Choice of Alternative Environmental Policies with Quadratic Costs under Uncertainty

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Abstract

In this paper, we investigate the environmental policy designed to reduce the emission of a pollutant under uncertainty. We consider that an economic agent benefits from an economic activity and suffers from the pollutant. So the agent implements the policy, which is irreversible. The costs to implement the policy are divided into the fixed cost, the proportional cost, and the quadratic adjustment cost. Further, we consider the agent has two policy options. Then, the agent must decide that which policy he implements and when he implements the policy in order to maximize his benefit. To solve the agent's problem, we formulate it as an optimal stopping problem. Furthermore, We present the numerical analysis and comparative-static analysis of the thresholds.

Keywords: alternative environmental policies, quadratic costs, uncertainty

1 Introduction

When we consider that the environmental problems like acid rain, global warming, we face many uncertainties, for example, demographic change, economic development, technological progress, and so on. Then, decision-makers consider uncertainties when they develop and implement the environmental policies. Pindyck (2000, 2002) investigate a environmental policy designed to reduce the emission of a pollutant under uncertainty. See also Barrieu and Chesney (2003), Ohyama and Tsujimura (2006, 2008), Wirl (2006a,2006b), and Lin, Ko and Yeh (2007)

In this paper, we also investigate the environmental policy under uncertainty. We consider that an economic agent benefits from an economic activity which emits a pollutant. Simultaneously the agent suffers from the pollutant. We assume that how much damage the agent suffers from the pollutant is uncertain. The uncertainty is represented by a stochastic differential equation. We also assume that implementing the policy is irreversible. Since there are

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uncertainty and irreversibility on implementing the environmental policy, it is important to decide the timing to implement the environmental policy. Furthermore, the agent has two policy options. The environmental policies are respectively indexed by 1 and 2 and characterized by the amount of emission reduction and theirs costs. The environmental policy 1 reduces less emission than policy 2 and costs less than policy 2. The costs to implement the environmental policy are assumed to be divided into the fixed cost, the proportional cost, and the quadratic adjustment cost. Therefore, the agent must decide that which policy he implements and when he implements the policy in order to maximize his benefit. To solve the agent's problem, we formulate it as an optimal stopping problem.

While Pindyck (2000, 2002) investigate the environmental policy when the agent has just one policy option, we investigate it when the agent has two policy options. We refer to the former as the single environmental policy and the latter as the alternative environmental policies.

As related work, Décamps, Mariotti and Villeneuve (2006) explore the investment decision problem of two alternative projects. Then, they show the value of flexibility that the agent can choose between the alternative projects.

The rest of this paper is organized as follows. In the next section, we investigate the single environmental policy under uncertainty. In Section 3, we investigate the alternative environmental policies under uncertainty. Section 4 presents numerical analysis. Section 5 concludes this paper.

2 Single Environmental Policy

Suppose that an economic agent benefits from an economic activity which emits a pollutant. At the same time, the agent suffers from the pollutant. Then, the agent considers when it is optimal to implement the environmental policy designed to reduce the emission of the pollutant. There are two environmental policy 1 and 2. The environmental policy 1 reduces less emission than policy 2 and costs less than policy 2. The agent considers that when it is optimal to implement the policy. In this section, we assume that the agent have either policy 1 or 2 as policy options. Pindyck (2000, 2002) also investigate the similar problems.

2.1 Agent's Problem

Let Q_t be the level of economic activity at time $t \geq 0$. The dynamics of the process of Q_t, $Q = \{Q_t\}_{t\geq 0}$, is given by

$$\mathrm{d}Q_t = \alpha Q_t \mathrm{d}t, \qquad Q_0 = q, \tag{2.1}$$

where $\alpha > 0$ is the constant growth rate of economic activity. The agent benefit is assumed to be given by pQ_t, where p is the parameter which converts the level of economic activity to money amount. If Q_t represents amount of production, P_t represents price of the product. Let $\gamma^0 Q_t$ be the emission flow of the pollutant when the agent has not implemented the policy. If the agent has implemented the policy i ($i = 1, 2$), it reduces the emission flow to $\gamma^i Q_t$ with $\gamma^0 > \gamma^1 > \gamma^2 > 0$. Then, the dynamics of the stock of the pollutant Y_t is given by

$$\mathrm{d}Y_t^i = (\gamma^i Q_t - \delta Y_t^i)\mathrm{d}t, \qquad Y_0^i = y, \tag{2.2}$$

where $\delta \in (0, 1)$ is the rate of natural decay of the stock of pollutant. Let $X_t Y_t^2$ be the damage which it suffers from the stock of the pollutant. X_t is a variable that stochastically shifts over time to reflect damage due to the pollutant and is assumed to be governed by

