} \pi(X_{t+s}, K_{t+s}) ds + \int_0^{\tau - t} e^{-rs} dK_{t+s} + e^{-r(\tau - t)} \left(aK_\tau + A \right) \right]. \quad (2)$$

The firm's problem involves two stochastic control problems, i.e. instantaneous control over a divestment path $\{K_{t+s}\}$ and optimal stopping at a stopping time τ.

3 Optimal Divestment Policy

3.1 Benchmark Cases and Linear Premium

In this subsection we consider two limit cases. In the first case, the firm has only gradual divestment available. In the second case, the firm can only downsize by firm sale. Both cases are straightforward simplifications of the more general optimization problem (2). This analysis is then used to study the case where both divestment modes are available and the firm-sale premium is linear in capital, i.e. $a \geq 1$ and $A = 0$.

Denote by $V^m(x, k)$ the value of the firm that follows optimal divestment policy in the case the firm can only sell partial capital. The optimal policy is characterized by a barrier function $X^m(k)$ that, for a given k, triggers infinitesimal divestment (see Pindyck (1988), Abel and Eberly (1996)). The standard arguments lead to the following Bellman equation that must be satisfied by V^m:

$$rV^m(x, k) = \frac{1}{2} \sigma^2 x^2 V^m_{XX}(x, k) + \mu x V^m_X(x, k) + \pi(x, k). \quad (3)$$

[4] The unit prices of capital are time constant in the current setup, but we relax this assumption in Section 5, where we allow for stochastic capital sale prices that are correlated with the market conditions variable.

The equation states that the required rate of return (the left-hand side) must be equal to the expected gain in firm value plus profit flow $\pi(x, k)$ (the right-hand side).

The divestment trigger $X^m(k)$ and the value V^m can be obtained by solving the differential equation (3) subject to appropriate boundary conditions. At the divestment trigger the firm sells the infinitesimal capital dk for a price of 1 per unit. It must hold that $V^m(X^m(k), k) = V^m(X^m(k), k - dk) + dk$. Writing this in derivative form, we obtain the smooth-pasting condition

$$V^m_K(X^m(k), k) = 1. \tag{4}$$

The condition requires that the marginal value of capital at the optimal divestment barrier $X^m(k)$ must be equal to its selling price.

The optimality condition for $X^m(k)$ requires that the slopes of the value function are equal at $X^m(k)$. The requirement in derivative form is known as the high-contact condition (see Dumas (1991)) and is written as

$$V^m_{XK}\left(X^m(k), k\right) = 0. \tag{5}$$

Finally, we also require that, as the demand shift increases to infinity, the option value to divest remains finite. This means that[5]

$$\lim_{x \to \infty} V^m(x, k) - \frac{\pi(x, k)}{r - \mu} < \infty. \tag{6}$$

In the second extreme case, the firm has only the option to phase out by firm sale. Denote by $V^e(x, k)$ the value function of the firm following an optimal firm sale policy at trigger $X^e(k)$. Given that the values in both cases are driven by the same stochastic process and the same payoff function, it is clear that before exit, V^e must satisfy the same type of Bellman equation as before:

$$rV^e(x, k) = \frac{1}{2}\sigma^2 x^2 V^e_{XX}(x, k) + \mu x V^e_X(x, k) + \pi(x, k). \tag{7}$$

In order to obtain the firm value and the optimal trigger strategy, we need to solve (7) subject to the appropriate boundary conditions. When the trigger $X^e(k)$ is reached, the firm sells k units of capital for unit price a and obtains a non-negative fixed premium A. The value function must be equal to the proceeds from sale, which means that the value-matching condition is

$$V^e\left(X^e(k), k\right) = ak + A. \tag{8}$$

[5] The discounted expected profit flow (the second term on the left-hand side) goes to infinity as $x \to \infty$, but the remaining value, i.e. the value of the option to divest, should be finite.

The firm maximizes its value by choosing the optimal $X^e(k)$ and this requires that the slopes of the value function are equal at the sale trigger. This translates into the smooth-pasting condition at $X^e(k)$:

$$V_X^e\left(X^e(k), k\right) = 0. \tag{9}$$

In addition, the value function should be finite as X raises to infinity, so that the firm-sale option remains finite:

$$\lim_{x\to\infty} V^e(x,k) - \frac{\pi(x,k)}{r-\mu} < \infty. \tag{10}$$

Using the above analysis, we prove the first result of the mixed case where both gradual divestment and firm sale are available, and the firm sells at a proportional premium. Before we state the result, let us define a^* by

$$a^* = \frac{1}{\gamma}\left[\frac{1-\beta\left(1-\gamma\right)}{\gamma}\right]^{\frac{1}{\beta-1}}.$$

