THE INFLUENCE OF THE PRICE DIFFUSION PROCESS IN THE VALUE AND IN THE OPTIMAL HARVESTING TIME OF A BRAZILIAN EUCALYPTUS FORESTRY INVESTMENT

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ABSTRACT

This paper examines a problem that is usual in investment theory: when to cut a stand of trees in an investment project of eucalyptus reforestation for cellulose production, under stochastic prices of wood. The real options approach that was used in this study has shown to be quite convenient for this type of analysis. The objective of this paper is to verify the influence of the price diffusion process in the value of a stand of trees and in the optimal cutting time. Two different processes were used, the Geometric Brownian Movement (GBM) and the process of geometric mean reversion (GMR), known as Geometric Ornstein-Uhlenbeck.

Key-words: Processes of price diffusion, real options, eucalyptus forest.

1. INTRODUCTION

This paper examines a problem that is usual in investment theory: when to cut a stand of trees in a project of eucalyptus reforestation for cellulose production. Such investment project was treated as a real option, similar in nature to an American call option, with the costs of crop as the exercise price of the option and under stochastic wood prices.

This modeling leads to the problem of evaluating the value of an American type call option on an asset that pays dividends and has an exercise price different from zero. As there is no known analytical solution for this problem, it is necessary to numerically solve the partial differential equations (PDE) that result from the modeling.

The numeric solution used in this paper is known as totally implicit finite difference method, which leads to a system of linear equations that was solved with an algorithm denominated projected successive over relaxation (PSOR), with the help of a software written in C especially for this purpose.

It is relevant of the academic point of view and of the practical point of view to quantify the managerial flexibility in the evaluation of projects of capital investment under uncertainty. This was done in this paper through a real option approach. We do not know about any other paper that has applied that model in the evaluation and in the determination of the optimal cutting time of a stand of eucalyptuses in Brazil, using two different price diffusion processes and comparing one another and solving the partial differential equations (PDE) with the totally implicit finite difference method and the resultant linear system of equations with the interactive technique known as projected successive over relaxation (PSOR).

The main objective of this paper is to verify the influence of the price diffusion process in the value and in the optimal cutting time of a eucalyptus forestry investment.

2. THEORETICAL FRAMEWORK

2.1. Real options

Flexibility has value. Although this statement is conceptually obvious, the practice has shown to be surprisingly complex. The presence of flexibility is not reflected entirely in the traditional techniques of investment analysis, as discounted cash flow, simulation of Monte Carlo and decision trees, but the models based on options, developed in the last decades, are capable to capture the impact of flexibility in the decisions of resource allocation (TRIGEORGIS, 2000).

The analysis of investment opportunities as options are the product of more than a decade of research of several economists and it is a topic still very active in the academic newspapers. The traditional rules ignore the irreversibility of an investment and the several options that exist, the option of postponing the investment, the option of abandonment, etc. For the same reason this theory also contradicts the orthodox economical vision of offer and demand, that goes back to Marshall, according to which a company enters the market and expands when the price exceeds the long term medium cost of and leaves or contracts when the price falls below the medium variable cost (DIXIT & PINDICK, 1994).

Thus many academics and managers already recognize that the traditional approach of the method of discounted cash flow for the evaluation of projects of capital investment is not capable to capture the flexibility of the administration to adapt appropriately and to review decisions in response to unexpected evolutions in the market and to the average flow of information (TRIGEORGIS, 2000).

The literature on the stochastic problem of cutting a stand of trees began in the eighties, with works such as Malliaris and Brock (1982), Brock et al. (1988), Brock and Rothschild (1984), and Miller and Voltaire (1983). In general these works present models that show how optimal harvest rules and asset value can be determined, when timber price P follows a Geometric Brownian Movement (GBM), given by:

$$dP = \mu P dt + \mu P dz \qquad (1)$$

where μ is the constant drift rate, σ is the constant variance rate, and dz the increment of a Wiener process.

Clarke & Reed (1989) and Reed & Clarke (1990) demonstrated that, if the price follows a GBM and if harvesting costs are ignored, then a barrier rule can be specified for the optimal cutting time. The barrier rule results in terms of optimal cutting age, when the growth in wood volume is specified as age dependent and it results in terms of optimal cutting size and when the growth in wood volume is specified as size dependent. Neither depends on the absolute level of wood prices. Given stochastic prices, the choice of the optimal time to cut a stand of trees represents a real option similar to an American type call option. The assumption of Clarke & Reed (1989) that the prices evolve according to a GBM and the exclusion of harvesting costs allows for an analytical solution of the real option. This is similar to valuing an American call option on a dividend-paying stock with a zero exercise price. Once harvesting costs are explicitly included, the option can no longer be solved analytically (INSLEY, 2002).

Other papers that studied optimal harvesting under stochastic prices are Morck et al. (1989), Yin and Newman (1997), and Thomson (1992), all assumed that the prices of the wood follow GBM. Conrad (1997) and Forsyth (2000) calculate the minimum additional income needed to preserve an old forest when that additional income is stochastic. Reed (1993) modeled both the income to cut the trees and the additional income from keeping the forest, as stochastic variables following a GBM. Saphores et al. (2000) examined the impact of jumps (jumps) in the harvesting decision. Haight and Holmes (1991) and Plantinga (1998) considered the implications of assuming that the prices follow a random walk without drift which was compared with mean reversion.