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \qquad X_0 = x, \tag{2.3}$$

where $\mu > 0$, $\sigma > 0$, and W_t is a standard Brownian motion given on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t\geq 0})$ satisfying the usual conditions[1]. Here $\mathcal{F}_t$ is generated by W_t in $\mathbb{R}$, i.e., $\mathcal{F}_t = \sigma(W_s, s \leq t)$. . The net benefit $B(Q_t, X_t, Y_t)$ from economic activity is given by

$$B^i(Q_t, X_t, Y_t^i) = pQ_t - X_t(Y_t^i)^2. \tag{2.4}$$

Let $K^i(Q_t)$ be the cost function of policy i and be given by

$$K^i(Q_t) = k_0 + k_1(\gamma^0 - \gamma^i)Q_t + k_2(\gamma^0 - \gamma^i)^2 Q_t^2, \tag{2.5}$$

where $k_0 > 0$ is the fixed cost, $k_1 > 0$ is the proportional cost parameter, and $k_2 > 0$ is the adjustment cost parameter. Since $\gamma^0 > \gamma^1 > \gamma^2$, we have $K^1 < K^2$. Then, the agent's expected total discounted benefit associated with policy i is given by

$$J^i(q, x, y; \tau_S^i) = \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt} B^i(Q_t, X_t, Y_t^i)\mathrm{d}t - \mathrm{e}^{-r\tau_S^i} K^i(Q_{\tau_S^i})\right], \tag{2.6}$$

where $r > 0$ is a discount rate, $\tau_S^i \in \mathcal{T}$ is the implementation time of the policy i, and $\mathcal{T}$ is the set of all admissible implementation times. Furthermore, we assume the following condition:

$$\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt} |B(Q_t, X_t, Y_t)| \mathrm{d}t\right] < \infty. \tag{AS.1}$$

Therefore, the agent's problem is to choose the timing of implementing the policy i to maximize J^i:

$$V^i(q, x, y) = \sup_{\tau_S^i \in \mathcal{T}} J^i(q, x, y; \tau_S^i) = J^i(q, x, y; \tau_S^{i*}), \tag{2.7}$$

where V^i is the value function of the agent's problem and τ_S^{i*} is an optimal timing to implement the policy i.

2.2 Optimal Environmental Policy

The agent's problem (2.7) is formulated as an optimal stopping problem. As is well known, optimal stopping problems are solved by the variational inequalities. See, for example, Hu and Øksendal (1998), Dupuis and Wang (2002), Øksendal (2003).

To define the variational inequalities, we rewrite (2.6) as

$$\begin{aligned} J^i(q, x, y; \tau_S^i) &= \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt} B(Q_t, X_t, Y_t)\mathrm{d}t - \mathrm{e}^{-r\tau_S^i} K^i(Q_t)\right] \\ &= \mathbb{E}\Bigg[\int_0^{\tau_S^i} \mathrm{e}^{-rt} B^0(Q_t, X_t, Y_t^0)\mathrm{d}t \\ &\qquad + \mathrm{e}^{-r\tau_S^i}\left(\int_{\tau_S^i}^\infty \mathrm{e}^{-r(t-\tau_S^i)} B^i(Q_t, X_t, Y_t^i)\mathrm{d}t - K^i(Q_t)\right)\Bigg] \\ &= \mathbb{E}\left[\int_0^{\tau_S^i} \mathrm{e}^{-rt} B^0(Q_t, X_t, Y_t^0)\mathrm{d}t + \mathrm{e}^{-r\tau_S^i} G^i(Q_{\tau_S^i}, X_{\tau_S^i}, Y_{\tau_S^i}^i)\right], \end{aligned} \tag{2.8}$$

[1]See, for example, Karatzas and Shreve (1991).

where $G^i(Q_t, X_t, Y_t^i)$ is given by:

$$G^i(Q_t, X_t, Y_t^i) = \int_t^\infty \mathrm{e}^{-r(s-t)} B^i(Q_s, X_s, Y_s^i)\mathrm{d}s - K^i(Q_t). \tag{2.9}$$

The region where the agent has not implemented the environmental policy i is defined by

$$H_S^i = \{(x,y); V^i(q,x,y) > G^i(q,x,y)\}. \tag{2.10}$$

That is, H_S^i is continuation region. It yields the timing of implementing the environmental policy i, τ_S^i, is given by

$$\tau_S^i = \inf\{t > 0; (x,y) \notin H_S^i\}. \tag{2.11}$$

We now define the variational inequalities of the agent's problem (2.7).