Proposition 1 *Suppose that $a \geq 1$, $A = 0$ and (X_0, K_0) is at or above the relevant triggers characterized below.*
(a) If $a < a^*$, the firm divests only via partial divestment at

$$X^m(k) = \frac{\beta}{\beta-1}\frac{1}{\gamma}\left(r-\mu\right)k^{1-\gamma},$$

and the firm value is

$$W(x,k) = B_1(k)x^\beta + \frac{1}{r-\mu}xk^\gamma,$$

where

$$B_1(k) = \frac{1}{1-\beta}\frac{k}{1-\beta\left(1-\gamma\right)}X^m(k)^{-\beta}$$

and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \leq 0.$$

(b) If $a \geq a^*$, then the firm sale trigger is given by

$$X^e(K_0) = \frac{\beta}{\beta-1}a\left(r-\mu\right)K_0^{1-\gamma}$$

and the firm value is

$$W(x,k) = B_2(k)x^\beta + \frac{1}{r-\mu}xk^\gamma,$$

where

$$B_2(k) = \frac{ak}{1-\beta} X^e(k)^{-\beta}.$$

The proposition characterizes the optimal divestment triggers and the firm values in two cases. When the proportional premium is sufficiently large, $a \geq a^*$, the whole firm is sold at once as soon as the market shock reaches $X^e(K_0)$. If $a < a^*$, the firm divests only gradually following the barrier control at $X^m(k)$. Figure 1 presents the optimal divestment policies in the two cases. The reason for this dichotomous outcome is that the proportionality of payoffs in the two alternative divestment modes translates into the proportionality of the value function. If the premium is sufficiently small, then flexibility of partial divestment always offsets the premium of firm sale. If a is sufficiently large, then the premium counterbalances the flexibility advantage of partial divestment at all levels of capital.[6]

3.2 Divestment with Non-linear Firm-sale Premium

In this section we consider a more general case of firm-sale premium and allow it to be affine in the level of capital. In other words, we assume that $a \geq 1$ and $A > 0$. The previous section shows that with $A = 0$, $a \geq a^*$ implies that $V^e(x,k) \geq V^m(x,k)$ and the firm is better off selling the whole entity. As we show next, this conveys to the affine case, but if $a < a^*$, it needs no longer be true that $V^e(x,k) < V^m(x,k)$ for all levels of capital.

Lemma 2 *Suppose that $a \geq 1$ and $A > 0$. If $a \geq a^*$, then $V^e(x,k) \geq V^m(x,k)$. If $a < a^*$, then there exists a level of capital $\tilde{k}$ that separates two regimes: $V^e(x,k) \leq V^m(x,k)$ for $k \geq \tilde{k}$, and $V^e(x,k) > V^m(x,k)$ for $k < \tilde{k}$.*

In the affine case, $V^e(x,k)$ exceeds $V^m(x,k)$ for sufficiently low k. The intuition is that at small levels of capital the benefit of achieving a positive fixed premium will

[6] The results and the conclusions presented here depend on the assumption that (X_0, K_0) is at or above the relevant triggers. The case is economically the most interesting. For the starting value to fall below the triggers, the firm must have deviated for some unmodeled reasons from the optimal policy before the initial date. Nevertheless, if $a < a^*$ and $X_0 \leq X^m(K_0)$ (in other words, the firm starts "too large" for a given market), the analysis resembles the model of Décamps, Mariotti and Villeneuve (2006) that studies an investment decision in one of two alternative projects. For a given x, there is a level of capital at which the firm is indifferent between partial divestment and whole-firm sale. Intuitively, if the firm has a high level of capital for the current (low) state of the market, it is better off selling all the capital with a premium than making a large partial adjustment at discounted prices and stay at the low market. If x falls below this indifference point, firm sale is preferable, if x rises, the value of partial divestment will exceed the value of firm sale. As in Décamps et al. (2006), it is possible to show that at the point of indifference the firm optimally does not make an divestment decision, and instead prefers to wait for the development of the market to decide for either partial adjustment, if x increases sufficiently, or firm sale, if x falls sufficiently and the market becomes unattractive for partial adjustment. The bottom line is that there is an inaction region at low levels of x for a given k, in which the firm does not make divestment decisions, but divest the whole firm if the market deteriorates and divests partially if the market *improves*.