The assumption of MBG adopted in many of the previous works lead to some unrealistic implications for the behavior of real commodity prices. On one side, the expected value and the variance of prices grow without limit. As observed by Schwartz (1997), basic microeconomic reasoning would suggest that when the price of a commodity is relatively high, the offer of this commodity increases as new producers with higher costs enter the market, putting a downward pressure on prices and vice-versa. Such behavior is more likely to result in a Mean Reverting price path instead of one that follows GBM. Besides, it can be seen by Equation (1) that if price under GBM reach zero by any chance, it will stay zero forever. It is not expected that the price of a commodity reach zero, but this characteristic of GBM also affects the value of the option when the price of the underlying asset is low, therefore the option value estimates will be incorrect for low prices of the underlying asset (INSLEY, 2002).

The paradigm of wood production in the world has been changing from harvesting existent trees, to harvesting reforestation trees (SEDJO, 1997). This paradigm change suggests that in the future the prices of the wood are more likely to follow a behavior similar to a Geometric Mean Reversion (GMR), with an average that contemplates in the long period the marginal cost of production, than a GBM. This provides an additional motivation to analyze the impact in the value of a forest investment with the prices reverting to the average.

2.2. Numeric solutions

Closed analytical solutions for valuing real options are seldom found in practice, thus it is necessary to make use of numerical solutions which are much more common.

The usual numerical solutions are binomial trees, trinomial trees, Monte Carlo simulation, finite difference methods, and more rarely the method of finite elements. Wilmott (2000) affirms that in practice he uses the method of the finite differences in 75% of the cases, Monte Carlo simulation in 20% of the cases, and explicit methods in the remaining 5% of the cases. These explicit methods, he explains, they are almost always the Black-Scholes formulas for pricing call options and put options and he adds that only once he used seriously a binomial method, even so the method was used to help modeling the option rather than for the numerical analysis. In Brazil there are several other papers such as Sato (2004) that have used binomial trees to solve real options, and some that have used trinomial trees such as Nogueira (2005).

2.3. Finite differences

Finite differences are approximations to derivatives. There are basically two sources of errors in finite difference numerical methods. One source of error is the truncation error that happens in the space discretization, the other error source is the truncation error that happens in the time variable discretization. It is necessary to deal with three fundamental aspects of numerical methods: 1 - consistence; 2 – stability; and 3-convergence.

Richtmyer (1957) mentions and demonstrates the Lax theorem that basically show that given a linear initial value problem, appropriately specified, and a solid finite difference method, stability is the necessary and sufficient condition for convergence. This result is important and it makes the care and concern with stability more relevant.

2.4. Fully Implicit Finite Differences

The differential equation that an option should satisfy (HULL, 2002) is given by:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \tag{2}$$

In the fully implicit finite difference method, the discretization assumes the following form:

$$a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j} \tag{3}$$

With: $a_j = \tfrac{1}{2} rj\delta t - \tfrac{1}{2}\sigma^2 j^2 \delta t$; $b_j = 1 + \sigma^2 j^2 \delta t + r\delta t$; $c_j = -\tfrac{1}{2} rj\delta t - \tfrac{1}{2}\sigma^2 j^2 \delta t$ (4)

Equation (4) is the discretized equation for the fully implicit finite difference method, which furnishes the relationship among three values of the option in time $i\delta t$, that is, $f_{i,j-1}$, $f_{i,j}$ and $f_{i,j+1}$, and a value of the option in the time $(1 + i)\delta t$, that is, $f_{i+1,j}$. As the problem is being solved backward (going back in time), then the fact of having only one known value which is suppose to determine three unknown values lead to a system of linear equations inside the mesh. In the ends of the mesh it is necessary to take into account the boundary conditions, in order to have the same number of equations and unknowns.

The greater complexity of this method is compensated thoroughly by its stability and convergence. The method is convergent and it is said to be unconditionally stable, not having limitations for the proportion between price interval and time interval in the discretization grid (BURDEN and FAIRES, 2003).

2.5. Interactive methods

The interactive method usually used to solve the system of linear equations produced by the fully implicit finite differences is the method known as Successive Over-Relaxation (SOR) or, in the case of American type options that have to respect the restriction of early exercise, the method of the Projected Successive Over-Relaxation (PSOR). Both are derived from the Gauss-Seidl method, which in turn is derived from the Jacobi method. The PSOR method consists of verifying the restriction of early exercise taking in each knot of the grid the maximum value between the calculated price and the exercise price, therefore this is the suitable method for American type options.

3. METHODOLOGY

The choice of the optimal time to cut of a stand of trees of homogeneous ages will be modeled as a real option, similar to an American type call option, with harvesting costs as the exercise price of the option.