Definition 2.1 (Variational Inequalities)**.** *The following relations are the variational inequalities of the agent's problem (2.7).*

$$\mathcal{L}V^i(q,x,y) + B^0(q,x,y) \leq 0, \tag{2.12}$$

$$V^i(q,x,y) \geq G^i(q,x,y), \tag{2.13}$$

$$[\mathcal{L}V^i(q,x,y) + B^0(q,x,y)][V^i(q,x,y) - G^i(q,x,y)] = 0, \tag{2.14}$$

where $\mathcal{L}$ is the partial differential operator defined by

$$\mathcal{L} := \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2} + \mu x \frac{\partial}{\partial x} + (\gamma^i q - \delta y)\frac{\partial}{\partial y} + \alpha q \frac{\partial}{\partial q} - r. \tag{2.15}$$

(2.14) is the complementary condition and can be rewritten as follows. If $(x,y) \in H_S^i$, then we have

$$\mathcal{L}V^i(q,x,y) + B^0(q,x,y) = 0. \tag{2.16}$$

On the other hand, if $(x,y) \notin H_S^i$, then we have

$$V^i(q,x,y) - G^i(q,x,y) = 0. \tag{2.17}$$

Let $\phi^i(q,x,y)$ be a candidate function of the value function $V^i(q,x,y)$. We can now prove that an environmental policy derived by the variational inequalities is optimal. The following theorem is the well-known verification theorem. See, for example, Theorem 10.4.1 in Øksendal (2003). The theorem also states if a candidate function satisfies the variational inequalities, the candidate function is equal to the value function. See also Hu and Øksendal (1998), Dupuis and Wang (2002).

Theorem 2.1. *1. Let $\phi^i(q,x,y)$ be a solution of the variational inequalities (2.12)-(2.14) that satisfies the following:*
The family $\{\phi^i(Q_{\tau_S^i}, X_{\tau_S^i}, Y_{\tau_S^i})\}_{\tau_S^i \in \hat{T}}$ is uniformly integrable with respect to $\mathbb{P}$, where $\hat{T}$ is the set of all bounded stopping times. Then we obtain that

$$\phi^i(q,x,y) \geq V^i(q,x,y). \tag{2.18}$$

2. When $(x,y) \in H_S^i$, we have (2.16). Furthermore, the timing of implementing the policy i, τ_S^i, is given by (2.11). Then, the candidate function ϕ^i is equal to the value function V^i:

$$\phi^i(q,x,y) = V^i(q,x,y). \tag{2.19}$$

In addition, τ_S^i is optimal.

Proof. Since the proof is similar to Øksendal (2003, Theorem 10.4.1), we omit it. □

Next, we investigate whether the candidate function $\phi^i(q,x,y)$ is a solution to the variational inequalities or not. From the formulation of the agent's problem (2.7), we conjecture the optimal environmental policy as follows. For a given pollutant stock level y, if the process of $X = \{X_t\}_{t\geq 0}$ reaches a threshold $x_S^i(y)$, the agent implements the environmental policy i, and otherwise it does not. Thus, the optimal timing of implementing the policy i is given by

$$\tau_S^i := \tau_S^i(y) = \inf\{t > 0; x \geq x_S^i(y)\}. \tag{2.20}$$

The variational inequalities implies that (2.16) holds for $x < x_S^i(y)$. We conjecture a solution to (2.16) is

$$\phi^i(q,x,y) = C_{S1}^i(y)x^{\beta_1} + C_{S2}^i(y)x^{\beta_2} + \frac{pq}{r-\alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2q^2}{\rho_1\rho_2\rho_3}, \tag{2.21}$$

where $C_{S1}^i(y)$ and $C_{S2}^i(y)$ are unknowns to be determined. $\rho_1 = r-\mu+2\delta$, $\rho_2 = r-\mu+\delta-\alpha$, and $\rho_3 = r-\mu-2\alpha$. β_1 and β_2 are the solutions to the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0, \tag{2.22}$$

and are calculated as

$$\begin{aligned} \beta_1 &= \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \\ \beta_2 &= \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \end{aligned} \tag{2.23}$$

If $X_t = 0$, the agent does not suffer from the pollutant. Then, we obtain the following boundary condition of the agent's problem:

$$\phi^i(q,0,y) = \frac{pq}{r-\alpha}. \tag{2.24}$$

It follows from (2.21) and (2.24) that we put $C_{S2}^i(y) = 0$. Then, (2.21) becomes

$$\phi^i(q,x,y) = C_{S1}^i(y)x^{\beta_1^i} + \frac{pq}{r-\alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2q^2}{\rho_1\rho_2\rho_3}. \tag{2.25}$$

The first term of the right-hand side of (2.25) represents the value that the agent can choose the timing of implementing the policy. From the second to fifth term represent the expected discounted value of B^0 in the case which the agent does not implement the environmental policy i forever. They are calculated as follows.