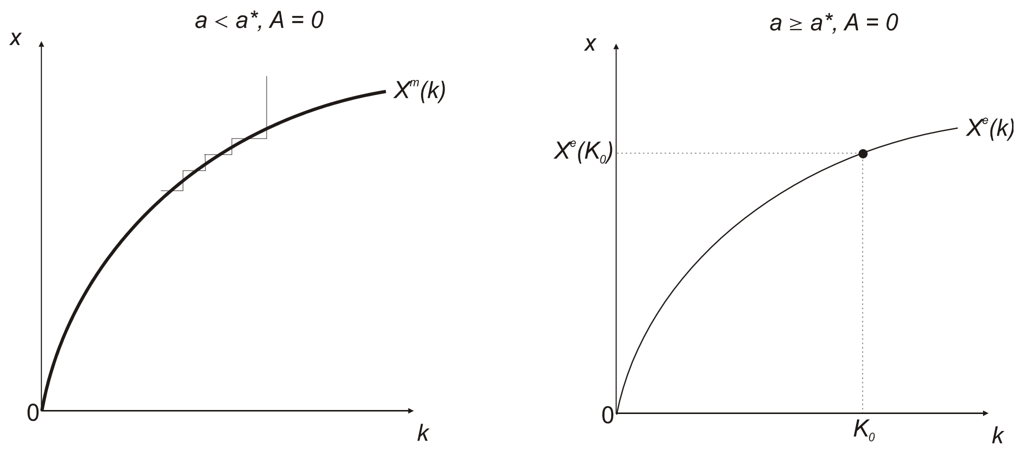

Figure 1: Divestment triggers with linear firm-sale premium. The left panel presents the case of $a < a^*$ and $A = 0$. In this case the firm divests only partially following barrier control at $X^m(k)$. The right panel presents the case $a \geq a^*$ and $A = 0$. In this case the firm divests only by firm sale at trigger $X^e(K_0)$.

offset the flexibility advantage of partial divestment. However, the inequality $V^e(x,k) > V^m(x,k)$ is only a necessary condition for whole-firm sale. Even if $V^e(x,k) > V^m(x,k)$ holds, the firm may still be better off selling some capital by partial divestment before selling the remaining capital at once. This will be the case as long as the marginal value of partial divestment exceeds the marginal value of capital sold with the whole firm. These arguments suggest that in the case of $a < a^*$, optimal divestment will take the form of a two-stage policy. If the capital level is relatively large, such that it exceeds a certain threshold on capital K^*, the firm will optimally divest partially. Below K^*, investors will be better off selling the whole firm. The aim of the remainder of this section is to characterize this policy and the corresponding firm value.

As before, it is standard to show that the value function $W(x,k)$ satisfies the following Bellman equation:

$$rW(x,k) = \frac{1}{2}\sigma^2 x^2 W_{XX}(x,k) + \mu x W_X(x,k) + \pi(x,k). \tag{11}$$

The optimal solution to the optimization problem (2) can be characterized using the differential equation (11) and the appropriate boundary conditions. As long as $k > K^*$, the marginal value of capital at the optimal divestment barrier $X^m(k)$ must be equal to its selling price. This means that the following holds

$$W_K\left(X^m(k), k\right) = 1. \tag{12}$$

The optimality condition for $X^m(k)$ requires the high-contact condition:

$$W_{KX}\left(X^m(k), k\right) = 0. \tag{13}$$

When the firm switches from the marginal divestment mode to the firm sale mode we require that the marginal values of capital from the respective policies are equal. Specifically, it must hold that

$$\lim_{k \downarrow K^*} W_K\left(X^m(k), k\right) = \lim_{k \uparrow K^*} W_K\left(X^m(k), k\right). \tag{14}$$

If the equality did not hold at K^*, the firm would increase its value by choosing another point to switch from partial to whole-firm divestment. The optimal firm sale is triggered at $X^e(k)$ and the value must satisfy the value matching condition:

$$W\left(X^e(k), k\right) = ak + A. \tag{15}$$

The condition means that the firm value must be equal to the proceeds from the sale. The optimality of the endogenous trigger requires that the value function is differentiable at the trigger, which leads to the smooth pasting condition:

$$W_X\left(X^e(k), k\right) = 0. \tag{16}$$

Before we characterize the solution of the divestment problem (2), let us define

$$R(k) \equiv \left[\gamma\left(a + \frac{A}{k}\right)\right]^{-\beta}\left[(1-\beta)\, a + \gamma\beta\left(a + \frac{A}{k}\right)\right] - 1. \tag{17}$$

Suppose $A > 0$ and $a < a^*$, and let K^* be the unique $k \geq \frac{\gamma A}{1-a\gamma}$ that satisfies $R(k) = 0$. If $a \geq a^*$, let $K^* = \infty$.

Proposition 3 *Suppose $A > 0$ and (X_0, K_0) is at or above the relevant triggers characterized below. The optimal divestment policy is characterized by the marginal divestment barrier*

$$X^m(k) = \frac{\beta}{\beta-1}\frac{1}{\gamma}\left(r-\mu\right)k^{1-\gamma} \quad \text{if } k > K^*$$

and the firm sale trigger is

$$X^e(k) = \frac{\beta}{\beta-1}\left(r-\mu\right)\left(ak+A\right)k^{-\gamma} \quad \text{if } k \leq K^*.$$

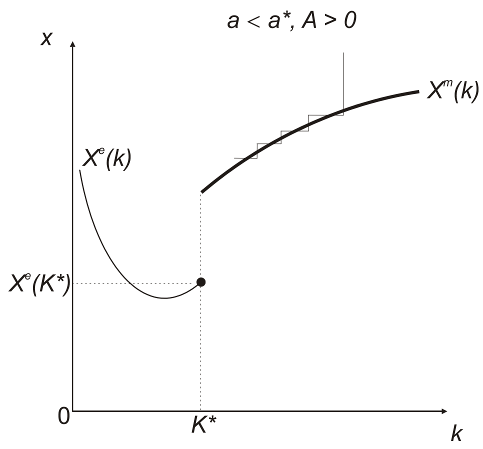

Figure 2: Divestment triggers with affine firm-sale premium ($A > 0$) and $a < a^*$. The firm divests partially following the barrier control at $X^m(k)$ as long as $k > K^*$. If $k \leq K^*$ the firm divests the remaining capital at trigger $X^e(k)$.