Harvesting the trees generates to the proprietor revenue from selling the logs but it also implies in several costs including harvesting costs, loss of future benefits from other incomes (liquid of the administration costs) that could occur if the forest is left standing, and loss of additional volume of wood that would occur if the the trees were allowed to grow for one more period. Under random prices, if the crop is postponed until the next period, the proprietor also faces uncertainty regarding prices that could be higher or lower than they are in the current period. Assuming that timber prices, P, follow some known stochastic process that is given in general terms by:

$$dP = a(P,t)dt + b(P,t)dz \tag{5}$$

where: a (P,t) is the drift term, and b(P,t) it is the variance term.

The terms a(P,t) and b(P,t), are non stochastic known functions, and dz is the increment of a Wiener process. It is assumed that wood volume in a stand of trees, X, grows according to a known deterministic function, dependent of age: dX(t) = g(t) dt.

The decision to harvest can be specified as an optimal stopping problem that is solved using the technique of dynamic programming (DIXIT and PINDYCK, 1994). The income is designated by R and the wood volume by X. Therefore R = X.P where P follows the stochastic process shown in the Equation (6). The Bellman equation is given below, where V(R, t) it is the value of the option of cutting the forest in the time t:

$$V(R,t) = \max\left[R(t) - K; A(t)\Delta t + (1 + r\Delta t)^{-1} E\left(V(R + \Delta R, t + \Delta t)\right)\right] \tag{6}$$

In the Equation (6), R(t) is the income when the trees are cut in time t, K is the harvesting costs, A(t) it is an eventual additional income (liquid of administration costs) originating from the standing forest in period t, if the trees are not cut (this income can be generated, for instance, by amenity and for that matter, in this paper, this eventual additional income will be from now on called amenity). The instantaneous discount interest rate is given by r. Given certain regularity conditions (DIXIT and PINDYCK, 1994), for each t there will be a critical value of income, R *, so that to preserve the forest it is optimal if R <R *, while cutting is optimal if R> R *. The solution for the problem of cutting the trees involves discovering the free border, R = R * (t).

Following Dixit and Pindyck (1994) standard arguments, taking the limit as $\Delta t \to 0$, and applying Ito's Lemma, from Equation (6) we derive a partial differential equation satisfied by the value function in the continuation region:

$$rV(R,t) = A(t) + V_t + \left[a(R,t)X + \frac{g(t)}{X(t)} R(t) \right] V_R + \frac{1}{2} b^2(R,t) X^2 V_{RR} \tag{7}$$

The optimal stopping problem (Equation 7) will now be respecified in the form that is more useful to valuing an American-type call option with a free boundary. The problem is formulated as a linear complementarity problem (WILMOTT ET. AL. 1993). There is a possible alternative formulation, the variational inequality, but both lead to the same result (INSLEY, 2002). Both formulations eliminate any explicit dependence on the free boundary; the free boundary can be recovered after the option problem valuation is solved (WILMOTT, 1998).

In order to formulate the tree harvesting problem as a linear complementarity problem, we write Equation (7), with τ defined as the remaining time, $\tau = T - t$:

$$HV = rV(R,\tau) + V_\tau - \frac{1}{2} b^2(R,\tau) X^2 V_{RR} - \left[a(R,\tau)X + \frac{g(\tau)}{X(\tau)} R(\tau) \right] V_R - \mathrm{A} \tag{8}$$

The problem of the linear complementarity can now be specified as:

(i) $HV \geq 0$

(ii) $V(R,t) - (R - \mathrm{K}) \geq 0$ (9)

(iii) $HV(V(R,t) - (R - K)) = 0$

This formulation can be seen intuitively as a description of the rational individual's strategy when holding an American-type option. Part (i) of the linear complementarity problem specifies that the required return (rV) less the actual return from delaying the harvest will not be negative. If HV = 0 then the required return from holding the option equals the actual return, and it is optimal to continue maintaining the option. If HV> 0, then the required return exceeds the actual return, impliying that the option should be exercised. The case when HV <0 implies in that the actual return exceeds the required return, a situation that one would not expect to persist in competitive markets. The part (ii) of the problem of linear complementarity establishes that the value of the option to harvest, V(R,t), can never go below the value of cutting the trees immediately, (R-K). This follows from the fact that the option to harvest can be exercised any time (American-type option). If the value of the option falls to the level of payout, it would be immediately exercised, therefore it would never fall below the payout value. The part (iii) establishes that either (i) or (ii) (or both) will hold as a strict equality. If HV = 0, then it is optimal to wait; if V(R,t) - (R - K) = 0, then it is optimal to cut. If both are identically null then the value of cutting is the same as the value of waiting and the owner would be theoretically indifferent to the two alternatives (INSLEY, 2002).

For a numerical solution of the equation system (9) it is necessary to specify the boundary conditions. It is noticed that because the linear complementarity problem does not depend explicitly on the free boundary it is not necessary to specify the value matching and smooth pasting conditions. Rather, these conditions are a consequence of this formulation (FRIEDMAN, 1988 APUD INSLEY, 2002).