$$\begin{aligned}
&\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}(pQ_t - X_t(Y_t^i)^2)\mathrm{d}t\right] \\
&= \int_0^\infty \mathrm{e}^{-rt} pq\mathrm{e}^{\alpha t}\mathrm{d}t - \int_0^\infty \mathrm{e}^{-rt} x\mathrm{e}^{\mu t}\left\{\mathrm{e}^{-\delta t}\left(y - \frac{\gamma^0 q}{\alpha+\delta}\right) + \mathrm{e}^{\alpha t}\frac{\gamma^0 q}{\alpha+\delta}\right\}^2 \mathrm{d}t \\
&= \frac{pq}{r-\alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0 q)^2}{\rho_1\rho_2\rho_3}.
\end{aligned} \tag{2.26}$$

The unknown $C_{S1}^i(y)$ and threshold $x_S^i(y)$ are calculated by the following simultaneous equations:

$$\phi^i(q, x_S^i(y), y) = G^i(q, x_S^i(y), y), \tag{2.27}$$

$$\phi_x^i(q, x_S^i(y), y) = G_x^i(q, x_S^i(y), y). \tag{2.28}$$

Theses equations are well-known as the value-matching condition and smooth-pasting condition, respectively. Then, we obtain that

$$C_{S1}^i(y) = \left(\frac{2(\rho_3 y\Gamma^i + \Upsilon^i)}{\beta_1\rho_1\rho_2\rho_3}\right)^{\beta_1}\left(\frac{\beta_1 - 1}{K^i(q)}\right)^{\beta_1 - 1}, \tag{2.29}$$

$$x_S^i(y) = \left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{\rho_1\rho_2\rho_3}{2(\rho_3 y\Gamma^i + \Upsilon^i)}\right)K^i(q), \tag{2.30}$$

where $\Gamma^i = (\gamma^0 - \gamma^i)q$, $\Upsilon^i = ((\gamma^0)^2 - (\gamma^i)^2)q^2$. In what follows, due to the tractability of notation, $C_{S1}^i := C_{S1}^i(y)$. From $\gamma^0 > \gamma^1 > \gamma^2$, $K^1 < K^2$ and (2.30), the threshold of the environmental policy 1 is smaller than the threshold of the environmental policy 2:

$$x_S^1(y) < x_S^2(y). \tag{2.31}$$

3 Alternative Environmental Policies

In this section, we consider that the agent has two environmental policy options. We assume that the agent implements either the environmental policy 1 or 2. We follow the framework of Décamps, Mariotti and Villeneuve (2006) which investigate the choice problem among two alternative investment projects.

Let τ_A be the implementation time of policy 1 or 2 and be given by

$$\tau_A = \min\left[\tau_A^1, \tau_A^2\right], \tag{3.1}$$

where τ_A^i $(i = 1, 2)$ is the timing of implementing the policy i in the case that the agent has two environmental policy options. Notice that these timings depend on the stock o pollutant y. Due to the tractability of notations, we omit the dependency on y. Then, the agent's expected total discounted benefit J is

$$
\begin{aligned}
J(q,x,y;\tau_A) = \mathbb{E}\bigg[&\int_0^{\tau_A^1\wedge\tau_A^2} \mathrm{e}^{-rt} B^0(Q_t,X_t,Y_t^0)\mathrm{d}t \\
&+ \mathbf{1}_{\{\tau_A^1\le\tau_A^2\}}\mathrm{e}^{-r\tau_A^1}\bigg(\int_{\tau_A^1}^{\infty} \mathrm{e}^{-r(t-\tau_A^1)} B^1(Q_t,X_t,Y_t^1)\mathrm{d}t - K^1(Q_t)\bigg) \\
&+ \mathbf{1}_{\{\tau_A^1>\tau_A^2\}}\mathrm{e}^{-r\tau_A^2}\bigg(\int_{\tau_A^2}^{\infty} \mathrm{e}^{-r(t-\tau_A^2)} B^2(Q_t,X_t,Y_t^2)\mathrm{d}t - K^2(Q_t)\bigg)\bigg] \\
= \mathbb{E}\bigg[&\int_0^{\tau_A^1\wedge\tau_A^2} \mathrm{e}^{-rt} B^0(Q_t,X_t,Y_t^0)\mathrm{d}t \\
&+ \mathbf{1}_{\{\tau_A^1\le\tau_A^2\}}\mathrm{e}^{-r\tau_A^1} G^1(Q_{\tau_A^1},X_{\tau_A^1},Y_{\tau_A^1}^1) + \mathbf{1}_{\{\tau_A^1>\tau_A^2\}}\mathrm{e}^{-r\tau_A^2} G^2(Q_{\tau_A^2},X_{\tau_A^2},Y_{\tau_A^2}^2)\bigg].
\end{aligned}
\tag{3.2}
$$

Therefore, the agent's problem is to choose the timing of implementing the policy to maximize the agent's expected total discounted benefit J:

$$
V(q,x,y) = \sup_{\tau_A\in\mathcal{T}} J(q,x,y;\tau_A) = J(q,x,y;\tau_A^*). \tag{3.3}
$$