The firm value is given by

$$W(x,k) = \begin{cases} B_3(k)x^\beta + \frac{1}{r-\mu}xk^\gamma & \text{if } k \geq K^* \text{and } x \geq X^e(k) \\ B_4(k)x^\beta + \frac{1}{r-\mu}xk^\gamma & \text{if } K^e \leq k \leq K^* \text{ and } x \geq X^e(k), \end{cases} \tag{18}$$

where

$$B_3(k) = \frac{1}{\beta-1}\frac{1}{\beta(\gamma-1)+1}\left(kX^m(k)^{-\beta} - K^*X^m(K^*)^{-\beta}\right) + B_4(K^*),$$

$$B_4(k) = \left(ak + A - \frac{1}{r-\mu}X^e(k)k^\gamma\right)X^e(k)^{-\beta},$$

and β is as characterized in Proposition 1.

4 Analysis and Implications

Proposition 3 characterizes the optimal divestment path. The optimal policy is illustrated in Figure 2 and can be described as follows. The firm divests marginally if the capital level is relatively high, above K^*, and whenever x reaches the divestment barrier $X^m(k)$. As soon as capital reaches K^*, the firm stops partial divestment. This is confirmed by Proposition 3, which states that partial divestment stops at $X^m(K^*)$ and firm sale is triggered by $X^e(K^*)$. As in general $X^m(K^*)$ will exceed $X^e(K^*)$, the optimal divestment path is characterized by an anticipation region, in which the firm does not divest marginally. Instead it waits until a sufficiently negative profitability shock occurs. This triggers firm sale and exit.

Figure 2 clearly illustrates the prediction of the model on the relationship between firm size and divestment policies. Large firms divest partially and small firms divest by firm sale. This prediction finds a strong confirmation in the evidence presented by Maksimovic and Phillips (2001). They find that the average firm that sells partial capital (partial divisions) has revenues of \$1.859 billion and operates 23.7 plants, and the average firm that sells in a merger has revenues of \$51 million and operates 1.78 plants.

An interesting special case is a premium with only the fixed component $A > 0$ and no proportional one, that is $a = 1$. In this case, K^* can be characterized explicitly by

$$K^* = \frac{\gamma A}{1 - \gamma}.$$

The firm size at which the firm is sold is increasing in the fixed premium A and in the level of returns to scale γ. The case of $a = 1$ is also special because the anticipation region $X^m(K^*) - X^e(K^*)$ disappears and the firm continuously moves from partial divestment to full-firm sale.

We are interested in the impact of parameters characterizing the firm and its environment on the choice between partial divestment and firm sale. We first consider the effects of uncertainty represented by the volatility parameter σ in the X_t process.

Proposition 4 *a^* decreases in σ. K^* increases in σ if $a \in (1, a^*)$.*

The proposition states that the effect of uncertainty on the preference between the flexibility of partial divestment and the premium of firm sale is unequivocal. The cutoff level of a that makes the firm to opt for full-firm sale decreases in the level of uncertainty. This means that in a more uncertain environment, the firm prefers full-firm sale for a larger set of parameters. This same kind of prediction is implied by the effect of σ on K^*: the firm exits with higher level of capital after some partial divestment.

These results may seem surprising at first. From the standard real options theory we know that higher uncertainty increases the value of waiting. One might expect that the flexibility advantage of partial divestment is more valuable in a more uncertain market. We find the opposite and the intuition for our result is the following. Firm sale is one irreversible real option and, as such, has a substantial value created by the value of waiting. Partial gradual divestment forms a sequence of real options, and despite the fact that these marginal divestment decisions are irreversible, the whole policy is, in a sense, less irreversible than firm sale. Hence the optimal gradual investment policy takes less into account the value of waiting and the value of the policy will be less responsive to the parameters affecting the value of flexibility.[7] Consequently, the value of firm sale

[7] These observations are similar to Malchow-Moeller and Thorsen (2005) who constrast repeated

is more responsive to the changes in uncertainty than is the value of gradual partial divestment and the former value increases more in σ making firm sale more attractive.

Proposition 5 *a^* increases in μ. K^* decreases in μ if $a \in (1, a^*)$.*

The result in the proposition implies that in a more declining market, the option to sell the whole firm and exit becomes more preferable over gradual divestment. In particular, with lower μ, the cutoff premium a^* decreases and the size of full-firm sale K^* increases. Intuitively, in a more declining market, there is less room to benefit from the flexibility of gradual divestment.