Boundary Condition 1. For nonzero values of X, as $R \to 0$, $P \to 0$, since R = X.P. From Equation (5), in order to avoid negative prices, it is necessary that $b \to 0$, when $P \to 0$ and $a \geq 0$ when $P \to 0$. Therefore when $R \to 0$ is possible to rewrite Equation (8) as:

$$HV \equiv rV\left(R,\tau\right)+V_{\tau}-a\left(R,\tau\right)XV_{R}-\mathrm{A} \tag{10}$$

Being supposed that part (i) of the System (9) it is a strict equality and the expression in part (ii) it is strictly positive, this means that HV = 0 so that:

$$V_{\tau}=a\left(R,\tau\right)XV_{R}-rV\left(R,\tau\right)+\mathrm{A} \tag{11}$$

Equation (11) it is a first order hyperbolic equation that has outgoing characteristics and therefore no further boundary conditions are required (HALL & PORSHING, 1990). More precisely it can be shown that, since $b(R,t) \to 0$ fastrer than $R^{1/2}$ no boundary condition is necessary at R = 0.

If instead harvesting is optimal then the part (ii) of the System (9) it will be an equality. The value of the option when $R \to 0$ is exactly equal to the payout, which will be the negative of harvesting costs (that is, V(R,t) = - K).

Boundary Condition 2. As revenue, R, gets very large, the boundary condition that seems intuitively reasonable is:

$$V(R,\tau)=\gamma(\tau)R \tag{12}$$

for some function $\gamma(\tau)$. As R tends to the infinite, it remains little potential of ascent for the option, due to capital gains on the forest. It is assumed, therefore, that the value of the option is proportional to R. This implicates that $V_R = \gamma(\tau)$ and $V_{RR} = 0$. Therefore:

$$\mathrm{H}V \equiv rV\left(R,\tau\right)+V_{\tau}-\left[\frac{a\left(R,\tau\right)X}{R}+\frac{g\left(\tau\right)}{X\left(\tau\right)}\right]V\left(R,t\right)-A\,,R\to\infty \tag{13}$$

Even for a very large R, it may be optimal to postpone harvesting if the trees are still growing quickly. If we are in the continuation region where is it is optimal to postpone harvesting, then HV = 0 and Equation (13) becomes:

$$V_{\tau}=V\left(R,\tau\right)\left[\frac{a\left(R,\tau\right)X}{R}+\frac{g\left(\tau\right)}{X}-r\right]+A\,,\ R\to\infty \tag{14}$$

If instead HV> 0, meaning that harvesting is optimal then the part (ii) of the System (9) it is a strict equality, and ignoring K (because R is large) we have:

$$\begin{gathered} V(R,\tau)-R=0 \\ (\gamma(\tau)-1)R=0,\Rightarrow \gamma(\tau)=1 \end{gathered} \tag{15}$$

Thus, as R increases, the function γ assumes value = 1, if harvesting is optimal. In the numeric solution of the problem, a number arbitrarily large is chosen for the maximum value of R. A verification is done to make sure that increasing the maximum R further does not change the results significantly.

Terminal Condition. As the remaining time tends to zero, the value of the option is either the income from cutting, or the amenity value, whichever is larger:

$$V(R, \tau = 0) = \max[R - \mathrm{K}, \mathrm{A}] \tag{16}$$

The numeric algorithm to determine the value of the option involves the discretization of the linear complementarity problem (System 9) using the fully implicit finite difference method (WILMOTT ET AL., 1993). The problem of linear complementarity was solved in this paper, at each time step, by the projected successive over-relaxation (PSOR) method, which is a modification of the traditional SOR method (WILMOTT ET AL., 1995). The problem could also be solved by the penalty method (ZVAN ET AL. ,1998) as was done by Insley (2002).

3.1. The harvest decision under GBM

Under GBM the parameters of the generalized stochastic, equation (5), are specified as a(R,τ) = μP = μ(R/X) for a constant drif rate μ and b(R,τ) = σP = σ(R/X) for a constant variance rate, σ. Then equation (8) becomes:

$$HV \equiv rV(R,\tau) + V_\tau - \tfrac{1}{2}\sigma^2 R^2 V_{RR} - \left[\mu + \tfrac{g(\tau)}{X}\right] R V_R - A \tag{17}$$

The solution of the equation (17) above provides the optimal harvesting time . Equation (17) will have to be discretized in order to use the fully implicit finite difference method, and the algorithm PSOR.

3.2. The harvest decision under GMR

As explained previously, it is quite reasonable to suppose that the wood prices for cellulose production, due to its quasi-commodity characteristic, follow a process of price diffusion that reverts to the mean. In this case, a known diffusion process will be used like Geometric Ornstein-Uhlenbeck that has the form:

$$dP = \eta\left(\overline{P} - P\right)dt + \sigma P dz \tag{18}$$

In this process P reverts to the mean $\overline{P}$ at a speed determined by the parameter η. Variance rate grows with P, so that the variance is zero if P goes to zero. This format is more appropriate than the simple Ornstein-Uhlenbeck process in which the variance rate is σdz. In the simpler process, as the price P approaches zero, the constant volatility could lead to negative prices.