From (2.10) the region where the agent implements neither the environmental policy 1 nor the environmental policy 2 is defined by

$$
H_A(y) = \{(x,y); V(q,x,y) > \max[G^1(q,x,y), G^2(q,x,y)]\}. \tag{3.4}
$$

That is, $H_A(y)$ is continuation region. Then, τ_A is given by:

$$
\tau_A = \inf\{t>0; x\notin H_A(y)\}. \tag{3.5}
$$

As in Section 2, the agent's problem is formulated as an optimal stopping problem and is solved via the variational inequalities. The variational inequalities of the agent's problem (3.3) are as follows.

$$
\mathcal{L}V(q,x,y) + B^0(q,x,y) \le 0, \tag{3.6}
$$

$$
V(q,x,y) \ge \max[G^1(q,x,y), G^2(q,x,y)], \tag{3.7}
$$

$$
\left[\mathcal{L}V(q,x,y) + B^0(q,x,y)\right]\left[V(q,x,y) - \max\left[G^1(q,x,y), G^2(q,x,y)\right]\right] = 0. \tag{3.8}
$$

Let $\tilde{x}$ be value of the shift variable such that $G^1(q,x,y) = G^2(q,x,y)$. Then, $\tilde{x}$ is calculated as

$$
\tilde{x} = (K^2(q) - K^1(q))\left[\frac{\rho_1\rho_2\rho_3}{2(y\rho_3\Gamma + \Upsilon)}\right], \tag{3.9}
$$

where $\Gamma = (\gamma^1-\gamma^2)q$, $\Upsilon = ((\gamma^1)^2 - (\gamma^2)^2)q^2$. The value function V smoothly pastes neither the function G^1 nor the function G^2 at $x=\tilde{x}$. Then, we obtain the following result.

Proposition 3.1. *When the shift variable is $\tilde{x}$, the agent implements neither the policy 1 nor the policy 2.*

Décamps, Mariotti and Villeneuve (2006) provide rigorous treatment in their Proposition 2.2. Furthermore, Décamps, Mariotti and Villeneuve (2006) obtain the following result in their Theorem 2.1.

Theorem 3.1. *Assume that*

$$\frac{(\rho_3 y \Gamma^1 + \Upsilon^1)^{\beta_1}}{K^1(q)^{\beta_1 - 1}} > \frac{(\rho_3 y \Gamma^2 + \Upsilon^2)^{\beta_1}}{K^2(q)^{\beta_1 - 1}}. \tag{3.10}$$

Let $x_A^i(y)$ $(i = 1, 2)$ be the threshold of implementing the policy i when the agent has two policy options 1 and 2. The timing of implementing the policy 1, τ_A^1, is given by

$$\tau_A^1 = \inf\{t > 0; x_S^1(y) \le X_t \le x_A^1(y)\}. \tag{3.11}$$

On the other hand, the timing of implementing the policy 2, τ_A^2, is given by

$$\tau_A^2 = \inf\{t > 0; X_t \ge x_A^2(y)\}. \tag{3.12}$$

The continuation region $H_A(y)$ is redefined by

$$H_A(y) = \{x; x < x_S^1(y),\ x_A^1(y) < x < x_A^2(y)\}. \tag{3.13}$$

Let $\phi(q, x, y)$ be a candidate function of the value function $V(q, x, y)$. From the variational inequalities (3.6)–(3.8), for $x \in H_A$ we have

$$\frac{1}{2}\sigma^2 x^2 \phi_{xx} + \mu x \phi_x + (\gamma^0 q - \delta y)\phi_y + \alpha q \phi_q - r\phi + B^0 = 0. \tag{3.14}$$

For $x < x_S^1$, when x reaches x_S^1, the agent implements the environmental policy 1. Then, we have ϕ^1 given by (2.25). For $x_A^1 < x < x_A^2$, when x reaches x_A^1 before x_A^2, the agent implements the environmental policy 1. On the other hand, when x reaches x_A^2 before x_A^1, the agent implements the environmental policy 2. Thus, the agent has two types of flexibility in this region. Then, the candidate function is

$$\phi(q, x, y) = C_{A1} x^{\beta_1} + C_{A2} x^{\beta_2} + \frac{pq}{r - \alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1 \rho_2} - \frac{2x(\gamma^0)^2 q^2}{\rho_1 \rho_2 \rho_3}, \tag{3.15}$$

where $C_{A1} := C_{A1}(y)$ and $C_{A2} := C_{A2}(y)$ are unknowns to be determined. The first term of the right-hand side is the value of the flexibility that the agent chooses the timing of implementing the environmental policy 1. The second term is the value of the flexibility for the environmental policy 2. Then, ϕ is divided by the level of x into as follows:

$$\phi(q, x, y) = \begin{cases} C_{S1}^1 x^{\beta_1} + \frac{pq}{r-\alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2 q^2}{\rho_1\rho_2\rho_3}, & x < x_S^1(y), \\ G^1(q, x, y), & x_S^1(y) \le x \le x_A^1(y), \\ C_{A1} x^{\beta_1} + C_{A2} x^{\beta_2} + \frac{pq}{r-\alpha} - \frac{xy^2}{\rho_1} - \frac{2xy\gamma^0 q}{\rho_1\rho_2} - \frac{2x(\gamma^0)^2 q^2}{\rho_1\rho_2\rho_3}, & x_A^1(y) < x < x_A^2(y), \\ G^2(q, x, y), & x \ge x_A^2(y). \end{cases} \tag{3.16}$$