5 Industry-specific Capital and Divestment

The price of capital has been fixed in the above formulation. Arguably, in a declining market the selling prices of capital are linked with the state of the market. One reason for prices changing together with market/profitability shocks is industry-specificity of capital. If capital is less productive outside industry, then, after a negative industry-related shock, demand for displaced capital falls and prices decrease. The argument is in line with the industry-equilibrium model of Shleifer and Vishny (1992). Their paper explicitly models potential buyers of displaced capital and predicts that negative industry-specific shocks and financing constraints will result in depressed prices of used capital.

We model these effects in a reduced form by linking the capital price P_t with the market/productivity process X_t. Specifically, we suppose that the evolution of X_t and P_t is given by

$$dX_t = \mu_X X_t dt + \sigma_X X_t (dZ_X)_t$$

and

$$dP_t = \mu_P P_t dt + \sigma_P P_t (dZ_P)_t,$$

where $\mathbb{E}[(dZ_X)_t (dZ_P)_t] = \rho dt$. We interpret the correlation coefficient ρ as the parameter measuring the industry-specificity of capital. A high positive ρ means that capital is industry specific and a decline in X_t results, on average, in a deflated capital price. To ensure that the problem is well defined and has a finite solution we assume that $\mu_X < r$ and $\sigma_X^2 - 2\rho\sigma_X\sigma_P + \sigma_P^2 > 0$.

The extension with variable capital price adds to the complexity of the model. In order to stay in a tractable environment we assume in this section that the whole firm sells only at a proportional premium, that is $A = 0$ and $a \geq 1$. To summarize, a unit

investment options and a single investment option.

of capital divested partially at time t sells at price P_t, and the firm holding K_t units of capital sells at aP_tK_t.

In this setup we are interested in the impact of industry-specificity of capital on the optimal divestment policy. We obtain the following result.

Proposition 6 *The more industry-specific is capital (the higher is ρ), the more preferable is gradual partial divestment over firm sale.*

The intuition for the result is related to the value of waiting created by the divestment options. The usual prediction of the real options theory is that in an environment as in this section, the value of waiting decreases if productivity and capital price are more correlated (see, e.g., Hartman and Hendrickson (2002)). As discussed in Section 4, the value of waiting is larger for the single option to sell the whole firm than for the sequence of marginal options to divest partially. Thus increasing ρ decreases the value of firm sale more than the value of gradual divestment. To put it differently, when capital is highly industry-specific (high ρ), then, after waiting for the market to deteriorate sufficiently to trigger full-firm sale, the firm will, with high probability, sell its capital at low prices. Consequently, the firm's preference moves towards gradual divestment.

6 Conclusions

The paper has studied divestment decisions and addressed directly the trade-off between the flexibility of gradual divestment and the price premium from full-firm sale. It provides analytical results for firm values and optimal divestment policies under alternative premium-discount structures. In particular, if the firm-sale premium is affine, the firm optimally divests marginal units of capital in a declining market until its size reaches a certain threshold. Subsequently, but after an anticipation phase in which the state of market falls to a sufficiently low level, the remaining capital is sold with the whole firm.

The model produces a number of novel predictions on the optimal choice of divestment policy and, specifically, on the choice between partial divestment and firm sale. We analyze the impact of displaced capital discount, firm sale premium, firm size, profit volatility, market growth and industry-specificity of capital. Future empirical research could directly test these predictions.

Future research should also explore if the same mechanisms that are described in this paper carry over when competition and potential buyers of capital are modeled explicitly. It may be particularly interesting to study a dynamic oligopoly model of a shrinking industry in which firms play a war of attrition as, for example, in Murto (2004), but then to allow firms to undertake partial divestment and takeovers.

The framework presented in the paper can be adapted to study the other side the capacity adjustment decision, namely investment. It will be interesting to consider a combination of gradual capital expansion and discrete technological change, analogously to capital downsizing and firm sale analyzed in this paper. The problem of capital accumulation and technology investment has received considerable attention in deterministic models (see, e.g., Feichtinger, Hartl, Kort and Veliov (2006)), but has not been addressed in the stochastic framework of real options.

A Appendix: Proofs

Proof of Proposition 1. Solving (3) subject to (4)-(6), we obtain that

$$X^m(k) = \frac{\beta}{\beta - 1} \frac{1}{\gamma} (r - \mu) k^{1-\gamma},$$

and, if $x \geq X^m(k)$,

$$V^m(x,k) = \frac{1}{1-\beta} \frac{k}{1-\beta(1-\gamma)} \left(\frac{x}{X^m(k)}\right)^\beta + \frac{1}{r-\mu} x k^\gamma.$$

The solution to (7) subject to (8)-(10) is

$$X^e(k) = \frac{\beta}{\beta - 1} (r - \mu) \left(a + \frac{A}{k}\right) k^{1-\gamma},$$

and, if $x \geq X^e(k)$, then

$$V^e(x,k) = \left[a - \frac{\gamma\beta}{\beta - 1} \left(a + \frac{A}{k}\right)\right] \left(\frac{x}{X^e(k)}\right)^\beta + \frac{1}{r-\mu} x k^\gamma.$$