Substituting equation (18) above in equation (8) gives:

$$V_\tau = \tfrac{1}{2}\sigma^2 R^2 V_{RR} + \left[\eta \overline{P} X(\tau) - \eta R(\tau) + \tfrac{g(\tau)}{X(\tau)} R(\tau)\right] V_R - rV(R,\tau) + A(\tau) \tag{19}$$

The solution of equation (19) will furnish the optimal harvesting in each instant t. This equation (19) models the tree cutting problem with the price diffusion process being a geometric mean reversion. The equation above will have to be discretized, as was done with the equation (17), in order to use the fully implicit finite diference method.

3.3. Hypotheses

The following Hypotheses were formulated in this paper:

HA,1: the value of the forest stand varies with the diffusion process;

HA,2: the optimal cutting time decision varies in function of the diffusion process;

HA,3: the value of the forestry investment, when the prices follow a GMR process, varies according to the speed of mean reversion;

HA,4: the optimal cutting time decision, when the prices follow a GMR process, varies according to the speed of mean reversion.

4. SAMPLE AND DATA

The data is secondary and it is necessary a historical series of wood prices, to estimate the parameters of the model. In the specific case of this paper, it is necessary a historical series of prices of reforested eucalyptus, for use in cellulose production, to estimate the value of the drift rate (μ) and the variance rate (σ^2), in the case that the diffusion process was GBM. In the case that the diffusion process was GMR the time series of timber prices was also used to estimate the speed of mean reversion (η) and the mean to which the prices supposedly revert ($\overline{P}$).

To check whether the prices revert to the mean, we used a unit root test known as augmented Dickey Fuller (ADF). However, as Dixit and Pindyck (1994) alert, it is necessary a quite long time series, with data from many years, in order to determine with some degree of confidence whether the variable in fact reverts to the mean.

It obtained a Brazilian time series of industrially planted eucalyptus prices, for use in cellulose production, in the Bauru region of São Paulo, Brazil, published by the Centro de Estudos Avançados em Economia Aplicada (CEPEA) da Escola Superior de Agricultura Luiz de Queiroz da Universidade de São Paulo (Esalq/USP). It is a monthly time series of prices beginning in October of 2002, and extending until August of 2007, covering a period of almost 5 years, therefore insufficient to be verify whether the prices revert to the mean. The prices were deflated and converted to a constant *Real* of August of 2007, using two different price indexes the Índice de Preços ao Consumidor Amplo (IPCA-IBGE) and the Índice Geral de Preços-Disponibilidade Interna (IGP-DI-FGV). There were not significant differences among them.

The model requires also an equation that supplies wood volume in eucalyptus stands in function of the age of the forest. The equation below was obtained in Rodriguez, Bueno and Rodrigues (1997).

$$V_t = 751,336e^{-6,0777t^{-1}} \tag{20}$$

where $\mathbf{V_t}$ is the wood volume produced measured in m^3/ha and **t** is the age of the forest in years. The domain of this function is $2 \leq t < 30$, therefore it was supposed that the forest does not produce marketable wood before 2 years and does not grow after the 30 years.

In the real option model the exercise price of a call option is represented by the harvesting cost. We estimated the harvesting cost in constant *Real* of November of 2007, with data obtained in Agrianual (2006) and in Agrianual (2007), indexed by the price index IGP-DI in the period from October 2002 to August 2007. The average harvesting cost in the period, in constant *Real*, was R$12,04/m3, which represents 19,34% of the average price of the wood in the period. Souza, Rezende and Oliveira (2001) mention that harvesting cost in Brazil is falling in proportion to the price of the wood, probably due to the mechanization of the crop, and in the 1060's harvesting costs represented about 50% of the revenue obtained by the sale of the logs and now this cost represents less than 20%.

5. DATA ANALYSIS

5.1. Data treatment

The real option model with prices following GBM requires as input data, among others, the drift rate (μ) and the variance rate (σ^2), which were obtained from the timber price time series. The historical prices were deflated by IGP-DI, then the natural logarithm of the deflated prices was obtained $\left(p_t = \ln P_t\right)$ and the average of the logarithm of the prices was obtained by:

$$\overline{p_t} = \frac{\sum_{t=2}^{n} p_t - p_{t-1}}{n} \tag{21}$$

If it is assumed that price follows a process of GBM, the maximum-likelihood estimates of the variance rate, σ^2, will be s^2, where s is the standard deviation of the time series $p_t - p_{t-1}$, and the estimates of maximum-likelihood of the drif rate μ is obtained by the equation (22) below after making the necessary adjustments to convert the estimates from a monthly to an annual basis:

$$\mu = m + \frac{1}{2}s^2 \tag{22}$$

where m is the average and s is the standard deviation of the series.

Beyond of those data, estimates of the annual risk free interest rate, capitalized continually (r), will be necessary. In this paper **r** was adopted = 10% a year.

It is also necessary to obtain the quotient of two equations $\left[\frac{g(\tau)}{X}\right]$. The equation that appears in the denominator is the equation of estimated timber volume as a function of age, already presented in the item 3.5.3, of this paper. The equation g (τ) is the first derivative of the timber volume equation (20), given below:

$$g(\tau) \equiv \frac{dV}{dt} = 4566.3948t^{-2}e^{\frac{-6.0777}{t}} \tag{23}$$

It can be verified that the timber volume growth rate, according to this model, is negligible after the 30 years.