As in Section 2, we have to determine the unknowns: C_{A1}, C_{A2} and the thresholds:$x_A^1(y)$, $x_A^2(y)$. They are calculated by simultaneous equations:

$$\phi(q, x_A^1(y), y) = G^1(q, x_A^1(y), y). \tag{3.17}$$

$$\phi(q, x_A^2(y), y) = G^2(q, x_A^2(y), y). \tag{3.18}$$

$$\phi_x(q, x_A^1(y), y) = G_x^1(q, x_A^1(y), y). \tag{3.19}$$

$$\phi_x(q, x_A^2(y), y) = G_x^2(q, x_A^2(y), y). \tag{3.20}$$

Unfortunately, they are not analytically derived. In the next section, we numerically calculate them.

4 Numerical Analysis

In this section, we numerically calculate the thresholds: $x_S^1(y)$, $x_S^2(y)$, $x_A^1(y)$, and $x_A^2(y)$ and investigate the effect of changes in parameters on the thresholds. The basic parameter values are set out in Table 1.

The value function V that the agent has two environmental policy options is illustrated in Figure 1. The threshold values are calculated as $x_S^1 = 0.1494$, $x_S^2 = 0.2423$, $x_A^1 = 0.2176$, $x_A^2 = 0.2793$ in the base case. The indifference value of shift variable is $\tilde{x} = 0.2476$.

We show the results of the comparative-static analysis for the thresholds of the environmental policies on r, μ, σ, y, γ^0, and k_0 in Figures 2–7. Figure 2 shows that the continuation region H_A is increasing in the discount rate r. Let $I_1(y) = \{x; x_S^1(y) \leq x \leq x_A^1(y)\}$ be the region that the environmental policy 1 is implemented. Let $I_2(y) = \{x; x \geq x_A^2(y)\}$ be the region that the environmental policy 2 is implemented. While the region I_1 is increasing in r, the region I_2 is decreasing in r.

Figure 3 shows that the continuation region is decreasing in the expected growth rate of the shift variable, μ. While the region I_1 is decreasing in μ, the region I_2 is increasing in μ. When μ goes to 0.02075, the assumption (3.10) does not hold. Further, the thresholds of the alternative environmental policies, x_A^1 and x_A^2, respectively equals to the thresholds x_S^1 and x_S^2 of the single environmental policy.

Figure 4 shows that the continuation region is increasing in the volatility of the shift variable, σ. The regions I_1 and I_2 are decreasing in σ. As in μ, when σ goes to 0.27923, the assumption (3.10) does not hold. Further, the thresholds x_A^1 and x_A^2 respectively equals to the thresholds x_S^1 and x_S^2.

Figure 5 shows the continuation region is decreasing in the stock of pollutant, y. While the region I_1 is decreasing in y, the region I_2 is increasing in y.

The continuation region H_A is divided into two regions. The one region is defined by $H_{A1}(y) = \{x; x < x_S^1(y)\}$. H_{A1} is the continuation region when the agent has only the environmental policy 1. The other region is defined by $H_{A12}(y) = \{x; x_A^1(y) < x < x_A^2(y)\}$. This region comes from the flexibility that the agent can choose between the environmental policy 1 and 2. Figure 6 shows that the region H_{A1} is increasing in the emission conversion factor γ^0. In contrast, H_{A12} is decreasing in γ^0. Combining these effects, H_A is increasing in γ^0. While the region I_1 is increasing in γ^0, the region I_2 is decreasing in γ^0.

Figure 7 shows that the continuation region is increasing in the fixed cost to implement the environmental policy, k_0. The regions I_1 and I_2 are decreasing in k_0.

5 Conclusion

In this paper, we investigate the environmental policy under uncertainty. We consider that an economic agent benefits from an economic activity and suffers from the pollutant. Since the agent has two policy options, the agent must decide that which policy he implements and when he implements the policy in order to maximize his benefit. To solve the agent's problem, we formulate it as an optimal stopping problem. We first investigate the single environmental policy and obtain the closed from of the threshold. Next we investigate the alternative environmental policies. Unfortunately, the thresholds of policies do not be derived explicitly. So we conduct numerical analysis and comparative-static analysis. As the representative result, the continuation region increases in volatility, that is, uncertainty. The policy implementing regions decreases in volatility. Further, the continuation region increases in the emission conversion factor that the agent has implemented neither policy 1 nor 2. While the region of implementing policy 1 increases in the conversion factor, the region of implementing policy 1 decreases in the conversion factor.