Now suppose that $A = 0$ and $x \geq \max\{X^e(k), X^m(k)\}$. Using the value functions characterized above, we have that

$$V^m(x,k) - V^e(x,k) = \frac{k}{1-\beta} \left(\frac{x}{X^e(k)}\right)^\beta \left[\frac{a^\beta \gamma^\beta}{1-\beta(1-\gamma)} - a\right].$$

The sign of the expression depends on the sign of the term in the square brackets. This means that if $a \geq a^*$ then $V^m(x,k) \leq V^e(x,k)$ and if $a < a^*$ then $V^m(x,k) > V^e(x,k)$.

In the case of $a < a^*$, the value of gradual divestment always exceeds the value of firm sale, so it is never optimal for the firm to choose the latter strategy. It follows that the optimal trigger policy of the firm with both divestment strategies available is given by $X^m(k)$ and its value W is equal to the value of the firm with marginal divestment $V^m(x,k)$.

In the case of $a \geq a^*$, the value of strategy comprising of *only* gradual divestment is always below the value of optimal firm sale. To conclude that the firm does not divest gradually, we still need to rule out a strategy consisting of some gradual divestment followed by firm sale. Suppose the firm divests a marginal unit of capital before the whole firm is sold. The marginal value of capital that is sold optimally by partial divestment is equal to $V_K^m(x, k)$ if $x > X^m(k)$ and equal to 1 if $x \leq X^m(k)$. In the first case, if $x > X^m(k)$, comparing this marginal value with the marginal value of capital from firm sale, we have that

$$V_k^m(x, k) - V_k^e(x, k) = \frac{1}{1-\beta}\left(\frac{x}{X^e(k)}\right)^\beta \left\{a^\beta \gamma^\beta - [1-\beta(1-\gamma)]\, a\right\} \leq 0,$$

which is non-positive because $a \geq a^*$. In the second case, if $x = X^m(k)$, the difference in marginal values is

$$1 - V_k^e(X^m(k), k) = \frac{1}{1-\beta}\left\{1 - [1-\beta(1-\gamma)]\, a^{1-\beta}\gamma^{-\beta}\right\} \leq 0.$$

The last inequality holds because $a \geq a^*$. It can be easily verified that for $X^e(k) \leq x \leq X^m(k)$, $V_k^e(X^m, k)$ is decreasing in x, so the difference $1 - V_k^e(x, k)$ remains non-positive (to see that $V_k^e(X^m, k)$ is decreasing in this interval, observe that $V_{xk}^e(X^m(k), k) < 0$ and that $V_{xk}^e(x, k)$ is a convex function on the relevant interval). It follows that the marginal value of capital sold by the firm sale always exceeds the marginal value of capital from partial divestment, so the maximizing firm never chooses to divest partially. ■

Proof of Lemma 2. The same steps that in the proof of Proposition 1 lead to the following formula for the difference between the values:

$$V^m(x, k) - V^e(x, k) = \frac{k}{1-\beta}\left(\frac{x}{X^e(k)}\right)^\beta \left[\xi - \frac{A}{k}\right],$$

where $\xi \equiv a^\beta \gamma^\beta [1-\beta(1-\gamma)]^{-1} - a$. It was also shown there that $\xi \leq 0$ is equivalent to $a \geq a^*$. It follows that $a \geq a^*$ implies that $\xi \leq A/k$ for all $k \geq 0$. Thus $a \geq a^*$ implies that $V^e(x, k) \geq V^m(x, k)$.

In the case of $a < a^*$, it holds that $\xi > 0$. So there exists $\tilde{k} > 0$ such that $\xi = A/\tilde{k}$. Moreover, $V^m(x, k) > V^e(x, k)$ if $k > \tilde{k}$, and $V^m(x, k) < V^e(x, k)$ if $k < \tilde{k}$. ■

Proof of Proposition 3. It is straightforward to verify that (18) satisfies (11)-(13) and (15)-(16) for a given K^*. Note that $\lim_{k \downarrow K^*} W_K(X^m(k), k) = 1$. Now we consider two cases to verify (14). First, if K^* is such that $X^e(K^*) > X^m(K^*)$, then the firm is sold at $X^m(K^*)$, and so $\lim_{k \uparrow K^*} W_K(X^m(k), k) = a$. It follows that, as long as $a > 1$, (14) cannot be satisfied if $X^e(K^*) > X^m(K^*)$. Second, we consider $X^e(K^*) \leq X^m(K^*)$, which can be shown to be equivalent to $k \geq \frac{\gamma A}{1-a\gamma}$. Applying then (14) to (18) we obtain that K^* must satisfy $R(K^*) = 0$. To verify that K^* is unique in the case of $a < a^*$, we show that there is a unique root to $R(k) = 0$ if $k \geq \frac{\gamma A}{1-a\gamma}$. It can be easily checked that $R'(k) < 0$ if $k > \frac{\gamma A}{1-a\gamma}$. Moreover, $R(\frac{\gamma A}{1-a\gamma}) = (1-\beta)^{-1}(a-1) \geq 0$. So $R(k)$ is monotonically decreasing starting from a positive value. Whether $R(k)$ has a root for $k > \frac{\gamma A}{1-a\gamma}$ depends on a. Note that $\lim_{k\to\infty} R(k) = \gamma^{-\beta} a^{1-\beta} [1-\beta(1-\gamma)] - 1$ is negative if $a < a^*$ and positive if $a > a^*$. We conclude that if $a < a^*$ there exists a unique finite K^* such that (14) holds. If $a \geq a^*$, the marginal value of capital sold with the whole firm always exceeds the marginal value of capital sold partially and $K^* = \infty$. ■