Lacking information on the value A, which represents other possible incomes, liquid of costs, that could occur for the standing forest, the value of A was considered zero.

If the price diffusion process follows GMR, it will also be necessary to estimate the average ($\overline{P}$) to which the prices revert and the parameter (η) which represents mean reversion speed.

Although it has not been possible to reject the hypothesis of random walk in the Brazilian series of prices, it is plausible that in the future the eucalyptus prices for cellulose in Brazil come to show a behavior compatible with a mean reversion process. The historical series, therefore, it is not reliable to estimate the above parameters.

The mean reversion process used in this paper known as geometric Ornstein-Uhlenbeck is given by the equation (24) below:

$$dP = \eta\left(\overline{P} - P\right)dt + \sigma P dz \tag{24}$$

A discreet approach of this equation, is given by:

$$\frac{P_t - P_{t-1}}{P_{t-1}} = c(1) + c(2)\frac{1}{P_{t-1}} + e_t \tag{25}$$

where: $c(1) \equiv -\eta\Delta t$, $c(2) \equiv \eta\overline{P}\Delta_t$, and $e_t \equiv \sigma\sqrt{\Delta_t}\varepsilon_t$

The regression was estimated for the time series of eucalyptus prices and the results are shown in the table 1 below.

Table 1 - Estimated Parameters with Brazilian Eucalyptus data

c_1	c_2	Δt	m
-0,037861	2,859194	0,083333	0,001745
η	σ	Pmed	μ
0,4543329	0,100718	62,25	0,006817

Source: the authors

6. RESULTS

The parameters presented above were used as input data to the software, in order to model the harvesting decision. The value of the option, that results from the model, represents the market value of the stand of trees in foot in the several ages. The table 2 below display the market values of the stand of Brazilian eucalyptus, with seven, twelve, seventeen and twenty-two years of age, for four different prices of wood, when the price diffusion model is GBM.

Table 2 - Value of the stand of Eucalyptus in Brazil - GBM

Timber price - R$	Value of the option			
	7 years	12 years	17 years	22 years
R$ 122,00	R$ 46.616,36	R$ 49.782,65	R$ 57.779,26	R$ 62.669,98
R$ 95,50	R$ 3.592,35	R$ 37.786,14	R$ 43.855,74	R$ 47.567,90
R$ 69,00	R$ 25.039,74	R$ 25.794,14	R$ 29.932,21	R$ 32.465,82
R$ 42,50	R$ 14.157,68	R$ 14.846,45	R$ 16.008,69	R$ 17.363,74

Source: the authors

Table 3 displays the market values of the stand of Brazilian eucalyptuses, with the same ages and prices shown in the table 2, but when the price diffusion model is GMR.

Table 3 - Value of the Stand of Eucalyptus in Brazil - GMR

Timber price - R$	Value of the option			
	7 years	12 years	17 years	22 years
R$ 122,00	R$ 34.670,83	R$ 49.782,65	R$ 57.779,26	R$ 62.669,98
R$ 95,50	R$ 26.315,93	R$ 37.786,14	R$ 43.855,74	R$ 47.567,90
R$ 69,00	R$ 19.823,51	R$ 25.789,62	R$ 29.932,21	R$ 32.465,82
R$ 42,50	R$ 16.487,79	R$ 18.450,18	R$ 20.006,45	R$ 20.955,26

Source: the authors

Hypothesis $H_{A,2}$ of this paper stipulates that the value of the forest stand varies as a function of the diffusion process and the hypothesis $H_{A,3}$ of this work stipulates that the optimal harvesting decision also varies as a function of the diffusion process. Both hypotheses are analyzed below.

Results show significant differences among the two diffusion processes. The values of the option for GBM, when timber prices are above average, tend to be larger than the values obtained with GMR. That was an expected result, once GBM incorporates a drift rate that can make the option worth more in the future. However, the values of the option for GBM, when timber prices are below average, tend to be smaller than the values which are obtained for the diffusion process RGM. This is also an expected result, once according to the mean reversion process, if prices are below average they will tend increase going back to the average, which makes the option value larger.

Results also demonstrate significant differences in the critical prices. Table 4 below display the critical prices, in several ages for the time series of eucalyptus prices in Brazil, with both price diffusion processes. Critical prices are the prices for which it is indifferent to exercise or not the option. In this modeling specifically, to exercise the option means to harvest the stand of trees.

Table 4 - Critical prices for both diffusion processes

Age in years	Crítical Prices R$	
	GBM	GMR
2	R$ 202,86	R$ 215,00
7	R$ 238,19	R$ 92,46
12	R$ 75,35	R$ 63,20
17	R$ 19,50	R$ 58,24
22	R$ 16,28	R$ 56,58

Source: the authors

Figure 1 below display the critical prices, when they exist, in all the years of the time series, for both diffusion processes.