To conclude the paper, we present a number of possible extensions for our model. First, we leave to examine the effect of technological progress. It plays important role of environmental policies. As in Second, in this paper, we assume that the dynamics of the economic activity is deterministic. Since economic development also is uncertain in real world, the dynamics of the economic activity or the price will be followed by stochastic differential equation. We leave these topics for future research.

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Table 1: The base case values of parameters and variables

Variable	Symbol	Value
Parameters		
Discount rate	r	0.05
Growth rate of economic activity	α	0.01
Price	p	10
Growth rate of shift variable	μ	0.01
Volatility of shift variable	σ	0.2
Rate of natural decay	δ	0.01
Emission conversion factor for policy 0	γ^0	0.05
Emission conversion factor for policy 1	γ^1	0.03
Emission conversion factor for policy 2	γ^2	0.02
Fixed cost	k_0	5
Proportional cost	k_1	100
Adjustment cost	k_2	10000
Variables		
Economic activity	q	5
Stock of pollutant	y	0.1

Policy 0 means that the agent has implemented neither the environmental policy 1 nor 2.

Figure 1: Value function of two environmental policy options.

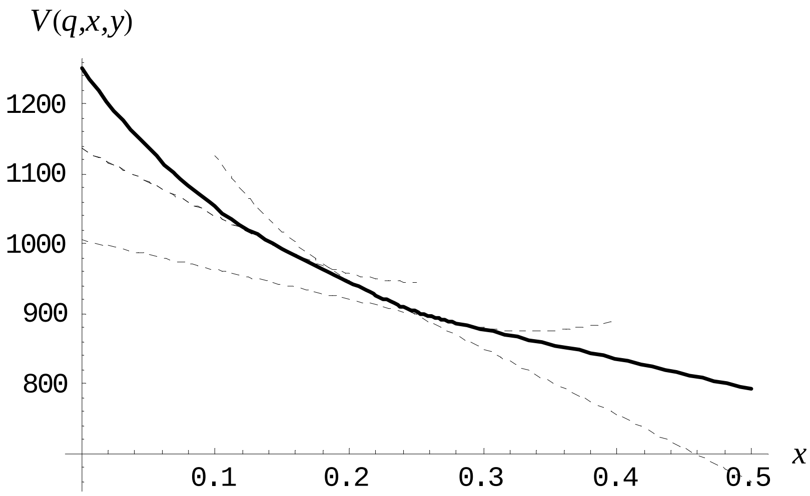

Figure 2: Comparative statics of thresholds with respect to r.

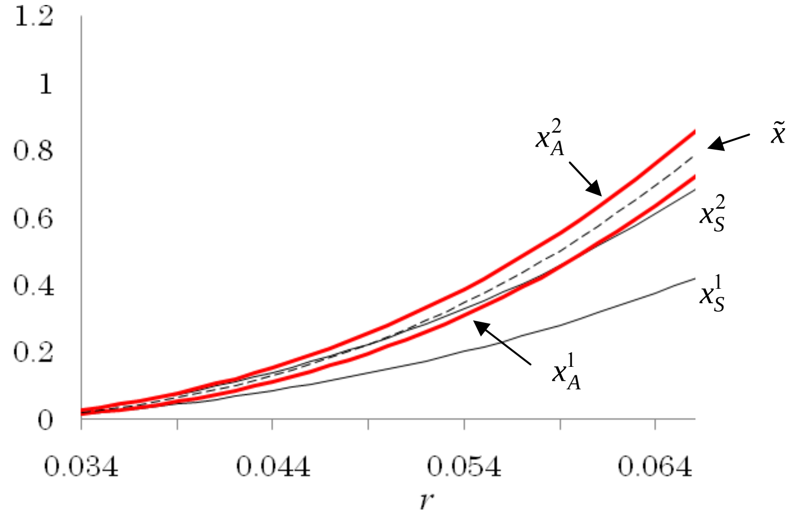

Figure 3: Comparative statics of thresholds with respect to μ.