Proof of Proposition 4. We first consider the effect on a^*. σ influences a^* via β. Taking the derivative of a^* with respect to β we have that

$$\frac{da^*}{d\beta} = \frac{a^* \eta_1}{(1-\beta)^2},$$

where

$$\eta_1 = \frac{(1-\gamma)(1-\beta)}{1-\beta(1-\gamma)} - \log\left[\frac{1-\beta(1-\gamma)}{\gamma}\right].$$

The sign of the derivative depends on the sign of η_1, which is a sum of a positive and negative term. We now show that η_1 is always less or equal to zero. Observe that η_1 increases in $\beta \leq 0$:

$$\frac{d\eta_1}{d\beta} = \frac{(1-\gamma)^2(1-\beta)}{[1-\beta(1-\gamma)]^2} \geq 0.$$

Moreover, $\lim_{\beta\to 0} \eta_1 = 1 - \gamma + \log\gamma < 0$ for all $\gamma \in (0,1)$. Thus η_1 is non-positive for all $\beta \leq 0$ and consequently $da^*/d\beta \leq 0$. Finally, it is straightforward to verify that $d\beta/d\sigma > 0$ so $da^*/d\sigma \geq 0$ as stated in the proposition.

Now consider the derivative of K^* with respect to σ. Recall that if $a \in (1, a^*)$, then K^* is the unique $k \geq \gamma A/(1-a\gamma)$ such that $R(k) = 0$. Thus

$$\frac{dK^*}{d\sigma} = -\frac{\partial R/\partial\sigma}{\partial R/\partial K^*}.$$

First, let $\eta_2 = [\gamma(a + A/k)]^{-\beta}$ and consider $\partial R/\partial\beta$:

$$\begin{aligned}\frac{dR}{d\beta} &= \eta_2 \left\{-\log\left[\gamma\left(a+\frac{A}{k}\right)\right]\left[(1-\beta)a + \gamma\beta\left(a+\frac{A}{k}\right)\right] - a + \gamma\left(a+\frac{A}{k}\right)\right\} \\ &= -\frac{1}{\beta}(a\eta_2 - 1 - \log\eta_2) > 0,\end{aligned}$$

where in the second equality we twice use substitutions implied by $R(k) = 0$, and the inequality follows from the observation that $\eta_2 - 1 \geq \log\eta_2$ for all positive η_2 with equality holding only at $\eta_2 = 1$. Combined with the previous observation that $d\beta/d\sigma > 0$, we have that $dR/d\sigma > 0$. Second, consider $\partial R/\partial K^*$:

$$\frac{\partial R}{\partial K^*} = -\beta\left(\beta - 1\right)\frac{\gamma A}{k^2}\left[\gamma\left(a + \frac{A}{k}\right)\right]^{-\beta-1}\left[\gamma\left(a + \frac{A}{k}\right) - a\right] < 0.$$

The inequality follows from the fact that $\gamma a \leq \gamma\left(a + A/k\right) \leq 1$ for $k \geq \gamma A/\left(1 - a\gamma\right)$. Combining the above observations we obtain that $dK^*/d\sigma > 0$. ■

Proof of Proposition 5. The proof is very similar to the proof of Proposition 4. μ affects a^* and K^* only via β. The only difference in comparision to the effect of σ in Proposition 4 is that—as can be readily verified—now we have that $d\beta/d\mu < 0$. Applying this to the derivatives in the proof of Proposition 4 we obtain the result. ■

Proof of Proposition 6. The firm optimization problem is now the following

$$W(X_t, P_t, K_t) = \sup_{\tau} \sup_{\{dK_{t+s}\}} \mathbb{E}_t \left[\int_0^{\tau-t} e^{-rs}\pi(X_{t+s}, P_{t+s}, K_{t+s})ds + \int_0^{\tau-t} e^{-rs}P_{t+s}dK_{t+s} + e^{-r(\tau-t)}aP_\tau K_\tau\right]. \quad (19)$$