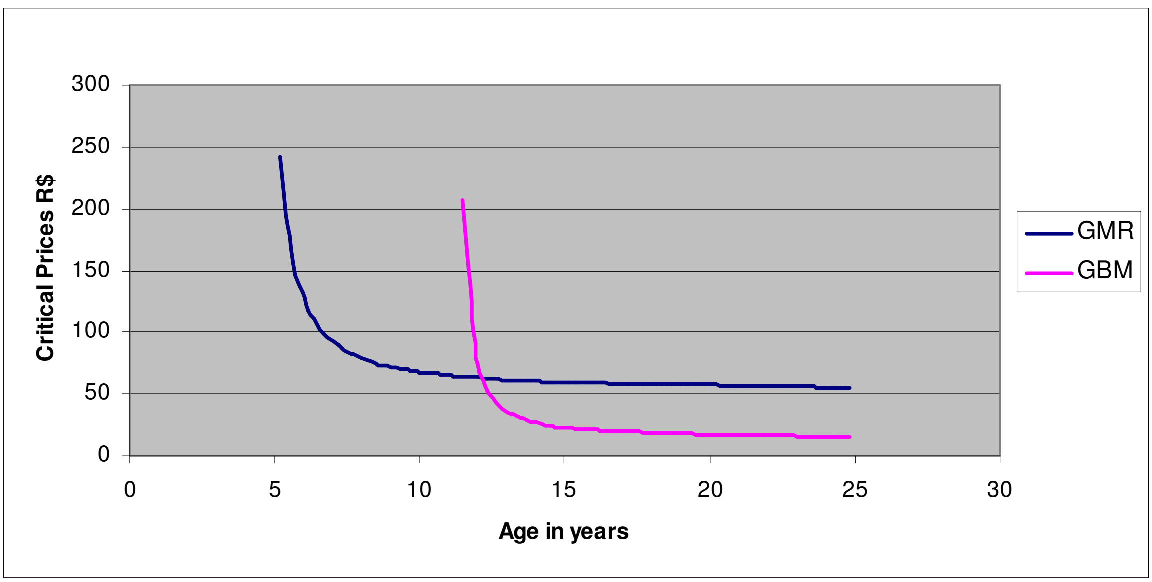

Figure 1 – Comparison of critical prices for the two diffusion processes.

Source: the authors.

It is interesting to notice that with GMR as the price diffusion process it is optimal to exercise the option (harvest the trees) in an earlier age. Critical prices, in both diffusion processes, are high in the beginning, falling fast and stabilizing after a few years, when the stand of trees begins to grow slower. The critical prices are larger for the GBM until twelve years of age; thereafter the critical prices for the mean reversion process are larger.

The maximum difference between critical prices for both price diffusion processes happen when the forest is between seven year and four months old and eleven year and six months old. Therefore, if prices follow a GMR diffusion process instead of GBM diffusion process, it is much more likely that the trees will be harvested in this time interval. This in fact happens in Brazil, paper such as Dias et al. (2005), Souza, Rezende and Oliveira (2001) and Bramucci (2001) show that the usual age of eucalyptus harvesting in Brazil is between seven and twelve years. These results suggest that, probably, timber prices in Brazil follow a GMR process, although it was not possible to prove that with the historical time series data obtained, due to the reasons already explained. Hypothesis $H_{A,1}$ and hypothesis $H_{A,2}$ of this paper were confirmed.

Hypothesis $H_{A,3}$ of this paper stipulates that the value of the forest, when prices follow GMR, varies as a function of the mean reversion speed $\boldsymbol{\eta}$. The hypothesis $H_{A,4}$ stipulates that the optimal harvesting decision, when the prices follow GMR process, varies as a function of the reversion speed $\boldsymbol{\eta}$. We examine bellow the influence of the reversion speed on the value of the settlement and on the optimal harvesting decision. First we triplicate the reversion speed to $\eta = 1.363$, and significant differences were verified in the values of the option and, mainly, in the critical exercise prices when timber prices are low. Next we used a mean reversion speed five times smaller than the original speed ($\eta = 0{,}0909$). The option values obtained with this slower speed were also significantly different from the original values. The same happened with the critical prices that define the optimal exercise boundary. The table 5 below displays the results of these sensibility analyses. The hypotheses $H_{A,3}$ and $H_{A,4}$ of this paper were also confirmed.

Table 5 - Values of the Brazilian forest as a function of age, timber prices, and mean reversion speeds.

Timber price (Reais)	η	Forest age in years			
		7	12	17	22
$ 42,50	$\eta = 0.091$	R$ 14.769,89	R$ 14.870,55	R$ 16.154,54	R$ 17.363,74
	$\eta = 0.454$	R$ 16.487,79	R$ 18.450,18	R$ 20.006,45	R$ 20.955,26
	$\eta = 1.363$	R$ 17.045,25	R$ 20.774,78	R$ 23.056,00	R$ 24.540,13
$ 69,00	$\eta = 0.091$	R$ 22.229,76	R$ 25.789,62	R$ 29.932,21	R$ 32.465,82
	$\eta = 0.454$	R$ 19.823,51	R$ 25.789,62	R$ 29.932,21	R$ 32.465,82
	$\eta = 1.363$	R$ 18.445,10	R$ 25.789,62	R$ 29.932,21	R$ 32.465,82
$ 95,50	$\eta = 0.091$	R$ 30.292,59	R$ 37.786,14	R$ 43.855,74	R$ 47.567,90
	$\eta = 0.454$	R$ 26.315,93	R$ 37.786,14	R$ 43.855,74	R$ 47.567,90
	$\eta = 1.363$	R$ 26.314,93	R$ 37.786,14	R$ 43.855,74	R$ 47.567,90
$ 122,00	$\eta = 0.091$	R$ 38.596,14	R$ 49.782,65	R$ 57.779,26	R$ 62.669,98
	$\eta = 0.454$	R$ 34.670,83	R$ 49.782,65	R$ 57.779,26	R$ 62.669,98
	$\eta = 1.363$	R$ 34.670,83	R$ 49.782,65	R$ 57.779,26	R$ 62.669,98