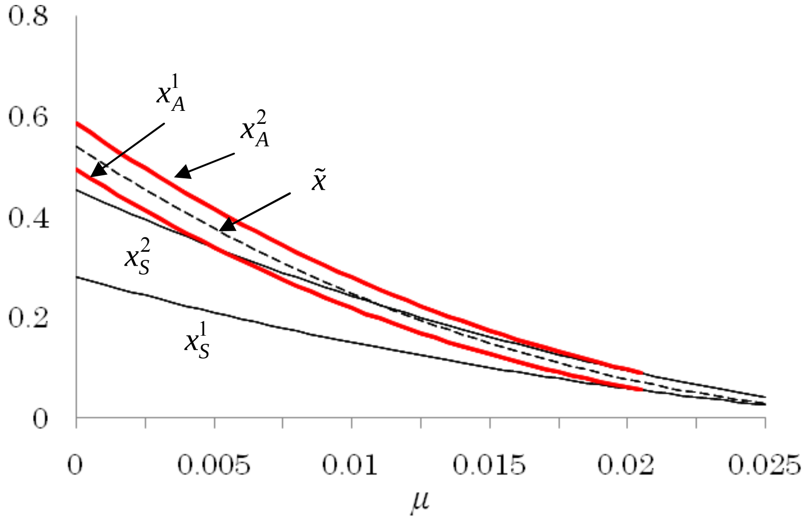

Figure 4: Comparative statics of thresholds with respect to σ.

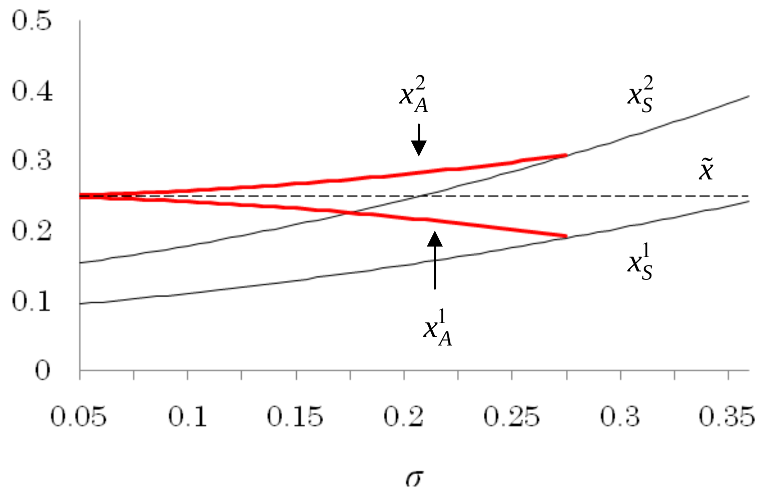

Figure 5: Comparative statics of thresholds with respect to y.

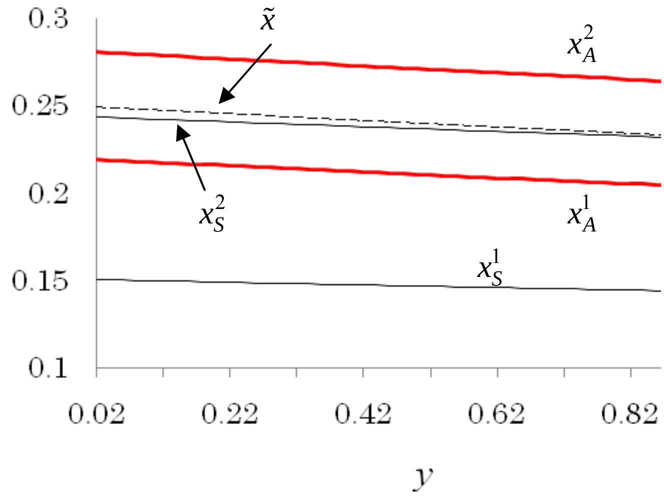

Figure 6: Comparative statics of thresholds with respect to γ^0.

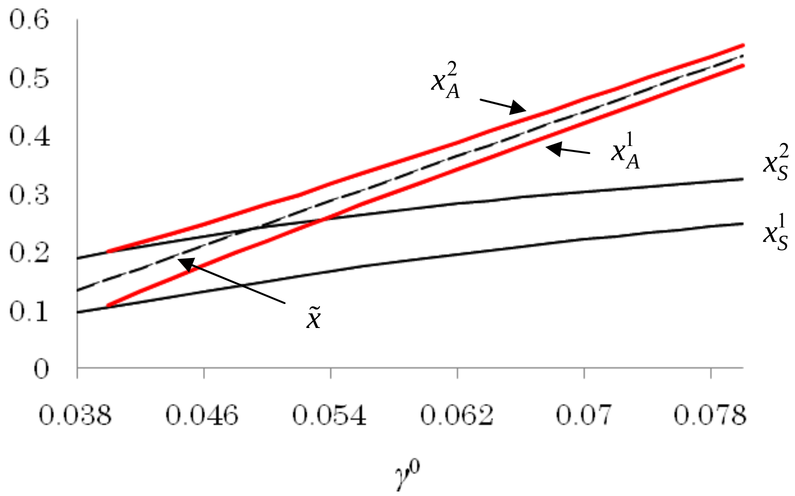

Figure 7: Comparative statics of thresholds with respect to k_0.