We take the same strategy as in Section 3.1 and Proposition 1. That is we suppose that (X_0, P_0, K_0) is at or above the relevant triggers and we consider two limit cases, one in which the firm has available only partial divestment and one in which the firm can only divest all capital at once. Both cases are straightforward simplifications of the more general optimization problem (19). Denote by $V^m(x, p, k)$ the value function of the firm following optimal partial divestment and by $V^e(x, p, k)$ the value function of the firm following optimal firm-sale policy. The value functions $V^\theta(x, p, k)$, $\theta \in \{m, e\}$, must satisfy the following partial differential equation (where we omit the function arguments for brevity):

$$rV^\theta = \frac{1}{2}\sigma_X^2 x^2 V_{XX}^\theta + \frac{1}{2}\sigma_P^2 p^2 V_{PP}^\theta + \rho\sigma_X\sigma_P xpV_{XP}^\theta + \mu_X xV_X^\theta + \mu_P pV_P^\theta + xk^\gamma. \quad (20)$$

Using that $V^\theta(x, p, k)$ is homogeneous of degree one in x and p, we can simplify the problem and reduce one state variable. Let $y = x/p$ and $v^\theta(y, k) = V^\theta(x/p, 1, k) = V^\theta(x, p, k)/p$. This implies that $V_X^\theta = v^\theta{}_Y$, $V_{XX}^\theta = v^\theta{}_{YY}/p$, $V_P^\theta = v^\theta - yv_Y^\theta$, $V_{PP}^\theta = y^2 v^\theta{}_{YY}/p$ and $V_{XP}^\theta = -yv_{YY}^\theta/p$. Then we can rewrite (20) in terms of v^θ:

$$\left(r - \mu_P\right)v^\theta = \left(\frac{1}{2}\sigma_X^2 - \rho\sigma_X\sigma_P + \frac{1}{2}\sigma_P^2\right)y^2 v^\theta{}_{YY} + \left(\mu_X - \mu_P\right)yv_Y^\theta + yk^\gamma.$$

The two ordinary differential equations for $\theta = m$ and $\theta = e$ have known general analytical solutions and are solved for the optimal value and divestment policy by setting appropriate boundary conditions. In the case of $\theta = m$, the optimal policy takes the form of barrier control at lower boundary $Y^m(k)$ in the space (y, k). We set the boundary conditions similar to conditions (4)-(6), i.e. $v_X^m(Y^m(k), k) = 1$, $v_{XK}^m(Y^m(k), k) = 0$ and the finiteness condition as y goes infinity. In the case of $\theta = e$, the optimal policy takes the form of an exit trigger $Y^e(k)$. The boundary conditions in this case are similar to the conditions (8)-(10), i.e. $v^e(Y^e(k), k) = ak$, $v_X^e(Y^e(k), k) = 0$ and the finiteness condition as y goes infinity.

Applying the boundary conditions we obtain in the case of $\theta = m$ that

$$Y^m(k) = \frac{\beta_1}{\beta_1 - 1}\frac{1}{\gamma}(r - \mu_X)\, k^{1-\gamma},$$

and, if $x/p \geq Y^m(k)$,

$$V^m(x, p, k) = pv^m(y, k) = \frac{1}{1-\beta_1}\frac{pk}{1-\beta_1(1-\gamma)}\left(\frac{x/p}{Y^m(k)}\right)^{\beta} + \frac{1}{r-\mu_X}xk^{\gamma},$$

where β_1 is the negative root of the quadratic equation:

$$\left(\frac{1}{2}\sigma_X^2 - \rho\sigma_X\sigma_P + \frac{1}{2}\sigma_P^2\right)\beta(\beta - 1) + (\mu_X - \mu_P)\beta + \mu_P - r = 0. \tag{21}$$

In the case of $\theta = e$, we have

$$Y^e(k) = \frac{\beta_1}{\beta_1 - 1}(r - \mu_X)\, ak^{1-\gamma},$$

and, if $x/p \geq Y^e(k)$, then

$$V^e(x, p, k) = pv^e(y, k) = ap\frac{1-\beta_1(1-\gamma)}{1-\beta_1}\left(\frac{x/p}{Y^e(k)}\right)^{\beta} + \frac{1}{r-\mu_X}xk^{\gamma}.$$

As in Proposition 1 we compare the values from the two limit policies, namely V^m and V^e. Straightforward calculations following the argument in Proposition 1 lead to the conclusion that there is a threshold level of a^* on a such that partial divestment is preferable over firm sale if $a < a^*$, and if $a \geq a^*$ the firm will optimally sell at once without partial divestment. It can be verified that

$$a^* = \frac{1}{\gamma}\left[\frac{1-\beta_1(1-\gamma)}{\gamma}\right]^{\frac{1}{\beta_1-1}}.$$

The derivative of a^* with respect to β_1 is the same as the one analyzed in the proof of Proposition 4, and it was shown there that $da^*/d\beta_1 \leq 0$. Differentiating (21) we obtain that $d\beta_1/d\rho < 0$. It follows that $da^*/d\rho \geq 0$, or in words, that with higher ρ the firm requires more premium to optimally choose firm sale over partial divestment. ■

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