Source: the authors

Figure 2 below displays the comparison among the optimal exercise boundary for the original mean reversion speed $\eta = 0{,}45$ and the reduced speed $\eta = 0{,}09$.

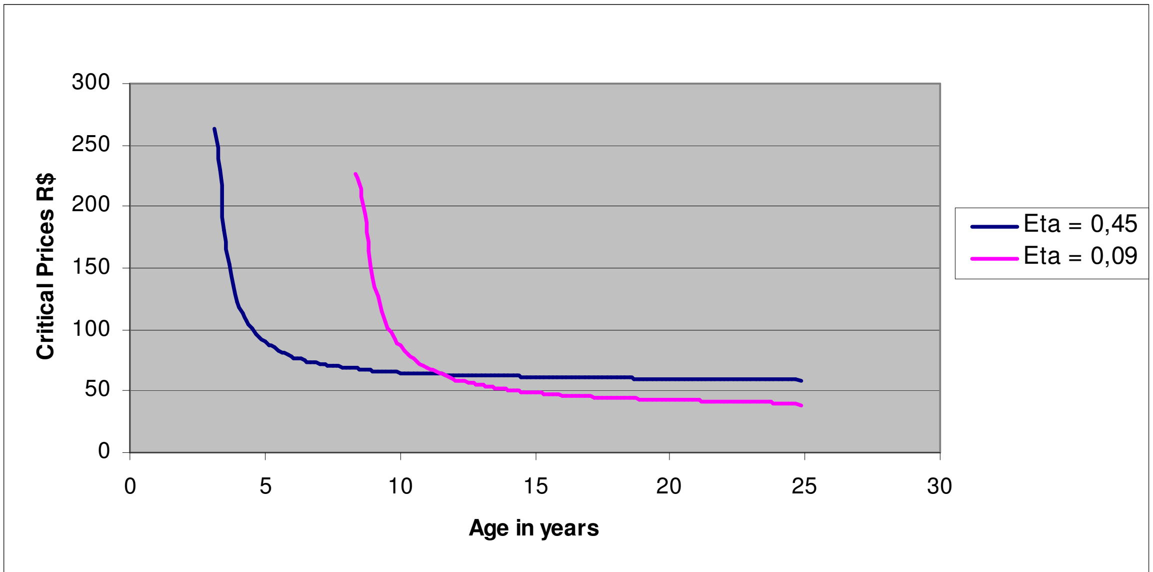

Figure 2 - Critical prices as a function of age with η = 0,45 and η = 0,09.

Source: the authors.

6.1. Limitations of this paper

We have chosen to use the one stochastic factor model, timber price, with two different price diffusion processes, GBM and GMR, and the results were compared. However, interest rate could also be considered stochastic and it would result in a model of two stochastic factors, but this is beyond the scope of this paper.

The eucalyptus price time series is unfortunately too short, but it was not possible to get a longer series, this fact prevented us to verify whether prices follow a mean reverting process or not.

It is recommended that in a next paper the modeling be done taking into account stochastic interest rates.

7. CONCLUSIONS

The problem of harvesting a stand of trees was treated, in this paper, like a real option, similar in nature to an American-type call option, with harvesting costs as the exercise price.

It is known that there is no closed analytical solution for the problem of evaluating an American-type call option, with exercise price different from zero, on an underlying asset that pays dividends. Then, when a forest investment is evaluated as a real option, it is necessary to solve numerically the differential inequations of the linear complementarity problem. This paper used the fully implicit finite difference method, with an interactive algorithm for the solution of the resultant linear system of simultaneous equations, denominated Projected Successive Over-Relaxation (PSOR). It is a robust technique to evaluate real options of this type, as it was demonstrated.

The main objective of this paper - which was to verifying the influence of the price diffusion process on the value and on the optimal harvesting time of an eucalyptus forest - was reached.

The impact of the timber price diffusion process on the value of the forest and on the optimal harvesting time of the forest was shown to be relevant. The results, assuming GBM, were compared with the results assuming GMR, and significative differences were verified among the results obtained with each one of the diffusion processes.

Specifically for the GMR price diffusion process, the influence of the speed of reversion on the value and the optimal harvesting time of the forest was verified. The speed of reversion showed to have significant influence on the value of the forest and on the optimal harvesting time. The influence of a decrease in speed, in this case, was shown to be more significant than the influence of an increase in the speed, probably because the original estimated speed was already high.

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